Three-Higgs-doublet models: the broad picture and hidden gems

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NHDMs

Is there life beyond the SM Higgs?

The minimal Higgs sector of the SM is overstretched. As a result:

- does not explain fermion masses and mixing, neutrino masses, *CP*-violation;
- has boring flavor properties: no tree-level FCNCs;
- does not help explain DM or baryon asymmetry.

The gauge structure of SM does not require the Higgs sector to be minimal It can well be rich and it can extend the idea of "generations" to scalars → *N*-Higgs-doublet models (NHDMs).

2HDM has been our playground for decades, time to move on!

Building blocks of NHDMs

Main ingredients in NHDMs:

- $N-1$ charged $H^\pm$ and $2N-1$ neutral Higgses → rich pheno;
- Higgs potential:

$$V = Y_{ab}(\phi_a^\dagger \phi_b) + Z_{ab,cd}(\phi_a^\dagger \phi_b)(\phi_c^\dagger \phi_d)\,,$$

  with N^2 quadratic, $N^2(N^2+1)/2$ quartic coefficients → challenges!
- Yukawa couplings with very rich interaction patterns;
- Symmetries (including CP-symmetries) as a way to bring order and obtain robust features.

Quark masses and mixing in SM

Yukawa couplings in the SM, $d_i = (d, s, b)$, $u_i = (u, c, t)$:

$$\bar{Q}_{Li}\Gamma_{ij}\phi d_{Rj} + \bar{q}_{Li}\Delta_{ij}\tilde{\phi}u_{Rj} + h.c. \to \bar{d}_{Li}(M_d)_{ij}d_{Rj} + \bar{u}_{Li}(M_u)_{ij}u_{Rj} + h.c.$$

where the 3×3 mass matrices are

$$M_d = \Gamma\frac{v}{\sqrt{2}}, \quad M_u = \Delta\frac{v^*}{\sqrt{2}}.$$

They can be diagonalized: $V_{dL}^\dagger M_d V_{dR} = D_d$, $V_{uL}^\dagger M_u V_{uR} = D_u$, but then the charged current matrix can become non-trivial:

$$\bar{u}_{Li}\gamma^\mu W_\mu^+ d_{Li} \to \bar{u}_{Li}\gamma^\mu W_\mu^+ V_{ij} d_{Lj}, \quad \text{where} \quad V_{ij}^{(CKM)} = V_{uL}^\dagger V_{dL} \neq \delta_{ij}.$$

No FCNC or LFV in Higgs exchanges.

Quark masses and mixing in NHDM

With N Higgs doublets ϕ_a, the procedure is the same:

$$\sum_a \left(\bar{Q}_{Li} \Gamma^{(a)}_{ij} \phi_a d_{Rj} + \bar{q}_{Li} \Delta^{(a)}_{ij} \tilde{\phi}_a u_{Rj} \right) + h.c.$$

Vacuum coresponds to a vev alignment $\langle \phi_a^0 \rangle = v_a/\sqrt{2}$, which gives

$$M_d = \frac{1}{\sqrt{2}} \sum_a \Gamma^{(a)} v_a \,, \quad M_u = \frac{1}{\sqrt{2}} \sum_a \Delta^{(a)} v_a^* \,.$$

- Tree-level FCNC can be avoided with natural flavour conservation via discrete symmetries [Weinberg, Glashow, 1977; Pachos, 1977].
- Individual $\Gamma^{(a)}$ and $\Delta^{(a)}$ can be very simple, symmetry-constrained. In M_d and M_u, this symmetry is ruined but can leave traces in masses/mixing.
- If complex vevs v_a get a relative phase, then CP-violation can appear spontaneously for real Γ and Δ [T.D.Lee, 1973, Branco, 1979].

Scalar dark matter

Some Higgs doublets can be inert: no coupling to fermions, and no role in W and Z masses. If inert scalars are protected by a new "parity" (do not get vev) $\to$ the lightest parity-odd scalar is stable $\to$ scalar DM without any fine-tuning.

