$h \to \gamma\gamma$ decay in the Standard Model Effective Field Theory

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Outline

1. **Introduction - Effective Field Theory approach**
2. **SMEFT construction and physical field basis**
3. **SMEFT quantization in R_ξ gauges**
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5. $h \to \gamma\gamma$ **decay in SMEFT**
6. **Conclusions**

Based on:
Dedes, Materkowska, Paraskevas, JR, Suxho, JHEP 1706 (2017) 143
Dedes, Paraskevas, JR, Suxho, Trifyllis, arXiv:1805.00302

Introduction

Since ∼ '1970, Standard Model (SM) works very well up to energy scales of few hundred GeV (LHC).

Theory of Everything? Probably not:

- dark matter and dark energy?
- origin of matter-antimatter asymmetry?

...

- the stability of the Higgs boson mass?
- number of fermion generations - why 3 (light) generations?
- hierarchy of fermion masses, $m_\nu/m_{top} \sim \mathcal{O}\left(10^{-13}\right)$?

...

Currently:

- no direct discoveries of new particles
- some experimental anomalies do appear – most fluctuations only ("new resonance" at $E_{CM} = 750$ GeV). **Anomalies in B physics persist!**

How to search for “Beyond SM” physics?

Very important feature of Nature: “decoupling”.

Interactions at lower energy scales are “screened” from higher energy phenomena:

- we can describe atomic physics (and chemistry) without knowing details of nuclear forces
- we can describe quantum electrodynamics without knowledge of the internal Standard Model structure
- SM seems to be working perfectly even if some unknown phenomena pointing to wider theory already observed
- ...

In Quantum Field Theory this feature could be formalized:

Theorem (Appelquist-Carazzone decoupling theorem:)

Assumption: *renormalizable quantum gauge theory is embedded into a larger renormalizable theory, with new particle mass scale* M

Conclusion: *effects of larger theory at scale* $E \ll M$ *suppressed by powers of* E/M

Consequences:

Experimental – two ways of discovering BSM physics:

- direct: production of new particles in colliders operating at $E > E_{SM}$
- indirect: detection of quantum effects of virtual new particles at current scales but precision at least E_{SM}^2/E^2

Theoretical – instead of studying a plethora of New Physics models, describe new effects in terms of the Effective Field Theory (EFT).

EFT description

Lets assume that SM is embedded in some larger theory (“Ultraviolet Completion”), with typical mass scales $\Lambda \gg v = 246$ GeV.

Then, at the energy scales $E \ll \Lambda$, effects of UV theory can be parametrized as new interaction terms for the SM fields

$$L_{EFT} = L_{SM}^{(4)} + \frac{1}{\Lambda} L^{(5)} + \frac{1}{\Lambda^2} L^{(6)} + \dots$$

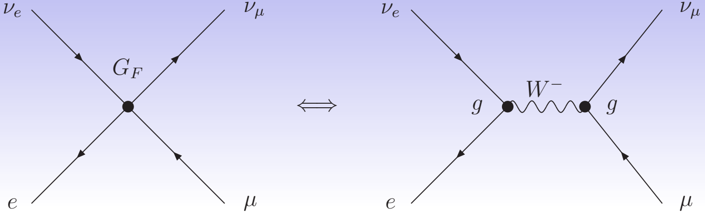
Upper index — “the mass dimension” of terms in the Lagrangian:

$\dim[\phi] = m$	scalar fields
$\dim[V] = m$	vector fields
$\dim[\psi] = m^{3/2}$	spin $\frac{1}{2}$ fermion fields

Example: $\dim[\bar{\psi}\gamma_\mu\psi\ \phi\partial^\mu\phi] = m^6$ (“dimension-6 term”).

SM Lagrangian: all terms dimension-4 (required by renormalizability).

EFT description well-known and successful since Fermi theory:



Before SM: weak interactions described as contact 4-fermion vertices of dimension-6.

After SM construction - 2 separate vertices of dimension-4 and

$$G_F = \frac{\sqrt{2}}{8}\frac{g^2}{M_W^2}$$

M_W plays the role of scale Λ for mesonic interactions at the energies $\mathcal{O}(1)$ GeV.

How general is the EFT description?

Theorem (Weinberg's "Folk Theorem"[a])

"If one writes down the most general possible Lagrangian, including all terms consistent with assumed symmetry principles, and then calculates matrix elements with this Lagrangian to any given order of perturbation theory, the result will simply be the most general possible S-matrix consistent with perturbative unitarity, analyticity, cluster decomposition, and the assumed symmetry properties."[b]

[a] S. Weinberg, "Effective Field Theory, Past and Future," arXiv:0908.1964
[b] S. Weinberg, "Phenomenological Lagrangians," Physica **A96**, 327 (1979)

First step - derive "the most general possible Lagrangian"!

$$L_{EFT} = L_{SM}^{(4)} + \frac{1}{\Lambda} C_{\nu\nu}^{(5)} O_{\nu\nu}^{(5)} + \frac{1}{\Lambda^2} \sum_i C_i^{(6)} O_i^{(6)} + \ldots$$

Grządkowski, Iskrzyński, Misiak, JR – JHEP 2010: derivation of irreducible basis of gauge invariant operators of dimension 5 and 6 constructed from the SM fields.

