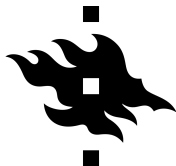


Froggatt-Nielsen mechanism in a model with $SU(3)_c \times SU(3)_L \times U(1)_X$ -gauge symmetry

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- K. Huitu and N. Koivunen, [arXiv:1706.09463 [hep-ph]] (PRD).
- K. Huitu and N. Koivunen, [arXiv:1904.xxxxx [hep-ph]] .

Osaka, 27.3.2019

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 - Number of fermion families
 - Fine-tuning of Yukawa couplings
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The flavour problem

The flavour problem

Number of fermion families

The SM has three generations of fermions

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \quad \begin{pmatrix} c \\ s \end{pmatrix}_L, \quad \begin{pmatrix} t \\ b \end{pmatrix}_L, \quad \begin{pmatrix} t' \\ b' \end{pmatrix}_L \cdots \cdots \\
 \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L, \quad \begin{pmatrix} \nu_4 \\ l_4 \end{pmatrix}_L \cdots \cdots$$

SM does not offer any explanation why there is only three generations. **In principle there could be more.**

⇒ 331-models offer explanation to the number of fermion generations.

SM Yukawa sector

SM has 3 generations of fermions:

Charged leptons: e, μ, τ .

$$\mathcal{L} \supset y_e \bar{L}_{e,L} H e_R + y_\mu \bar{L}_{\mu,L} H \mu_R + y_\tau \bar{L}_{\tau,L} H \tau_R + h.c.$$

$$y_e = 2.94 \times 10^{-6}, \quad y_\mu = 6.07 \times 10^{-4}, \quad y_\tau = 1.02 \times 10^{-2}$$

Up-type quarks: u, c, t .

$$\mathcal{L} \supset y_u \bar{Q}_{u,L} \tilde{H} u_R + y_c \bar{Q}_{c,L} \tilde{H} c_R + y_t \bar{Q}_{t,L} \tilde{H} t_R + h.c.$$

$$y_u = 1.32 \times 10^{-5}, \quad y_c = 7.33 \times 10^{-3}, \quad y_t = 1$$

Down-type quarks: d, s, b .

$$\mathcal{L} \supset y_d \bar{Q}_{d,L} H d_R + y_s \bar{Q}_{s,L} H s_R + y_b \bar{Q}_{b,L} H b_R + h.c.$$

$$y_d = 2.76 \times 10^{-5}, \quad y_s = 5.46 \times 10^{-4}, \quad y_b = 2.4 \times 10^{-2}$$

Fine-tuning in the SM fermion sector

Up-type quarks: u, c, t .

$$\mathcal{L} \supset y_u \bar{Q}_{u,L} \tilde{H} u_R + y_c \bar{Q}_{c,L} \tilde{H} c_R + y_t \bar{Q}_{t,L} \tilde{H} t_R + h.c.$$

$$y_u = 1.32 \times 10^{-5}, \quad y_c = 7.33 \times 10^{-3}, \quad y_t = 1$$

Each generation treated perfectly equally.

⇒ Expect couplings of similar magnitude.

But the couplings span over 5 orders of magnitude!!! ⇒ Fine-tuning!!!

Unacceptable!!! This needs an explanation.

⇒ Froggatt-Nielsen mechanism

FN-mechanism

Froggatt-Nielsen mechanism

(The way to explain fermion mass hierarchy)

Froggatt-Nielsen mechanism

Introduce new $U(1)_{FN}$ flavour symmetry that **forbids SM Yukawa couplings**.

Higgs and SM fermions charged under it.

Also introduce heavy **messenger fermions** χ_a and new complex scalar **Flavon** ϕ . These also charged under $U(1)_{FN}$.

So no SM Yukawa couplings!

