Primordial black holes in the era of gravitational wave astronomy

Teruaki Suyama

(Tokyo Institute of Technology)

Refs.

- **M.Sasaki, TS, T.Tanaka, S.Yokoyama, PRL 117, 061101(2016) [arXiv:1603.08338]**
- **B.Kocsis, TS, T.Tanaka, S.Yokoyama, ApJ 854, 41 (2018) [arXiv:1709.09007]**
- **M.Sasaki, TS, T.Tanaka, S.Yokoyama, CQG 35, 063001(2018) [arXiv:1801.05235]**
- 日本物理学会誌７２巻１０月号(pp. 723-727)、
「LIGOで検出された重力波は原始ブラックホールから？」


平成28年2月12日（金）

産経新聞

号外

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重力波 初検出

米チーム

宇宙誕生の謎に迫る

相対性理論 裏付け

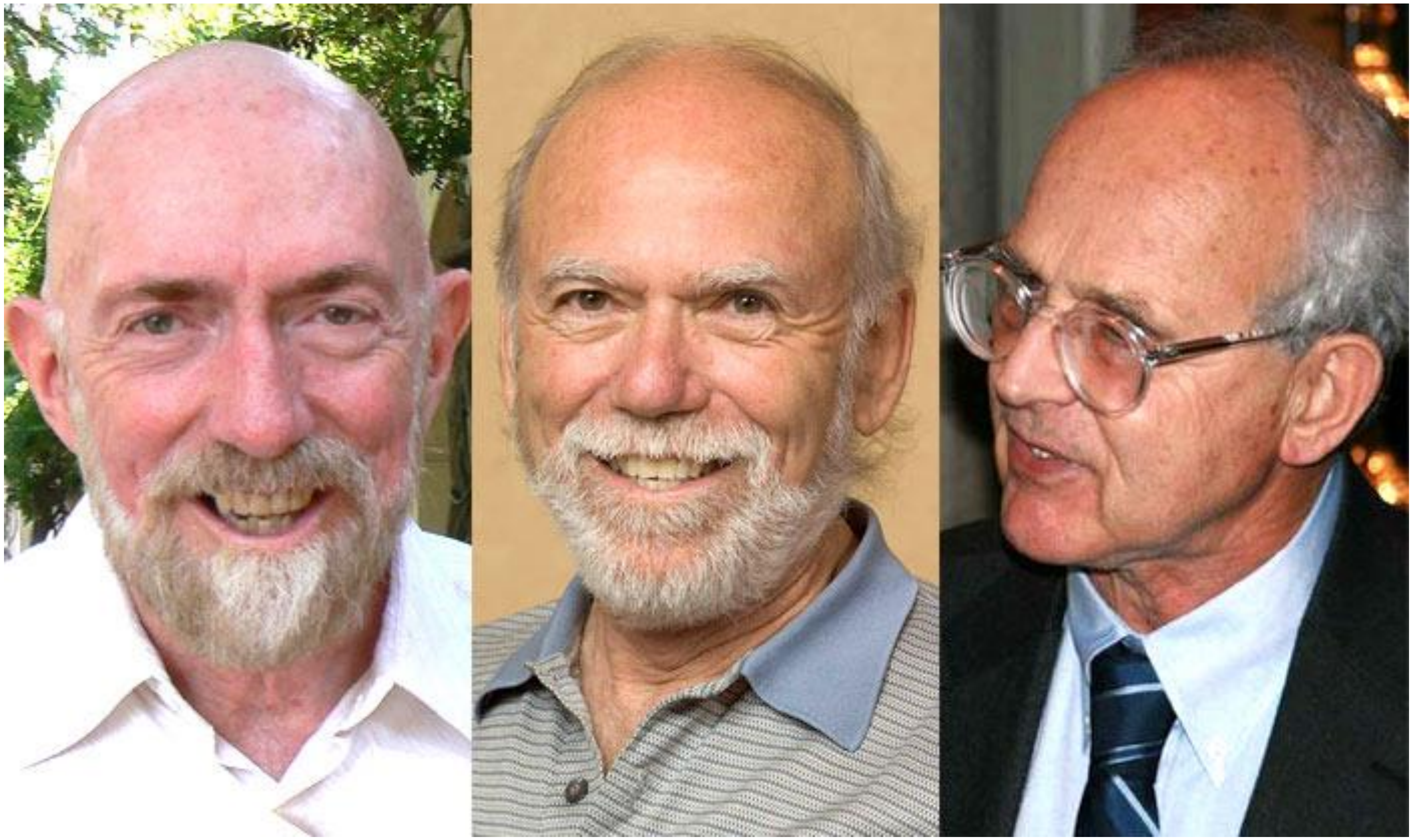
11日、米ワシントンで開かれた記者会見で喜ぶ研究チーム「LIGO」の関係者（AP）

広がっていく現象。全ての物質を光速で貫通し、減衰しないとされる。人が腕を回しても発生するが、振幅が小さすぎて検出できない。非常に重い中性子星同士

波による空間のゆがみをとらえる装置は、太陽と地球の距離（約1.5億キロ）が水素原子1個分伸び縮みするわずかな変化を検知できるほどの高感度が求められる。

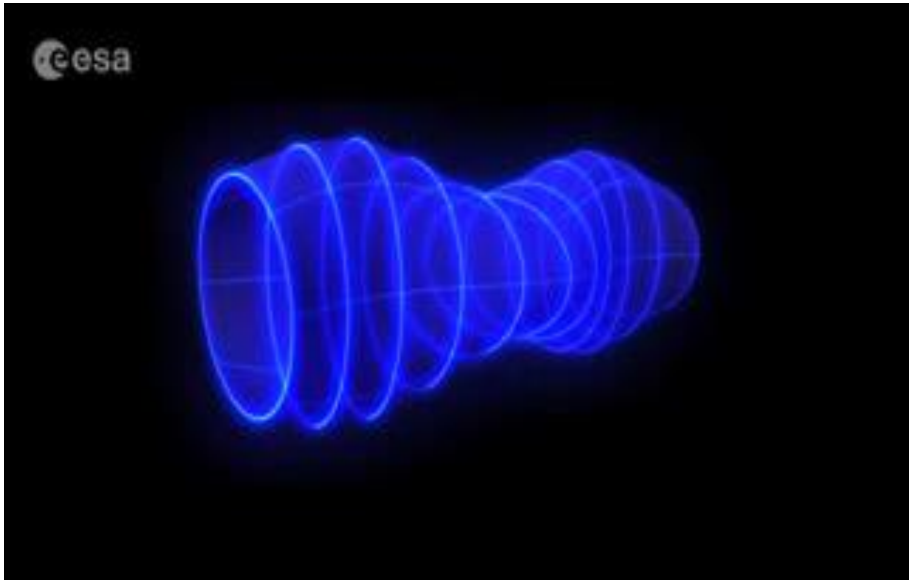
Gravitational waves detected!!
Greatest achievement in 21 century.
GW astronomy has finally started.

The 2017 Nobel Prize in Physics



"for decisive contributions to the LIGO detector and the observation of gravitational waves"

What is GWs？



Prediction of general relativity

Propagation of distortion of spacetime

Propagate at c, 2 DOFs, transverse

Detected 100 years after the birth of GR!!

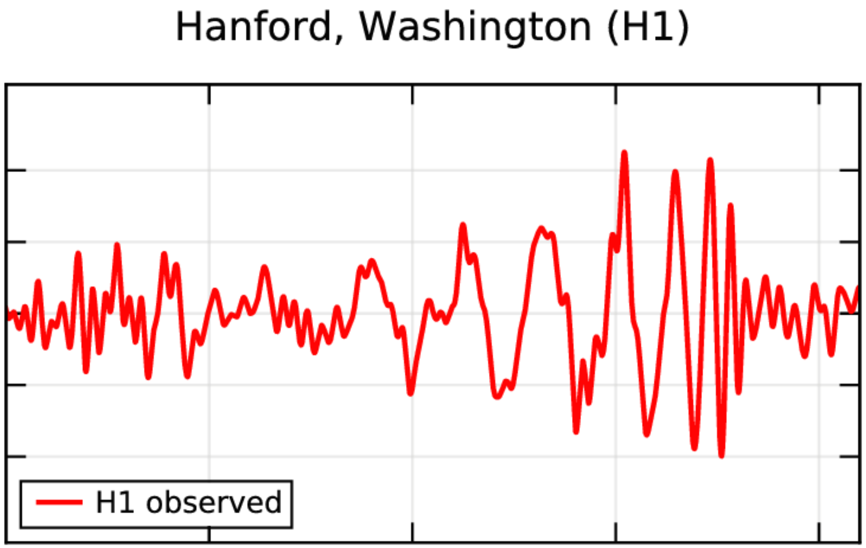
I think over the coming decades we will see enormous numbers of things. Just as electromagnetic astronomy was begun in essence, at least modern astronomy, by Galileo pointing his telescope in the sky and discovering Jupiter's moons. This is the same thing but for gravitational waves.

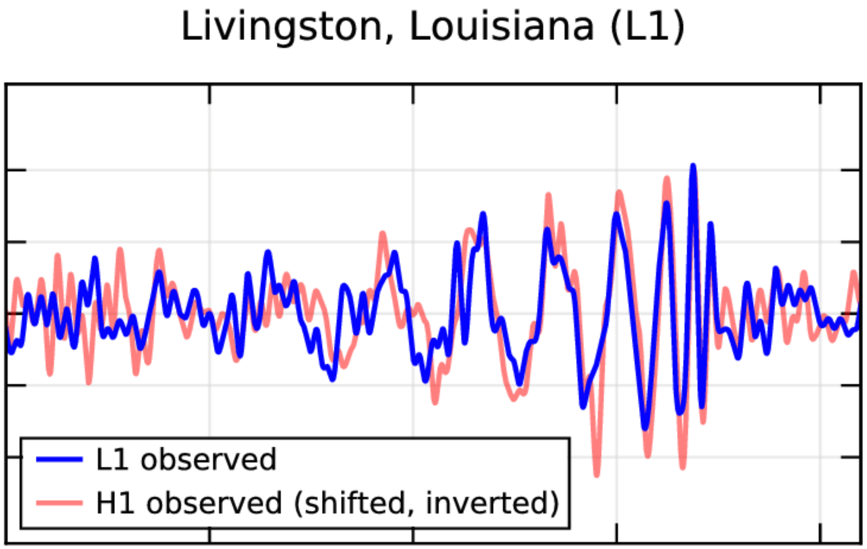
–Kip Thorne-

「ガリレオガリレイが自作の望遠鏡で初めて月を見たことに対応する。その後の電磁波によって我々が得た宇宙の知見は膨大である」

物理学会誌71巻4号　中村卓史（京大名誉教授）

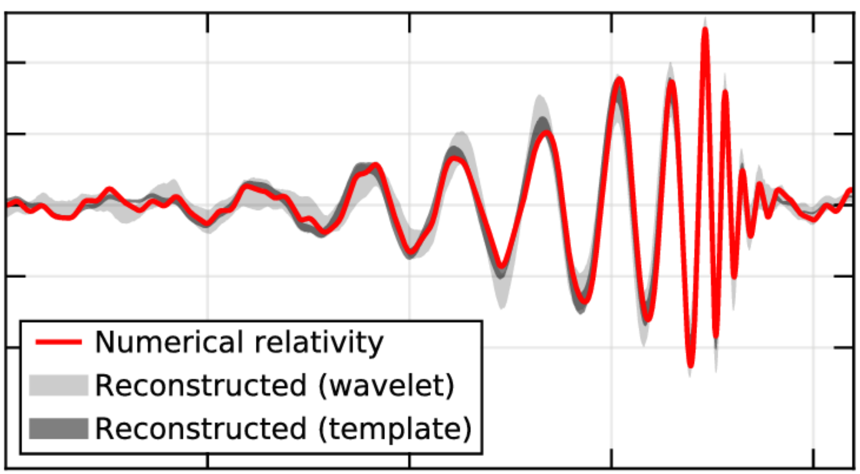
GWs detected by LIGO





What emitted GWs?

Predicted by theory



GWs from black hole binary!!

GWs show that BH-BH binaries exit and they merge in the age of the Universe.

(Until LIGO, we didn't know if they exist.)

So far, 5 merger events have been detected.

Selected for a Viewpoint in Physics
PRL 116, 061102 (2016) PHYSICAL REVIEW LETTERS week ending 12 FEBRUARY 2016

Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott et al.*
(LIGO Scientific Collaboration and Virgo Collaboration)
(Received 21 January 2016; published 11 February 2016)

On September 14, 2015 at 09:50:45 UTC the two detectors of the Laser Interferometer Gravitational-Wave Observatory simultaneously observed a transient gravitational-wave signal. The signal sweeps upwards in frequency from 35 to 250 Hz with a peak gravitational-wave strain of 1.0×10^{-21}. It matches the waveform predicted by general relativity for the inspiral and merger of a pair of black holes and the ringdown of the resulting single black hole. The signal was observed with a matched-filter signal-to-noise ratio of 24 and a false alarm rate estimated to be less than 1 event per 203 000 years, equivalent to a significance greater than 5.1σ. The source lies at a luminosity distance of 410^{+160}_{-180} Mpc corresponding to a redshift $z = 0.09^{+0.03}_{-0.04}$. In the source frame, the initial black hole masses are $36^{+5}_{-4} M_\odot$ and $29^{+4}_{-4} M_\odot$, and the final black hole mass is $62^{+4}_{-4} M_\odot$, with $3.0^{+0.5}_{-0.5} M_\odot c^2$ radiated in gravitational waves. All uncertainties define 90% credible intervals. These observations demonstrate the existence of binary stellar-mass black hole systems. This is the first direct detection of gravitational waves and the first observation of a binary black hole merger.