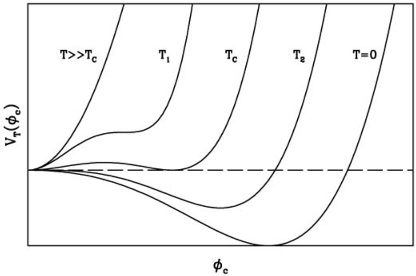
Example: Inert doublet model = 2HDM with exact $\mathbb{Z}_2$-symmetry [Deshpande, Ma, 1978; Barbieri et al, 2006, Lopez Honorez et al, 2006].

Two Higgs doublets ϕ and ϕ_D, with $\langle\phi_D\rangle = 0$. Extra Higgses from ϕ_D: $H^\pm$, H, $A \to$ DM + interesting collider pheno.

Cosmological phase transition

Baryon asymmetry requires a strong first-order thermal electroweak phase transition in early Universe.

$$\frac{v(T_c)}{T_c} \gtrsim 1 \,. \quad \text{[Shaposhnikov, 1986]}$$



The SM could satisfy it only for light Higgs ($m_h \lesssim 50$ GeV) [Kajantie et al, 1996; Csikor, 1999].

Extra scalars modify the Higgs potential and can produce strong EWPT, from [Turok, Zadrozny, 1992]. Can be probed in future GW observatories.

3HDM

What's new in 3HDM compared to 2HDM:

- richer pheno (both scalar and fermion sectors);
- sophisticated potential $\to$ many minima, many options for phase transitions;
- combining nice features of 2HDM, e.g. NFC + CPV [Weinberg, 1976; Branco, 1979], scalar DM + CPV [Grzadkowski et al, 2009];
- new options for *CP* violation, e.g. geometrical CPV [Branco, Gerard, Grimus, 1984], and for *CP* symmetry, such as CP4 [Ivanov, Silva, 2015];
- symmetries, lots of symmetries in the 3HDM scalar sector!

Symmetries in 3HDM: flavour physics connection

- The original idea from 1970's:
  - extent G to fermion sector,
  - arrange for spontaneous violation $G \to G_v$,
  - derive masses/mixing/CPV;
- Early examples:
  - permutation groups S_3 or S_4: [Pakvasa, Sugawara, 1978, 1979, + Yamanaka, 1982] → perfectly (for early 80's!) reproduced CKM;
  - rephasing + permutations: $\Delta(54)$ which makes $\Gamma^{(a)}$ very simple [Segre, Weldon, Weyers, 1979]: mass hierarchy may come from $v_1 \ll v_2 \ll v_3$.
  - typical prediction for top mass: 20–40 GeV; decline of activity in 90's.
- Many combinations of G + irreps + vevs were tested, but
  - if G is large → severe problems in the quark sector; A_4/S_4 illustrations in [Gonzales Felipe et al, 1302.0861, 1304.3468];
  - if G is small → too many free parameters, no predictive power.

Symmetries in 3HDM: flavour physics connection

The obstacle [Leurer, Nir, Seiberg, 1993; Gonzales Felipe et al, 1401.5807]

- If the (active) Higgs sector is equipped with G, then vevs must break G completely in order to produce physical m_q's and CKM.
- But for large G, this is algebraically impossible [Ivanov, Nishi, 1410.6139].

Perhaps, soft breaking of G is a solution? No systematic study exists.

Symmetries in 3HDM

Full classification of 3HDM symmetries in the scalar sector appeared only recently.

