Black hole information
and
Reeh-Schlieder theorem

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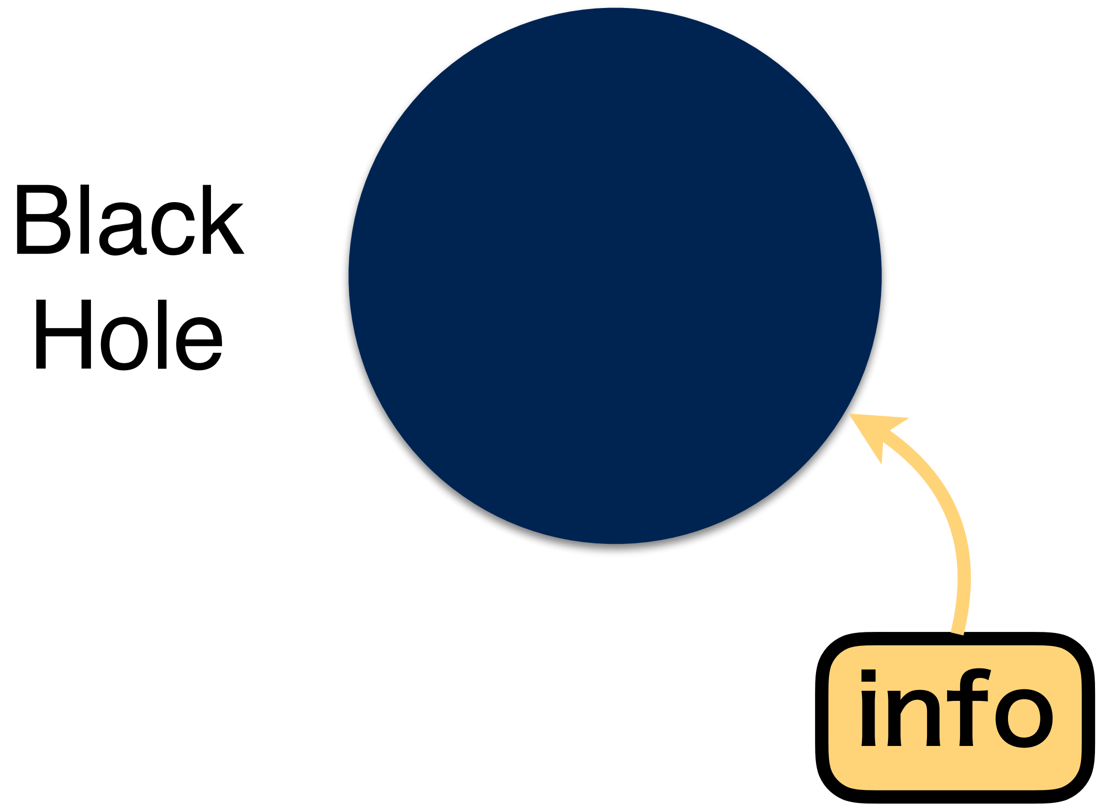
Based on [1807.05399]

Introduction

Information problem in the most basic and naive form:



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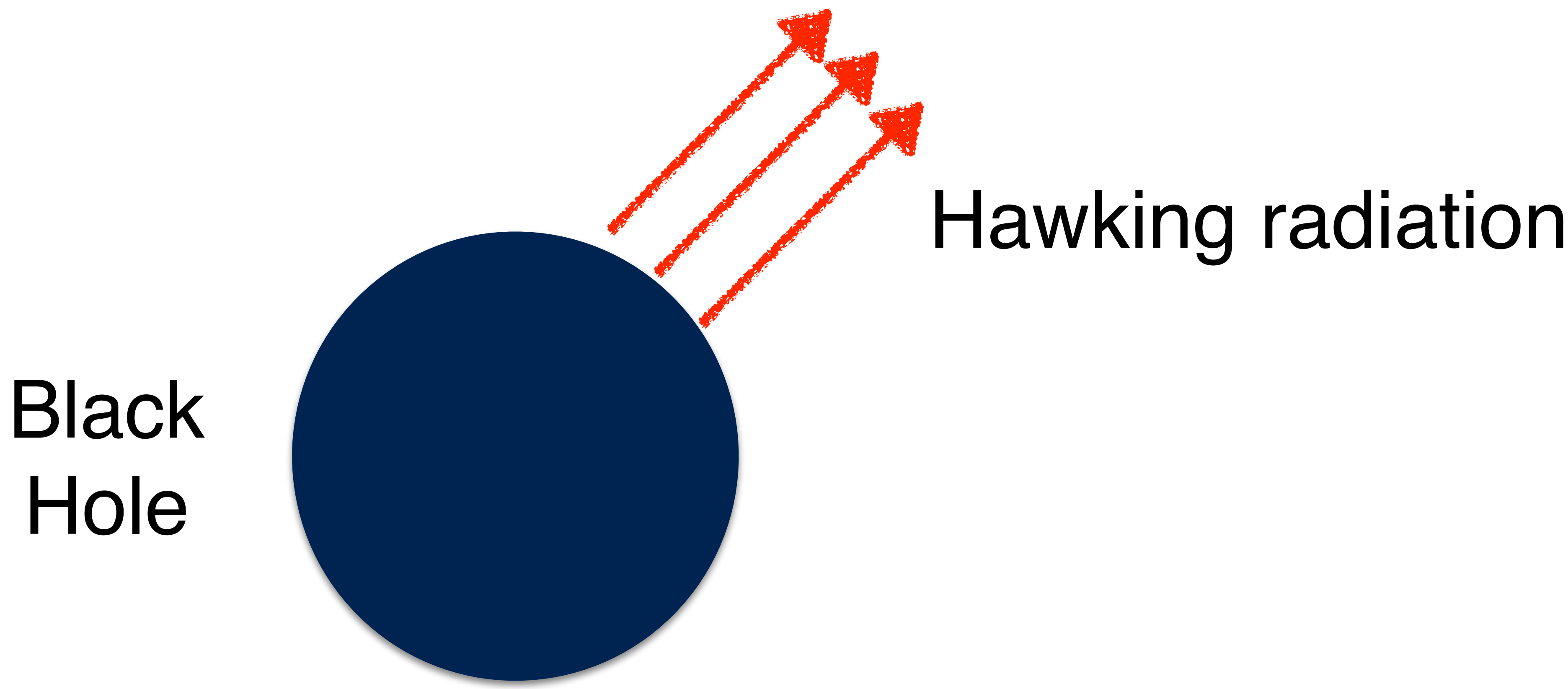

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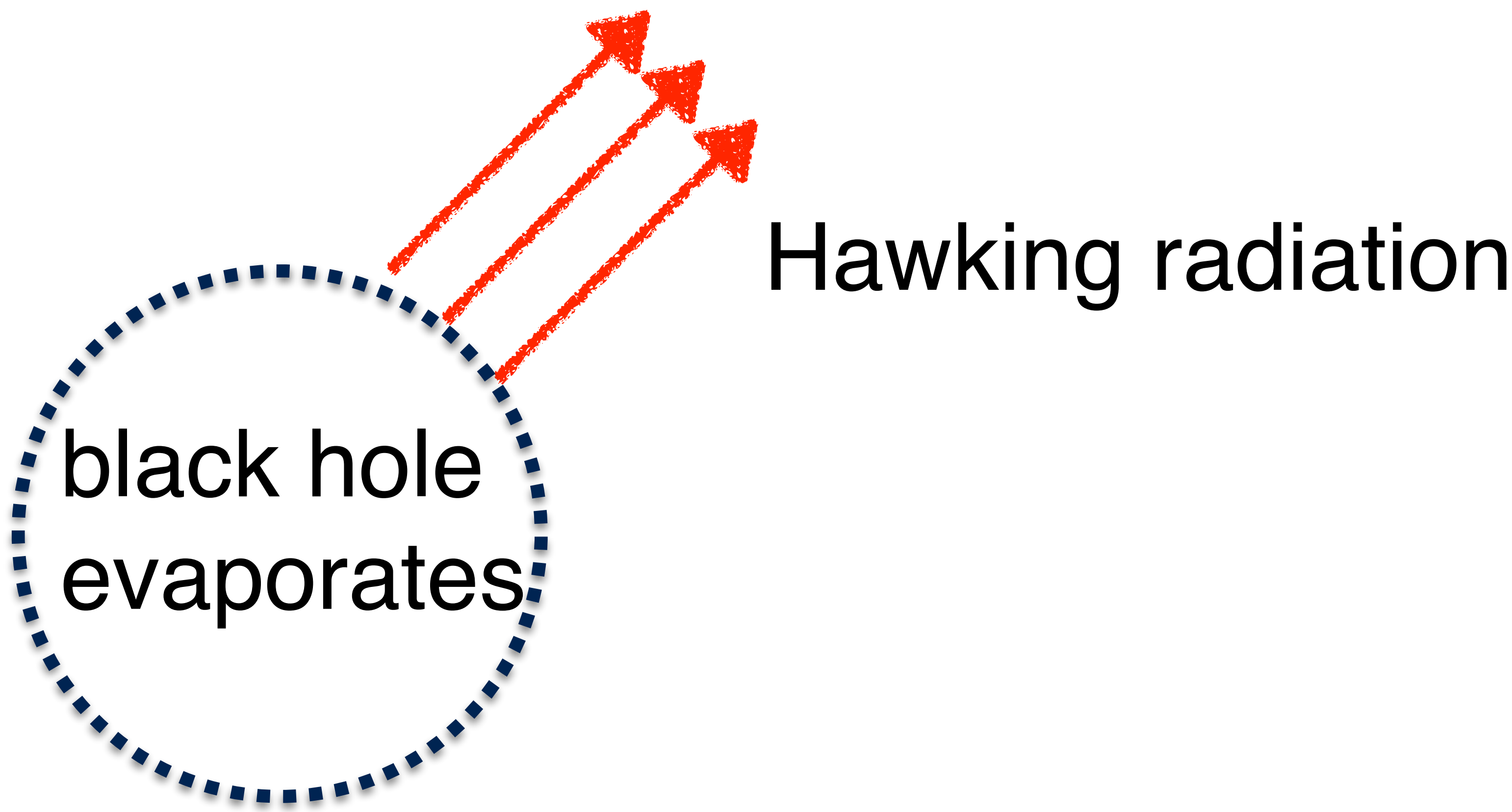
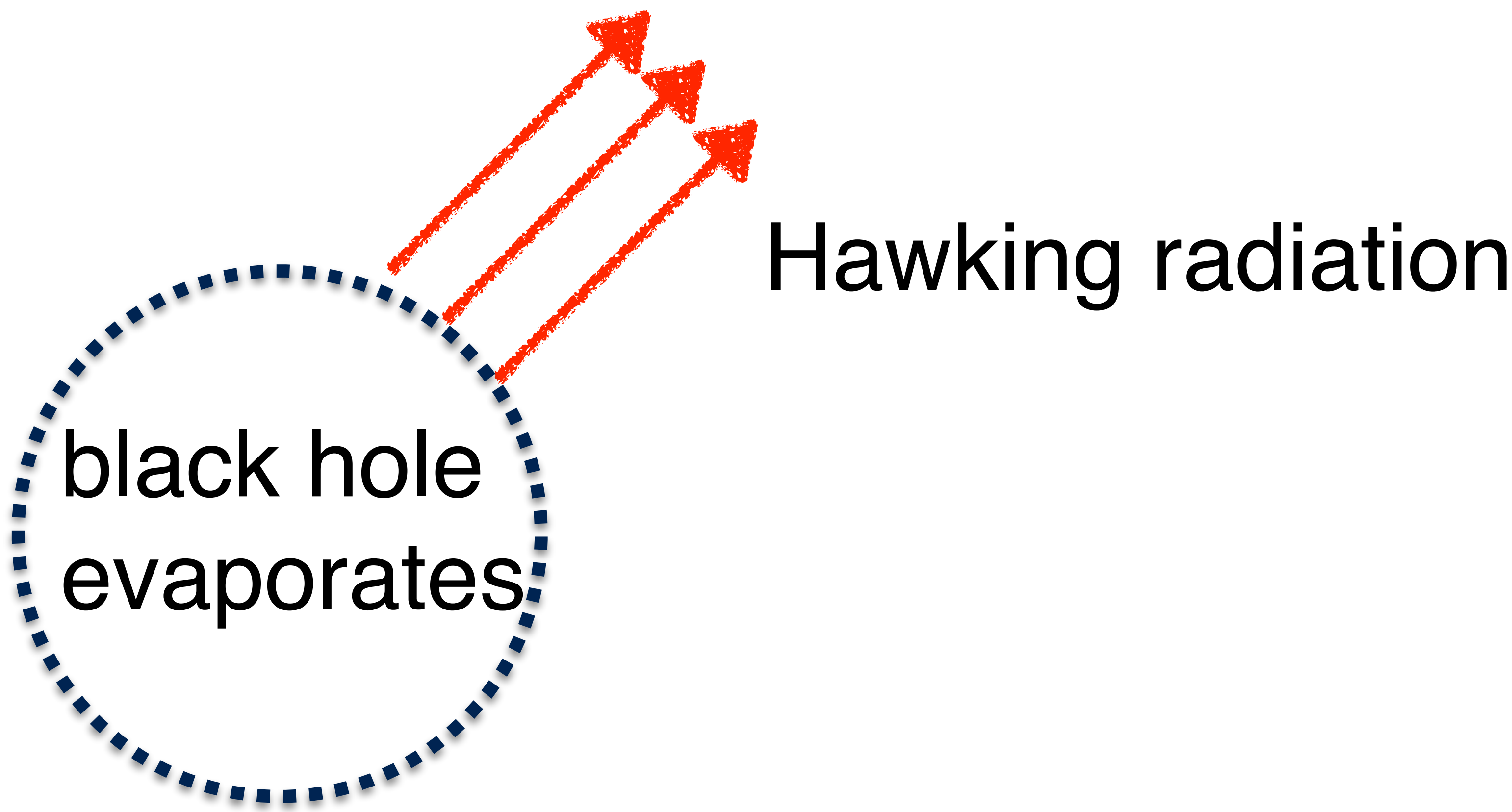


Introduction

Information problem in the most basic and naive form:



Naively, Hawking radiation seems to be independent of the information thrown into the black hole.
Information lost?

