Decay Rate of the Electroweak Vacuum in the Standard Model and Beyond

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Endo, TM, Nojiri, Shoji, PLB 771 ('17) 281 [1703.09304]
Endo, TM, Nojiri, Shoji, JHEP 1711 ('17) 074 [1704.03492]
Chigusa, TM, Shoji, PRL 119 ('17) 211801 [1707.09301]
Chigusa, TM, Shoji, PRD (to appear) [1803.03902]

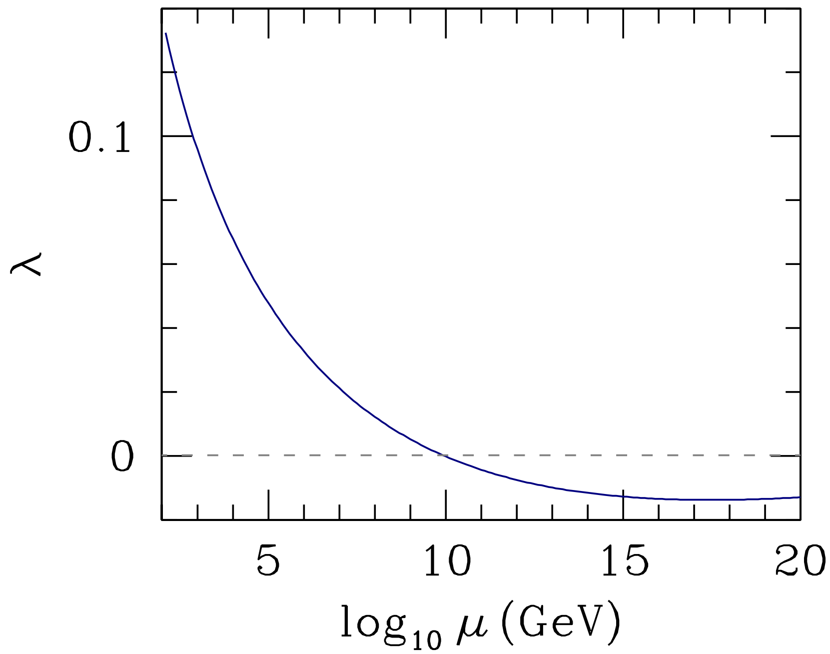
Seminar at Osaka University, '18.06.08

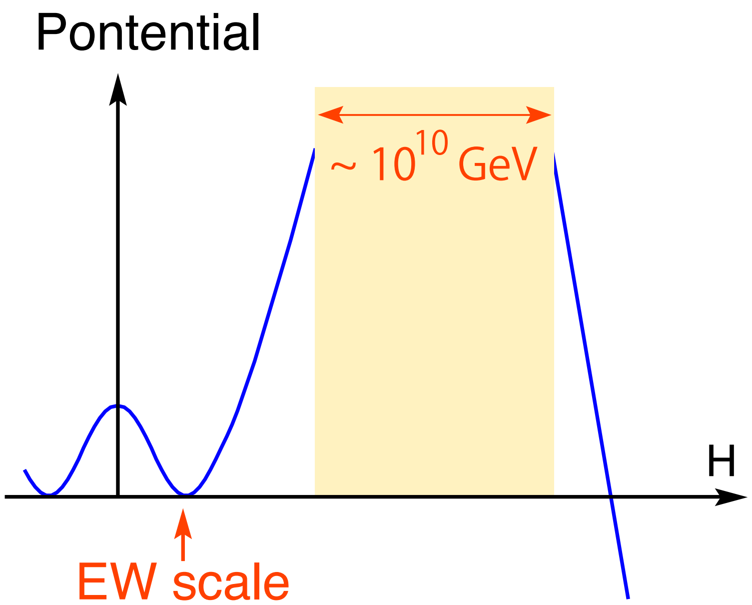
1. Introduction

What we learn from the Higgs mass

$$m_h \simeq 125 \text{ GeV} \quad \Rightarrow \quad V = \lambda(|H|^2 - v^2)^2 \text{ with } \lambda(m_h) \simeq 0.13$$

λ becomes negative at a very high scale





- EW vacuum is not stable in the standard model (SM)
- λ is minimized at $\mu \sim 10^{17}$ GeV

Is the decay rate small enough so that $t_{\text{now}} \simeq 13.6$ Gyr?

⇒ Many previous works said "yes"

[Isidori, Ridolfi & Strumia; Degrassi et al.; Alekhin, Djouadi & Moch; Espinosa et al.; Plascencia & Tamarit; Lalak, Lewicki & Olszewski; Espinosa, Garny, Konstandin & Riotto; ···]

How precisely can we estimate the decay rate?

