K理論とトポロジカル結晶物質

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Outline

1. Topological (crystalline) insulators/superconductors

2. K-group classification

3. Nonsymmorphic symmetry and new topological phases

Mobius twisted surface states

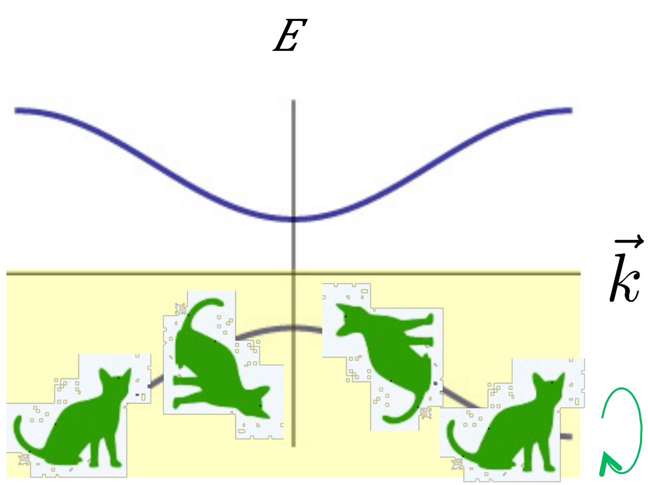
Shiozaki-MS, Phys. Rev. B90, 166114 (2014). [41pages]
Shiozaki-MS-Gomi, Phys. Rev. B91, 155120 (2015).
Shiozaki-MS-Gomi, Phys. Rev. B93, 195413 (2016).
Shiozaki-MS-Gomi, Phys. Rev. B95, 235425 (2017). [Editor's suggestion, 54pages]

Introduction

The idea of topological insulator/superconductor (TI/TSC) has been successfully established with many experimental supports for surface states

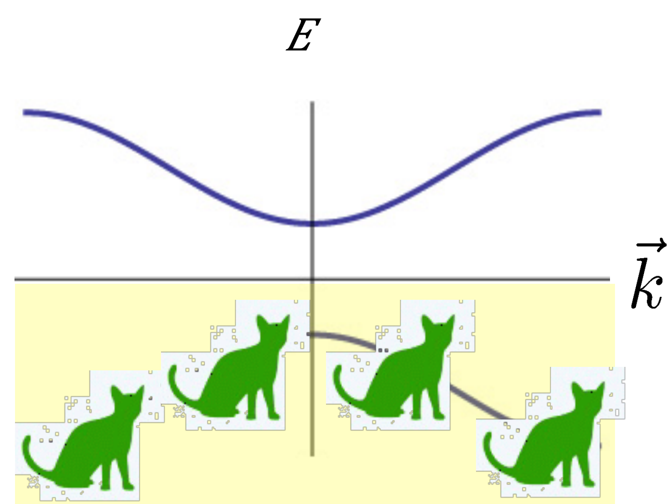
TI/TSC = Non-trivial topological # of occupied state

TI/TSC



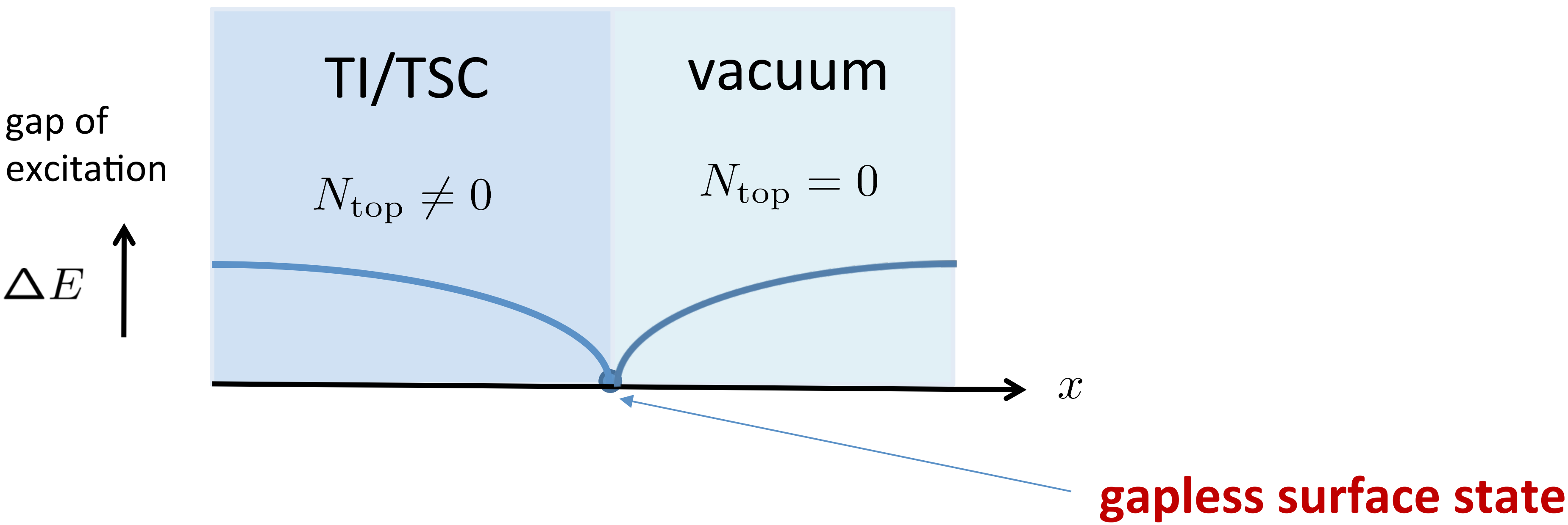
$N_{\rm top} \neq 0$

Ordinary insulator/SC



$N_{\rm top} = 0$

The non-trivial topological structure predicts the existence of gapless surface states



TIs/TSCs have gapless boundary states ensured by bulk topological numbers

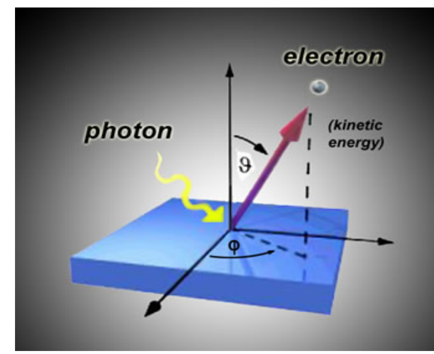
bulk-boundary correspondence

Topological insulator (TI)

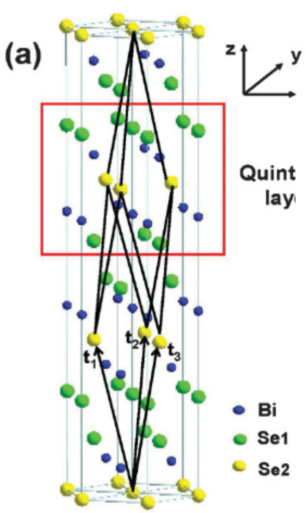
insulator supporting surface Dirac fermions

$Bi_{1-x}Sb_x$

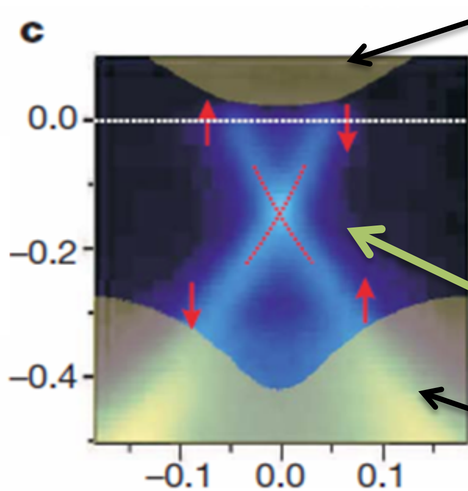
Angle resolved photoemission spectroscopy (ARPES) (photoelectron effect)



Bi_2Se_3



conduction band



Energy(eV)

wave number ($Å^{-1}$)

Fermi level

surface Dirac fermions

valence band

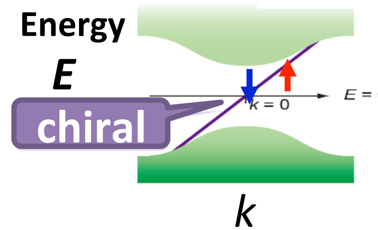
$$H(\boldsymbol{k}) = v(k_x\sigma_y - k_y\sigma_x)$$

Topological SCs

spin-triplet (odd-parity) SC

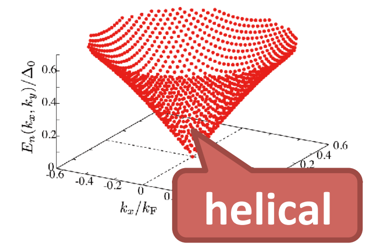
[MS (09, 10), Fu-Berg (10)]

T-breaking topological SC — Sr_2RuO_4 [Kashiwaya et al (11)]



T-invariant topological SC — ^{3}He-B [Murakawa, Nomura et al (09)]

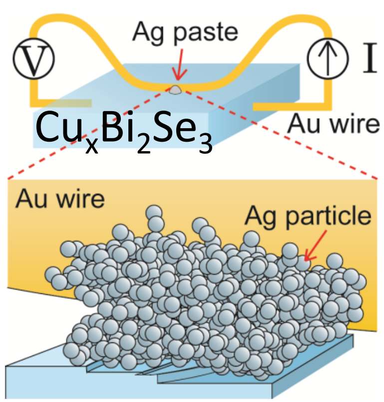
$Cu_xBi_2Se_3$ [Sasaki et al (09)]