Introduction

I’m going to argue that the problem at this most naive level does not exist, because the concept of

“Information localized inside a black hole”

is not well-defined at the full quantum level.

Remark 1

I believe that black hole information problem is a very deep problem and teaches us important lessons.

(E.g. global symmetries do not exist in quantum gravity)

But my claim is that we shouldn't discuss it too naively. Some "contradictions" originate just from the conflict between naive intuition and quantum (field) theory which has nothing to do with black holes.

We must take counter-intuitive facts of quantum (field) theory seriously.

Remark 2

I'm **not** going to talk about measurement at all.
(Don't ask me about measurement!)

The question

" Is the black hole evaporation unitary?"

can be asked without considering measurement.
It is a problem about dynamics. This is the problem
I'm going to discuss.

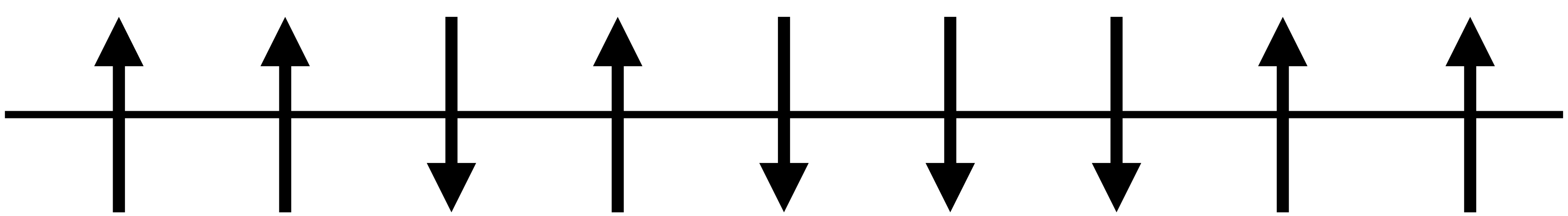
Contents

Information?

What is “information”?

- Classically, things contain classcial information such as position, momentum, etc.

- Semi-quantum mechanically, things contain so-called quantum information such as $|\uparrow\rangle$, $|\downarrow\rangle$
  The positions of “things” are still classical.

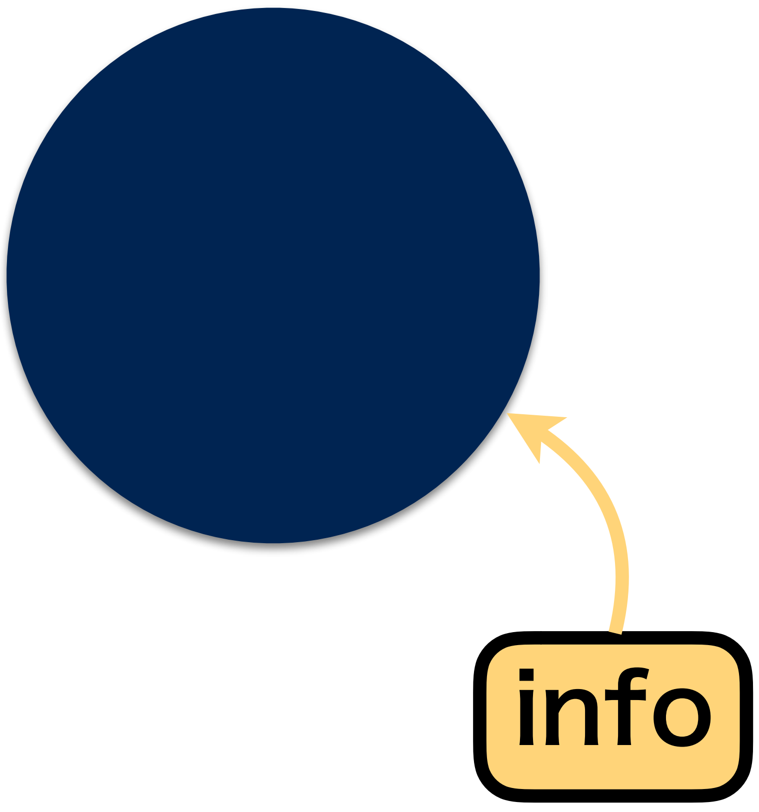


: qubits

- Quantum mechanically, positions of things are described by wave functions.

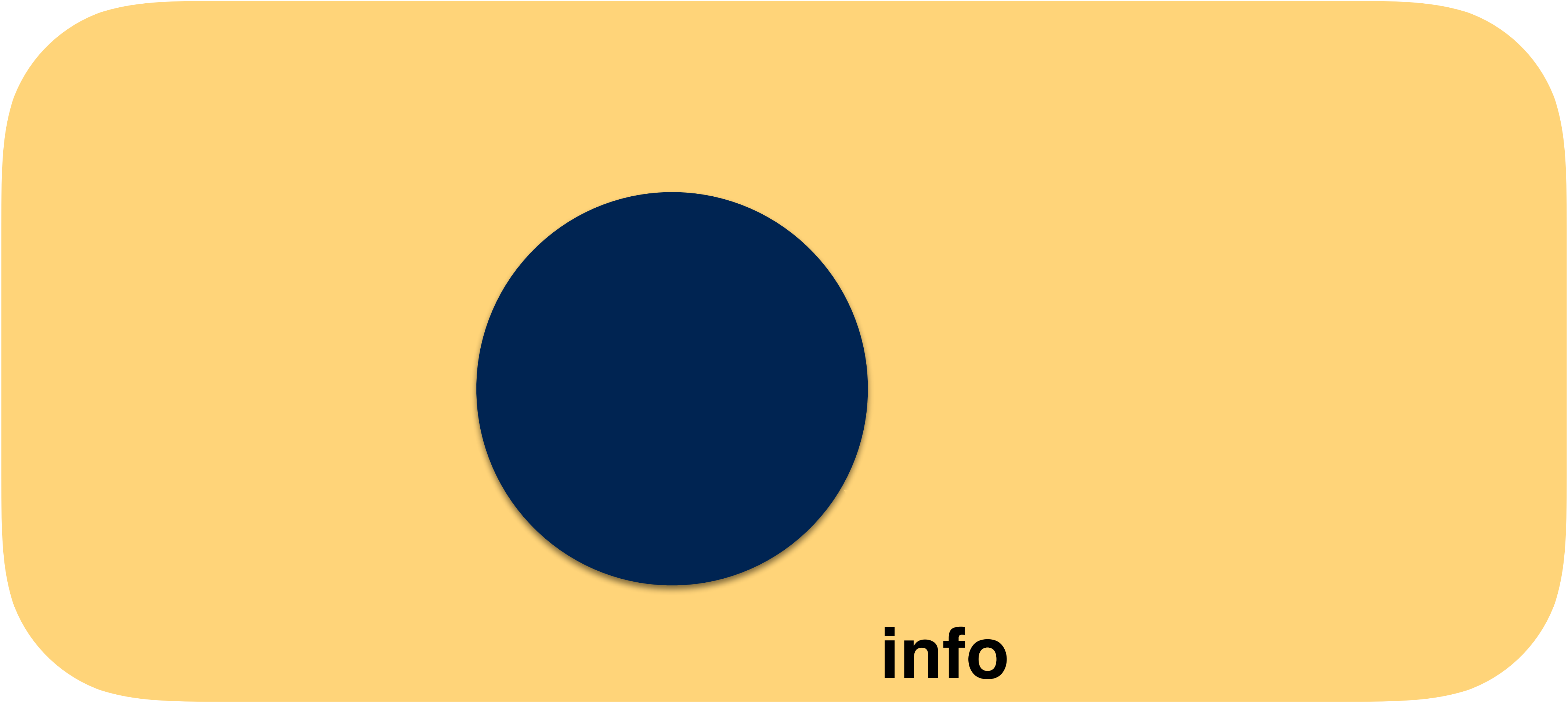
Information?

Classically or semi-quantum mechamically…



Information?

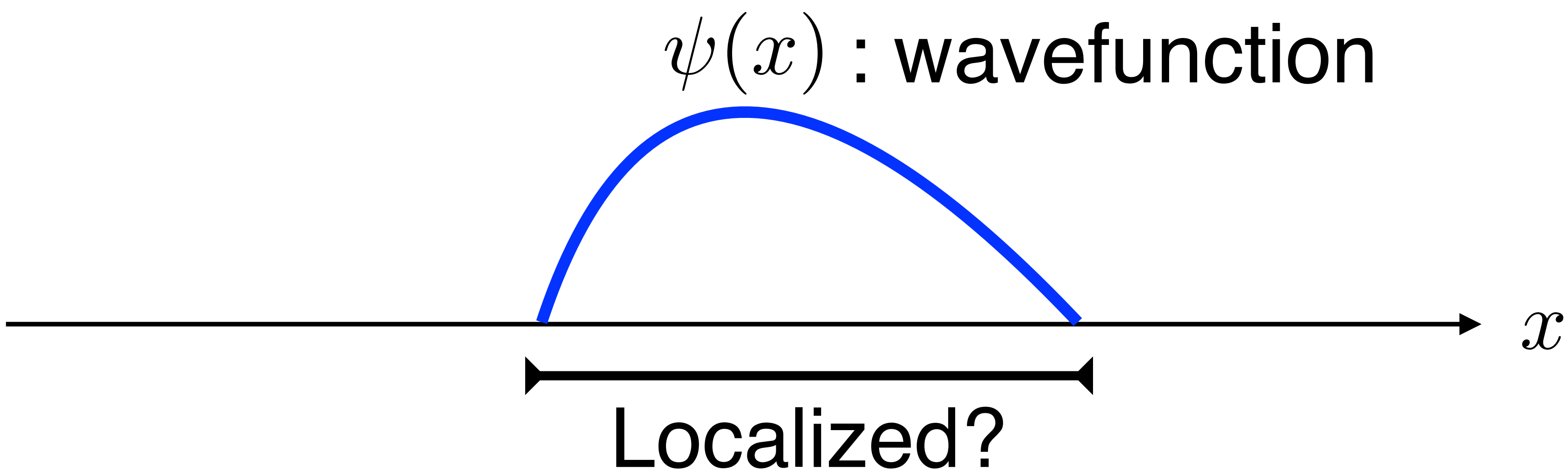
Quantum mechamically…



- Wave functions spread over entire spacetime.
- Information is not definitely inside or outside black hole.

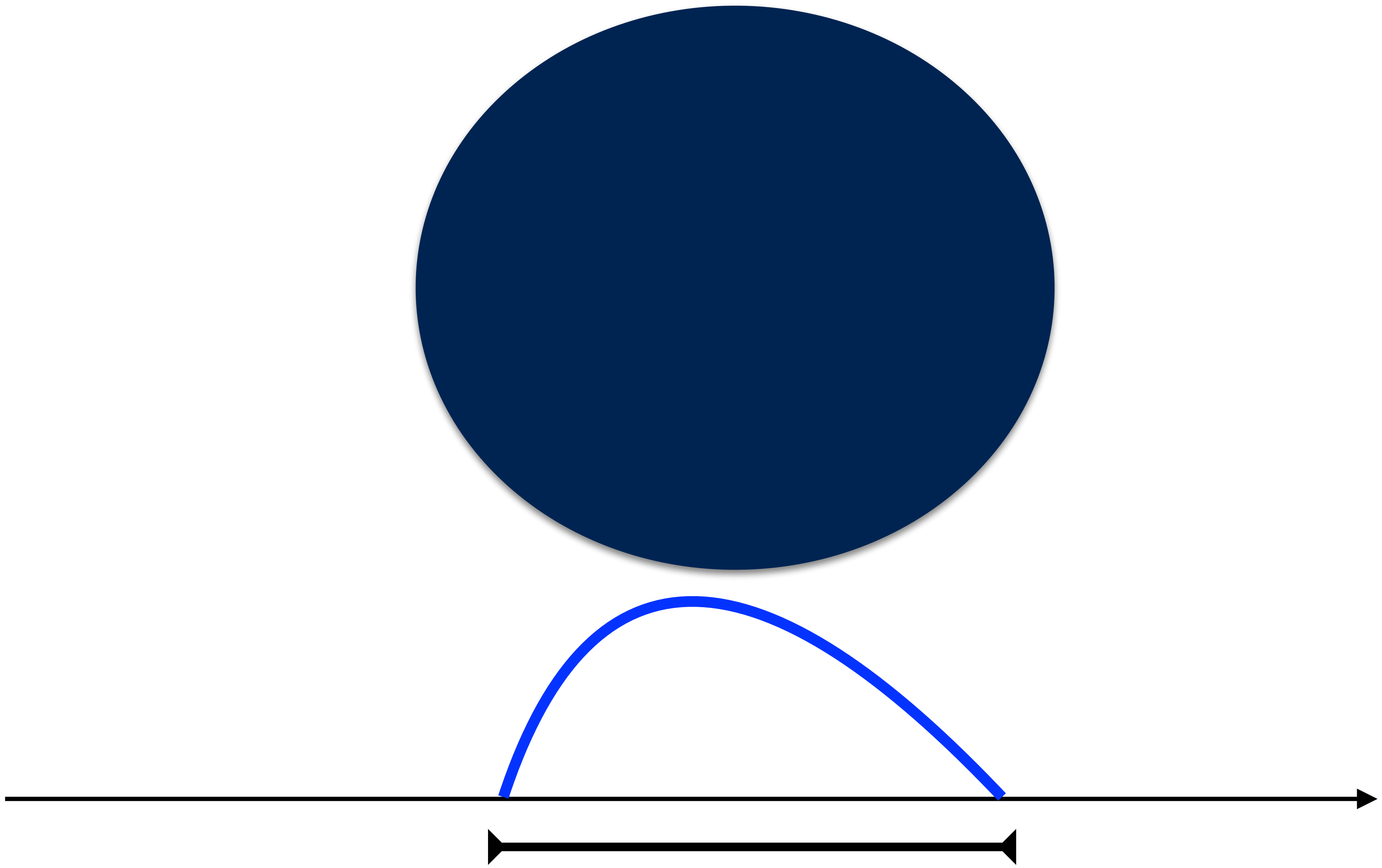
Localized information?