DOI: 10.1103/PhysRevLett.116.061102

PRL 116, 241103 (2016) PHYSICAL REVIEW LETTERS week ending 17 JUNE 2016

GW151226: Observation of Gravitational Waves from a 22-Solar-Mass Binary Black Hole Coalescence

B. P. Abbott et al.*
(LIGO Scientific Collaboration and Virgo Collaboration)
(Received 31 May 2016; published 15 June 2016)

We report the observation of a gravitational-wave signal produced by the coalescence of two stellar-mass black holes. The signal, GW151226, was observed by the twin detectors of the Laser Interferometer Gravitational-Wave Observatory (LIGO) on December 26, 2015 at 03:38:53 UTC. The signal was initially identified within 70 s by an online matched-filter search targeting binary coalescences. Subsequent off-line analyses recovered GW151226 with a network signal-to-noise ratio of 13 and a significance greater than 5σ. The signal persisted in the LIGO frequency band for approximately 1 s, increasing in frequency and amplitude over about 55 cycles from 35 to 450 Hz, and reached a peak gravitational strain of $3.4^{+0.7}_{-0.9} \times 10^{-22}$. The inferred source-frame initial black hole masses are $14.2^{+8.3}_{-3.7} M_\odot$ and $7.5^{+2.3}_{-2.3} M_\odot$, and the final black hole mass is $20.8^{+6.1}_{-1.7} M_\odot$. We find that at least one of the component black holes has spin greater than 0.2. This source is located at a luminosity distance of 440^{+180}_{-190} Mpc corresponding to a redshift of $0.09^{+0.03}_{-0.04}$. All uncertainties define a 90% credible interval. This second gravitational-wave observation provides improved constraints on stellar populations and on deviations from general relativity.

PRL 118, 221101 (2017) PHYSICAL REVIEW LETTERS week ending 2 JUNE 2017

GW170104: Observation of a 50-Solar-Mass Binary Black Hole Coalescence at Redshift 0.2

B. P. Abbott et al.*
(LIGO Scientific and Virgo Collaboration)
(Received 9 May 2017; published 1 June 2017)

We describe the observation of GW170104, a gravitational-wave signal produced by the coalescence of a pair of stellar-mass black holes. The signal was measured on January 4, 2017 at 10:11:58.6 UTC by the twin advanced detectors of the Laser Interferometer Gravitational-Wave Observatory during their second observing run, with a network signal-to-noise ratio of 13 and a false alarm rate less than 1 in 70 000 years. The inferred component black hole masses are $31.2^{+8.4}_{-6.0} M_\odot$ and $19.4^{+5.3}_{-5.9} M_\odot$ (at the 90% credible level). The black hole spins are best constrained through measurement of the effective inspiral spin parameter, a mass-weighted combination of the spin components perpendicular to the orbital plane, $\chi_{\rm eff} = -0.12^{+0.21}_{-0.30}$. This result implies that spin configurations with both component spins positively aligned with the orbital angular momentum are disfavored. The source luminosity distance is 880^{+450}_{-390} Mpc corresponding to a redshift of $z = 0.18^{+0.08}_{-0.07}$. We constrain the magnitude of modifications to the gravitational-wave dispersion relation and perform null tests of general relativity. Assuming that gravitons are dispersed in vacuum like massive particles, we bound the graviton mass to $m_g \leq 7.7 \times 10^{-23}$ eV/c^2. In all cases, we find that GW170104 is consistent with general relativity.

DOI: 10.1103/PhysRevLett.118.221101

PRL 119, 141101 (2017) PHYSICAL REVIEW LETTERS week ending 6 OCTOBER 2017

GW170814: A Three-Detector Observation of Gravitational Waves from a Binary Black Hole Coalescence

B. P. Abbott et al.*
(LIGO Scientific Collaboration and Virgo Collaboration)
(Received 23 September 2017; published 6 October 2017)

On August 14, 2017 at 10:30:43 UTC, the Advanced Virgo detector and the two Advanced LIGO detectors coherently observed a transient gravitational-wave signal produced by the coalescence of two stellar mass black holes, with a false-alarm rate of $\lesssim 1$ in 27 000 years. The signal was observed with a three-detector network matched-filter signal-to-noise ratio of 18. The inferred masses of the initial black holes are $30.5^{+5.7}_{-3.0} M_\odot$ and $25.3^{+2.8}_{-4.2} M_\odot$ (at the 90% credible level). The luminosity distance of the source is 540^{+130}_{-210} Mpc, corresponding to a redshift of $z = 0.11^{+0.03}_{-0.04}$. A network of three detectors improves the sky localization of the source, reducing the area of the 90% credible region from 1160 deg^2 using only the two LIGO detectors to 60 deg^2 using all three detectors. For the first time, we can test the nature of gravitational-wave polarizations from the antenna response of the LIGO-Virgo network, thus enabling a new class of phenomenological tests of gravity.

DOI: 10.1103/PhysRevLett.119.141101

The Astrophysical Journal Letters, 851:L35 (11pp), 2017 December 20 https://doi.org/10.3847/2041-8213/aa9f0c

GW170608: Observation of a 19 Solar-mass Binary Black Hole Coalescence

LIGO Scientific Collaboration and Virgo Collaboration
(See the end matter for the full list of authors.)

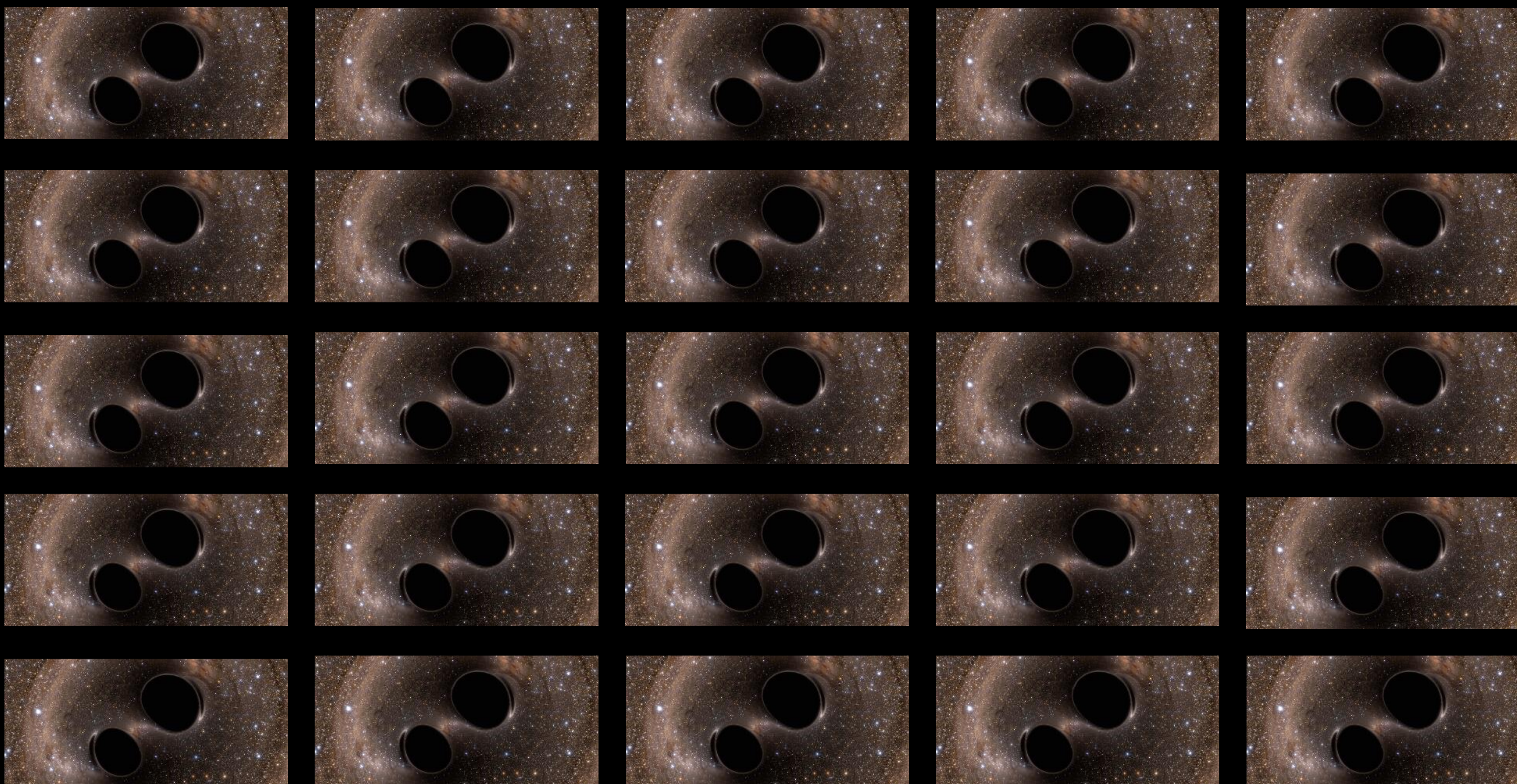
Received 2017 November 14; revised 2017 November 30; accepted 2017 December 2; published 2017 December 18

Abstract

On 2017 June 8 at 02:01:16.49 UTC, a gravitational-wave (GW) signal from the merger of two stellar-mass black holes was observed by the two Advanced Laser Interferometer Gravitational-Wave Observatory detectors with a network signal-to-noise ratio of 13. This system is the lightest black hole binary so far observed, with component masses of $12^{+7}_{-2} M_\odot$ and $7^{+2}_{-2} M_\odot$ (90% credible intervals). These lie in the range of measured black hole masses in low-mass X-ray binaries, thus allowing us to compare black holes detected through GWs with electromagnetic observations. The source's luminosity distance is 340^{+140}_{-140} Mpc, corresponding to redshift $0.07^{+0.03}_{-0.03}$. We verify that the signal waveform is consistent with the predictions of general relativity.

Key words: binaries: general – gravitational waves – stars: black holes

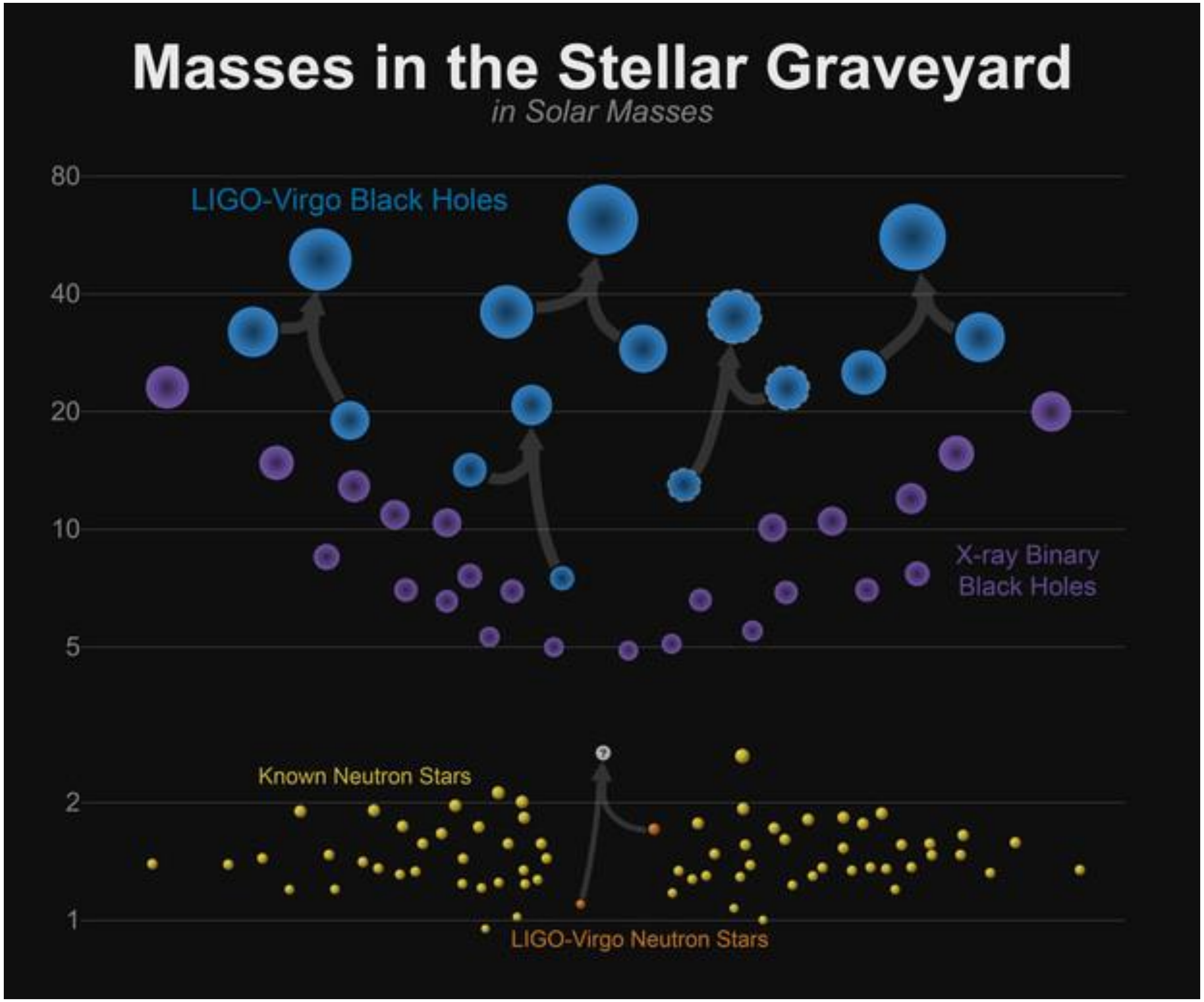
We now know that many BH-BH mergers are occurring in the Universe.



When and where did those BHs form and how did they form binaries?

