Fast scrambling in holographic Einstein-Podolsky-Rosen pair

Osaka University,
Keiju Murata

特任助教 from Jan.

"Fast scrambling in holographic Einstein-Podolsky-Rosen pair”,
arXiv:1708.09493 [hep-th], JHEP 1711 (2017) 049

My career

1982 Born in Tokyo.

2001 – 2005 (4year s) Tokyo institute of technology (undergraduate)

2005 – 2010 (5year s) Kyoto University, Tentaikaku group (Ph.D student)

2010 – 2013 (3year s) University of Cambridge (Postdoc)

2013 – 2018 (~5years) Keio University (有期助教)

General relativity (Stability of higher dimensional BHs)

Non-equilibrium process in AdS/CFT

My research interest

Classical and quantum aspects
of black holes,
and their applications.

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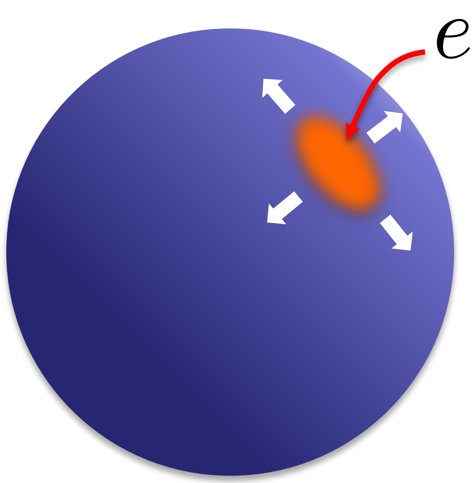
1. Introduction

Fast scrambling conjecture

"Black holes are the fastest scramblers in nature"

Sekino&Susskind, 08



The electric charge spreads out all over the horizon.

Its time scale:

$$t_* \sim \beta \ln S \sim \beta \ln N$$

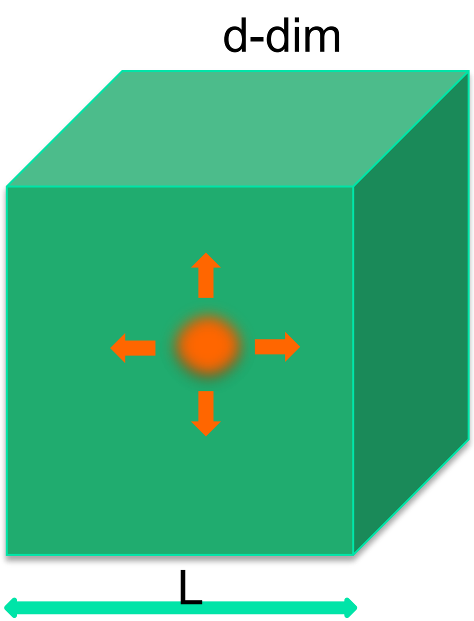
inverse Hawking temperature

BH entropy

The number of microscopic degree of freedom of BH.

Scrambling = Delocalization of local quantum excitation

What is scrambling time of a conventional matter?



diffusion coefficient

Scale of the excitation: $\langle \Delta x^2 \rangle \sim \kappa t$

Scrambling time: $t_* \sim \frac{1}{\kappa} L^2$

Number of DOF $\propto$ Volume $L^d \propto N$

$$\Rightarrow \quad t_* \propto N^{2/d}$$

BH's scrambling is much faster than conventional matters.

$$\ln N \ll N^{2/d}$$

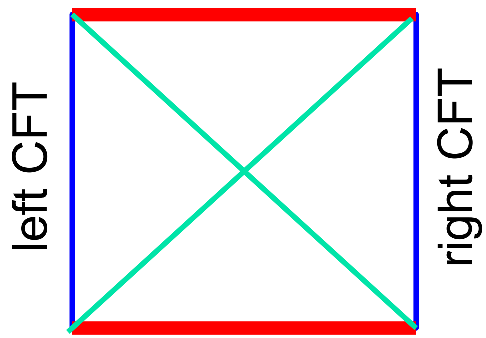
$\Downarrow$

Quantum DOF of BH would be strongly chaotic.

Fast scrambling in AdS/CFT

Shenker&Stanford, 13

What is dual state of eternal black hole?



pure state

$$\left|\Psi\right\rangle \; ?$$

Thermofield double state: $$|\Psi\rangle = \frac{1}{\sqrt{Z}} \sum_n e^{-\beta E_n/2} |n\rangle_L |n\rangle_R \; .$$

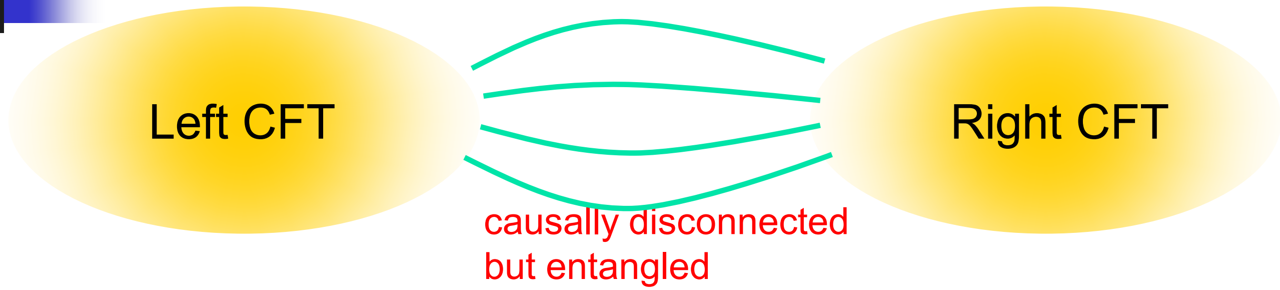
Maldacena, 03

Thermal property is explained by the entanglement.

$$\langle\Psi|\mathcal{O}_R|\Psi\rangle \; = Z^{-1}\mathrm{tr}[e^{-\beta H_R}\mathcal{O}_R]$$

Correlation is a measure of scrambling.

Shenker&Stanford, 13



Assume that CFT is “chaotic”.

Give a perturbation on Left CFT.

- Left CFT is scrambled by “chaos”.
- Destroy the correlation between L and R

The correlation between left and right CFTs quantifies the scrambling.

Example: non-integrable spin system

Shenker&Stanford, 13

Hamiltonian:

$$H_L = \sum_{i=1}^{10} \left\{ \sigma_z^{(i)} \sigma_z^{(i+1)} - 1.05\ \sigma_x^{(i)} + 0.5\ \sigma_z^{(i)} \right\}.$$

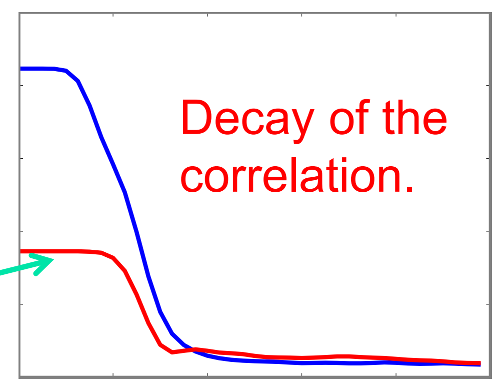
Thermofield double:

$$|\Psi\rangle = \frac{1}{\sqrt{Z}} \sum_n e^{-\beta E_n/2} |n\rangle_L |n\rangle_R\ .$$