- Gauge-invariance of the result was unclear
- Effects of zero modes were not properly taken into account
- There has been progresses in the calculation of the decay rate of false vacuum

  [Endo, TM, Nojiri & Shoji; Chigusa, TM & Shoji; see also Andreassen, Frost & Schwartz]

Today, I discuss

- A precise calculation of the decay rate of EW vacuum
- Effects of extra matters

<u>Outline</u>

1. Introduction
2. Bounce in the SM
3. Effects of Higgs Mode
4. Comment on Gauge and NG Contributions
5. Total Decay Rate
6. Case with Extra Matters
7. Summary

2. Bounce in the SM

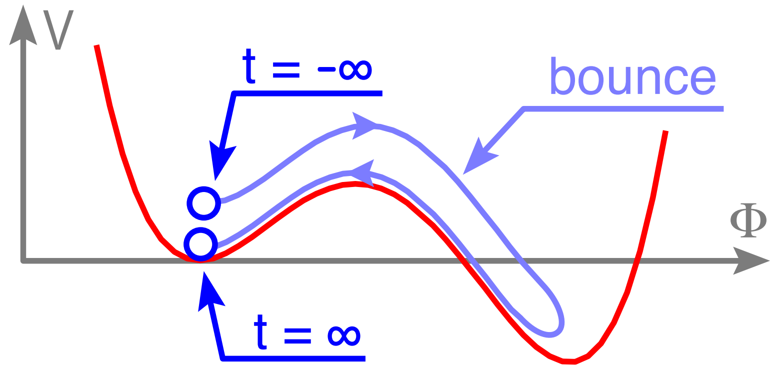
The decay rate is related to 4D Euclidean partition function

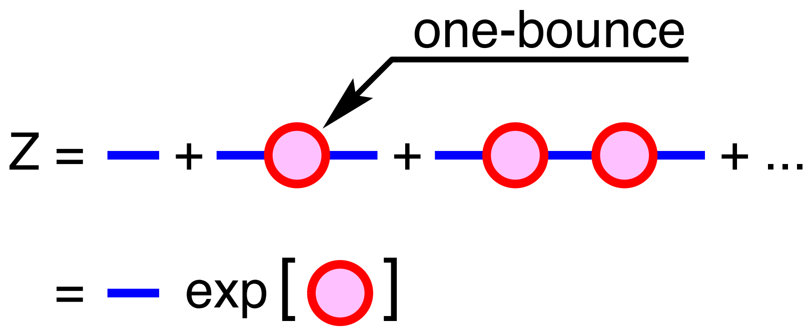
[Coleman; Callan & Coleman]

$$Z = \langle \mathrm{FV} | e^{-HT} | \mathrm{FV} \rangle \propto \exp(i\gamma VT)$$

The path integral is dominated by the "bounce"

Bounce: a saddle-point solution of classical EoM





$$\gamma \simeq \frac{1}{VT} \mathrm{Im} \left[\frac{\int_{\text{1-bounce}} \mathcal{D}\Psi \, e^{-S_{\mathrm{E}}}}{\int_{\text{0-bounce}} \mathcal{D}\Psi \, e^{-S_{\mathrm{E}}}} \right] \equiv \mathcal{A} e^{-\mathcal{B}} \quad \text{with } \mathcal{B} = S_{\mathrm{E}}(\text{Bounce})$$

Main concern of this talk: calculation of the prefactor $\mathcal{A}$

$\Leftrightarrow$ $\mathcal{A}$ takes account of loop effects

We expand the action around the classical EoM

$$S_{\mathsf{E}}[\bar{\phi}+\Psi] = S_{\mathsf{E}}[\bar{\phi}] + \frac{1}{2}\int d^4x \Psi \mathcal{M} \Psi + O(\Psi^3)$$

$$S_{\mathsf{E}}[v+\Psi] = S_{\mathsf{E}}[v] + \frac{1}{2}\int d^4x \Psi \widehat{\mathcal{M}} \Psi + O(\Psi^3)$$

Prefactor $\mathcal{A}$ (for bosonic contribution)

$$\mathcal{A} \simeq \frac{1}{VT}\left|\frac{\mathsf{Det}\mathcal{M}}{\mathsf{Det}\widehat{\mathcal{M}}}\right|^{-1/2} \propto \prod_n \sqrt{\frac{\widehat{\omega}_n}{\omega_n}} \quad \text{with} \begin{cases} \omega_n = \text{eigenvalue of } \mathcal{M} \\ \widehat{\omega}_n = \text{eigenvalue of } \widehat{\mathcal{M}} \end{cases}$$

Sometimes $\mathcal{M}$ has zero eigenvalue

$\Rightarrow$ A careful treatment is needed

Higgs potential in the SM: $V = -m^2 H^\dagger H + \lambda (H^\dagger H)^2$

- We consider very large Higgs amplitude for which $\lambda < 0$
- We will neglect quadratic term because $\lambda < 0$ occurs at a scale much higher than the EW scale

We use the following potential (choosing $\mu \gg 10^{10}$ GeV):

$$V = -|\lambda| (H^\dagger H)^2$$

The "bounce solution" (Fubini-Lipatov instanton)

$$H_{\text{bounce}} = \frac{1}{\sqrt{2}} e^{i\sigma^a \theta^a} \begin{pmatrix} 0 \\ \bar{\phi} \end{pmatrix} \quad \text{with } \partial_r^2 \bar{\phi} + \frac{3}{r} \partial_r \bar{\phi} + 3|\lambda| \bar{\phi}^2 = 0$$

