Entanglement Entropy for 2D Gauge Theories with Matters

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Collaboration with

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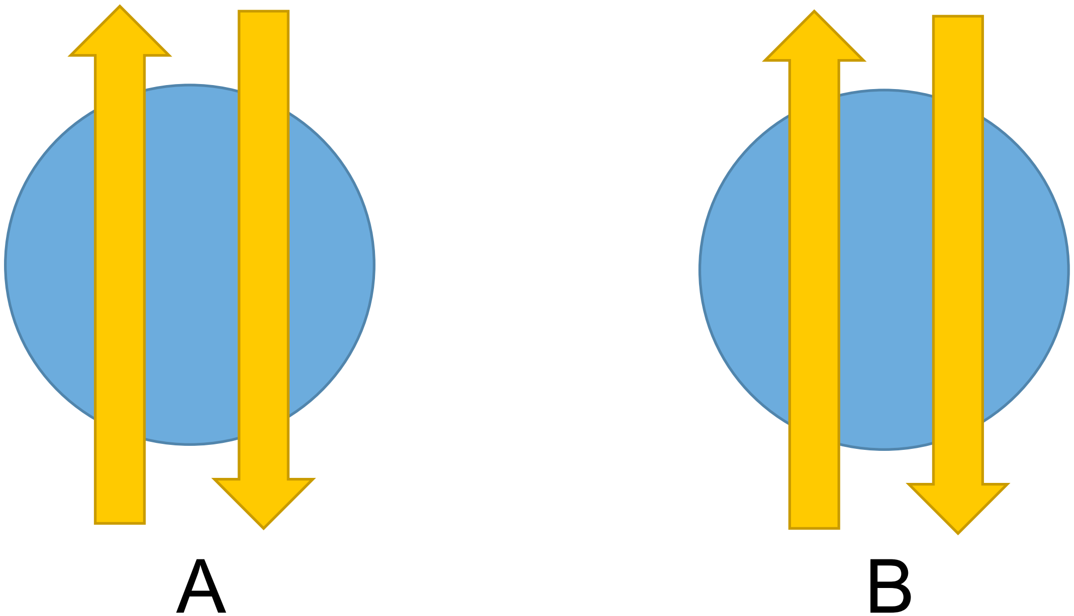
What was entanglement?

Bipartite System



$|\uparrow\rangle_A$ or $|\downarrow\rangle_A$

$|\uparrow\rangle_B$ or $|\downarrow\rangle_B$

Basis $|\uparrow\rangle := \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $|\downarrow\rangle := \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$\sigma^z |\uparrow\rangle = |\uparrow\rangle$$
$$\sigma^z |\downarrow\rangle = -|\downarrow\rangle$$

Not Tensor Product = Entanglement

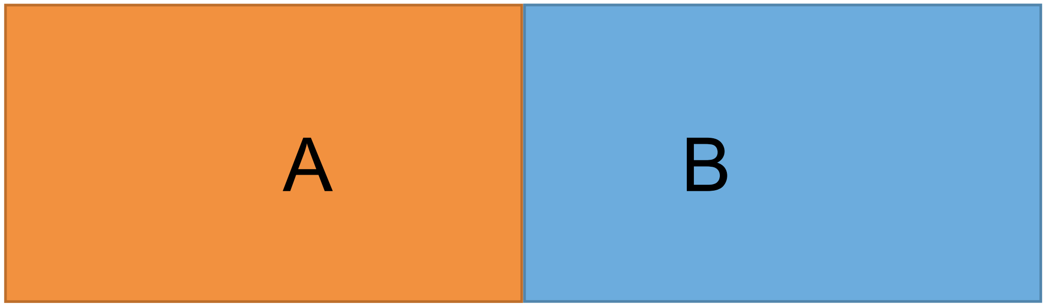
$$|\psi_1\rangle = \frac{1}{2}(|\uparrow\rangle_A + |\downarrow\rangle_A)(|\uparrow\rangle_B + |\downarrow\rangle_B)$$

Tensor Product $\rightarrow$ no Entanglement

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}|\uparrow\rangle_A|\uparrow\rangle_B + \frac{1}{\sqrt{2}}|\downarrow\rangle_A|\downarrow\rangle_B$$

Correlation $\rightarrow$ Entanglement

How Much The State is Entangled? $\rightarrow$ EE



Dividing System $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$ $B = \bar{A}$