$30M_{\odot}$

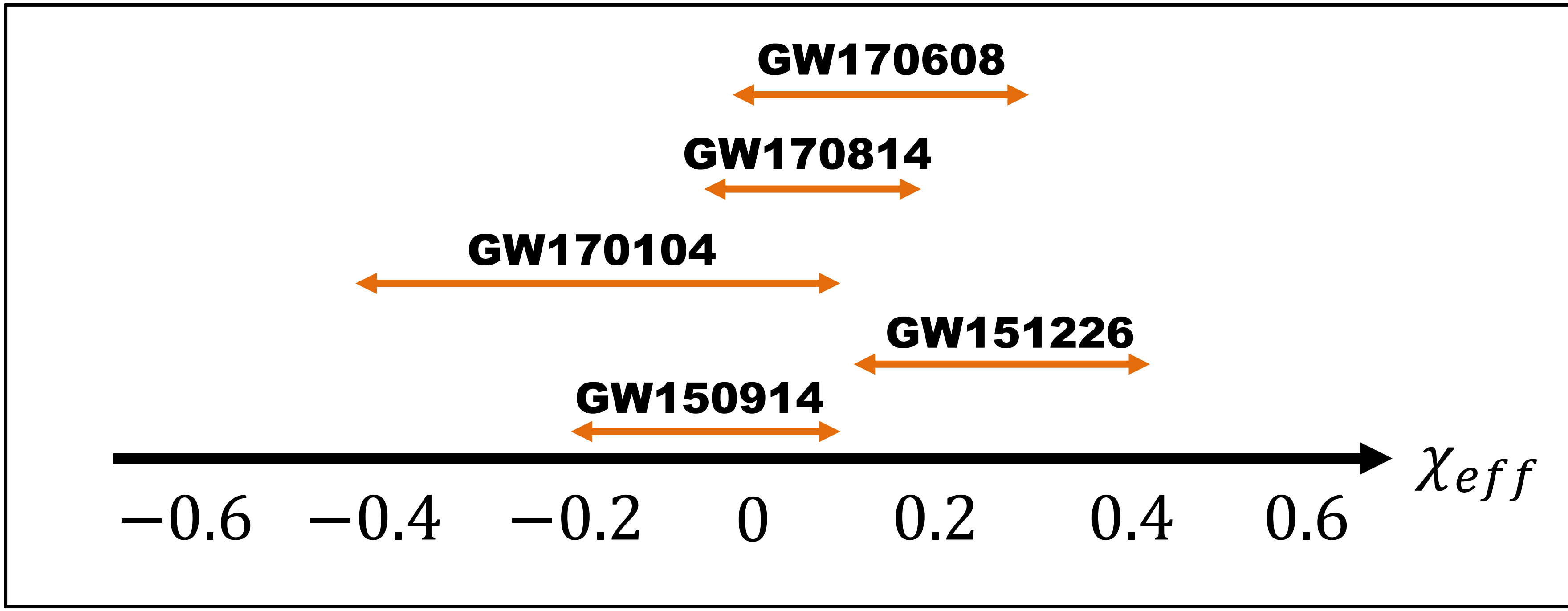
$\sim$0.1AU



LIGO BHs are anomalously heavy?

BH spin

$$\chi_{\text{eff}} = \frac{c}{GM}\left(\frac{\boldsymbol{S}_1}{m_1} + \frac{\boldsymbol{S}_2}{m_2}\right) \cdot \frac{\boldsymbol{L}}{|\mathbf{L}|} \qquad -1 \leq \chi_{eff} \leq 1$$



BHs have low spin?

BH spins are misaligned?

What is the origin of LIGO BHs?

- list possible scenarios as many as we can.
- propose many ideas of how to test and distinguish them observationally.

The seemingly unusual features may suggest that those BHs are new population.

Maybe, primordial black holes!

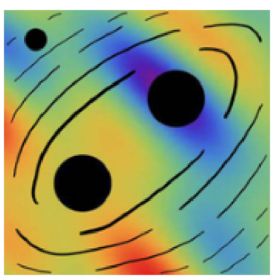
Editors' Suggestion

PDF HTML

Primordial Black Hole Scenario for the Gravitational-Wave Event GW150914

Misao Sasaki, Teruaki Suyama, Takahiro Tanaka, and Shuichiro Yokoyama

Phys. Rev. Lett. **117**, 061101 (2016) – Published 2 August 2016



A theoretical analysis examines the possibility that the gravitational wave signal (GW150914) detected by LIGO was due to the coalescence of primordial black holes created by the extremely dense matter present in the early Universe.

Show Abstract +

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Primordial Black Hole Scenario for the Gravitational-Wave Event GW150914

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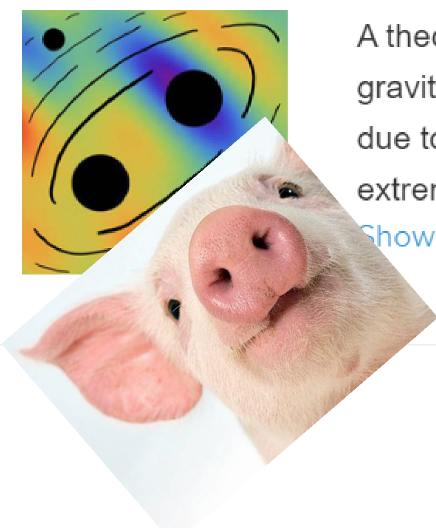


A theoretical analysis examines the possibility that the gravitational wave signal (GW150914) detected by LIGO was due to the coalescence of primordial black holes created by the extremely dense matter present in the early Universe.

Show Abstract +

What are primordial BHs?

PBHs=BHs that formed in the very early Universe

S.Hawking 1971

PBHs might comprise all/fraction of dark matter.

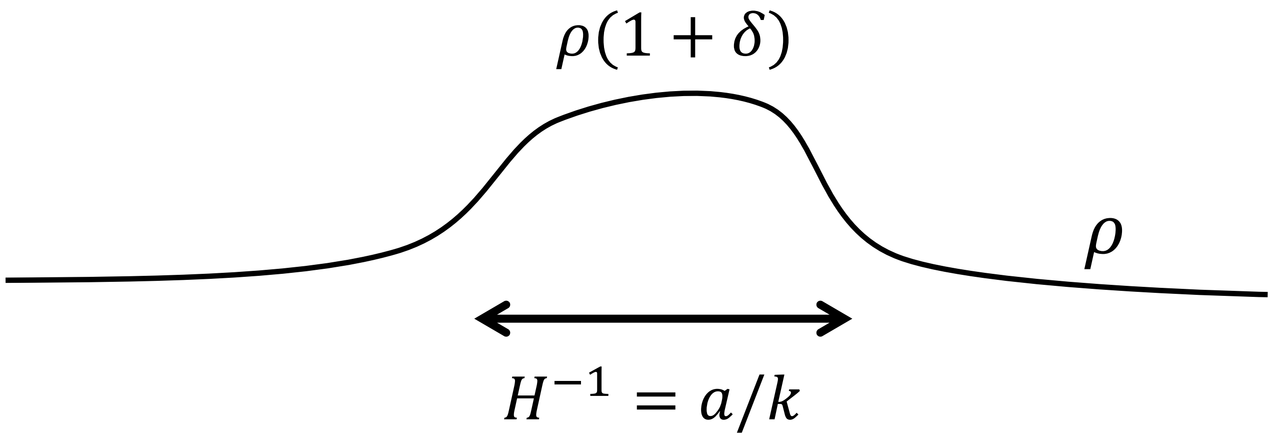
Several formation mechanism of PBHs

- Direct gravitational collapse of primordial density perturbation. (widely investigated)
- Collapse of cosmic strings
- Creation of vacuum bubbles
- ....

PBHs are fossils of high energy physics such as inflation.

Formation of PBHs

Collapse of primordial density perturbation



If density contrast is $\sim$1 at the horizon reentry, the overdense region collapses to BH.

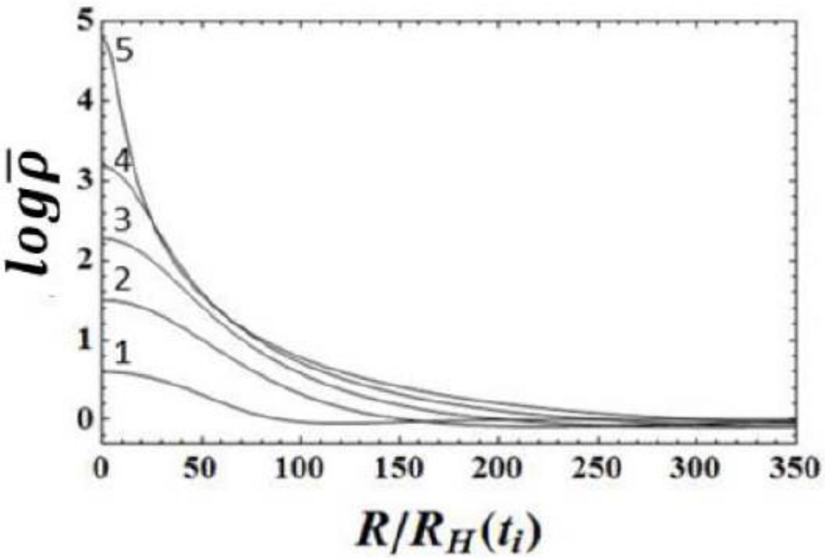
$$r_{\mathrm{SCH}} \sim GM \sim G\rho H^{-3} \sim H^{-1} \sim t$$

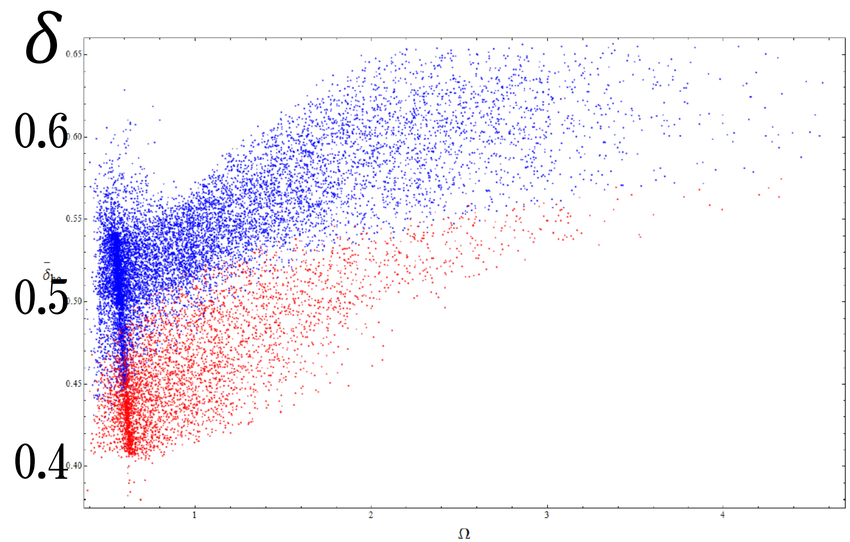
Shortly after the overdensity starts to contract, it falls within its Schwarzschild radius.
So the mass is roughly determined by the horizon mass:

$$M_{\mathrm{PBH}} \sim \rho H^{-3} \sim \frac{1}{GH} \sim 10 M_{\odot} \left(\frac{t}{0.1 ms} \right) \sim 10 M_{\odot} \left(\frac{k}{1\mathrm{pc}^{-1}} \right)^{-2}$$

Formation of PBH

It has been confirmed by simulations that BH forms out of primordial overdense region.