$\Rightarrow$ Explicit form of the bounce:

$$\bar{\phi}(r) = \sqrt{\frac{8}{|\lambda|}} R^{-1} \frac{1}{1 + R^{-2} r^2} \quad \text{with } R = \text{(free parameter)}$$

Bounce action for the SM

$$\mathcal{B} = \frac{8\pi^2}{3|\lambda|}$$

Possible deformations of the bounce

- Dilatation: parameterized by R
- $SU(2)$ transformation: parameterized by θ^a

  Effects of zero modes in association with these transformations were not properly taken into account before
- Translation

  [Callan & Coleman]

Expansion around the bounce:

$$H = \frac{1}{\sqrt{2}} e^{i\sigma^a\theta^a} \begin{pmatrix} \varphi^1 + i\varphi^2 \\ \bar{\phi} + h - i\varphi^3 \end{pmatrix}, \quad W^a_\mu = w^a_\mu, \quad B_\mu = b_\mu$$

3. Effects of the Higgs Mode

We need to calculate the functional determinant of $\mathcal{M}^{(h)}$

$$\mathcal{L} \ni \frac{1}{2} h \left(-\partial^2 - 3|\lambda|\bar{\phi}^2\right) h = \frac{1}{2} h \, \mathcal{M}^{(h)} \, h$$

Expansion of h w.r.t. 4D spherical harmonics $\mathcal{Y}_{J,m_A,m_B}$

$$h(x) = \sum_{J,m_A,m_B,n} c_{n,J,m_A,m_B} \, \mathcal{G}_{n,J}(r) \, \mathcal{Y}_{J,m_A,m_B}(\hat{\mathbf{r}})$$

$J = 0, \ 1/2, \ 1, \ 3/2, \ \cdots$

$\mathcal{G}_{n,J}$: radial mode function

c_{n,J,m_A,m_B}: expansion coefficient

Fluctuation operator for angular-momentum eigenstates:

$$\mathcal{M}_J^{(h)} \equiv -\left(\Delta_J + 3|\lambda|\bar{\phi}^2\right) \equiv -\left[\partial_r^2 + \frac{3}{r}\partial_r - \frac{4J(J+1)}{r^2} + 3|\lambda|\bar{\phi}^2\right]$$

Radial mode function $\mathcal{G}_{n,J}$

- $\mathcal{M}_J^{(h)} \mathcal{G}_{n,J}(r) = \omega_{n,J} \mathcal{G}_{n,J}(r)$
- $\mathcal{G}_{n,J}(r = 0) < \infty$
- $\mathcal{G}_{n,J}(r \to \infty) = 0$

Higgs-mode contribution to the prefactor $\mathcal{A}$

$$\mathcal{A}^{(h)} = \prod_J \left[\frac{\text{Det}\mathcal{M}_J^{(h)}}{\text{Det}\widehat{\mathcal{M}}_J^{(h)}} \right]^{-(2J+1)^2/2} \simeq \prod_{n,J} \left[\frac{\omega_{n,J}}{\widehat{\omega}_{n,J}} \right]^{-(2J+1)^2/2}$$

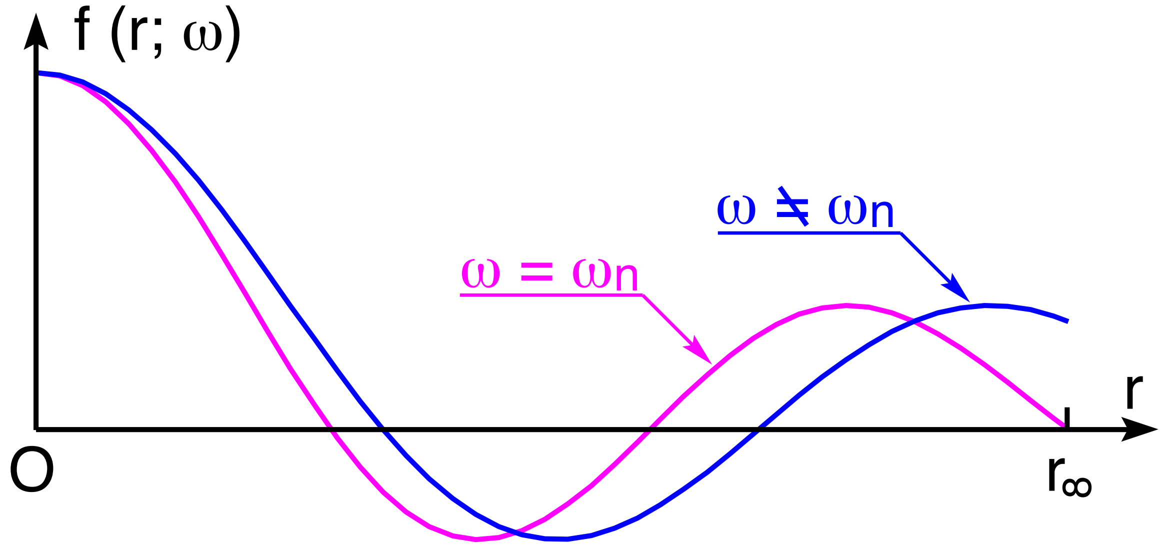
The ratio of the functional determinants can be evaluated with so-called Gelfand-Yaglom theorem