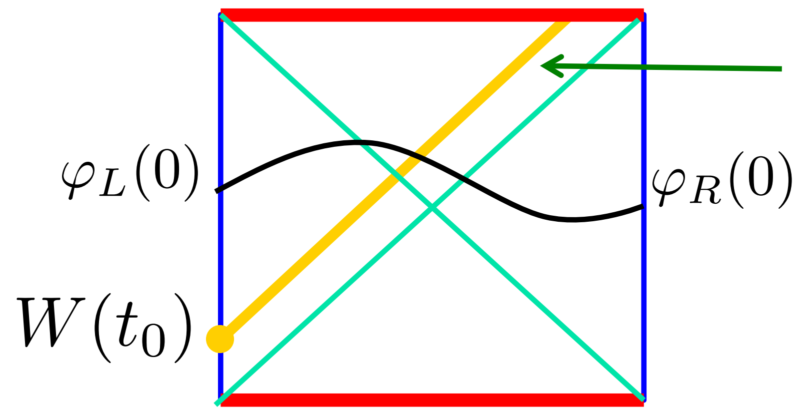
Give a perturbation at t = - tw :

$$|\Psi'\rangle = \sigma_z^{(5,L)}(-t_w)|\Psi\rangle$$

Measure the correlation at t=0:



AdS/CFT calculation of scrambling



perturbation on Left CFT

injected energy

Geometry is deformed.

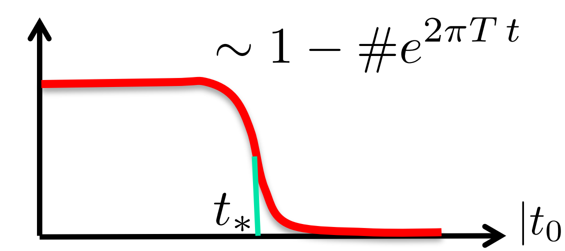
Correlation function in this geometry

$$\langle \varphi_L(0) \varphi_R(0) \rangle_W \sim e^{-\Delta d}$$

conformal weight

geodesic distance

$$\sim \left(\frac{1}{1 + \frac{\beta \delta E}{S} \exp(\frac{2\pi}{\beta}|t_0|)} \right)^{2\Delta}$$



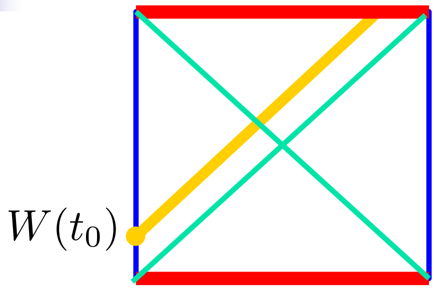
Scrambling time

$$t_* \sim \beta \ln S$$

Bound on “Lyapunov” exponent

Maldacena, Shencker&Stanford, 16

Correlation function ~ OTOC



perturbed thermofield double

$$\Longleftrightarrow \quad W_L(t_0)|\Psi\rangle$$

$$\langle\varphi_L(0)\varphi_R(0)\rangle_W \sim \langle\Psi|W_L^\dagger(t_0)\varphi_L(0)\varphi_R(0)W_L(t_0)|\Psi\rangle$$

$$= \frac{1}{Z}\sum_{n,m} e^{-\beta(E_n+E_m)/2}\langle n|\langle n|W_L(t_0)\varphi_L(0)\varphi_R(0)W_L(t_0)|m\rangle|m\rangle$$

$$= \frac{1}{Z}\mathrm{tr}\left[e^{-\beta H/2}\varphi(0)e^{-\beta H/2}W(t_0)\varphi(0)W(t_0)\right]$$

$$= \langle W(t_0)\varphi(0)W(t_0)\varphi(i\beta/2)\rangle_\beta$$

$$\sim \langle W(t_0)\varphi(0)W(t_0)\varphi(0)\rangle_\beta$$

Out-of-time-order correlator (OTOC)

Lyapunov bound

Maldacena, Shenker&Stanford, 16

1. Quantum Lyapunov exponent is defined by the OTOC.

$$\langle W(t)V(0)W(t)V(0)\rangle_\beta \sim 1 - \# e^{\lambda_L t}$$

2. The Lyapunov exponent has upper bound. $\lambda_L \leq 2\pi T$

AdS/CFT $\Rightarrow$ $\lambda_L = 2\pi T$

"refinement of the fast scrambling conjecture"

3. If a QFT saturates the bound, it has an Einstein dual.

most chaotic $\Leftrightarrow$ black hole dual

(ex. SYK model)

In this talk

We propose one of the simplest model which shows fast scrambling.

quark-antiquark in N=4 SYM

Holographic EPR pair.

q  $\bar{q}$

entangled
causally disconnected

$$\lambda_L = 2\pi T \quad \tau_* \sim \beta \ln S$$

Dual picture of the EPR pair is just a probe string.

Fast scrambling or Lyapunov bound do not directly imply the existence of an Einstein dual.

Holographic EPR pair

Setup

We consider a probe classical string in AdS5.

$$ds^2 = \frac{\ell^2}{z^2}(-dt^2 + dz^2 + dx^2 + dy_1^2 + dy_2^2) \ ,$$

$$S_{\rm NG} = -\frac{1}{2\pi\alpha'}\int d^2\sigma\sqrt{-h} \ ,$$

For simplicity, we consider the string in y1=y2=0 plane.
(Effectively AdS3)

Mikhailov solution

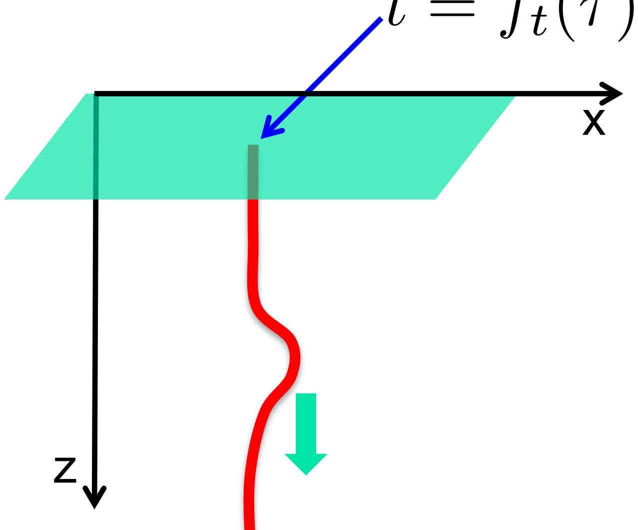
Mikhailov, 03

Worldsheet coordinates: (τ, σ)

$$t = \frac{\dot{f}_t(\tau)}{\sigma} + f_t(\tau)\ , \quad x = \frac{\dot{f}_x(\tau)}{\sigma} + f_x(\tau)\ , \quad z = \frac{1}{\sigma}\ .$$

where $-\dot{f}_t^2 + \dot{f}_x^2 = -1\ .$

$t = f_t(\tau),\ x = f_x(\tau)$



AdS boundary: $\sigma = \infty$.