Instead $U(1)_{FN}$ -invariant Yukawa-like couplings:

$$c_{ia} \bar{f}_i \chi_a \frac{h}{\sqrt{2}}, \quad b_{ia} \bar{f}_i \chi_a \frac{\phi}{\sqrt{2}}$$

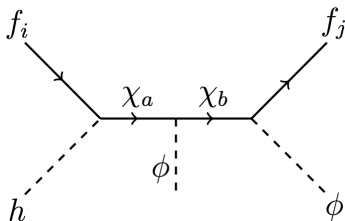
Couplings c_{ia} and b_{ia} are order-one numbers ($|c| \in [0.1, 9]$)

⇒ **No fine-tuning!**

Froggatt-Nielsen mechanism

But how to generate the SM Yukawa-couplings $\bar{f}_{L,i} f_{R,j} h$?

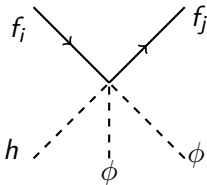
⇒ One can draw diagrams of the following type (one can add more virtual χ s and real ϕ s)



There are extra ϕ s and χ s. How to get rid of them?

⇒ Integrate out heavy FN-messengers χ .

Froggatt-Nielsen mechanism



This corresponds to the following effective operator:

$$c_{ij}^f \left(\frac{\phi}{\sqrt{2}\Lambda} \right)^{n_{ij}} \bar{f}_{L,i} f_{R,j} \frac{h}{\sqrt{2}}, \quad q_{\bar{L},i} + q_{R,j} + q_h + q_\phi n_{ij} = 0$$

$\Lambda =$ mass scale of χ s, $c_{ij}^f =$ order-one number.

Particle	$\bar{f}_{L,i}$	$f_{R,i}$	h	ϕ
U(1) charge	$q_{\bar{L},i}$	$q_{R,i}$	q_h	q_ϕ

Froggatt-Nielsen mechanism

Particle	$\bar{f}_{L,i}$	$f_{R,i}$	h	ϕ
$U(1)$ charge	$q_{\bar{L},i}$	$q_{R,i}$	q_h	q_ϕ

The SM Yukawa couplings are generated as effective couplings, when the flavon gets VEV:

$$\mathcal{L}_{Yukawa} \supset \underbrace{c_{ij}^f \left(\frac{v_\phi}{\sqrt{2}\Lambda} \right)^{-\frac{1}{q_\phi}(q_{\bar{L},i} + q_{R,j} + q_h)}}_{Y_{ij}^f} \bar{f}_{L,i} f_{R,j} \frac{h}{\sqrt{2}}$$

$$\epsilon = \frac{v_\phi}{\sqrt{2}\Lambda} < 1$$

Fermion mass hierarchy is generated naturally!

Froggatt-Nielsen mechanism

Fermionic part of the Lagrangian

$$\begin{aligned} \mathcal{L}_{eff.} &\supset c_{ij}^f \left(\frac{\phi + v_\phi}{\sqrt{2}\Lambda} \right)^{n_{ij}} \bar{f}_{L,i} f_{R,j} \frac{h + v_h}{\sqrt{2}} \\ &= \frac{v_h}{\sqrt{2}} \bar{f}'_{L,i} Y_{ij} f'_{R,j} \left(1 + \frac{h}{v_h} \right) + \bar{f}'_{L,i} \left(\frac{v_h}{v_\phi} n_{ij} Y_{ij} \right) f'_{R,j} \phi + \dots \end{aligned}$$

Diagonalize fermion mass matrix: $Y_{diag} = U_L Y_{ij} U_R^\dagger$
 $\Rightarrow \kappa_{ij} = U_L v_h n_{ij} Y_{ij} U_R^\dagger$ will not be diagonal.

$$\mathcal{L}_{eff.} \supset \bar{f}_L M_{diag} f_R + \bar{f}_L \frac{Y_{diag}}{\sqrt{2}} f_R h + \frac{\kappa_{ij}}{v_\phi} \bar{f}_{L,i} f_{R,j} \phi + h.c.$$

Flavon coupling, κ_{ij} , violates flavour!