Taking a state $\rho = |\psi\rangle \langle\psi|$

EE $$\boxed{S_{EE}(\mathcal{H}, A, |\psi\rangle) = -\mathrm{Tr}_{\mathcal{H}_A} \rho_A \log \rho_A}$$

$$\rho_A = \mathrm{Tr}_{\mathcal{H}_B} \rho$$

EE ~ #Bell pairs

$$|\psi_1\rangle = \frac{1}{2}(|\uparrow\rangle_A + |\downarrow\rangle_A)(|\uparrow\rangle_B + |\downarrow\rangle_B)$$

$$\Rightarrow \quad S_{EE} = 0$$

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}\,|\uparrow\rangle_A\,|\uparrow\rangle_B + \frac{1}{\sqrt{2}}\,|\downarrow\rangle_A\,|\downarrow\rangle_B$$

$$\Rightarrow \quad S_{EE} = \log 2$$

Entanglement is Extractable

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}|\uparrow\rangle_A|\uparrow\rangle_B + \frac{1}{\sqrt{2}}|\downarrow\rangle_A|\downarrow\rangle_B$$

$$|\psi_2'\rangle = \frac{1}{\sqrt{2}}|\downarrow\rangle_A(|\uparrow\rangle_B + |\downarrow\rangle_B)$$

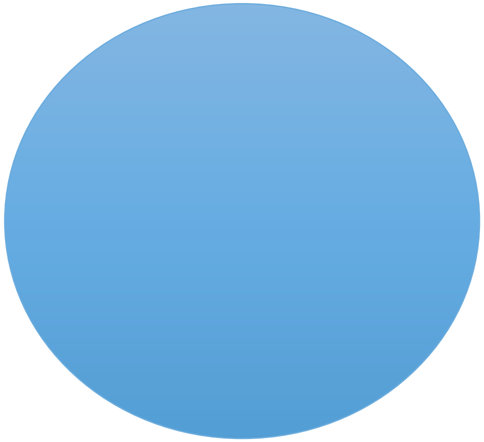
We can break entanglement by local operation

Entanglement is the source of quantum communication

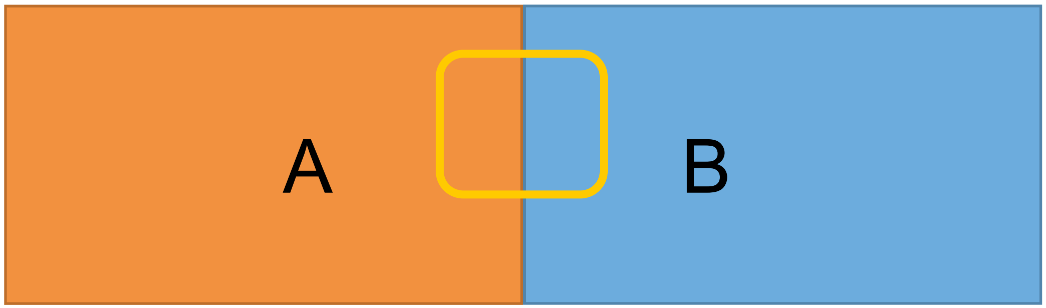
EE for Gauge Theories

Gauge/Gravity duality (Boundary/Bulk duality)

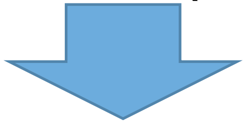
EE in boundary theory = geometric quantity of bulk theory (c.f. Ryu-Takayanagi formula)



A Difficulty



Non-local dof(Ex. Wilson loop)



Physical (i.e. gauge invariant)Hilbert space cant be spacely decomposed separately.

$$\mathcal{H}^{phys} \neq \mathcal{H}_A^{phys} \otimes \mathcal{H}_B^{phys}$$

How can we define EE?