Can't we still have localized wave packet?



What happens to a wave packet which is localized completely inside a black hole?

Localized information?



Localized
info inside
black hole?

Localized information?

Theorem

(Baby version of Reeh-Schlieder theorem)

Suppose that the Fourier transform

$$\tilde{\psi}(k) = \int dx e^{-ikx} \psi(x)$$

of a wave function $\psi(x)$ satisfies the condition that its energy is finite in the sense that

$$\exists \epsilon > 0 \text{ such that}$$

$$|\tilde{\psi}(k)| e^{\epsilon |k|} \to 0 \quad (|k| \to \infty)$$

Then $\psi(x)$ is an analytic function.

Localized information?

Proof

$$\psi(x) = \int dk\; e^{ikx}\tilde{\psi}(k)$$

$$= \int dk\; e^{ikx-\epsilon|k|}\;[\tilde{\psi}(k)e^{\epsilon|k|}]$$

convergent at $|k| \to \infty$

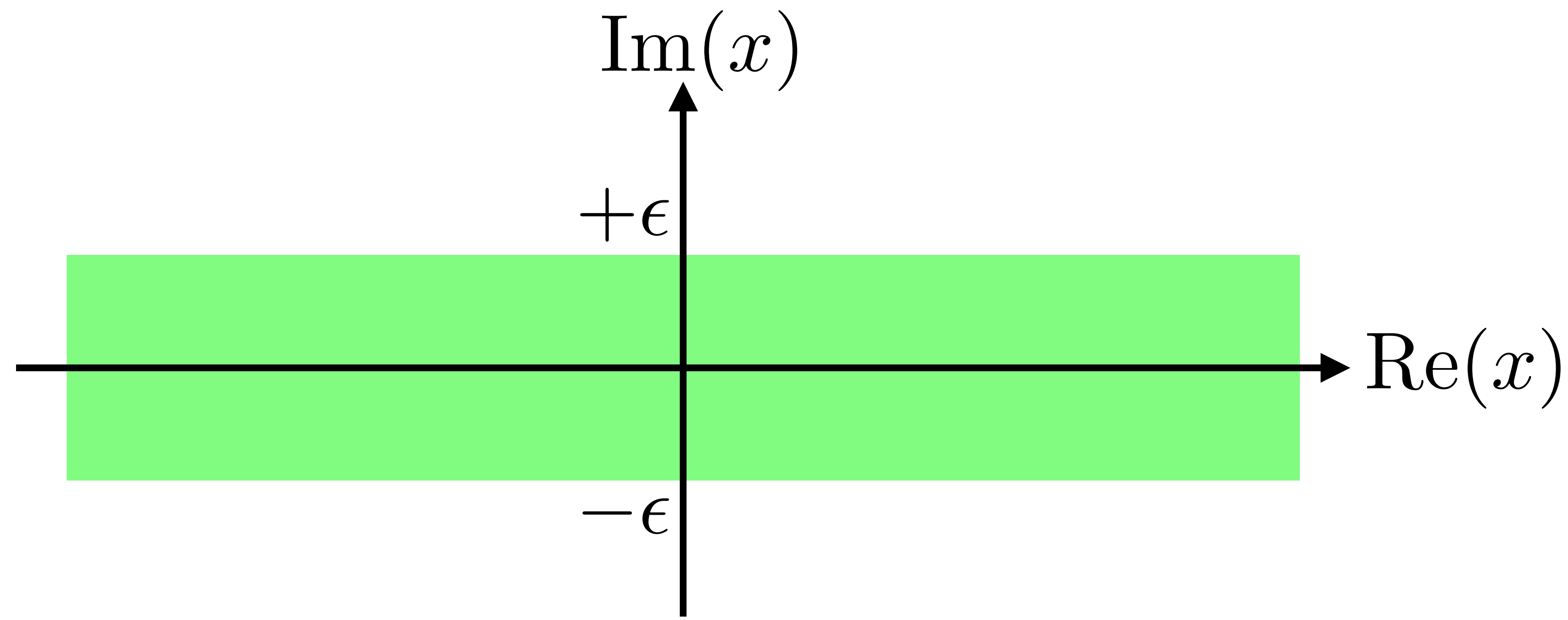
convergent if

$$-\epsilon < \mathrm{Im}(x) < \epsilon$$

Therefore, we can analytically continue to the region

$$-\epsilon < \mathrm{Im}(x) < \epsilon$$

Localized information?

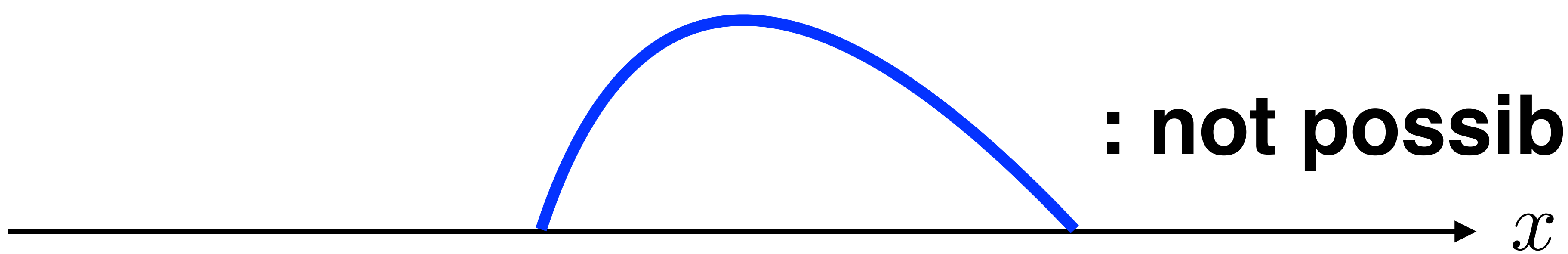


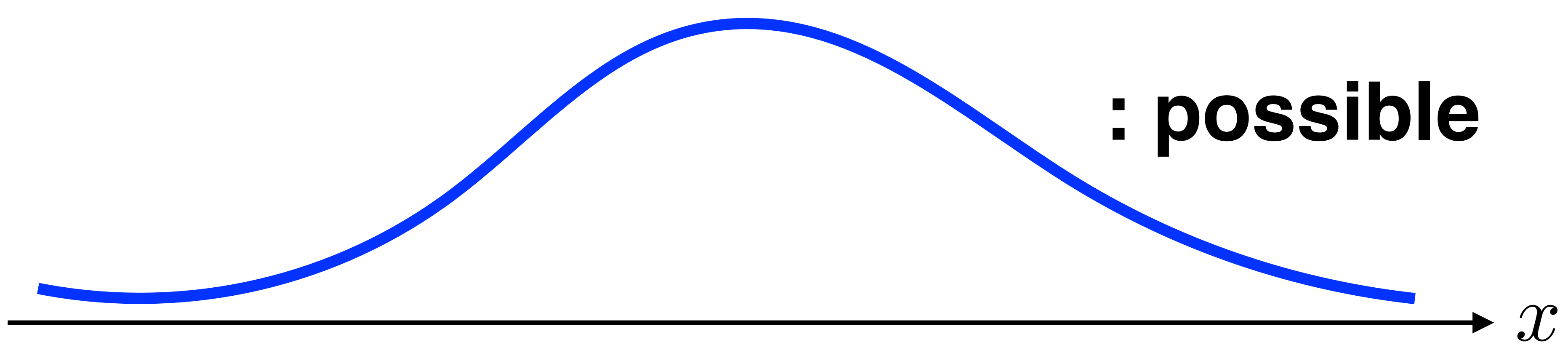
$\psi(x)$ is analytic in this region. This completes the proof.

Finite energy $\longrightarrow$ **Analyticity**

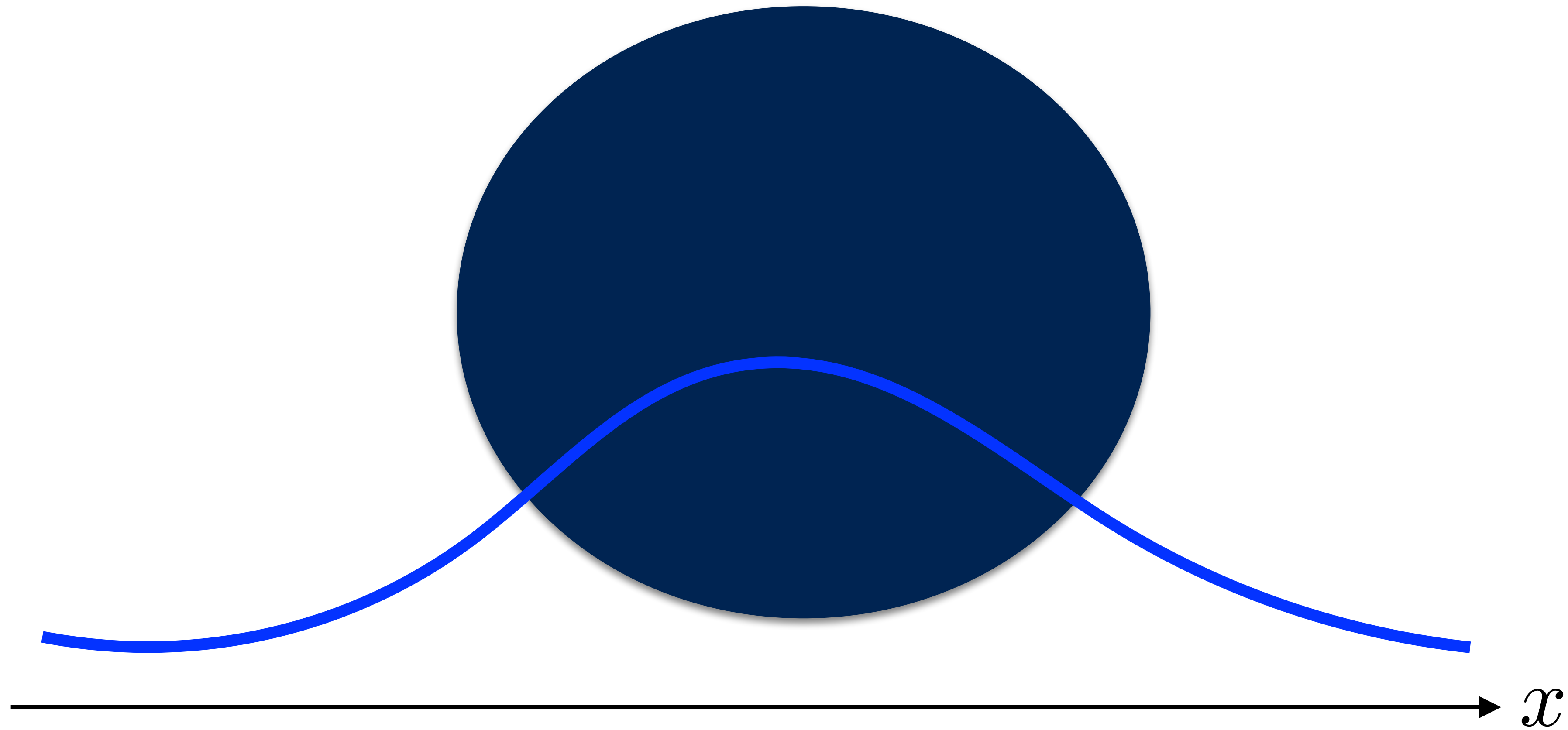
No localized information

Analytic function cannot be localized:





No localized information



If the wavefunction outside the black hole is known, its form inside the black hole is just determined by analytic continuation. No information loss.

Finite energy

I have assumed the condition of finite energy.
How do we justify it?