Higgs-flavon mixing

The scalar potential

$$V(H, \Phi) = -\mu_h^2(H^\dagger H) + \lambda_h(H^\dagger H)^2 - \mu_\phi^2(\Phi^\dagger \Phi) + \lambda_\phi(\Phi^\dagger \Phi)^2 + \lambda_{h\phi}(H^\dagger H)(\Phi^\dagger \Phi)$$

with

$$H = \begin{pmatrix} 0 \\ \frac{v_h + h}{\sqrt{2}} \end{pmatrix}, \quad \Phi = \begin{pmatrix} v_\phi + \phi \\ \sqrt{2} \end{pmatrix}$$

The physical states

$$\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} \equiv \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} h \\ \phi \end{pmatrix}$$

Physical Higgs H_1 acquires CLFV coupling through mixing with the flavon!

New Yukawa couplings

The physical states

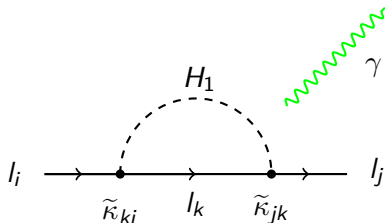
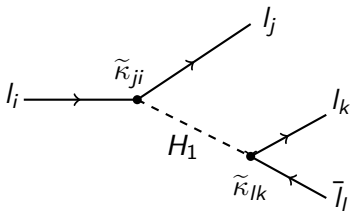
$$\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} \equiv \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} h \\ \phi \end{pmatrix}$$

both contribute to CLFV terms

$$\begin{aligned} \mathcal{L} \supset & \left[\cos \theta \frac{y_{ij}^e}{\sqrt{2}} + \sin \theta \frac{\kappa_{ij}}{v_\phi} \right] \bar{l}_i P_R l_j H_1 + \left[\cos \theta \frac{y_{ij}^e}{\sqrt{2}} + \sin \theta \frac{\kappa_{ji}^*}{v_\phi} \right] \bar{l}_i P_L l_j H_1 \\ & + \left[\cos \theta \frac{\kappa_{ij}}{v_\phi} - \sin \theta \frac{y_{ij}^e}{\sqrt{2}} \right] \bar{l}_i P_R l_j H_2 + \left[\cos \theta \frac{\kappa_{ji}^*}{v_\phi} - \sin \theta \frac{y_{ij}^e}{\sqrt{2}} \right] \bar{l}_i P_L l_j H_2 \end{aligned}$$

Higgs mediated flavour violation

SM Higgs can mediate dangerous lepton flavour violating processes like:
 $\mu \rightarrow e\gamma$ and $\mu \rightarrow eee$



331-model

331-models

(The way to explain number of fermion families)

331-gauge extension

331-models replace the SM gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ with $SU(3)_c \times SU(3)_L \times U(1)_X$.

⇒ 5 more generators = 5 more gauge bosons.

⇒ Scalar sector has to be extended in order to give masses to the new gauge bosons.

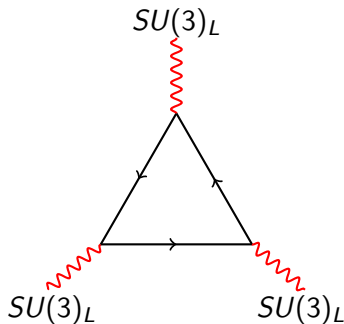
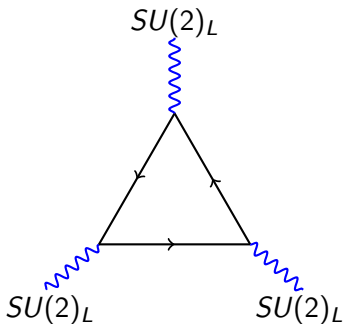
Fundamental representation is now a $SU(3)_L$ -triplet for left-handed fermions. ⇒ Fermion sector needs to be extended

$$Q_{L,i} = \begin{pmatrix} u_i \\ d_i \\ q_{\text{exotic}} \end{pmatrix}_L, \quad L_{L,i} = \begin{pmatrix} \nu_i \\ e_i \\ l_{\text{exotic}} \end{pmatrix}_L.$$

