Quantum entanglement between bubble universes

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Based on

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arXiv:1802.08794 [hep-th]; PRD 97, 083520 (2018)

Inflationary cosmology/String landscape

Inflationary cosmology/String landscape suggest that our universe may not be the only universe but is part of a vast complex of universes that we call the multiverse.

These universes may be highly entangled initially.

Is our bubble universe entangled with some other universe initially?

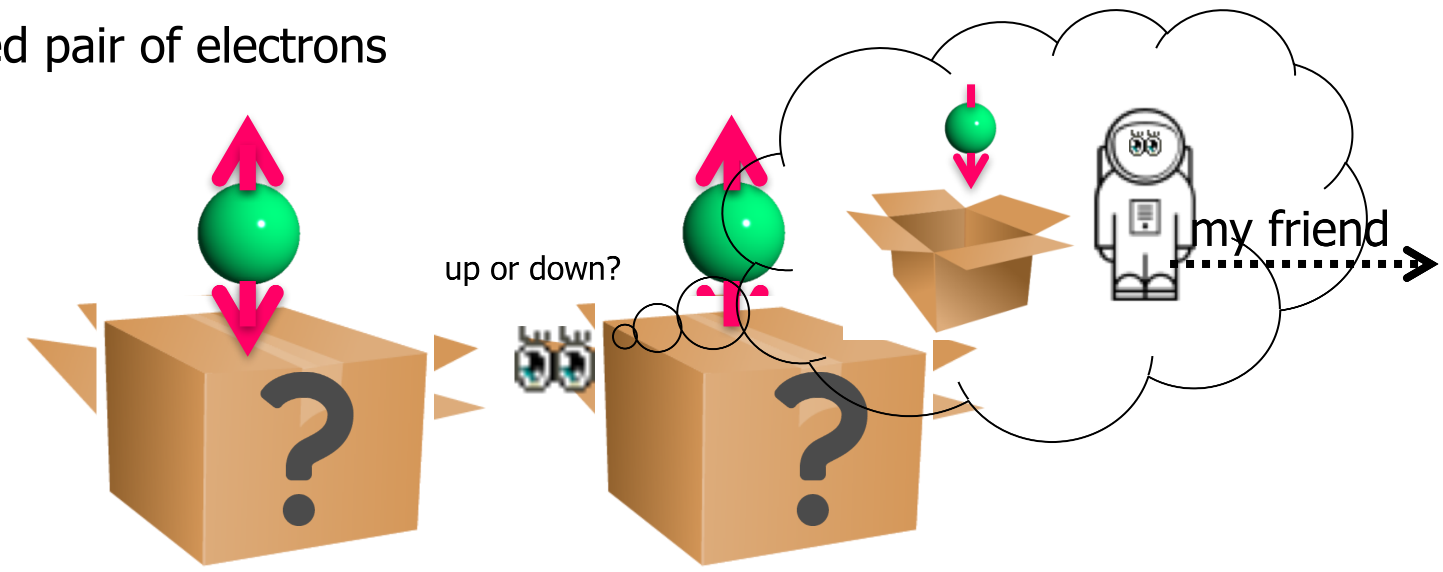
Quantum entanglement?

The most fascinating aspect: *Einstein-Podolsky-Rosen paradox*

To affect the outcome of local measurements instantaneously once a local measurement is performed.

The information travels faster than the speed of light?

Eg) An entangled pair of electrons



The instant I measure it, I immediately *know* which spin my friend out there will find.

But my friend doesn't know it until he measures it by himself.

Quantum entanglement was confirmed by experiments. Aspect et al. (1981)

Causality remains intact.

Entanglement exists in arbitrary large distances

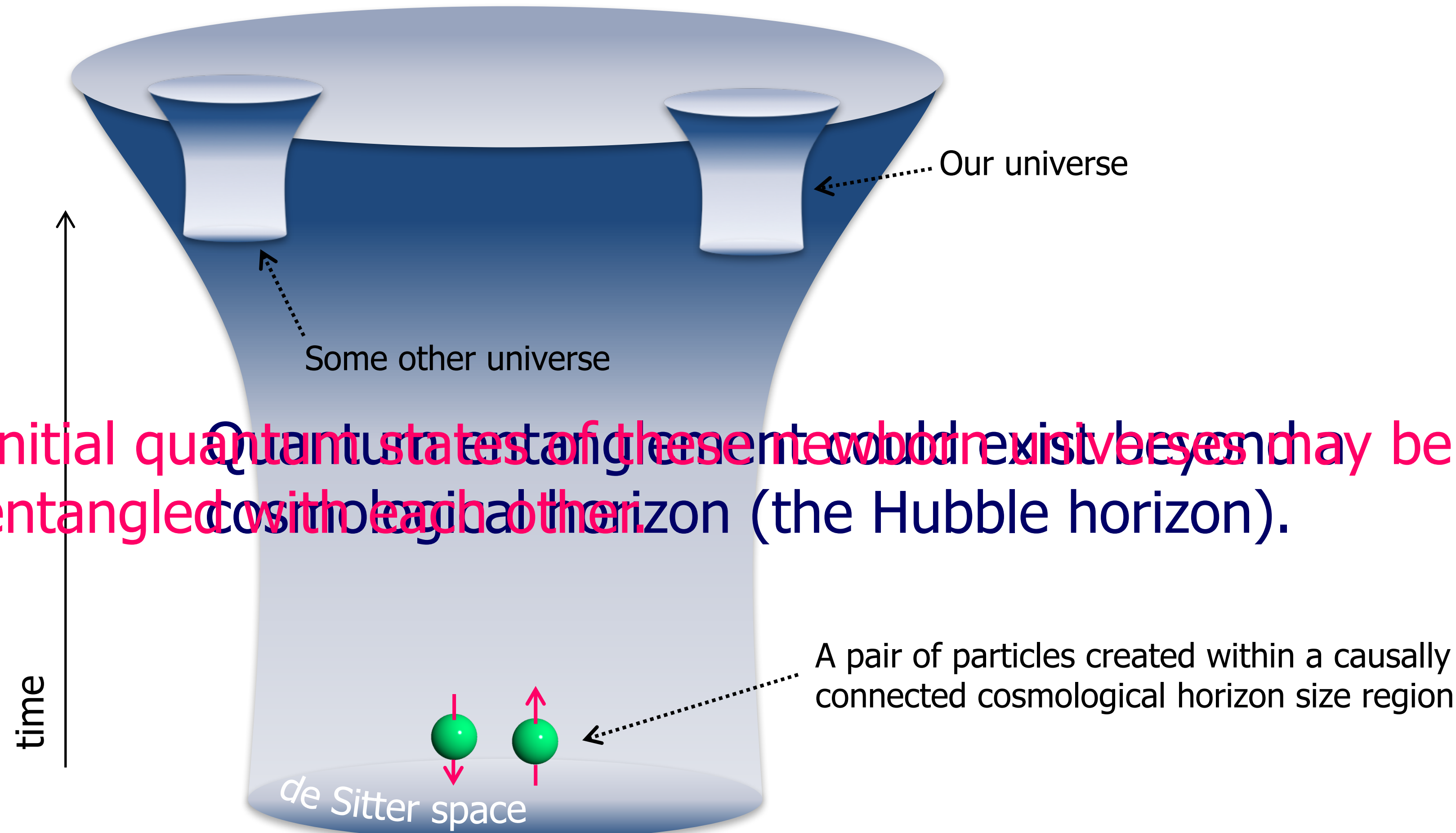
In principle, you gain information about the partner particle by measuring your own particle wherever the partner particle goes, if they are entangled.

a causally disconnected different universe

Naive expectation

Inflationary universe is approximated by a de Sitter space.

In quantum mechanics, vacuum state is full of virtual particles in entangled pairs.

