Simplest Little Higgs Revisited:
Hidden Mass Relation, Unitarity and Naturalness

Chen Zhang (NCTS)

@Osaka University

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References

1. Shi-Ping He, Ying-nan Mao, <u>CZ</u> & Shou-hua Zhu

   ArXiv: 1709.08929

   Journal: PRD 97(2018)075005

   Title: ZHη vertex in the simplest Little Higgs model

This talk

2. **Kingman Cheung, Shi-Ping He, Ying-nan Mao, <u>CZ</u> & Yang Zhou**

   ArXiv: 1801.10066

   Title: Simplest Little Higgs Revisited: Hidden Mass Relation, Unitarity and Naturalness

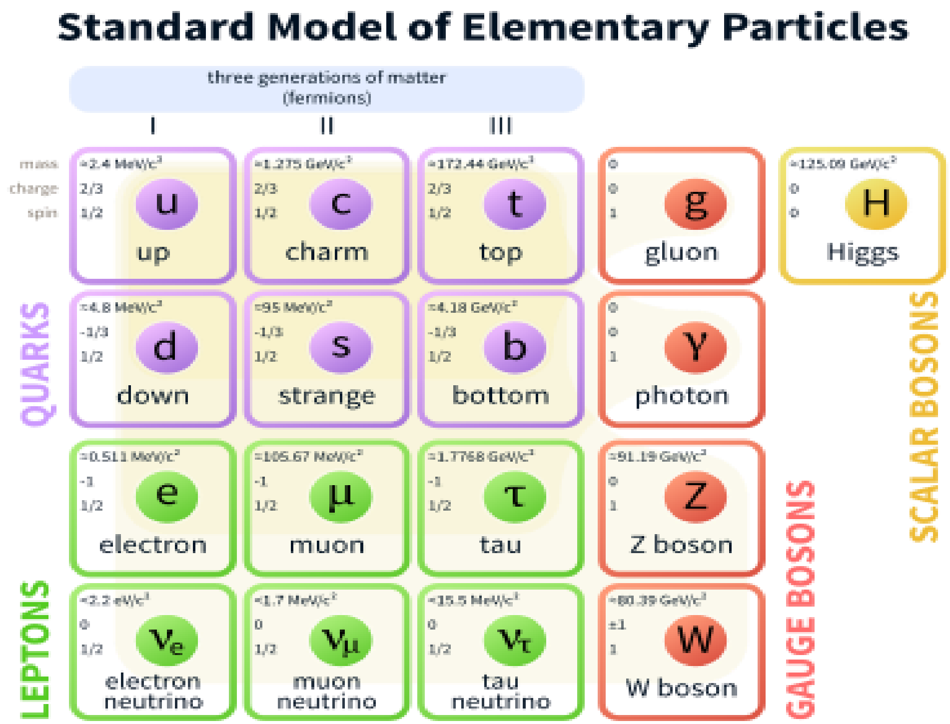
3. Kingman Cheung, Shi-Ping He, Ying-nan Mao, Po-Yan Tseng, <u>CZ</u> & Yang Zhou

   Work in progress on η phenomenology

Outline

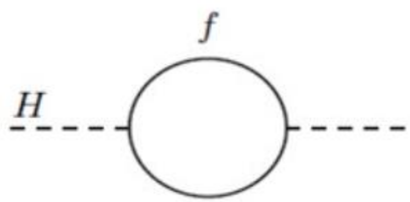
1. Naturalness and the Little Higgs
2. Approaches for EWSB predictions
3. Introduction to the Simplest Little Higgs (SLH)
4. Hidden Mass Relation from Scalar Potential Analysis
5. Naturalness in the SLH
6. Concluding remarks

Naturalness and the Little Higgs



The SM particle content is complete with the discovery of the 125 GeV Higgs boson, which however, could exacerbate the concern of the infamous naturalness/fine-tuning/hierarchy problem.

Naturalness and the Little Higgs



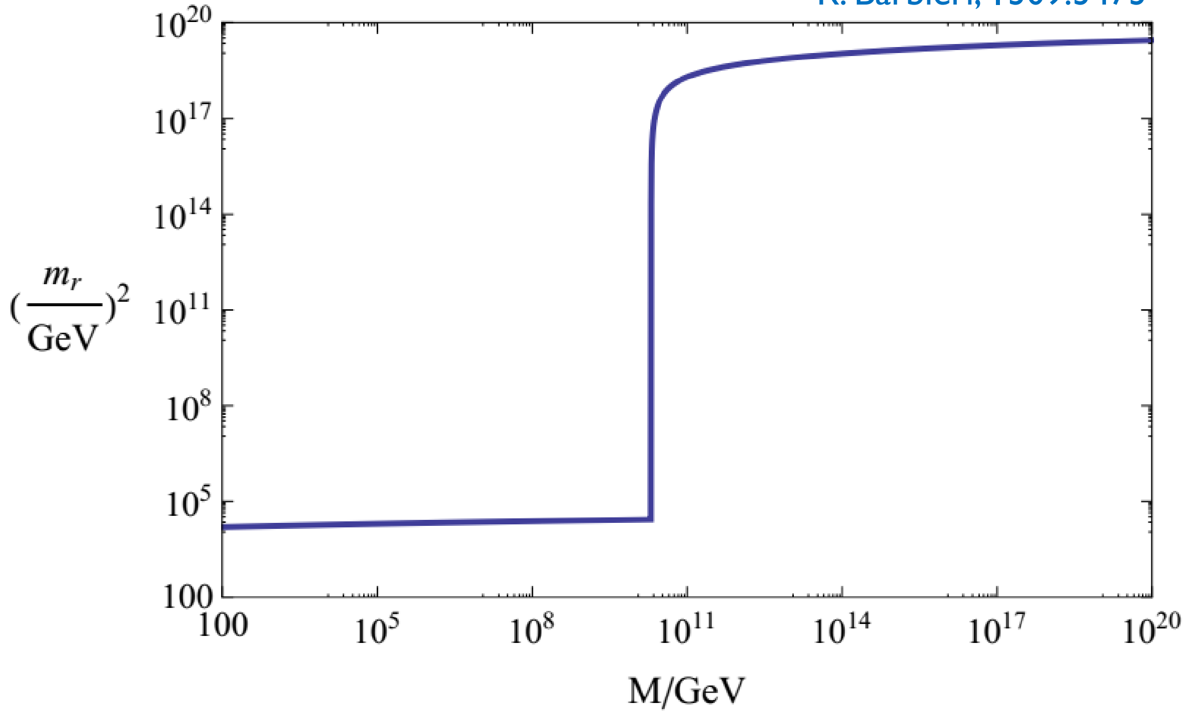
$$\Delta m_H^2 = -\frac{|\lambda_f|^2}{8\pi^2}\Lambda^2 + \ldots$$

?