Functional determinant for operators defined in $0 \le r \le r_\infty$

$$\mathrm{Det}\mathcal{M} \simeq \prod_n \omega_n \text{ with } \begin{cases} \mathcal{M}\mathcal{G}_n = \omega_n \mathcal{G}_n \text{ with } \mathcal{M} = -\Delta_J + \delta W(r) \\ \mathcal{G}_n(0) < \infty \\ \mathcal{G}_n(r_\infty) = 0 \end{cases}$$

We introduce a function f which obeys: $\mathcal{M} f(r;\omega) = \omega f(r;\omega)$

- $f(r = r_\infty; \omega)|_{\omega=\omega_n} = 0$
- $\mathrm{Det}(\mathcal{M} - \omega)|_{\omega=\omega_n} = 0$



Gelfand-Yaglom theorem

[Gelfand & Yaglom; Coleman; Dashen, Hasslacher & Neveu; Kirsten & McKane]

$$\frac{\text{Det}(\mathcal{M}-\omega)}{\text{Det}(\widehat{\mathcal{M}}-\omega)} = \frac{f(r=r_\infty;\omega)}{\hat{f}(r=r_\infty;\omega)} \text{ with } \begin{cases} \mathcal{M}f(r;\omega) = \omega f(r;\omega) \\ \widehat{\mathcal{M}}\hat{f}(r;\omega) = \omega \hat{f}(r;\omega) \\ f(r=0) = \hat{f}(r=0) < \infty \end{cases}$$

$\Rightarrow$ Notice: LHS and RHS have the same analytic behavior

- LHS and RHS have same zeros and infinities
- LHS and RHS becomes equal to 1 when $\omega \to \infty$

We need:

$$\frac{\text{Det}\mathcal{M}}{\text{Det}\widehat{\mathcal{M}}} = \frac{f(r=\infty;0)}{\hat{f}(r=\infty;0)} \quad \text{with} \quad \mathcal{M}f(r;0) = \widehat{\mathcal{M}}\hat{f}(r;0) = 0$$

Zero modes exist for $\mathcal{M}^{(h)}$

- Dilatational zero mode (for $J = 0$)

$$\mathcal{G}_{\mathsf{D}}(r) \propto \frac{\partial \bar{\phi}}{\partial R} \quad \Leftrightarrow \quad \mathcal{M}_0^{(h)} \mathcal{G}_{\mathsf{D}}(r) = 0 \text{ and } \mathcal{G}_{\mathsf{D}}(r \to \infty) = 0$$

- Translational zero modes (for $J = 1/2$)

[Callan & Coleman]

$$\mathcal{G}_{\mathsf{T}}(r) \propto \frac{\partial \bar{\phi}}{\partial r} \quad \Leftrightarrow \quad \mathcal{M}_{1/2}^{(h)} \mathcal{G}_{\mathsf{T}}(r) = 0 \text{ and } \mathcal{G}_{\mathsf{T}}(r \to \infty) = 0$$

Gauge zero modes are in gauge and NG sector

- A gauge transformation of the bounce gives the gauge zero mode

- A global $SU(2) \times U(1)$ symmetry remains after gauge fixing

Path integral over dilatational zero mode = integral over R

$$H \ni \bar{\phi} + h = \bar{\phi} + c_{\mathsf{D}} \mathcal{N}_{\mathsf{D}} \frac{\partial \bar{\phi}}{\partial R} + \cdots \simeq \bar{\phi}\Big|_{R \to R + c_{\mathsf{D}} \mathcal{N}_{\mathsf{D}}} + \cdots$$

$$\Rightarrow \int \mathcal{D}h^{(\mathsf{dilatation})} \equiv \int dc_{\mathsf{D}} \to \int \frac{dR}{\mathcal{N}_{\mathsf{D}}}$$

$$\Rightarrow \left[\frac{\mathsf{Det}\mathcal{M}_0^{(h)}}{\mathsf{Det}\widehat{\mathcal{M}}_0^{(h)}} \right]^{-1/2} \to \int \frac{dR}{\mathcal{N}_{\mathsf{D}}} \left[\frac{\mathsf{Det}'\mathcal{M}_0^{(h)}}{\mathsf{Det}\widehat{\mathcal{M}}_0^{(h)}} \right]^{-1/2}$$

Det': zero eigenvalue is omitted from the Det

Higgs-mode contribution:

[Chigusa, TM & Shoji; Andreassen, Frost & Schwartz]

$$\mathcal{A}^{(h)} \to \int d \ln R \left(\frac{16\pi}{|\lambda|} \right)^{1/2} \prod_{J \geq 1/2} \left[\frac{\mathsf{Det}\mathcal{M}_J^{(h)}}{\mathsf{Det}\widehat{\mathcal{M}}_J^{(h)}} \right]^{-(2J+1)^2/2}$$

