Spherical M5-branes from the plane wave matrix model

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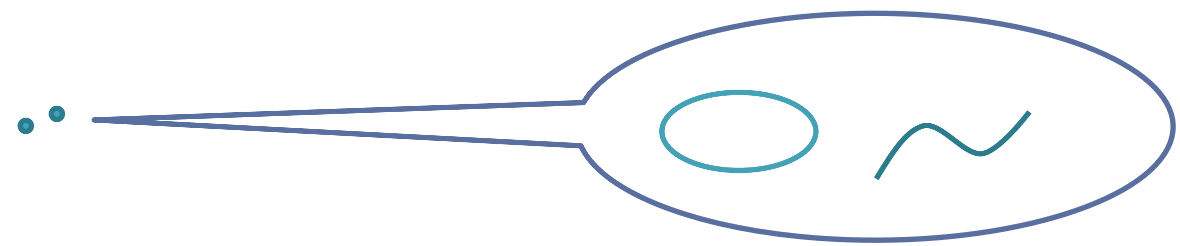
Contents

Superstring theory

Field theory:
Theory of point particles

In string theory, we assume that the fundamental particles are not point particles but strings



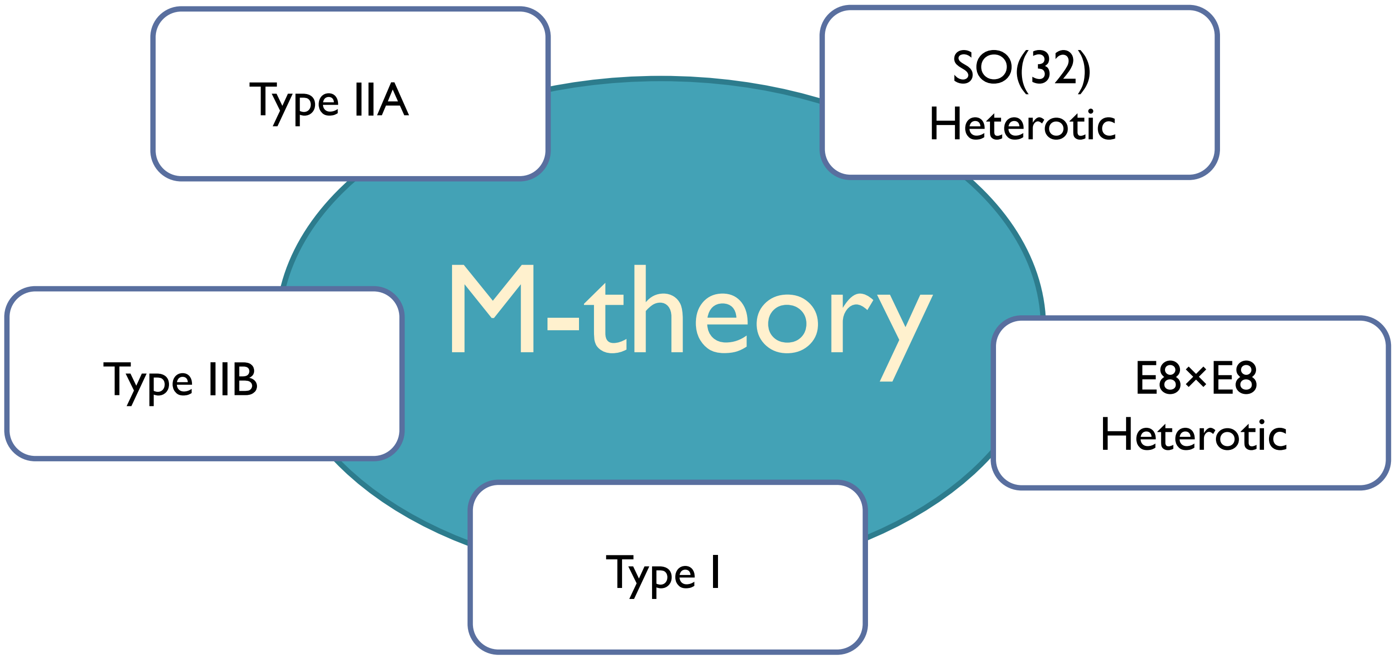
○ Electromagnetic interaction
○ Strong interaction
○ Weak interaction
× Gravitational interaction