This infamous problem has been the major driving force of the development of TeV scale BSM physics for decades (~40 years).

Naturalness and the Little Higgs

R. Barbieri, 1309.3473



Aren't we supposed to talk only of physical renormalized quantities, with all divergences suitably reabsorbed?

Naturalness and the Little Higgs

- The latter formulation is not simply a refinement of the former one. Just consider, what is the origin of the radiative instability? SM itself or new physics?
- It is widely believed that new physics must exist (not necessarily at TeV scale) for plenty of reasons.
- Then the real issue is: how could the SM remains stable against quantum corrections when embedded into a larger theory?

Naturalness and the Little Higgs

- The radiative instability is closely related to the renormalization structure of the theory.
  - Scale could be dangerous.
- Safe way to generate the scale?
  - Dimensional transmutation?
  - Control of the RG structure
  - EFT spirit
- Note: I totally avoid the use of "quadratic divergence".

Naturalness and the Little Higgs

N. Arkani-Hamed et al., JHEP 07(2002)034

- One way to stabilize the SM is the Little Higgs mechanism, based on Collective Symmetry Breaking (CSB): Higgs is realized as a Goldstone boson of some spontaneous global symmetry breaking. The global symmetry is also explicitly broken in a collective manner.

	Coupling 1	Coupling 2	Explicit Breaking	Higgs Mass
Case 1	On	Off	No	No
Case 2	Off	On	No	No
Case 3	On	On	Yes	Yes

- Example of CSB couplings

$$\mathcal{L}_{gk} = (D_\mu \Phi_1)^\dagger (D^\mu \Phi_1) + (D_\mu \Phi_2)^\dagger (D^\mu \Phi_2)$$

$$\mathcal{L}_{tY} = i\lambda_1^t \bar{u}_{R3}^1 \Phi_1^\dagger Q_3 + i\lambda_2^t \bar{u}_{R3}^2 \Phi_2^\dagger Q_3$$

Naturalness and the Little Higgs

- With CSB, more operators are needed to renormalize the Higgs mass.
  - Renormalization of dimensionful quantities => Renormalization of dimensionless quantities
- The global symmetry breaking scale itself is supposed to be natural, e.g. generated via dimensional transmutation.
- The central physics issue in this kind of models should be its EWSB predictions (which is related to naturalness analysis).

Approaches for EWSB predictions

- Two approaches for EWSB predictions in the literature

  1. Assumption of "**N**o large **D**irect **C**ontribution to the scalar potential from the physics at the **C**utoff" (NDCC assumption)

     [Tree level potential assumed to vanish apart from a μ term. One-loop divergence is cut off by some fixed scale, usually the NDA cutoff.]

$$V_{\text{total}} = \left(\mu^2 \frac{f^2}{f_1 f_2} + \delta m^2\right) h^\dagger h + \left(-\frac{1}{12}\frac{\mu^2 f^4}{f_1^3 f_2^3} + \delta\lambda\right)(h^\dagger h)^2 .$$

$$\delta m^2 = \frac{-3}{8\pi^2}\left[\lambda_t^2 m_T^2 \,\text{Log}\left(\frac{\Lambda^2}{m_T^2}\right) - \frac{g^2}{4} m_{W'}^2 \,\text{Log}\left(\frac{\Lambda^2}{m_{W'}^2}\right) - \frac{g^2}{8}(1+t^2) m_{Z'}^2 \,\text{Log}\left(\frac{\Lambda^2}{m_{Z'}^2}\right)\right]$$

$$\delta\lambda = \frac{|\,\delta m^2|}{3}\frac{f^2}{f_1^2 f_2^2} + \frac{3}{16\pi^2}\left[\lambda_t^4 \,\text{Log}\left(\frac{m_T^2}{m_t^2}\right) - \frac{g^4}{8}\,\text{Log}\left(\frac{m_{W'}^2}{m_W^2}\right) - \frac{g^4}{16}(1+t^2)^2 \,\text{Log}\left(\frac{m_{Z'}^2}{m_Z^2}\right)\right].$$

M. Schmaltz, JHEP 08(2004)056

Approaches for EWSB predictions

- Two approaches for EWSB predictions in the literature

    2. Abandon the NDCC assumption and treat all the parameters as free

In the SLH, we take f, t_β, m_T, m_D, m_S, and m_η as new free parameters. Reference [4] shows that the LEP-II data excludes η with a mass below 5–7 GeV [35]. So we scan over the following parameter space:

$$\begin{aligned} &2\ \mathrm{TeV} < f < 6\ \mathrm{TeV}, \qquad 0.5\ \mathrm{TeV} < m_T < 3\ \mathrm{TeV}, \\ &0.5\ \mathrm{TeV} < m_D(m_S) < 3\ \mathrm{TeV}, \qquad 1 < t_\beta < 30, \\ &10\ \mathrm{GeV} < m_\eta < 500\ \mathrm{GeV}. \end{aligned} \tag{24}$$

X-F. Han et al., PRD 87(2013)055004

Approaches for EWSB predictions

- Problems with the first approach: Unclear conceptual foundation
  a) Interpretational ambiguity: w.r.t. the meaning of the cutoff in the literature. (A tendency to obscure the meaning of the cutoff?)
  b) Multiple conceptual problems depending on the interpretation of the cutoff.
  c) The cutoff value to be used is, to a large extent, uncertain.
- Problem with the second approach: Predictivity is basically lost.

Approaches for EWSB predictions

These two approaches are not peculiar to SLH. They are widely adopted for other Little Higgs and Twin Higgs models as well.

Why not use the standard renormalization procedure taught in QFT textbooks and the original Coleman-Weinberg paper?