τ : proper time of the end point

Non-linear wave from AdS boundary

$$t = \frac{\dot{f}_t(\tau)}{\sigma} + f_t(\tau) \ , \quad x = \frac{\dot{f}_x(\tau)}{\sigma} + f_x(\tau) \ , \quad z = \frac{1}{\sigma} \ .$$

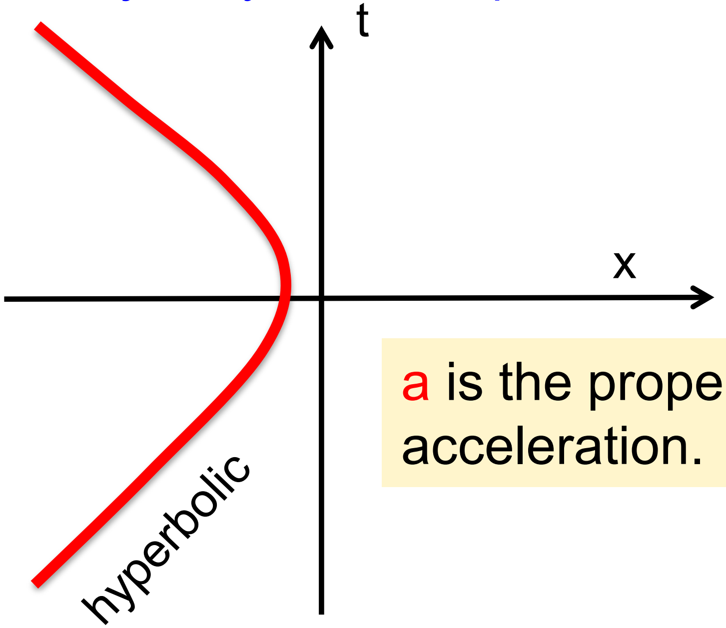
Holographic EPR solution

As a special case

$$f_t(\tau) = \frac{1}{a} \sinh a\tau \ , \quad f_x(\tau) = -\frac{1}{a} \cosh a\tau \ .$$

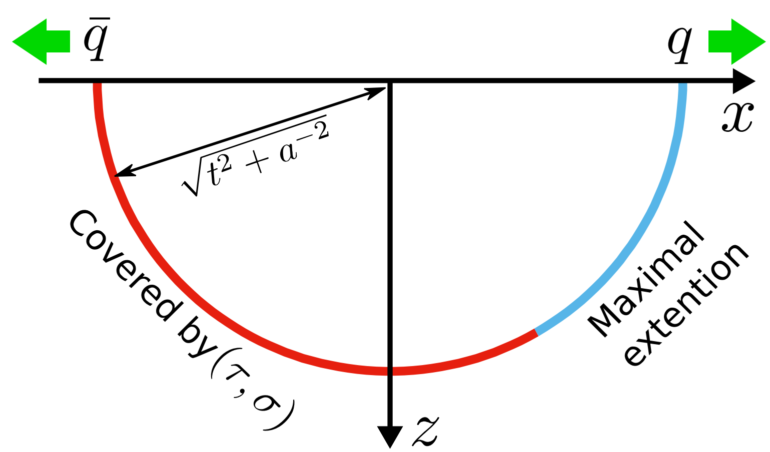
$$x^2 + z^2 = t^2 + \frac{1}{a^2}$$

trajectory of the endpoint



a is the proper acceleration.

string profile



Causal structure of the worldsheet

Induced metric

$$ds_h^2 = -[\sigma^2 - a^2]d\tau^2 + 2d\tau d\sigma \ .$$

$$U = e^{a\tau} \ , \quad V = -\frac{\sigma - a}{\sigma + a} e^{-a\tau}$$

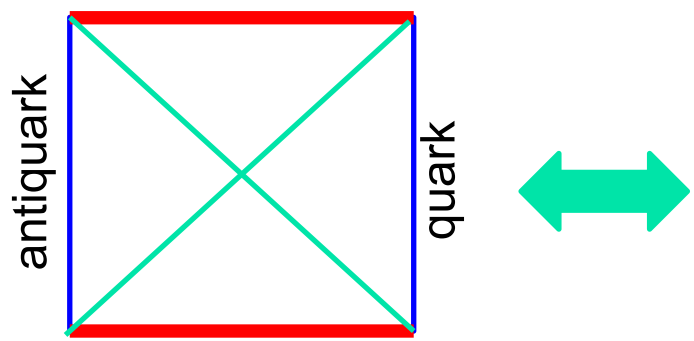
$$ds_h^2 = -\frac{4dUdV}{(1+UV)^2} \ .$$

$$t = \frac{1}{a}\frac{U+V}{1-UV} \ , \quad x = -\frac{1}{a}\frac{U-V}{1-UV} \ , \quad z = \frac{1}{a}\frac{1+UV}{1-UV} \ .$$

Right quark "appears" by the maximal extension.

Why is this solution called EPR pair?

Worldsheet = eternal BH



Quark-antiquark is causally disconnected, but in thermofield double state.

$$|\Psi\rangle = \frac{1}{\sqrt{Z}} \sum_n e^{-\beta E_n/2} |n\rangle_L |n\rangle_R \ .$$

"Causally disconnected" & "entanglement" = "EPR pair"

Unruh temperature and Entropy

Accelerating particle feels thermal bath. Unruh effect

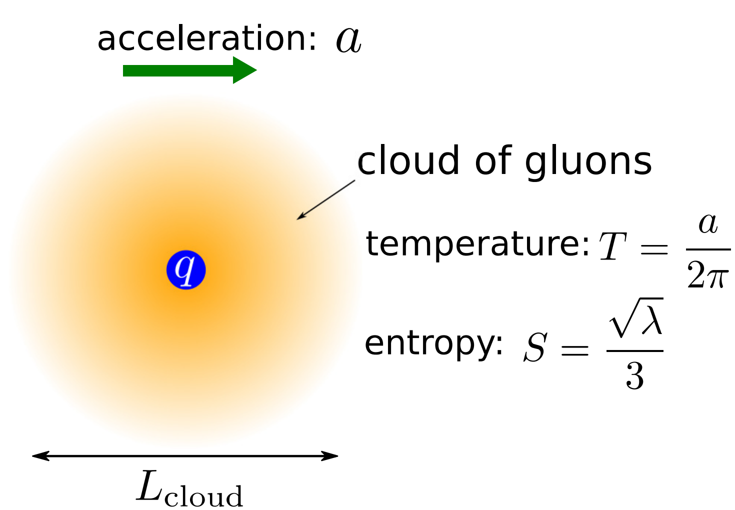
$$T = \beta^{-1} = \frac{a}{2\pi}$$ Unruh temperature

Accelerating quark has entropy & energy. Jensen&Karch, 14

$$E = \frac{4}{3}\sqrt{\lambda}T \ .$$

λ: 't hooft coupling

$$\begin{aligned} F &= -TS_{\text{on-shell}} \\ S &= dF/dT \\ E &= F + TS \end{aligned}$$

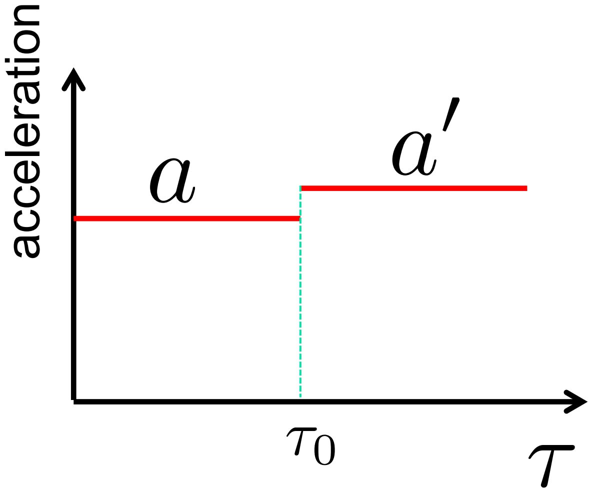


Holographic EPR with a shock

$$t = \frac{\dot{f}_t(\tau)}{\sigma} + f_t(\tau) \ , \quad x = \frac{\dot{f}_x(\tau)}{\sigma} + f_x(\tau) \ , \quad z = \frac{1}{\sigma} \ .$$

Changing the acceleration

As a perturbation, we slightly change the acceleration at $\tau = \tau_0$.