Anomaly cancellation in 331-model

$$\text{Standard model} = SU(3)_C \times SU(2)_L \times U(1)_Y$$

$$\text{331-model} = SU(3)_C \times SU(3)_L \times U(1)_X$$



Pure $SU(3)_L$ anomaly cancels only if number of fermion triplets equals the number of antitriplets.

Number of families = 3

Assume that extra fermions are $SU(2)_L$ -singlets:

$$f_L = \begin{pmatrix} \text{SM-fermion} \\ \text{SM-fermion} \\ \text{New fermion} \end{pmatrix}_L.$$

⇒ The number of families is integer multiple of 3 (number of colors):
number of families = 3, 6, 9, ...

Further assume that QCD is confining.

⇒ number of families is 3.

Flavour changing neutral currents

The cancellation of anomalies predicts the number of fermion families.
This is good.

In order to cancel the anomalies, one quark family has to be in a different representation than the other two.

⇒ **Scalar mediated flavour changing neutral currents (FCNC) at tree-level!**
(and no natural suppression mechanism!!!)

Traditional 331-models are plagued by this in general (maybe this can be cured by Froggatt-Nielsen mechanism...).

331-model

The $SU(3)_L$ -gauge group has one additional diagonal generator compared to the $SU(2)_L$:

⇒ Freedom in electric charge definition: $Q = T_3 + \beta T_8 + X$ (compare to SM: $Q = T_3 + Y/2$).

Two types of models:

- $\beta = \pm\sqrt{3}$:
F. Pisano and V. Pleitez, Phys. Rev. D **46**, 410 (1992).
P. H. Frampton, Phys. Rev. Lett. **69**, 2889 (1992).
- $\beta = \pm\frac{1}{\sqrt{3}}$:
M. Singer, J. W. F. Valle and J. Schechter, Phys. Rev. D **22**, 738 (1980).

$$\beta = \begin{pmatrix} + \\ - \end{pmatrix} \frac{1}{\sqrt{3}}$$

M. Singer, J. W. F. Valle and J. Schechter, Phys. Rev. D **22**, 738 (1980).

Fermion representations:

$$\text{Triplets: } L_{L,i} = \begin{pmatrix} \nu_i \\ e_i \\ N_i \end{pmatrix}_L \sim (1, 3, -\frac{1}{3}), \quad Q_{L,1} = \begin{pmatrix} u \\ d \\ U \end{pmatrix}_L \sim (3, 3, \frac{1}{3})$$

$$\text{Antitriplets: } Q_{L,2} = \begin{pmatrix} s \\ c \\ D_1 \end{pmatrix}_L \sim (3, 3^*, 0), \quad Q_{L,3} = \begin{pmatrix} b \\ t \\ D_2 \end{pmatrix}_L \sim (3, 3^*, 0)$$

$N_{L,i}$ = new neutrino-like fields, U = new up-type quark, D_1, D_2 = new down-type quarks.