Approaches for EWSB predictions

H. Georgi, EFT Review (1993)

- Continuum Effective Field Theory (CEFT)

  (Adopted in this work)

  - Regularization cutoff is removed by renormalization, and thus is unphysical
  - Renormalization requires the introduction of an unphysical renormalization scale
  - Requiring physics be independent of the renormalization scale leads to Gell-Mann-Low type RG running

- Wilsonian Effective Field Theory (WEFT)

Introduction to the SLH

- To build a little Higgs model
  - Enlargement of scalar sector is necessary.
  - Enlargement of EW gauge group is necessary.
  - Nonlinearly-realized scalar sector is preferable.
- Simplest Little Higgs (SLH): based on $SU(3)_L \times U(1)_X$ electroweak gauge group and the global symmetry breaking pattern

$$[SU(3)_1 \times U(1)_1] \times [SU(3)_2 \times U(1)_2] \to [SU(2)_1 \times U(1)_1] \times [SU(2)_2 \times U(1)_2]$$

nonlinearly realized through two scalar triplets. M. Schmaltz, JHEP 08(2004)056

Introduction to the SLH

- Parametrization of the scalar triplets

F. del Aguila et al., JHEP 03(2011)080

10-8=2 degrees of freedom will ultimately be physical.

$$\Phi_1 = \exp\left(\frac{i\Theta'}{f}\right)\exp\left(\frac{it_\beta\Theta}{f}\right)\begin{pmatrix}0\\0\\fc_\beta\end{pmatrix} \qquad \Phi_2 = \exp\left(\frac{i\Theta'}{f}\right)\exp\left(-\frac{i\Theta}{ft_\beta}\right)\begin{pmatrix}0\\0\\fs_\beta\end{pmatrix}$$

η: pseudo-axion

$$\Theta = \frac{\eta}{\sqrt{2}} + \begin{pmatrix}\mathbf{0}_{2\times 2} & h\\ h^\dagger & 0\end{pmatrix}, \quad \Theta' = \frac{\zeta}{\sqrt{2}} + \begin{pmatrix}\mathbf{0}_{2\times 2} & k\\ k^\dagger & 0\end{pmatrix}$$

$$h = \begin{pmatrix}h^0\\h^-\end{pmatrix}, \quad h^0 = \frac{1}{\sqrt{2}}(v + H - i\chi) \qquad \hat{h} \equiv (h^\dagger h)^{1/2}$$

$$k = \begin{pmatrix}k^0\\k^-\end{pmatrix}, \quad k^0 = \frac{1}{\sqrt{2}}(\sigma - i\omega)$$

CP-even scalars: H, σ

CP-odd scalars: η, ζ, χ, ω

$$s_\beta \equiv \sin\beta, c_\beta \equiv \cos\beta, t_\beta \equiv \tan\beta$$

$\boxed{t_\beta \geq 1 \text{ assumed}}$

Introduction to the SLH

- Gauge kinetic terms for the scalar triplets (automatic CSB from gauge symmetry)

$$\mathcal{L}_{gk} = (D_\mu \Phi_1)^\dagger (D^\mu \Phi_1) + (D_\mu \Phi_2)^\dagger (D^\mu \Phi_2)$$

$$D_\mu = \partial_\mu - i g A^a_\mu T^a + i g_x Q_x B^x_\mu, \quad g_x = \frac{g t_W}{\sqrt{1 - t_W^2/3}} \quad Q_x = -\tfrac{1}{3}$$

$$A^a_\mu T^a = \frac{A^3_\mu}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \frac{A^8_\mu}{2\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & W^+_\mu & Y^0_\mu \\ W^-_\mu & 0 & X^-_\mu \\ Y^{0\dagger}_\mu & X^+_\mu & 0 \end{pmatrix}$$

$$Y^0_\mu \equiv \frac{1}{\sqrt{2}}(Y_{R\mu} + i Y_{I\mu}), \quad Y^{0\dagger}_\mu \equiv \frac{1}{\sqrt{2}}(Y_{R\mu} - i Y_{I\mu})$$

First order gauge boson mixing

$$\begin{pmatrix} A^3 \\ A^8 \\ B_x \end{pmatrix} = \begin{pmatrix} 0 & c_W & -s_W \\ \sqrt{1 - \frac{t_W^2}{3}} & \frac{s_W t_W}{\sqrt{3}} & \frac{s_W}{\sqrt{3}} \\ -\frac{t_W}{\sqrt{3}} & s_W \sqrt{1 - \frac{t_W^2}{3}} & c_W \sqrt{1 - \frac{t_W^2}{3}} \end{pmatrix} \begin{pmatrix} Z' \\ Z \\ A \end{pmatrix}$$

Introduction to the SLH

- Fermion field content and Yukawa Lagrangian (anomaly-free embedding)

$$L_m = (\nu_L, \ell_L, iN_L)^T_m \text{ with } Q_x = -\tfrac{1}{3}$$

$$\ell_{Rm} \text{ with } Q_x = -1 \text{ and } N_{Rm} \text{ with } Q_x = 0.$$

$$\begin{aligned} Q_1 &= (d_L, -u_L, iD_L)^T, \quad d_R, \quad u_R, \quad D_R \\ Q_2 &= (s_L, -c_L, iS_L)^T, \quad s_R, \quad c_R, \quad S_R \\ Q_3 &= (t_L, b_L, iT_L)^T, \quad t_R, \quad b_R, \quad T_R \end{aligned}$$

$$\mathcal{L}_{LY} = i\lambda_N^m \bar{N}_{Rm} \Phi_2^\dagger L_m + \frac{i\lambda_\ell^{mn}}{\Lambda} \bar{\ell}_{Rm} \epsilon_{ijk} \Phi_1^i \Phi_2^j L_n^k + \text{h.c.}$$

$$\begin{aligned} \mathcal{L}_{QY} = & \boxed{i\lambda_1^t \bar{u}_{R3}^1 \Phi_1^\dagger Q_3 + i\lambda_2^t \bar{u}_{R3}^2 \Phi_2^\dagger Q_3} + i\frac{\lambda_b^m}{\Lambda} \bar{d}_{Rm} \epsilon_{ijk} \Phi_1^i \Phi_2^j Q_3^k \\ & + i\lambda_1^{dn} \bar{d}_{Rn}^1 Q_n^T \Phi_1 + i\lambda_2^{dn} \bar{d}_{Rn}^2 Q_n^T \Phi_2 + i\frac{\lambda_u^{mn}}{\Lambda} \bar{u}_{Rm} \epsilon_{ijk} \Phi_1^{*i} \Phi_2^{*j} Q_n^k + \text{h.c.} \end{aligned}$$

- CSB violation is suppressed formally to higher dimensional operators and by light fermion Yukawa

Hidden Mass Relation from Scalar Potential Analysis

- We adopt an approach consistent with CEFT. The bare potential includes the quartic term required by renormalization

$$V_B = -\mu_B^2(\Phi_{1B}^\dagger \Phi_{2B} + \Phi_{2B}^\dagger \Phi_{1B}) + \lambda_B |\Phi_{1B}^\dagger \Phi_{2B}|^2$$