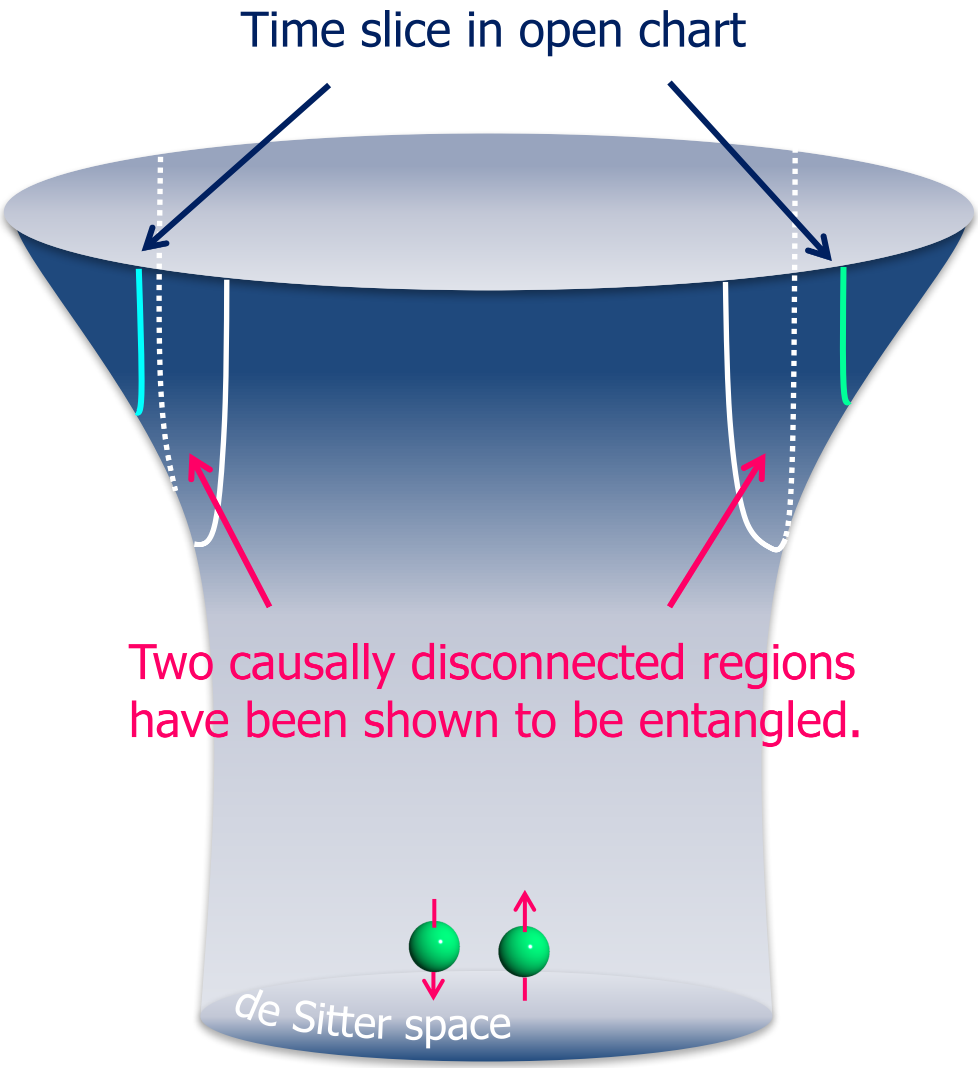


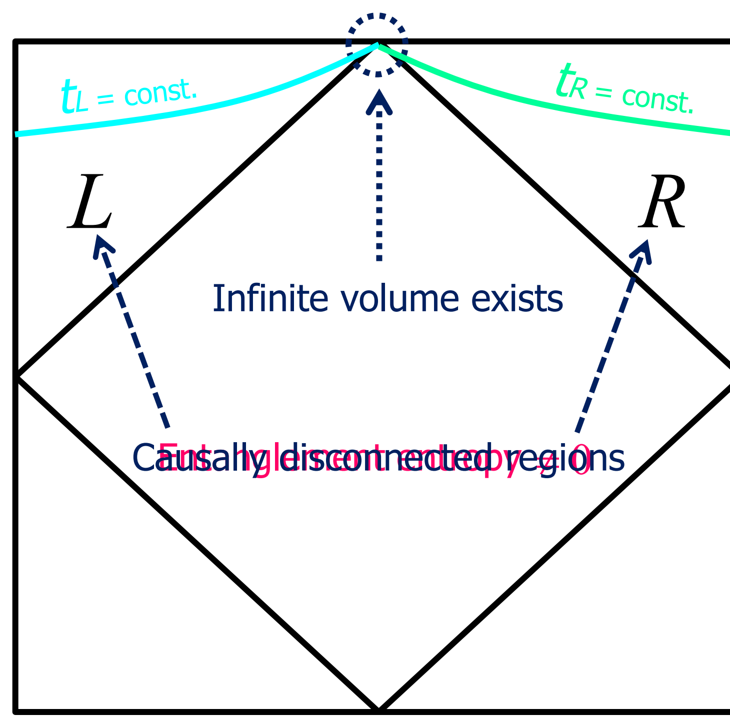
Initial quantum states of these newborn universes may be entangled with each other.

Quantum entanglement could exist beyond a cosmological horizon (the Hubble horizon).

Good news

Maldacena & Pimentel (2013)





Open chart

Review of Maldacena & Pimentel's computation

Action (No bubble wall)

$$S=\int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-\frac{m^2}{2}\phi^2\right]$$

Metric in each *R* and *L* region

$$ds^2=H^{-2}\left[-dt_R^2+\sinh^2 t_R\left(dr_R^2+\sinh^2 r_R\,d\Omega^2\right)\right]$$

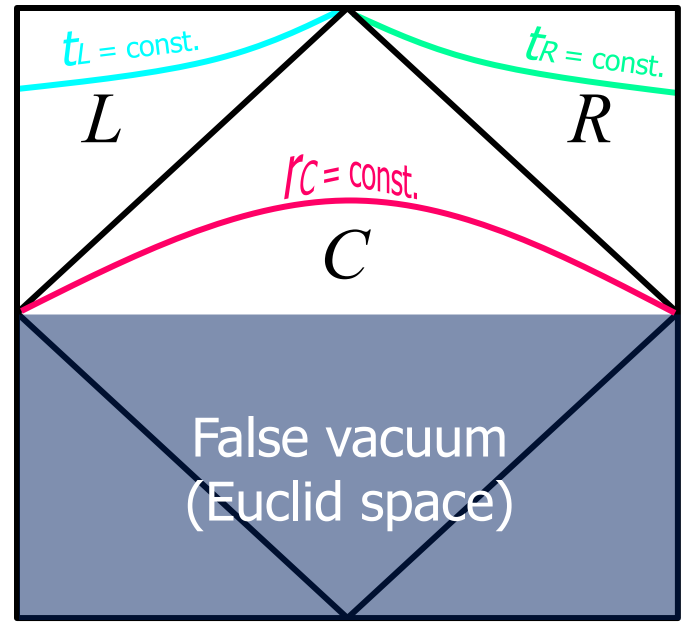
The Hubble radius² of de Sitter space

and *C* region

$$ds^2=H^{-2}\left[dt_C^2+\cos^2 t_C\left(-dr_C^2+\cosh^2 r_C\,d\Omega^2\right)\right]$$

can be obtained by analytic continuation from the Euclidean metric

$$ds^2=H^{-2}\left[-d\tau^2+\cos^2\tau\,\left(d\rho^2+\sin^2\rho\,d\Omega^2\right)\right]$$



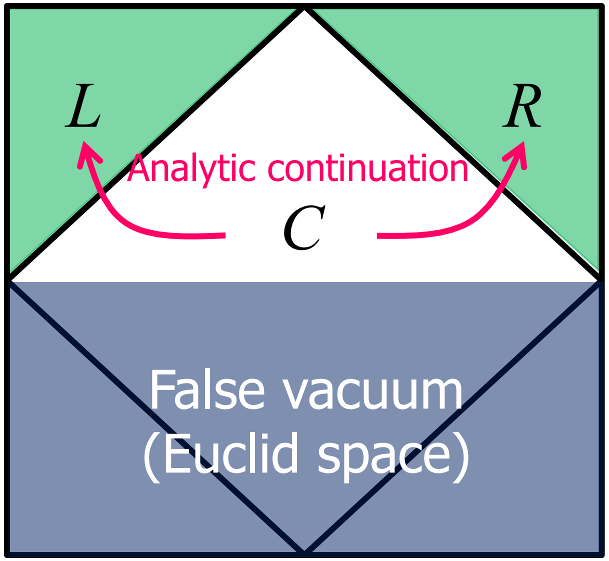
Open chart

The positive freq. mode in the Euclidean vacuum

Separation of variables

$$\phi_{p\ell m}\left(t_C, r_C, \Omega\right) \sim \frac{H}{\cos t_C}\chi_p\left(t_C\right)\underbrace{Y_{p\ell m}\left(r_C, \Omega\right)}_{}$$

Harmonic functions on the 3-dim hyperbolic space



The solutions of the mode function in the *C* region

$$\chi_p(t_C) = \boxed{P^{ip}_{\nu-1/2}(\sin t_C)} \qquad \nu = \sqrt{\frac{9}{4} - \frac{m^2}{H^2}}$$

Mode

The associated Legendre function

Mass parameter

We want the positive frequency mode functions supported both on *R* and *L* regions (two causally disconnected regions).