Minimal Scalar Sector:

$$\eta = \begin{pmatrix} \eta^+ \\ \eta^0 \\ \eta'^+ \end{pmatrix} \sim (1, 3, \frac{2}{3}), \quad \rho = \begin{pmatrix} \rho^0 \\ \rho^- \\ \rho'^0 \end{pmatrix}, \quad \chi = \begin{pmatrix} \chi^0 \\ \chi^- \\ \chi'^0 \end{pmatrix} \sim (1, 3, -\frac{1}{3})$$

$$\beta = (+)\sqrt{3}$$

F. Pisano and V. Pleitez, Phys. Rev. D **46**, 410 (1992).

P. H. Frampton, Phys. Rev. Lett. **69**, 2889 (1992).

$$\text{Triplets: } L_{L,i} = \begin{pmatrix} \nu_{L,i} \\ e_{L,i} \\ (e_{R,i})^c \end{pmatrix} \sim (1, 3, 0), \quad Q_{L,1} = \begin{pmatrix} u \\ d \\ q_1^{+5/3} \end{pmatrix} \sim (1, 3, \frac{2}{3}),$$

$$\text{Antitriplets: } Q_{L,2} = \begin{pmatrix} c \\ s \\ q_2^{-4/3} \end{pmatrix}, \quad Q_{L,3} = \begin{pmatrix} t \\ b \\ q_3^{-4/3} \end{pmatrix} \sim (1, 3^*, -\frac{1}{3}).$$

$q_1^{+5/3}$, $q_2^{-4/3}$ and $q_3^{-4/3}$ are new quarks with exotic electric charges!

$\beta = (+)\sqrt{3}$, scalar sector

$$\mathcal{L}_m = \epsilon_{\alpha\beta\gamma} G_{ij} \bar{L}_{L,i}^\alpha (L_{L,j}^\beta)^c \langle \eta^* \rangle^\gamma = \left(\epsilon_{\alpha\beta\gamma} G_{ij} \langle \eta^{0*} \rangle \right) \bar{e}_{L,i} e_{R,j}$$

⇒ Antisymmetric charged lepton mass matrix! Need to add scalar sextet.

Minimal Scalar Sector:

$$\eta = \begin{pmatrix} \eta^0 \\ \eta^- \\ \eta'^+ \end{pmatrix} \sim (1, 3, 0), \quad \rho = \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^{++} \end{pmatrix} \sim (1, 3, 1),$$

$$\chi = \begin{pmatrix} \chi^- \\ \chi^{--} \\ \chi^0 \end{pmatrix} \sim (1, 3, -1), \quad S = \begin{pmatrix} \sigma_1^0 & h_2^- & h_1^+ \\ h_2^- & H_1^{--} & \sigma_2^0 \\ h_1^+ & \sigma_2^0 & H_2^{++} \end{pmatrix} \sim (1, 6, 0).$$

FN-mechanism in 331-model

Let us combine Froggatt-Nielsen mechanism and 331-models

(And do it in the most economical way)

K. Huitu and N. Koivunen, [arXiv:1706.09463 [hep-ph]] (PRD),
K. Huitu and N. Koivunen, [arXiv:1904.xxxxx [hep-ph]] .

The Model

$$Q = T_3 - \frac{1}{\sqrt{3}} T_8 + X$$

Fermion representations:

$$\text{Triplets: } L_{L,i} = \begin{pmatrix} \nu_i \\ e_i \\ N_i \end{pmatrix}_L \sim (1, 3, -\frac{1}{3}), \quad Q_{L,1} = \begin{pmatrix} u \\ d \\ U \end{pmatrix}_L \sim (3, 3, \frac{1}{3})$$

$$\text{Antitriplets: } Q_{L,2} = \begin{pmatrix} s \\ c \\ D_1 \end{pmatrix}_L \sim (3, 3^*, 0), \quad Q_{L,3} = \begin{pmatrix} b \\ t \\ D_2 \end{pmatrix}_L \sim (3, 3^*, 0)$$

$N_{L,i}$ = new neutrino-like fields, U = new up-type quark, D_1, D_2 = new down-type quarks.

- 6 triplets and 6 antitriplets \Rightarrow Gauge anomalies cancel!
- **Generic feature of 331-models:** Quark generations in different representations \Rightarrow Flavour Changing Neutral Currents!