String theory can describe gravity
(But still incomplete)

The various different fundamental particles can be described as different oscillation modes of strings

Problems of string theory

- Only perturbation theory is known. Quantizable only on simple space-times (flat, pp-wave)
- Theory of everything? But there exist 5 consistent theories…

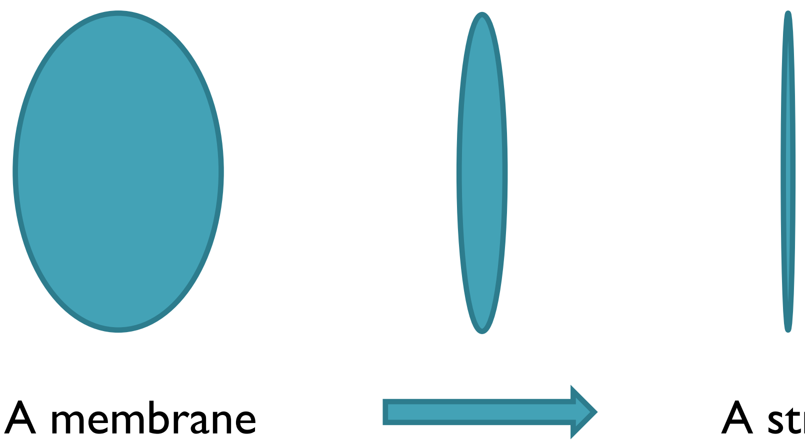


Which one is describing our universe?

It was conjectured that these theories are different aspects of a single theory[Witten]

M-theory

- A theory in 11-dimension, with the maximal supersymmetry
- Compactifying 11th dimension, M-theory reduces to type IIA superstring
- Fundamental objects are membranes (1+2 dim objects)



A membrane → A string

compactification

- Low energy effective theory is 11D SUGRA（uniquely determined from SUSY）

Does such 11D theory really exist?

Then it was conjectured that M-theory is defined by the matrix model

Theory of a single membrane

Nambu-Goto action

$$S = -T\int d^3\sigma \sqrt{-\det h_{\alpha\beta}}$$

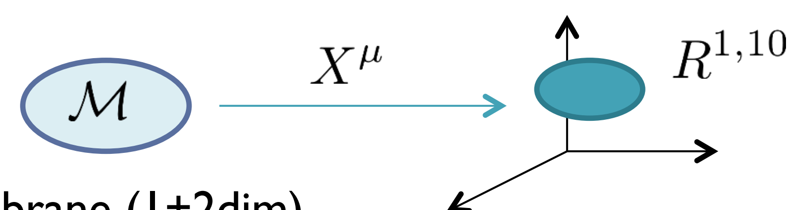
Volume of the world line of membrane (World volume)

$$h_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X_\mu$$

Induced metric

$$X^\mu : \mathcal{M} \to R^{1,10}$$

Embedding function



Membrane (1+2dim)

Polyakov action

Symmetric auxiliary field

$$S = -\frac{T}{2}\int d^3\sigma \sqrt{-\gamma}(\gamma^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X_\mu - 1)$$

Gauge fixing

By using
(1) EOM of the auxiliary field
(2) Diffeomorphism in 1+2D
we can simplify the theory

$$\gamma_{0a} = 0$$

$$\gamma_{00} = -4\det h_{ab}$$

$$X^+(\tau, \sigma_1, \sigma_2) = \tau \qquad \left[X^\pm = (X^0 \pm X^{10})/\sqrt{2} \right]$$

$$S = \frac{T}{4} \int d^3\sigma \left((\dot{X}^i)^2 - 2\{X^i, X^j\}^2 \right) \qquad i, j = 1, 2, \cdots, 9$$

constraint $\{\dot{X}^i, X_i\} = 0$

The theory is written in terms of the canonical Poisson bracket on the membrane

$$\{f, g\} = \epsilon^{ab} \partial_a f \partial_b g$$

[Hoppe, De Witt-Hoppe-Nicolai]

Matrix regularization

M-theory must have SUSY → Supermembrane

$$S \to S + S_{\text{fermions}}$$

Matrix regularization : Regularization preserving the all SUSYs.