- abelian groups [Ivanov, Keus, Vdovin, 1112.1660]

$$\mathbb{Z}_2, \quad \mathbb{Z}_3, \quad \mathbb{Z}_4, \quad \mathbb{Z}_2 \times \mathbb{Z}_2, \quad U(1), \quad U(1) \times \mathbb{Z}_2, \quad U(1) \times U(1)\,.$$

- discrete non-abelian groups [Ivanov, Vdovin, 1210.6553]

$$S_3, \quad D_4, \quad A_4, \quad S_4, \quad \Delta(54), \quad \Sigma(36)\,.$$

- symmetry breaking patterns $G \to G_v$ [Ivanov, Nishi, 1410.6139]
- interplay between G and CP [many classical works].

Digression on symmetry group classification in 3HDM

Two recipes for building symmetry-based bSM models:

- choose your favorite group G, impose it on $\mathcal{L}$, add extra fields at will;
- fix the set of fields and derive all symmetry groups that are allowed.

Strategy for symmetry groups in the scalar sector of NHDM:

- find all abelian groups A_i;
- build G by combining various A_i but avoid generating abelian groups not in the list;
- for each G, check that it does not produce accidental symmetry.

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Symmetry breaking in 3HDM

Strongest and weakest breaking of discrete symmetries in 3HDM and spontaneous CPV [Ivanov, Nishi, 1410.6139].

group	$\|G\|$	$\|G_v\|_{min}$	$\|G_v\|_{max}$	sCPv possible?
abelian	2, 3, 4, 8	1	$\|G\|$	yes
$\mathbb{Z}_3 \rtimes \mathbb{Z}_2^*$	6	1	6	yes
S_3	6	1	6	—
$\mathbb{Z}_4 \rtimes \mathbb{Z}_2^*$	8	2	8	no
$S_3 \times \mathbb{Z}_2^*$	12	2	12	yes
$D_4 \times \mathbb{Z}_2^*$	16	2	16	no
$A_4 \rtimes \mathbb{Z}_2^*$	24	4	8	no
$S_4 \times \mathbb{Z}_2^*$	48	6	16	no
CP-violating $\Delta(27)$	18	6	6	—
CP-conserving $\Delta(27)$	36	6	12	yes
$\Sigma(36)$	72	12	12	no

$\Sigma(36)$ 3HDM

Σ(36) 3HDM

The largest discrete group in 3HDM scalar sector is

$$\Sigma(36) \simeq (\mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes \mathbb{Z}_4$$

generated by:

$$a = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \quad b = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad d = \frac{i}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix},$$

where $\omega = \exp(2\pi i/3)$. Orders of generators:

$$a^3 = \mathbf{1}, \quad b^3 = \mathbf{1}, \quad d^4 = \mathbf{1}.$$

This possibility was overlooked in previous studies of 3HDMs.

Σ(36) 3HDM

The scalar potential

$$
\begin{aligned}
V \quad = \quad & -m^2\left[\phi_1^\dagger\phi_1+\phi_2^\dagger\phi_2+\phi_3^\dagger\phi_3\right]+\lambda_1\left[\phi_1^\dagger\phi_1+\phi_2^\dagger\phi_2+\phi_3^\dagger\phi_3\right]^2 \\
& +\lambda_2\left[|\phi_1^\dagger\phi_2|^2+|\phi_2^\dagger\phi_3|^2+|\phi_3^\dagger\phi_1|^2-(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2)-(\phi_2^\dagger\phi_2)(\phi_3^\dagger\phi_3)-(\phi_3^\dagger\phi_3)(\phi_1^\dagger\phi_1)\right] \\
& +\lambda_3\left(|\phi_1^\dagger\phi_2-\phi_2^\dagger\phi_3|^2+|\phi_2^\dagger\phi_3-\phi_3^\dagger\phi_1|^2+|\phi_3^\dagger\phi_1-\phi_1^\dagger\phi_2|^2\right),
\end{aligned}
$$

where terms in blue are $U(3)$-invariant and λ_3 term selects out $\Sigma(36)$ subgroup.

- At $\lambda_3 = 0$: continuous $SU(3)$ symmetry, spontaneously broken to $SU(2) \times U(1)$, 4 NG bosons and residual mass degeneracy;
- non-zero λ_3 gives masses to scalars keeping some degeneracy $\to$ very rigid spectrum and couplings.