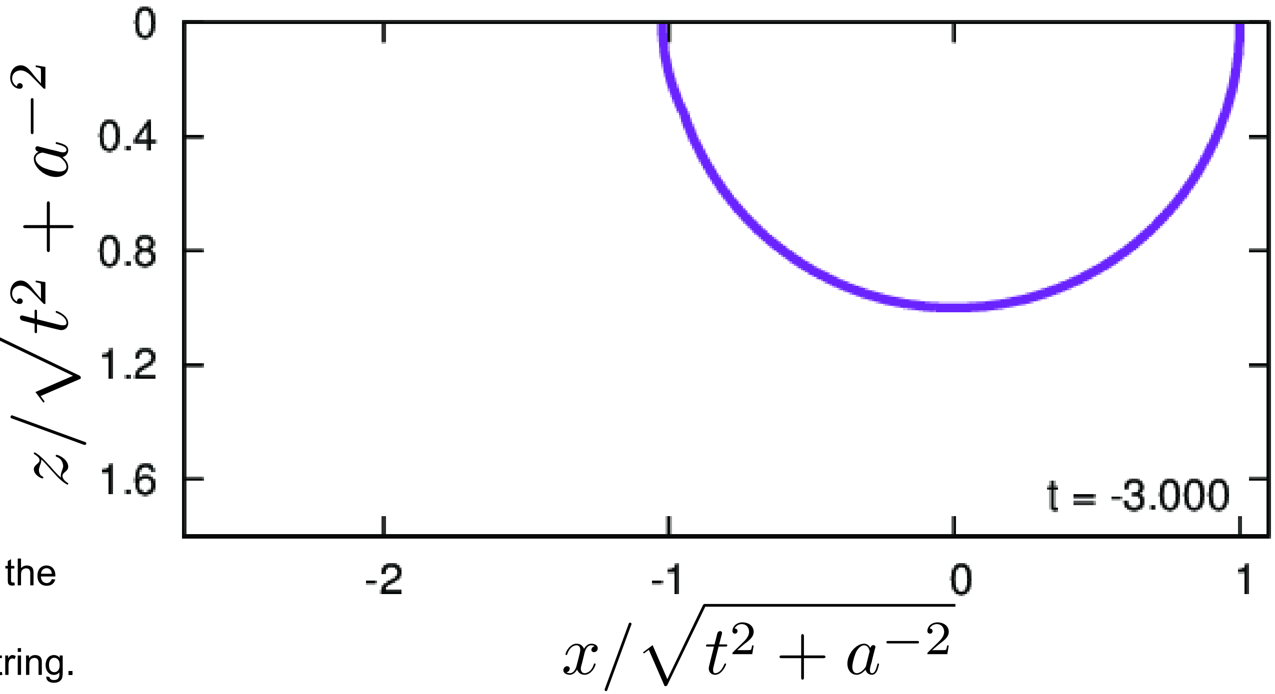


$$\begin{cases} \frac{1}{a}\cosh a\tau & (\tau < \tau_0) \\ \frac{1}{a'}\cosh[a'\tau + c_1] + c_2' & (\tau > \tau_0) \end{cases} ,$$

junction condition:

$$c_1 = -(a' - a)\tau_0 \ , \quad c_2 = \left(\frac{1}{a} - \frac{1}{a'}\right)\sinh a\tau_0 \ , \quad c_2' = \left(\frac{1}{a} - \frac{1}{a'}\right)\cosh a\tau_0 \ .$$

String dynamics



normalized by the radius of the unperturbed string.

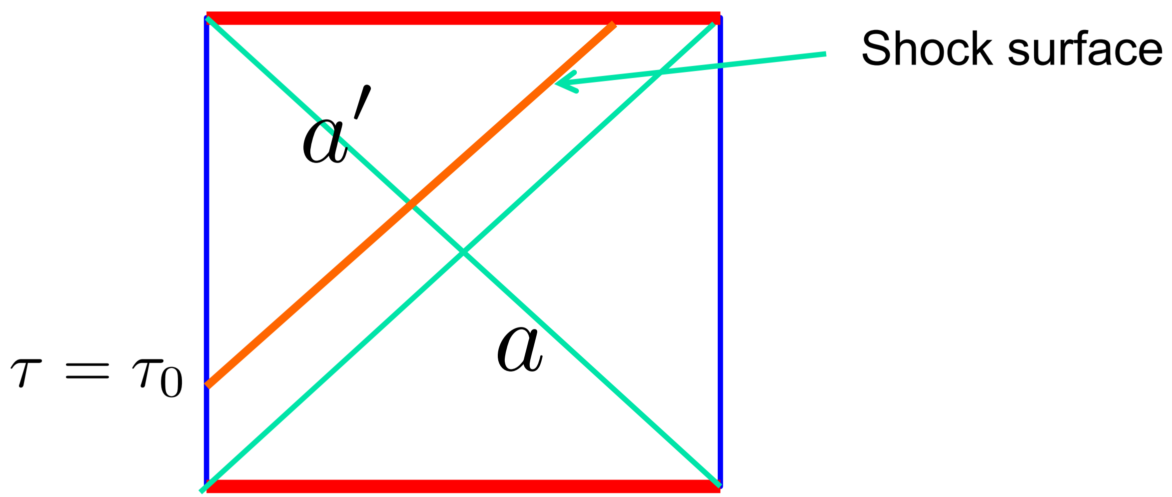
Even for tiny change of the acceleration, the string profile is significantly deformed.

induced metric of the string

$$ds_h^2 = -[\sigma^2 - M(\tau)]d\tau^2 + 2d\tau d\sigma \ ,$$

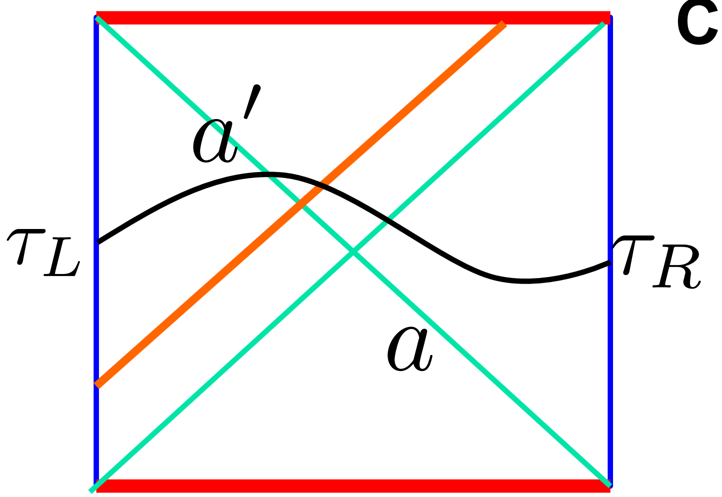
$$M(\tau) = a^2 + (a'^2 - a^2)\theta(\tau - \tau_0) \ .$$

Vaidya-like geometry with infalling null shock.