We need to extend the solution analytically from the *C* region to *R* and *L* regions by requiring regularity in the lower hemisphere of the Euclidean de Sitter space.

Euclidean vacuum (Bunch-Davies vacuum) solutions

Sasaki, Tanaka & Yamamoto (1995)

Solutions supported both on the *R* and *L* regions

$$\chi_p^R(t) = \begin{cases} P_{\nu-1/2}^{ip}(\cosh t_R) \\ \dfrac{\cos \pi\nu}{i \sinh \pi p} P_{\nu-1/2}^{ip}(\cosh t_L) + e^{-\pi p} \dfrac{\cos \pi(ip+\nu)}{i \sinh \pi p} \dfrac{\Gamma(\nu+1/2+ip)}{\Gamma(\nu+1/2-ip)} P_{\nu-1/2}^{-ip}(\cosh t_L) \end{cases}$$

Mode

$$\chi_p^L(t) = \begin{cases} \dfrac{\cos \pi\nu}{i \sinh \pi p} P_{\nu-1/2}^{ip}(\cosh t_R) + e^{-\pi p} \dfrac{\cos \pi(ip+\nu)}{i \sinh \pi p} \dfrac{\Gamma(\nu+1/2+ip)}{\Gamma(\nu+1/2-ip)} P_{\nu-1/2}^{-ip}(\cosh t_R) \\ P_{\nu-1/2}^{ip}(\cosh t_L) \end{cases}$$

Mode

These factors come from the requirement of analyticity of Euclidean hemisphere

The Euclidean vacuum (Bunch-Davies vacuum) is selected as the initial state.

Looks the same as a Minkowski vacuum at short distances

Bogoliubov transformation and entangled state

The Fourier mode field operator is

Positive freq. mode in the past in the Euclidean vacuum

$$\phi(t) = a_\sigma \chi^\sigma + a_\sigma^\dagger \chi^{\sigma *} \qquad a_\sigma \left|0\right\rangle_{\mathrm{ED}} = 0 \qquad \left[a_\sigma, a_{\sigma'}^\dagger\right] = \delta_{\sigma\sigma'}$$

$\sigma = R, L$

$$\varphi^R \sim P_{\nu-1/2}^{ip}\left(\cosh t_R\right)$$
$$\varphi^L \sim P_{\nu-1/2}^{ip}\left(\cosh t_L\right)$$

Positive freq. mode in the past in each *R* or *L* vacuum

$$= b_q \varphi^q + b_q^\dagger \varphi^{q*} \qquad b_R \left|0\right\rangle_R = 0\,, \qquad b_L \left|0\right\rangle_L = 0, \qquad \left[b_q, b_{q'}^\dagger\right] = \delta_{qq'}$$

$q = R, L$ $\qquad$ $q = R, L$

Then the operators $\left(a_\sigma, a_\sigma^\dagger\right)$ and $\left(b_q, b_q^\dagger\right)$ are related by a Bogoliubov transformation.

We are looking at analytic properties for each value of *p*

$$\chi_p^\sigma \qquad \varphi_p^q \qquad a_{\sigma,\,p} \qquad b_{q,\,p} \qquad \left|0_p\right\rangle$$

although we omit the label *p* for simplicity

The essence of the entanglement

The Euclidean vacuum can be constructed from the *R*, *L* vacua as

$$|0\rangle_{\mathrm{ED}} \propto \exp\left(\frac{1}{2}\sum_{i,j=R,L} m_{ij}\, b_i^\dagger b_j^\dagger\right)|0\rangle_R |0\rangle_L \sim |0\rangle_R|0\rangle_L + m_{RL}|1\rangle_R|1\rangle_L + m_{RL}^2|2\rangle_R|2\rangle_L + \cdots$$

: Entangled state of the $\mathcal{H}_R \otimes \mathcal{H}_L$ Hilbert space

Symmetric matrix which consists of the Bogoliubov coefficients

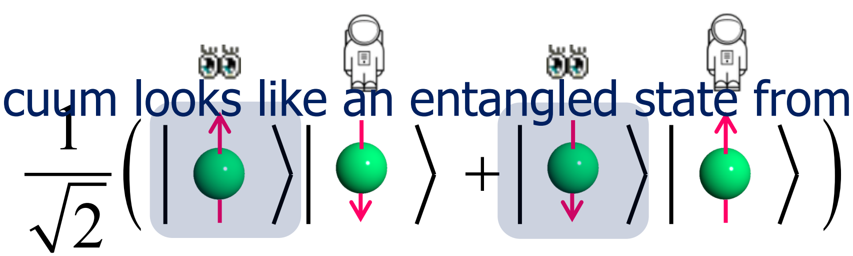
Unimporatant phase factor

$$m_{ij} = e^{i\theta}\frac{\sqrt{2}e^{-p\pi}}{\sqrt{\cosh 2\pi p + \cos 2\pi\nu}}\begin{array}{cc} \begin{array}{cc} R & L \end{array} & \\ \begin{pmatrix} \cos\pi\nu & i\sinh p\pi \\ i\sinh p\pi & \cos\pi\nu \end{pmatrix} & \begin{array}{c} R \\ L \end{array} \end{array}$$

The off-diagonal components make the operators in *R* and *L* regions correlated.

Eg) An entangled pair of electrons

The Euclidean vacuum looks like an entangled state from the point of view of *R*, *L* vacua.



The quantum entanglement is observer dependent !?

Entangled state of $\mathcal{H} \otimes \mathcal{H}$ Hilbert space

The density matrix

The Euclidean vacuum can be constructed from the *R*, *L* vacua as

$$|0\rangle_{\mathrm{ED}} \propto \exp\left(\frac{1}{2}\sum_{i,j=R,L} m_{ij}\, b_i^\dagger b_j^\dagger\right)|0\rangle_R |0\rangle_L$$

Symmetric matrix which consists of the Bogoliubov coefficients

Unimporatant phase factor

$$m_{ij} = e^{i\theta}\frac{\sqrt{2}e^{-p\pi}}{\sqrt{\cosh 2\pi p + \cos 2\pi\nu}}\begin{pmatrix} \cos\pi\nu & i\sinh p\pi \\ i\sinh p\pi & \cos\pi\nu \end{pmatrix}$$

Conformal invariance ($\nu = 1/2$)
Masslessness ($\nu = 3/2$)

The density matrix $\rho = |0\rangle_{\mathrm{ED}\ \mathrm{ED}}\langle 0|$ is not diagonal in the $|0\rangle_R |0\rangle_L$ basis.

It is difficult to trace out the degree of freedom in, say, the *L* region later in order to calculate the entanglement entropy in the *R* region.

We perform a further Bogoliubov transformation to get a diagonalized form.

Bogoliubov transformation 2

We perform a further Bogoliubov transformation in each *R* and *L* region

R region: $c_R = u\, b_R + v\, b_R^\dagger$

L region: $c_L = u^* b_L + v^* b_L^\dagger$

$$|u|^2 - |v|^2 = 1 \qquad \left[c_i, c_j^\dagger\right] = \delta_{ij}$$

This transformation does not mix the operators in $\mathcal{H}_R$ space and those in $\mathcal{H}_L$ space and thus does not affect the entangled state between $\mathcal{H}_R$ and $\mathcal{H}_L$.