$\mathcal{H}$: Hilbert space

$\mathcal{H}_{\rm finite}$: Subspace of finite energy states

$\mathcal{H}_{\rm finite}$ is dense in $\mathcal{H}$

For example, let $P_{<E}$ be projection to states with energy engenvalue $< E$. Then any state is written as:

$$|\Psi\rangle \;=\; \lim_{E\to\infty} P_{<E}|\Psi\rangle$$

Finite energy

Suppose we have a linear map

$$U : \mathcal{H}_{\text{finite}} \to \mathcal{H}'$$

If it is a bounded operator (e.g. unitary operator), it is uniquely extended to

$$U : \mathcal{H} \to \mathcal{H}'$$

Implication to time evolution:

$$\boxed{\text{If black hole evaporation is unitary for the subspace } \mathcal{H}_{\text{finite}}\text{, it is unitary for all } \mathcal{H}}$$

Finite energy

Therefore, imposing the finite energy condition
does not lose generality for the present purposes.

(Also intuitively, we never create “infinite energy”
by physical processes, so finite energy should be enough.

Summary of the point

Finite energy

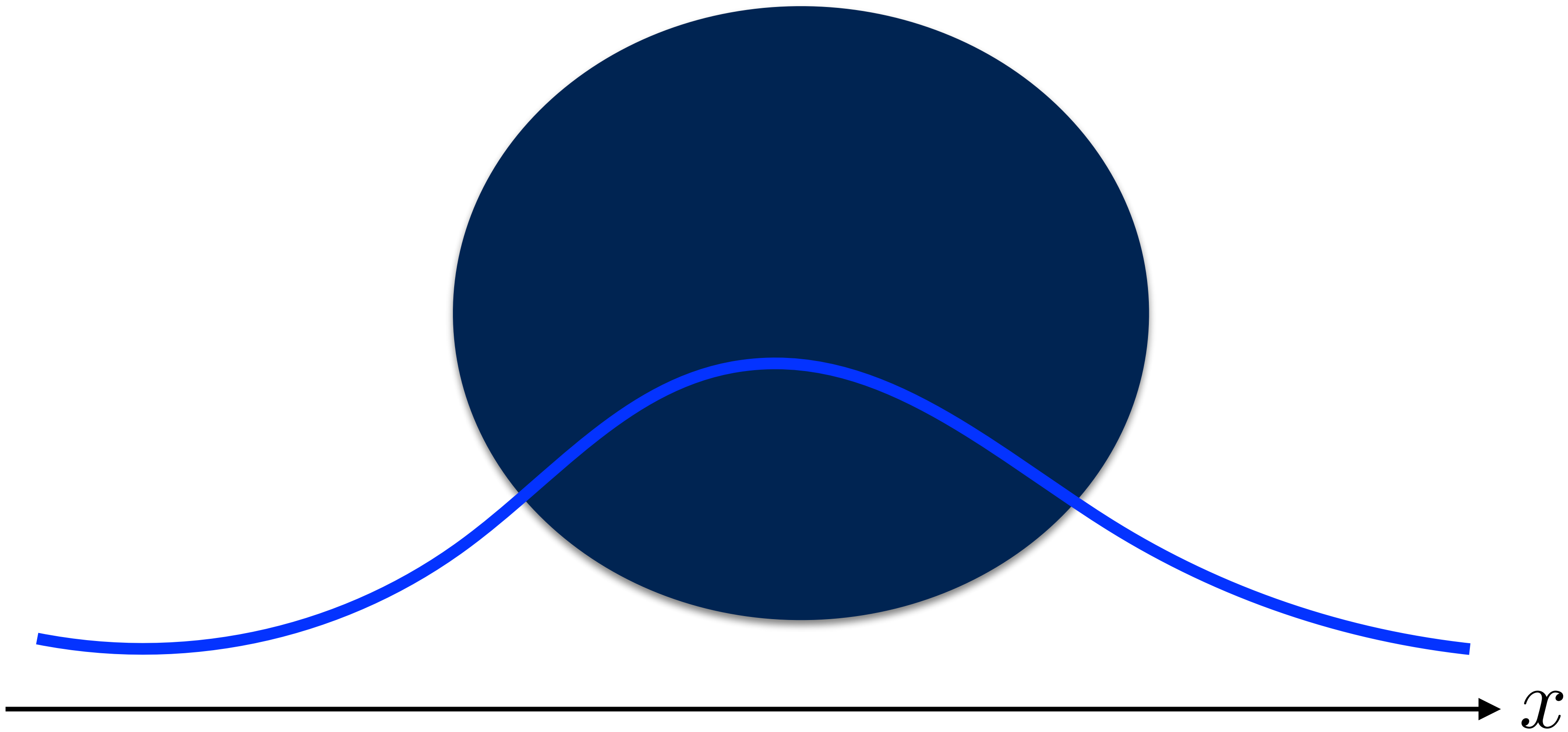
$\downarrow$

Analyticity

$\downarrow$

Non-locality of information

Summary of the point



Analytic continuation implies no information loss.

Summary of the point

This concludes the explanation of the essential point.

The rest of the talk is about some details of how to make the argument more concrete in quantum field theory.

Remark:
Gravity is treated as effective field theory and I assume standard facts about field theory applies to it.

Contents

Local operators

Let us consider all operators which are polynomials of operators

$$a = \cdots + \phi(x_1)\phi(x_2)\cdots\phi(x_n) + \cdots$$

$x_i \in A$: a region in spacetime

I denote the set of all operators in the region-A as

$$\mathcal{A}_A$$

Local operators

$\mathcal{A}_A$: the set (actually algebra) of operators localized in a spacetime region A

If M is the entire spacetime,

$$\mathcal{A}_M = \text{the set of all operators}$$
$$:= \mathcal{A}_{\mathrm{all}}$$

Axiomatic QFT

An axiom (and a theorem) of QFT state the following :

See [Streater-Wightman]

Let us take an (almost) arbitrary state $|\Psi\rangle$. Then, states of the form

$$a|\Psi\rangle \qquad a \in \mathcal{A}_{\mathrm{all}} = \text{set of all operators}$$

are dense in the Hilbert space.

Axiomatic QFT

In other words, any state $|\Phi\rangle$ can be approximated as

$$|\Phi\rangle \simeq a|\Psi\rangle$$

(More precisely, there exists a sequence of operators a_n such that $|\Phi\rangle = \lim_{n\to\infty} a_n|\Psi\rangle$.)

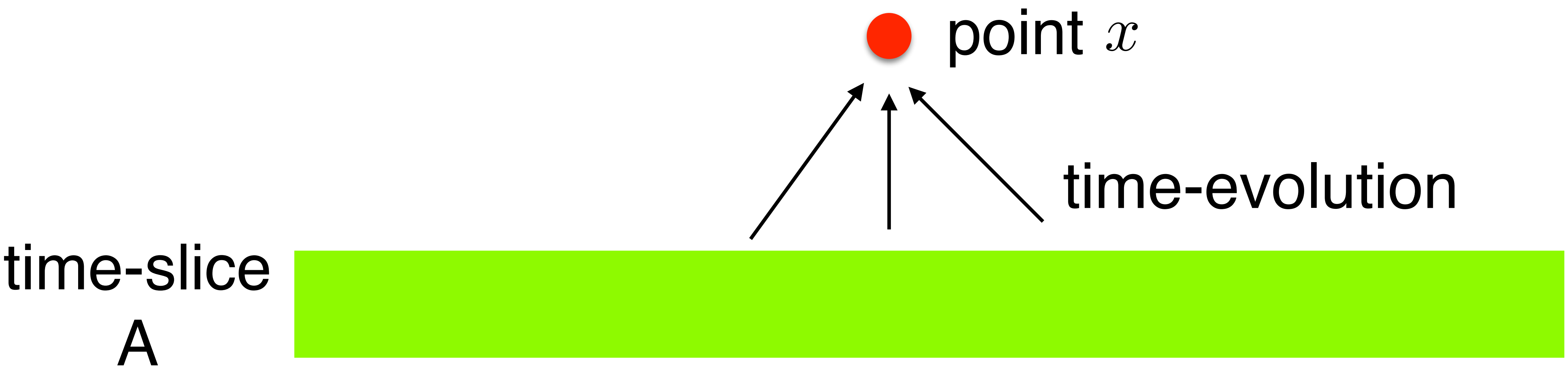
This just means that any state can be created by acting some operators.

Time-slice axiom

An reasonable extension of the axiom: time-slice axiom

See [Haag]

It is reasonable to think that local operators at an arbitrary point are determined by time-evolution from some time-slice



$\phi(x)$ is determined by $\mathcal{A}_A$

Time-slice axiom

Time-slice axiom

Let us take an (almost) arbitrary state $|\Psi\rangle$.

Any state $|\Phi\rangle$ can be approximated as

$$|\Phi\rangle \simeq a|\Psi\rangle \quad a \in \mathcal{A}_A$$

A : a region containing time-slice

Time-slice axiom is a physically natural axiomatization of time-evolution (i.e. Heisenberg equations of motion).

Reeh-Schlieder theorem

What is surprising is the Reeh-Schlieder theorem, which I state now.

Reeh-Schlieder theorem

Reeh-Schlieder theorem

Take a state $|\Psi\rangle$ satisfying the finite energy condition

$$\langle\Psi|e^{2\epsilon H}|\Psi\rangle < \infty \quad \text{for some } \epsilon > 0$$

Any state $|\Phi\rangle$ can be approximated as

$$|\Phi\rangle \simeq a|\Psi\rangle \quad a \in \mathcal{A}_A$$

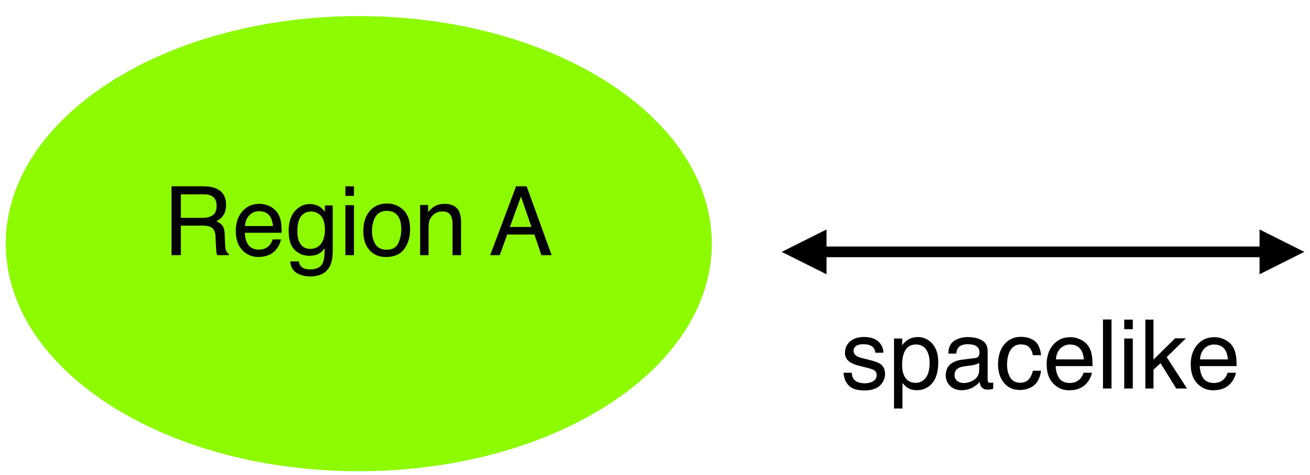
A : an arbitrary (open) region in spacetime

In other words, states of the form $a|\Psi\rangle$ span a dense subspace of the Hilbert space.

Reeh-Schlieder theorem

Borrowing Witten's words…

We can create Planet Jupiter by operators in a region-A which is spacelike separated from it.



$$|\text{Jupiter}\rangle \simeq a|\text{vacuum}\rangle$$
$$a \in \mathcal{A}_A$$

Reeh-Schlieder theorem

I don't discuss a proof of the Reeh-Schlieder theorem.

See [Streater-Wightman], [Witten,2018]
Or watch video of [Talk by Witten@Strings2018]

However, let me just mention the relation with a theorem on quantum mechanics which I discussed before.

Reeh-Schlieder theorem

Proof sketch

Suppose the theorem is not true. Then there exists a state $|\Phi\rangle$ which is orthogonal to any state of the form

$$a|\Psi\rangle \qquad a \in \mathcal{A}_A$$

So we have

$$\langle\Phi|\phi(x_1)\cdots\phi(x_n)|\Psi\rangle = 0$$

$$x_i \in A$$

Reeh-Schlieder theorem

The finite energy condition $\langle\Psi|e^{2\epsilon H}|\Psi\rangle < \infty$ means that the state vector $e^{\epsilon H}|\Psi\rangle$ is well-defined. We write

$$\underbrace{[\langle\Phi|\phi(x_1)\cdots\phi(x_n)e^{-\epsilon H}]}_{\text{exponential damping}}\underbrace{[e^{\epsilon H}|\Psi\rangle]}_{\text{well-defined}}$$

Thanks to exponential damping, the operators

$$\phi(x_i) = e^{iP\cdot x_i}\phi(0)e^{-iP\cdot x_i}$$

can be analytically continued to imaginary x_i

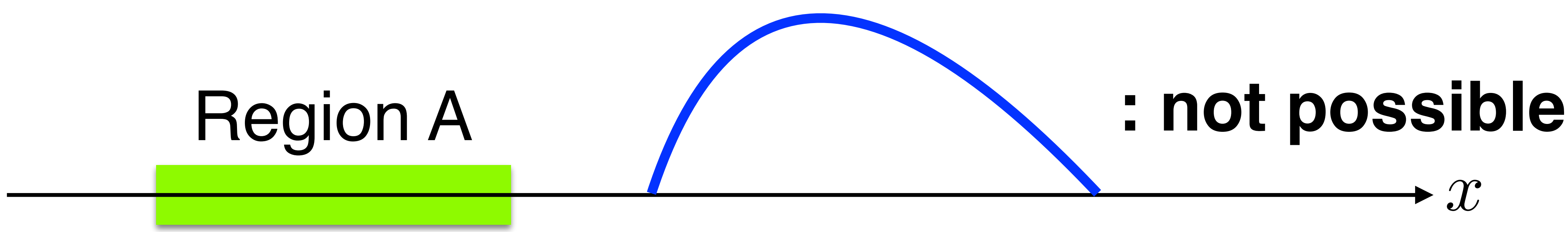
(Detail omitted.)