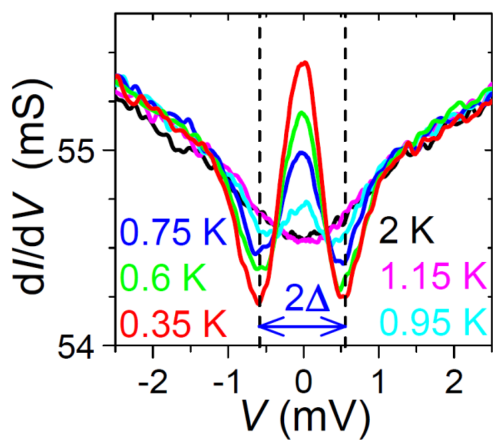


$Cu_xBi_2Se_3$

[Fu-Berg (10)]

[MS (10)]

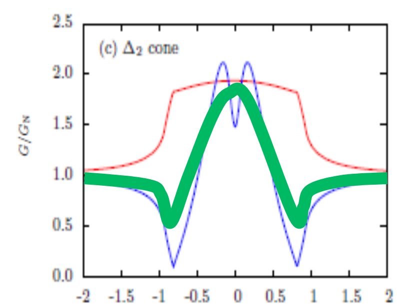




Experiment

[Sasaki-Kriener-Segawa-Yada-Tanaka-MS –Ando (11)]

Theory

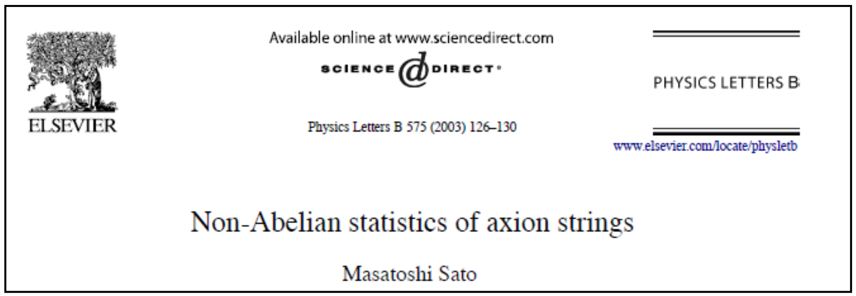


[Yamakage-Yada-MS-Tanaka (12)]

S-wave SCs can host topological superconductivity if a spinless system is realized effectively

- Dirac fermion + s-wave condensate

MS(03)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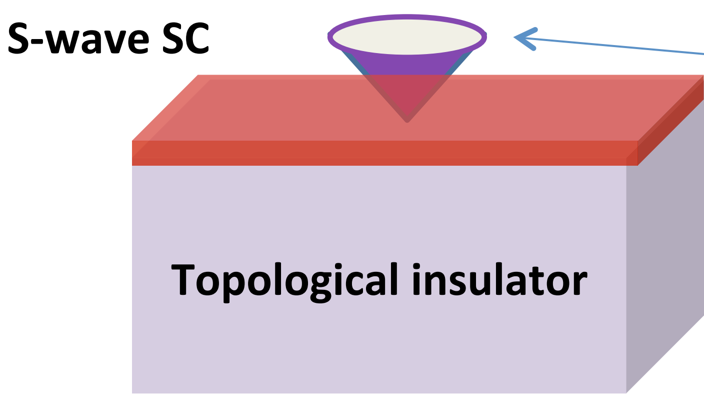
Non-Abelian statistics of axion strings

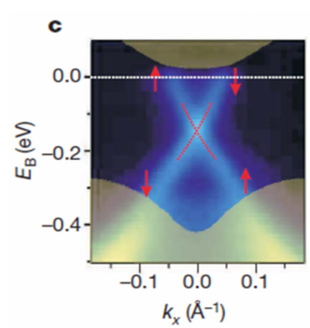
Masatoshi Sato

$$\mathcal{H} = \begin{pmatrix} -i\sigma_i\partial_i & \Phi^* \\ \Phi & i\sigma_i\partial_i \end{pmatrix}$$

Fu-Kane (08)



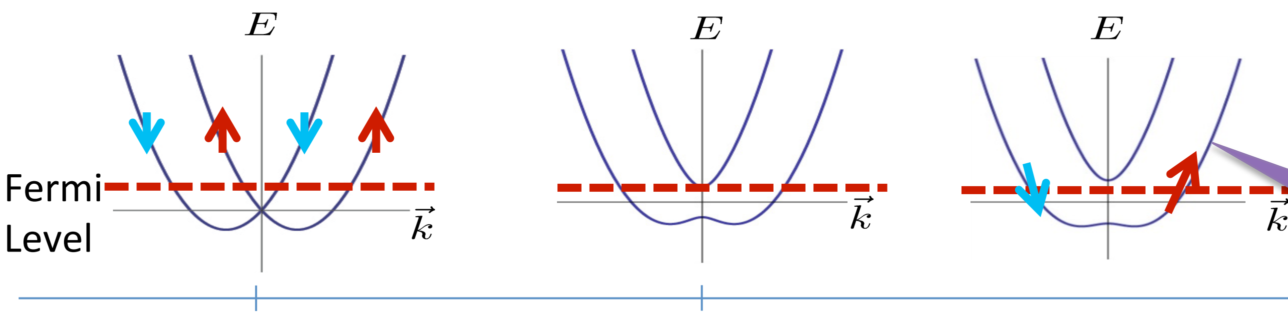
Dirac fermion + s-wave SC



Hsieh et al

- S-wave superconducting state with Rashba SO + Zeeman field

[MS-Takahashi-Fujimoto (09), J. Sau et al (10)]



Non spin-degenerate single Fermi surface

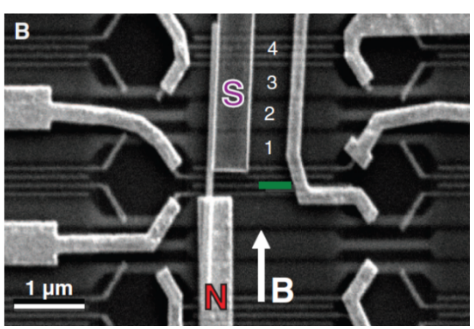
Topolgiocal SC

1D Nanowire

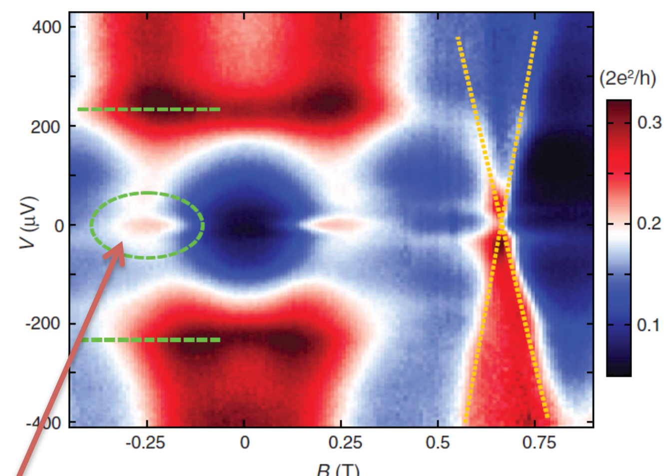
Zeeman field

Lutchyn et al (10), Oreg et al (10)

Mourik *et al.*, Science (2012)



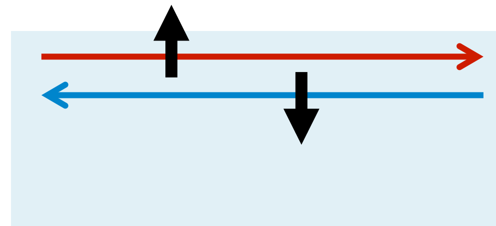
InSb/
NbTiN



Majorana Fermion

Key to realize TI/TSC = Symmetry

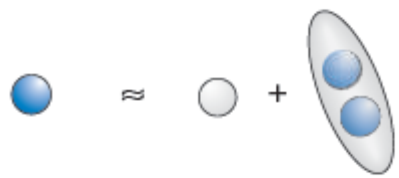
Time-reversal symmetry (TRS)



Kramers pair

- No back scattering
- topologically stable

Particle-hole symmetry (PHS)



Majorana fermion

[Schnyder-Ryu-Furusaki-Ludwig (12)] [Avron-Seiler-Simon (83)]

	TRS	PHS	CS	d=1	d=2	d=3
A	0	0	0	0	**Z**	0
AIII	0	0	1	**Z**	0	**Z**
AI	1	0	0	0	0	0
BDI	1	1	1	**Z**	0	0
D	0	1	0	Z_2	**Z**	0
DIII	-1	1	1	Z_2	Z_2	**Z**
AII	-1	0	0	0	Z_2	Z_2
CII	-1	-1	1	**2Z**	0	Z_2
C	0	-1	0	0	**2Z**	0
CI	1	-1	1	0	0	**2Z**

IQHS (A, d=2)

Majorana nanowire (D, d=1)

p+ip chiral p Sr_2RuO_4, ^3He-A (D, d=2)

^3He-B (DIII, d=3)

$Cu_xBi_2Se_3$ (DIII, d=3)

