$\theta = \pi$ in $\mathrm{SU}(N)/\mathbb{Z}_N$ Theory

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Based on Collaboration with
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Introduction

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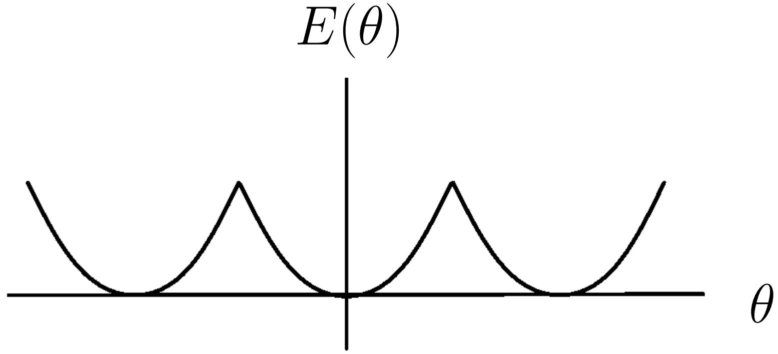
Their claim for Yang-Mills theory:

- $\exists$ spontaneous CP violation at $\theta = \pi$
- for gauge group $\mathrm{SU}(N)$ with finite N
- without SUSY
- using mixed anomaly for CP and center symmetry,

among various interesting claims.

SSB of CP occurs in large N Yang-Mills. **[Witten 80]**

The ground state energy behaves as



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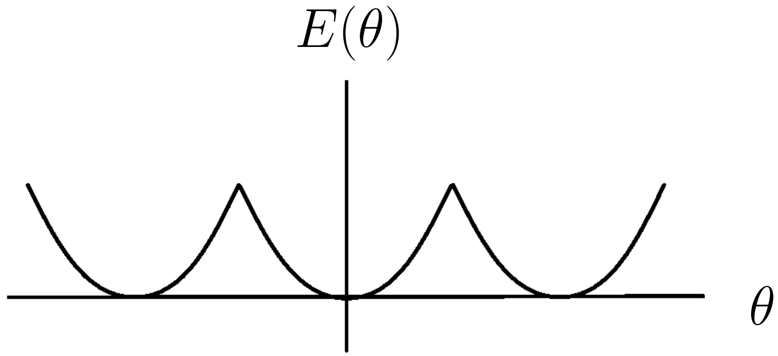

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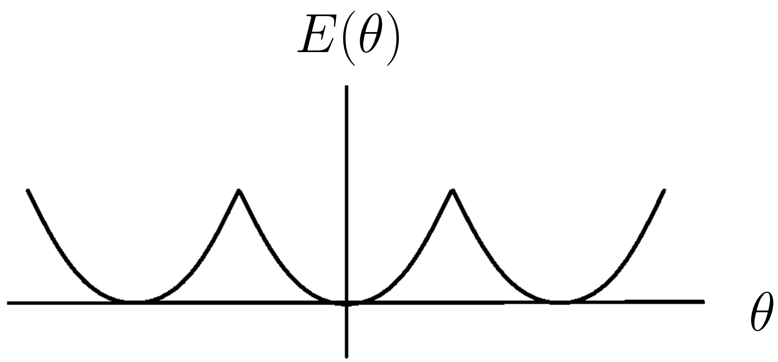

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Also for finite N?

Consider softly broken $\mathcal{N} = 1$ super Yang-Mills theory.

See e.g. [Dine et al. 17]

SUSY is broken by gaugino mass term $m\lambda\lambda + \mathrm{h.c.}$ which induces

$$F(\theta) \;=\; -2m\mu^3 e^{-8\pi^2/g(\mu)^2 N + i\theta/N} + \mathrm{c.c.}$$

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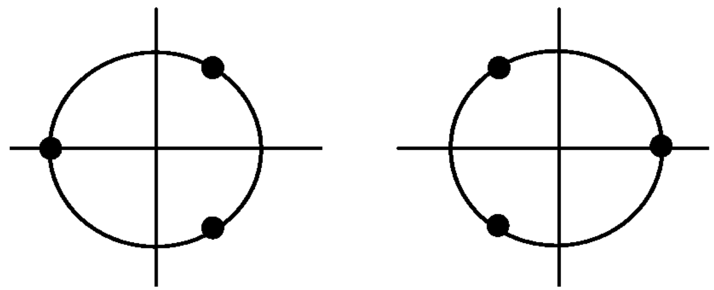

$\theta=0$ $\theta=\pi$

$\leftarrow -\langle\lambda\lambda\rangle$ are plotted.