Reeh-Schlieder theorem

The result of the finite energy condition:

$$\langle \Phi | \phi(x_1) \cdots \phi(x_n) | \Psi \rangle$$

is an analytic function (or more precisely a boundary of an analytic function).



If it is zero in Region A, it is zero in the entire spacetime by analyticity.

Reeh-Schlieder theorem

$$\boxed{\begin{array}{c}\langle\Phi|\phi(x_1)\cdots\phi(x_n)|\Psi\rangle = 0 \\ x_i \in A\end{array}}$$

$\Downarrow$ analytic continuation

$$\boxed{\begin{array}{c}\langle\Phi|\phi(x_1)\cdots\phi(x_n)|\Psi\rangle = 0 \\ \text{Any } x_i\end{array}}$$

$\Downarrow$ the original axiom or time-slice axiom

$$\boxed{|\Phi\rangle = 0}$$

There is no state orthogonal to all of $a|\Psi\rangle$ $(a \in \mathcal{A}_A)$

Relation between QM and QFT

Finite energy in QM

↓

Analyticity of wave function

$$\psi(x)$$

Finite energy in QFT

↓

Analyticity of

$$\langle\Psi|\phi(x_1)\cdots\phi(x_n)|\Phi\rangle$$

Roughly speaking, in second quantization, "many body wave function" is given by

$$\psi(x_1,\cdots,x_n)\sim\langle\Psi|\phi(x_1)\cdots\phi(x_n)|\Phi\rangle$$

Contents

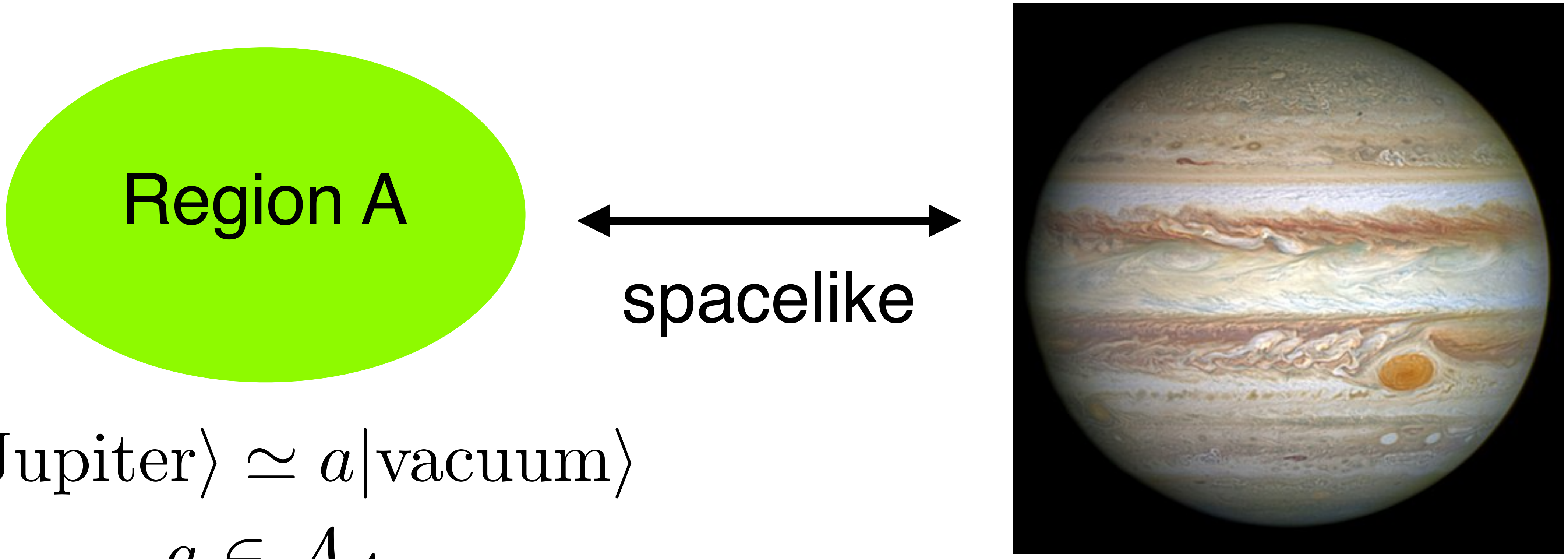
Important remark

I am **NOT** going to prove that black hole evaporation is unitary. That is impossible unless we know full quantum gravity.

Information problem is a problem that macroscopic amount of information seems to be lost, no matter what assumption we make about UV quantum gravity.

I argue that there exist a set of assumptions by which there is no information loss.

Basic idea

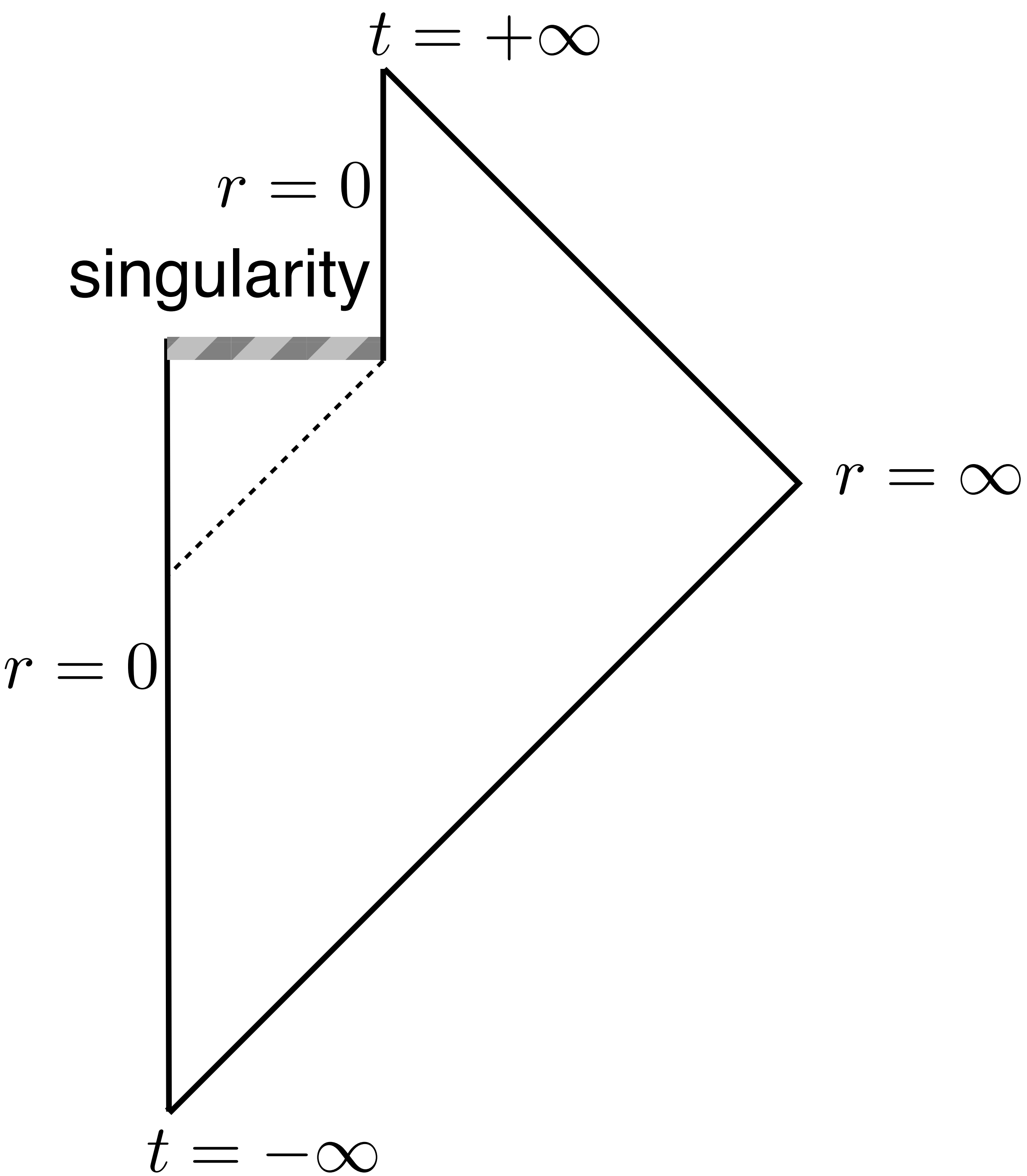


$$|\mathrm{Jupiter}\rangle \simeq a|\mathrm{vacuum}\rangle$$
$$a \in \mathcal{A}_A$$

If this is possible, why don't we create things inside a black hole by operators outside the black hole?

Aim

Penrose diagram of black hole evaporation



I'm going to argue that there is no information loss in this diagram.

Assumption 1

Assumption 1

> I assume that Reeh-Schlieder theorem is valid in curved analytic manifold, and especially in evaporating black hole geometry outside singularity.

The Reeh-Schlieder theorem was prove in flat space, and there are some mathematical works on curved manifolds.

Assumption 1

Why do I think that the Reeh-Schlieder theorem is applicable to black hole states?

Black holes may behave like a thermal object.

$$|\Psi\rangle\langle\Psi| \xrightarrow[\text{in energy eigenstates}]{\text{dropping off-diagnal components}} \rho \sim e^{-\beta H}$$

dropping
off-diagnal components
in energy eigenstates

β : inverse temperature

Assumption 1

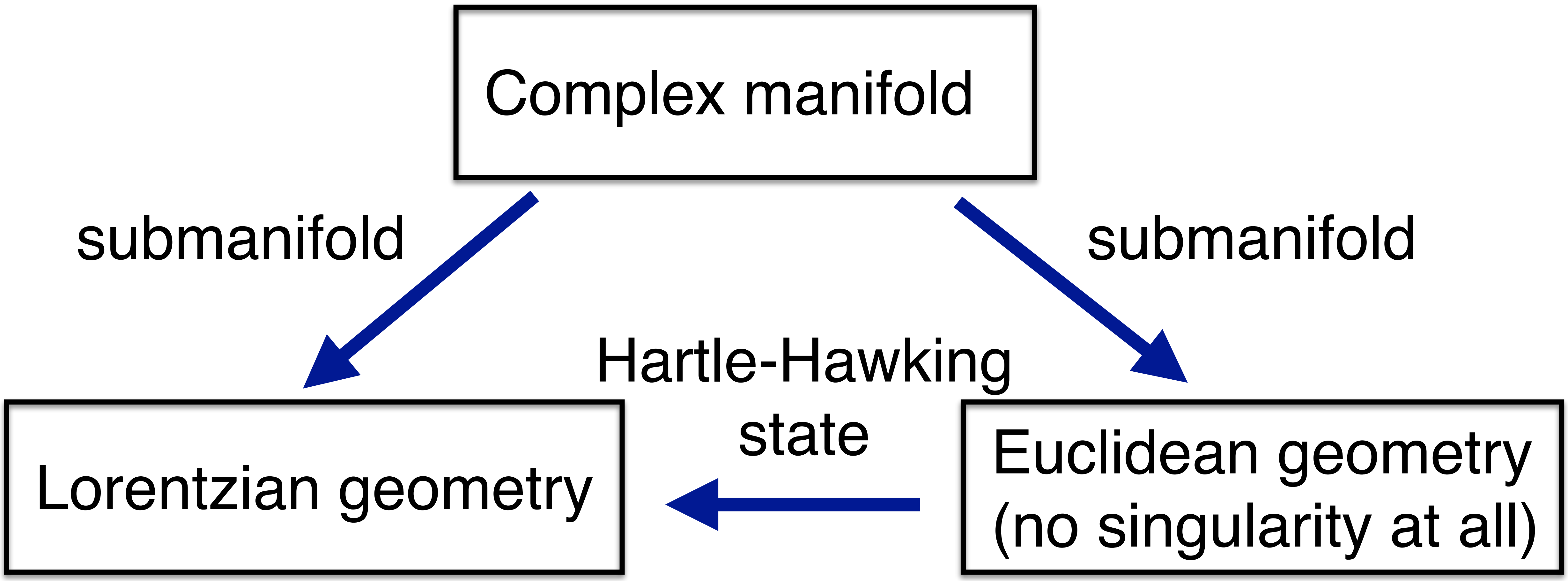
$$\langle\Psi|e^{2\epsilon H}|\Psi\rangle = \mathrm{tr}(e^{2\epsilon H}|\Psi\rangle\langle\Psi|) \sim \mathrm{tr}(e^{2\epsilon H}e^{-\beta H})$$

This is finite as long as $2\epsilon < \beta$

Anything which looks thermal satisfy the finite energy condition.