For example, let us consider the case in which the world volume is $R \times S^2$

$$X^i(\tau, \sigma^a) = \sum_{J=0}^{\infty} \sum_{m=-J}^{J} X^i_{Jm}(\tau) Y_{Jm}(\sigma^a)$$ ← spherical harmonics

$$Y_{Jm} = \sum_{l=0}^{J} c^{lm}_{i_1 i_2 \cdots i_l} x^{i_1} x^{i_2} \cdots x^{i_l} \quad (x_i x_i = 1)$$

The theory of supermembrane = a theory of SUSY field theory on $R \times S^2$

Momentum cutoff regularization

Let us first consider the sharp momentum cutoff

$$J = 0, 1, 2, \cdots, \infty \quad \Longrightarrow \quad J = 0, 1, 2, \cdots, 2\Lambda$$

$$X^i = \sum_{J=0}^{2\Lambda} \sum_{m=-J}^{J} X^i_{Jm} Y_{Jm}$$

Such functions with cutoff do not close under the multiplication

$$[2\Lambda] \otimes [2\Lambda] = 0 \oplus 1 \oplus \cdots \oplus [4\Lambda]$$

Multiplication of functions yields modes beyond the cutoff

In most theories, this would break symmetries.
For the supermembrane, this regularization would break SUSY.

Is there any good regularization which is compatible with the algebraic structure?

[Madore]

Matrix reguraization (fuzzy sphere)

Nicer way of introducing the cutoff

$$Y_{Jm} = \sum_{l=0}^{J} c^{lm}_{i_1 i_2 \cdots i_l} x^{i_1} x^{i_2} \cdots x^{i_l} \xrightarrow[\text{map}]{} \hat{Y}_{Jm} = \sum_{l=0}^{J} c^{lm}_{i_1 i_2 \cdots i_l} \hat{x}^{i_1} \hat{x}^{i_2} \cdots \hat{x}^{i_l}$$

$$\left\{ \begin{array}{l} \hat{x}^i = \dfrac{1}{\sqrt{\Lambda(\Lambda+1)}} L_i \\ \\ L_i \ \text{: SU(2) generators in spin } \Lambda \text{ representation.} \quad [L_i, L_j] = i\epsilon_{ijk} L_k \end{array} \right.$$

Matrix size: $(2\Lambda+1) \times (2\Lambda+1)$

$\hat{x}^i$ are noncommutative counterpart of coordinates on S^2

$$\hat{x}_i \hat{x}_i = \mathbf{1}$$

$$[\hat{x}_i, \hat{x}_j] = \frac{i}{\sqrt{\Lambda(\Lambda+1)}} \epsilon_{ijk} \hat{x}_k \to 0 \ \ (\Lambda \to \infty)$$

The limit of large matrix size = The commutative limit

$$Y_{Jm} = \sum_{l=0}^{J} c^{lm}_{i_1 i_2 \cdots i_l} x^{i_1} x^{i_2} \cdots x^{i_l} \xrightarrow[\text{map}]{} \hat{Y}_{Jm} = \sum_{l=0}^{J} c^{lm}_{i_1 i_2 \cdots i_l} \hat{x}^{i_1} \hat{x}^{i_2} \cdots \hat{x}^{i_l}$$

Cutoff !

$\{\hat{Y}_{Jm} | J = 0, \cdots, 2\Lambda\}$ gives a basis of $(2\Lambda+1)\times(2\Lambda+1)$ matrices

Algebra is obviously closed!

Furthermore, the structure constants agree in the commutative limit $\Lambda \to \infty$

$$Y_1 Y_2 = C^3_{12} Y_3 \qquad C^3_{12} = \int (Y_3)^* Y_1 Y_2$$

$$\hat{Y}_1 \hat{Y}_2 = \hat{C}^3_{12} \hat{Y}_3 \qquad \hat{C}^3_{12} = \mathrm{Tr}(\hat{Y}_3)^\dagger \hat{Y}_1 \hat{Y}_2 \qquad \hat{C}^3_{12} \to C^3_{12} \ \ (\Lambda \to \infty)$$

The cutoff is introduced as the matrix size

This cutoff is compatible with SUSY because of its nice algebraic structure!