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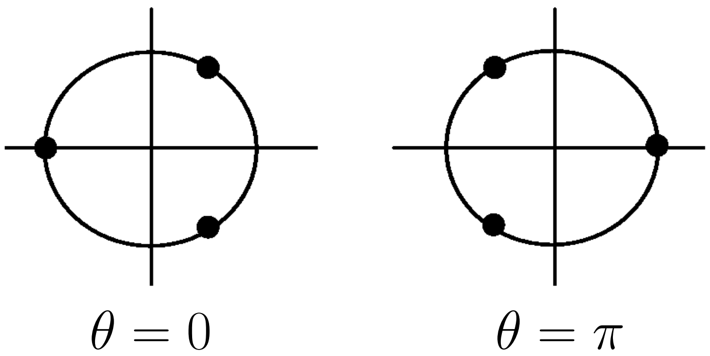

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$m\to\infty$ limit?

Recall the chiral anomaly:

$$\partial_\mu J_A^\mu \;\propto\; \mathrm{Tr} F_{\mu\nu} F_{\rho\sigma} \epsilon^{\mu\nu\rho\sigma}.$$

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chiral symm. $\Rightarrow$ CP symm.

global symm. $\Rightarrow$ center symm.

They showed CP anomaly at $\theta = \pi$ if $\exists$ a non-trivial background for center symmetry.

$\Rightarrow$ **CP is spontaneously broken at $\theta = \pi$.**

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- **1st order phase transition**

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For this, we investigate θ-dependence of free energy density.

Our claim can possibly be verified numerically.

Contents:

$\mathrm{SU}(N)/\mathbb{Z}_N$ bundles and center symmetry

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$\tilde{g}_{ij}$ **are determine up to** $C_{ij} \in \mathbb{Z}_N \subset \mathrm{SU}(N)$**. They satisfy**

$$\tilde{g}_{ij}\tilde{g}_{jk}\tilde{g}_{ki} \;=\; C_{ijk} \;\in\; \mathbb{Z}_N \;\subset \mathrm{SU}(N).$$

We identify

$$C_{ijk} \;\sim\; C_{ijk}C_{ij}^{-1}C_{jk}^{-1}C_{ki}^{-1}.$$

$\tilde{g}_{ij}$ **define "twisted"** $\mathrm{SU}(N)$ **bundles in general.**

The twists C_{ijk} **define an element of** $H^2(M, \mathbb{Z}_N)$**. For** $M = T^4$**,**

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This classifies $\mathrm{SU}(N)/\mathbb{Z}_N$ **bundles. More explicitly,** **[van Baal 82]**

$$\frac{1}{16\pi^2}\int \mathrm{Tr} F \wedge F \ = \ \nu + \frac{N-1}{N}\frac{n_{\mu\nu}\tilde{n}_{\mu\nu}}{4},$$

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where ν **is instanton number and** $n_{\mu\nu} = -n_{\nu\mu}$**.**

It is convenient to define

$$k_i \ := \ n_{i4}, \qquad m_i \ := \ \frac{1}{2}\epsilon_{ijk}n_{jk}.$$

They are integers modulo N**, labeling** $\mathrm{SU}(N)/\mathbb{Z}_N$ **bundles.**

C_{ijk} **are summed over, but suppose they are fixed.**

There is still a "symmetry"

$$\tilde{g}_{ij} \;\rightarrow\; C_{ij}\tilde{g}_{ij},$$

as long as C_{ij} **must satisfy**

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This is the center symmetry.

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Summation of C_{ijk} **amounts to gauge the center symmetry.**

Fixed C_{ijk} **determines a background gauge field.**

$$\mathrm{SU}(N)/\mathbb{Z}_N \textbf{ theory} \quad = \quad \mathrm{SU}(N) \textbf{ theory with gauged center.}$$

To see that it is really center symmetry, consider $W(C)$.

If C goes through several patches,

$$W(C) \ = \ \mathrm{Tr}\left(\cdots \mathrm{P}\exp\left[i\int_{p_i}^{q_i} A^{(i)}\right]\tilde{g}_{ij}\,\mathrm{P}\exp\left[i\int_{p_j}^{q_j} A^{(j)}\right]\cdots\right).$$

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$\Rightarrow$ **$W(C)$ may transform non-trivially.**

For contractible loops, $W(C)$ is invariant due to cocycle condition for C_{ij}.