22 May 2018: 691 citations in SPIRES.

"Warsaw basis":

- 1 dimension-5 operator (L-violating, Weinberg 1979):

$$O^{(5)IJ}_{\nu\nu} = \varepsilon_{jk}\varepsilon_{mn}\varphi^j\varphi^m l^k_{LI}\,\mathbf{C}\,l^n_{LJ}$$

- 59 (63 if lepton and/or baryon number not conserved) independent dimension-6 operators.

Notation: φ — Higgs doublet, B, W, G — $U(1), SU(2), SU(3)$ gauge bosons, q, u, d, l, e - left and right fermion fields.

Bosonic and 2-fermion dimension-6 operators:

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				

$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I\varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar l_p\gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I \varphi)(\bar l_p\tau^I\gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde\varphi\, G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar e_p\gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r)\tau^I\widetilde\varphi\, W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar q_p\gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I \varphi)(\bar q_p\tau^I\gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r)\varphi\, G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar u_p\gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r)\tau^I\varphi\, W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar d_p\gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu\varphi)(\bar u_p\gamma^\mu d_r)$

4-fermion operators:

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(l_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(l_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$		

Worth noting: all operators Q_X including fermionic fields carry flavor indices. The same for their respective Wilson coefficients C_X.

$\rightarrow$ many Wilson coefficients are 3×3 or $3 \times 3 \times 3 \times 3$ matrices in flavor space.

Including symmetry properties under flavor permutations - still over 2000 free parameters in dim-6 SMEFT!

New papers exist on classification of higher order operators:

- dim-7 operators always B or L violating and not numerous
- hundreds of dim-8 operators, even not counting flavor structure

SMEFT predictive power at higher orders limited...

Physical fields in SMEFT

"Warsaw basis" constructed as a set of **gauge invariant operators** before the Spontaneous Symmetry Breaking!

Operators given in terms of unphysical fields - "electroweak basis".

To compare with experiment, one should calculate observables in physical field basis $\rightarrow$ mass eigenstates basis after the SSB.

To do list:

- perform the SSB and Higgs mechanism
- find physical fields and effective couplings
- quantize the theory - gauge fixing choice
- derive the Feynman rules

Spontaneous Symmetry Breaking

Operators contributing to the Higgs potential:

$$\begin{aligned}\mathcal{L}_{\mathrm{H}} &= (D_\mu\varphi)^\dagger(D^\mu\varphi) + m^2(\varphi^\dagger\varphi) - \frac{\lambda}{2}(\varphi^\dagger\varphi)^2 \\ &+ C^{\varphi}(\varphi^\dagger\varphi)^3 + C^{\varphi\Box}(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi) + C^{\varphi D}(\varphi^\dagger D_\mu\varphi)^*(\varphi^\dagger D^\mu\varphi)\end{aligned}$$

Higgs doublet expansion:

$$\varphi = \begin{pmatrix} Z_{G^+}^{-1}G^+ \\ \frac{1}{\sqrt{2}}(v + Z_h^{-1}h + iZ_{G^0}^{-1}G^0) \end{pmatrix}$$

Corrected vacuum expectation value:

$$v = \sqrt{\frac{2m^2}{\lambda}} + \frac{3m^3}{\sqrt{2}\lambda^{5/2}}C^{\varphi}$$

Bilinear terms of the scalar fields:

$$\begin{aligned}\mathcal{L}_H &= \frac{1}{2}Z_h^{-2}\left(1+\frac{1}{2}C^{\varphi D}v^2 - 2C^{\varphi\Box}v^2\right)(\partial_\mu h)^2 \\ &+ Z_h^{-2}\left(\frac{1}{2}m^2 - \frac{3}{4}\lambda v^2 + \frac{15}{8}v^4 C^{\varphi}\right)h^2 \\ &+ \frac{1}{2}Z_{G^0}^{-2}\left(1+\frac{1}{2}C^{\varphi D}v^2\right)(\partial_\mu G^0)^2 + Z_{G^+}^{-2}(\partial_\mu G^-)(\partial^\mu G^+).\end{aligned}$$

Canonically normalized kinetic terms - choose:

$$\begin{aligned}Z_h &\equiv 1+\frac{1}{4}C^{\varphi D}v^2 - C^{\varphi\Box}v^2 \\ Z_{G^0} &\equiv 1+\frac{1}{4}C^{\varphi D}v^2 \\ Z_{G^+} &\equiv 1\end{aligned}$$

Physical Higgs mass:

$$M_h^2 = \lambda v^2 - (3\,C^{\varphi} - 2\,\lambda\, C^{\varphi\Box} + \frac{\lambda}{2}\,C^{\varphi D})v^4$$

Gauge boson masses

Contributing Lagrangian terms:

$$\begin{aligned}\mathcal{L}_{\mathrm{EW}} &= -\frac{1}{4}W^I_{\mu\nu}W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + (D_\mu\varphi)^\dagger(D^\mu\varphi) \\ &+ C^{\varphi W}(\varphi^\dagger\varphi)W^I_{\mu\nu}W^{I\mu\nu} + C^{\varphi B}(\varphi^\dagger\varphi)B_{\mu\nu}B^{\mu\nu} + C^{\varphi WB}(\varphi^\dagger\tau^I\varphi)W^I_{\mu\nu}B^{\mu\nu} \\ &+ C^{\varphi D}(\varphi^\dagger D_\mu\varphi)^*(\varphi^\dagger D^\mu\varphi)\end{aligned}$$