Matrix regularization (general)

Matrix regularization is a family of linear maps $(T_N)_N \ (N = 1, 2, \cdots \infty)$

$T_N : C^\infty(\mathcal{M}) \to M_N(C)$ is a linear map satisfying

$$\begin{cases} \lim_{N\to\infty} ||T_N(f)T_N(g) - T_N(fg)|| = 0 \\ \lim_{N\to\infty} ||iN[T_N(f), T_N(g)] - T_N(\{f,g\})|| = 0 \\ \lim_{N\to\infty} \mathrm{Tr}T_N(f) = \int f \end{cases}$$

Preserving algebra

Poisson bracket
⇒ Commutator

Integral
⇒ Trace

The existence of $(T_N)_N$ is proved for any closed compact 2D surface

Supermembrane as a matrix model

$$S = \frac{T}{4}\int d^3\sigma \left((\dot{X}^i)^2 - 2\{X^i, X^j\}^2 + \text{fermions} \right) \qquad X^i(t,\sigma) \to X^i_{ab}(t)$$

$$\to \int dt \mathrm{Tr}\left[\frac{1}{2}(\dot{X}^i)^2 + \frac{1}{4}[X^i, X^j]^2 + \text{fermions} \right]$$

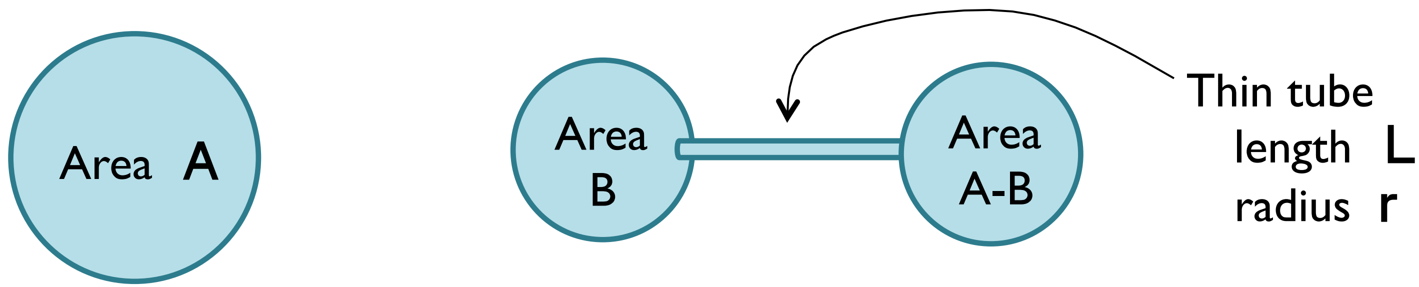
Matrix model

ALL SUSY is kept

Shape of membrane ⇔ Configuration of matrices

Problem of instability and BFSS conjecture

If we consider the matrix model as a regularized theory of a single membrane, there is the problem of the instability



Energy $\sim TA$ Energy $\sim T(A+2\pi rL)$

If the tube is thin enough ($r \ll 1/(TL)$), the above two configurations have almost the same energy. $\Rightarrow$ Configurations like multiple membranes also contribute…

Banks et al. studied the same matrix model from a different viewpoint and conjectured that the model is actually the second-quantized M-theory (describing the many-body system of supermembranes)

Furthermore, they demonstrated that the gravitational interaction in the low energy limit (SUGRA) can be correctly derived from this interpretation.

Summary so far

BFSS matrix model

$$S = \frac{1}{g^2} \int dt \mathrm{Tr} \left[\frac{1}{2} (DX^i)^2 + \frac{1}{4} [X^i, X^j]^2 + \text{fermions} \right]$$

$$\begin{cases} X^i_{ab}(t) : N \times N & \text{Hermitian} \\ DX^i = \dot{X} - i[A, X^i] \end{cases}$$

In the A=0 gauge,
Gauss Law (EOM of A) reproduces
the constraint $\{\dot{X}^i, X_i\} = 0$

This model is the second quantized M-theory in 11D with light-like compactification

To realize M-theory, we need to take the large-N limit (DLCQ)

M5-branes?