Particle Content

Minimal Scalar Sector:

$$\eta = \begin{pmatrix} \eta^+ \\ \eta^0 \\ \eta'^+ \end{pmatrix} \sim (1, 3, \frac{2}{3}), \quad \rho = \begin{pmatrix} \rho^0 \\ \rho^- \\ \rho'^0 \end{pmatrix}, \quad \chi = \begin{pmatrix} \chi^0 \\ \chi^- \\ \chi'^0 \end{pmatrix} \sim (1, 3, -\frac{1}{3})$$

The most general vacuum is:

$$\langle \eta \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v' \\ 0 \end{pmatrix}, \quad \langle \rho \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 \\ 0 \\ v_2 \end{pmatrix}, \quad \langle \chi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ u \end{pmatrix}$$

The symmetry breaking pattern is:

$$SU(3)_L \times U(1)_X \xrightarrow{u, v_2} SU(2)_L \times U(1)_Y \xrightarrow{v_1, v'} \times U(1)_{em}, \quad (1)$$

where $v_2, u \gtrsim 1\text{TeV}$ and $v_1, v' = \mathcal{O}(100\text{GeV})$

Minimal scalar sector admits Froggatt-Nielsen mechanism!

$$\rho = \begin{pmatrix} \rho^0 + \langle \rho^0 \rangle \\ \rho^- \\ \rho'^0 + \langle \rho'^0 \rangle \end{pmatrix}, \quad \chi = \begin{pmatrix} \chi^0 \\ \chi^- \\ \chi'^0 + \langle \chi'^0 \rangle \end{pmatrix} \sim (1, 3, -\frac{1}{3}).$$

- The combination $\rho^\dagger \chi$ is a gauge singlet.
- It can carry a non-zero $U(1)$ charge.
- $\rho^\dagger \chi$ has non-zero VEV:

$$\langle \rho^\dagger \chi \rangle = \frac{v_2 U}{2} \neq 0$$

The combination $\rho^\dagger \chi$ can play the role of the flavon in the Froggatt-Nielsen mechanism!

331 Froggatt-Nielsen mechanism

Froggatt-Nielsen mechanism works the usual way:

$$(c_s^f)_{ij} \left(\frac{\rho^\dagger \chi}{\Lambda^2} \right)^{(n_f^s)_{ij}} \bar{\psi}_{L,i}^f S f_{R,j} \rightarrow \underbrace{(c_s^f)_{ij} \left(\frac{v_2 U}{2\Lambda^2} \right)}_{(y_s^f)_{ij}} \overbrace{\quad}^{\epsilon} (n_f^s)_{ij} \bar{\psi}_{L,i}^f (S + \langle S \rangle) f_{R,j} + \dots$$

The minimal scalar content of the 331-models with $\beta = \pm \frac{1}{\sqrt{3}}$ is enough for Froggatt-Nielsen mechanism. **No new scalars required!**

Models with $\beta = \pm \frac{1}{\sqrt{3}}$ thus explain both the number of fermion families and their hierarchy simultaneously!

Up-type quark mass matrix

$$m_u = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 y_{11}^u & v_1 y_{12}^u & v_1 y_{13}^u & v_1 y_{14}^u \\ -v' y_{21}^u & -v' y_{22}^u & -v' y_{23}^u & -v' y_{24}^u \\ -v' y_{31}^u & -v' y_{32}^u & -v' y_{33}^u & -v' y_{34}^u \\ v_2 y_{11}^u + u y_{11}^u & v_2 y_{12}^u + u y_{12}^u & v_2 y_{13}^u + u y_{13}^u & v_2 y_{14}^u + u y_{14}^u \end{pmatrix}$$

Two sources of hierarchy:

- Usual Froggatt-Nielsen hierarchy:

$$y_{ij} = c_{ij} \left(\frac{v_2 u}{2\Lambda^2} \right)^{n_{ij}} = c_{ij} \epsilon^{n_{ij}}, \quad c_{ij} = \mathcal{O}(1) \quad \text{and} \quad \epsilon < 1$$