We obtain the relation

$$|0\rangle_{\mathrm{ED}} = N_{\gamma_p}^{-1} \exp\left(\gamma_p\, c_R^\dagger\, c_L^\dagger\right)|0\rangle_{R'}|0\rangle_{L'} \qquad N_{\gamma_p}^2 = \left|\exp\left(\gamma_p\, c_R^\dagger\, c_L^\dagger\right)\right|^2 = \left(1 - |\gamma_p|^2\right)^{-1}$$

$$\gamma_p = i\frac{\sqrt{2}}{\sqrt{\cosh 2\pi p + \cos 2\pi\nu} + \sqrt{\cosh 2\pi p + \cos 2\pi\nu + 2}}$$

Conformal invariance ($\nu = 1/2$)
Masslessness ($\nu = 3/2$)

$$\xrightarrow{\nu = 1/2\,,\,3/2} \sim e^{-\pi p}$$

Reduced density matrix

Finally, the reduced density matrix after tracing out *L* region is found to be diagonalized as

$$\rho_R = \mathrm{Tr}_L \left|0\right\rangle_{\mathrm{ED}\ \mathrm{ED}}\left\langle 0\right|$$

Conformal invariance ($\nu = 1/2$)
Masslessness ($\nu = 3/2$)

$$= \left(1 - \underbrace{|\gamma_p|^2}_{\sim e^{-2\pi p}}\right) \sum_{n=0}^{\infty} \underbrace{|\gamma_p|^{2n}}_{\sim e^{-2\pi p n}} \left|n\right\rangle\left\langle n\right|$$

: Thermal state $T = \dfrac{H}{2\pi}$

$$\left|n_p\right\rangle = \frac{1}{\sqrt{n!}}\left(c_R^\dagger\right)^n \left|0_p\right\rangle_{R'}$$

For each mode

: *n* particle excitation states

Thermal state: $\dfrac{1}{e^{\varepsilon/T} - 1}$

The de Sitter space has some peculiar property for the conformal and massless cases.

Entanglement entropy between R and L regions

Maldacena & Pimentel (2013)

They calculated the entanglement entropy between those R and L regions.

Entanglement entropy for each mode:

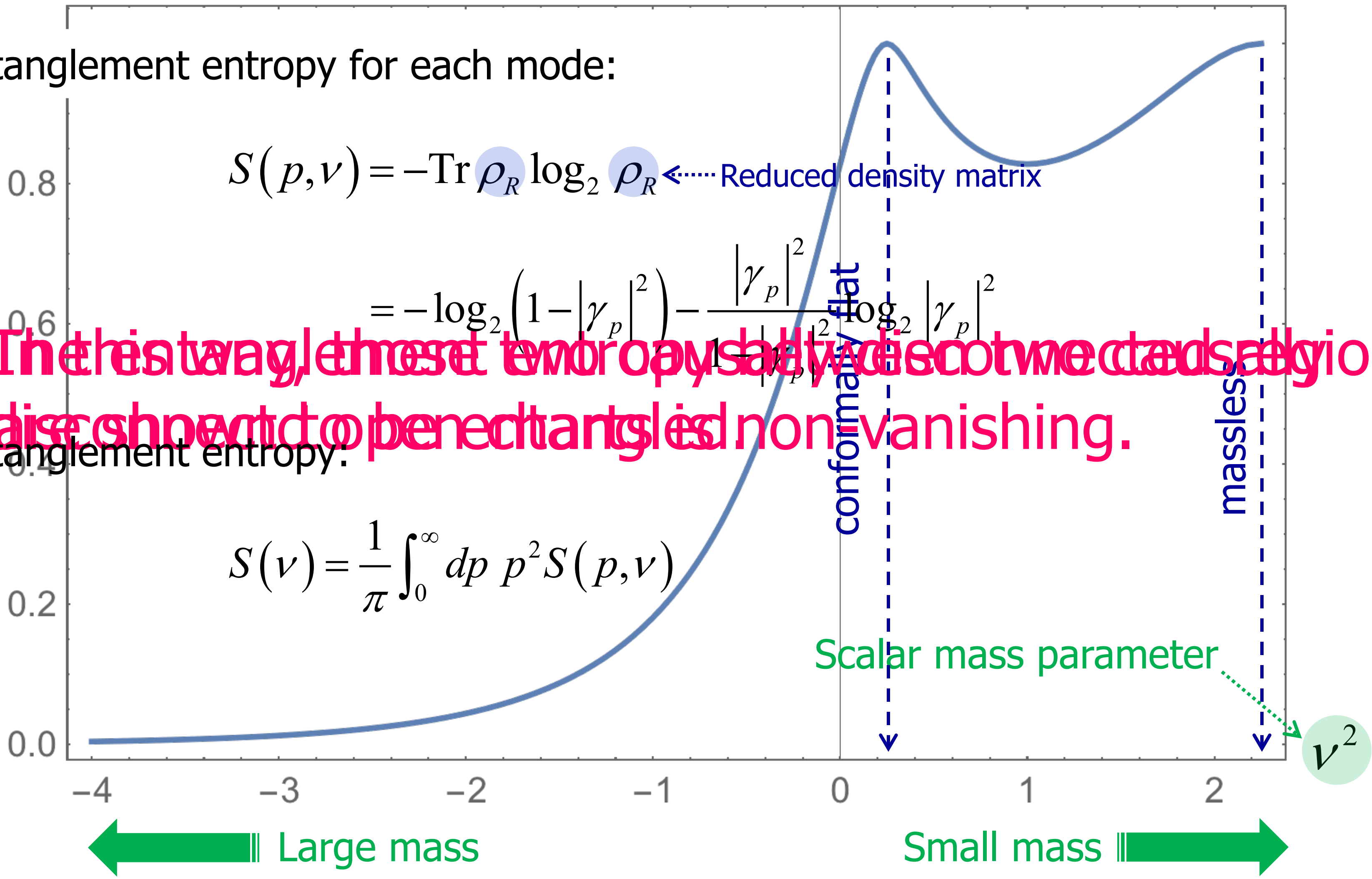
$$S(p,\nu) = -\mathrm{Tr}\,\rho_R \log_2 \rho_R$$

Reduced density matrix

$$= -\log_2\left(1-|\gamma_p|^2\right) - \frac{|\gamma_p|^2}{1-|\gamma_p|^2}\log_2 |\gamma_p|^2$$

Entanglement entropy:

$$S(\nu) = \frac{1}{\pi}\int_0^\infty dp\, p^2 S(p,\nu)$$



The entanglement entropy between two causally disconnected regions is non-vanishing.

In this way, these two regions are shown to be entangled.

Now going back to the original question

Is our bubble universe entangled with some other universe initially?

We found that two causally disconnected regions were entangled but bubble universes through quantum tunneling at false vacuum decay may cause decoherence…