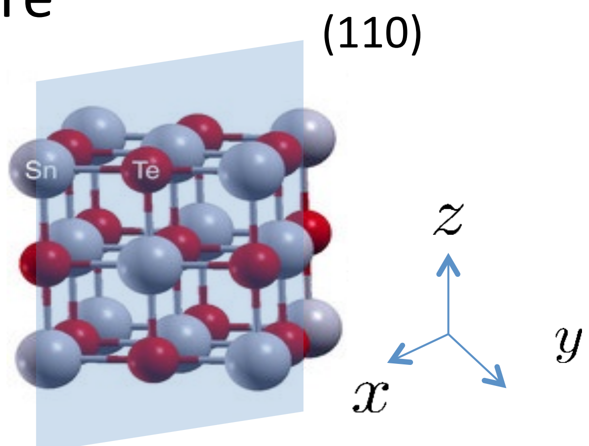
3D TI (AII, d=3)

QSH (AII, d=2)

Topological Crystalline Insulator [L. Fu (11), Hsieh et al (12)]

Recently, it has been recognized that point group symmetry also provides novel topological surface states

SnTe

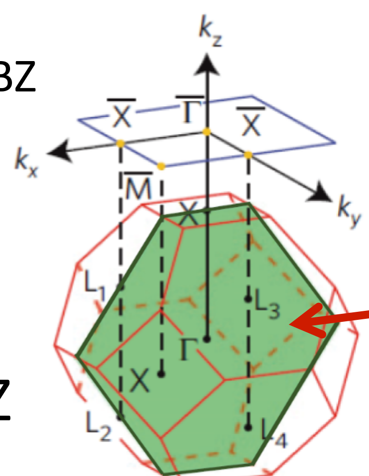


Mirror reflection $\mathcal{M}_{zx}$

$$y \to -y$$

$$\mathcal{M}_{zx}\mathcal{H}(k_x, k_y, k_z)\mathcal{M}_{zx}^{-1} = \mathcal{H}(k_x, -k_y, k_z)$$

surface BZ



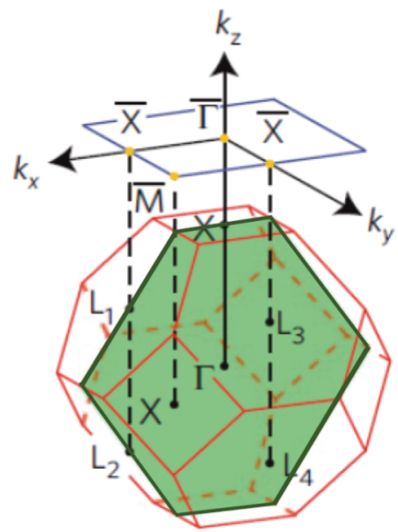
BZ

$$k_y = 0$$

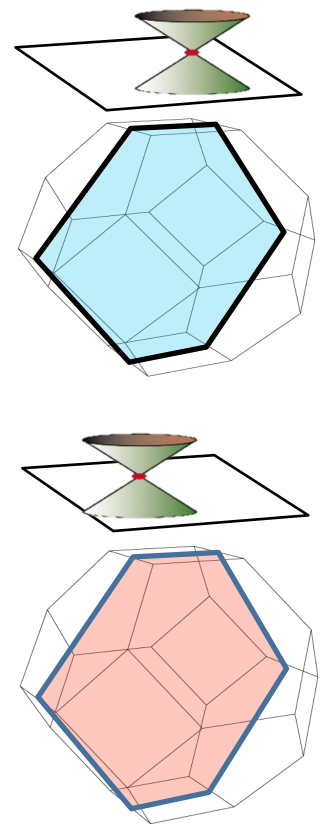
$$[\mathcal{M}_{zx}, \mathcal{H}(k_x, 0, k_z)] = 0$$

Idea

Using the eigen value of mirror operator, ky=0 plane can be separated into two QH states.



$$[\mathcal{M}_{zx}, \mathcal{H}(k_x, 0, k_z)] = 0$$



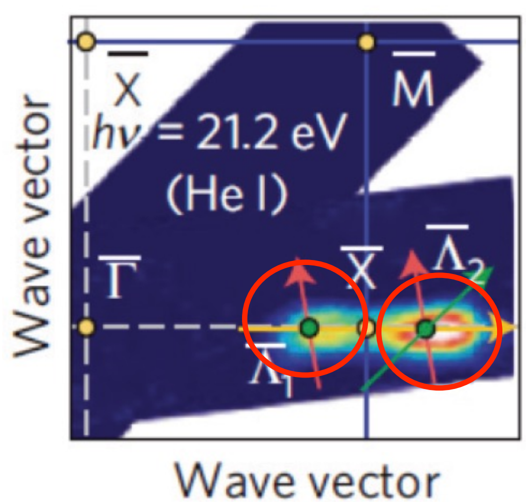
$\nu_{\rm mCh} = 1$

$\mathcal{M}_{zx} = i$

$\nu_{\rm mCh} = -1$

$\mathcal{M}_{zx} = -i$

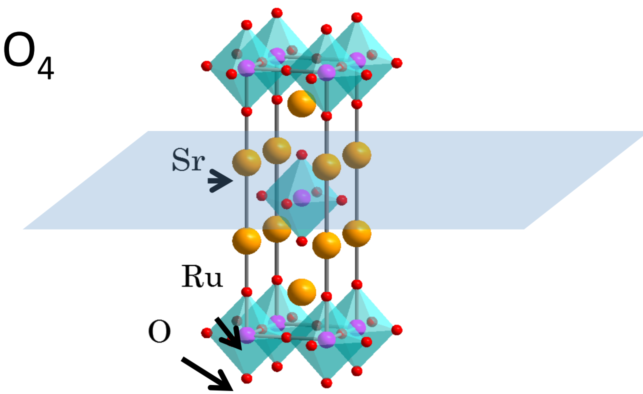
[Y. Tanaka et al (12)]



two Dirac fermions

Topological Crystalline superconductor [Ueno-MS et al (13), Zhang-Kane-Mele (13)]

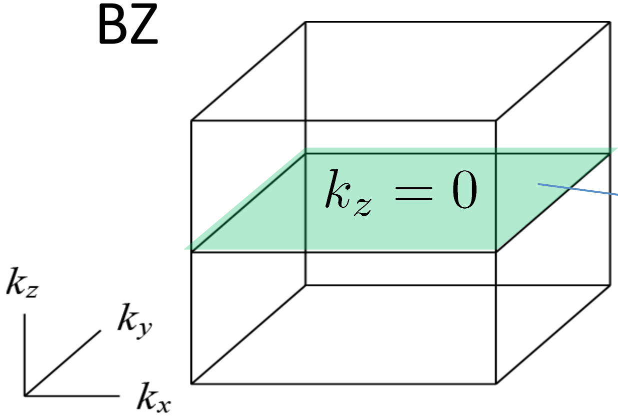
Sr_2RuO_4



mirror reflection symmetry $\mathcal{M}_{xy}$

$$(x, y, z) \to (x, y, -z)$$
$$(\sigma_x, \sigma_y, \sigma_z) \to (-\sigma_x, -\sigma_y, \sigma_z)$$

BZ



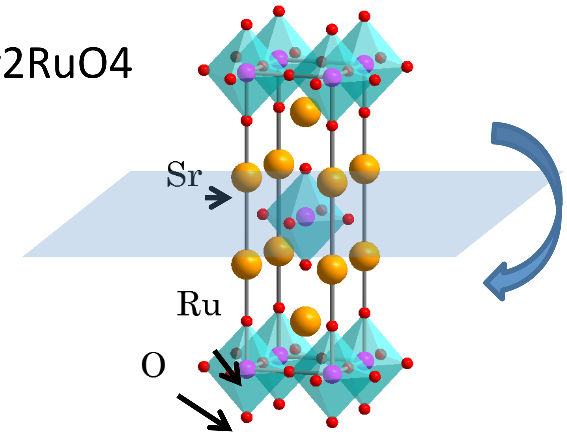
$$\mathcal{M}_{xy}\mathcal{H}(k_x, k_y, k_z)\mathcal{M}_{xy}^{-1} = \mathcal{H}(k_x, k_y, -k_z)$$

$$[\mathcal{M}_{xy}, \mathcal{H}(k_x, k_y, 0)] = 0$$

Application to Sr_2RuO_4

[Ueno-MS et al (13)]

Sr2RuO4

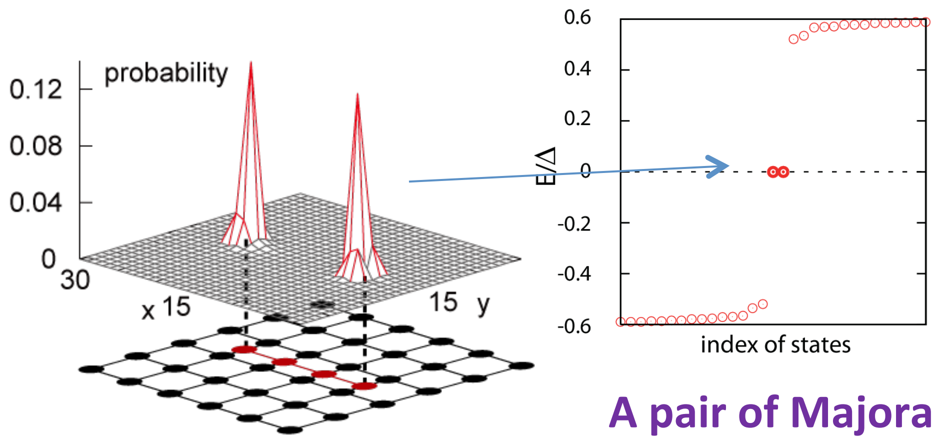


In the presence of magnetic fields along the z-direction, NMR date suggests

$$\mathcal{M}_{xy}\Delta(k_x,k_y,k_z)\mathcal{M}_{xy}^t = -\Delta(k_x,k_y,-k_z)$$

Odd

We can expect Majorana Fermions !!