Assumption 1

In fact, Hartle-Hawking discussed
analytic continuation of black hole geometry.



Correlation functions are determined by analyticity,
at least for two-sided eternal black holes.

Assumption 2

Assumption 2

I assume that there exists a state, which I denote

$$|\Psi\rangle$$

in which black hole evaporation is unitary.

Assumption 2

Why do I assume the existence of a unitary state?

As I mentioned, it is not possible to prove unitarity unless we know UV complete quantum gravity.
So I assume that a small (e.g. Planck size) black hole has no information loss.

Then I will argue that there is no information loss in other states (e.g. big black holes).

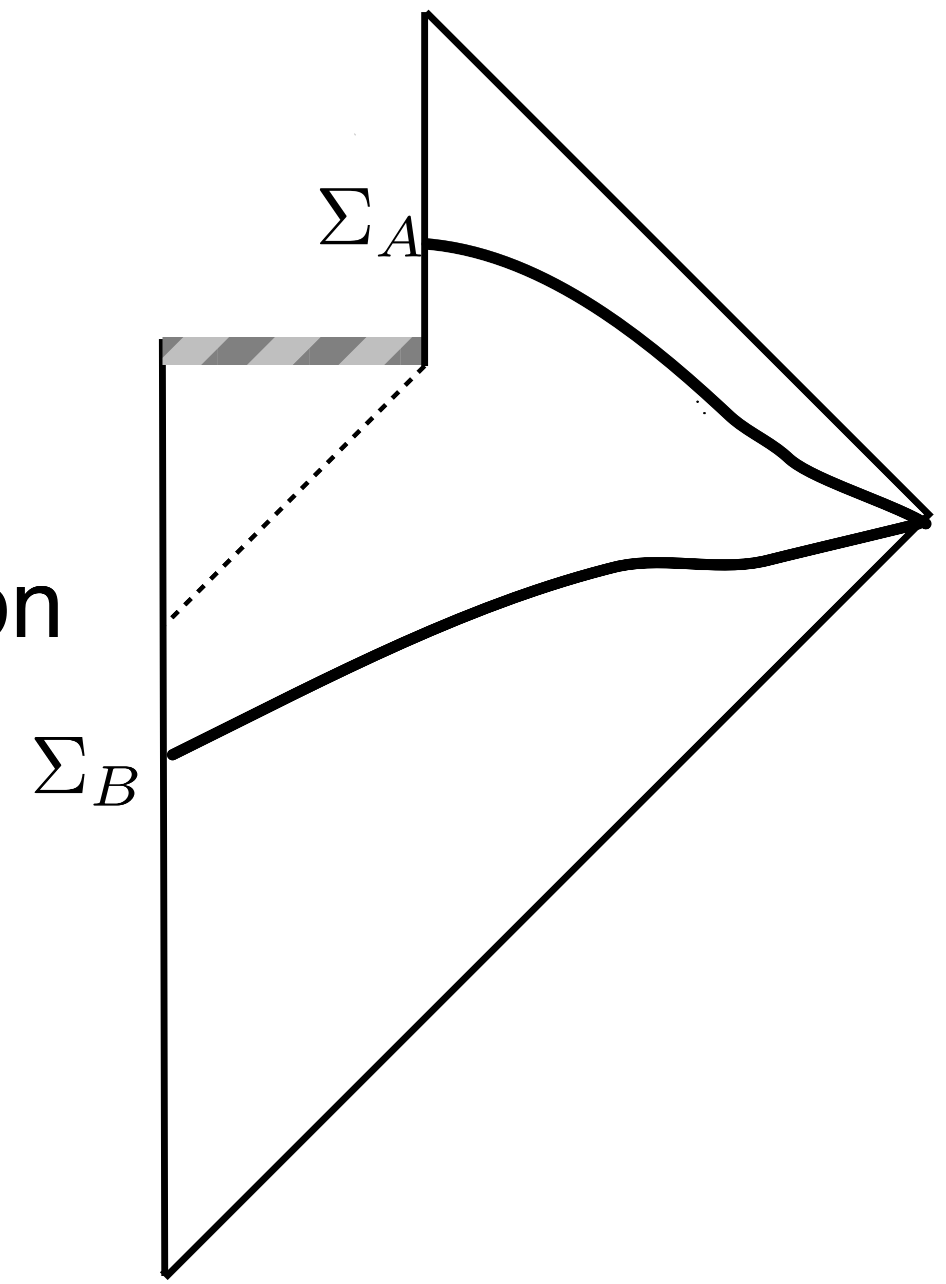
Assumption 2

Σ_A : time-slice After evaporation

Σ_B : time-slice Before evaporation

$\mathcal{H}_A$: Hilbert space After evaporation

$\mathcal{H}_B$: Hilbert space Before evaporation



Unitary evolution :

$\mathcal{H}_B \ni |\Psi\rangle_B \mapsto |\Psi\rangle_A \in \mathcal{H}_A$

The pure state $|\Psi\rangle_B$ goes to a pure state $|\Psi\rangle_A$.

Remark

I use “half Schrodinger, half Heisenberg picture”
in the following sense:

1. Each of the physics before and after evaporation is described by Heisenberg picture.

2. The relation between states before and after evaporation is described by Schrodinger picture.

(In the original paper I discussed completely in Heisenberg picture. I’m not sure which cause less confusion…)

The unitary evolution

We have assumed that for a certain (small) black hole

$$|\Psi\rangle$$

the evaporation is unitary.

Now I describe a scenario of unitary evaporation for other (maybe big) black hole states

$$|\Phi\rangle$$

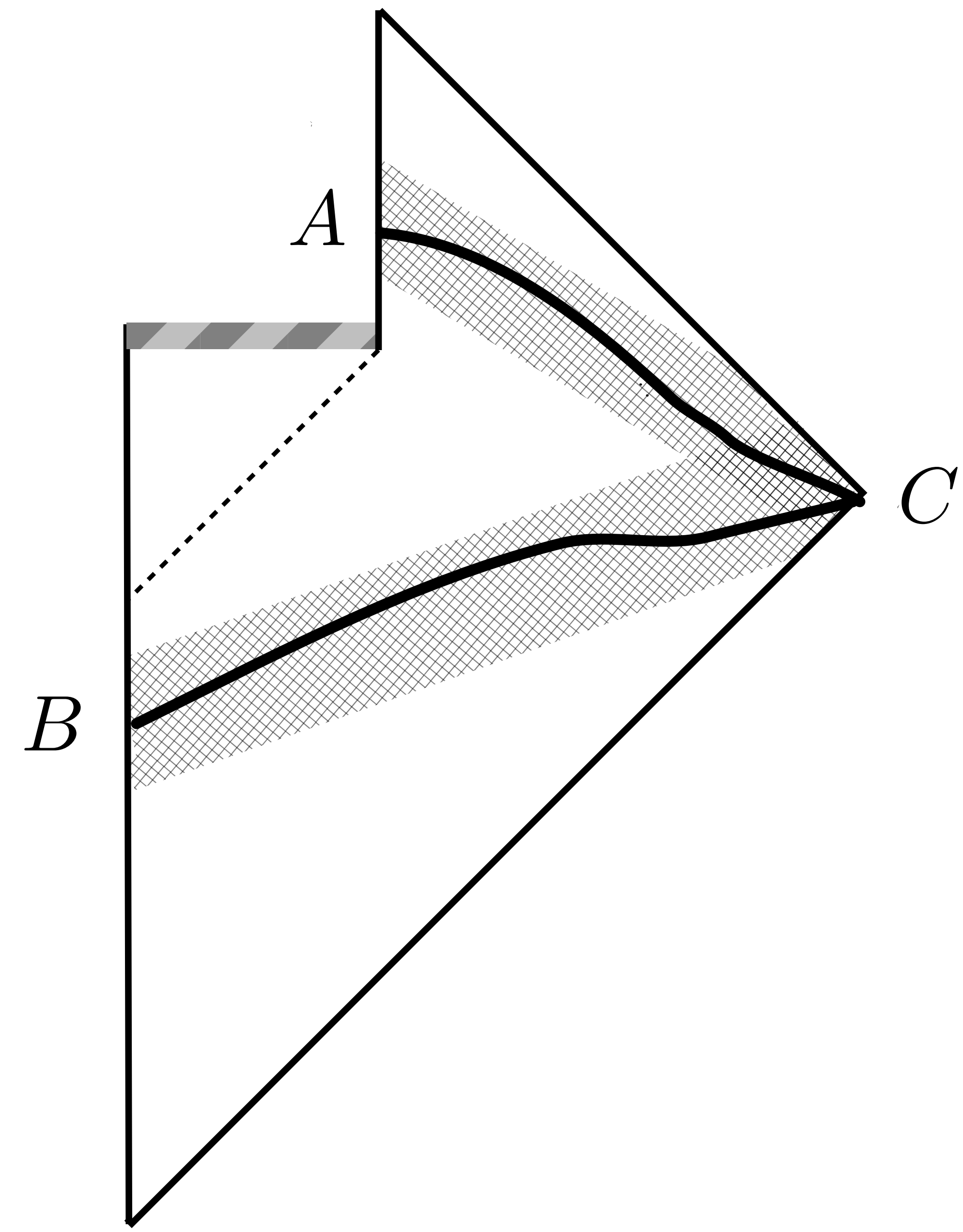
The unitary evolution

<u>Geometrical setup</u>

A : neighborhood of a time-slice after evaporation

B : neighborhood of a time-slice before evaporation

$C = A \cap B$



The unitary evolution

Step 1:

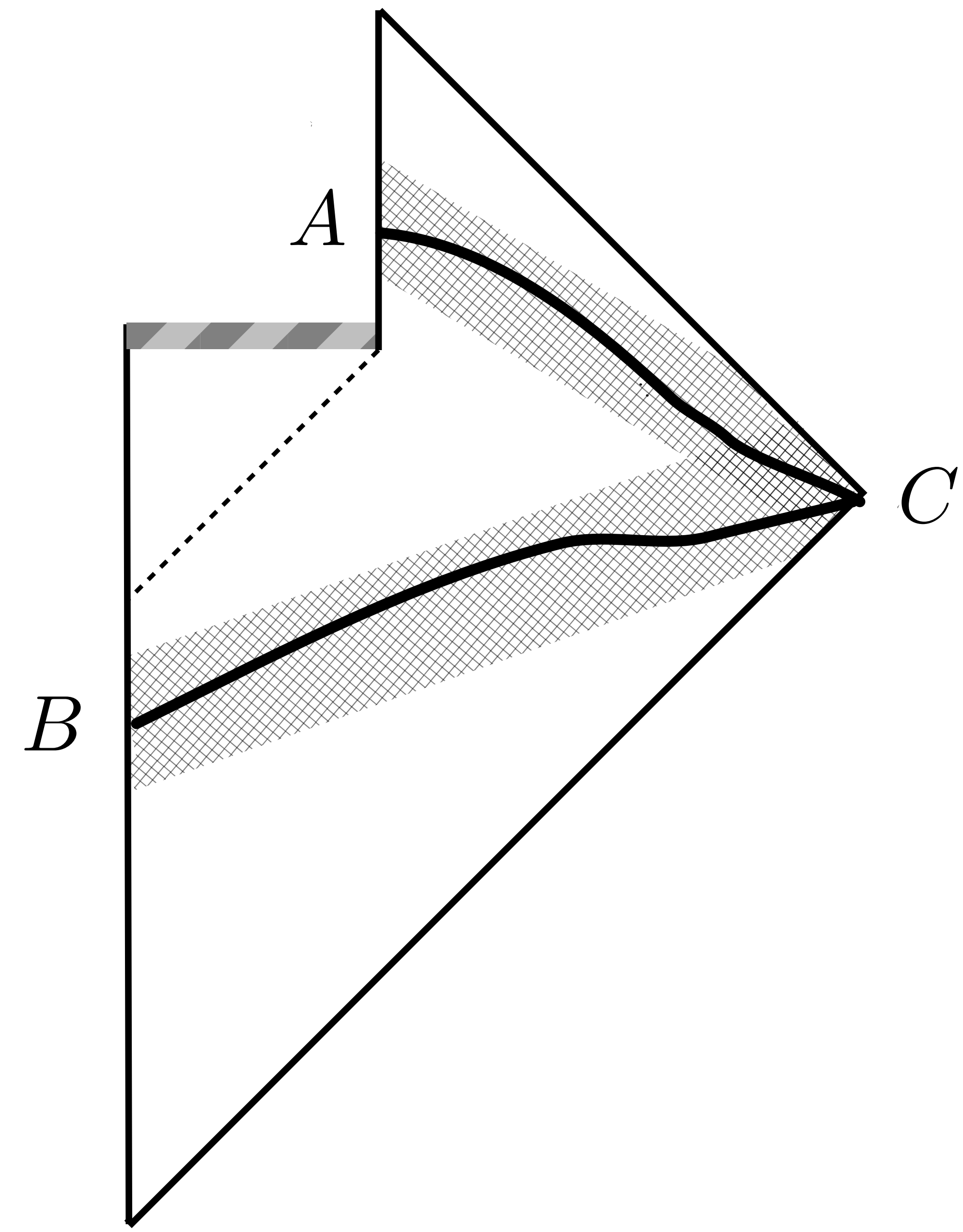
Take any $|\Phi\rangle_B \in \mathcal{H}_B$
: state before evaporation

Step 2:

Reeh-Schlieder theorem in the regions B and C implies:

$$|\Phi\rangle_B \simeq c|\Psi\rangle_B$$

$c \in \mathcal{A}_C$: operator in region C



The unitary evolution

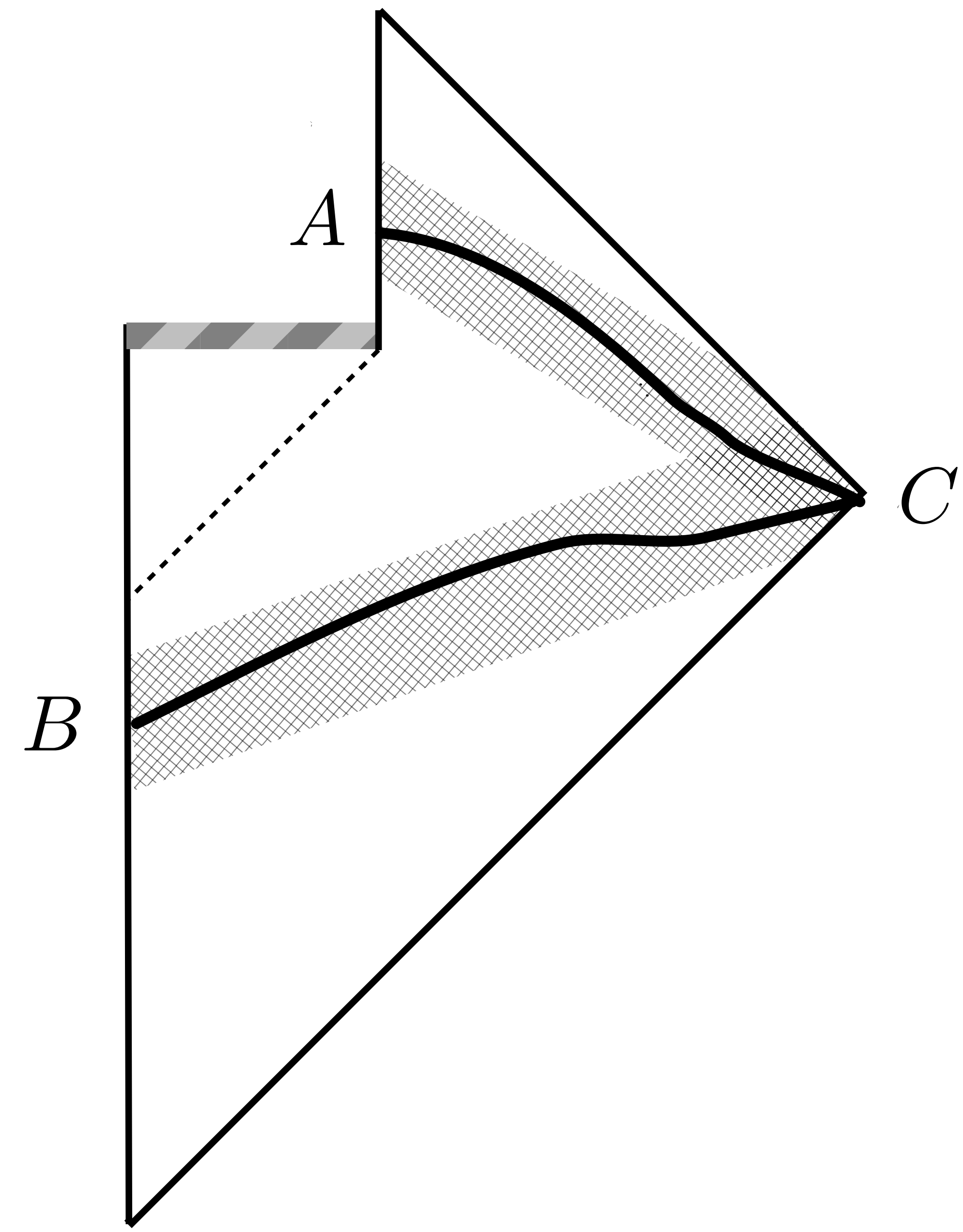
Step 3:

$$c \in \mathcal{A}_C \subset \mathcal{A}_A$$

Therefore,

$$c|\Psi\rangle_A \in \mathcal{H}_A$$

makes sense as a state after black hole evaporation



The unitary evolution

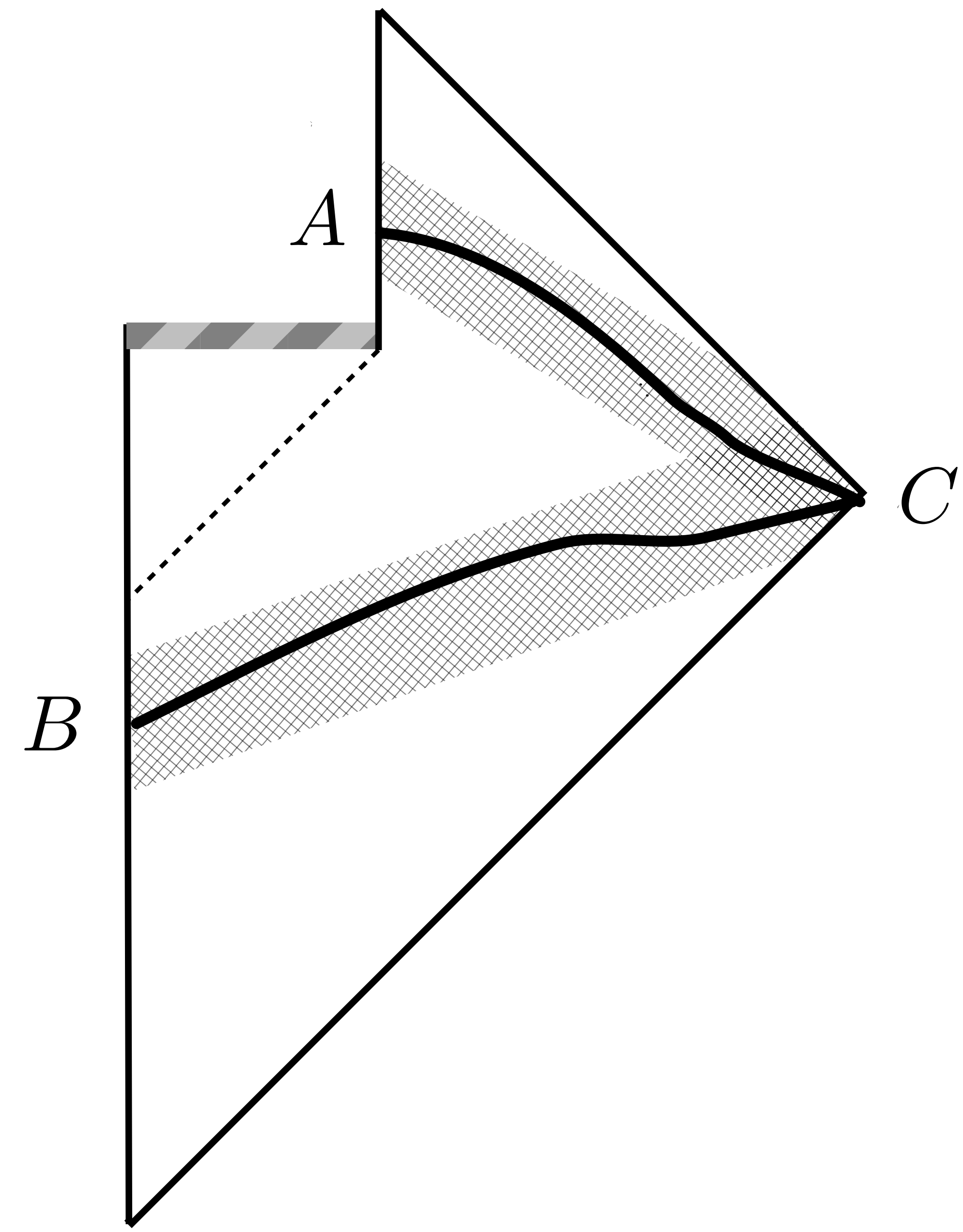
Step 4:

Define a linear map

$$U : \mathcal{H}_B \to \mathcal{H}_A$$

By

$$U : c|\Psi\rangle_B \mapsto c|\Psi\rangle_A$$



Remark:

U is defined only on a dense subspace of $\mathcal{H}_B$.

If U is unitary, it can be uniquely extended to the entire $\mathcal{H}_B$.

Assumption 3

Assumption 3

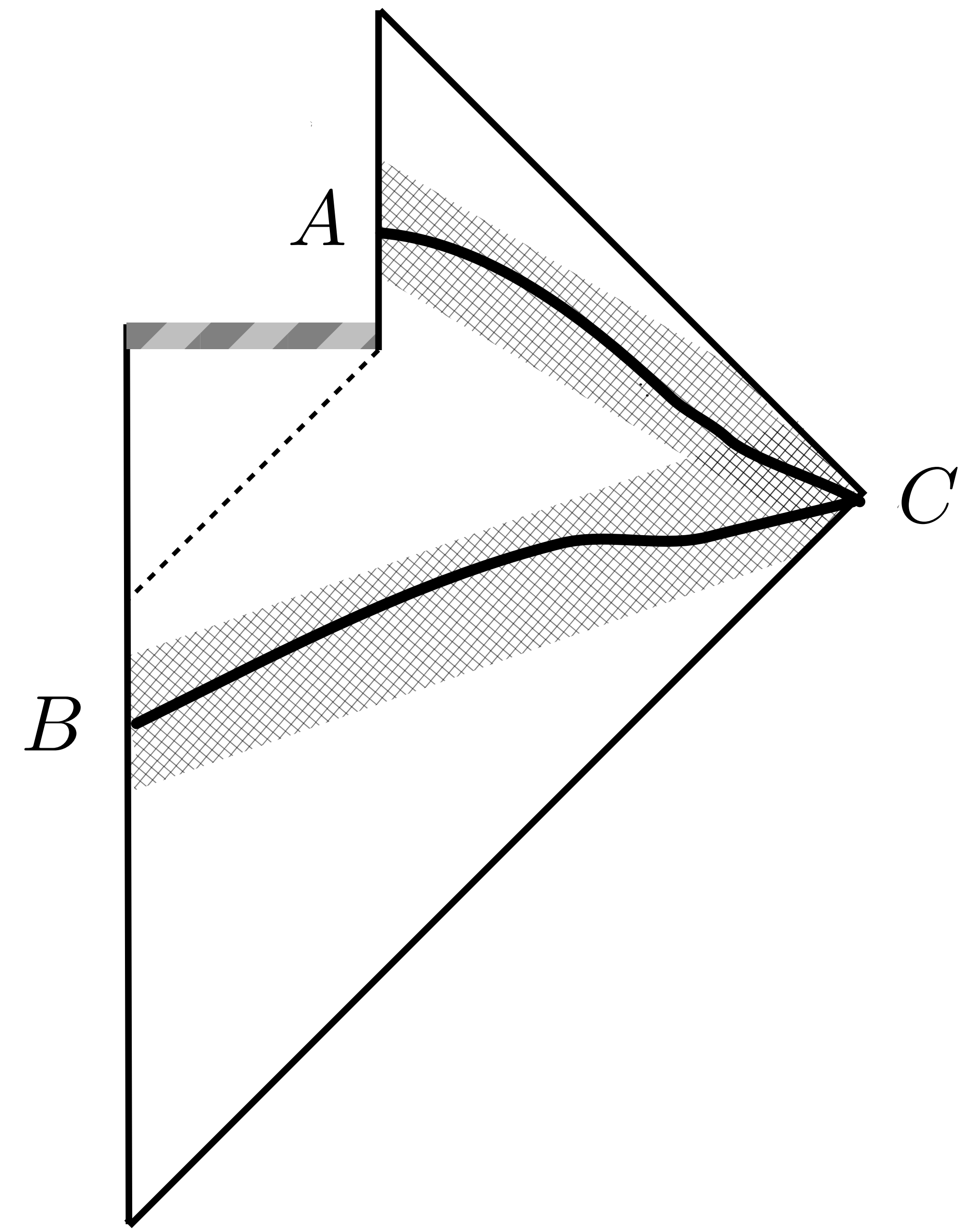
The linear map

$$U : c|\Psi\rangle_B \mapsto c|\Psi\rangle_A$$

is a unitary map.
(i.e. it preserves inner products.)

This is regarded as an assumption about the state $|\Psi\rangle$.

In other words, it is an assumption about the UV quantum gravity.