- Scalar effective potential is calculated via renormalized perturbation theory (see Peskin & Schroeder, Section 11.4)

$$V_{\text{tree}} = -\mu^2(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1) + \lambda_R |\Phi_1^\dagger \Phi_2|^2$$

- One-loop potential at small field value is expressed as (recall $\hat{h} \equiv (h^\dagger h)^{1/2}$)

$$V_{\text{1-loop}} = V_{\text{1-loop}}^{\text{s}} + V_{\text{1-loop}}^{\text{ns}} \quad V_{\text{1-loop}}^{\text{s}} = \bar{\lambda} |\Phi_1^\dagger \Phi_2|^2 \quad V_{\text{1-loop}}^{\text{ns}} = \Delta(\hat{h})\hat{h}^4$$

Hidden Mass Relation from Scalar Potential Analysis

- Combining tree and one-loop, the scalar effective potential at small field value is given by $\boxed{V = -\mu^2(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1) + \lambda|\Phi_1^\dagger\Phi_2|^2 + \Delta(\hat{h})\hat{h}^4 \quad \lambda \equiv \lambda_R + \bar{\lambda}}$
- Taking into account gauge boson and top sector Yukawa contributions, in Landau gauge and $\overline{\mathrm{MS}}$ renormalization scheme, we have

$$\bar{\lambda} = -\frac{3}{8\pi^2}\left[\lambda_t^2\frac{M_T^2}{f^2}\left(\ln\frac{M_T^2}{\mu_R^2} - 1\right) - \frac{1}{4}g^2\frac{M_X^2}{f^2}\left(\ln\frac{M_X^2}{\mu_R^2} - \frac{1}{3}\right)\right.$$
$$\left. - \frac{1}{8}g^2(1+t_W^2)\frac{M_{Z'}^2}{f^2}\left(\ln\frac{M_{Z'}^2}{\mu_R^2} - \frac{1}{3}\right)\right]$$

$$\Delta(\hat{h}) = \frac{3}{16\pi^2}\left\{\lambda_t^4\left[\ln\frac{M_T^2}{m_t^2(\hat{h})} - \frac{1}{2}\right] - \frac{1}{8}g^4\left[\ln\frac{M_X^2}{m_W^2(\hat{h})} - \frac{1}{2}\right]\right.$$
$$\left. - \frac{1}{16}g^4(1+t_W^2)^2\left[\ln\frac{M_{Z'}^2}{m_Z^2(\hat{h})} - \frac{1}{2}\right]\right\}$$

Parameter Definition

$$\lambda_t \equiv \frac{\lambda_1^t\lambda_2^t}{\sqrt{\lambda_1^{t2}c_\beta^2 + \lambda_2^{t2}s_\beta^2}}$$

$$m_t^2(\hat{h}) = \lambda_t^2\hat{h}^2$$
$$m_W^2(\hat{h}) = \frac{1}{2}g^2\hat{h}^2$$
$$m_Z^2(\hat{h}) = \frac{1}{2}g^2(1+t_W^2)\hat{h}^2$$

$$M_T^2 \equiv (\lambda_1^{t2}c_\beta^2 + \lambda_2^{t2}s_\beta^2)f^2$$
$$M_X^2 \equiv \frac{1}{2}g^2f^2$$
$$M_{Z'}^2 \equiv \frac{2}{3-t_W^2}g^2f^2$$

$$M_T^2 = m_T^2 + m_t^2$$
$$M_X^2 = m_X^2 + m_W^2$$
$$M_{Z'}^2 = m_{Z'}^2 + m_Z^2$$

Hidden Mass Relation from Scalar Potential Analysis

- Counting the parameters: 5→3

$$f, t_\beta, M_T, \mu, \lambda \xrightarrow{\text{vev=246, Higgs mass=125}} f, t_\beta, M_T$$

Hidden relation between pseudo-axion mass and top partner mass

- The hidden mass relation (as will be shown) does not depend on renormalization scale manifestly, since it can be viewed alternatively as a zeroth-order natural relation.

Hidden Mass Relation from Scalar Potential Analysis

- The derivation of hidden mass relation is based on stationary point condition and the associated Hessian matrix. The result is

$$\boxed{m_\eta^2 = [m_h^2 - v^2\Delta_A(3 - 2\theta t_{2\theta}^{-1}) + v^2 A(5 - 2\theta t_{2\theta}^{-1})] s_\theta^{-2}}$$

$$\theta \equiv \frac{v}{\sqrt{2} f s_\beta c_\beta} \qquad A \equiv \frac{3}{16\pi^2}\left[\lambda_t^4 - \frac{g^4}{8} - \frac{g^4}{16}(1+t_W^2)^2\right]$$

$$\Delta_A \equiv \frac{3}{16\pi^2}\left[\lambda_t^4 \ln\frac{M_T^2}{m_t^2} - \frac{g^4}{8}\ln\frac{M_X^2}{m_W^2} - \frac{g^4}{16}(1+t_W^2)^2 \ln\frac{M_{Z'}^2}{m_Z^2}\right]$$

- Basic feature: pseudo-axion mass and top partner mass are anti-correlated.
- The positivity of pseudo-axion mass squared leads to constraint on parameter space.

Hidden Mass Relation from Scalar Potential Analysis

- Lower bound on M_T from $M_T^2 \equiv (\lambda_1^{t2} c_\beta^2 + \lambda_2^{t2} s_\beta^2) f^2$