So we consider the effect of

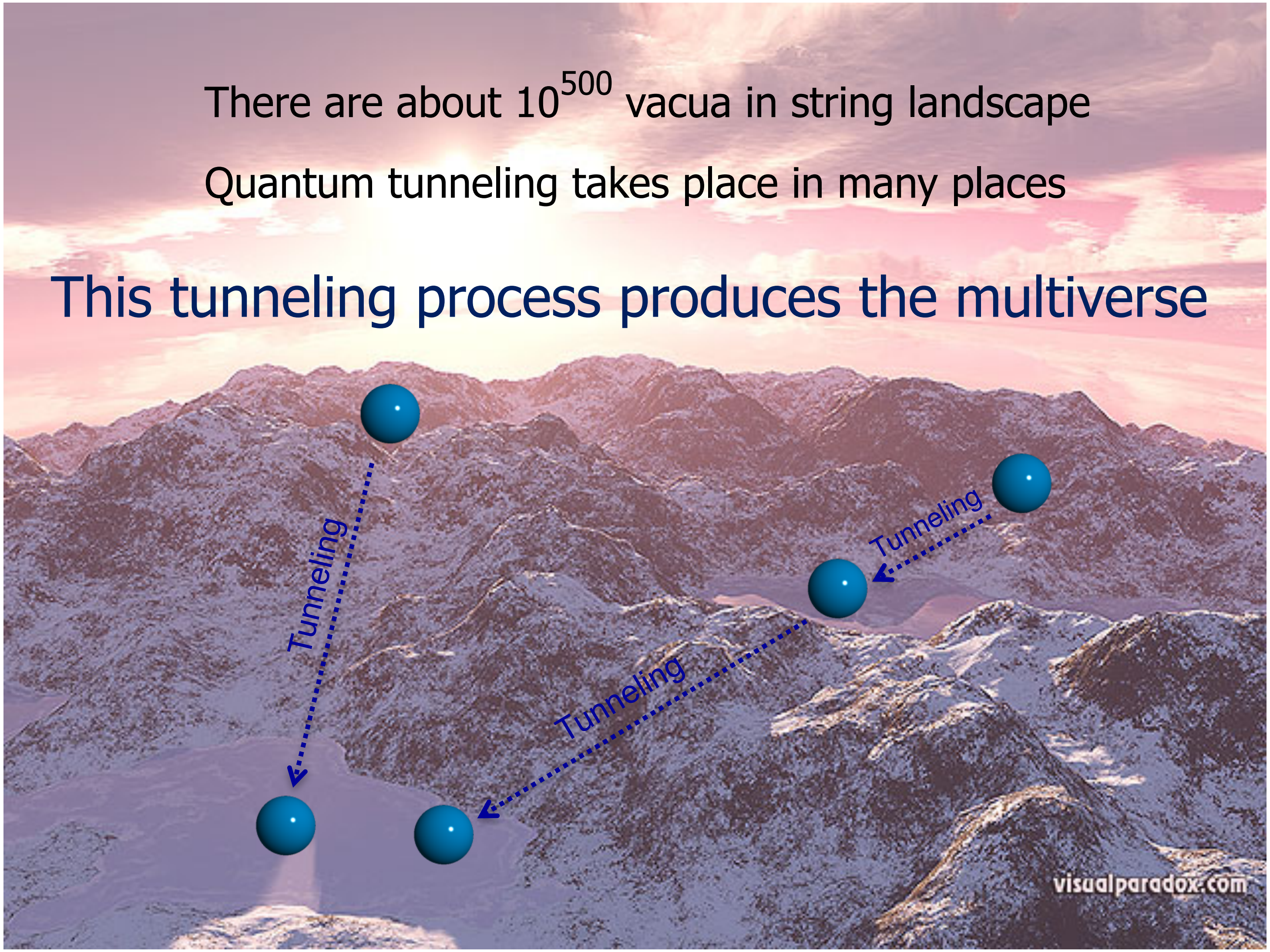
a bubble wall

on the entanglement entropy.

String/cosmic landscape

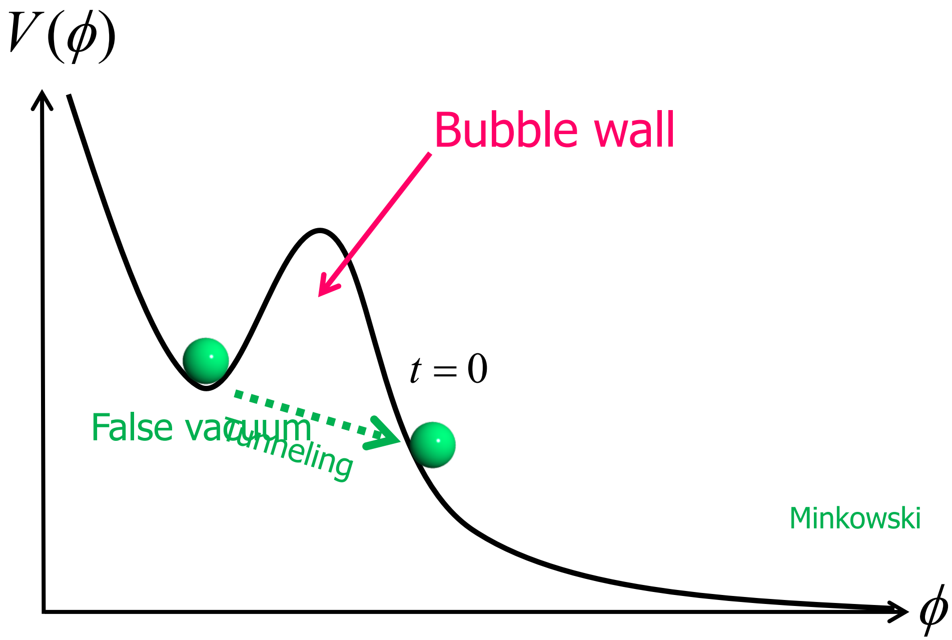
Sato et al. (1981), Vilenkin (1983), Linde (1986), Bousso & Polchinski (2000), Susskind (2003)

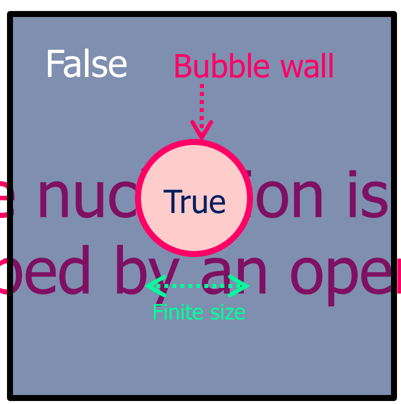
The configuration space of all possible values of scalar fields with all possible potentials.



Open chart describes bubble nucleation

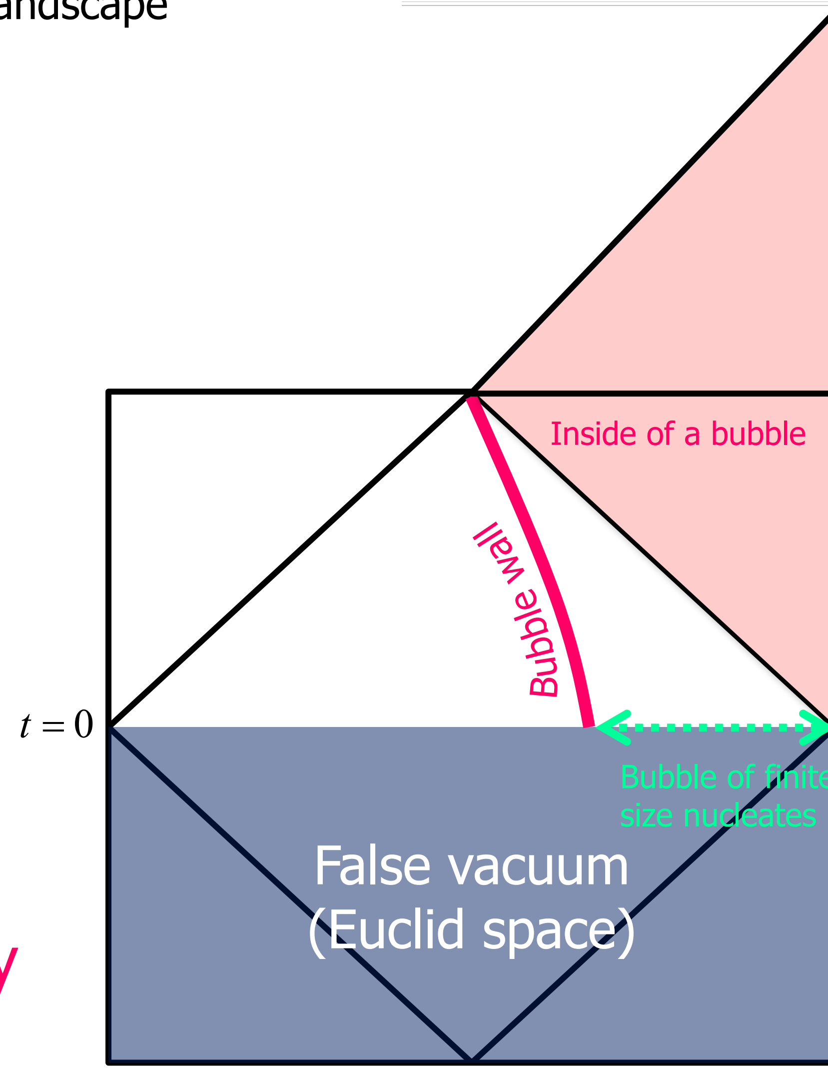
Eg) One type of potential of a scalar field in the landscape