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} + C^{\varphi G}(\varphi^\dagger\varphi)G^A_{\mu\nu}G^{A\mu\nu}$$

Technical complications:

- non-canonical bilinear kinetic terms
- kinetic term mixing from $C^{\varphi WB}$ operator

"Barred" fields and couplings:

$$\begin{aligned}
\bar{W}^I_\mu &\equiv Z_g W^I_\mu & \bar{g} &\equiv Z_g^{-1} g & Z_g &\equiv 1 - C^{\varphi W} v^2 \\
\bar{B}_\mu &\equiv Z_{g'} B_\mu & \bar{g}' &\equiv Z_{g'}^{-1} g' & Z_{g'} &\equiv 1 - C^{\varphi B} v^2 \\
\bar{G}^A_\mu &\equiv Z_{g_s} G^A_\mu & \bar{g}_s &\equiv Z_{g_s}^{-1} g_s & Z_{g_s} &\equiv 1 - C^{\varphi G} v^2
\end{aligned}$$

Covariant derivative form preserved:

$$\begin{aligned}
D_\mu &= \partial_\mu + i g' Y_q B_\mu + \frac{ig}{2}\tau^I W^I_\mu + i g_s T^A G^A_\mu \\
&= \partial_\mu + i \bar{g}' Y_q \bar{B}_\mu + \frac{i\bar{g}}{2}\tau^I \bar{W}^I_\mu + i \bar{g}_s T^A \bar{G}^A_\mu = \bar{D}_\mu
\end{aligned}$$

Gauge kinetic terms for "barred" fields canonical up to kinetic mixing.

Gluon field already physical, no mixing:

$$g_\mu^A \equiv \bar{G}_\mu^A$$

Electroweak gauge bosons bilinears:

$$\begin{aligned}\mathcal{L}_{EW} &= -\frac{1}{4}(\bar{W}^1_{\mu\nu}\bar{W}^{1\mu\nu} + \bar{W}^2_{\mu\nu}\bar{W}^{2\mu\nu}) - \frac{1}{4}\begin{pmatrix}\bar{W}^3_{\mu\nu}\\ \bar{B}_{\mu\nu}\end{pmatrix}^\top \begin{pmatrix}1 & \epsilon\\ \epsilon & 1\end{pmatrix}\begin{pmatrix}\bar{W}^{3\mu\nu}\\ \bar{B}^{\mu\nu}\end{pmatrix}\\ &+\frac{\bar{g}^2 v^2}{8}(\bar{W}^1_\mu \bar{W}^{1\mu} + \bar{W}^2_\mu \bar{W}^{2\mu})\\ &+\frac{v^2}{8}Z^2_{G^0}\begin{pmatrix}\bar{W}^3_\mu\\ \bar{B}_\mu\end{pmatrix}^\top \begin{pmatrix}\bar{g}^2 & -\bar{g}\bar{g}'\\ -\bar{g}\bar{g}' & \bar{g}'^2\end{pmatrix}\begin{pmatrix}\bar{W}^{3\mu}\\ \bar{B}^{\mu}\end{pmatrix}\end{aligned}$$

where

$$\epsilon \equiv C^{\varphi WB}\ v^2$$

Simultaneous diagonalization of mass matrix and canonicalization of kinetic terms necessary ("congruent transformation"):

$$W^{\pm}_{\mu} = \frac{1}{\sqrt{2}}(\bar{W}^1_{\mu} \mp i\bar{W}^2_{\mu}) \qquad \begin{pmatrix} \bar{W}^3_{\mu} \\ \bar{B}_{\mu} \end{pmatrix} = \mathbb{X} \begin{pmatrix} Z_{\mu} \\ A_{\mu} \end{pmatrix}$$

with

$$\mathbb{X} = \begin{pmatrix} 1 & -\frac{\epsilon}{2} \\ -\frac{\epsilon}{2} & 1 \end{pmatrix} \begin{pmatrix} \cos\bar{\theta} & \sin\bar{\theta} \\ -\sin\bar{\theta} & \cos\bar{\theta} \end{pmatrix}$$

and

$$\tan\bar{\theta} = \frac{\bar{g}'}{\bar{g}} + \frac{\epsilon}{2}\left(1 - \frac{\bar{g}'^2}{\bar{g}^2}\right)$$

Gauge boson masses:

$$\begin{aligned} M_W &= \frac{1}{2}\,\bar{g}\,v \\ M_Z &= \frac{1}{2}\sqrt{\bar{g}^2 + \bar{g}'^2}\,v\,\left(1 + \frac{\epsilon\bar{g}\bar{g}'}{\bar{g}^2 + \bar{g}'^2}\right) Z_{G^0} \\ M_A &= 0 \end{aligned}$$

Gauge-Goldstone mixing ("unwanted terms")

$$\mathcal{L}_H \supset \ (\bar{D}_\mu\varphi)^\dagger(\bar{D}^\mu\varphi) \ + \ C^{\varphi D}(\varphi^\dagger\bar{D}_\mu\varphi)^*(\varphi^\dagger\bar{D}^\mu\varphi)$$

After SSB:

$$\mathcal{L}_{G-EW} \ = \ iM_W(W_\mu^+\partial^\mu G^- - W_\mu^-\partial^\mu G^+) - M_Z\, Z_\mu\, \partial^\mu G^0$$

Very important! In terms of physical fields: Gauge-Goldstone mixing identical as in the SM.