M-theory contains not only membranes but also M5-branes (1+5 dim objects)

4D gauge field A_μ $\left\{ \begin{array}{l} \text{electrons} \\ \text{monopoles} \end{array} \right.$

$$F_2 = dA_1 \qquad \int A_1$$

$$\tilde{F}_2 = *F_2 = d\tilde{A}_1 \qquad \int \tilde{A}_1$$

11D SUGRA
3-form gauge field $C_{\mu\nu\rho}$ $\left\{ \begin{array}{l} \text{M2-brane (membrane)} \\ \text{M5-brane} \end{array} \right.$

$$F_4 = dC_3 \qquad \int C_3$$

$$\tilde{F}_7 = *F_4 = d\tilde{C}_6 \qquad \int \tilde{C}_6$$

Does the matrix model contain not only M2-branes but also M5-branes?

⇒ To understand this problem, we consider M-theory on the pp-wave geometry, which admits nice stable configurations of M5-branes.

Contents

- Introduction (M theory・Matrix model)
- M-theory on the PP-wave geometry
  - Spherical M2-brane
  - Spherical M5-brane
- A conjecture on M5-branes in the matrix model
- Our analysis based on the localization
- Summary

2. M-theory on pp-wave background

PP-wave background

- Maximally supersymmetric pp-wave solution in 11D SUGRA

$$ds^2 = -2dx^+dx^- + dx^M dx^M - \left(\frac{\mu^2}{9}x^i x^i + \frac{\mu^2}{36}x^a x^a\right) dx^+ dx^+$$

$$F_{123+} = \mu$$

$$\begin{cases} M = 1, \cdots, 9 \\ i = 1, 2, 3 \\ a = 4, \cdots, 9 \end{cases}$$

Spatial rotational symmetry: SO(3)×SO(6)

- Let us consider M2- and M5-branes on this background

M2-brane

- Nambu-Goto action

$$S = -\int d^3\sigma \sqrt{-h} + \int C_3$$

$$h = \det h_{\alpha\beta}$$

$$h_{\alpha\beta} = g_{\mu\nu}(X)\partial_\alpha X^\mu \partial_\beta X^\nu$$: induced metric

X : Embedding function

$$dC_3 = F_4 = \mu\, dx^1 \wedge dx^2 \wedge dx^3 \wedge dx^+$$

- Polyakov action

$$S = -\frac{1}{2}\int d^3\sigma \sqrt{-\gamma}\left(\gamma^{\alpha\beta} g_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu - 1\right) + \int C_3$$

$\gamma_{\alpha\beta}$: auxiliary field

Gauge-fixing

$$\begin{cases} \gamma_{01} = \gamma_{02} = 0 \\ \gamma_{00} = -4\,\det h_{ab} \quad (a, b = 1, 2) \end{cases} \longrightarrow \text{Fixing Diffeomorphism}$$

$$X^+ = \sigma^0 \quad \longrightarrow \text{EOM for } \gamma_{ab} \text{ is trivialized}$$

$$\begin{cases} M = 1, \cdots, 9 \\ i = 1, 2, 3 \\ a = 4, \cdots, 9 \end{cases}$$

Light-cone hamiltonian

Poisson bracket

$$H = \frac{2\pi}{p^+}\left(P_M^2 + \frac{1}{2}\{X_M, X_N\}^2\right) + \frac{p^+}{8\pi}\left(\frac{\mu^2}{9}X_i^2 + \frac{\mu^2}{36}X_a^2\right)$$
$$-\frac{\mu}{6}\left(\epsilon^{ijk}X_i\{X_j, X_k\}\right)$$

p^+: Light-cone momentum

Potential for $X_i (i = 1, 2, 3)$ becomes a perfect square

$$V(X_i) = \frac{p^+\mu^2}{72\pi}\left(X_i + \frac{6\pi}{p^+}\epsilon_{ijk}\{X^j, X^k\}\right)^2$$

Spherical M2-brane

$$V(X_i) = \frac{p^+\mu^2}{72\pi}\left(X_i + \frac{6\pi}{p^+}\epsilon_{ijk}\{X^j, X^k\}\right)^2$$