$t = 0$

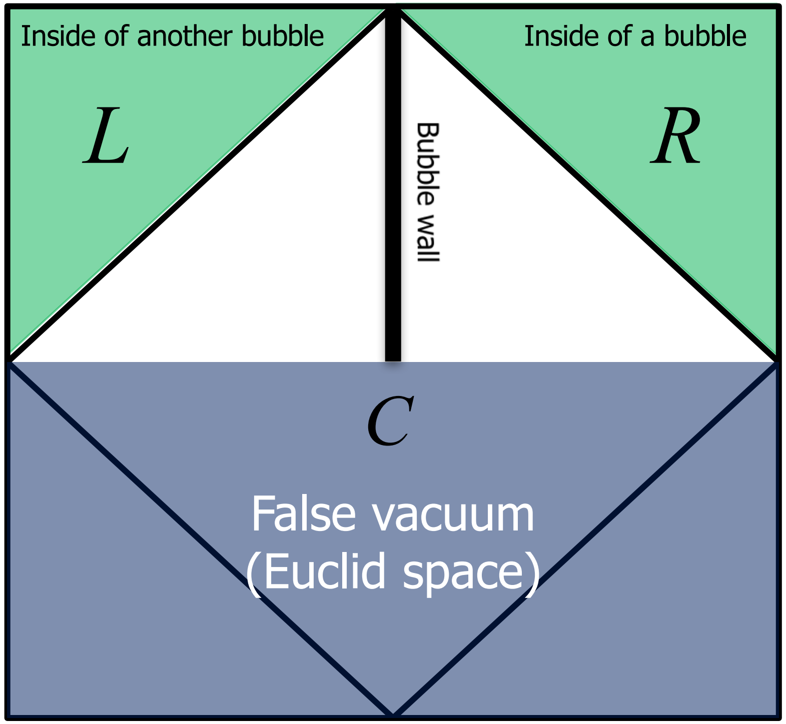
Bubble nucleation is conveniently described by an open chart.

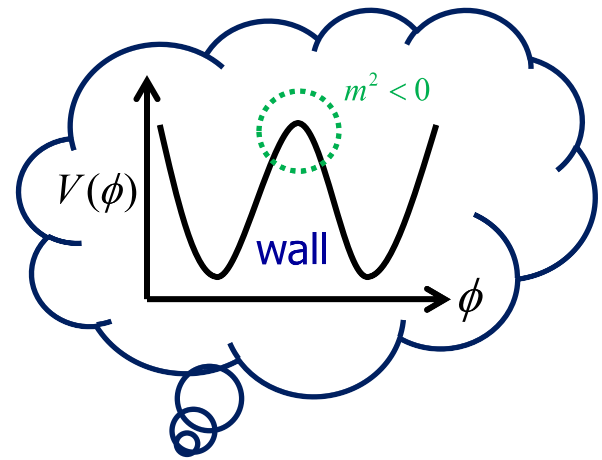


Open chart

Our setup

We assume there is a delta-functional wall between two open charts *R* and *L*.





Action

A delta-functional wall of hight (depth) Λ

This can be thought of as a model of pair creation of identical bubble universes separated by a bubble wall.

$$S=\int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-\frac{m^2-\Lambda\delta(t_C)}{2}\phi^2\right]$$

The ED vacuum solutions in the presence of a wall

The positive frequency mode functions for the Euclidean vacuum in the presence of the bubble wall

These factors come from the requirement of analyticity of Euclidean hemisphere

$$\chi_p^R(t) = \begin{cases} P^{ip}_{\nu-1/2}(\cosh t_R) \\ \left(A_p C_p + B_p D_{-p}\right) P^{ip}_{\nu-1/2}(\cosh t_L) + e^{\pi p}\left(A_p D_p + B_p C_{-p}\right) P^{-ip}_{\nu-1/2}(\cosh t_L) \end{cases}$$

$$\chi_p^L(t) = \begin{cases} \left(A_p C_p + B_p D_{-p}\right) P^{ip}_{\nu-1/2}(\cosh t_R) + e^{\pi p}\left(A_p D_p + B_p C_{-p}\right) P^{-ip}_{\nu-1/2}(\cosh t_R) \\ P^{ip}_{\nu-1/2}(\cosh t_L) \end{cases}$$

Wall effect

$$A_p = 1 + \frac{\pi}{2i\sinh \pi p}\frac{\Lambda}{H^2} P^{ip}_{\nu-1/2}(0) P^{-ip}_{\nu-1/2}(0) \qquad C_p = \frac{\cos \pi\nu}{i\sinh \pi p}$$

$$B_p = -\frac{\pi}{2i\sinh \pi p}\frac{\Lambda}{H^2}\left(P^{ip}_{\nu-1/2}(0)\right)^2 \qquad D_p = -e^{-2\pi p}\frac{\cos(\nu + ip)\pi}{i\sinh \pi p}\frac{\Gamma(1/2+\nu+ip)}{\Gamma(1/2+\nu-ip)}$$

We can expect the effect of the wall would appear in the entanglement entropy.

Bogoliubov transformation and entangled state

The Euclidean vacuum can be constructed from the *R*, *L* vacua as

$$|0\rangle_{\rm ED} \propto \exp\left(\frac{1}{2}\sum_{i,j=R,L} m_{ij}\, b_i^\dagger b_j^\dagger\right)|0\rangle_R |0\rangle_L$$

: Entangled state of the $\mathcal{H}_R \otimes \mathcal{H}_L$ Hilbert space

Symmetric matrix $\begin{pmatrix} m_{RR} & m_{RL} \\ m_{LR} & m_{LL} \end{pmatrix}$

$$m_{RR} = m_{LL} = -\frac{g_p^*}{1-f_p^{*2}}\frac{1}{E}\left[\left(f_p + f_p^*\right)\left(1-\frac{|g_p|^2}{1-f_p^{*2}}\right) - \left(1+|f_p|^2-|g_p|^2\right)\left(f_p - \frac{f_p^*|g_p|^2}{1-f_p^{*2}}\right)\right]$$

$$m_{RL} = m_{LR} = -\frac{g_p^*}{1-f_p^{*2}}\frac{1}{E}\left[\left(f_p + f_p^*\right)\left(f_p - \frac{f_p^*|g_p|^2}{1-f_p^{*2}}\right) - \left(1+|f_p|^2-|g_p|^2\right)\left(1-\frac{|g_p|^2}{1-f_p^{*2}}\right)\right]$$

Wall effect

$$f_p = A_p C_p + B_p D_{-p} \qquad g_p = e^{\pi p}\left(A_p D_p + B_p C_{-p}\right) \qquad E = \left(1-\frac{|g_p|^2}{1-f_p^{*2}}\right)^2$$

We perform another Bogoliubov transformation to make the density matrix diagonal as we did without bubble wall.

The Euclidean vacuum is defined by Bogoliubov transformation that mixes the operator b_R with b_L.

Bogoliubov transformation 2

The second Bogoliubov transformation to make the density matrix diagonal and that does not mix the operators in *R* and *L* anymore is

$$|0\rangle_{\mathrm{ED}} = N_{\gamma_p}^{-1} \exp\left(\gamma_p c_R^\dagger c_L^\dagger\right) |0\rangle_{R'} |0\rangle_{L'} \qquad N_{\gamma_p}^2 = \left| \exp\left(\gamma_p c_R^\dagger c_L^\dagger\right) \right|^2 = \left(1 - |\gamma_p|^2\right)^{-1}$$