Allows to use R_ξ-gauge fixing also for SMEFT loop calculations.

Corrections to the SM vertices

Corrections to the interactions in the SM Lagrangian

- genuine new vertices from higher order operators
- modifications of dimension-4 vertices from shifts in the fields and parameters ("oblique" corrections)

"Oblique" corrections: can be described expressing covariant derivative by physical fields and parameters:

$$\bar{D}_\mu^{EW} = \partial_\mu + i\frac{\bar{g}}{\sqrt{2}}\left(T^+ W_\mu^+ + T^- W_\mu^-\right) + i\bar{g}_Z\left(T^3 - \sin^2\bar{\theta}\, Q\right) Z_\mu + i\bar{e} Q A_\mu$$

with corrected effective couplings

$$\bar{e} = \frac{\bar{g}\bar{g}'}{\sqrt{\bar{g}^2 + \bar{g}'^2}}\left(1 - \frac{\epsilon\bar{g}\bar{g}'}{\bar{g}^2 + \bar{g}'^2}\right)$$

$$\bar{g}_Z = \sqrt{\bar{g}^2 + \bar{g}'^2}\left(1 + \frac{\epsilon\bar{g}\bar{g}'}{\bar{g}^2 + \bar{g}'^2}\right)$$

Worth noting: 7 Wilson coefficients contribute to bilinear terms in gauge and Higgs sector: C^{φ}, $C^{\varphi\Box}$, $C^{\varphi D}, C^{\varphi W}, C^{\varphi B}, C^{\varphi WB}, C^{\varphi G}$.

In physical basis, only 2 parameters — $C^{\varphi D}$ and $C^{\varphi WB}$ — not absorbed in redefinitions!

$C^{\varphi D}$ coefficient breaks the custodial invariance:

$$\rho = \frac{|J_{C.C}|^2}{|J_{N.C.}|^2} = \frac{\bar{g}^2 \, M_Z^2}{\bar{g}_Z^2 M_W^2} = 1 + \frac{1}{2} C^{\varphi D} v^2$$

Experiment – $|\rho - 1|$ constrained at the level of 0.1%, strong upper bound on $C^{\varphi D}$.

Sizable "oblique" corrections in the gauge sector possible only from the gauge boson kinetic mixing term $C^{\varphi WB}$.

New Physics effects mostly "screened" from the SM structure!

Fermionic sector

4 operators contribute to fermion masses: $C^{\nu\nu}, C^{e\varphi}, C^{u\varphi}, C^{d\varphi}$

3×3 mass matrices:

$$M'_\nu = -v^2 C^{\nu\nu}\ , \qquad M'_e = \frac{v}{\sqrt{2}} \left(Y_e - C^{e\varphi}\frac{v^2}{2} \right),$$
$$M'_u = \frac{v}{\sqrt{2}} \left(Y_u - C^{u\varphi}\frac{v^2}{2} \right), \quad M'_d = \frac{v}{\sqrt{2}} \left(Y_d - C^{d\varphi}\frac{v^2}{2} \right)\ .$$

Diagonalization:

$$\psi'_X = U_{\psi_X}\, \psi_X\ ,$$

with $\psi = \nu, e, u, d,\ X = L, R$

$$U^\dagger_{\psi_L}\, M'_\psi\, U_{\psi_R} = \ M_\psi \qquad U^T_{\nu_L}\, M'_\nu\, U_{\nu_L} = M_\nu$$

Physical Higgs-fermion interactions in general not diagonal, e.g.:

$$\mathcal{L}_{huu} = \ - \ \bar{u} \left[\frac{M_u}{v} \left(1 - \frac{1}{4} C^{\varphi D} v^2 + C^{\varphi\Box} v^2 \right) - C^{u\varphi}\, \frac{v^2}{\sqrt{2}} \right] P_R\, u\, h + \text{H.c.}$$

"Oblique" corrections to flavor couplings

Right charged currents at the tree level exist!

$$\mathcal{L}_{c.c.} = -\frac{\bar{g}}{\sqrt{2}} W_\mu^+ \bar{u}_p \gamma^\mu \left\{ \left[U_{u_L}^\dagger U_{d_L} + v^2 C^{\varphi q(3)} \right]_{pr} P_L + \frac{v^2}{2} C_{pr}^{\varphi ud} P_R \right\} d_r$$
$$- \frac{\bar{g}}{\sqrt{2}} W_\mu^+ \bar{\nu}_p \gamma^\mu \left[U_{e_L}^\dagger U_{\nu_L} + v^2 C^{\varphi l(3)} \right]_{pr}^\dagger P_L e_r + \text{H.c.}$$

Physical CKM and PMNS matrices no longer unitary:

$$K_{\text{CKM}} \equiv K \equiv U_{u_L}^\dagger U_{d_L} + v^2 C^{\varphi q(3)}$$
$$U_{\text{PMNS}} \equiv U \equiv U_{e_L}^\dagger U_{\nu_L} + v^2 C^{\varphi l(3)}$$

SMEFT quantization

Quantization of the gauge theory - gauge fixing required.