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The action is given as [Banks, Seberg, 11]

$$S_{\rm GC} \;=\; \int F \wedge (NB - dA),$$

where A and F are auxiliary (gauge) fields which transform as

$$A \;\to\; A + N\Lambda, \qquad F \;\to\; F.$$

Mixed anomaly [Gaiotto et al. 17]

Introducing a background gauge field for center symmetry

$\Leftrightarrow$ Gauging center symmetry and fixing the gauge field.

How to gauge center symmetry?

Mixed anomaly

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Introducing a background gauge field for center symmetry

$\Leftrightarrow$ **Gauging center symmetry and fixing the gauge field.**

How to gauge center symmetry?

A trick: Recall the global structure of $\mathrm{U}(N)$**.**

$$\mathrm{U}(N) \simeq (\mathrm{SU}(N) \times \mathrm{U}(1))/\mathbb{Z}_N.$$

(Center of $\mathrm{SU}(N)$ **is shared with** $\mathrm{U}(1)$**.)**

$\Rightarrow$ **Replace** $\mathrm{U}(1)$ **part with** S_{GC}**.**

Λ**-transf. also acts on** $\mathrm{SU}(N)$ **transition functions.**

Explicitly, replace $A_{\mathrm{SU}(N)}$ with

$$\mathcal{A} \;:=\; A_{\mathrm{SU}(N)} + A.$$

The field strength $\mathcal{F}$ is not invariant. We replace

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The θ-term becomes

$$i\theta \left[\textbf{(integer)} - \frac{N}{8\pi^2} \int B \wedge B \right].$$

$\Rightarrow \theta \to \theta + 2\pi$ **is not a symmetry. $\exists$ Mixed anomaly.**

CP at $\theta = \pi$ is broken.

$\mathrm{SU}(N)/\mathbb{Z}_N$ Theory on T^4

[KSY 17]

Recall the spontaneous CP violation in large N Yang-Mills.

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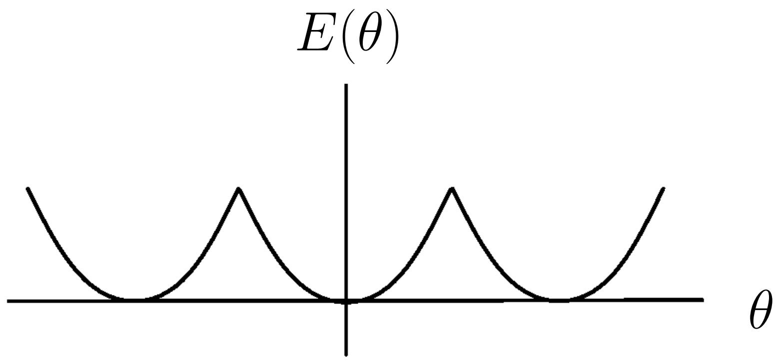

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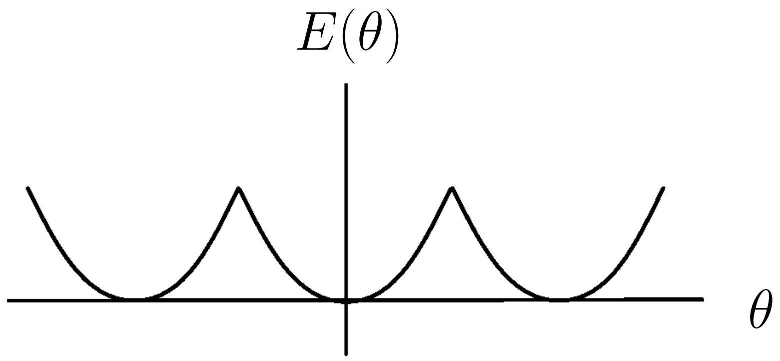