- VEVs of two scales: $v_2, u \gtrsim 1\text{TeV}$ and $v_1, v' = \mathcal{O}(100\text{GeV})$

Higgs mediated FCNCs

Higgs mediated Flavour Changing Neutral Currents

(And how to suppress them)

Higgs mediated FCNCs at tree-level

Up-type quark Yukawa couplings:

$$\mathcal{L}_{up} = \sum_{\gamma=1}^4 (y_{\rho}^u)_{1\gamma} \bar{Q}'_{L,1} \rho u'_{R,\gamma} + \sum_{\gamma=1}^4 (y_{\chi}^u)_{1\gamma} \bar{Q}'_{L,1} \chi u'_{R,\gamma} + \sum_{\alpha=2}^3 \sum_{\gamma=1}^4 (y_{\eta^*}^u)_{\alpha\gamma} \bar{Q}'_{L,\alpha} \eta^* u'_{R,\gamma}$$

Down-type quark Yukawa couplings:

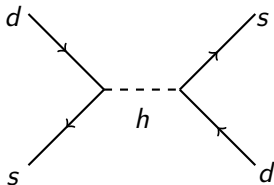
$$\begin{aligned} \mathcal{L}_{down} &= \sum_{\gamma=1}^5 (y_{\eta}^d)_{1\gamma} \bar{Q}'_{L,1} \eta d'_{R,\gamma} \\ &+ \sum_{\alpha=2}^3 \sum_{\gamma=1}^5 (y_{\rho^*}^d)_{\alpha\gamma} \bar{Q}'_{L,\alpha} \rho^* d'_{R,\gamma} + \sum_{\alpha=2}^3 \sum_{\gamma=1}^5 (y_{\chi^*}^d)_{\alpha\gamma} \bar{Q}'_{L,\alpha} \chi^* d'_{R,\gamma}. \end{aligned}$$

Both up- and down-type quarks couple to three different scalar triplets.

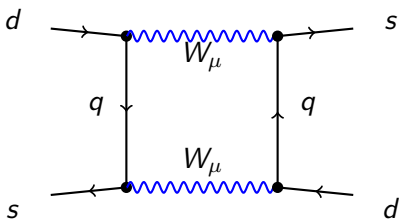
⇒ **FCNCs!**

Neutral meson mixing at tree-level

Higgs mediates neutral meson mixing at tree-level in 331-models.
Neutral Kaon mixing ($K^0 = d\bar{s}$):



Compare to SM:



Up-type quark couplings to Higgs

Higgs is the linear combination of real parts of the neutral fields η^0 , ρ^0 , ρ'^0 , χ^0 and χ'^0 .

$$\mathcal{L}_{Yukawa} = \bar{u}'_L \frac{\Gamma'^u}{\sqrt{2}} u'_R h = \bar{u}_L \underbrace{(U_L^u) \frac{\Gamma^u}{\sqrt{2}} (U_R^{d\dagger})}_{\Gamma_h^u / \sqrt{2}} u_R h$$

The physical Up-type Higgs-Yukawa coupling can be written as:

$$(\Gamma_h^u)_{ij} = \sqrt{2} \frac{m_j}{v_{SM}} \left[\delta_{ij} + \alpha_1 (U_L^u)_{i1} (U_L^{u\dagger})_{1j} - (U_L^u)_{i4} (U_L^{u\dagger})_{4j} \right. \\ \left. + \alpha_2 (U_L^u)_{i1} (U_L^{u\dagger})_{4j} + \alpha_3 (U_L^u)_{i4} (U_L^{u\dagger})_{1j} \right],$$

where

$$\alpha_j = \frac{SU(2)_L\text{-breaking scale}}{SU(3)_L\text{-breaking scale}} \ll 1 \Rightarrow \text{Suppression!}$$