Consistent and convenient choice of gauge fixing conditions and ghost sector:

- Cancel the unwanted Goldstone-gauge boson bilinear mixing.
- Lead to SM-like propagators in terms of the effective mass basis parameters and fields.
- Preserve the BRST invariance of the full Lagrangian.

SM preferred choices:

- unitary gauge for tree level calculations
- R_ξ gauges for loop calculations

Can R_ξ gauges be used in SMEFT? Non-trivial but yes, due to SM-like gauge-Goldstone mixing terms!

EW gauge fixing terms choice:

$$\mathcal{L}_{GF} = -\frac{1}{2}\boldsymbol{F}^{\top}\hat{\boldsymbol{\xi}}^{-1}\boldsymbol{F}$$

with

$$\boldsymbol{F} = \begin{pmatrix} F^1 \\ F^2 \\ F^3 \\ F^0 \end{pmatrix} = \begin{pmatrix} \partial_\mu \bar{W}^{1\mu} \\ \partial_\mu \bar{W}^{2\mu} \\ \partial_\mu \bar{W}^{3\mu} \\ \partial_\mu \bar{B}^{\mu} \end{pmatrix} - \frac{v\hat{\boldsymbol{\xi}}}{2} \begin{pmatrix} -i\bar{g}\frac{G^+ - G^-}{\sqrt{2}} \\ \bar{g}\frac{G^+ + G^-}{\sqrt{2}} \\ -\bar{g}Z_{G^0}G_0 \\ \bar{g}' Z_{G^0}G_0 \end{pmatrix}$$

4×4 symmetric matrix $\hat{\xi}$ is

$$\hat{\boldsymbol{\xi}} = \begin{pmatrix} \xi_W & & 0 \\ & \xi_W & \\ 0 & & \mathbb{X}\begin{pmatrix} \xi_Z & \\ & \xi_A \end{pmatrix}\mathbb{X}^{\top} \end{pmatrix}$$

($\mathbb{X}$ is 2×2 mixing matrix of the neutral electroweak gauge bosons)

Gauge fixing expressed in physical basis:

$$\mathcal{L}_{GF} = \; - \; \frac{1}{\xi_W}(\partial^\mu W_\mu^+ + i\xi_W M_W G^+)(\partial^\nu W_\nu^- - i\xi_W M_W G^-) \\ - \; \frac{1}{2\xi_Z}(\partial^\mu Z_\mu + \xi_Z M_Z G^0)^2 - \frac{1}{2\xi_A}(\partial^\mu A_\mu)^2$$

Identical to the SM with the standard linear R_ξ gauges fixing!

Ghost terms necessary to restore BRST invariance:

$$\mathcal{L}_{FP} = \bar{\boldsymbol{N}}^\top \begin{pmatrix} \mathbb{1}_{2\times 2} & 0_{2\times 2} \\ 0_{2\times 2} & (\mathbb{X}^\top)^{-1}\mathbb{X}^{-1} \end{pmatrix} (\hat{\boldsymbol{M}}_F N)$$

where ghosts $N^i = (N^1, N^2, N^3, N^0)$ and antighosts $\bar{N}^i = (\bar{N}^1, \bar{N}^2, \bar{N}^3, \bar{N}^0)$

Matrix $\hat{M}_F$ - obtained by applying the BRST-operator, $\boldsymbol{s}$ on gauge-fixing functionals F^i:

$$\hat{M}_F^{ij} N^j = \boldsymbol{s} F^i$$

On “barred” fields:

$$\begin{aligned}
\boldsymbol{s}\varphi &= -i\bar{g}' Y \varphi \, N^0 - i\bar{g} T^I \varphi \, N^I \, , \\
\boldsymbol{s}\varphi^\dagger &= +i\bar{g}' \varphi^\dagger \, Y N^0 + i\bar{g} \varphi^\dagger T^I \, N^I \, , \\
\boldsymbol{s}\bar{B}_\mu &= \partial_\mu N^0 \, , \\
\boldsymbol{s}\bar{W}_\mu^I &= \partial_\mu N^I - \bar{g}\epsilon^{IJK} \bar{W}_\mu^J N^K \, .
\end{aligned}$$

Result: in mass basis canonical bilinear ghost terms with squared masses

$$M_{\eta^A}^2 = 0, \quad M_{\eta^W}^2 = \xi_W M_W^2, \quad M_{\eta^Z}^2 = \xi_Z M_Z^2.$$

QCD sector trivial, ghost interactions identical as in the SM:

$$\mathcal{L}_{GF} + \mathcal{L}_{FP} = -\frac{1}{2\xi_G}(\partial_\mu g^{A\mu})^2 + \bar{\eta}_G^A \partial^2 \eta_G^A + (\partial_\mu \bar{\eta}_G^A)\bar{g}_s f^{ABC} g^{B\mu} \eta_G^C$$

Further checks: explicit BRST invariance of SMEFT action

Main Lagrangian: BRST invariance follows from the gauge invariance.

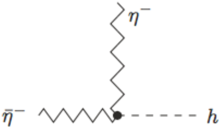
After explicit calculation:

- Gauge fixing + ghost terms also invariant under BRST
- As required, BRST transformation nilpotent, $\boldsymbol{s}^{\mathbf{2}}(any\ field) = 0$

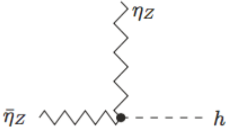
Proves consistency of our approach

Ghost propagators identical as in SM $\rightarrow$ corrections from dimension-6 operators appear in ghost vertices.