Several Works on this issue

- Casini, Huerta, Rosabal(‘13) – General discussion
- Ghosh, Soni, Trivedi(‘15), Aoki, Iritani, Nozaki, Numasawa(‘15) – Extended(Exd) Hilbert space
- Aoki, Itou, Nagata(‘16) – Explicit calculation for 1+1 pure lattice gauge theory

Exd Hilbert Space $\mathcal{H}_{exd}$

- Recipe : "Forget gauge invariance for a while"

$$\mathcal{H}^{exd} = \mathcal{H}^{phys} \oplus \mathcal{H}^{unphys} \qquad \mathcal{H}^{unphys} = \mathcal{H}^{\perp}_{phys}$$

- Dof become localized $\quad \mathcal{H}^{exd} = \mathcal{H}^{exd}_A \otimes \mathcal{H}^{exd}_B$

  $\rightarrow$ We can define EE!

$$\boxed{S_{EE}(\mathcal{H}^{exd}, A, |\psi\rangle) = -\mathrm{Tr}_{\mathcal{H}^{exd}_A} \rho_A \log \rho_A}$$

$$\rho_A = \mathrm{Tr}_{\mathcal{H}^{exd}_B} \rho$$

Deference? → New Contributions

As an consequence of using exd Hilbert space,

$$S_{EE} = \underline{S_{Shannon}} + \underline{S_{color}} + S_{Bell}$$

In the Work

- We estimated how these 3 types of EE emerge in the ground state for 1+1D Lattice Gauge Theories with Matters

- We found all of 3 EE contribute

EE for 1+1D Lattice Gauge Theories with Matters

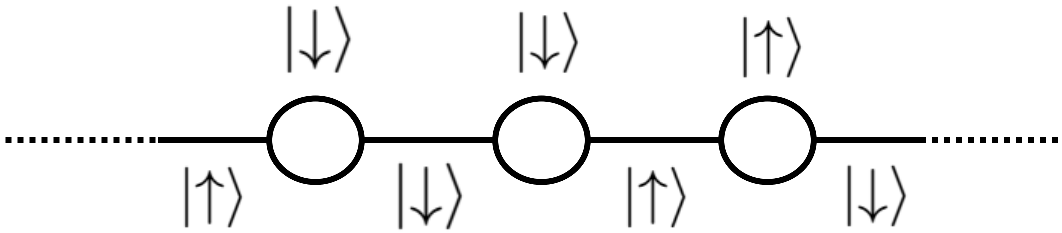
- Lattice → Explicit calculation
  (we have to consider continuum limit)
- 1+1D → don't need any gauge coupling approximation
- Matters → the source of EE in the ground state
- Dimensional reducted theory
- We will use perturbation about mass of matter(Hopping parameter expansion)

Contents

- Introduction
- Z2 gauge theory – Shannon part
- SU(N) gauge theory – Color part
- Ground state