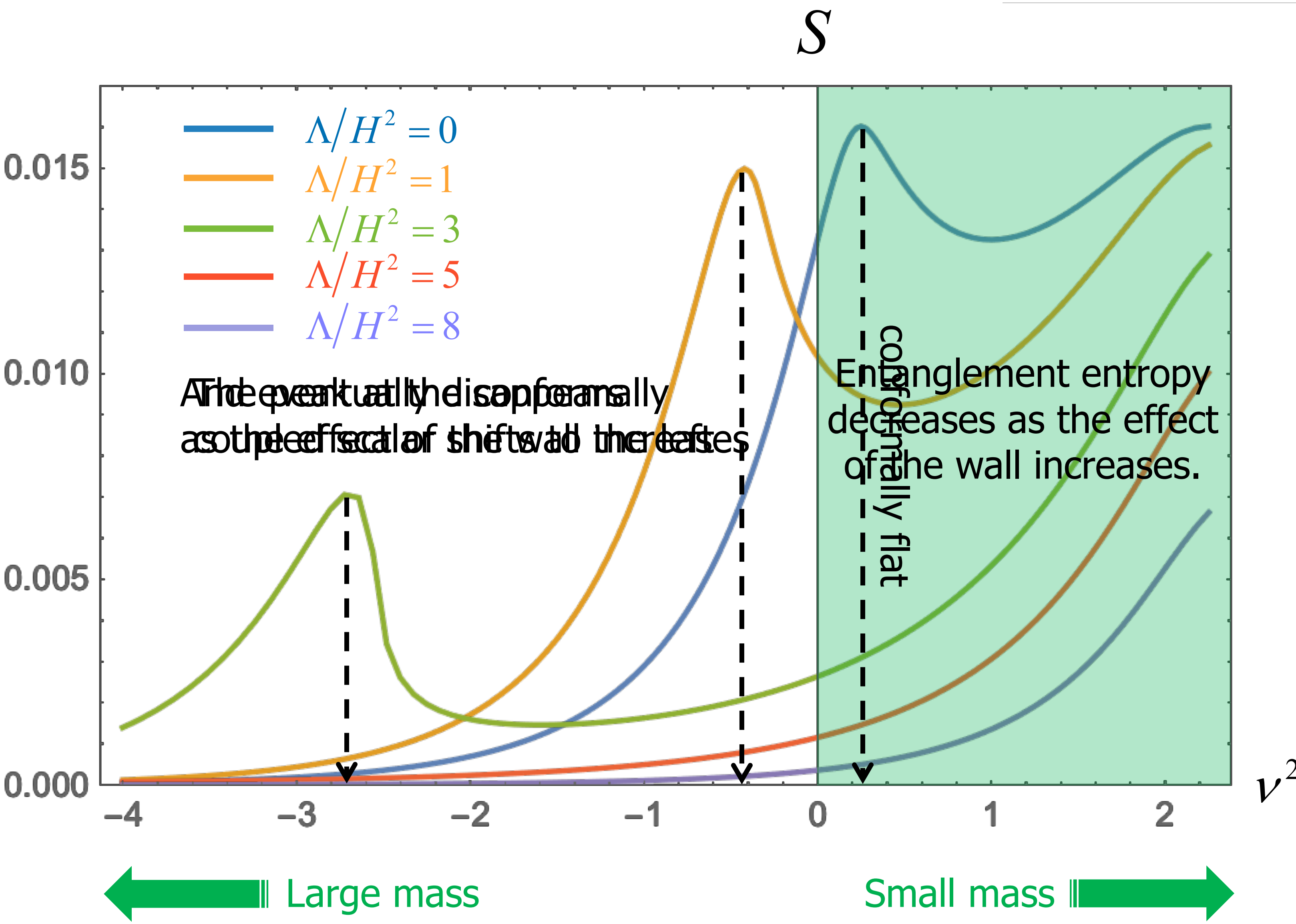
we want to know

If we write $m_{RR} = m_{LL} \equiv \omega$ and $m_{RL} = m_{LR} \equiv \varsigma$

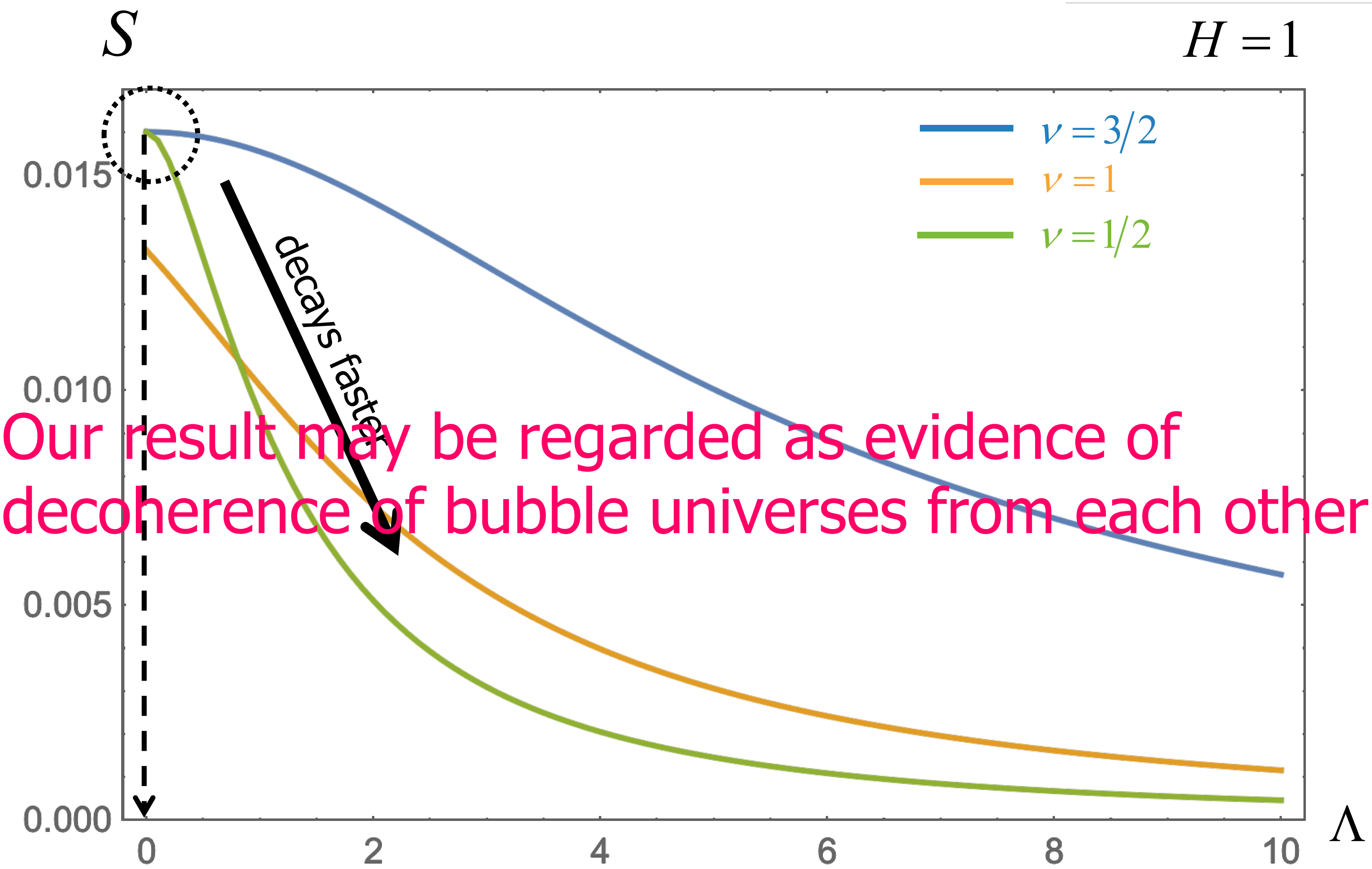
$$|\gamma_p|^2 = \frac{1}{2|\varsigma|^2}\Big[-\omega^2\varsigma^{*2} - \omega^{*2}\varsigma^2 + |\omega|^4 - 2|\omega|^2 + 1 + |\varsigma|^4 \\ - \sqrt{\left(\omega^2\varsigma^{*2} + \omega^{*2}\varsigma^2 - |\omega|^4 + 2|\omega|^2 - 1 - |\varsigma|^4\right)^2 - 4|\varsigma|^4}\Big]$$

We can expect the effect of the wall would appear in the entanglement entropy.