Fast scrambling of EPR

Correlation between quark and antiquark



geodesic distance

Correlation:

Force on left and right quarks

$$\langle F_L(\tau_L) F_R(\tau_R) \rangle_W \sim e^{-d}$$

geodesic distance

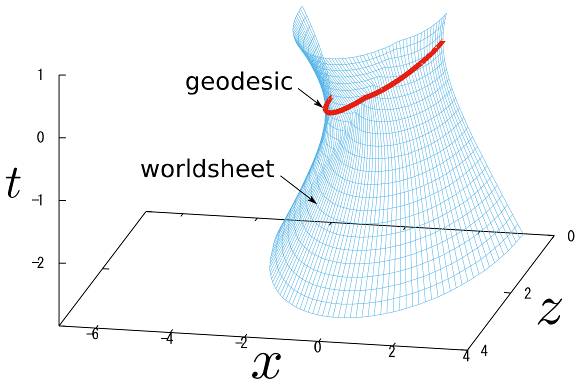
Field on worldsheet is the target space coordinate. $x(\tau, \sigma)$

Its conjugate operator is the force.

$$\mathcal{L}_{\text{quark}} \sim -m\sqrt{1-v^2} + V(x)$$
$$\partial \mathcal{L}_{\text{quark}} / \partial x \sim F$$

$$\left[\delta a \to 0 \ , \qquad \tau_0 \to -\infty \ , \qquad \gamma \equiv \frac{\delta a}{2a} e^{-a\tau_0} \ : \ \text{fixed} \ . \right]$$

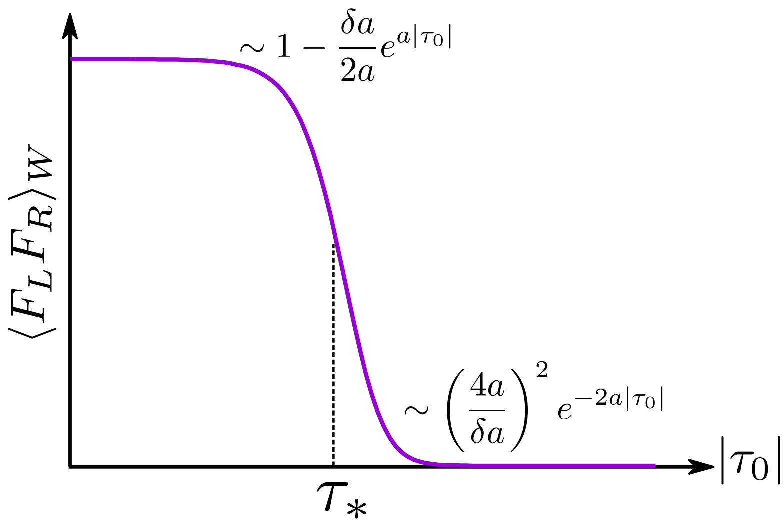
Visualizing the correlation



earlier change of acceleration

Scrambling time

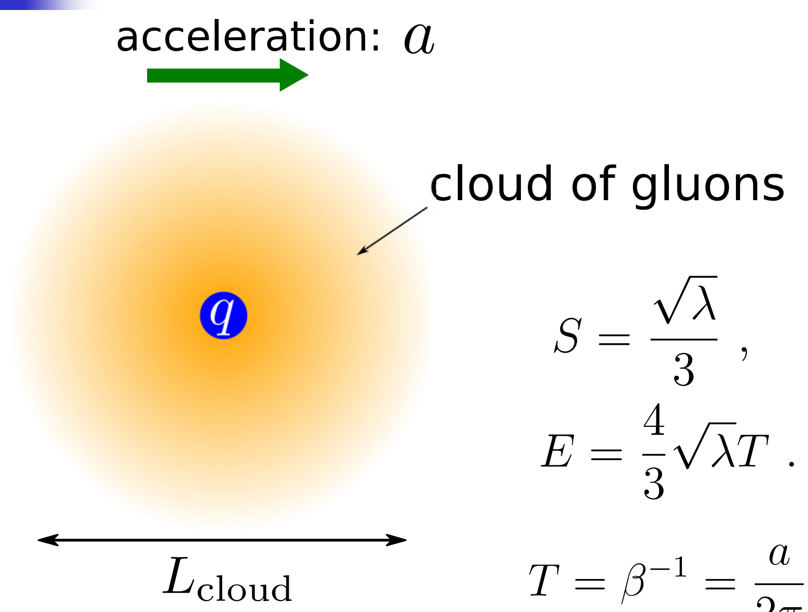
$$\langle F_L(0) F_R(0) \rangle_W \sim \left(1 + \frac{\delta a}{4a} e^{a|\tau_0|}\right)^{-2}$$



Scrambling time:

$$\tau_* \sim \frac{1}{a} \ln\left(\frac{a}{\delta a}\right) .$$

Fast scrambling



$$S = \frac{\sqrt{\lambda}}{3}\,,$$

$$E = \frac{4}{3}\sqrt{\lambda}T\,.$$

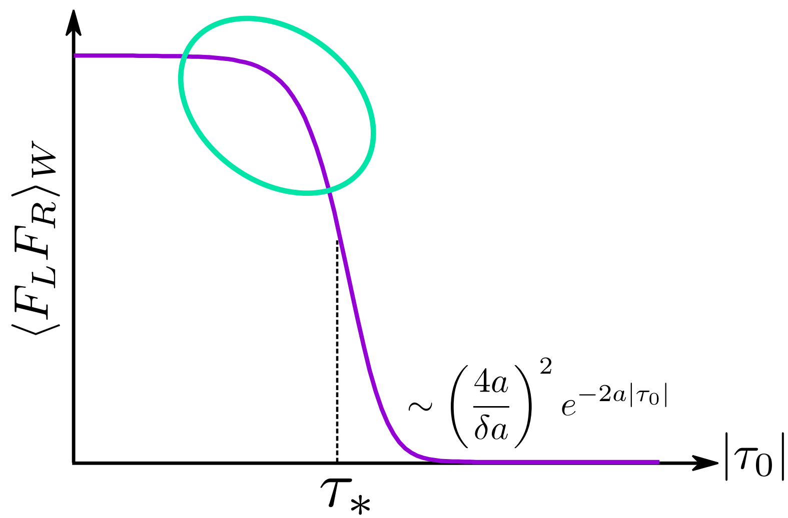
$$T = \beta^{-1} = \frac{a}{2\pi}$$

$$\tau_* \sim \frac{1}{a}\ln\left(\frac{a}{\delta a}\right)\,.$$

$$\tau_* \sim \frac{\beta}{2\pi}\ln\left(\frac{S}{\beta\delta E}\right)$$

Holographic EPR is a fast scrambler.

Saturate Lyapunov bound



$$\langle F_L F_R \rangle_W \sim \left(1 + \frac{\delta a}{4a} e^{a|\tau_0|}\right)^{-2}$$

saturates the bound.

Saturation of the Lyapunov bound

$\not\Rightarrow$ Einstein dual.