The potential is vanishing when

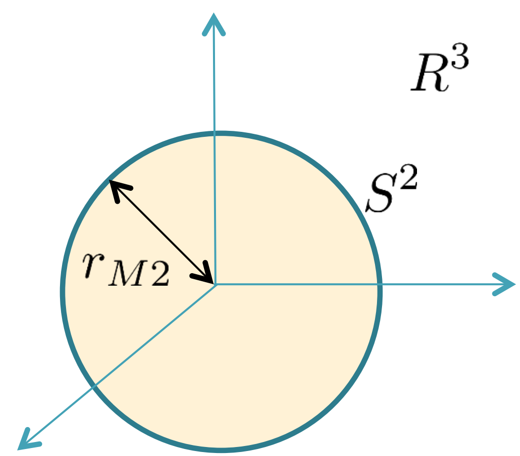
$$X_i + \frac{6\pi}{p^+}\epsilon_{ijk}\{X^j, X^k\} = 0 \qquad X_a = 0\ (a = 4, \cdots, 9)$$

This is solved by a spherical M2-brane

$$X_1^2 + X_2^2 + X_3^2 = r_{M2}^2$$

$$r_{M2} = \frac{\mu p^+}{12\pi}$$

(in the unit of $T_{M2} = 1$)



Spherical M5-brane

We can repeat the same computation for an M5-brane

- Action

$$S = -\int d^6\sigma\sqrt{-h} + \int C_6$$

- Hamiltonian

$$\{f_1, \cdots, f_5\} = \epsilon^{a_1\cdots a_5}\partial_{a_1} f_1 \cdots \partial_{a_5} f_5$$

$$H = \frac{\pi^3}{p^+}\left(P_M^2 + \frac{1}{5!}\{X_{M_1}, \cdots, X_{M_5}\}^2\right) + \frac{p^+}{2\pi^3}\left(\frac{\mu^2}{9}X_i^2 + \frac{\mu^2}{36}X_a^2\right)$$
$$-\frac{\mu}{6!}\left(\epsilon^{a_1\cdots a_6}X_{a_1}\{X_{a_2}, \cdots, X_{a_6}\}\right)$$

Potential for $X_a (a = 4, \cdots, 9)$ forms a perfect square

$$V(X_a) = \frac{p^+\mu^2}{72\pi^3}\left(X_{a_1} + \frac{6\pi^3}{5!\mu p^+}\epsilon_{a_1 a_2\cdots a_6}\{X^{a_2}, \cdots, X^{a_6}\}\right)^2$$

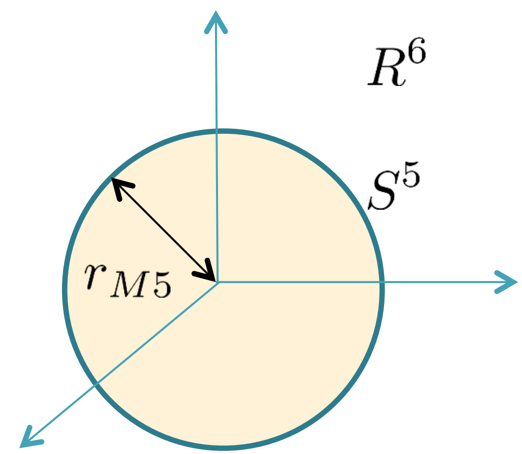
Spherical M5-brane

Light-cone hamiltonian is zero for

$$\begin{cases} X_i = 0 \ (i = 1, 2, 3) \\ \displaystyle\sum_{a=4}^{9} X_a^2 = r_{M5}^2 \end{cases}$$

$$r_{M5} = \left(\frac{\mu p^+}{6\pi^3} \right)^{1/4}$$

(in the unit of $T_{M5} = 1$)



Spherical M5-brane

Summary so far

In M-theory on pp-wave, there exist spherical M2- and M5-branes with vanishing light cone energy.