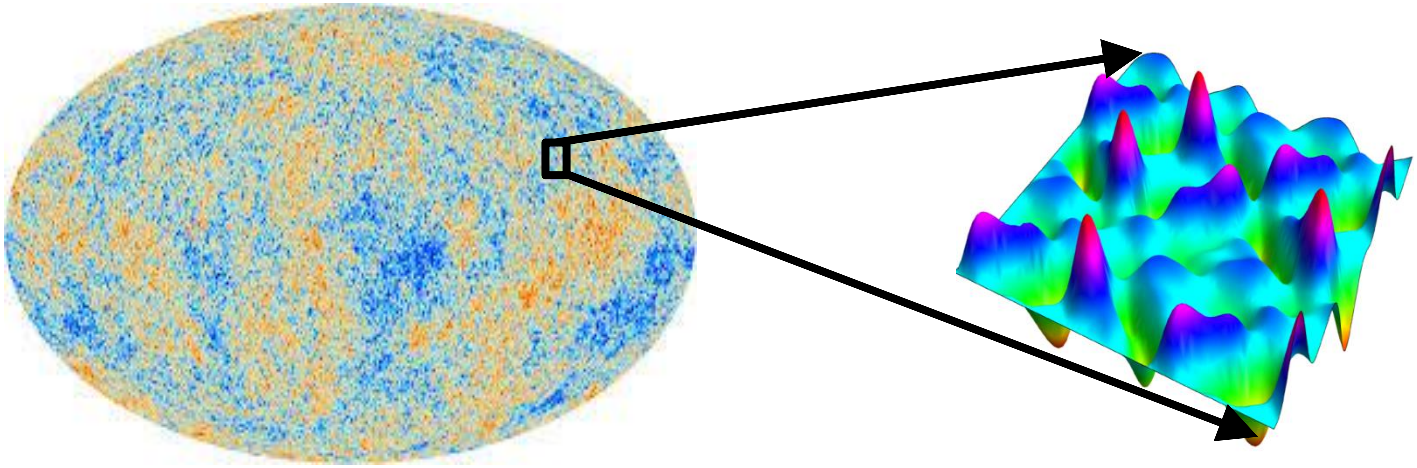


T.Nakama et al. 2014

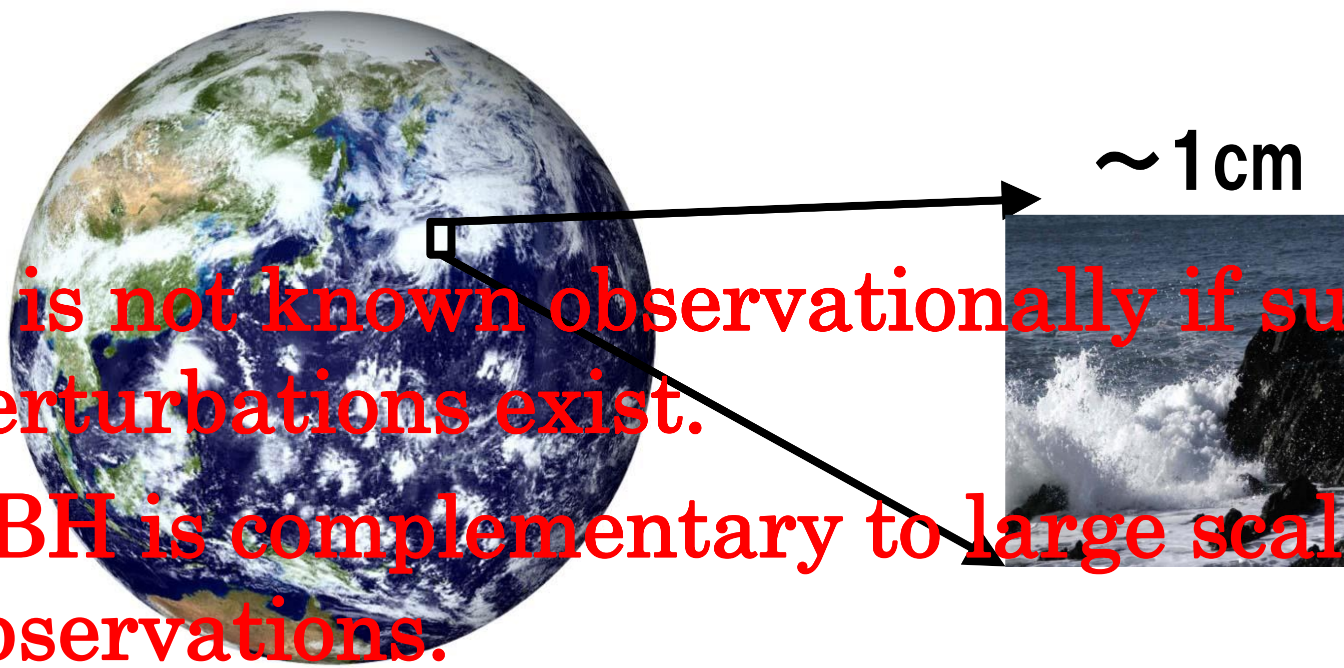
BH forms at around $\delta = 0.4{\sim}0.5$

Consistent with analytic estimation T.Harada, C.Yoo, K.Kohri 2013

Is δ~1 allowed observationally?



PBHs originate from very small-scale perturbations.

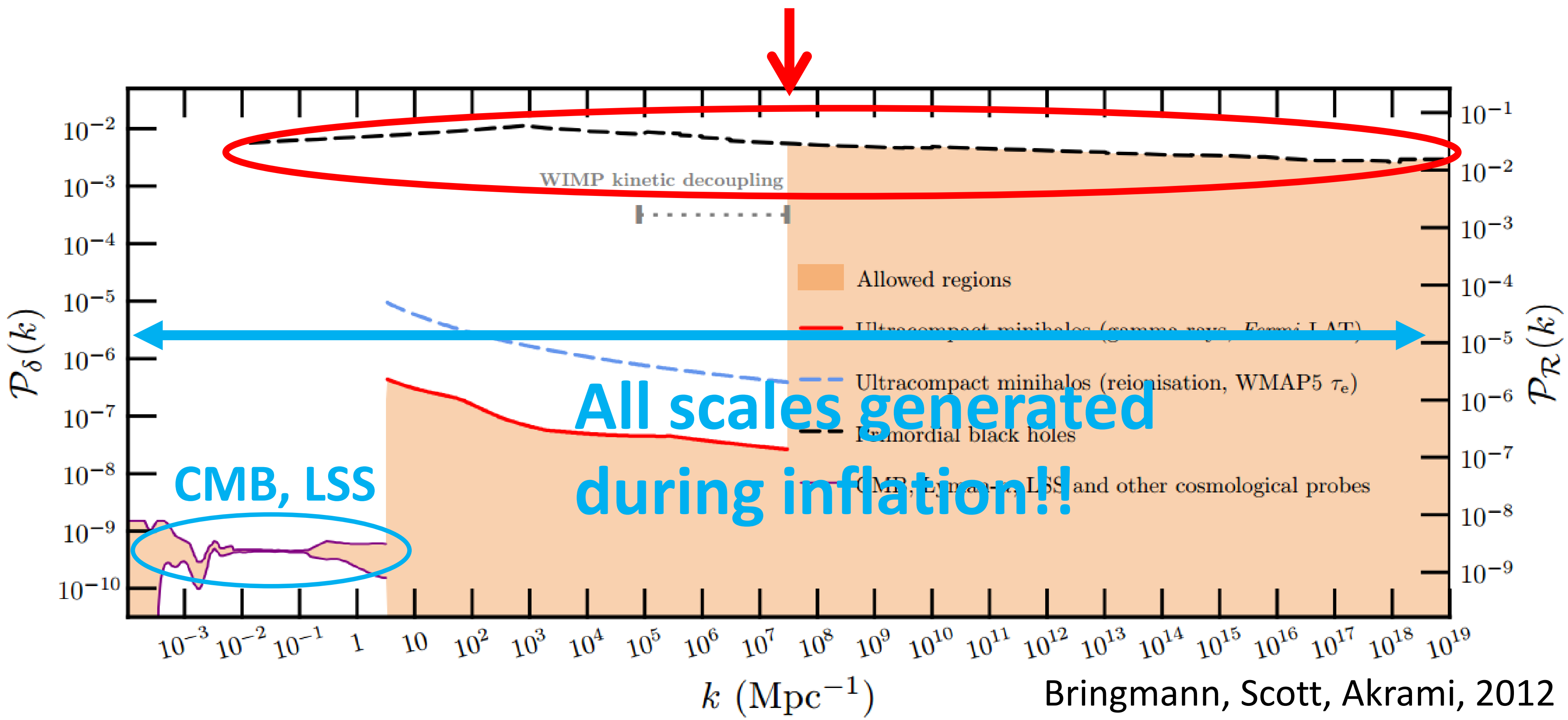


It is not known observationally if such perturbations exist.

PBH is complementary to large scale observations.

PBH can constrain primordial perturbation on small scales.

$$\sigma^2(\sim\mathcal{P}_\zeta) \lesssim (0.4/10)^2 \simeq 2\times 10^{-3}$$



Bringmann, Scott, Akrami, 2012

Even non-detection of PBH provides useful information about primordial perturbation.

An inflation model predicting PBHs

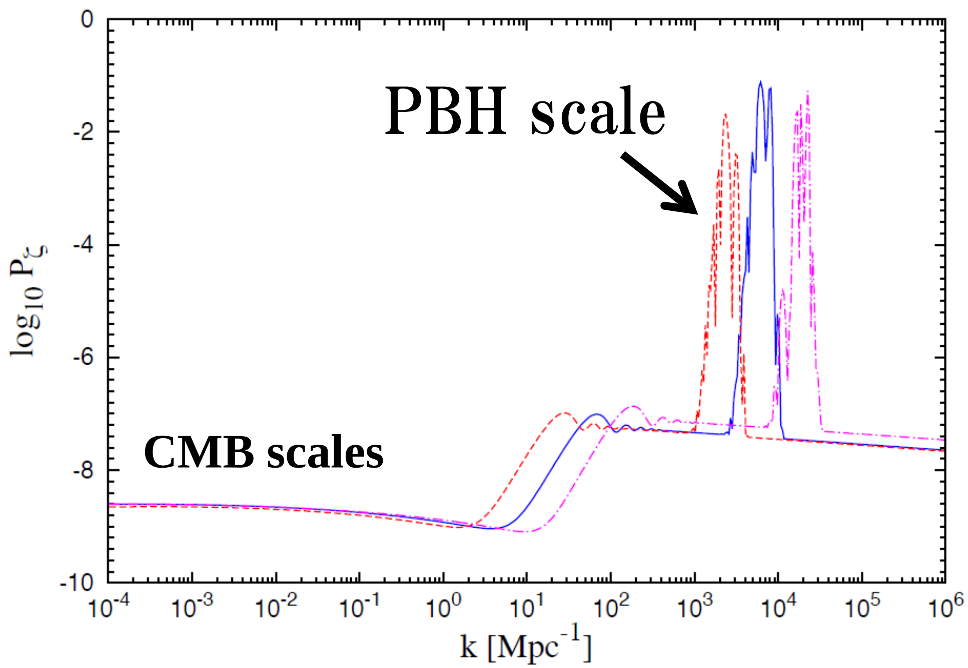
(Kawasaki, Kusenko, Yanagida, 2012)

$$V = V_{\rm H} + V_{\rm N} + V_{\rm HN},$$

$$V_{\rm H}(\phi,\psi) = \left(1 + \frac{\phi^4}{8} + \frac{\psi^2}{2}\right)\left(-\mu^2 + \frac{\psi^4}{16M^2}\right)^2 + \frac{\phi^2\psi^6}{16M^4},$$

$$V_{\rm N}(\varphi) = v^4\left(1 - \frac{\kappa}{2}\varphi^2\right) - \frac{g}{2}v^2\varphi^4 + \frac{g^2}{16}\varphi^8,$$

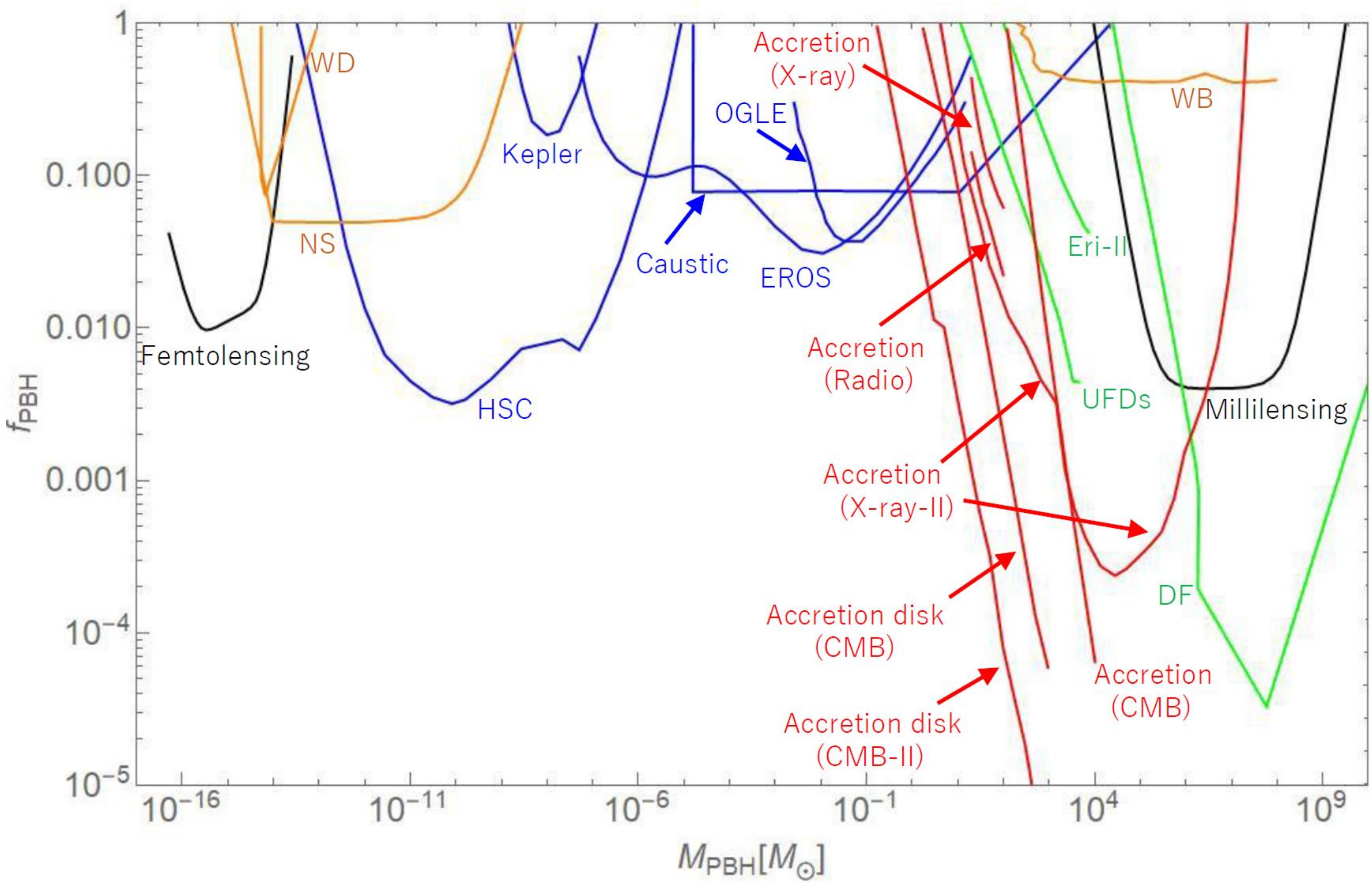
$$V_{\rm HN}(\phi,\psi,\varphi) = \left(-\mu^2 + \frac{\psi^4}{16M^2}\right)^2\frac{\varphi^2}{2} - \left(-\mu^2 + \frac{\psi^4}{16M^2}\right)v^2\phi\varphi,$$



Since its original proposal in 70s, PBH has been studied both theoretically and observationally.

- **PBH with any mass can be produced in the early Universe.**
- **Small PBH ($\lesssim 10^{15} g$) evaporates by now by the Hawking radiation.**
- **Large PBH ($\gtrsim 10^{15} g$) behaves as (cold) dark matter.**
- **No detection of PBHs and only upper limits on PBH abundance exist.**

Observational limits on $f_{PBH} = \Omega_{PBH}/\Omega_{DM}$



Coming back to LIGO BHs, two things need to be explained before including the PBH as a possible explanation of LIGO events.

- How PBHs formed binaries?
- Do their mergers explain the observed merger rate?