Summary

We proposed one of the simplest model showing fast scrambling: holographic EPR pair.

scrambling time

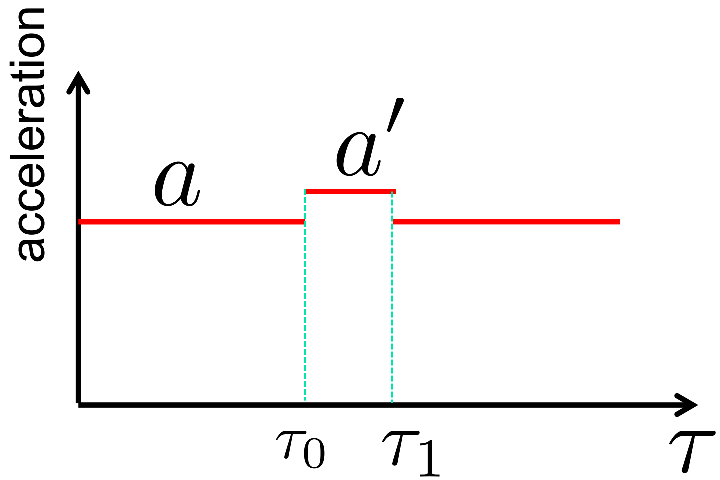
Lyapunov exponent $\lambda_L = 2\pi T$

Fast scrambler
Saturation of the Lyapunov bound

$\not\Rightarrow$ Einstein dual.

(SYK is really Einstein gravity?)

Two shock case



$$\langle F_L F_R \rangle_{ww'} \sim \left(1+\frac{\delta a}{4a}e^{a|\tau_0|}\left(1-e^{-a(\tau_1-\tau_0)}\right)\right)^{-2}$$

Fast scrambling if tau1-tau2 is not too small.

Decreasing the accerelation

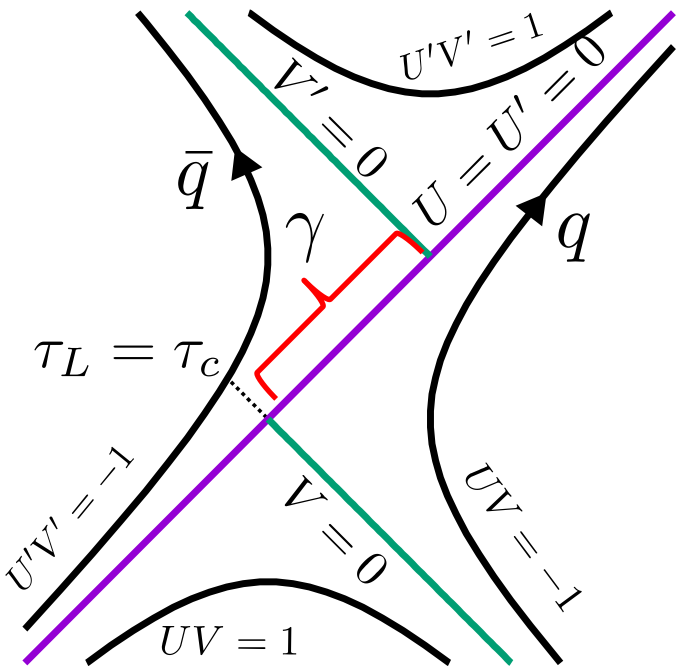
$$\delta a < 0 \quad \Rightarrow \quad \gamma < 0$$

$$\langle F_L F_R \rangle_W \sim \left(1 + \frac{\gamma}{2}\right)^{-2} = \left(1 - \frac{|\delta a|}{4a} e^{a|\tau_0|}\right)$$

Diverging？

What is happening?

One-way traverse wormhole



Causally connected.
"One-way traversable wormhole"

Divergence of the correlation or OTOC.

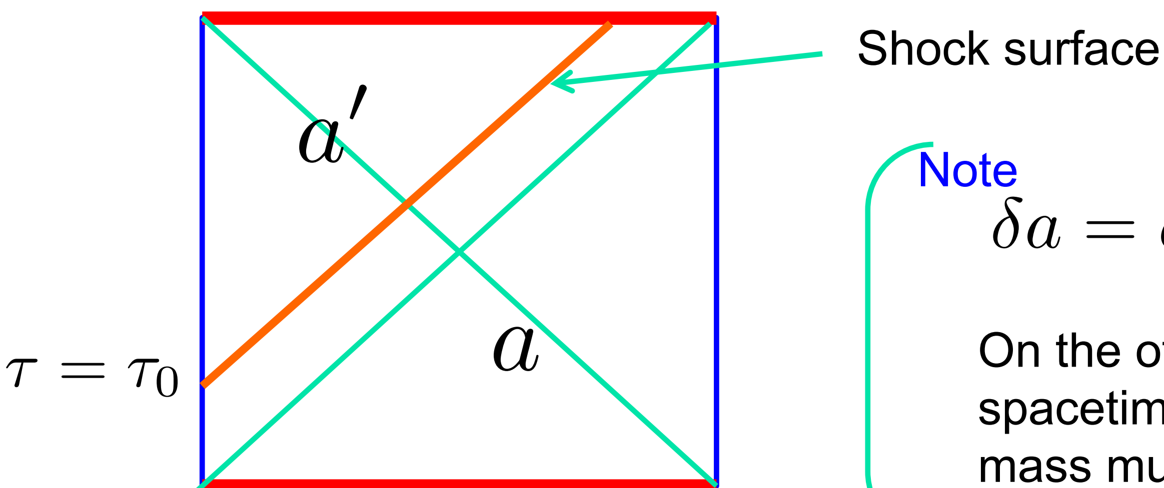
δa > 0

induced metric

$$ds_h^2 = -[\sigma^2 - M(\tau)]d\tau^2 + 2d\tau d\sigma \ ,$$

$$M(\tau) = a^2 + (a'^2 - a^2)\theta(\tau - \tau_0) \ .$$

Vaidya-like geometry with infalling null shock.

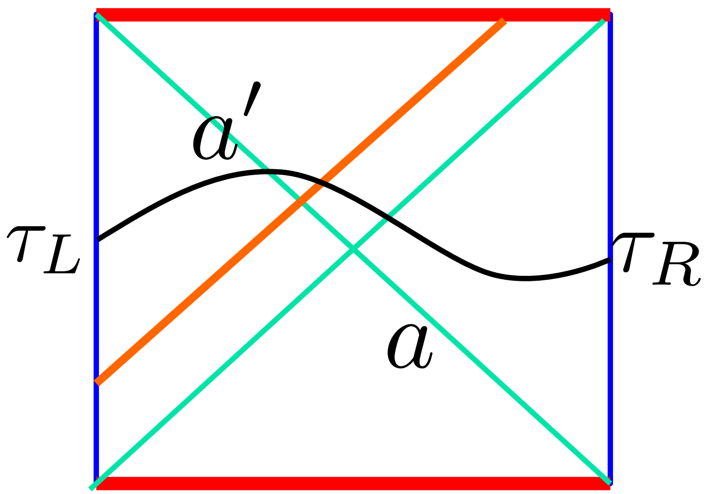


Note

$\delta a = a' - a$ can be negative.

On the other hand, for Vaidya spacetime, the deviation of the mass must be positive.