$$M_T \geq \sqrt{2}\frac{m_t}{v} f s_{2\beta} \approx f s_{2\beta}$$

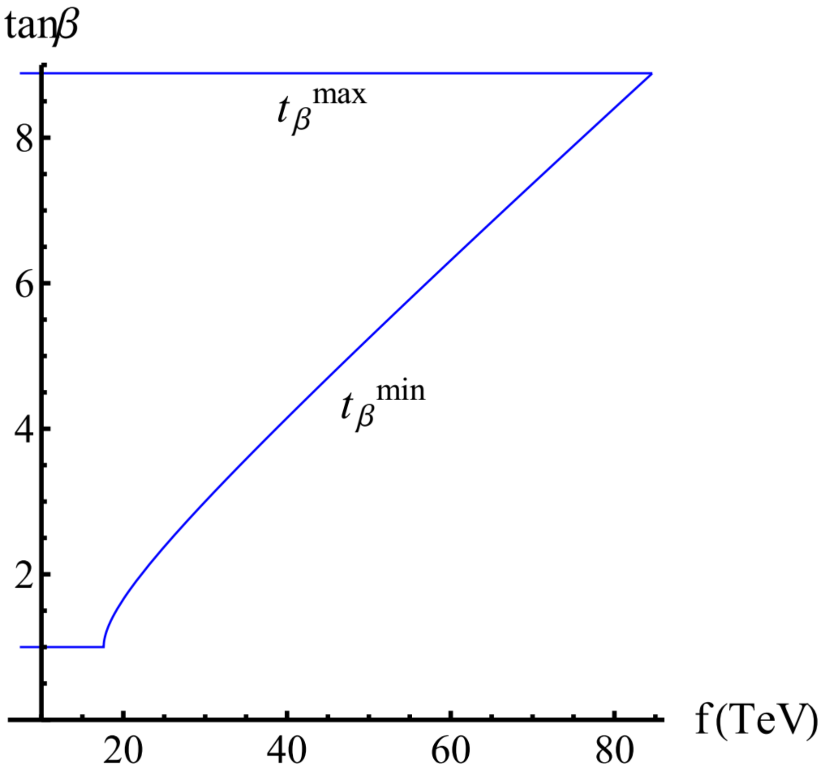
- Constraint from partial wave unitarity: using methods from S. Chang & H-J. He, PLB 586(2004)95. As a result we require

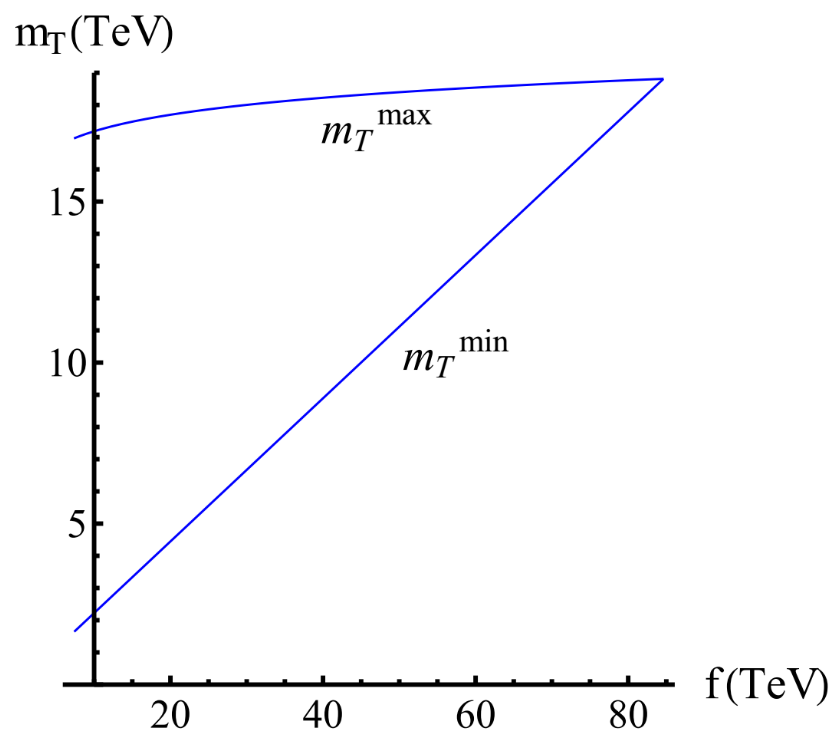
$$M_{Z'} \leq \sqrt{8\pi} f c_\beta \qquad M_T \leq \sqrt{8\pi} f c_\beta$$