[Ueno-MS et al(13)]

A pair of Majorana fermions

Questions

- Is it possible to classify such topological phases systematically?
- How many new topological phases can we obtain in the presence of additional symmetry?

To answer these questions, we employ the K-theory.

Shiozaki-MS, Phys. Rev. B90, 166114 (2014).
Shiozaki-MS-Gomi, Phys. Rev. B91, 155120 (2015).
Shiozaki-MS-Gomi, arXiv:1511.01463.

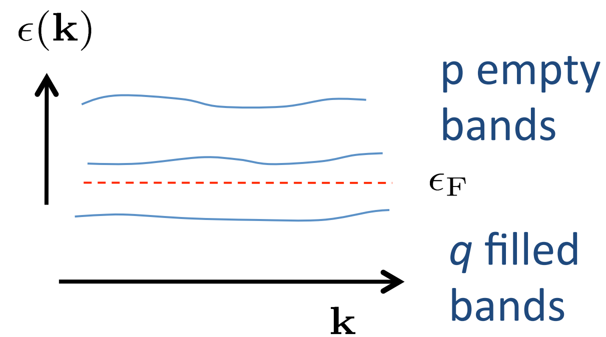
Our setting

In stead of occupied states, we classify flattened Hamiltonians

$$\mathcal{H}(\boldsymbol{k}) = U(\boldsymbol{k})^\dagger \begin{pmatrix} E_1(\boldsymbol{k}) & & \\ & \ddots & \\ & & E_n(\boldsymbol{k}) \end{pmatrix} U(\boldsymbol{k})$$

wave fn.

empty band occupied band

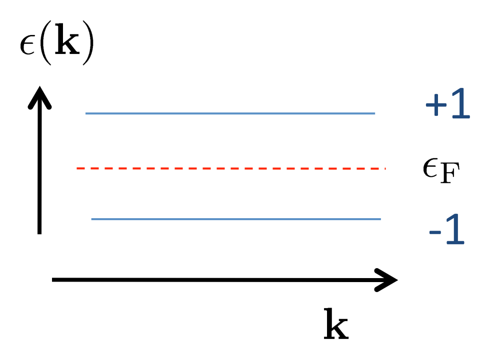


only distinction between empty and occupied band is important

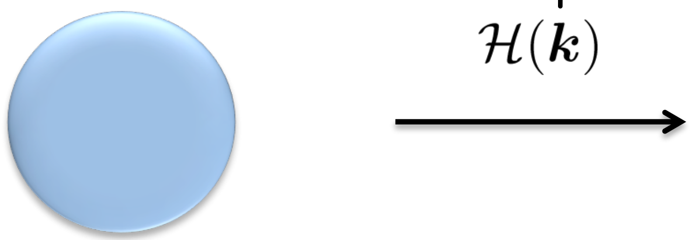
$$\mathcal{H}(\boldsymbol{k}) = U(\boldsymbol{k})^\dagger \begin{pmatrix} \mathbf{1}_{p\times p} & 0 \\ 0 & -\mathbf{1}_{q\times q} \end{pmatrix} U(\boldsymbol{k})$$

$$[\mathcal{H}(\boldsymbol{k})]^2 = 1$$

flattened Hamiltonian



The flattened Hamiltonian defines a map from momentum space to Hilbert space



The problem is how symmetry affects classifying space

If the map defines a non-trivial homotopy, we may have a non-trivial topological phase

Why we use K-theory?

Adding topologically trivial bands makes the classification simpler

[Kitaev(09)]

In addition to simple deformation of Hamiltonians, the K-theory approach allows us to add topologically trivial bands during the deformation of Hamiltonians

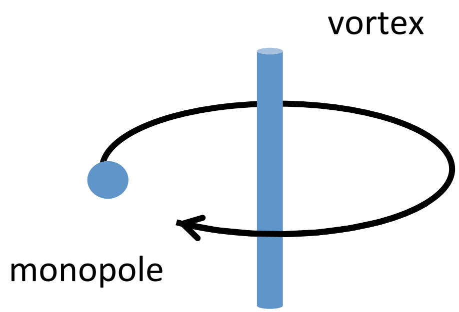
stable equivalence $\mathcal{H}_1 \sim \mathcal{H}_2$

deformable by adding extra trivial bands

→ **The stable-equivalence classes defines topological phases**

Importantly, using stable equivalence, we can avoid annoying interference between topological charges

$\mathcal{H}_{\text{vortex+monopole}}$



interference due to AB effect

But if we use stable equivalence …

pair-annihilation of monopoles

trivial band

$$\mathcal{H}_{\text{vortex+monopole}} \sim \begin{pmatrix} \begin{pmatrix} \mathcal{H}_{\text{vortex+monopole}} \quad \mathcal{H}_{\text{vortex}} & \\ & \mathcal{H}_{\text{monopole}} \end{pmatrix} & & \\ & \mathcal{H}_{\text{antimonopole}} & \\ & & \mathcal{H}_{\text{monopole}} \end{pmatrix}$$

The vortex and monopole belong to different sectors of Hamiltonian

No interference

The K-theory simplifies the classification of topological phases

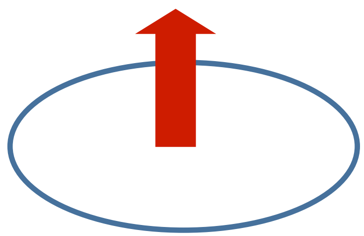
Classification of TCIs and TCSCs: K-theory approach

In general, topological phases can be understood as the existence of topological objects in the momentum space

1 dim top. phase

1dim BZ

= 1dim circle



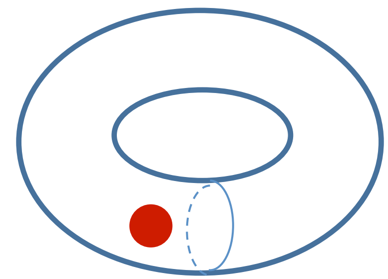
"vortex"

winding number

$$N_w = \oint_{1\mathrm{dBZ}} \frac{dk}{2\pi} \mathrm{tr}\left[\Gamma H^{-1}(\boldsymbol{k})\partial H(\boldsymbol{k})\right]$$

$$\{\Gamma, H(\boldsymbol{k})\} = 0$$ chiral symmetry

2 dim top. phase



2dim BZ

= 2dim torus

"monopole"

Chern number

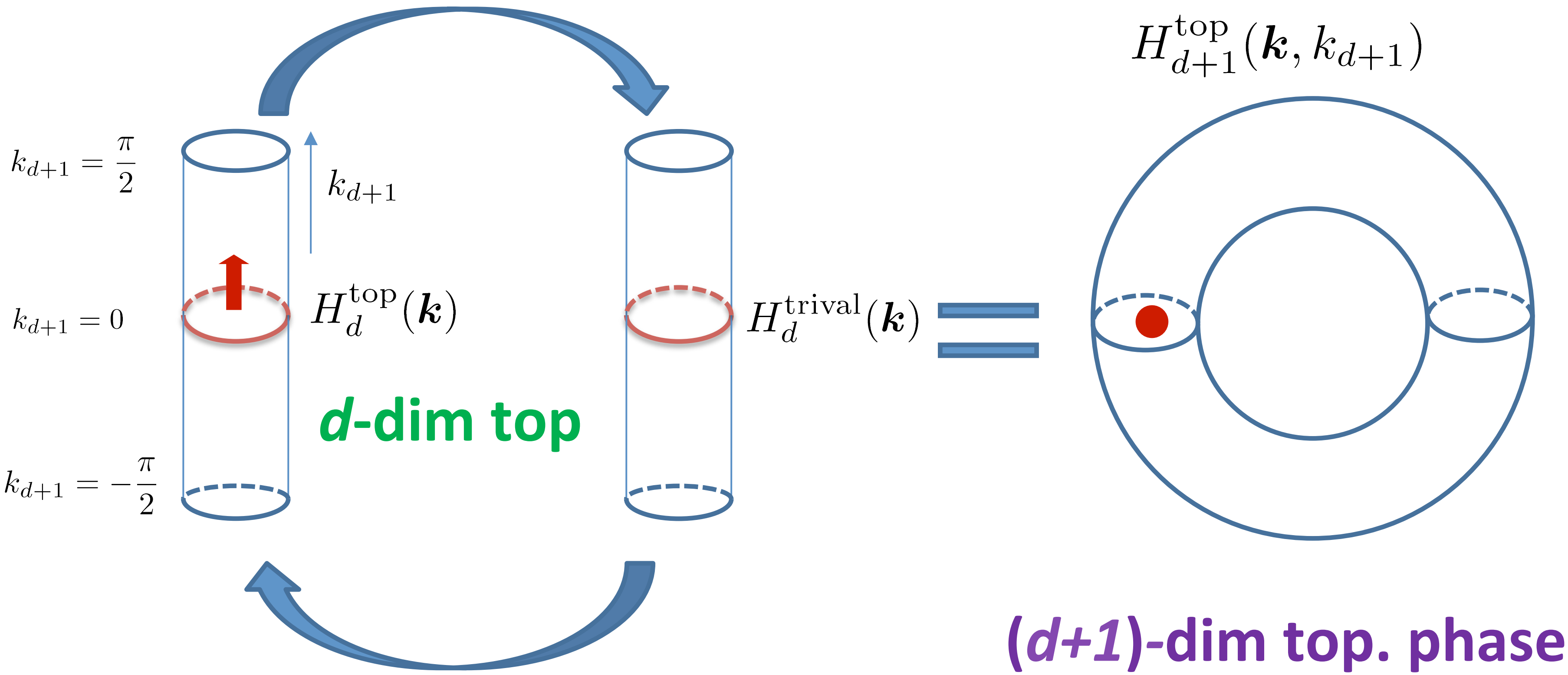
$$Ch_1 = \int_{2\mathrm{dBZ}} \left(\frac{d\boldsymbol{S}}{2\pi}\right) \cdot \left[\boldsymbol{\nabla} \times \mathcal{A}(\boldsymbol{k})\right]$$

$$\mathcal{A}(\boldsymbol{k}) = i \sum_{n \in \mathrm{occ}} \langle u_n(\boldsymbol{k}) | \boldsymbol{\nabla} u_n(\boldsymbol{k}) \rangle$$