Correlation between quark and antiquark



geodesic distance

Correlation:

geodesic distance

$$\langle F_L(\tau_L) F_R(\tau_R) \rangle_W \sim e^{-d}$$

Field on worldsheet is the target space coords. $x(\tau, \sigma)$

Its conjugate operator is the force.

$$\mathcal{L}_{\text{quark}} \sim -m\sqrt{1-v^2} + V(x)$$

$$\partial \mathcal{L}_{\text{quark}} / \partial x \sim F$$

$$\left[\delta a \to 0 \ , \qquad \tau_0 \to -\infty \ , \qquad \gamma \equiv \frac{\delta a}{2a} e^{-a\tau_0} \ : \ \text{fixed} \ . \right]$$

Comment on the singularity

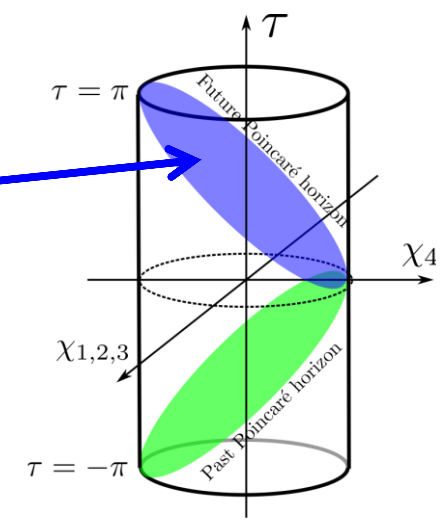
The induced metric is regular here.

$$ds_h^2 = -\frac{4dUdV}{(1+UV)^2} \ .$$

But the string solution diveges

$$t = \frac{1}{a}\frac{U+V}{1-UV} \ , \quad x = -\frac{1}{a}\frac{U-V}{1-UV} \ , \quad z = \frac{1}{a}\frac{1+UV}{1-UV} \ .$$

The "singularity" corresponds to
the Poincare horizon in the target space.



If we take the global coordinates for the target space,
there is no divergence.

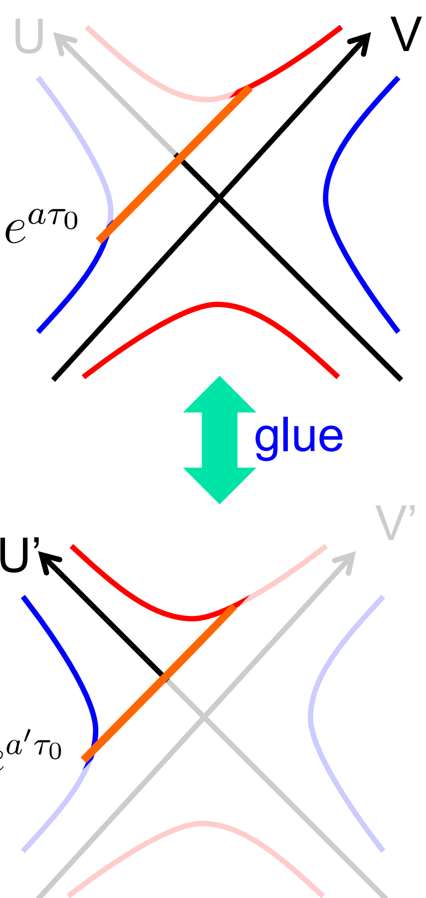
Spacetime structure

For $\tau < \tau_0$

we introduced (U,V)-coordinates.

$$U = e^{a\tau}\ , \quad V = -\frac{\sigma - a}{\sigma + a} e^{-a\tau} \iff \tau = \frac{1}{a}\ln U\ , \quad \sigma = a\,\frac{1-UV}{1+UV}\ .$$

$$U = e^{a\tau_0}$$



$$U' = e^{a'\tau_0}$$

For $\tau > \tau_0$

we introduce (U',V')-coordinates as

$$U' = e^{a'\tau}\ , \quad V' = -\frac{\sigma - a'}{\sigma + a'} e^{-a'\tau}$$

$$\iff \quad \tau = \frac{1}{a'}\ln U'\ , \quad \sigma = a'\,\frac{1-U'V'}{1+U'V'}\ .$$

Matching condition at shock surface?

Matching condition

$$U = e^{a\tau} \; , \quad V = -\frac{\sigma - a}{\sigma + a} e^{-a\tau} \quad \Longleftrightarrow \quad \tau = \frac{1}{a} \ln U \; , \quad \sigma = a \, \frac{1 - UV}{1 + UV} \; .$$

$$U' = e^{a'\tau} \; , \quad V' = -\frac{\sigma - a'}{\sigma + a'} e^{-a'\tau} \quad \Longleftrightarrow \quad \tau = \frac{1}{a'} \ln U' \; , \quad \sigma = a' \, \frac{1 - U'V'}{1 + U'V'} \; .$$

(τ, σ) is common in (U,V) and (U',V') patches.

$$a \, \frac{1 - UV}{1 + UV}\bigg|_{\tau = \tau_0} = a' \, \frac{1 - U'V'}{1 + U'V'}\bigg|_{\tau = \tau_0} \; .$$

$$\frac{a + a') \, e^{a\tau_0} \, V}{\delta a \, e^{a\tau_0} \, V + (a + a')} e^{-a'\tau_0} \; .$$

Double scaling limit

$$\delta a \to 0 \ , \qquad \tau_0 \to -\infty \ , \qquad \gamma \equiv \frac{\delta a}{2a} e^{-a\tau_0} \ : \ \text{fixed} \ .$$

$$\frac{a + a') \, e^{a\tau_0} \, V}{\delta a \, e^{a\tau_0} \, V + (a + a')} e^{-a'\tau_0} \ . \qquad \Rightarrow \qquad V' = V + \gamma \ ,$$

$$U = e^{a\tau_0} \qquad U' = e^{a'\tau_0}$$

OTOC measures chaos?

Maldacena, Shenker & Stanford, 16

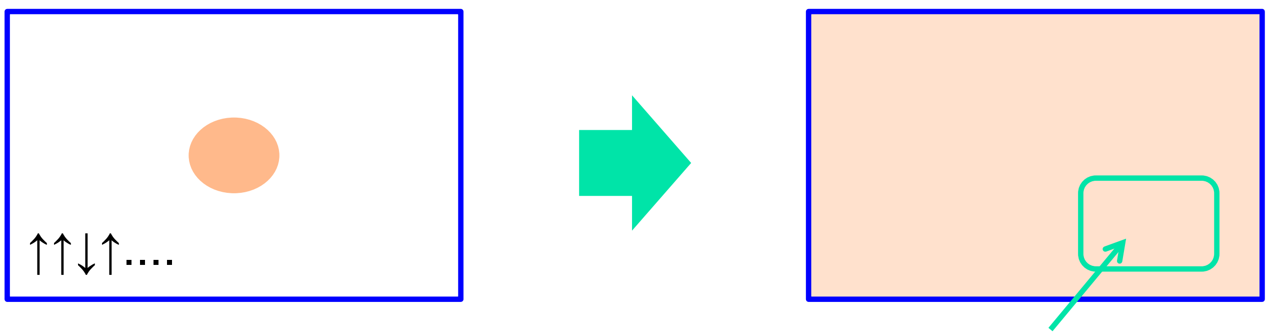
$$C_T = -\langle [\hat{x}(t), \hat{p}(0)]^2 \rangle \sim \{x(t), p(0)\}_{\mathrm{PB}}^2$$

$$= \left(\frac{\delta x(t)}{\delta x(0)} \right)^2 \quad \to e^{2\lambda t}$$

λ :Lyapunov

What is scrambling

Scrambling = delocalization of local quantum information



$$\mathrm{tr}_{\mathrm{outside}}[|\psi\rangle\langle\psi|] \simeq \rho_{\mathrm{thermal}}$$

Scrambling ~ Thermalization

~ Quantum chaos, Butterfly effect

Important in the context of black hole physics or AdS/CFT.