Σ(36) 3HDM

Minimization [Ivanov, Nishi, 1410.6139]: up to cyclic permutations, the minimum can only be

$$A: \ (\omega, 1, 1), \quad A': \ (\omega^2, 1, 1), \quad B: \ (1, 0, 0),$$

and

$$C: \ (1, 1, 1), \quad (1, \omega, \omega^2), \quad (1, \omega^2, \omega).$$

- These points do not depend on λ's;
- In each case, there are 6 degenerate minima: $A + A'$ or $B + C$.
- no CP-violation is possible due to the $\mathbb{Z}_4$ subgroup.

Σ(36) 3HDM

There exists no detailed phenomenological study of Σ(36) 3HDM.

- Exact Σ(36) symmetry → exact alignment without decoupling, mass-degenerate Higgses and other relations.
- Extending to quark sector: the exactly Σ(36)-symmetric will produce unphysical quark masses or V_{CKM} → what about softly broken?
- Soft breaking will lead to:
  - splitting in Higgs spectra,
  - departure from the exact scalar alignment,
  - 6 degenerate minima will split → additional phase transitions during thermal evolution of the Universe.
  - all these features are correlated!

CP4 3HDM

Freedom of defining CP

In QFT, CP is not uniquely defined *a priori*.

- phase factors $\phi(\vec{r},t) \xrightarrow{CP} e^{i\alpha}\phi^*(-\vec{r},t)$ [Feinberg, Weinberg, 1959],
- with N scalar fields ϕ_i, the general CP transformation is

$$J: \quad \phi_i \xrightarrow{CP} X_{ij}\phi_j^*\,, \quad X \in U(N)\,.$$

  If $\mathcal{L}$ is invariant under such J with whatever fancy X, it is explicitly CP-conserving [Grimus, Rebelo, 1997; Branco, Lavoura, Silva, 1999].

- NB: The "standard" convention $\phi_i \xrightarrow{CP} \phi_i^*$ is basis-dependent!

Freedom of defining CP

$$J: \quad \phi_i \xrightarrow{CP} X_{ij}\phi_j^*\,, \quad X \in U(N)\,,$$

Applying J twice leads to family transformation $J^2 = XX^*$ which may be non-trivial. It may happen than only $J^k = \mathbb{I}$ (k = power of 2).

CP-symmetry does not have to be of order 2

The usual CP = CP2, the first non-trivial is CP4, then CP8, CP16, etc.

Models with higher-order GCP were known in 2HDM [Ferreira, Haber, Maniatis, Nachtmann, Silva, 2011] but they always led to accidental symmetries including the usual CP.

The question

What is the minimal multi-Higgs-doublet model

realizing CP4 without accidental symmetries?

CP4 3HDM

The answer was given in [Ivanov, Keus, Vdovin, 2012; Ivanov, Silva, 2016].

Consider 3HDM with the following potential $V = V_0 + V_1$ (notation: $i \equiv \phi_i$):

$$V_0 = -m_{11}^2(1^\dagger 1) - m_{22}^2(2^\dagger 2 + 3^\dagger 3) + \lambda_1(1^\dagger 1)^2 + \lambda_2\left[(2^\dagger 2)^2 + (3^\dagger 3)^2\right]$$
$$+\lambda_3(1^\dagger 1)(2^\dagger 2 + 3^\dagger 3) + \lambda_3'(2^\dagger 2)(3^\dagger 3) + \lambda_4\left[(1^\dagger 2)(2^\dagger 1) + (1^\dagger 3)(3^\dagger 1)\right] + \lambda_4'(2^\dagger 3)(3^\dagger 2)\,,$$

with all parameters real, and

$$V_1 = \frac{\lambda_6}{2}\left[(2^\dagger 1)^2 - (3^\dagger 1)^2\right] + \lambda_8(2^\dagger 3)^2 + \lambda_9(2^\dagger 3)\left[(2^\dagger 2) - (3^\dagger 3)\right] + h.c.$$

with real λ_6 and complex $\lambda_{8,9}$. It is invariant under CP4 $J:\ \phi_i \xrightarrow{CP} X_{ij}\phi_j^*$ with

$$X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & i \\ 0 & -i & 0 \end{pmatrix}, \quad J^2 = \mathrm{diag}(1, -1, -1)\,, \quad J^4 = \mathbb{I}\,.$$