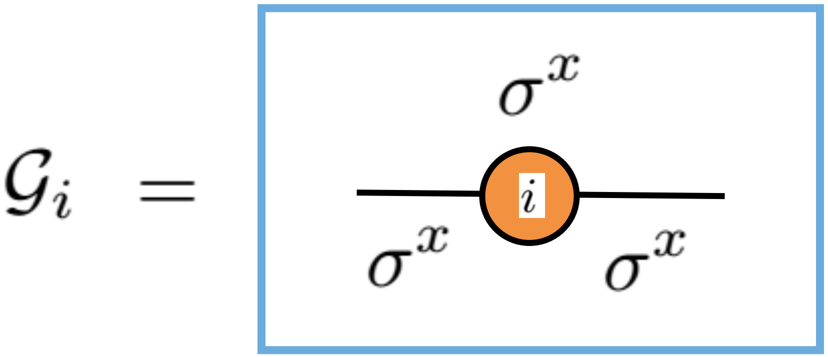
Z2 Gauge Theory

Lattice



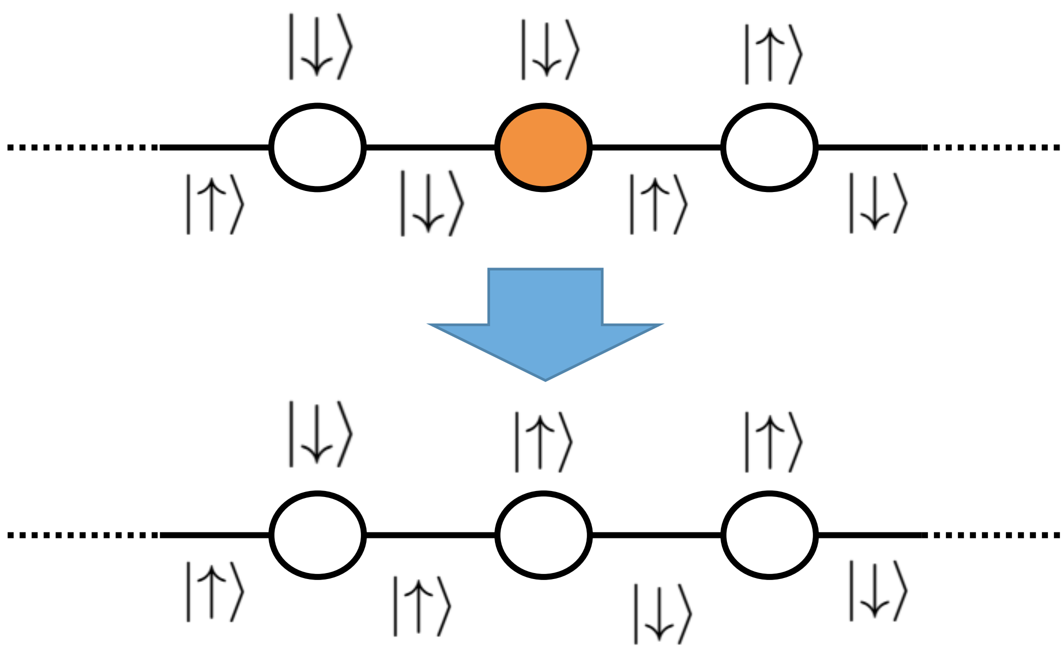
- Temporal gauge
- Gauge field on links with $|\uparrow\rangle$ or $|\downarrow\rangle$
- Matter field(Ising spins) on sites with $|\uparrow\rangle$ or $|\downarrow\rangle$

Z2 Gauge Transformation



$$\mathcal{G}_i \, |phys\rangle = |phys\rangle$$

Gauge constraint



Unphysical state

Using Basis $|\pm\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle \pm |\downarrow\rangle)$

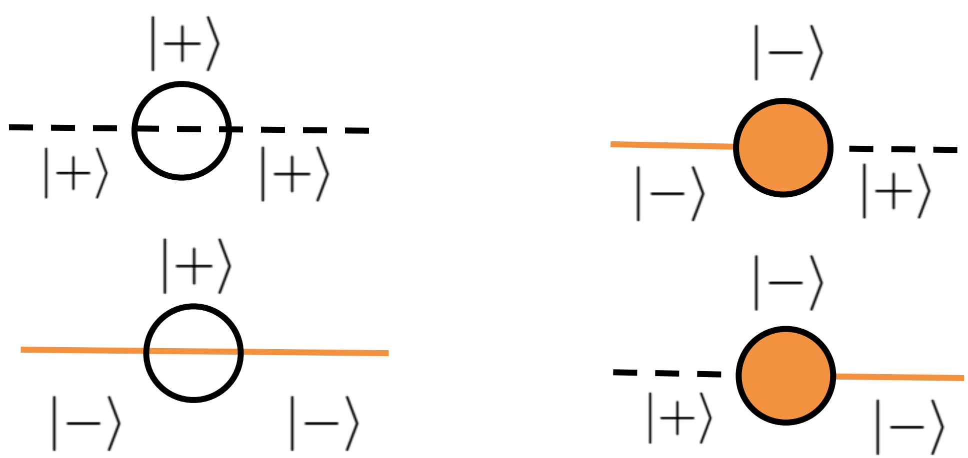
σ^x

$|+\rangle \rightarrow |+\rangle$ Trivial rep.

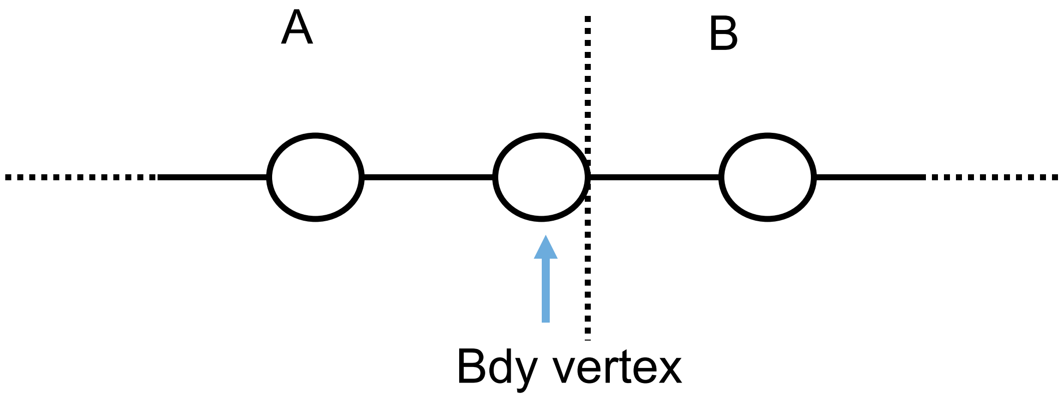
$|-\rangle \rightarrow -|-\rangle$ Non-trivial rep.