4. Comment on gauge and NG contribution

Gauge fixing is important

- Gauge fixing function in previous analysis (for $U(1)$)

$$\mathcal{F} = \partial_\mu B_\mu - 2\xi g(\mathrm{Re}H)(\mathrm{Im}H) \;\Rightarrow\; \mathcal{L} \ni \frac{1}{2\xi}\mathcal{F}^2 + \cdots$$

- The gauge-fixing terms depend on Higgs field

With such a choice, gauge and NG fields couple in the EoM

⇒ Bounce configuration

$$H = \frac{1}{\sqrt{2}}\bar{\phi}e^{i\Theta(r)}, \qquad A_\mu = \frac{1}{g}\partial_\mu\Theta(r)$$

$$\partial_r^2\Theta + \frac{3}{r}\partial_r\Theta - \frac{1}{2}\xi g^2\bar{\phi}^2 \sin 2\Theta = 0$$

⇒ Gauge zero modes were not properly treated (and I don't know how to deal with them with this gauge fixing)

Gauge fixing function in our calculation

[Kusenko, Lee & Weinberg]

$$\mathcal{F} = \partial_\mu B_\mu, \;\; \mathcal{F}^a = \partial_\mu W_\mu^a$$

General form of the bounce

$$H_{\text{bounce}} = \frac{1}{\sqrt{2}} e^{i\sigma^a \theta^a} \begin{pmatrix} 0 \\ \bar{\phi} \end{pmatrix}, \qquad B_\mu = W_\mu = 0$$

$\theta^a =$ constants

Path integral over gauge zero modes = integral over θ^a

$$\left[\frac{\text{Det}\mathcal{M}^{(W,Z,\text{NG})}}{\text{Det}\widehat{\mathcal{M}}^{(W,Z,\text{NG})}} \right]^{-1/2} \to \mathcal{V}_{SU(2)} \left(\frac{16\pi}{|\lambda|} \right)^{3/2} \prod_{J \geq 1/2} \left[\frac{\text{Det}\mathcal{M}_J^{(W,Z,\text{NG})}}{\text{Det}\widehat{\mathcal{M}}_J^{(W,Z,\text{NG})}} \right]^{-1/2}$$

5. Total Decay Rate

Decay rate:

$$\gamma = \int d\ln R \left[I^{(h)} I^{(W,Z,\mathrm{NG})} I^{(t)} e^{-\mathcal{S}_{\mathrm{C.T.}}} e^{-\mathcal{B}} \right]$$

We derived complete and gauge-invariant expressions of $I^{(X)}$

$I^{(h)}$: Higgs contribution

$I^{(W,Z,\mathrm{NG})}$: gauge and NG contribution

$I^{(t)}$: top contribution

We are calculating the effective action at one-loop

⇒ Renormalization is necessary

⇒ We subtract the divergence with $\overline{\mathrm{MS}}$ scheme

⇒ The result has μ-dependence

Choice of μ?

$$\gamma^{(\text{one-loop})} \propto \int d\ln R \frac{1}{R^4} \exp\left[-\frac{8\pi^2}{3|\lambda(\mu)|} - \frac{8\pi^2 \beta_\lambda^{(1)}(\mu)}{3|\lambda(\mu)|^2} \ln(\mu R) \right]$$

⇒ The μ-dependence vanishes at the leading-log level

[Endo, TM, Nojiri & Shoji]

We take the renormalization scale as $\mu \sim 1/R$

$$\gamma = \int d\ln R \left[I^{(h)} I^{(W,Z,\text{NG})} I^{(t)} e^{-\mathcal{S}_{\text{C.T.}}} e^{-\mathcal{B}} \right]_{\mu \sim 1/R}$$

⇒ The effects of μ-dependent terms from higher loops, i.e., $\sim \ln^p(\mu R)$, are expected to be minimized

⇒ This is important for the convergence of the integral

We use:

- $m_h = 125.09 \pm 0.24$ GeV
- $m_t = 173.1 \pm 0.6$ GeV
- $\alpha_s(m_Z) = 0.1181 \pm 0.0011$
- 3-loop RGEs (with relevant threshold corrections)

Decay rate of the EW vacuum (taking $\mu = 1/R$)