- Lower bound on Z' mass from LHC constraint is translated into

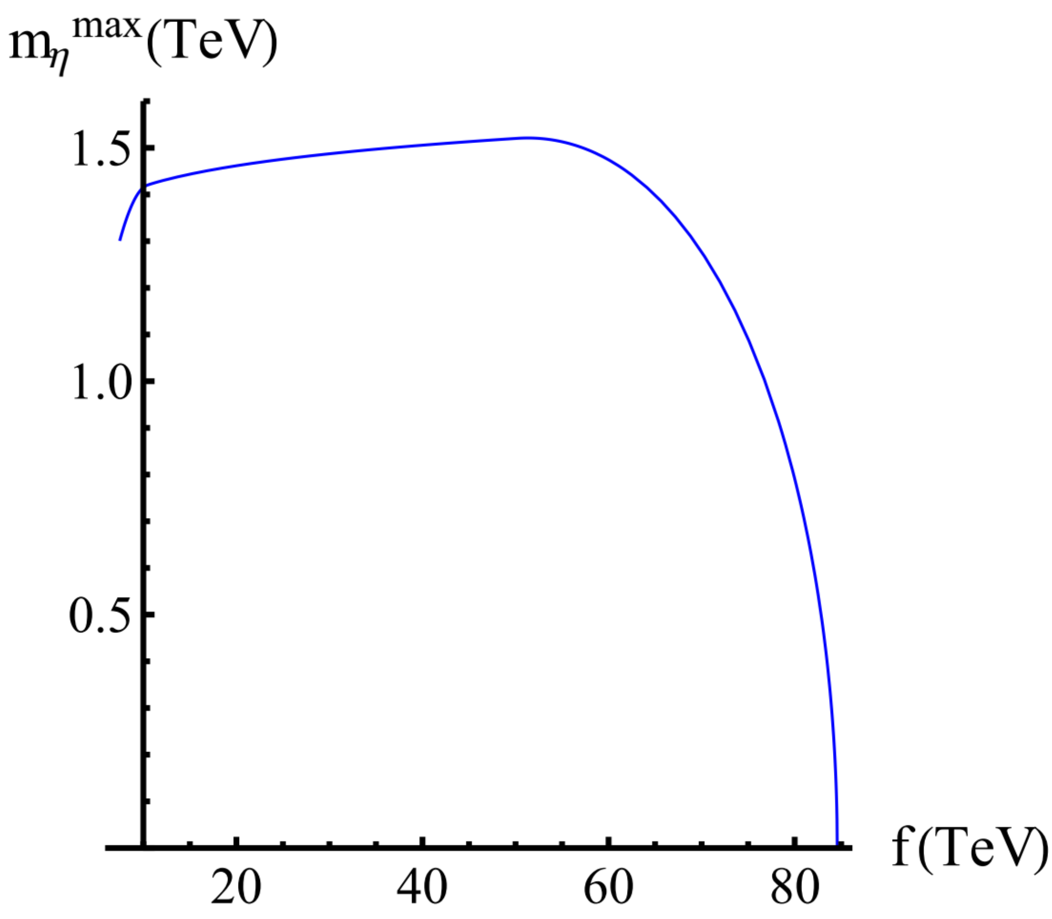
$$f > 7.5 \text{ TeV}$$

Hidden Mass Relation from Scalar Potential Analysis



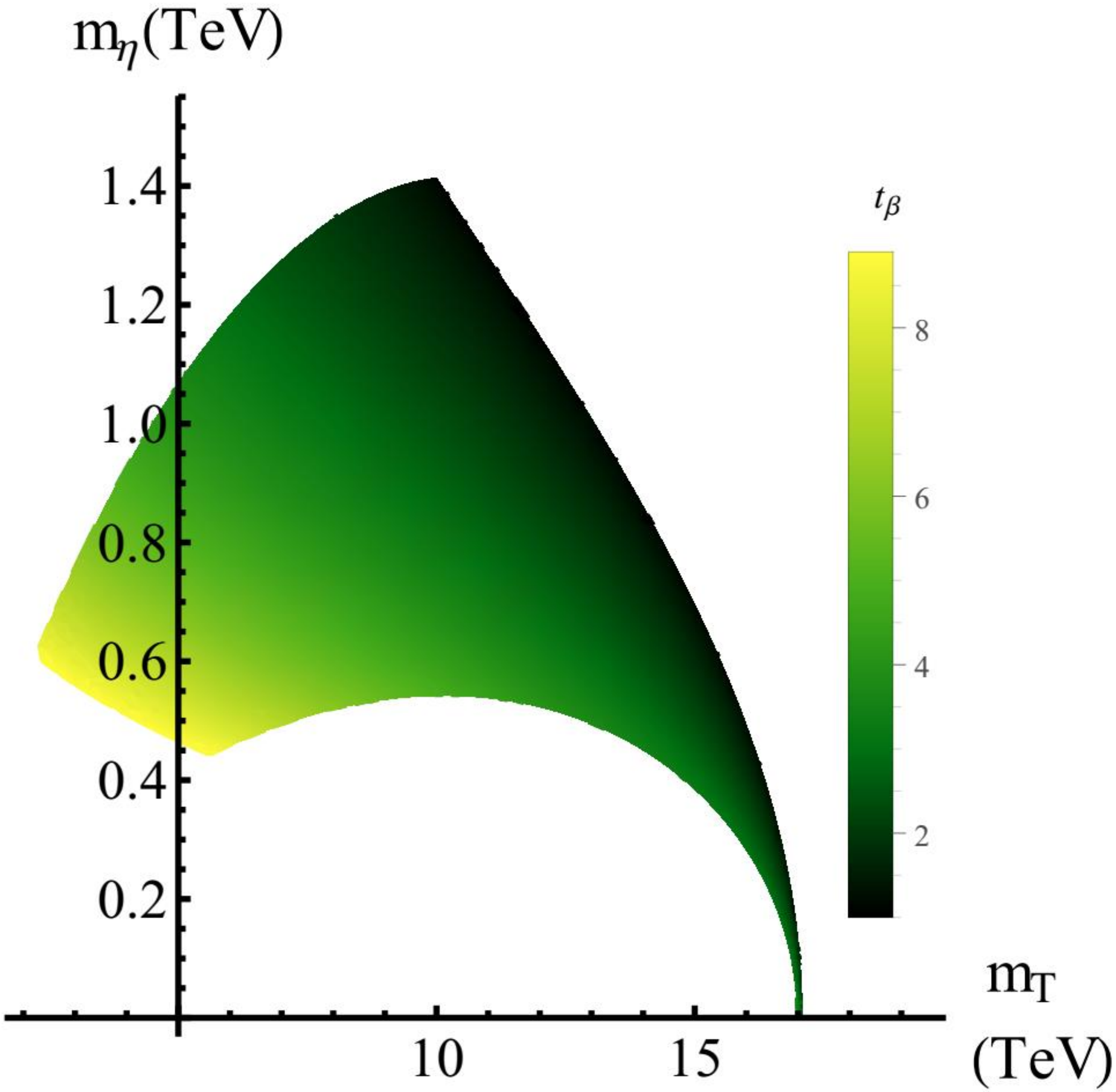


Hidden Mass Relation from Scalar Potential Analysis



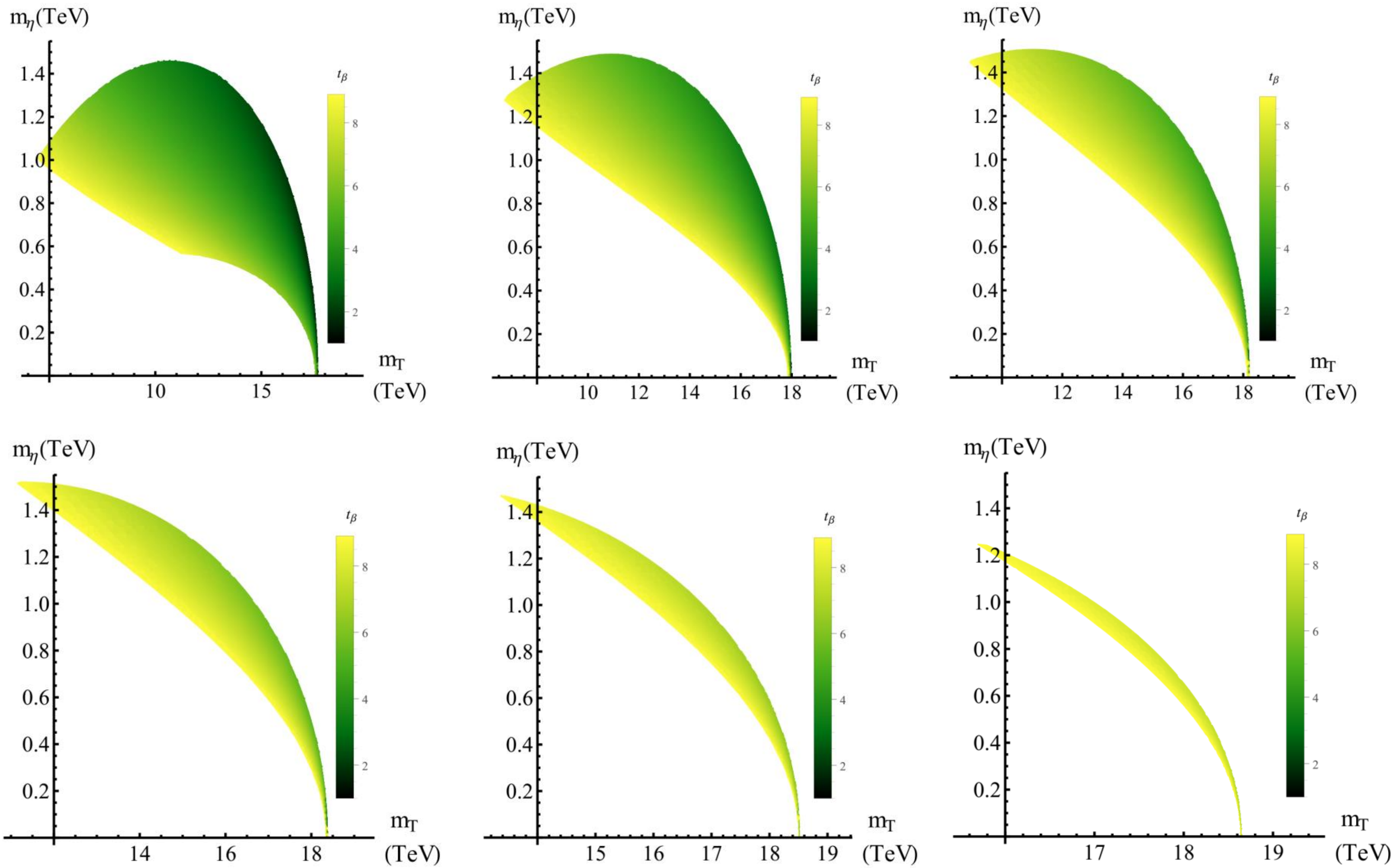
Hidden Mass Relation from Scalar Potential Analysis