Entanglement entropy between R and L regions



Wall dependence of the entanglement entropy

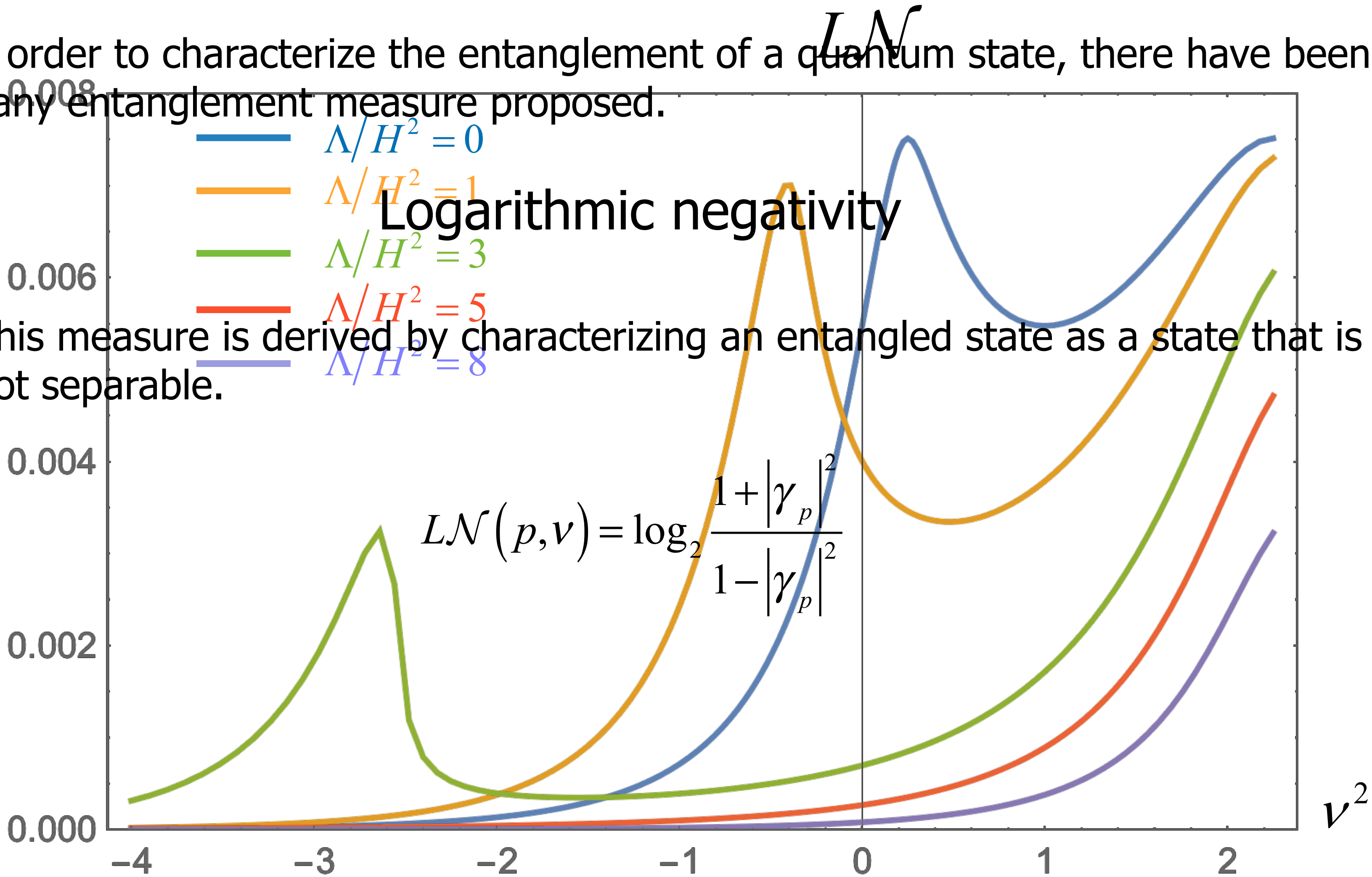


Our result may be regarded as evidence of decoherence of bubble universes from each other !?

No bubble wall

Logarithmic negativity between *R* and *L* regions

In order to characterize the entanglement of a quantum state, there have been many entanglement measure proposed.



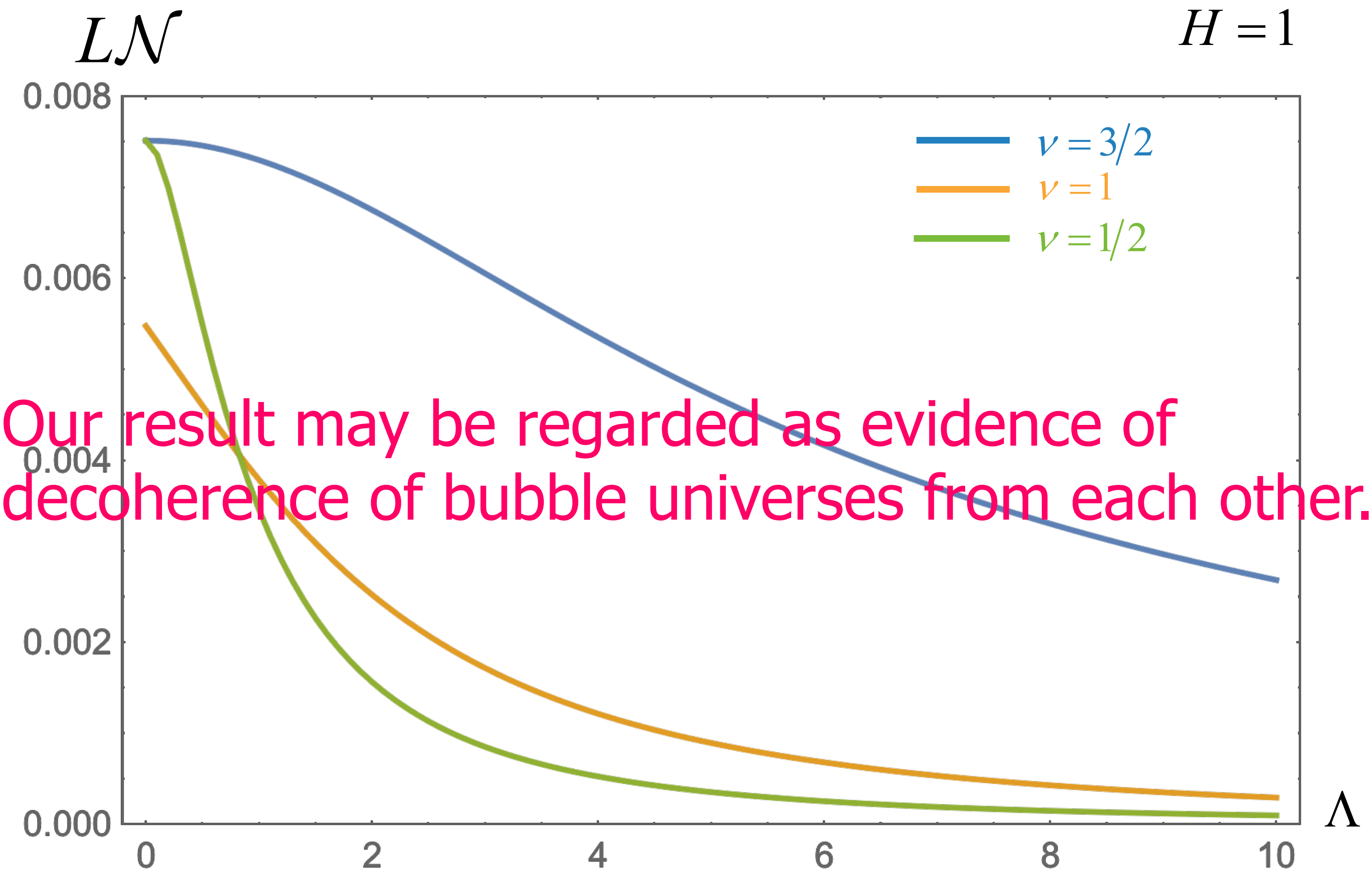
Logarithmic negativity

This measure is derived by characterizing an entangled state as a state that is not separable.

$$L\mathcal{N}(p,\nu) = \log_2 \frac{1+|\gamma_p|^2}{1-|\gamma_p|^2}$$

Qualitative features are the same as the result of entanglement entropy.

Wall dependence of the logarithmic negativity



Our result may be regarded as evidence of decoherence of bubble universes from each other.

Qualitative features are the same as the result of entanglement entropy.

Summary

We studied the effect of a bubble wall on the quantum entanglement of a free massive scalar field between two causally disconnected open charts in de Sitter space.

We assumed there is a delta-functional wall between them.

Our model may be regarded as a model describing the pair creation of identical bubble universes separated by a bubble wall.

We computed the entanglement entropy and logarithmic negativity of the scalar field and compared the result with the case of no bubble wall.

We found that larger the wall leads to less entanglement.

Our result may be regarded as evidence of decoherence of bubble universes from each other.