The unitary evolution

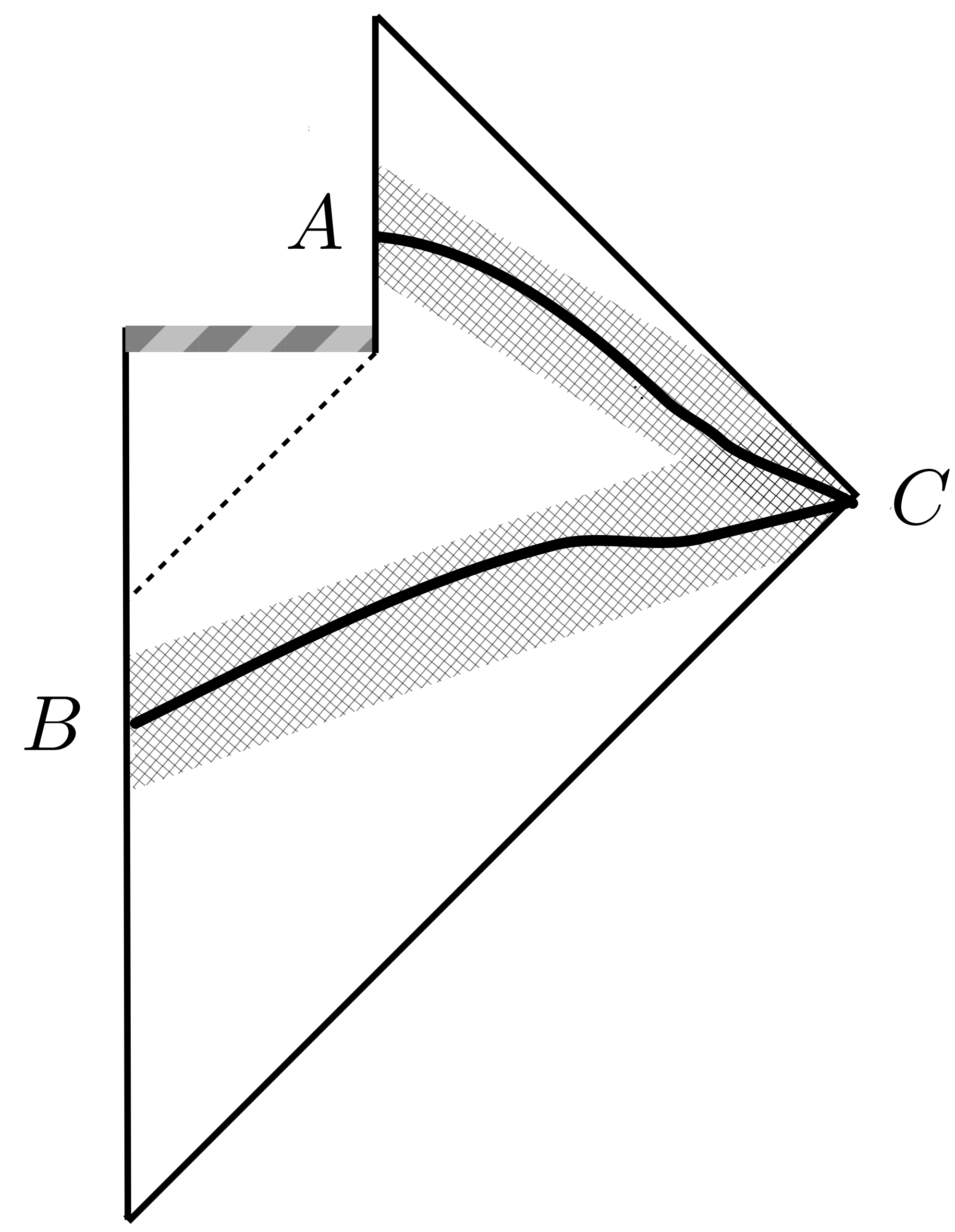
Proposal

The linear map

$$U : \mathcal{H}_B \to \mathcal{H}_A$$
$$U : c|\Psi\rangle_B \mapsto c|\Psi\rangle_A$$

describes black hole evaporation.

Pure states go to pure states.



Meaning of the proposal

If there is no spacetime singularity, the proposal reduces to a very trivial statement as follows.

Take global Heisenberg picture in entire spacetime:

$$\mathcal{H}_A = \mathcal{H}_B := \mathcal{H}$$
$$|\Psi\rangle_A = |\Psi\rangle_B := |\Psi\rangle$$

Then the map $U : c|\Psi\rangle \mapsto c|\Psi\rangle$ is just given by

$$U = 1$$

This is a trivial statement that Heisenberg picture states do not evolve with time.

Meaning of the proposal

The linear map

$$U : \mathcal{H}_B \to \mathcal{H}_A$$
$$U : c|\Psi\rangle_B \mapsto c|\Psi\rangle_A$$

describes black hole evaporation.

This proposal

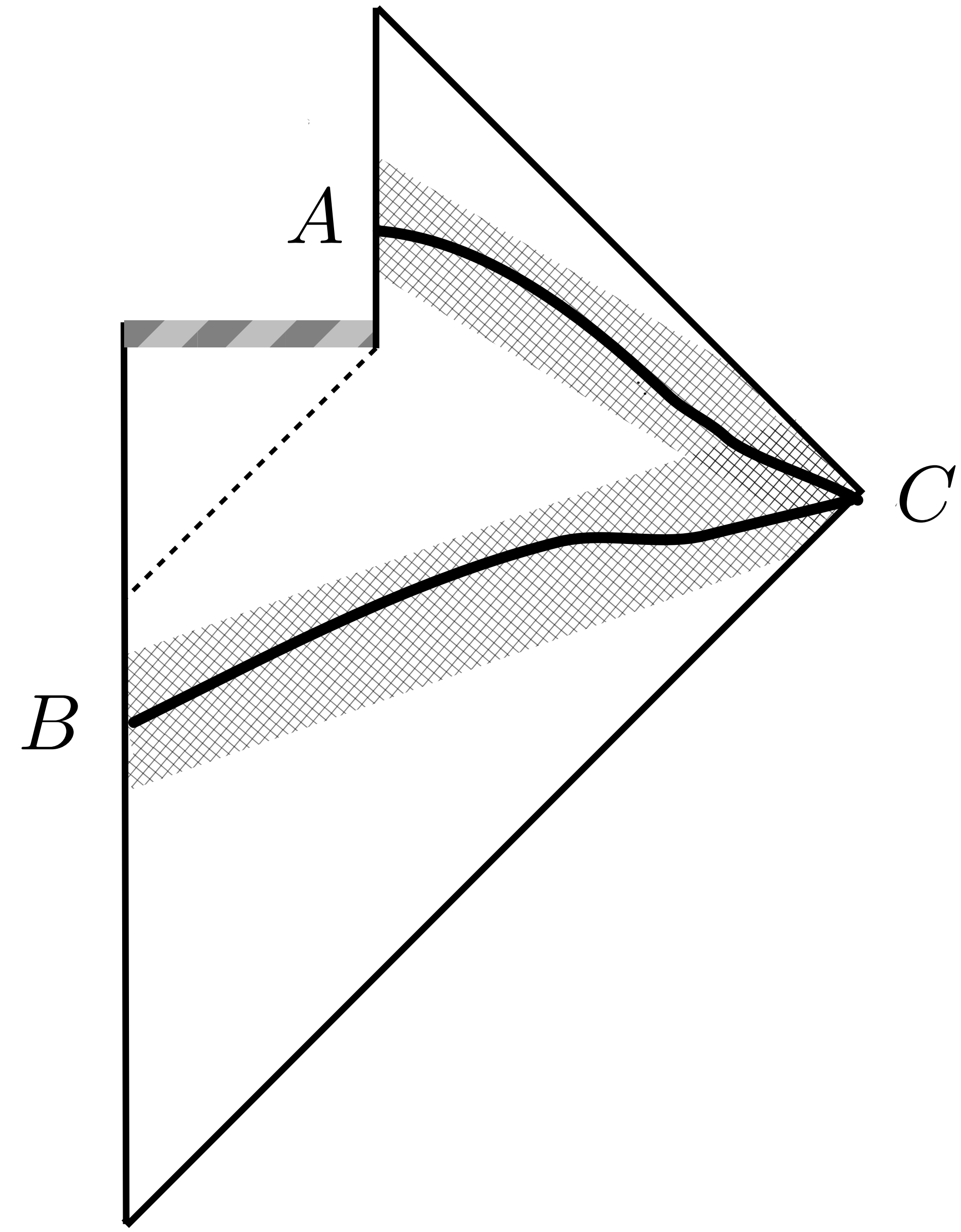
(1) is a realization of the trivial statement $U = 1$ in the Heisenberg picture,

(2) makes sense even in the presence of black holes.

AdS/CFT

$$|\Phi\rangle \simeq c|\Psi\rangle$$

$$c \in \mathcal{A}_C \qquad C = A \cap B$$



Region-C is a neighborhood of spatial infinity.

All (or dense) states are created by operators near spatial infinity:
Agrees very well with the idea of AdS/CFT!

Reeh-Schlider partially explains why AdS/CFT is possible.

Contents

Summary

Finite energy $\longrightarrow$ Analyticity of
- wave functions
- quantum fields

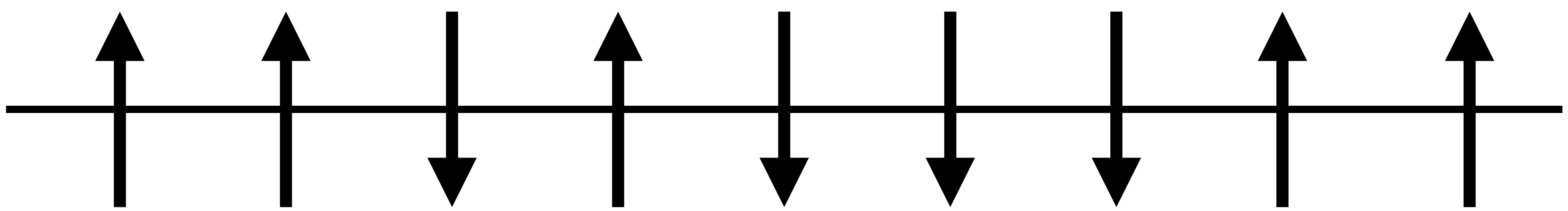
- States inside and outside black holes are related by analytic continuation. There is no such thing as "information localized inside/outside black holes".

- Reeh-Schlieder theorem can be used to make precise how information is preserved in black hole evaporation by "storing information at spatial infinity".

Backup

Hilbert space

Some information-theoretic considerations rely on finite dimensional Hilbert spaces as toy models.



: qubits

However, I remark that the Reeh-Schlieder theorem essentially uses infinite dimensionality of the Hilbert space.

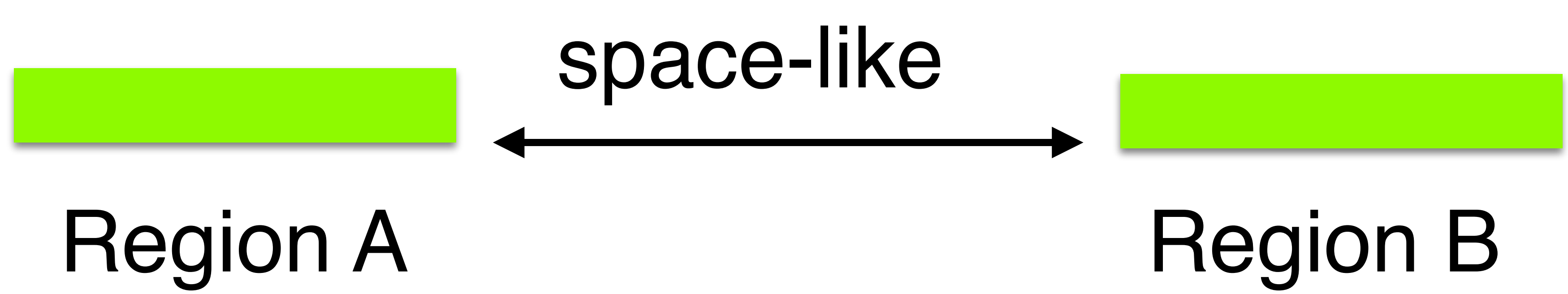
I would like to demonstrate this point.

Corollary of Reeh-Schlieder

Reeh-Schlieder theorem implies a corollary:

Suppose there are regions A and B which are spacelike to each other. Then

$$a|\Psi\rangle \neq 0 \qquad \text{for nonzero} \quad a \in \mathcal{A}_A$$



Corollary of Reeh-Schlieder

Proof

States of the form

$$b|\Psi\rangle \qquad b \in \mathcal{A}_B$$

spans a dense subspace of the Hilbert space by the Reeh-Schlieder theorem.
If $a|\Psi\rangle = 0$, then we get

$$\begin{aligned} ab|\Psi\rangle &= \pm ba|\Psi\rangle \qquad \text{(A, B spacelike)} \\ &= 0 \end{aligned}$$

a annihilates dense subspace, and hence

$$a = 0$$

Finite Hilbert space?

For a finite dimensional Hilbert space,

dense subspace = Hilbert space itself

The Reeh-Schlieder theorem, if true for finite dimensional Hilbert space, becomes

$$\forall |\Phi\rangle \in \mathcal{H}, \; \exists a \in \mathcal{A}_A \; \text{ such that } \; |\Phi\rangle = a|\Psi\rangle$$

I now show a contradiction.

Finite Hilbert space?

Let's consider three regions space-like to each other:

Region A Region B Region C

For arbitrary $|\Phi\rangle$, we have

$$|\Phi\rangle = a|\Psi\rangle = b|\Psi\rangle \qquad a \in \mathcal{A}_A \quad b \in \mathcal{A}_B$$

$$(a-b)|\Psi\rangle = 0 \qquad (a-b) \in \mathcal{A}_{A\cup B}$$

Finite Hilbert space?

$$(a-b)|\Psi\rangle = 0 \qquad (a-b) \in \mathcal{A}_{A\cup B}$$

Corollary of the Reeh-Schlieder applied to $A \cup B$ and C

$$a = b$$

It is impossible that an operator in Region A and an operator in Region B are equal except for the identity.
(*Proof sketch*: such an operator commutes with all operators near a time-slice which goes through A and B.)

Contradiction.

We must be careful about how to interpret results based on finite dimensional Hilbert spaces.