Different versions of CP4 3HDM

- DM CP4 3HDM: unbroken CP4 with scalar DM candidates, similar to the inert doublet model in 2HDM. We assume that ϕ_2, ϕ_3 don't get vevs $\to$ scalar DM candidates (stabilized by CP4!) with peculiar properties [Ivanov, Silva, 2016; Ivanov, Laletin, 2018].

  Scotogenic model for radiative neutrino masses based on CP4 rather than $\mathbb{Z}_2$ [Ivanov, 1712.02101].
- flavored CP4 3HDM: CP4 is extended to the Yukawa sector and must be spontaneously broken $\to$ patterns in the flavor sector. [Ferreira, Ivanov, Jimenez, Pasechnik, Serodio, 2017]

DM CP4 3HDM

CP4-conserving minimum: $v_i = (v, 0, 0)$. Expand the doublets as

$$\phi_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + h_{SM} + iG^0) \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} H_2^+ \\ \frac{1}{\sqrt{2}}(H + ia) \end{pmatrix}, \quad \phi_3 = \begin{pmatrix} H_3^+ \\ \frac{1}{\sqrt{2}}(h + iA) \end{pmatrix}.$$

These fields are mass eigenstates; h and a are the DM candidates.

But they are not CP-eigenstates:

$$H \xrightarrow{CP} A, \quad A \xrightarrow{CP} -H, \quad h \xrightarrow{CP} -a, \quad a \xrightarrow{CP} h.$$

They can be combined into neutral complex CP-eigenstate fields

$$\Phi = \frac{1}{\sqrt{2}}(H - iA), \quad \varphi = \frac{1}{\sqrt{2}}(h + ia), \quad \Phi \xrightarrow{CP} i\Phi, \quad \varphi \xrightarrow{CP} i\varphi.$$

Conserved quantum number: not CP-parity but CP-charge q defined mod 4.

Asymmetric DM regime

Suppose due to some non-thermal process the starting values $n(\varphi) \neq n(\varphi^*) \to$ asymmetric DM. Will this asymmetry survive?

- Unlike in typical asymmetric DM models, $n(\varphi) - n(\varphi^*)$ is not fixed.
- In the φ-dominated situation, there is no direct annihilation: $\varphi\varphi \not\to$ SM.
- But there exists regeneration $\varphi\varphi \to \varphi^*\varphi^*$, so that $\varphi\varphi^* \to$ SM is now possible $\to$ two-stage annihilation.
- We explored the competition between the two processes in [Ivanov, Laletin, 1812.05525].

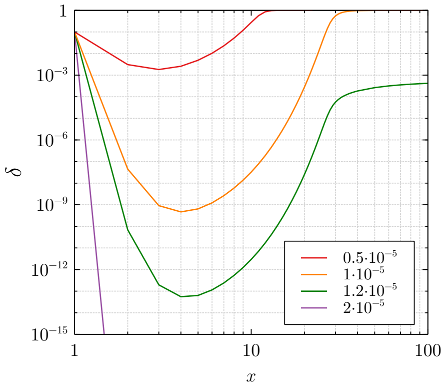
Asymmetric DM regime

The competition between the annihilation $\varphi\varphi \not\to$ SM and conversion $\varphi\varphi \leftrightarrow \varphi^*\varphi^*$ due to

$$\frac{\lambda_{\rm conv}}{4!}(\varphi^4 + (\varphi^*)^4)$$

affects the thermal evolution of the asymmetry:

$$\delta = \frac{n_\varphi - n_{\varphi^*}}{n_\varphi + n_{\varphi^*}}\,.$$



Whether the asymmetry surivives or not sharply depends on $\lambda_{\rm conv}$!