In the framework of K-theory, one can increase the dimension of the system systematically

Teo-Kane (10)

$$H_{d+1}(\boldsymbol{k}, k_{d+1}) = \begin{cases} H_d(\boldsymbol{k}) \cos k_{d+1} + \Gamma \sin k_{d+1} & \text{chiral case} \\ (H_d(\boldsymbol{k}) \otimes \sigma_z) \cos k_{d+1} + (1 \otimes \sigma_y) \sin k_{d+1} & \text{non-chiral case} \end{cases}$$



This map keeps the topological number but it shifts the symmetry of the system

class DIII (**TRI SCs**)

$$C H_d(\boldsymbol{k}) C^{-1} = -H_d(-\boldsymbol{k}) \qquad C = \tau_x K \qquad C^2 = 1$$

$$T H_d(\boldsymbol{k}) T^{-1} = H_d(-\boldsymbol{k}) \qquad T = i s_y K \qquad T^2 = -1$$

$$H_{d+1}(\boldsymbol{k}, k_{d+1}) = H_d(\boldsymbol{k}) \cos k_{d+1} + \Gamma \sin k_{d+1} \qquad \Gamma = iTC$$

This term breaks PHS

class AII (**TRI Insulators**)

$$T H_{d+1}(\boldsymbol{k}, k_{d+1}) T^{-1} = H_{d+1}(-\boldsymbol{k}, -k_{d+1})$$

hierarchy of top #

	TRS	PHS	CS	d=1	d=2	d=3
AI	1	0	0	0	0	0
BDI	1	1	1	**Z**	0	0
D	0	1	0	**Z_2**	**Z**	0
DIII	-1	1	1	**Z_2**	**Z_2**	**Z**
AII	-1	0	0	0	**Z_2**	**Z_2**
CII	-1	-1	1	**2Z**	0	**Z_2**
C	0	-1	0	0	**2Z**	0
CI	1	-1	1	0	0	**2Z**

We generalize this idea to systems with additional symmetry

Shiozaki-MS(14)

class DIII (TRI SCs) + **mirror reflection**

$$CH_d(\boldsymbol{k})C^{-1} = -H_d(-\boldsymbol{k}) \qquad C = \tau_x K \qquad C^2 = 1$$

$$TH_d(\boldsymbol{k})T^{-1} = H_d(-\boldsymbol{k}) \qquad T = is_y K \qquad T^2 = -1$$

$$UH_d(\boldsymbol{k})U^{-1} = H_d(-k_1, k_2, \dots, k_d) \qquad U = is_x$$

mirror sym.

$$H_{d+1}(\boldsymbol{k}, k_{d+1}) = H_d(\boldsymbol{k}) \cos k_{d+1} + \underline{\Gamma \sin k_{d+1}} \qquad \Gamma = iTC$$

This term keeps mirror sym.

class AII (TRI Insulators) + **mirror reflection**

$$TH_{d+1}(\boldsymbol{k}, k_{d+1})T^{-1} = H_{d+1}(-\boldsymbol{k}, -k_{d+1})$$

$$UH_{d+1}(\boldsymbol{k})U^{-1} = H_{d+1}(-k_1, k_2, \dots, k_d, k_{d+1})$$

mirror sym.

However, this is not the only possibility ….

The mapped Hamiltonian also has a different additional symmetry

$$(U\Gamma)H_{d+1}(\boldsymbol{k})(U\Gamma)^{-1} = -H_{d+1}(-k_1, k_2, \ldots, k_d, \underline{-k_{d+1}}) \qquad \Gamma = iTC$$

This symmetry flips k_{d+1}

= 2-fold rotation

***d*-dim**

class DIII

$H_d(\boldsymbol{k})$

$T \quad C \quad U$

(*d+1*)-dim

class AII

$H_{d+1}(\boldsymbol{k})$

$T \quad U$

mirror reflection

The same top #

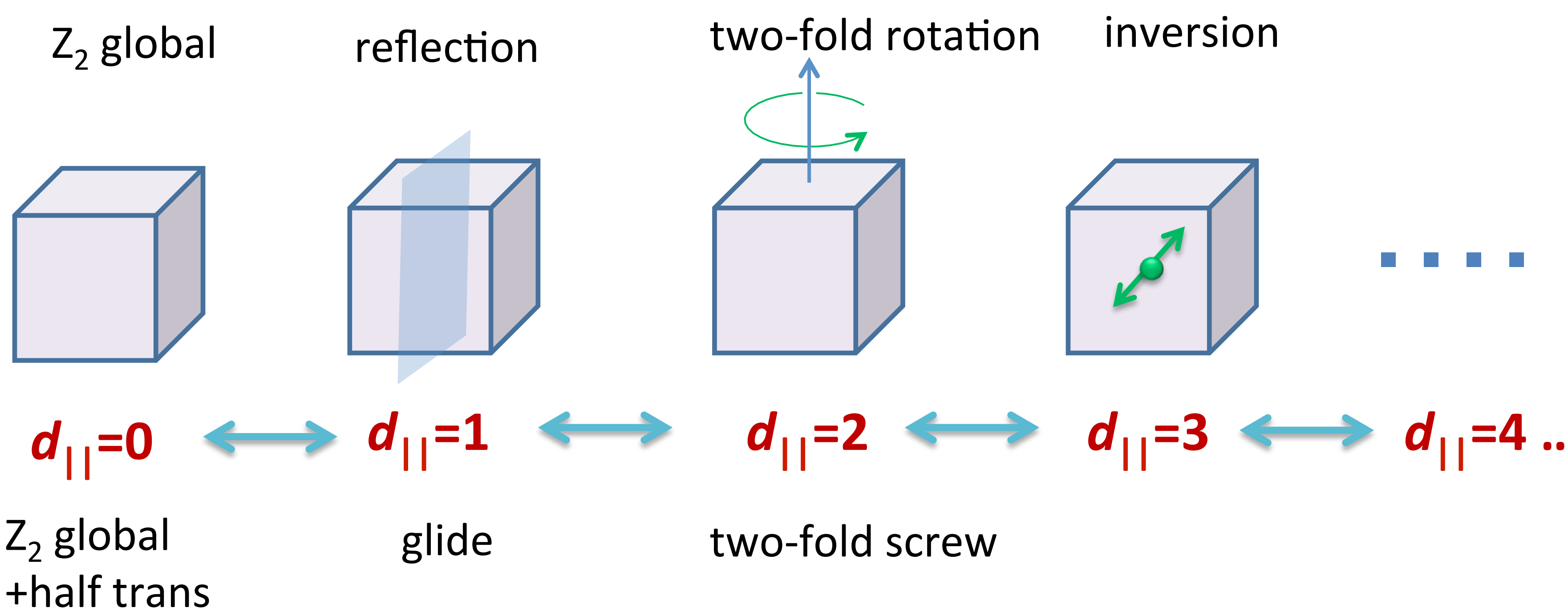
$H_{d+1}(\boldsymbol{k})$

$T \quad U\Gamma$

2-fold rotation

Different additional symmetry but the same topological structure

In this manner, **we can change the number of flipped coordinates $d_{||}$ under the symmetry,** with keeping the topological structure



Using these relations, we complete the classification of TCIs and TCSCs protected by order-two space (and magnetic space) groups

Extended topological Table

[Shiozaki-MS (14),
Shiozaki-MS-Gomi (15)]

A single periodic table with 10 different topological class

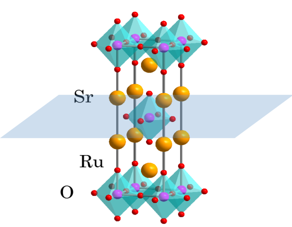
[Schnyder et al (08)]