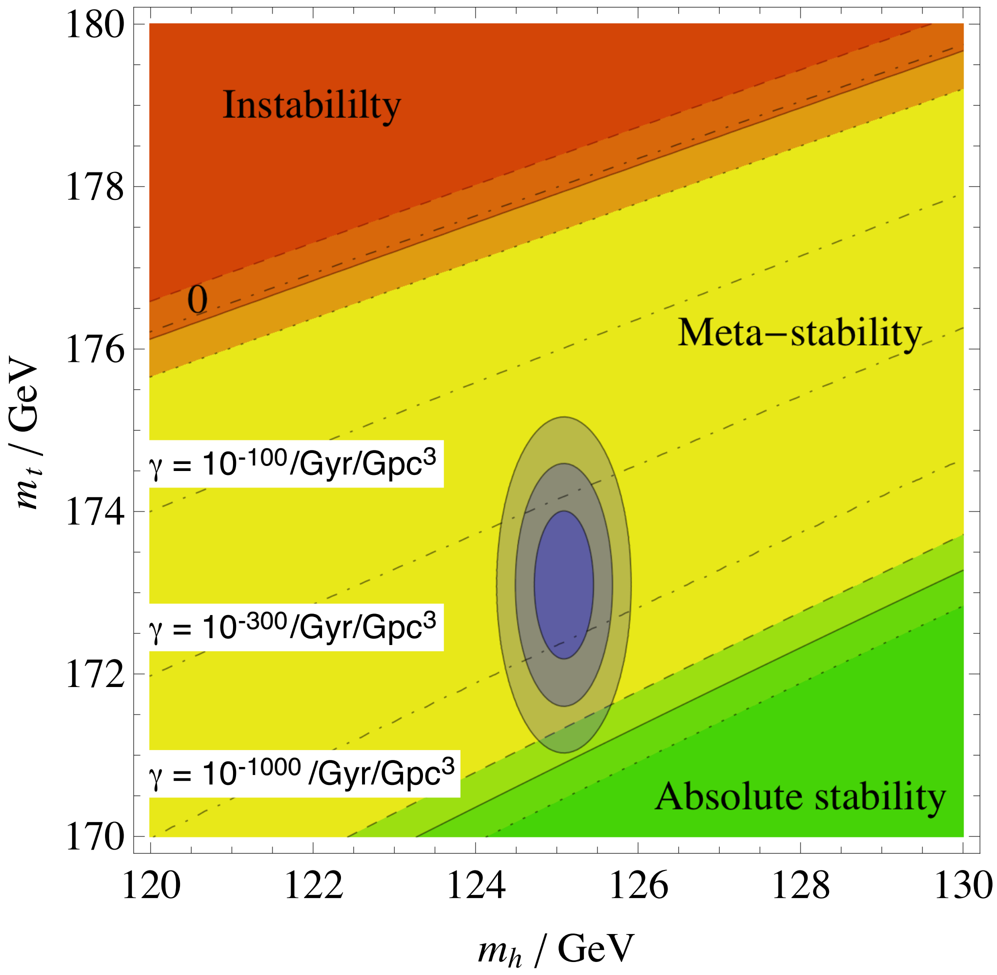
- $\log_{10}[\,\gamma\ (\text{Gyr}^{-1}\text{Gpc}^{-3})\,] \simeq -564^{+38+173+137}_{-43-312-208}$

For the present universe:

- Cosmic age: $t_0 \simeq 13.6$ Gyr
- Horizon scale: $H_0^{-1} \simeq 4.5$ Gpc

$\log_{10}[\gamma\ (\text{Gyr}^{-1}\text{Gpc}^{-3})]$ on m_h vs. m_t plane (with $\mu = 1/R$)

[Chigusa, TM & Shoji]



- Instability: $\gamma > H_{\text{now}}^4$
- Metastability: $\gamma < H_{\text{now}}^4$
- Absolute stability: $\lambda > 0$

6. Case with Extra Matters

Extra particles may affect the stability of the EW vacuum

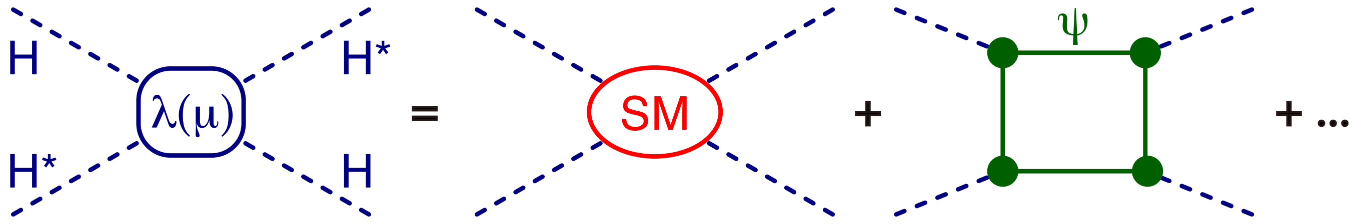
[Espinosa, Garny, Konstandin & Riotto; Casas, Di Clemente, Ibarra & Quiros; Gogoladze, Okada & Shafi; He, Okada & Shafi; Rodejohann & Zhang; Chakrabortty, Das & Mohanty; Chao, Gonderinger & Ramsey-Musolf; Masina; Khan, Goswami & Roy; Bhupal Dev, Ghosh, Okada & Saha; Kobakhidze & Spencer-Smith; Datta, Elsayed, Khalil & Moursy; Chakrabortty, Konar & Mondal; Xiao & Yu; Hamada, Kawai & Oda; Khan & Rakshit; Bambhaniya, Khan, Konar & Mondal; Khan & Rakshit; Salvio; Lindner, Patel & Radovcic; Rose, Marzo & Urbano; Haba, Ishida, Okada & Yamaguchi; $\cdots$]