4 parts

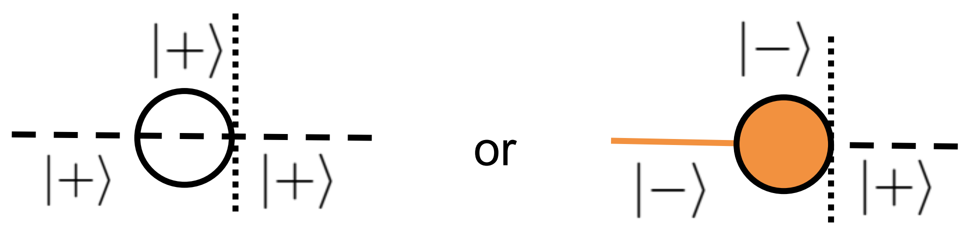
Gauge invariant(physical) combination



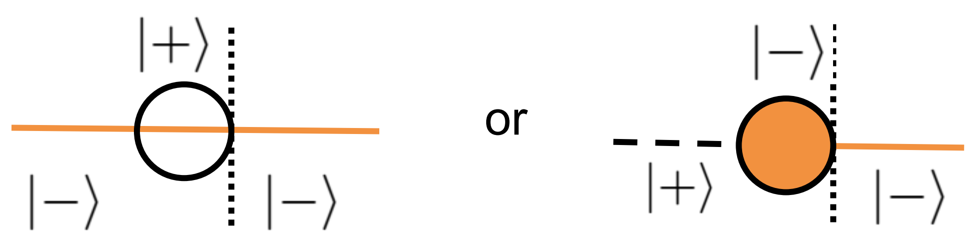
Flux Penetrating



Non penetrating



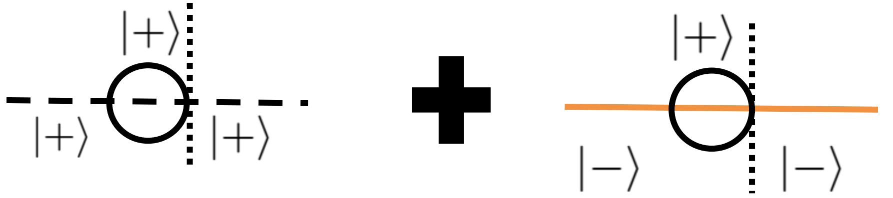
Penetrating



Origin of Shannon Part

$$|\psi\rangle = \sqrt{p_{NP}}\,|NP\rangle + \sqrt{p_P}\,|P\rangle$$

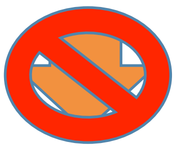
Ex.

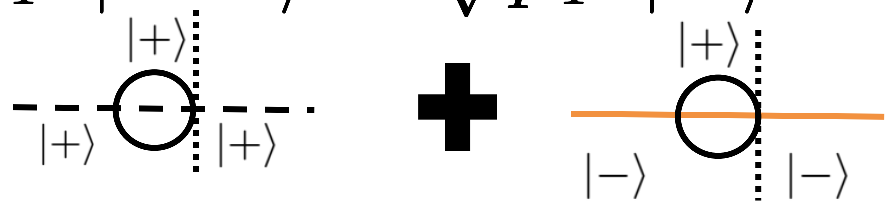


$|NP\rangle$ and $|P\rangle$ belong to different superselection sector (different rep. of gauge group)

Shannon Part is Not Extractable

$$|\psi\rangle = \sqrt{p_{NP}}\,|NP\rangle + \sqrt{p_P}\,|P\rangle$$





$$|\psi'\rangle = |NP'\rangle$$

We cannot break entanglement by local operation (it will break gauge sym.)