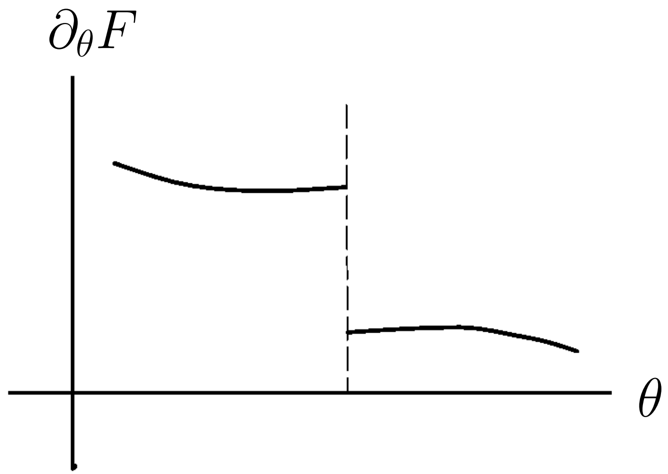
Spontaneous CP violation

$\Leftrightarrow$ **A cups in $E(\theta)$ (or free energy)**

$\Leftrightarrow$ **1st order phase transition**

It seems difficult to show $\exists$ a cusp directly.

Instead, we look for a large finite volume corrections to $\partial_\theta F$.



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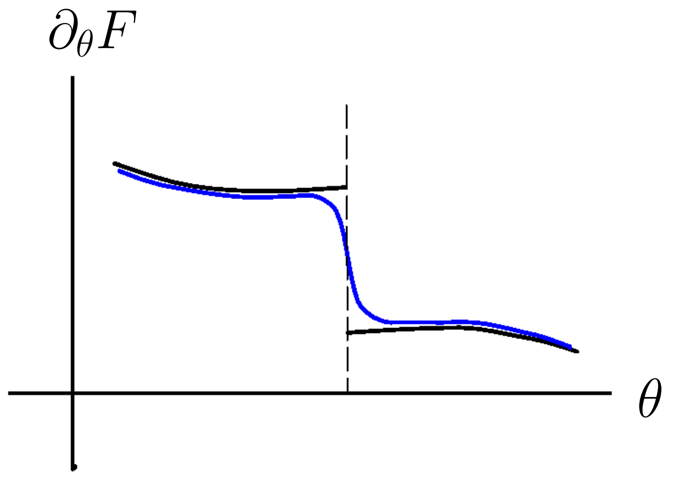

Physical quantities are analytic for a finite volume.

⇒ Finite V correction becomes large near transition points.

cf. In ordinary cases, **[Lüscher 86]**

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If $g(\theta, V)$ varies more than exponential, then

1. $\exists$ a phase transition $\Rightarrow$ CP violation, or

2. $\Delta = 0$ somewhere in $[0, 2\pi]$.

In either case, it is surprising!

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Boundary condition becomes irrelevant in $V \to \infty$.

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Share the same features of the free energy.

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Share the same features of the free energy.

Estimate $g(2\pi, V) - g(0, V)$ **for** $\mathrm{SU}(N)/\mathbb{Z}_N$ **theory.**

Partition function:

For a fixed $\mathrm{SU}(N)/\mathbb{Z}_N$ bundle labeled by k_i and m_i,

$$Z_{k,m}(\theta, V) \quad = \quad \sum_{\nu \in \mathbb{Z}} Z(\nu, k, m, V) e^{i\nu\theta},$$

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The partition function is

$$Z(\theta, V) \;:=\; \sum_{k,m} Z_{k,m}(V) e^{i(N-1)k\cdot m\theta/N}.$$

Note: This is $2N\pi$ periodic.

Note: k_i **are not suitable for electric flux number. Instead**

$$e^{-V\cdot F(e,m,\theta,V)} \;:=\; \frac{1}{N^3}\sum_k e^{-2\pi i k\cdot e/N}\cdot e^{i(N-1)k\cdot m\theta/N} Z_{k,m}(\theta,V)$$

defines the free energy density $F(e,m,\theta,V)$ **for electric flux** e_i **and magnetic flux** m_i**.** **['t Hooft 79][Witten 00]**

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(Acting $W(C)$ **on a state changes** e_i **by one.)**

This implies

$$Z(\theta,V) = N^3\sum_m e^{-V\cdot F(0,m,\theta,V)}.$$

Due to Witten effect, **[Witten 79]**

$$Z(\theta+2\pi,V) = N^3\sum_m e^{-V\cdot F(m,m,\theta,V)}.$$

What we want to estimate is

$$
\begin{aligned}
g(2\pi, V) - g(0, V) &= \big(F(2\pi, V) - F(0, V)\big) - \big(F(2\pi, \infty) - F(0, \infty)\big) \\
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($F(\theta, \infty)$ is 2π periodic.)

Assume the confinement at $\theta = 0$. This implies **['t Hooft 79]**

$$
F(e, m, 0, V) \;\to\; \begin{cases} \infty, & (e \neq 0) \\ 0. & (e = 0) \end{cases} \qquad (V \to \infty)
$$

I.e. electric fluxes are heavy, while magnetic fluxes are screened.