f=10TeV



Hidden Mass Relation from Scalar Potential Analysis

f=20, 30, 40, 50, 60, 70TeV



Naturalness in the SLH

- The crucial difference between two fine-tuning formulations can be traced back to whether the cutoff has physical significance as opposed to only acting as a mathematical regulator for the functional integral.
- In the following we opt to consider the cutoff as unphysical and only discuss the physical fine-tuning in the theory.
- CEFT naturally offers a framework for analyzing fine-tuning, i.e. how much IR parameters are sensitive to UV parameters. Two sources of fine-tuning will be identified: threshold tuning & RG tuning.

Naturalness in the SLH

- Recall $\boxed{V = -\mu^2(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1) + \lambda|\Phi_1^\dagger\Phi_2|^2 + \Delta(\hat{h})\hat{h}^4 \quad \lambda \equiv \lambda_R + \bar{\lambda}}$
- The electroweak vev and Higgs mass can be calculated once the following set of parameters are given at the unitarity cutoff

$$f, t_\beta, M_T, \lambda_R, \mu^2$$

- We need to follow the RG flow to

$$\text{a scale } M_L \text{ which satisfies } \bar{\lambda} = 0$$

So that we are able to use the result of scalar potential analysis to obtain electroweak vev and Higgs mass.

Naturalness in the SLH

- Expression of Higgs mass

$$\boxed{m_h^2 = 2\left(\lambda f^2 - \frac{\mu^2}{s_\beta c_\beta}\right) - 2Av^2}$$

$$v^2 = \frac{4\left(\lambda f^2 - \frac{\mu^2}{s_\beta c_\beta}\right)}{\frac{1}{3f^2 s_\beta^2 c_\beta^2}\left(4\lambda f^2 - \frac{\mu^2}{s_\beta c_\beta}\right) + 4\Delta_0 - 2A}$$

$$\Delta_0 = \frac{3}{16\pi^2}\left[\lambda_t^4\left(\ln\frac{M_T^2}{m_t^2} - \frac{1}{2}\right) - \frac{g^4}{8}\left(\ln\frac{M_X^2}{m_W^2} - \frac{1}{2}\right) - \frac{g^4}{16}(1+t_W^2)^2\left(\ln\frac{M_{Z'}^2}{m_Z^2} - \frac{1}{2}\right)\right]$$

- Threshold tuning definition $\Delta_{\mathrm{TH}}^{\lambda} \equiv \left|\frac{\lambda}{m_h^2}\frac{\partial m_h^2}{\partial\lambda}\right|$ $\quad \Delta_{\mathrm{TH}}^{\mu^2} \equiv \left|\frac{\mu^2}{m_h^2}\frac{\partial m_h^2}{\partial\mu^2}\right|$
- Results $\Delta_{\mathrm{TH}}^{\lambda} = 1 + \frac{2m_\eta^2}{m_h^2}$ $\quad \Delta_{\mathrm{TH}}^{\mu^2} = \frac{2m_\eta^2}{m_h^2}$
- Threshold tuning is simply dictated by pseudo-axion mass!

Naturalness in the SLH

- Beta function of λ (neglecting wave function renormalization)

$$\beta_\lambda = -\frac{3\lambda_t^2}{4\pi^2}\frac{M_T^2}{f^2} + \frac{3g^4}{32\pi^2}\frac{5+t_W^2}{3-t_W^2}$$

- IR-UV parameter relation $\lambda = \lambda_U - \beta_\lambda \ln\frac{\Lambda_U}{M_L}$ $\quad \Lambda_U \equiv \sqrt{8\pi} f c_\beta$

- RG tuning of λ is defined as (Note: RG tuning of μ^2 is trivially, one.)

$$\Delta_{\rm RG}^{\lambda} \equiv \left|\frac{\lambda_U}{\lambda}\frac{\partial\lambda}{\partial\lambda_U}\right| = \left|1 + \frac{1}{\lambda}\beta_\lambda \ln\frac{\Lambda_U}{M_L}\right|$$

- Results

$$\Delta_{\rm RG}^{\lambda} = \left|1 - \frac{3}{2\pi^2}\frac{\lambda_t^2 M_T^2 - \frac{g^4 f^2}{8}\frac{5+t_W^2}{3-t_W^2}}{m_h^2 + 2m_\eta^2}\ln\frac{\sqrt{8\pi} f c_\beta}{M_T} - \frac{3}{4\pi^2}\frac{B}{m_h^2+2m_\eta^2}\right|$$

$$B \equiv \lambda_t^2 M_T^2 + \frac{1}{4}g^2 M_X^2\left(\ln\frac{M_X^2}{M_T^2} - \frac{1}{3}\right) + \frac{1}{8}g^2(1+t_W^2)M_{Z'}^2\left(\ln\frac{M_{Z'}^2}{M_T^2} - \frac{1}{3}\right)$$