→This correlation is classical

Shannon Part

$$|\psi\rangle = \sqrt{p_{NP}}\,|NP\rangle + \sqrt{p_P}\,|P\rangle$$

$$\begin{aligned} S_{EE}(\mathcal{H}_{exd}, |\psi\rangle) = & -p_{NP}\log p_{NP} - p_P \log p_P \\ & + p_{NP} S_{EE}(\mathcal{H}_{NP}^{exd}, |NP\rangle) + p_P S_{EE}(\mathcal{H}_P^{exd}, |P\rangle) \\ = & S_{Shannon} + S_{Bell} \end{aligned}$$

Summary of Z2

- Use eigenstate of gauge trans. as basis → rep. of Z2
- Classify the state by the rep. of flux penetrating the boundary

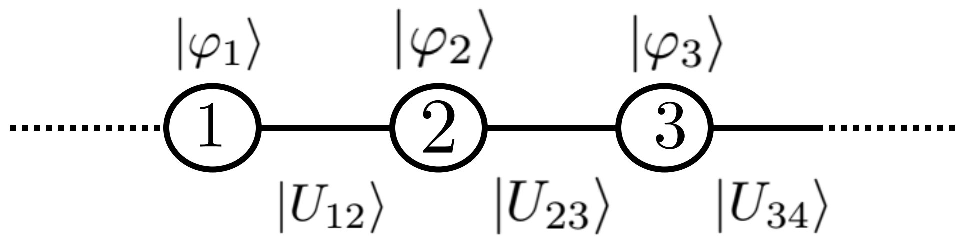
→ Shannon part

- For each sector we may have Bell pair part

SU(N) Gauge Theory

SU(N)+Fundamental Scalar Field

Lattice



- Temporal gauge
- Gauge field on links with link variable U_{ij}

$$U_{ij} \in SU(N) \text{ group}$$

- Matter field on sites with φ_i

Rule of the Game is Almost Same

- Use eigenstate of Casimir op. as basis → irre. rep. of SU(N)
- Classify the state by the rep. of flux penetrating the boundary

→ Shannon part

- For each sector we may have Bell pair part + color part

Using Basis $|R\rangle$

labeled by eigenvalues of Casimir operator

$$|R\rangle \xrightarrow{\;g\;} g(R)\,|R\rangle$$

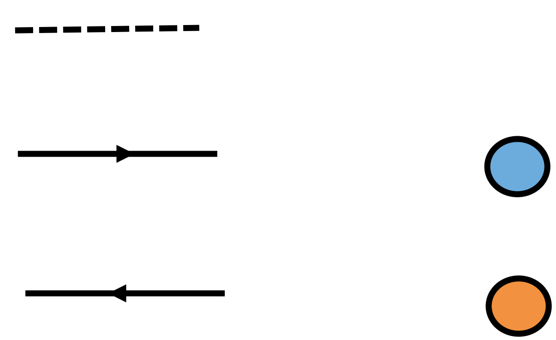
R irre. rep. for SU(N) group

link　　Site (Example)

$R = 1$ trivial rep.

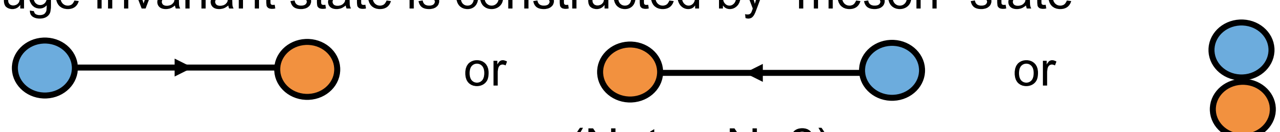
$R = F$ fund rep.

$R = \bar{F}$ anti-fund rep.