Validity of the geodesic approximation

$$\langle F_L F_R \rangle_W \sim e^{-\Delta d}$$

is effective for large Δ.

Our case is Δ=1...

We use the geodesic approximation
just for the rough estimation of the correlation.

(For pure AdS, the geodesic approimation is exact for any Δ.)

Geodesic approximation

$$\langle F_L F_R \rangle_W \sim e^{-\Delta d}$$

conformal weight **geodesic distance**

Conjugate bulk field for F: $x(\tau, \sigma)$

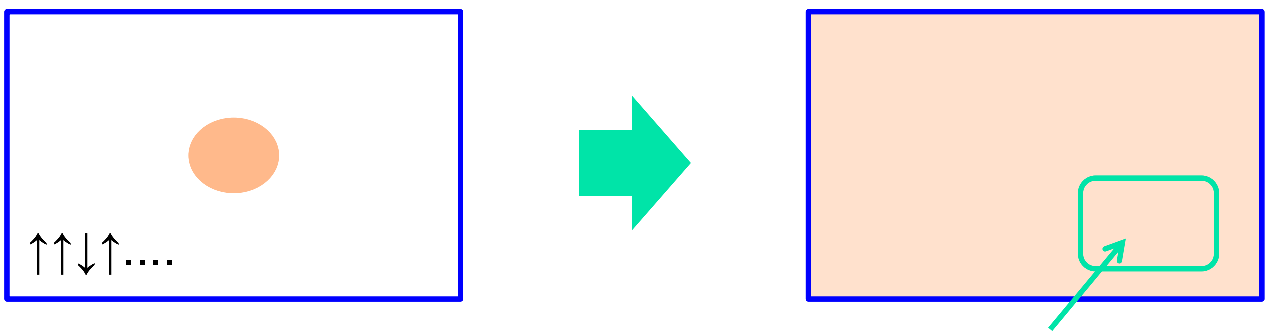
$$\begin{pmatrix} \mathcal{L}_{\text{quark}} \sim -m\sqrt{1-v^2} + V(x) \\ \partial \mathcal{L}_{\text{quark}} / \partial x \sim F \end{pmatrix}$$

massless

$\Rightarrow$ $\Delta = 1$

What is scrambling

Scrambling = delocalization of local quantum information



$$\mathrm{tr}_{\mathrm{outside}}[|\psi\rangle\langle\psi|] \simeq \rho_{\mathrm{thermal}}$$

Scrambling ~ Thermalization

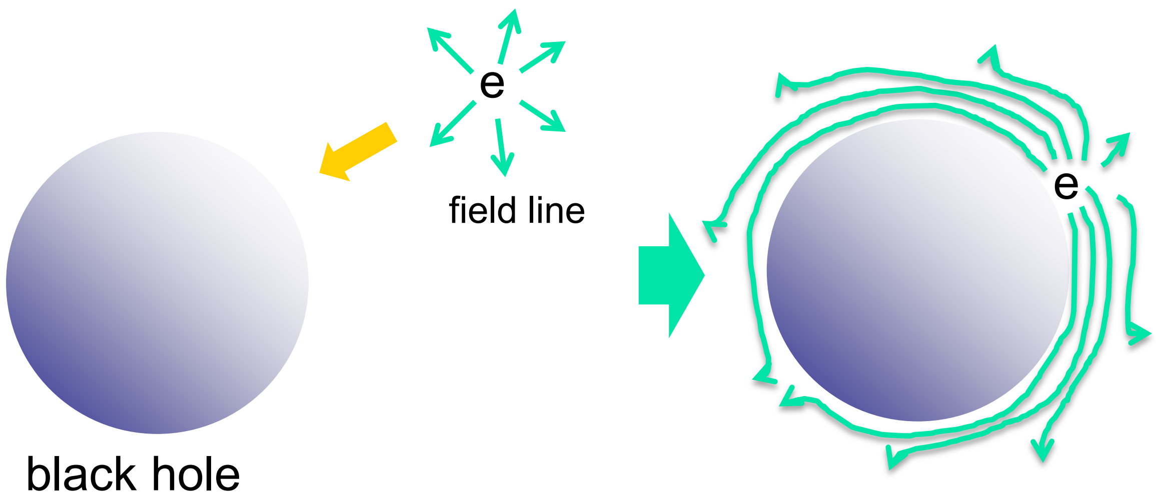
~ Quantum chaos, Butterfly effect

Important in the context of black hole physics or AdS/CFT.

Fast scrambling conjecture

"Black holes are the fastest scramblers in nature"

Sekino&Susskind, 08



In Schwarzshild coordinates, the field line surrounds the BH.

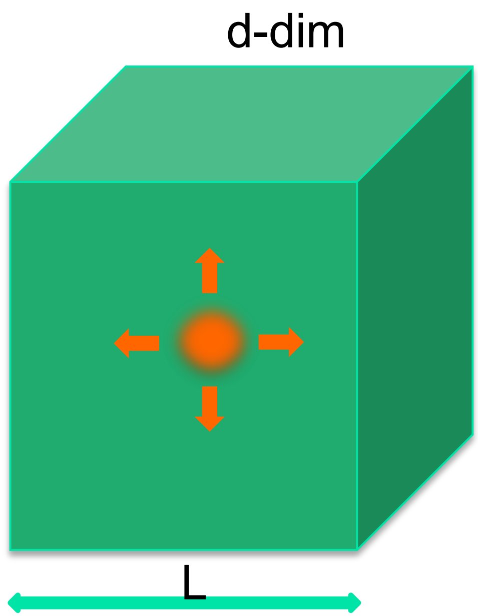
We cannot distinguish BH and charge

$$t_* \sim \beta \ln S \sim \beta \ln N$$

What is scrambling time of a conventional matter?

diffusion coefficient

Scale of the excitation: $\langle \Delta x^2 \rangle \sim \kappa t$



Scrambling time: $t_* \sim \frac{1}{\kappa} L^2$

Number of DOF $\propto$ Volume $L^d \propto N$

$$\Rightarrow \quad t_* \propto N^{2/d}$$

BH's scrambling is much faster than conventional matters.

$$\ln N \ll N^{2/d}$$

Quantum DOF of BH would be strongly chaotic.

$$t_* \sim \beta \ln S$$: Fast scrambler

Lyapunov bound

Maldacena, Shencker&Stanford, 16

1. Quantum Lyapunov exponent is defined by the OTOC.

$$\langle W(t)V(0)W(t)V(0)\rangle_\beta \sim 1 - \# e^{\lambda_L t}$$

$$\left[C_T = -\langle [\hat{x}(t), \hat{p}(0)]^2 \rangle \sim \{x(t), p(0)\}_{\mathrm{PB}}^2 = \left(\frac{\delta x(t)}{\delta x(0)}\right)^2 \to e^{2\lambda t} \right]$$

2. The Lyapunov exponent has upper bound. $\lambda_L \leq 2\pi T$

AdS/CFT $\Rightarrow$ $\lambda_L = 2\pi T$

"refinement of the fast scrambling conjecture"

3. If a QFT saturates the bound, it has an Einstein dual.

most chaotic $\Rightarrow$ black hole dual

(ex. SYK model)

Lyapunov bound

OTOC

saturates the bound.

Saturation of the Lyapunov bound

Einstein dual.