Additional suppression from U_L^u

$$(\Gamma_h^u)_{ij} = \sqrt{2} \frac{m_j}{v_{SM}} \left[\delta_{ij} + \alpha_1 (U_L^u)_{i1} (U_L^{u\dagger})_{1j} - (U_L^u)_{i4} (U_L^{u\dagger})_{4j} \right. \\ \left. + \alpha_2 (U_L^u)_{i1} (U_L^{u\dagger})_{4j} + \alpha_3 (U_L^u)_{i4} (U_L^{u\dagger})_{1j} \right]$$

Example Assume **hierarchy**:

$$m_{i,j}^u \leq m_{i+1,j}^u.$$

FN charges: $q(Q_{L,1}^c) = 3$, $q(Q_{L,2}^c) = 2$, $q(Q_{L,3}^c) = 0$,
expansion parameter: $\epsilon = 0.23$.

$$U_L^u \sim \begin{pmatrix} 1 & \epsilon^1 & \epsilon^3 & \epsilon^{1+L} \\ \epsilon^1 & 1 & \epsilon^2 & \epsilon^L \\ \epsilon^3 & \epsilon^2 & 1 & \epsilon^{L-2} \\ \epsilon^{1+L} & \epsilon^L & \epsilon^{L-2} & 1 \end{pmatrix} \approx 1,$$

where $L = (\log \epsilon)^{-1} \log(v'/v_2) \geq 2$. \Rightarrow **More Suppression!**

The Γ_h^u texture becomes:

$$\Gamma_h^u \sim \begin{pmatrix} y_u & y_c[\alpha\epsilon^1] & y_t[\alpha\epsilon^x] & \epsilon^2 \\ y_u[\alpha\epsilon^1] & y_c & y_t[\alpha\epsilon^x] & \epsilon^2 \\ y_u[\alpha\epsilon^x] & y_c[\alpha\epsilon^x] & y_t & 1 \\ y_u[\alpha] & y_c[\alpha] & y_t[\epsilon^x] & \epsilon^x \end{pmatrix}, \quad \text{where } x \geq 0,$$

$$\alpha = \frac{SU(2)_L\text{-breaking scale}}{SU(3)_L\text{-breaking scale}} \ll 1, \quad \epsilon = 0.23$$

- The off-diagonal elements over the diagonal are largest. They are suppressed by α and ϵ .
- The suppression is enough when $SU(3)_L$ -breaking scale $= v_2, u \gtrsim 4\text{TeV}$

Quark Flavour Constraints

The most stringent bounds on flavour changing couplings comes from neutral meson mixing: $K^0 - \bar{K}^0$, $B_d^0 - \bar{B}_d^0$, $B_s^0 - \bar{B}_s^0$ and $D^0 - \bar{D}^0$.

Process	coupling	Current bound
$K^0 - \bar{K}^0$	$\sqrt{\text{Re}(\Gamma_{ds}^{d\ 2})}$	$([-6.3, 6.1] \times 10^{-2}) \times y_s$
$B^0 - \bar{B}^0$	$ \Gamma_{db}^d $	$< (8.8 \times 10^{-3}) \times y_b$
$B_s^0 - \bar{B}_s^0$	$ \Gamma_{sb}^d $	$< (7.8 \times 10^{-2}) \times y_b$
$D^0 - \bar{D}^0$ (Tree-level)	$ \Gamma_{uc}^u $	$< (1.4 \times 10^{-2}) \times y_c$
$D^0 - \bar{D}^0$ (Loop-level)	$ \Gamma_{ut}^u \Gamma_{ct}^u $	$< (1.5 \times 10^{-2}) \times y_t^2$

Summary

- Our model **economically** combines 331-model and Froggatt-Nielsen mechanism.
- The minimal 331-scalar sector is sufficient in housing Froggatt-Nielsen mechanism. **No additional scalars required!**
- Froggatt-Nielsen mechanism naturally suppresses Higgs mediated FCNCs.