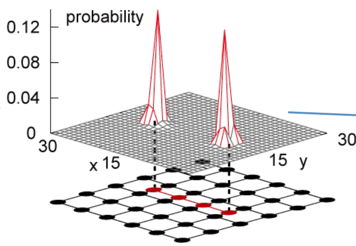
→

6 periodic tables with 27 classes
+
6 periodic tables with 10 classes

222 class

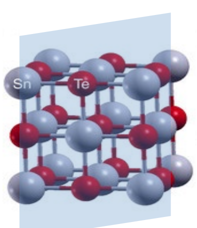
Sr_2RuO_4

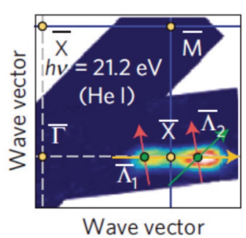




Symmetry	Class	$\mathcal{C}_q$ or $\mathcal{R}_q$	$\delta=0$	$\delta=1$	$\delta=2$	$\delta=3$	$\delta=4$	$\delta=5$	$\delta=6$	$\delta=7$
U	A	$\mathcal{C}_1$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
U_+	AIII	$\mathcal{C}_0$	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
U_-	AIII	$\mathcal{C}_1\times\mathcal{C}_1$	0	$\mathbb{Z}\oplus\mathbb{Z}$	0	$\mathbb{Z}\oplus\mathbb{Z}$	0	$\mathbb{Z}\oplus\mathbb{Z}$	0	$\mathbb{Z}\oplus\mathbb{Z}$
	AI	$\mathcal{R}_1$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
	BDI	$\mathcal{R}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0
	D	$\mathcal{R}_3$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$
$U^+_+,U^-_-,U^+_{++},U^-_{--}$	DIII	$\mathcal{R}_4$	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
	AII	$\mathcal{R}_5$	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
	CII	$\mathcal{R}_6$	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
	C	$\mathcal{R}_7$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
	CI	$\mathcal{R}_0$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
U^+_{+-},U^-_{-+}	BDI	$\mathcal{R}_1\times\mathcal{R}_1$	$\mathbb{Z}_2\oplus\mathbb{Z}_2$	$\mathbb{Z}\oplus\mathbb{Z}$	0	0	0	$2\mathbb{Z}\oplus2\mathbb{Z}$	0	$\mathbb{Z}_2\oplus\mathbb{Z}_2$
U^+_{-+},U^-_{+-}	DIII	$\mathcal{R}_3\times\mathcal{R}_3$	0	$\mathbb{Z}_2\oplus\mathbb{Z}_2$	$\mathbb{Z}_2\oplus\mathbb{Z}_2$	$\mathbb{Z}\oplus\mathbb{Z}$	0	0	0	$2\mathbb{Z}\oplus2\mathbb{Z}$
U^+_{+-},U^-_{-+}	CII	$\mathcal{R}_5\times\mathcal{R}_5$	0	$2\mathbb{Z}\oplus2\mathbb{Z}$	0	$\mathbb{Z}_2\oplus\mathbb{Z}_2$	$\mathbb{Z}_2\oplus\mathbb{Z}_2$	$\mathbb{Z}\oplus\mathbb{Z}$	0	0
U^+_{-+},U^-_{+-}	CI	$\mathcal{R}_7\times\mathcal{R}_7$	0	0	0	$2\mathbb{Z}\oplus2\mathbb{Z}$	0	$\mathbb{Z}_2\oplus\mathbb{Z}_2$	$\mathbb{Z}_2\oplus\mathbb{Z}_2$	$\mathbb{Z}\oplus\mathbb{Z}$
	AI	$\mathcal{R}_7$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
	BDI	$\mathcal{R}_0$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
	D	$\mathcal{R}_1$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
$U^+_-,U^-_+,U^+_{--},U^-_{++}$	DIII	$\mathcal{R}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0
	AII	$\mathcal{R}_3$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$
	CII	$\mathcal{R}_4$	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
	C	$\mathcal{R}_5$	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
	CI	$\mathcal{R}_6$	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
U^+_{-+},U^-_{+-}	BDI, CII	$\mathcal{C}_1$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
U^+_{+-},U^-_{-+}	DIII, CI	$\mathcal{C}_1$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$

SnTe





Question

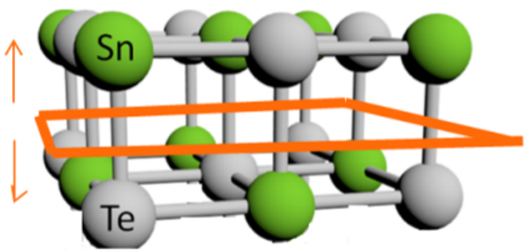
- Is it possible to realize essentially new structure of surface states?

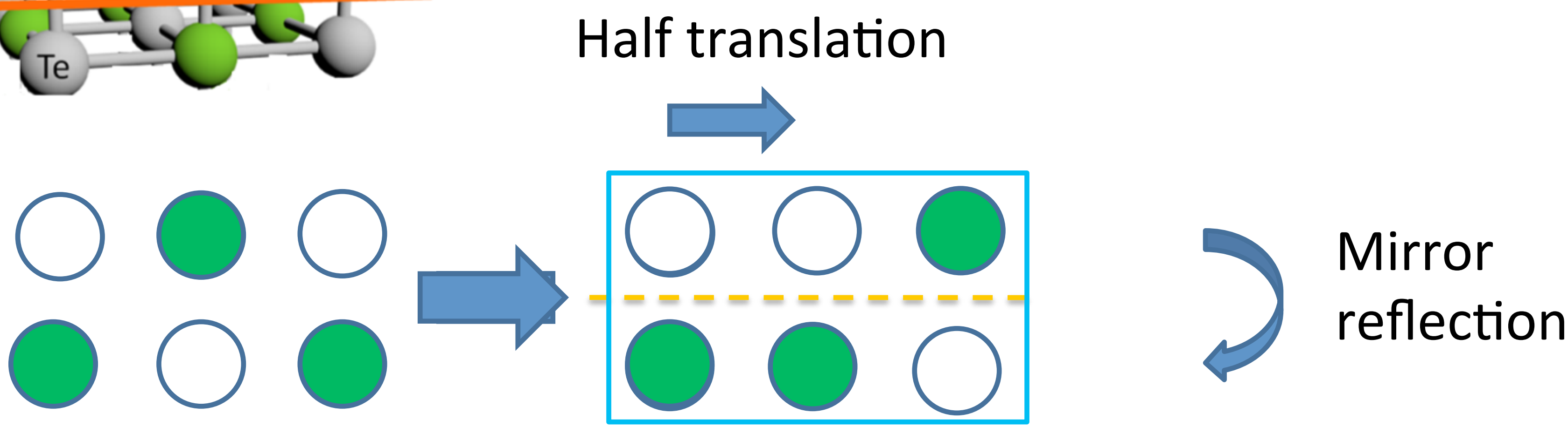
⇒ Glide symmetry gives rise to distinct topological states

Mobius twist in surface states

Shiozaki-MS-Gomi(15)

glide = mirror reflection + non-primitive lattice translation





Nonsymmorphic Symmetry

Glide symmetry gives essentially new structures in topological table

	d=1	d=2	d=3
A	0	**Z**	0
AIII	**Z**	0	**Z**
AI	0	0	0
BDI	**Z**	0	0
D	Z_2	**Z**	0
DIII	Z_2	Z_2	**Z**
AII	0	Z_2	Z_2
CII	**2Z**	0	Z_2
C	0	**2Z**	0
CI	0	0	**2Z**

[Schnyder-Ryu-Furusaki-Ludwig (08)]

with glide symmetry

	d=1	d=2	d=3
A	Z_2	0	Z_2
AIII	0	Z_2	0
AI	Z_2	0	0
BDI	Z_2	Z_2	0
D	Z_4	Z_2	Z_2
DIII	Z_2	Z_4	Z_2
AII	Z_2	Z_2	Z_4
CII	0	Z_2	Z_2
C	0	0	Z_2
CI	0	0	0

Z_2# without anti-unitary symmetry

With anti-unitary symmetry, we have Z_4#

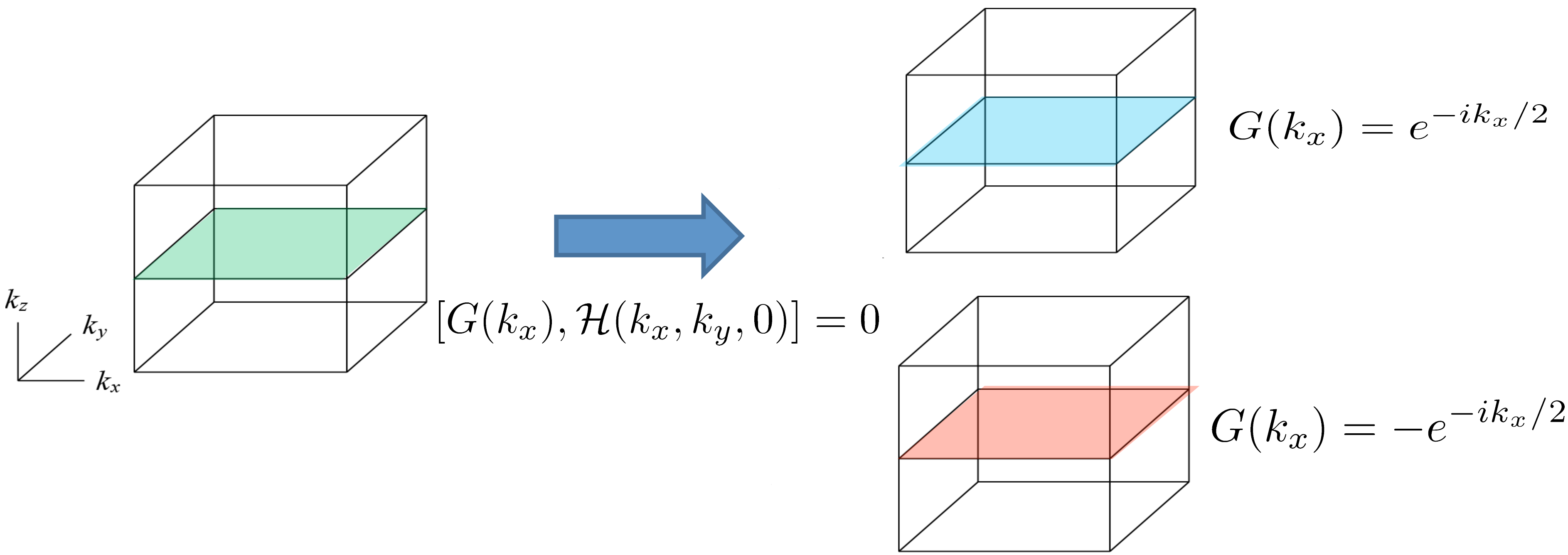
[Shiozaki-MS-Gomi (15)]