Examples:



$$+\frac{1}{4}i\bar{g}^2 v\xi_W + \frac{1}{4}i\bar{g}^2 v^3 \xi_W C^{\varphi\Box} - \frac{1}{16}i\bar{g}^2 v^3 \xi_W C^{\varphi D}$$



$$+\frac{1}{4}iv\xi_Z\left(\bar{g}^2+\bar{g}'^2\right) + \frac{1}{4}iv^3\xi_Z\left(\bar{g}^2+\bar{g}'^2\right)C^{\varphi\Box} + \frac{1}{16}iv^3\xi_Z\left(\bar{g}^2+\bar{g}'^2\right)C^{\varphi D} + \frac{1}{2}i\bar{g}\bar{g}'v^3\xi_Z C^{\varphi WB}$$

Mini-summary:

- physical fields identified
- coupling shifts defined
- gauge fixing terms and ghost Lagrangian derived

Everything ready for amplitude calculations?

Not so simple: $\sim$ 100 complicated vertices in unitary gauge, $\sim$ 500 vertices in R_ξ gauges, up to 6-tuple field interactions appear!

Full ready-to-use list of Feynman rules in 2017 JHEP paper

Still: calculations needs to be automatized...

SMEFT package

Dedicated **Mathematica/FeynRules** package developed to calculate Feynman rules.

Package structure:

- starting point: SM Lagrangian + extra operators/parameters encoded using **FeynRules** syntax
- evaluation of bilinears and canonically normalized physical fields
- derivation of SMEFT Lagrangian in mass-eigenstates basis, consistently to order $1/\Lambda^2$
- evaluation of Feynman rules in **Mathematica** format
- translation of Feynman rules to printable **LaTeX** format
- to be published soon: exporting Feynman rules in **UFO** (Universal File) format), so they can be directly imported to Monte Carlo generators.

Application: $h \to \gamma\gamma$ decay in SMEFT

SM prediction: finite and precise, small theoretical uncertaininities.

Experiments report results relative to SM prediction:

$$\mathcal{R}_{h\to\gamma\gamma} \equiv 1 + \delta\mathcal{R}_{h\to\gamma\gamma} = \frac{\Gamma(\text{EXP}, h \to \gamma\gamma)}{\Gamma(\text{SM}, h \to \gamma\gamma)} \,.$$

Current constraints:

$$\begin{aligned} \text{ATLAS:} \qquad \mathcal{R}_{h\to\gamma\gamma} &= 0.99^{+0.15}_{-0.14}\,, \\ \text{CMS:} \qquad \mathcal{R}_{h\to\gamma\gamma} &= 1.18^{+0.17}_{-0.14}\,, \end{aligned}$$

Radiative process, small in SM - sensitive to contributions from new operators.

Prior to our work the most complete calculation: C. Hartmann and M. Trott, arXiv:1507.03568, 1505.02646

Improvements in our analysis:

- exploiting linear R_ξ-gauges (previously technically complicated BFM gauge fixing).
- analytic proof of gauge invariance
- simple renormalization framework
- complete analytical and semi-numerical expressions for $\delta R_{h\to\gamma\gamma}$
- bounds on Wilson coefficients

"Serious" 1-loop calculation:

- 17 CP conserving dim-6 operators contribute, not counting flavor and H.c. (neglected: 10 CP violating ones, strongly suppressed by CP observables like EDM etc.)
- complicated structure of interaction vertices, 3-, 4- and 5-tuple, some momentum dependent, many include scalar and tensor Dirac structures.
- non-trivial multi-parameter renormalization procedure.
- calculation performed in general R_ξ gauges with independent ξ_W, ξ_Z parameters - difficult but ξ cancellation provides very strict cross checks.

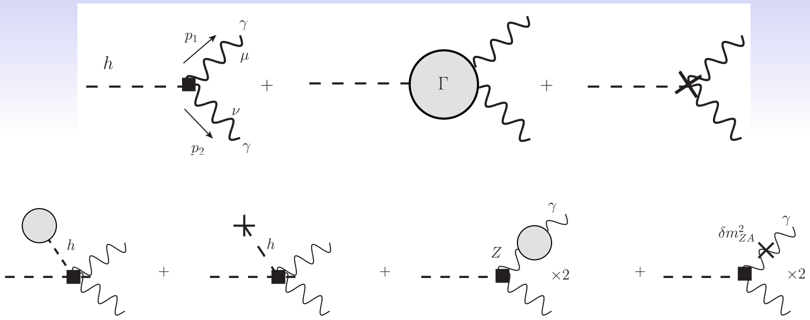
Contributing CP-conserving operators:

$Q_W = \varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{e\varphi} = (\varphi^\dagger\varphi)(\bar{l}'_p e'_r \varphi)$
$Q_{\varphi\Box} = (\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi} = (\varphi^\dagger\varphi)(\bar{q}'_p u'_r \widetilde{\varphi})$
$Q_{\varphi D} = \left(\varphi^\dagger D^\mu\varphi\right)^* \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi} = (\varphi^\dagger\varphi)(\bar{q}'_p d'_r \varphi)$
$Q_{\varphi B} = \varphi^\dagger\varphi\, B_{\mu\nu}B^{\mu\nu}$	$Q_{ll} = (\bar{l}'_p\gamma_\mu l'_r)(\bar{l}'_s\gamma^\mu l'_t)$
$Q_{\varphi W} = \varphi^\dagger\varphi\, W^I_{\mu\nu}W^{I\mu\nu}$	$Q^{(3)}_{\varphi l} = (\varphi^\dagger i\overleftrightarrow{D}^I_\mu\,\varphi)(\bar{l}'_p\tau^I\gamma^\mu l'_r)$
$Q_{\varphi WB} = \varphi^\dagger\tau^I\varphi\, W^I_{\mu\nu}B^{\mu\nu}$	
$Q_{eB} = (\bar{l}'_p\sigma^{\mu\nu}e'_r)\varphi B_{\mu\nu}$	$Q_{eW} = (\bar{l}'_p\sigma^{\mu\nu}e'_r)\tau^I\varphi W^I_{\mu\nu}$
$Q_{uB} = (\bar{q}'_p\sigma^{\mu\nu}u'_r)\widetilde{\varphi}\, B_{\mu\nu}$	$Q_{uW} = (\bar{q}'_p\sigma^{\mu\nu}u'_r)\tau^I\widetilde{\varphi}\, W^I_{\mu\nu}$
$Q_{dB} = (\bar{q}'_p\sigma^{\mu\nu}d'_r)\varphi\, B_{\mu\nu}$	$Q_{dW} = (\bar{q}'_p\sigma^{\mu\nu}d'_r)\tau^I\varphi\, W^I_{\mu\nu}$

Q_φ - present in vertices but cancels out completely in the amplitude.

Q_{ll} and $Q^{(3)}_{\varphi l}$ enter indirectly through corrections to Fermi constant.

On-shell S-matrix amplitude



plus external wave function renormalizations for the photon and the Higgs required by the LSZ reduction formula.

Renormalization

We assume perturbative renormalization. We are working at 1-loop and up to $1/\Lambda^2$ in EFT expansion.

- hybrid renormalization scheme: on-shell in SM-quantities (a la Sirlin 1980) and $\overline{MS}$ in Wilson coefficients
- all infinities absorbed by SM and EFT counterterms as normal
- a closed expression for the amplitude that respects the Ward-Identities

The renormalized parameters are translated to well measured ones

$$\{g', g, v, \lambda, y_t\} \longrightarrow \{M_Z, M_W, G_F, M_h, m_t\}$$

Fermi constant G_F - derived from the muon lifetime (tree level):

$$\frac{G_F}{\sqrt{2}} = \frac{\bar{g}^2}{8\sqrt{2}M_W^2}\left[1 + v^2(C_{11}^{\varphi l(3)} + C_{22}^{\varphi l(3)}) - v^2 C_{1221}^{ll}\right]$$

Wilson coefficients renormalized in MS-bar scheme $\rightarrow$ renormalization scale dependent.

$$C - \delta C = \bar{C}(\mu) - \delta\bar{C}$$

Full renormalization scheme - complicated (as the model itself) but fairly standard procedure, in spite of working with non-renormalizable theory.

Nothing special w.r.t. textbook renormalization technics !!

Renormalized amplitude:

$$i\mathcal{A}^{\mu\nu}(h\to\gamma\gamma) = \langle \gamma(\epsilon^\mu, p_1),\, \gamma(\epsilon^\nu, p_2) \,|\, S \,|\, h(q)\,\rangle = 4i\,\left[p_1^\nu\, p_2^\mu - (p_1\cdot p_2)\, g^{\mu\nu} \right]\, \mathcal{A}_{h\to\gamma\gamma}$$

where

$$\begin{aligned}
\mathcal{A}_{h\to\gamma\gamma} \quad &= \quad \Bigg\{ c^2\, v\, \bar{C}^{\varphi B}(\mu) \left[1 + \Gamma^{\varphi B} - \frac{\delta v}{v} + \frac{1}{2}\Pi'_{HH}(M_h^2) - \Pi_{\gamma\gamma}(0) + 2\tan\theta_W\, \frac{\Pi_{Z\gamma}(0) + \delta m^2_{Z\gamma}}{M_Z^2} \right] \\
&+ \quad s^2\, v\, \bar{C}^{\varphi W}(\mu) \left[1 + \Gamma^{\varphi W} - \frac{\delta v}{v} + \frac{1}{2}\Pi'_{HH}(M_h^2) - \Pi_{\gamma\gamma}(0) - \frac{2}{\tan\theta_W}\, \frac{\Pi_{Z\gamma}(0) + \delta m^2_{Z\gamma}}{M_Z^2} \right] \\
&- \quad sc\, v\, \bar{C}^{\varphi WB}(\mu) \left[1 + \Gamma^{\varphi WB} - \frac{\delta v}{v} + \frac{1}{2}\Pi'_{HH}(M_h^2) - \Pi_{\gamma\gamma}(0) - \frac{2}{\tan 2\theta_W}\, \frac{\Pi_{Z\gamma}(0) + \delta m^2_{Z\gamma}}{M_Z^2} \right] \\
&+ \quad \frac{1}{M_W}\overline{\Gamma}^{\rm SM} + \sum_{X\neq\varphi B,\varphi W,\varphi WB} v\, C^X(\mu)\, \Gamma^X \Bigg\}_{\rm finite}
\end{aligned}$$

Analytical result given in the paper. We checked that $\mathcal{A}_{h\to\gamma\gamma}$ is:

- finite
- gauge invariant (ξ indepedent)
- renormalisation scale invariant, in the sense $\frac{d}{d\mu}\mathcal{A}_{h\to\gamma\gamma}(\mu) = 0$ (proven using RGE for $C's$ by Alonso, Jenkins, Manohar, Trott).