- RG evolution of λ may change
- A new particle much heavier than the EW scale may affect the decay rate

Let us consider vector-like fermions coupled to H

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + y_\psi H \psi_L \psi_R + y_{\bar{\psi}} H^* \bar{\psi}_L \bar{\psi}_R + M_\psi \bar{\psi}_L \psi_L + M_\psi \bar{\psi}_R \psi_R + \cdots$$

RGE for λ



$$\frac{d\lambda}{d\ln\mu} = \left[\frac{d\lambda}{d\ln\mu}\right]_{\text{SM}} - \frac{1}{4\pi^2} \sum_\psi N^{(\psi)} y_\psi^4 + \cdots$$

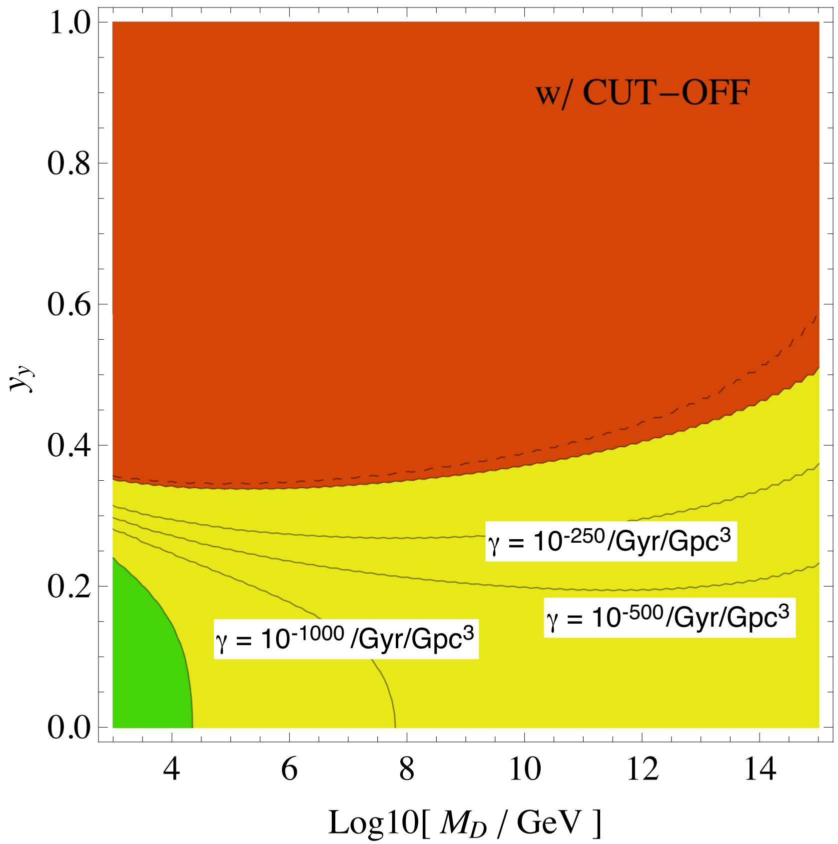
With extra fermions, λ may become smaller (at high scale)

$\Rightarrow$ Enhancement of the decay rate

C.f., $\gamma = \mathcal{A} e^{-\mathcal{B}}$ with $\mathcal{B} = \dfrac{8\pi^2}{3|\lambda|}$

Case 1: Down-quark-like colored fermions

$\Rightarrow \psi_L(\mathbf{3}, \mathbf{2}, 1/6)$ and $\psi_R(\bar{\mathbf{3}}, \mathbf{1}, -1/3)$