Then, in the limit $V \to \infty$,

$$Z(0,V) \;=\; N^3 \sum_m e^{-V\cdot F(0,m,\theta,V)} \;\sim\; N^6,$$

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This implies

$$\boxed{g(2\pi,V) - g(0,V) \;\sim\; \frac{1}{V}\log N^3.}$$

$\Rightarrow$ **Spontaneous CP violation!**

(Or $\Delta = 0$.)

Assume further that there is only one transition in $[0, 2\pi]$ in both $\mathrm{SU}(N)$ theory and $\mathrm{SU}(N)/\mathbb{Z}_N$ theory.

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$\mathrm{SU}(N)$ theory has 2π periodicity, and is CP invariant at $\theta = 0$.

$$F(\theta + 2\pi, \infty) = F(\theta, \infty), \qquad F(-\theta, \infty) = F(\theta, \infty).$$

$\Rightarrow$ **Allowed transition point is $\theta = \pi$.**

Equally exciting if there are multiple transitions in $[0, 2\pi]$.

Proposals for Numerical Simulation

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$\mathrm{SU}(N)/\mathbb{Z}_N$ **theory can be realized on a lattice by using link variables** U_{ij} **in the adjoint representation of** $\mathrm{SU}(N)$**.**

Interesting observables:

1. Ratio of partition functions

$$Z(2l\pi, V)/Z(0, V) \;\sim\; N^{-3} \qquad (V \to \infty)$$

in the confining phase.

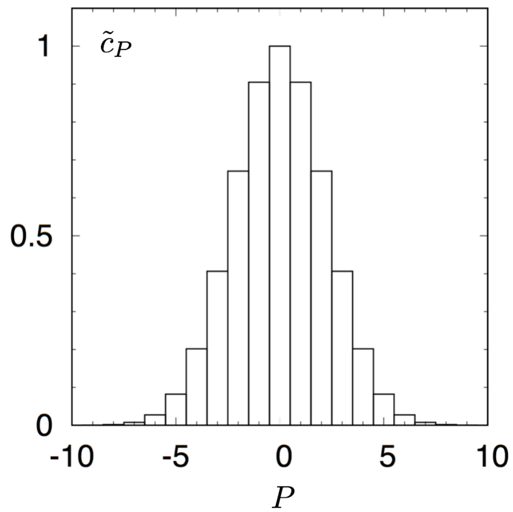
Note: Polyakov loop is not gauge invariant in $\mathrm{SU}(N)/\mathbb{Z}_N$ **theory.**

2. Histogram for instanton numbers

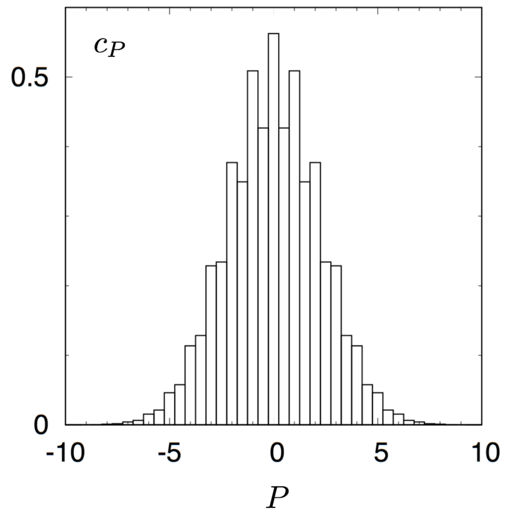
The partition function can be written as

$$Z(\theta, V) = \sum_{P \in \mathbb{Z}/N} c_P(V) e^{iP\theta}.$$

Because of the specific behavior of $g(\theta, V)$, $c_P(V)$ sould behave as



SU(2)



$SU(2)/\mathbb{Z}_2$

Summary

- $\mathrm{SU}(N)/\mathbb{Z}_N$ **theory has a rich topological structure.**
- **Spontaneous CP breaking at** $\theta = \pi$ **is seen from finite volume effects.**
- **Possibly verified by numerical simulation.**

Open issues

- **Numerical simulation of** $\mathrm{SU}(N)/\mathbb{Z}_N$ **theory.**
- **Detailed investigation of** $\mathbf{CP}^N$ **model.**
- **Adding matter, phase diagram.**
- **etc.**