Glide symmetric Hamiltonian

[Shiozaki-MS-Gomi (15),Fang-Fu (15)]

$$G(k_x)\mathcal{H}(k_x,k_y,k_z)G^{-1}(k_x)=\mathcal{H}(k_x,k_y,-k_z),\quad G^2(k_x)=e^{-ik_x}$$

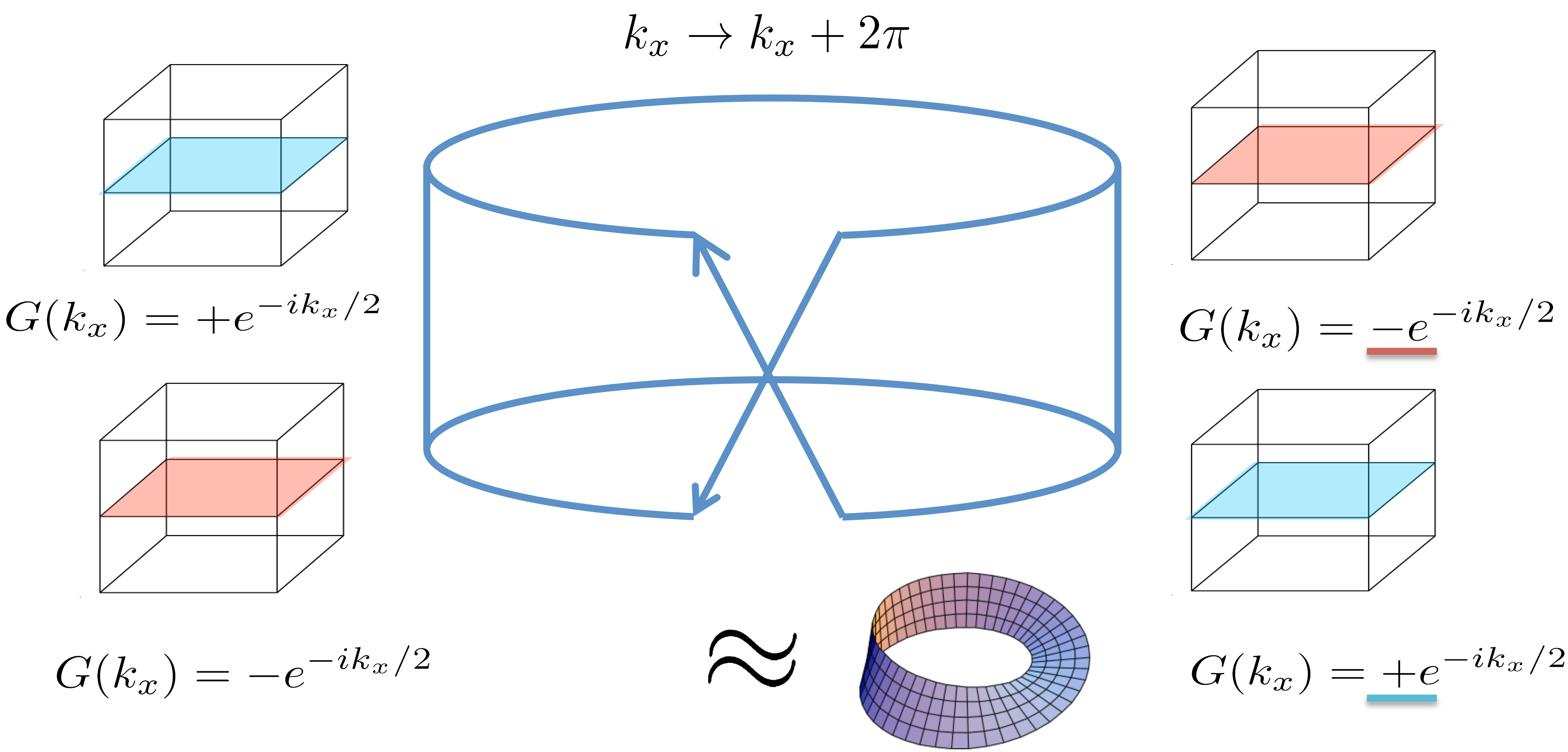
Like topological crystalline insulators, k_z=0 plane can be separated into two glide subsectors



But, there is an important difference

Key point

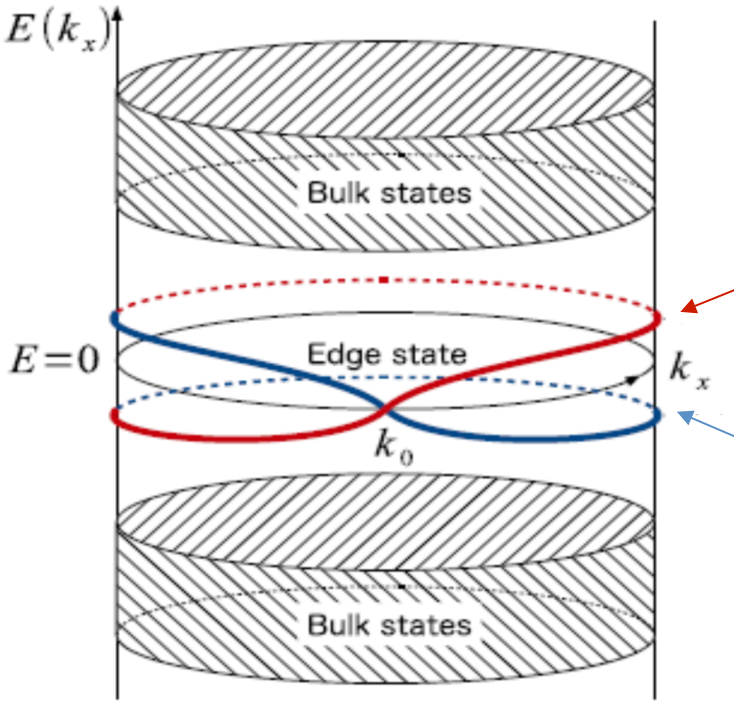
Eigenvalues of glide operator do not have 2π periodicity in k_x



As a result, these two subsectors are connected with a twist

Mobius strip structure

Correspondingly, a surface state also should have a Mobius twist structure when k_z=0,



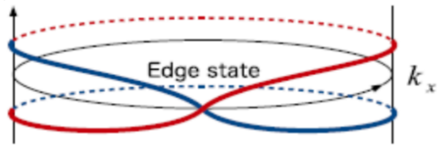
$G(k_x) = e^{-ik_x/2}$ **Chiral state**

$G(k_x) = -e^{-ik_x/2}$ **Anti-chiral state**

Mobius twist

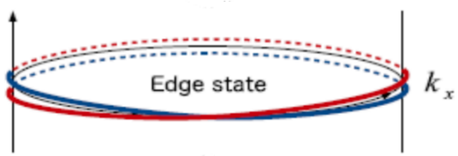
Along the k_x direction, chiral edge state in $G(k_x)=e^{ik_x/2}$ sector is smoothly connected to anti-chiral one in $G(k_x)=-e^{ik_x/2}$ sector.

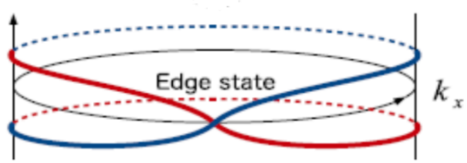
In addition, from the Mobius twist structure, the two glide subsectors can be exchanged adiabatically



$$G(k_x) = e^{-ik_x/2}$$ **Chiral state**

$$G(k_x) = -e^{-ik_x/2}$$ **Anti-chiral state**





$$G(k_x) = -e^{-ik_x/2}$$ **Chiral state**

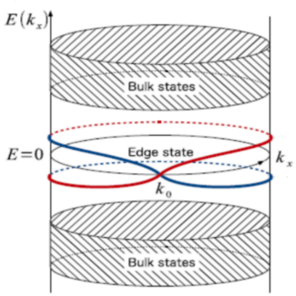
$$G(k_x) = e^{-ik_x/2}$$ **Anti-chiral state**

Chiral mode (N=1) = Anti-chiral mode (N=-1)

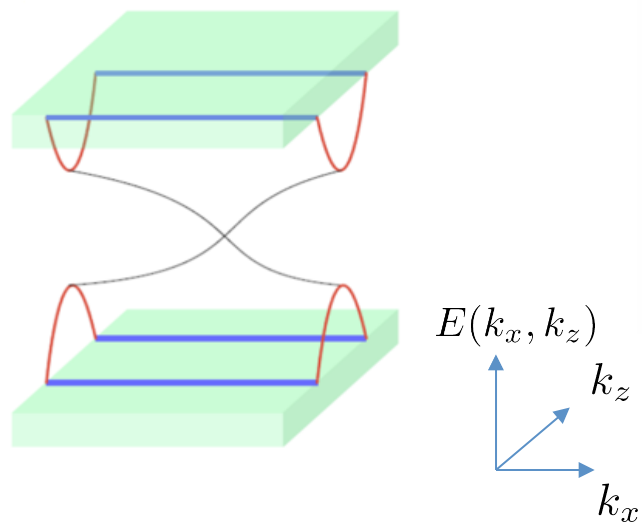
Z_2 topological phase

Interestingly, the surface state with Mobius twist can be open in the k_x-direction