Binary formation in the RD era

(candidate of my master thesis)

THE ASTROPHYSICAL JOURNAL, 487:L139–L142, 1997 October 1

GRAVITATIONAL WAVES FROM COALESCING BLACK HOLE MACHO BINARIES

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Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606, Japan
MISAO SASAKI AND TAKAHIRO TANAKA
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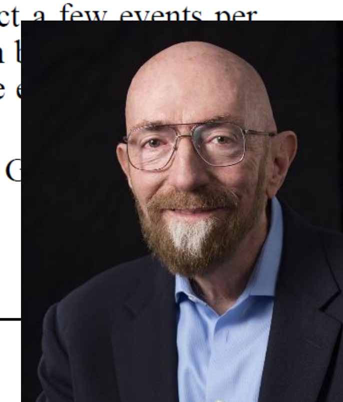
Received 1997 April 11; accepted 1997 July 23; published 1997 September 2

ABSTRACT

HOs are black holes of mass $\boxed{\sim 0.5\ M_\odot}$ they must have been formed in the early uni
e was ~1 GeV. We estimate that in this case in our Galaxy's halo out to ~ 50 kpc there exist ~5 ×
hole binaries the coalescence times of which are comparable to the age of the universe, so that the
e rate will be ~5 × 10^{-2} events yr^{-1} per galaxy. This suggests that we can expect a few events per
15 Mpc. The gravitational waves from such coalescing black hole MACHOs can b
tion of interferometers in the LIGO/VIRGO/TAMA/GEO network. Therefore, the
HOs can be tested within the next 5 yr by gravitational waves.

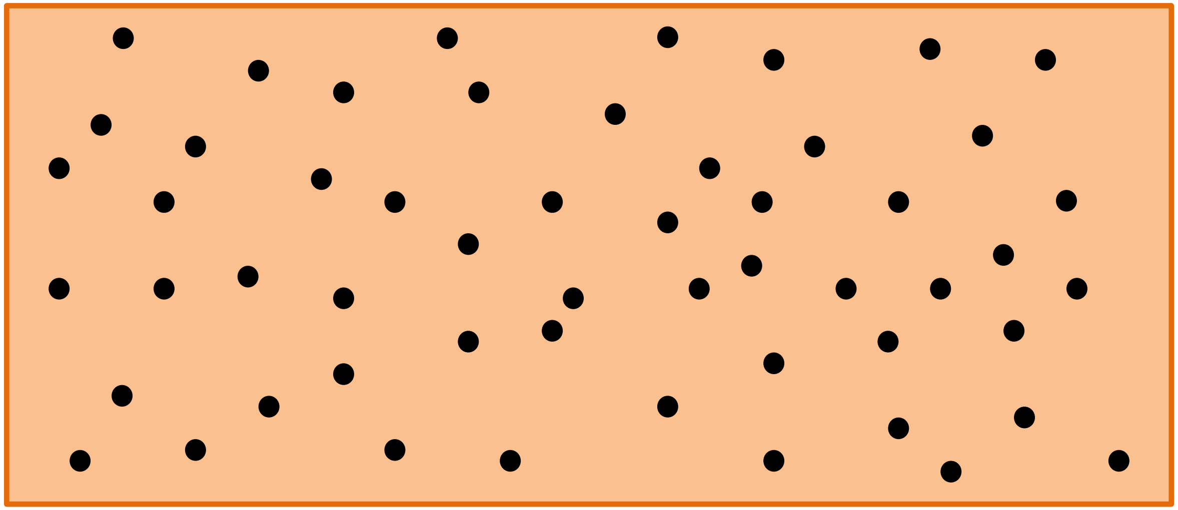
dings: black hole physics — dark matter — gravitation — gravitational lensing — G



Nobel Prize again?

Two assumptions **(Nakamura et al. 1997)**

1. After PBHs are formed (by some mechanism), they distribute uniformly in space (Poisson).



Initially, PBHs are on the flow of the cosmic expansion.

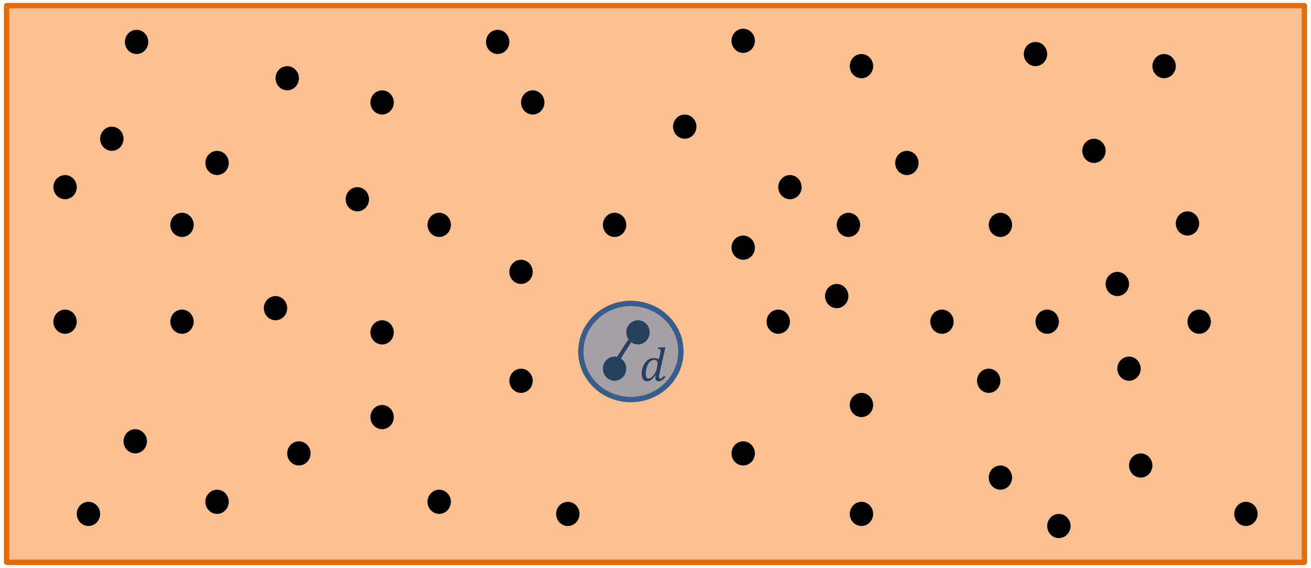
2. All PBHs have the same mass

Binary formation in RD era (Nakamura et al. 1997)

(The rest is not assumption but physical consequence.)

As the universe expands, distance between PBHs becomes smaller than the Hubble horizon.

When the PBH energy $2\,M_{BH}$ in the volume $\sim d^3$ exceeds $\rho_{rad} d^3$, the PBHs in pair decouple from cosmic expansion and start to come closer by the gravitational force. (Ioka et al. 1998, Ali-Haimoud et al. 2017)

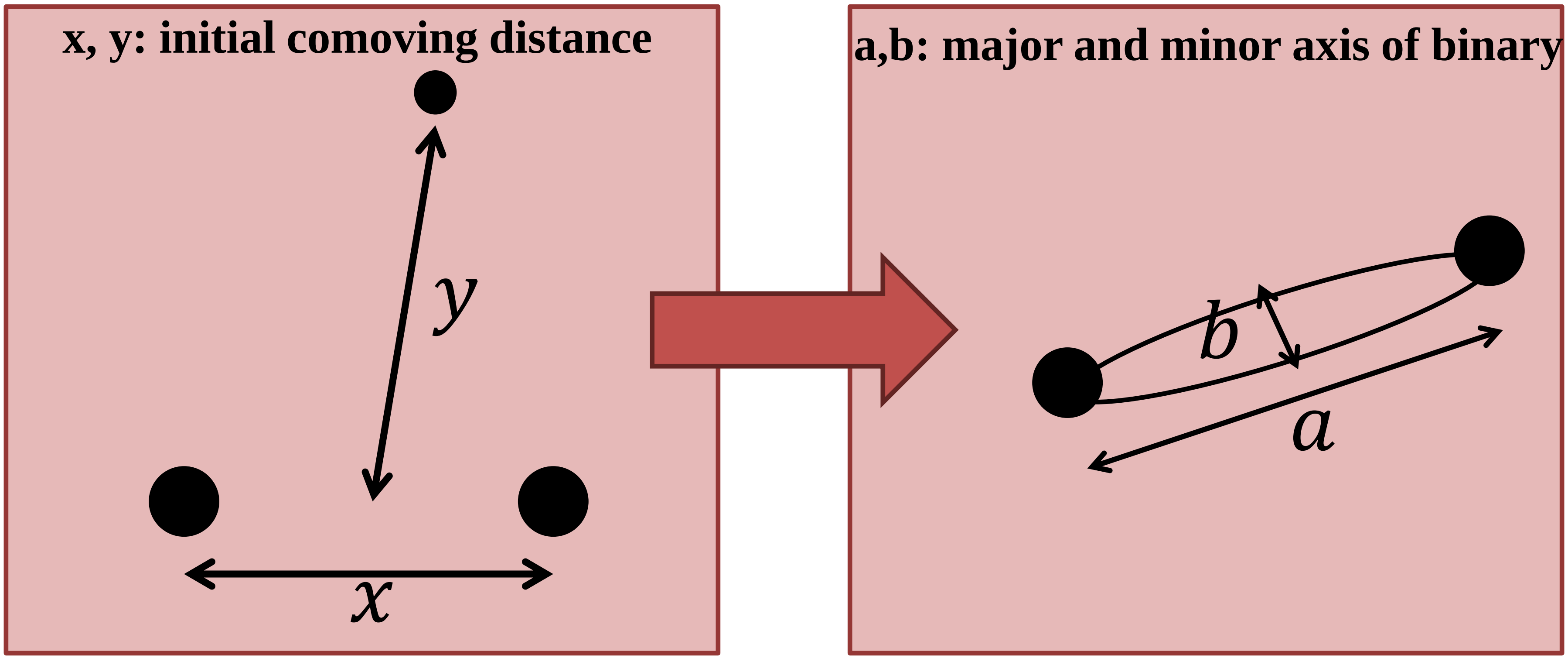


This can happen only in the RD era.

Binary formation in RD era (Nakamura et al. 1997)

The surrounding PBHs (especially the nearest one) exert torque and the bound system acquires the angular momentum.

(Binary formation!)

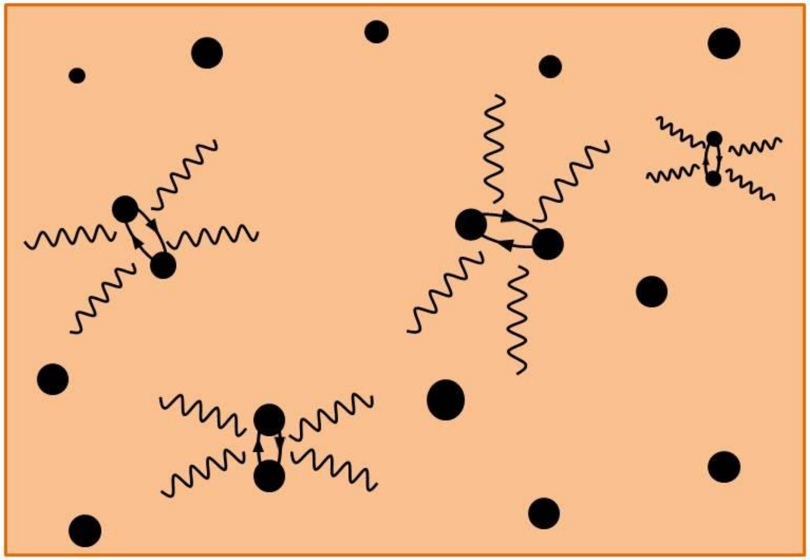


Once x and y are fixed, a and b are determined as

$$a = \frac{1}{f_{\mathrm{PBH}}} \frac{x^4}{\bar{x}^3} \qquad b = \frac{x^3}{y^3} a$$

Binary formation in RD era (Nakamura et al. 1997)

We can compute probability distribution of (a,e).



Uniform distribution

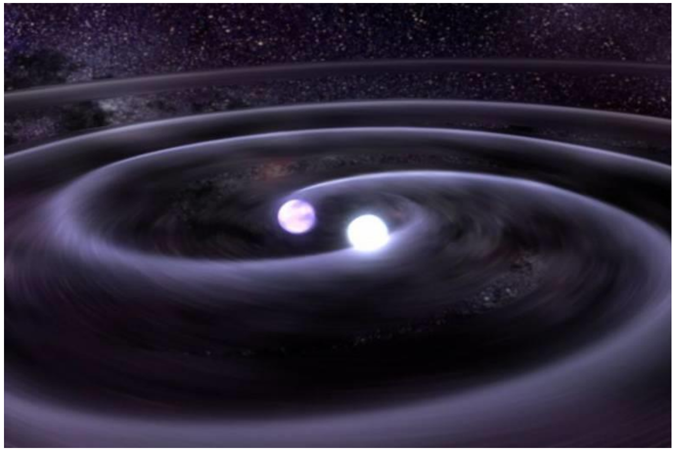
$$dP = \frac{9}{\bar{x}^6} x^2 y^2 dx dy \qquad 0 < x < y < \bar{x}$$

Probability in $(a, a + da)$ and $(e, e + de)$

$$\boxed{dP = \frac{3}{4} f^{3/2} \bar{x}^{-3/2} a^{1/2} e (1 - e^2)^{-3/2} da\, de.}$$

Life time of the binary

The next thing to do is to convert the probability in (a, e) to the merger probability in $(t, t + dt)$.