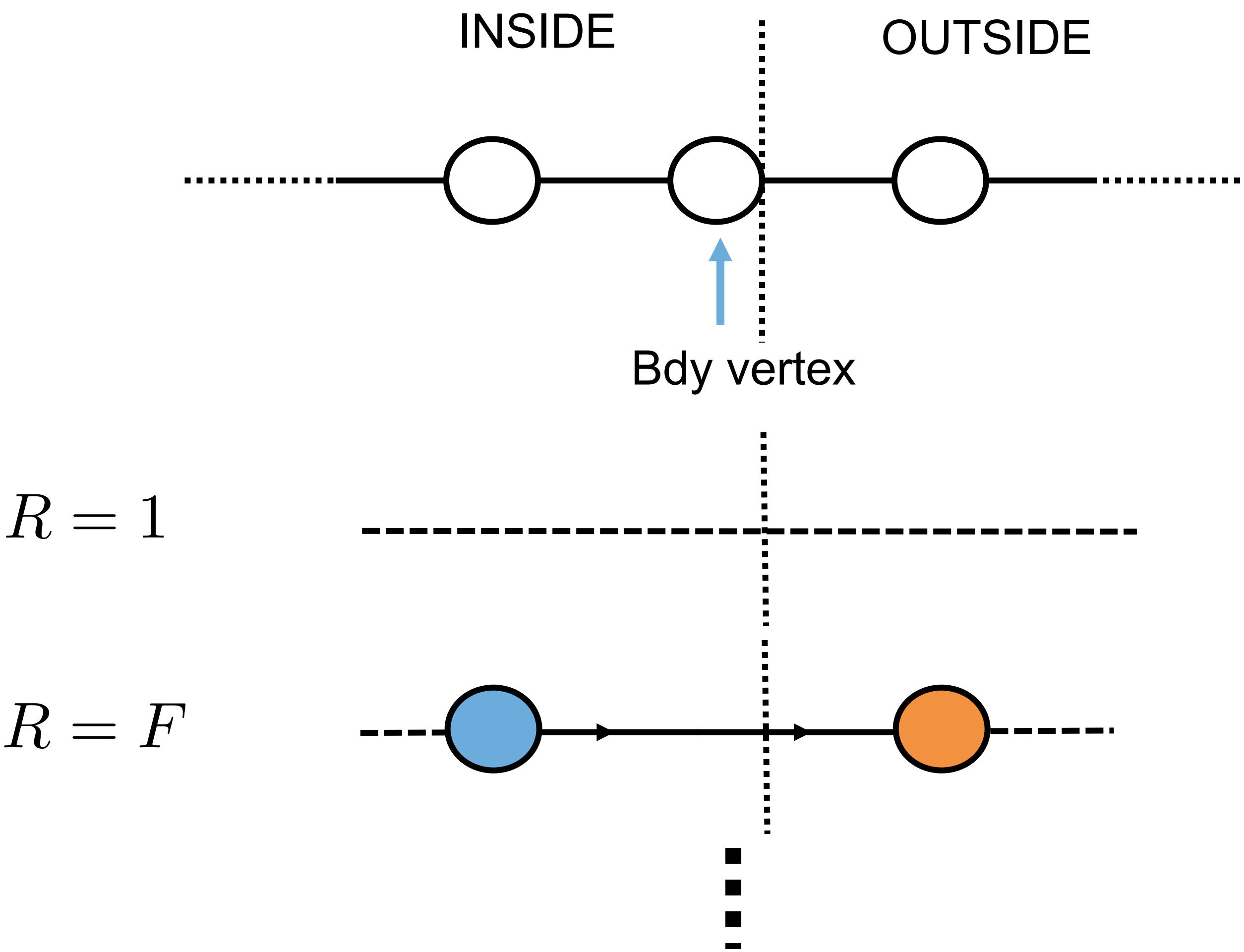
⋮

Gauge invariant state is constructed by “meson” state



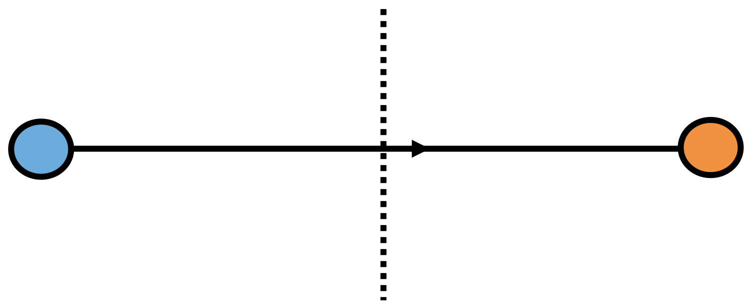
(Note : N>2)

Classifying at boundary



Each representation → Superselection sector

Single meson State



$$S_{EE} = \log N$$

Superselection Sector is fixed(Fundamental)→No Shannon Part

Is this Bell Pair (extractable) part? → No.