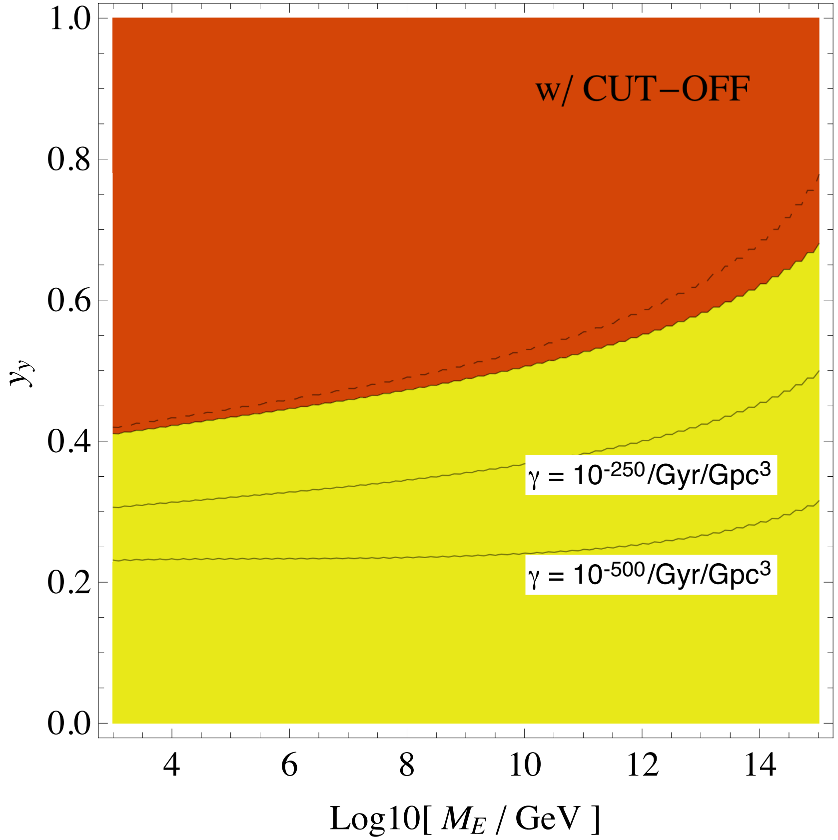


- $\bar{\phi}_C^{(\mathrm{max})} = M_{\mathrm{Pl}}$

$\Rightarrow$ Yukawa coupling larger than $\sim 0.4 - 0.5$ is dangerous

Case 2: Charged-lepton-like fermions

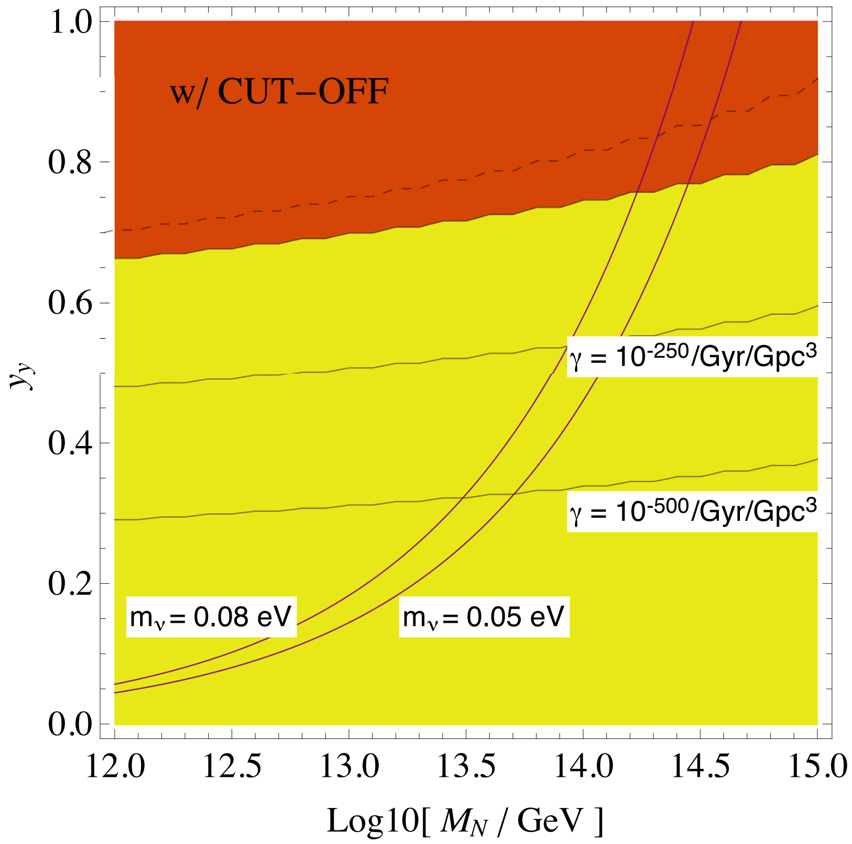
$\Rightarrow \psi_L(\mathbf{1}, \mathbf{2}, 1/2)$ and $\psi_R(\mathbf{1}, \mathbf{1}, -1)$



- $\bar{\phi}_C^{(\mathrm{max})} = M_{\mathrm{Pl}}$

Case 3: Right-handed neutrino

$$\mathcal{L} = \mathcal{L}_{\mathsf{SM}} + y_\nu H \ell_L \nu_R^c + \frac{1}{2} M_\nu \nu_R^c \nu_R^c + \cdots$$



- $\bar{\phi}_C^{(\mathsf{max})} = M_{\mathsf{Pl}}$

7. Summary

We calculated the decay rate of the EW vacuum at one-loop

- Zero modes are properly treated
- We performed a gauge-invariant calculation

Numerical result

$$\log_{10}[\,\gamma\ (\mathsf{Gyr}^{-1}\mathsf{Gpc}^{-3})\,] \simeq -564^{+38+173+137}_{-43-312-208}$$

$\Rightarrow$ In the SM, the EW vacuum decays if we wait $\sim 10^{562}$ Gyr

Extra fermions may change the above conclusion

$\Rightarrow$ $y \gtrsim 0.4 - 0.6$ is dangerous

`ELVAS`: C++ package to study `EL`ectroweak `VA`cuum `S`tability [`https://github.com/YShoji-HEP/ELVAS/`]

$\Rightarrow$ Decay rate is calculated once the RG evolutions of the coupling constants are provided