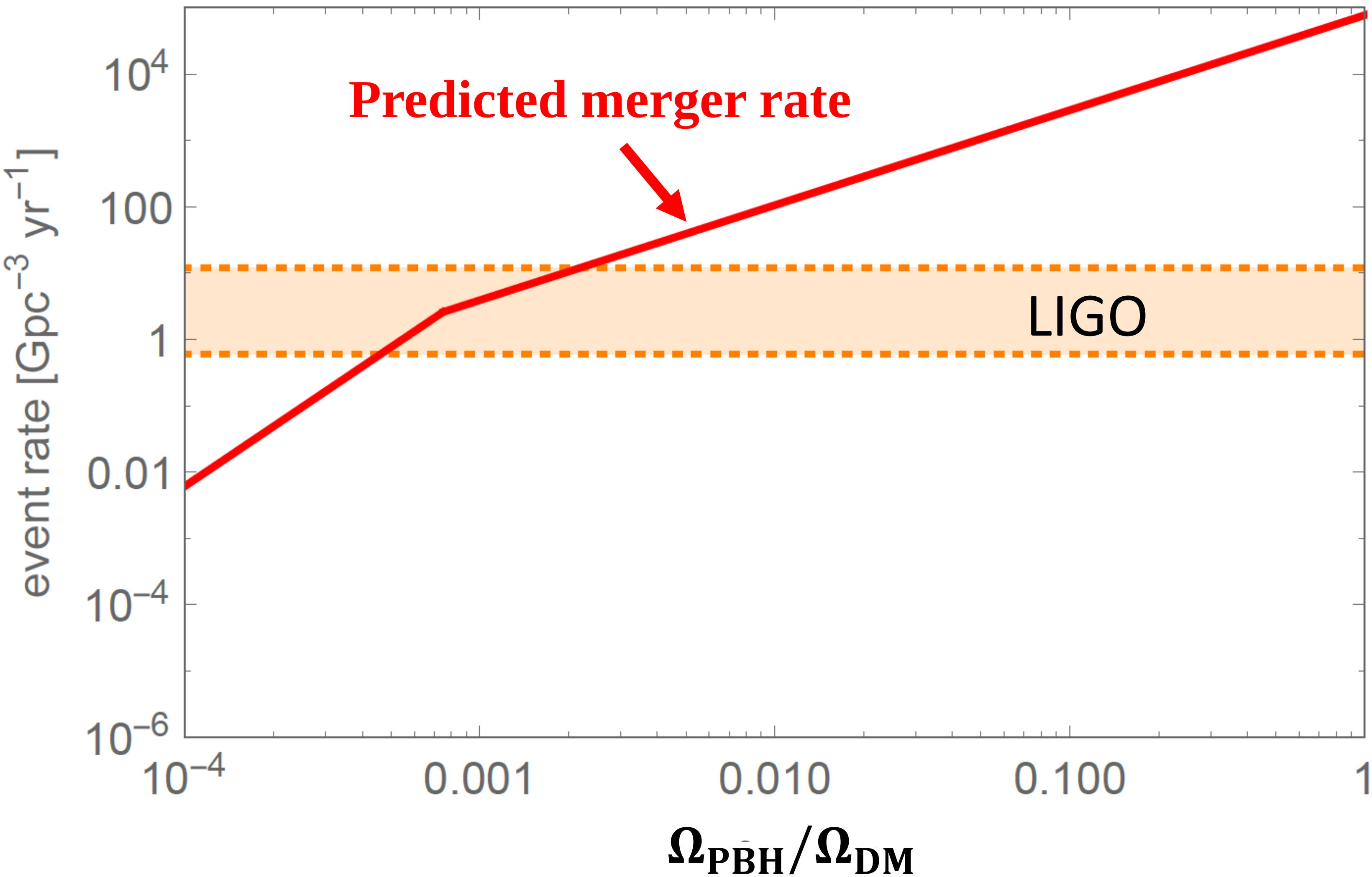
Life time of the binary is a function of major axis a and eccentricity e.

$$t = Qa^4(1 - e^2)^{7/2}, \qquad Q = \frac{3}{170}(GM_{\rm BH})^{-3}$$

In the paper by Nakamura et al. 1997, $M_{BH} = 0.5 M_{\odot}$ and $\Omega_{PBH} = \Omega_{DM}$ was considered.

In the paper by Sasaki et al. 2016, $M_{BH} = 30 M_{\odot}$ and the formula was extended to the case $\Omega_{PBH} < \Omega_{DM}$.

Merger event rate (Sasaki et al. 2016)

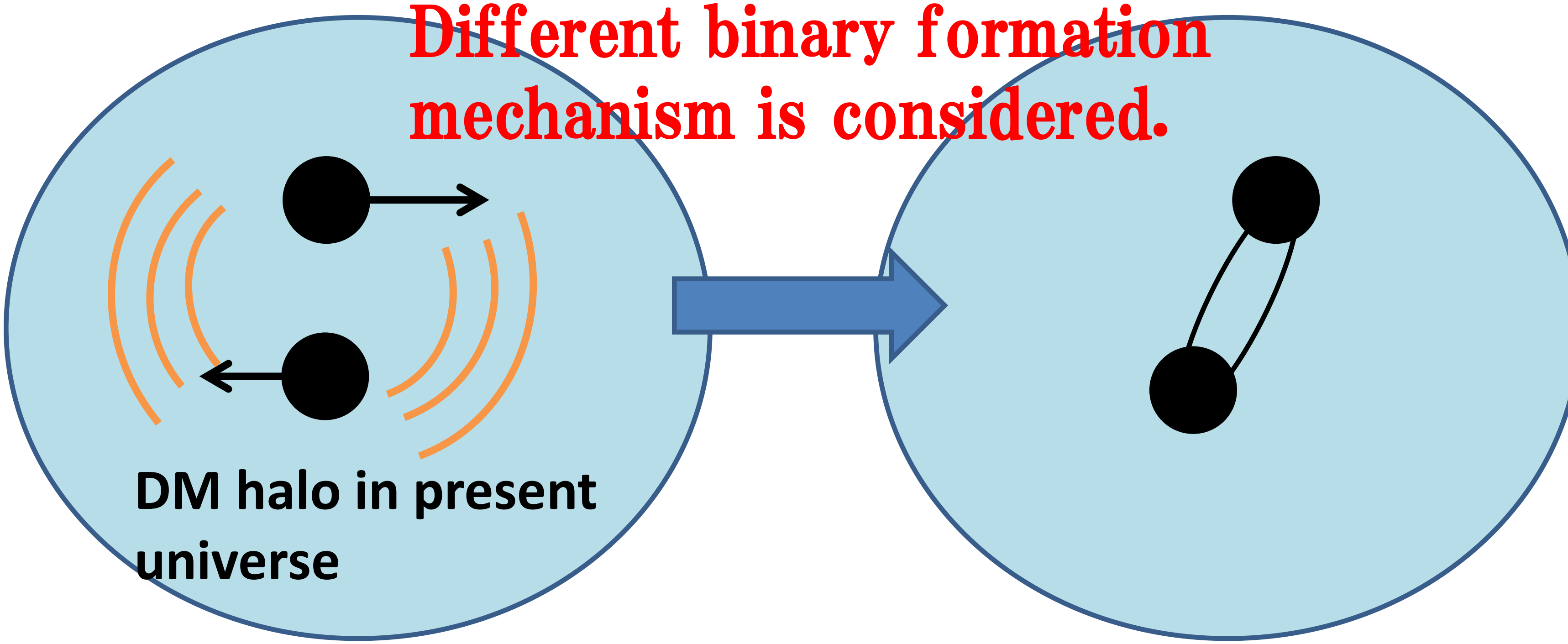


Consistent with LIGO if $30M_\odot$ PBHs constitute about 0.1% of dark matter.

Remark

PBH paper S.Bird et al. PRL 116, 201301(2016).

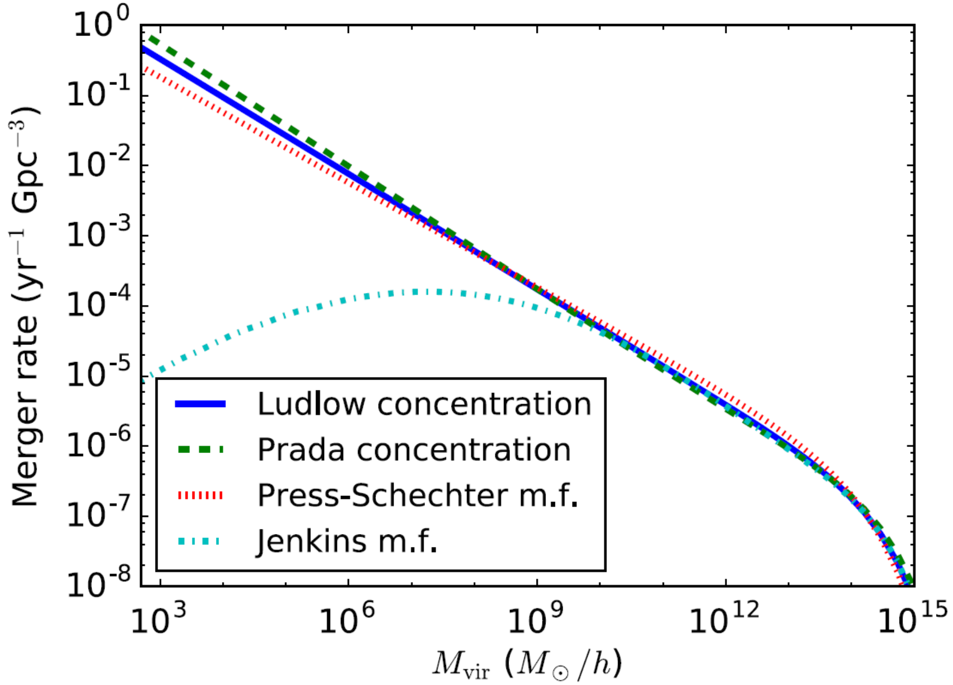
Different binary formation mechanism is considered.



$$\sigma = \pi \left(\frac{85\pi}{3}\right)^{2/7} R_s^2 \left(\frac{v_{\mathrm{pbh}}}{c}\right)^{-18/7}$$
$$= 1.37 \times 10^{-14} M_{30}^2 v_{\mathrm{pbh}-200}^{-18/7}\ \mathrm{pc}^2$$

Remark

PBH paper S.Bird et al. PRL 116, 201301(2016).

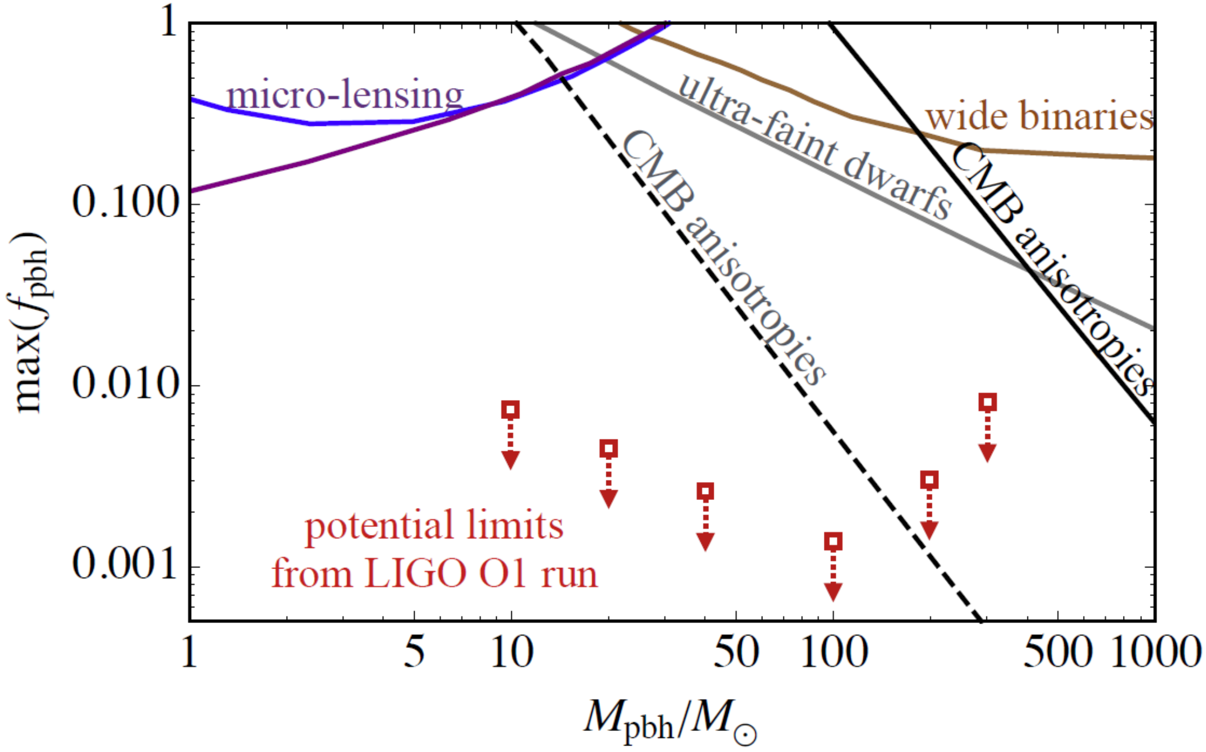


$$\boxed{\mathcal{V} = 2f(M_c/400M_\odot)^{-11/21}\ \mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1},}$$

Consistent with LIGO if PBH is whole dark matter

Recently, the same formula has been used to place upper limit on Ω_{PBH} from the LIGO observation.

(Ali-Haimoud, Kovetz, Kamionkowski 2017)



It is important to keep in mind that monochromatic mass function is assumed.
Additional consideration is necessary for the extended mass function. **(Carr et al. 2017)**

PBH hypothesis

LIGO BHs could be primordial BHs.

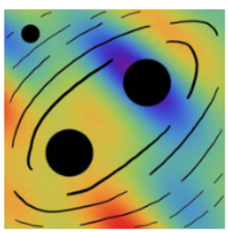
Editors' Suggestion

PDF HTML

Primordial Black Hole Scenario for the Gravitational-Wave Event GW150914

Misao Sasaki, Teruaki Suyama, Takahiro Tanaka, and Shuichiro Yokoyama

Phys. Rev. Lett. **117**, 061101 (2016) – Published 2 August 2016



A theoretical analysis examines the possibility that the gravitational wave signal (GW150914) detected by LIGO was due to the coalescence of primordial black holes created by the extremely dense matter present in the early Universe.