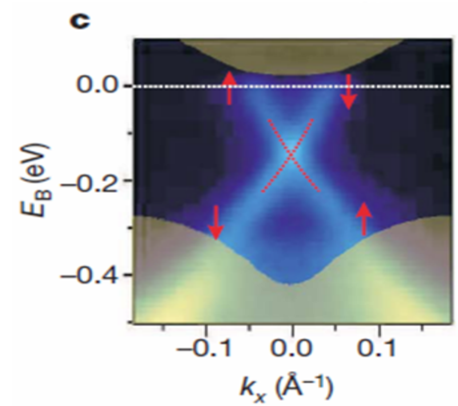
The present case



at k_z=0



Topological Insulator



Bi_2Se_3

[Hsieh et al (09)]

Surface state is always attached to the bulk state

Surface state is detached from the bulk state at k_z=0

This feature can be used to identify this phase by ARPES experiments

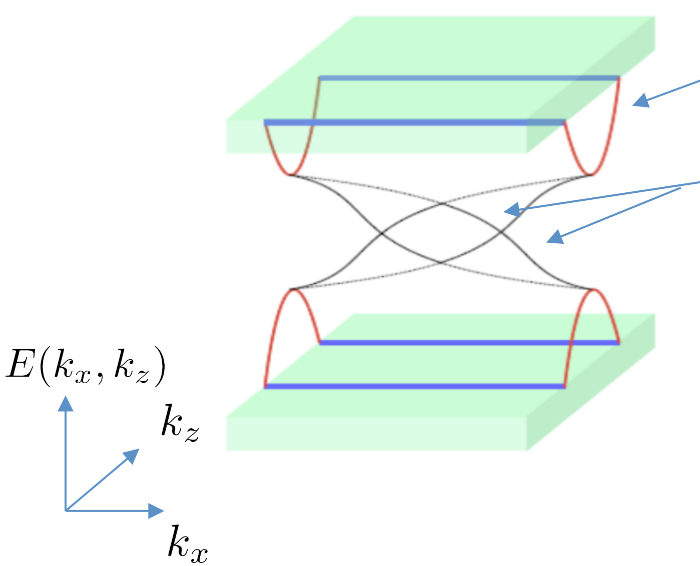
Z_4 phase (TRS+glide)

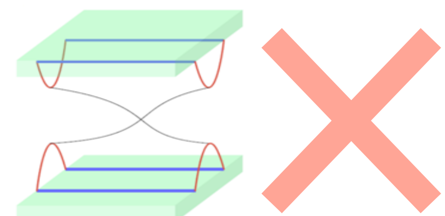
Shiozaki-Gomi-MS, arXiv:1511.01463 (15)

$$G(k_x)\mathcal{H}(k_x,k_y,k_z)G^{-1}(k_x)=\mathcal{H}(k_x,k_y,-k_z),\quad G^2(k_x)=e^{-ik_x}$$

$$T\mathcal{H}(\boldsymbol{k})T^{-1}=\mathcal{H}(-\boldsymbol{k})$$

In the presence of TRS, the Kramers degeneracy at the zone boundary requires two helical modes to have the open structure.





At k_z=0

Two chiral state $G(k_x)=e^{-ik_x/2}$

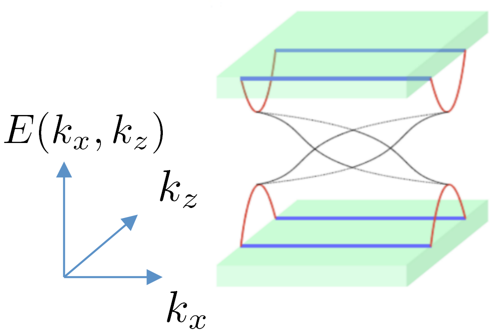
Two anti-chiral edge state $G(k_x)=-e^{-ik_x/2}$

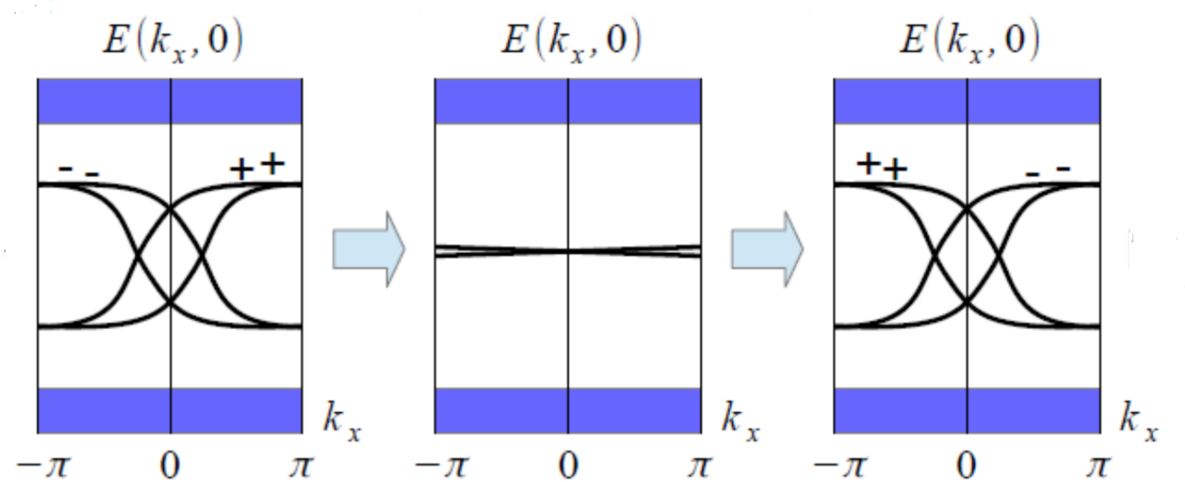
Using a similar argument as before,

Shiozaki-Gomi-MS, arXiv:1511.01463 (15)

Two chiral mode (N=2) = Two anti-chiral mode (N=-2)

Z_4 topological phase





$G(k_x) = e^{-ik_x/2}$ **Two chiral state** **Two anti-chiral state**

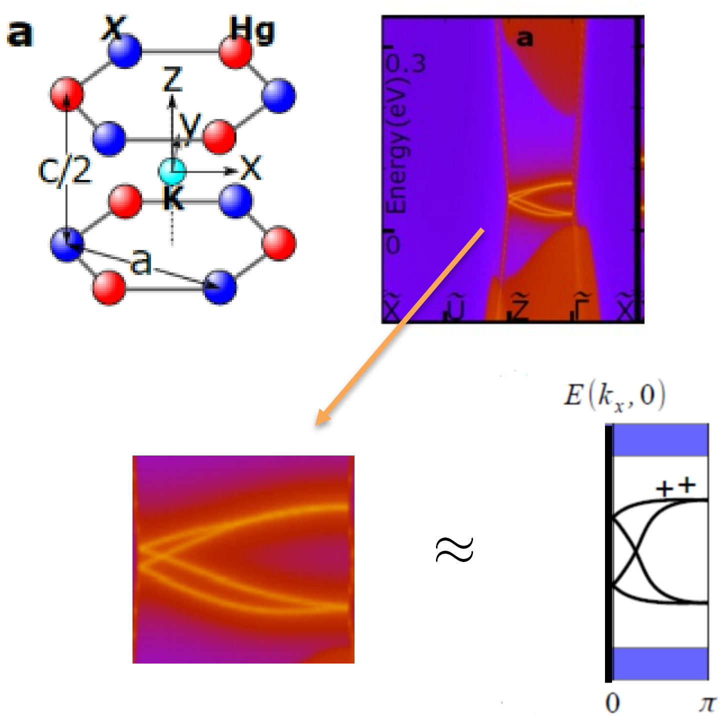
$G(k_x) = -e^{-ik_x/2}$ **Two anti-chiral state** **Two chiral state**

Material realization

Material realizations were quickly proposed after our finding

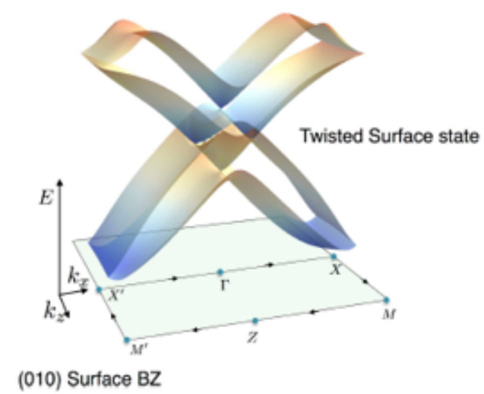
- Hourglass fermion

KHgSb Wang et al, Nature (16)



- Mobius Kondo Insulator

CeNiSn Po-Yao Chang et al, arXiv:1603.03435



We can expect other candidates ..

- Heavy fermion SCs

Summary

- Space group as well as time-reversal and particle-hole (=charge conjugation) symmetries give novel topological phases.
- Using the Hamiltonian mapping shifting symmetry, a systematic classification of TCIs and TCSCs can be done
- The nonsymmorphic symmetry provides an interesting Mobius twist structure of the surface state.