Comparison with only existing full calculation (Hartmann & Trott 2015) - we found some errors, missing 1/4 in normalization and Yukawa coupling factors.

Semi-analytic formula

$$
\begin{aligned}
\delta\mathcal{R}_{h\to\gamma\gamma} \quad \simeq \quad & 0.06\left(\frac{C^{\ell\ell}_{1221}-C^{\varphi\ell(3)}_{11}-C^{\varphi\ell(3)}_{22}}{\Lambda^2}\right)+0.12\left(\frac{C^{\varphi\Box}-\frac{1}{4}C^{\varphi D}}{\Lambda^2}\right)\\
- \quad & 0.01\left(\frac{C^{e\varphi}_{22}+4C^{e\varphi}_{33}+5C^{u\varphi}_{22}+2C^{d\varphi}_{33}-3C^{u\varphi}_{33}}{\Lambda^2}\right)\\
- \quad & \left[48.04-1.07\log\frac{\mu^2}{M_W^2}\right]\frac{C^{\varphi B}}{\Lambda^2}-\left[14.29-0.12\log\frac{\mu^2}{M_W^2}\right]\frac{C^{\varphi W}}{\Lambda^2}\\
+ \quad & \left[26.62-0.52\log\frac{\mu^2}{M_W^2}\right]\frac{C^{\varphi WB}}{\Lambda^2}+\left[0.16-0.22\log\frac{\mu^2}{M_W^2}\right]\frac{C^{W}}{\Lambda^2}\\
+ \quad & \left[2.11-0.84\log\frac{\mu^2}{M_W^2}\right]\frac{C^{uB}_{33}}{\Lambda^2}+\left[1.13-0.45\log\frac{\mu^2}{M_W^2}\right]\frac{C^{uW}_{33}}{\Lambda^2}\\
- \quad & \left[0.03+0.01\log\frac{\mu^2}{M_W^2}\right]\frac{C^{uB}_{22}}{\Lambda^2}-\left[0.01+0.00\log\frac{\mu^2}{M_W^2}\right]\frac{C^{uW}_{22}}{\Lambda^2}\\
+ \quad & \left[0.03+0.01\log\frac{\mu^2}{M_W^2}\right]\frac{C^{dB}_{33}}{\Lambda^2}-\left[0.02+0.01\log\frac{\mu^2}{M_W^2}\right]\frac{C^{dW}_{33}}{\Lambda^2}\\
+ \quad & \left[0.02+0.00\log\frac{\mu^2}{M_W^2}\right]\frac{C^{eB}_{33}}{\Lambda^2}-\left[0.01+0.00\log\frac{\mu^2}{M_W^2}\right]\frac{C^{eW}_{33}}{\Lambda^2}+\ldots,
\end{aligned}
$$

Only 5 operators contribute significantly and can be bounded by the current LHC experimental measurement. Taking $\mu = M_W$, one has

$$\frac{|C^{\varphi B}|}{\Lambda^2} \lesssim \frac{0.003}{(1\ \text{TeV})^2}, \qquad \frac{|C^{\varphi W}|}{\Lambda^2} \lesssim \frac{0.011}{(1\ \text{TeV})^2},$$

$$\frac{|C^{\varphi WB}|}{\Lambda^2} \lesssim \frac{0.006}{(1\ \text{TeV})^2},$$

$$\frac{|C^{uB}_{33}|}{\Lambda^2} \lesssim \frac{0.071}{(1\ \text{TeV})^2}, \qquad \frac{|C^{uW}_{33}|}{\Lambda^2} \lesssim \frac{0.133}{(1\ \text{TeV})^2}.$$

Competing constraints on $C^{\varphi B}, C^{\varphi W}, C^{\varphi WB}$ from EW precision measurements - similar order of magnitude.

Constraints on C^{33}_{uB} and c^{33}_{uW} the $\bar{t}tZ$ and single top production measurements at LHC: *more than an order of magnitude weaker*.

Conclusions

1. Effective Field Theory extension of the SM is the important phenomenological tool for the New Physics description.
2. "Warsaw basis" derivation revived interest in SMEFT analyses. Very complicated model, requires systematic step by step development.
3. Recent progress - expressing the SMEFT Lagrangian in terms of physical fields, correct and technically workable gauge fixing, public **Mathematica** code calculating full set of Feynman rules both in symbolic and printable formats (download at `www.fuw.edu.pl/smeft`).
4. Starting point for systematic and consistent analyses of physical processes
5. Example of a non-trivial loop calculation: $h \to \gamma\gamma$ decay. Testing field for Feynman rules, gauge invariance and renormalization procedures.
6. In progress, technically even more complicated but soon to be measured: $h \to Z\gamma$ decay.