Show Abstract +

By testing this scenario, we get more knowledge about the condition of the extremely early universe.

Various effects that are ignored have been evaluated in other papers.

(Ioka et al. 1998, Hayasaki et al. 2009, Sasaki et al. 2016, Eroshenko 2016, Ali-Haimoud et al. 2017)

- Tidal force from outer BHs
- Initial peculiar velocity of PBHs
- Three body collisions
- Additional tidal force from dark matter perturbations
- Encounters of other PBHs (later time effect)
- Tidal force from halos (later time effect)
- Dynamical friction from DM and baryon (later time effect)

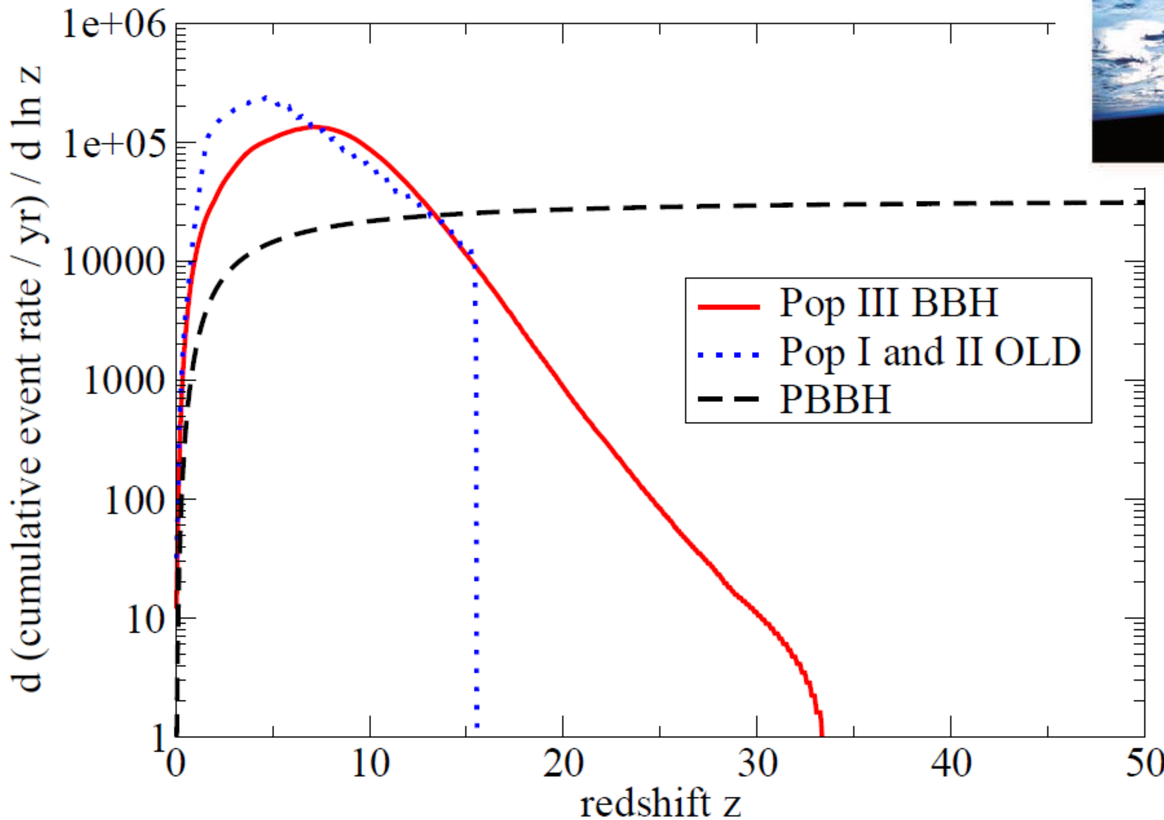
Simple analytical estimation suggest that those effects do not lead to the significant change of the result.

We have to keep in mind that these studies adopt the two assumptions.

How do we test the PBH scenario?

- Cosmic evolution of merger rate T.Nakamura et al. 2016
- Spin distribution T.Chiba and S.Yokoyama 2016
- Stochastic GWs K.Ioka et al 1999, S.Wang et al. 2016, M.Raidal et al. 2017
- Event distribution in BH mass plane B.Kocsis, TS, T.Tanaka, S.Yokoyama 2017

PBH merger rate at high-z



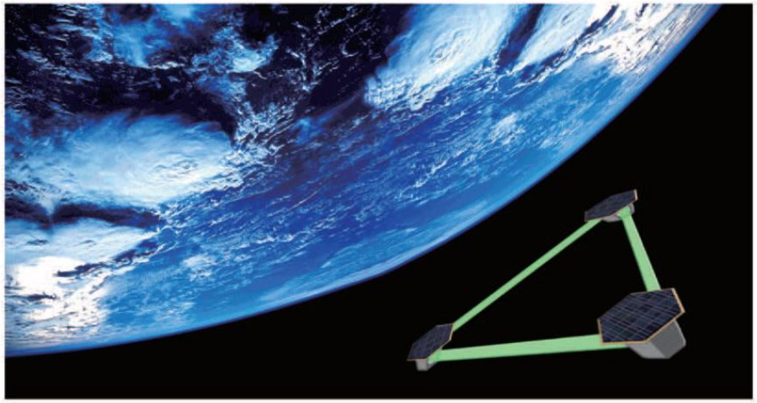


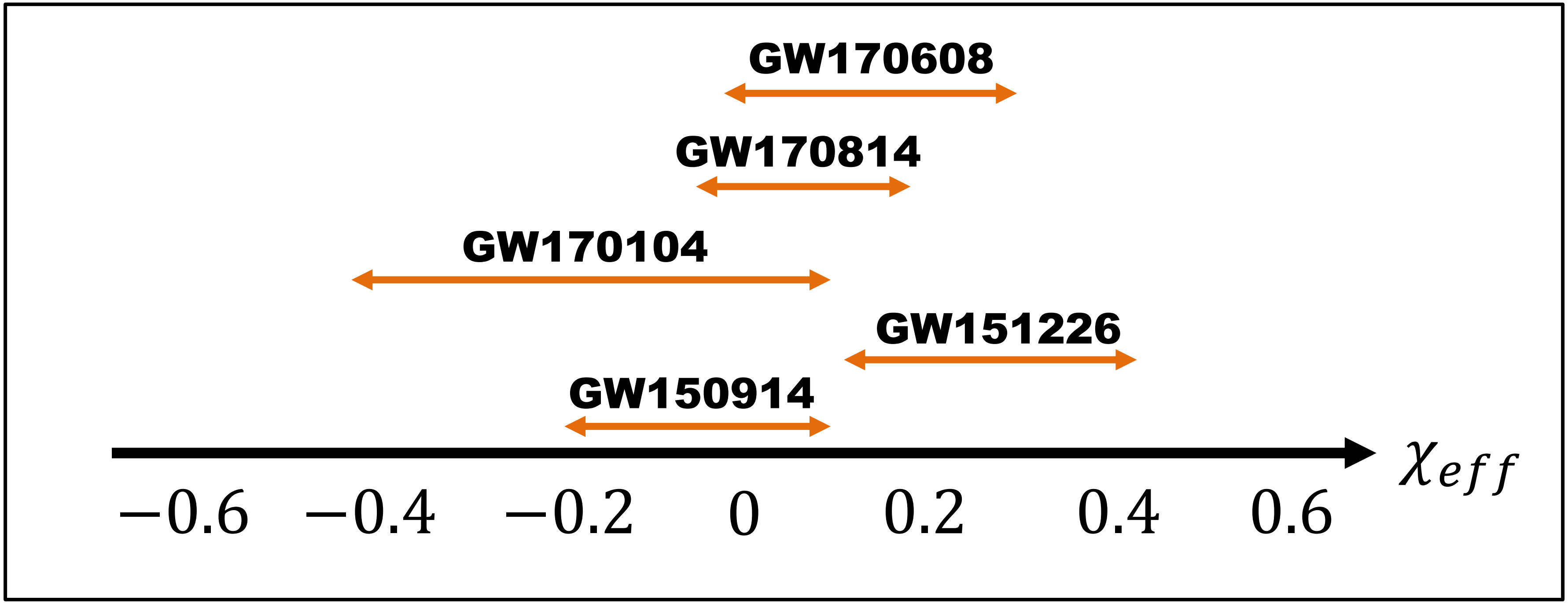
Image of Pre-DECIGO

T.Nakamura et al. 1607.00897

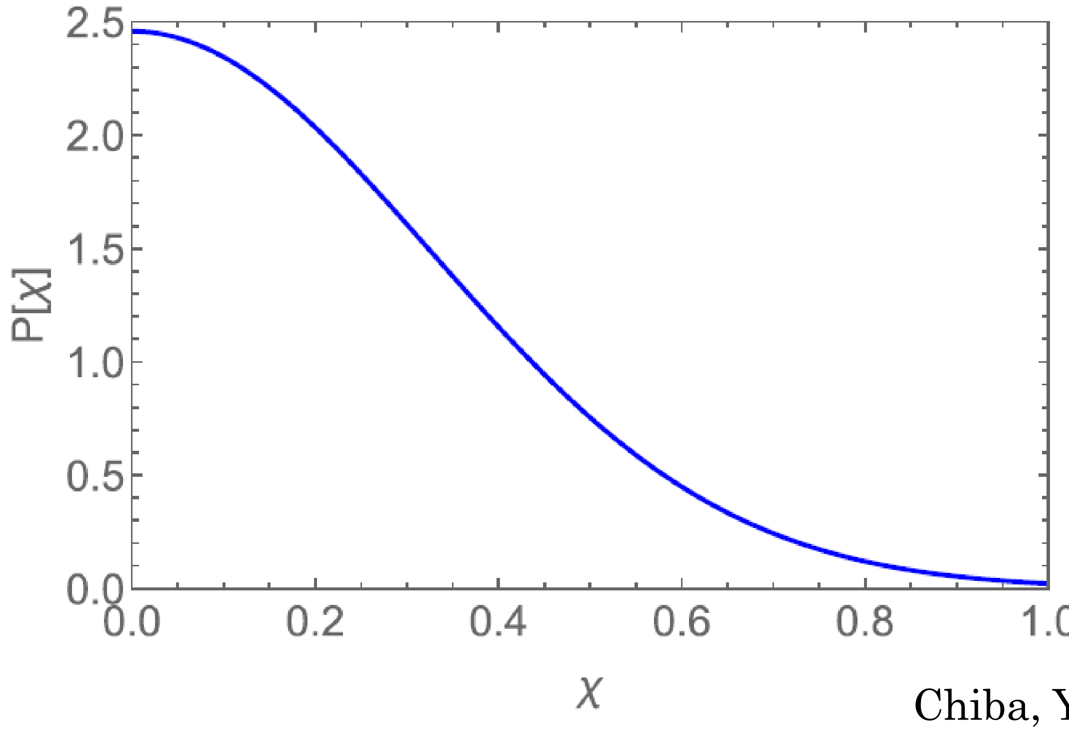
We can test the PBH scenario by observing the merger rate at high-z.

Spin distribution

$$\chi_{\mathrm{eff}} = \frac{c}{GM}\left(\frac{\boldsymbol{S}_1}{m_1} + \frac{\boldsymbol{S}_2}{m_2}\right)\cdot\frac{\boldsymbol{L}}{|\mathbf{L}|}$$



Spin distribution of PBHs (formed in RD)



Chiba, Yokoyama 2017

$$\mathrm{d}P \propto \exp\left[-\left(\frac{\chi}{\chi_*}\right)^2\right]\mathrm{d}\chi, \quad \chi_* \approx 0.46.$$

PBHs are expected to be slowly rotating.

(simply because rotation requires more δ to form PBH)

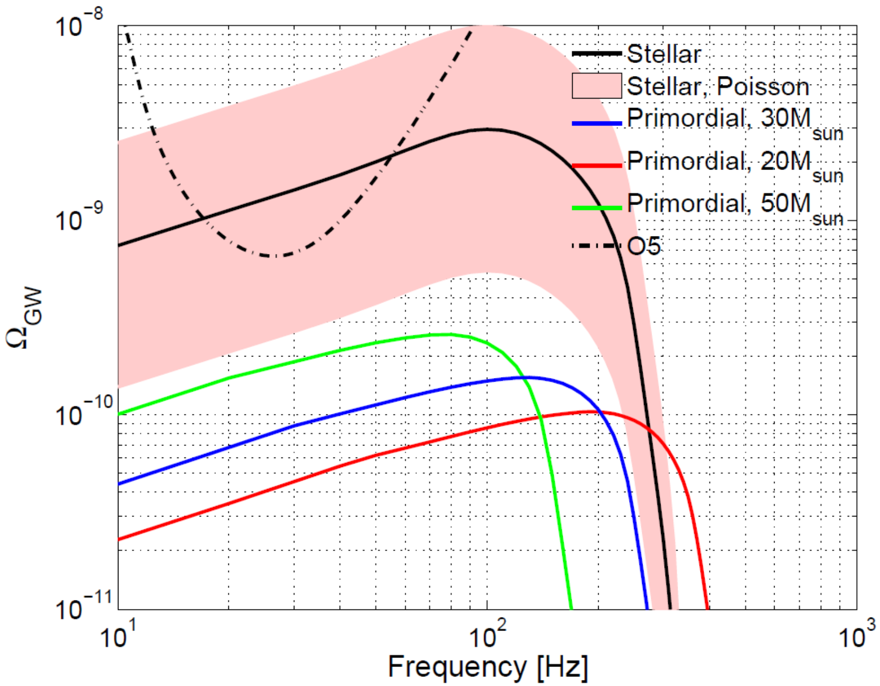
Positive χ_{eff} and negative one are equally likely.