If evolution starts with $x = 1$, the boundary is $\lambda_{\rm conv} \sim 10^{-5}$.

CP4-symmetric quark sector

Extending CP4 to the Yukawa sector: $\psi_i \to Y_{ij}\psi_j^{CP}$, where $\psi^{CP} = \gamma^0 C \bar{\psi}^T$.

$$\bar{Q}_L \Gamma_a d_R \phi_a + \bar{Q}_L \Delta_a u_R \tilde{\phi}_a + h.c.$$

is invariant under CP4 with known X_{ab} if

$$(Y^L)^\dagger \Gamma_a Y^d X_{ab} = \Gamma_b^* \,, \quad (Y^L)^\dagger \Delta_a Y^u X_{ab}^* = \Delta_b^* \,.$$

We solved these equations: cases A, B_1, B_2, B_3.

CP4-symmetric quark sector

case B_1

$$\Gamma_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ g_{31} & g_{31}^* & g_{33} \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ 0 & 0 & 0 \end{pmatrix}, \quad \Gamma_3 = \begin{pmatrix} -g_{22}^* & -g_{21}^* & -g_{23}^* \\ g_{12}^* & g_{11}^* & g_{13}^* \\ 0 & 0 & 0 \end{pmatrix}.$$

case B_2

$$\Gamma_1 = \begin{pmatrix} 0 & 0 & g_{13} \\ 0 & 0 & g_{13}^* \\ 0 & 0 & g_{33} \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} g_{11} & g_{12} & 0 \\ g_{21} & g_{22} & 0 \\ g_{31} & g_{32} & 0 \end{pmatrix}, \quad \Gamma_3 = \begin{pmatrix} g_{22}^* & -g_{21}^* & 0 \\ g_{12}^* & -g_{11}^* & 0 \\ g_{32}^* & -g_{31}^* & 0 \end{pmatrix}.$$

case B_3

$$\Gamma_1 = \begin{pmatrix} g_{11} & g_{12} & 0 \\ -g_{12}^* & g_{11}^* & 0 \\ 0 & 0 & g_{33} \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} 0 & 0 & g_{13} \\ 0 & 0 & g_{23} \\ g_{31} & g_{32} & 0 \end{pmatrix}, \quad \Gamma_3 = \begin{pmatrix} 0 & 0 & -g_{23}^* \\ 0 & 0 & g_{13}^* \\ g_{32}^* & -g_{31}^* & 0 \end{pmatrix}.$$

Numerical scan

1. Scalar sector scan:
   - stick to the scalar alignment, take h_{125} to be the lightest scalar, vary 9 free parameters: v_3/v_2, u/v_1, and 7 λ's;
   - simplified checks of boundedness from below and perturbativity;
   - check that S, T, U parameters are within 3σ of expt.
2. Yukawa sector scan
   - fit all quark masses, mixing, and CPV phase (easy);
   - add K and B oscillation parameters $|\epsilon_K|$, Δm_K, Δm_{B_d}, Δm_{B_s} via expressions from [Buras et al, 2013] (tree-level contributions from neutral Higgses only).
3. Results: many good points found, but they lead to rather light $H^\pm$ $\to$ probably ruled out by $t \to H^+ q'$ and $H^+ \to \bar{q}q'$ LHC results.

Conclusions

- 3HDMs offer a richer list of opportunities than 2HDM: flavour, CPV, scalar pheno, astroparticle, cosmology.
- (Approximate) symmetries and their breaking play crucial role.
- Classification of symmetry-related situation and basis-invariant methods now exist $\rightarrow$ it's time to explore 3HDM phenomenology systematically.