Naturalness in the SLH

- Total tuning $\Delta^{\lambda}_{\rm TOT} \equiv \Delta^{\lambda}_{\rm TH} \times \Delta^{\lambda}_{\rm RG}$ $\quad \Delta^{\mu^2}_{\rm TOT} \equiv \Delta^{\mu^2}_{\rm TH} \times \Delta^{\mu^2}_{\rm RG} = \Delta^{\mu^2}_{\rm TH}$
- The total tuning for λ

$$\Delta^{\lambda}_{\rm TOT} = \left| 1 + \frac{2m_\eta^2}{m_h^2} - \frac{3}{2\pi^2} \frac{\lambda_t^2 M_T^2 - \frac{g^4 f^2}{8} \frac{5+t_W^2}{3-t_W^2}}{m_h^2} \ln \frac{\sqrt{8\pi} f c_\beta}{M_T} - \frac{3}{4\pi^2} \frac{B}{m_h^2} \right|$$

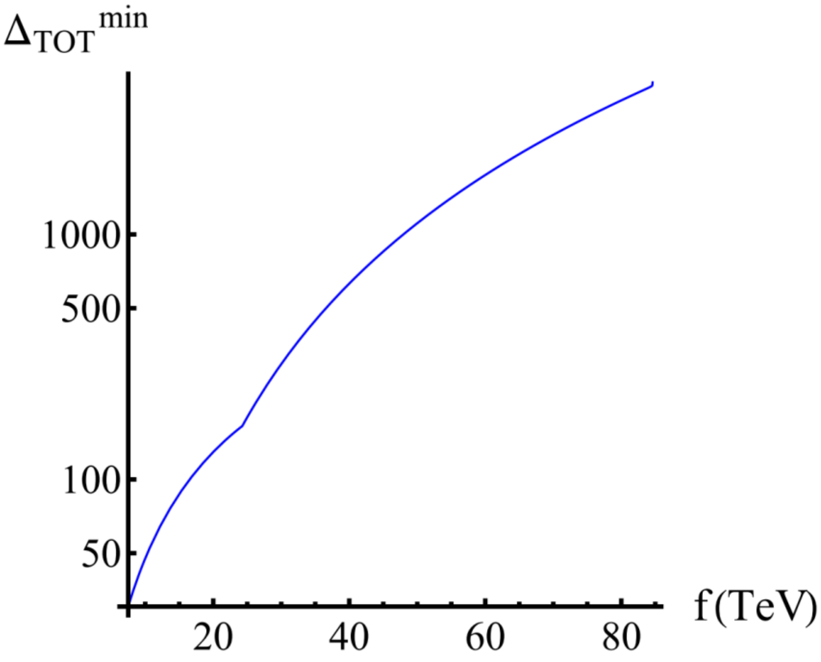
- Overall degree of tuning in the SLH is defined for simplicity as

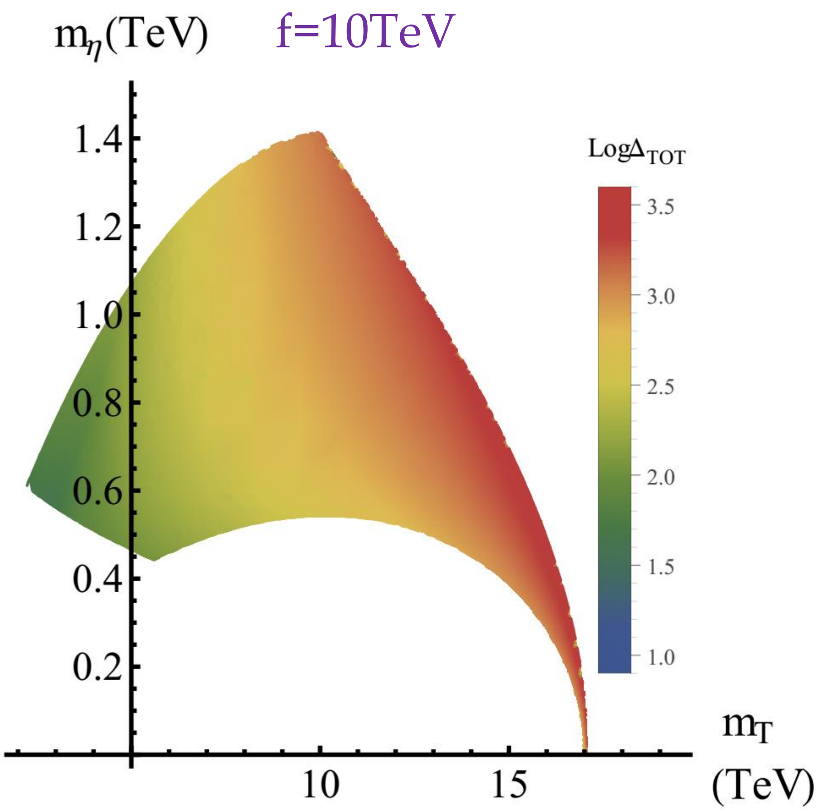
$$\Delta_{\rm TOT} = \max\{\Delta^{\mu^2}_{\rm TOT}, \Delta^{\lambda}_{\rm TOT}\}$$

Naturalness in the SLH

- NDCC assumption, if used properly, corresponds to a hypersurface in CEFT-analysis-allowed parameter space, which has the "good" property of vanishing RG (and total) tuning of λ.
- However, it would be misleading to think parameter points satisfying NDCC assumption as particularly good, mainly for two reasons:
    1. There is still fine-tuning of μ^2 parameter.
    2. The vanishing of RG (and total) tuning of λ is illusory if we consider UV completion.

Naturalness in the SLH





Concluding Remarks

1. What's the virtue of our CEFT approach compared to using a "floating cutoff" in previous analyses?

    (1) CEFT approach is of course, more conceptually solid and clear.

    (2) CEFT approach allows us to identify which predictions are reliable and which are not. This is why conventional cutoff-based approach fails to reveal the hidden mass relation.

2. What's the implication for other models?

    Most perturbative Little Higgs and Twin Higgs models could/should be reanalyzed via the CEFT approach, to make solid and clear predictions (e.g. mass relations) that can be tested experiments and serve as discriminating tests between different EWSB mechanisms.

Concluding Remarks

- **"But I was always irritated at Dirac for complaining that renormalization was sweeping the infinities under the rug. In fact you have to renormalize whether the theory is finite or infinite. It's simply a matter of recognizing what it is that you observe."**

----Steven Weinberg

Thank you!