GWs from many PBH binaries (Stochastic GWs)

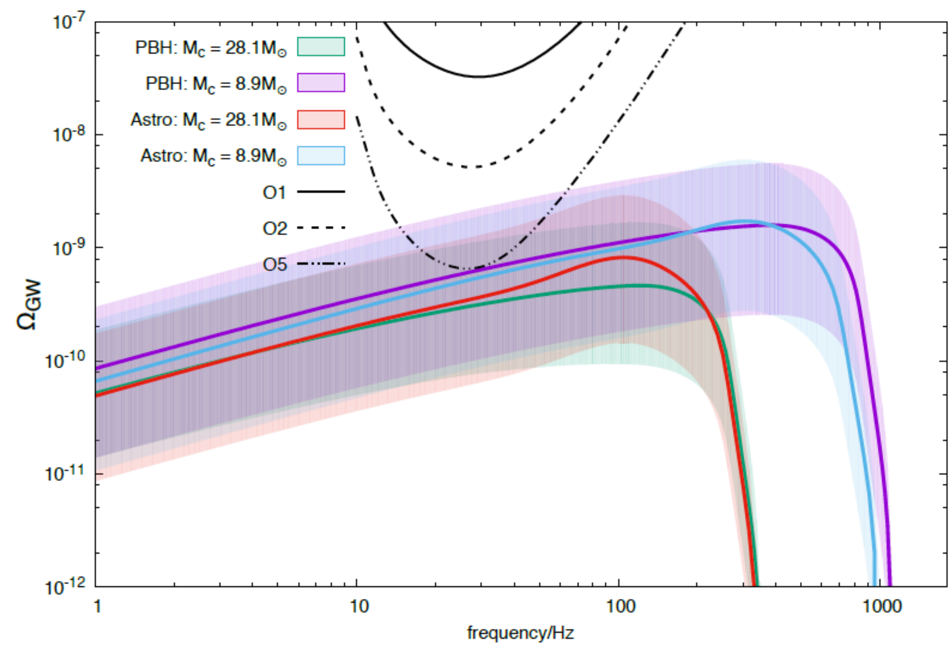
V.Mandic, S.Bird, I.Cholis, 1608.06699 I.Cholis, 1609.03565

S.Clesse, J.Garcia-Bellido, 1609.03565

S.Wang et al., 1610.08725



V.Mandic et al., 1608.06699



S.Wang et al., 1610.08725

Distinguishing from stellar-origin BHs is crucial.

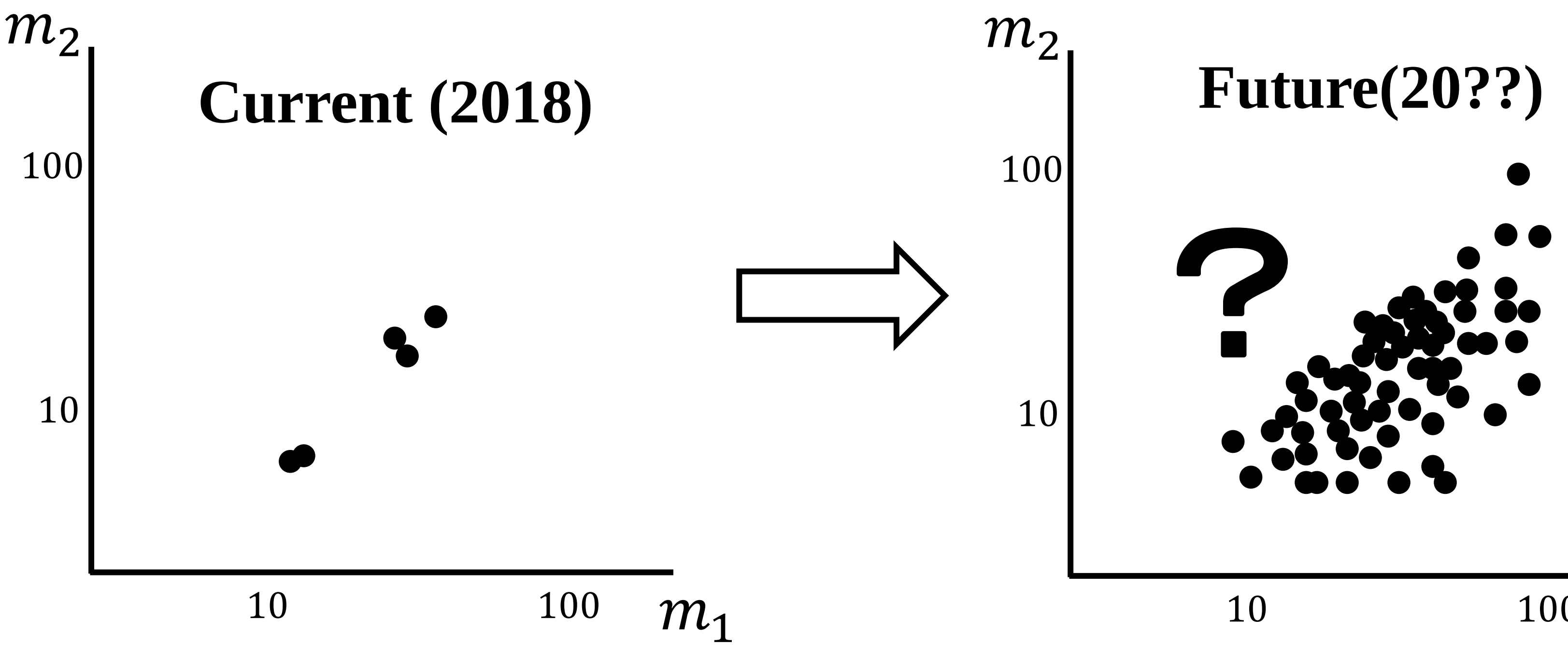
How do we test the PBH scenario?

-Merger event distribution in mass plane-

Main message

Merger rate distribution in the PBH mass plane has hidden universality and is an interesting observable to address this question.

B.Kocsis, TS, T.Tanaka, S.Yokoyama 2017



In the future, we will have observed many merger events and will be able to discuss about the distribution in the PBH mass plane (m_1,m_2).

In order to investigate what kind of feature appears in the distribution in the mass plane in the PBH scenario, we first generalized the formula to the case of the extended PBH mass function $\boldsymbol{f(m_{BH})}$.

Assumption

In Kocsis et al.2017, we considered the extended mass function which is not so broad ($\lesssim 10$) since it is not clear at all if the same mechanism of the binary formation can still work dominantly for very broad mass function.

No correlation between different PBH masses.

Apart from this, we do not assume a specific form of $f(m)$.

Merger event rate distribution in (m_1, m_2) plane

Mass function

$$\mathcal{R}(m_1, m_2, t) = \frac{1}{2n_{\mathrm{PBH}}}\mathcal{R}_{\mathrm{intr}}(m_1, m_2, t) f(m_1) f(m_2)$$

Observable in the future

Probability that given BH pair (m_1, m_2) form a binary and merge at time t.

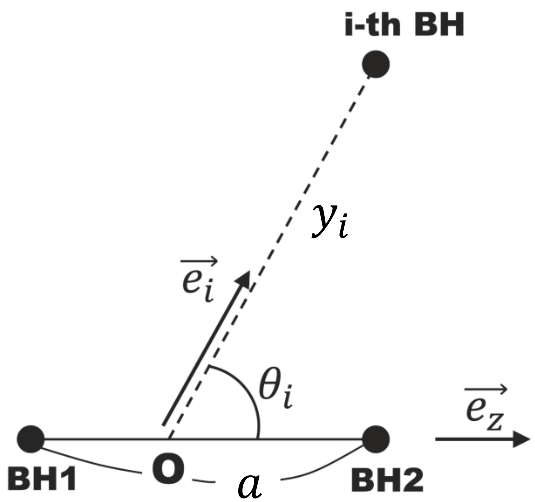
Non-trivial task is to evaluate $\mathcal{R}_{intr}$.

To derive the merger rate for given (m_1, m_2), we need to know the probability distribution of (a, e).

Distribution of (a, e) is determined by statistical variables: $x, y_i, M_i, \boldsymbol{e}_i$

$$a = Ax^4, A = \frac{1}{1+z_{eq}} \frac{\rho_c \Omega_m}{m_1 + m_2}$$

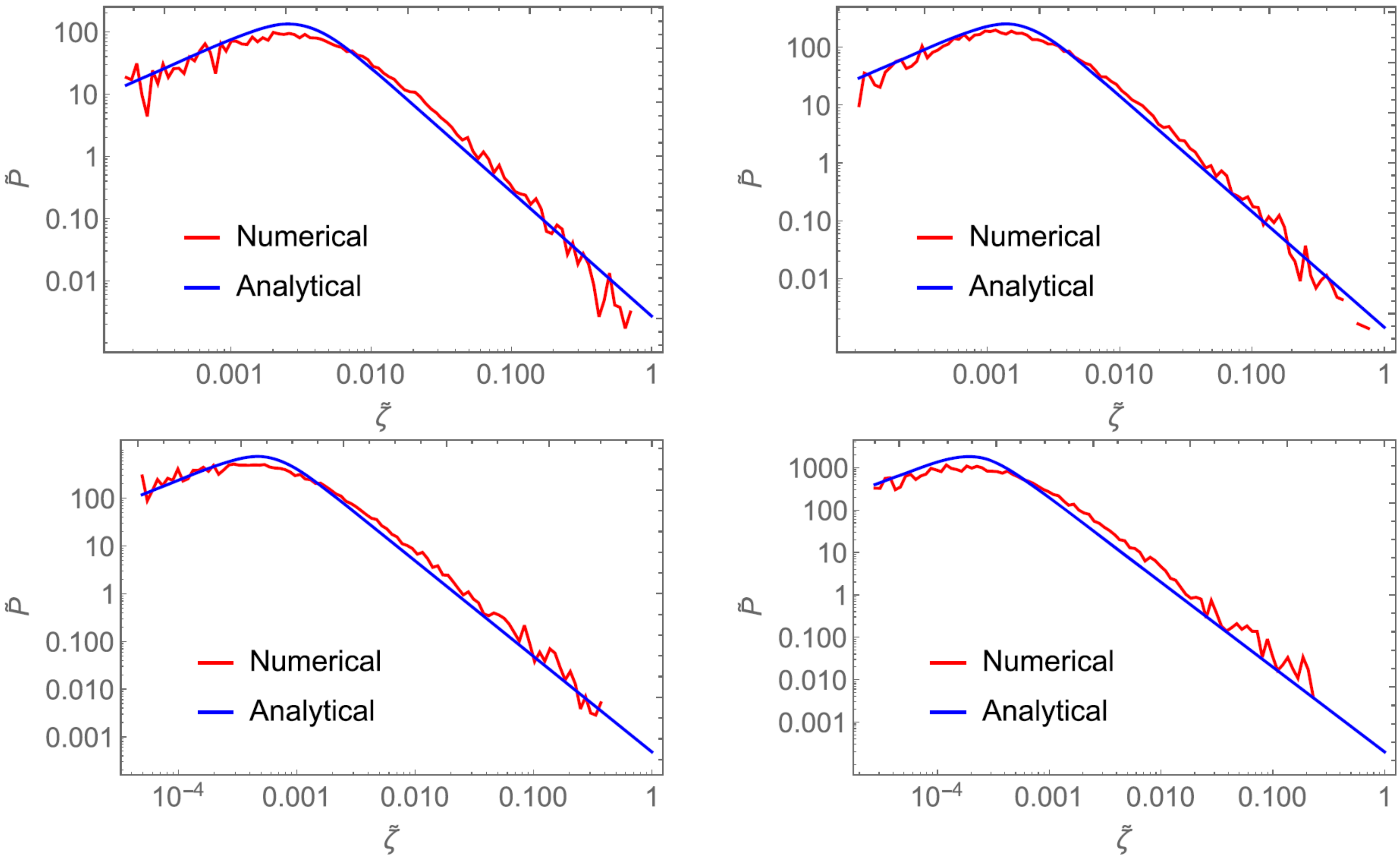
$$1 - e^2 = \frac{9}{4}\zeta^2, \quad \zeta = \sum_{i=1}^{N} \frac{x^3}{y_i^3} \frac{M_i}{m_t} \sin(2\theta_i) \frac{(\boldsymbol{e}_z \times \boldsymbol{e}_i)}{|\boldsymbol{e}_z \times \boldsymbol{e}_i|},$$



We evaluated the merger rate under two different approximations.

- Nearest BH only ($N = 1$), analytically
- Flat mass function (numerically)

Flat mass function



We found an approximate fitting formula for the probability distribution of the eccentricity.

In both cases, we found that the merger rate distribution is given by

$$\mathcal{R}(m_1, m_2, t) = C\tilde{f}(m_1)\tilde{f}(m_2)(m_1 + m_2)^{\alpha}$$

$C, \tilde{f}(m)$: sensitive to the PBH mass function

Dependence on the total mass is not sensitive to the mass function!!

$$0.97 < \alpha < 1.05$$

$$\ln\mathcal{R} = \ln C + \ln\tilde{f}(m_1) + \ln\tilde{f}(m_2) + \alpha\ln(m_1 + m_2)$$

$$\Rightarrow \quad \frac{\partial^2}{\partial m_1 \partial m_2}\ln\mathcal{R} = -\frac{\alpha}{(m_1 + m_2)^2}$$

Hidden Universality of $R(m_1, m_2, t)$

Statement

Construct a quantity α out of the distribution $R(m_1, m_2, t)$ as

$$\alpha(m_1, m_2, t) \equiv -(m_1 + m_2)^2 \frac{\partial^2}{\partial m_1 \partial m_2} \ln \mathcal{R}(m_1, m_2, t)$$

Then, the PBH mergers in the present mechanism predict

$$0.97 \lesssim \alpha \lesssim 1.05$$

for any PBH mass function (as long as it is not broad).

Different formation mechanisms predict different value

$\alpha \approx 1.43$ **PBH binary formation at low redshift.**
(Bird et al. 2016, Clesse, Garcia-Bellido 2016)

$\alpha \sim 4$ **Dynamical formation scenario (astrophysics BHs)**

Summary

GW astronomy has just begun.

LIGO might have detected PBHs for the first time.

The PBH scenario can be tested in the future by GW data.