Color Part

In non-abelian case, we have color dof (this is unphysical)

correlation of color dof → color part of EE

$$S_{EE} = S_{Shannon} + S_{color} + S_{Bell}$$

Color Part

$$|\psi\rangle = \sum_R \sqrt{p_R}\, |\psi_R\rangle$$

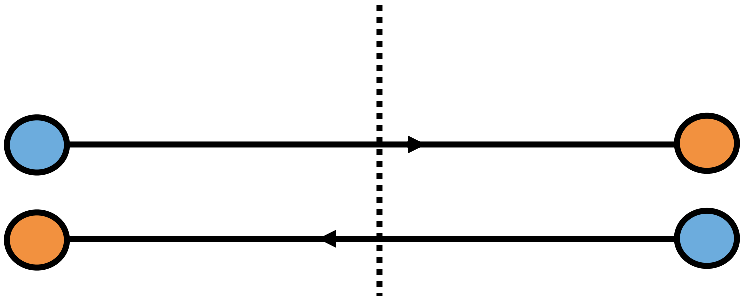
$$S_{EE}(\mathcal{H}_{exd}, |\psi\rangle) = -\sum_R p_R \log p_R$$

$$+\sum_R p_R \log d_R$$

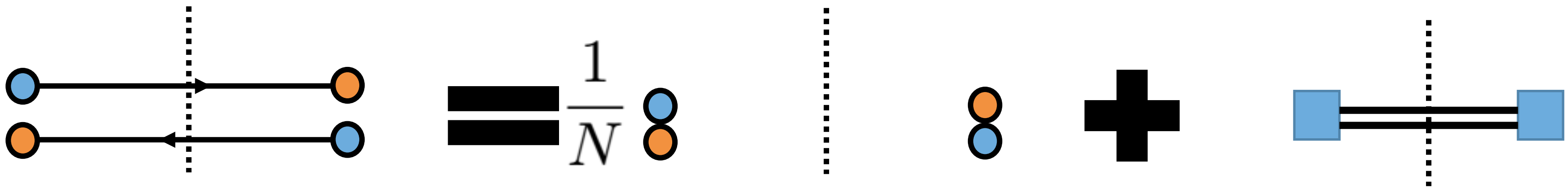
$$+\sum_R p_R S_{EE}(\mathcal{H}_R^{exd}, |\psi_R\rangle)$$

$$= S_{Shannon} + S_{color} + S_{Bell}$$

Multiple meson State



Multiple meson State



Fundamental $\otimes$ Anti-Fundamental   Singlet $\oplus$ Adjoint

We have to decompose the state to ire. Reps.

$$S_{EE} = -\left\{ \frac{N+1}{2N} \log\left(\frac{N+1}{2N}\right) + \frac{N-1}{2N} \log\left(\frac{N-1}{2N}\right) \right\} + \frac{N-1}{2N} \log(N^2-1) \,.$$

Summary of SU(N)

- Use eigenstate of Casimir op. as basis → irre. rep. of SU(N)
- Classify the state by the rep. of flux penetrating the boundary

→ Shannon part

- For each sector we may have Bell pair part + color part

Ground State

Some Tools

- Analysis for massless matter is difficult
- We used Hopping Parameter Expansion(HPE) with $K = 1/(m^2a^2 + 2)$
- By using Transfer Matrix method, we can derive the ground state order by order of K

Result – All Parts appear

- No Entanglement up to 2nd order
- Shannon part and color part first appear at 3rd order(with fundamental rep.)
- Bell pair part first appear at 6th order(with multiple meson excitation)
- Bell pair part is N^0 order

(Fundamental matter)

Connection with Continuum vacuum

- Higher order correction must be included
- EE is positive definite

→ In the continuum vacuum all three contributions exist

- The continuum vacuum is filled with lattice meson states and it causes entanglement

Summary

- In gauge theory we generically have 3 types of EE

$$S_{EE} = S_{Shannon} + S_{color} + S_{Bell}$$

- For 1+1d SU(N) lattice gauge theory with fundamental matter, the ground state have all 3 contributions (Bell pair part is N^0 order)

Future Direction

- Large N limit
- Corresponding gravity dual?
- Higher dimensional case