3. A conjecture for M5-branes in PWMM

Matrix model for M-theory on the pp-wave geometry

M-theory on the flat space-time ⇒ BFSS matrix model

M-theory on the PP-wave geometry ⇒ plane wave matrix model (PWMM)

PWMM is given as a mass deformation of the BFSS matrix model

Matrix model formulation of M-theory

- PWMM is obtained by matrix regularization of M2-brane on pp-wave.

$$X^M(t,\sigma^1,\sigma^2) \to X^M(t) : \; N \times N \text{ Hermitian matrix}$$

$$\{\ ,\ \} \to iN[\ ,\ ]$$

$$S_{\rm PWMM} = \frac{1}{g^2}\int dt\, {\rm tr}\left[\frac{1}{2}(D_t X_M)^2 - \frac{1}{4}[X_M, X_N]^2 + \frac{1}{2}\Psi^\dagger D_t \Psi - \frac{1}{2}\Psi^\dagger \gamma_M [X_M, \Psi] \right.$$
$$\left. + \frac{\mu^2}{2}(X_i)^2 + \frac{\mu^2}{8}(X_a)^2 + i\mu\epsilon_{ijk}X_i X_j X_k + i\frac{3\mu}{8}\Psi^\dagger\gamma_{123}\Psi\right],$$

$$\begin{cases} X_i : \text{SO(3) scalar} \\ X_a : \text{SO(6) scalar} \end{cases}$$

- PWMM is conjectured to realize the second quantization of M-theory on pp-wave background [BFSS, BMN].

- It must contain all states in M-theory. In particular, states with zero light-cone energy (spherical M2/M5) must be realized as the vacuum states of PWMM.

Vacua of PWMM

$$S_{\rm PWMM} = \frac{1}{g^2}\int dt\,\mathrm{tr}\left[\frac{1}{2}(D_t X_M)^2 - \frac{1}{4}[X_M, X_N]^2 + \frac{1}{2}\Psi^\dagger D_t \Psi - \frac{1}{2}\Psi^\dagger \gamma_M [X_M, \Psi] + \frac{\mu^2}{2}(X_i)^2 + \frac{\mu^2}{8}(X_a)^2 + i\mu\epsilon_{ijk}X_i X_j X_k + i\frac{3\mu}{8}\Psi^\dagger\gamma_{123}\Psi\right],$$

Potential for SO(3) scalars

$$V(X_i) = \frac{1}{g^2}\mathrm{tr}\left(\mu X_i + \frac{i}{2}\epsilon_{ijk}[X^j, X^k]\right)^2$$

Any N dimensional representation of SU(2) generator gives a vacuum

$X_i = \mu L_i \qquad [L_i, L_j] = i\epsilon_{ijk}L_k \qquad$ (Fuzzy sphere)

Are only M2-branes visible in the vacua of PWMM ???
Where are M5-branes ???

The conjecture

"Trivial vacuum $(X_i = 0)$ corresponds to a single spherical M5". [BMN]

More generally, if one makes an irreducible decomposition,

$$X_i = \mu L_i = \mu \bigoplus_{s=1}^{\Lambda} \left(L_i^{[N_5^{(s)}]} \otimes \mathbf{1}_{N_2^{(s)}} \right)$$

$N_5^{(s)}$: dim of irrep ⇔ M5 charge

$N_2^{(s)}$: multiplicity of irrep ⇔ M2 charge

[Maldacena-Sheiki-Jabbari-Raamsdonk]

$N_2 \to \infty$ with N_5 fixed ⇒ N_5 M5-branes are realized (M5-brane limit)

$N_5 \to \infty$ with N_2 fixed ⇒ N_2 M2-branes are realized (M2-brane limit)

(commutative limit of fuzzy sphere)

For the trivial vacuum, they checked the conjecture by showing that the mass spectra of PWMM and a single M5-brane agree with each other.

Trivial Vacuum = Single M5 ?

Trivial vacuum does not look like a spherical 1+5 dimensional object.

However, recall that M-theory is supposed to be realized in the limit,

$N \to \infty$ with $p^+ = N/R$ fixed. [BFSS]

This corresponds to a very strong coupling limit in PWMM $(g^2 N \sim (N/p^+)^3)$

- SO(6) scalars may form the spherical M5-brane at strong coupling.

$$X^a = \underbrace{0 + \text{Large quantum fluctuation}}_{S^5?}$$

We checked this by using the localization!

4. M5-branes from PWMM

Localization in PWMM

$$\begin{cases} M,N = 1,\cdots,9 \\ i,j,k = 1,2,3 \\ a,b = 4,\cdots,9 \end{cases}$$

$$S_{\mathrm{PWMM}} = \frac{1}{g^2}\int dt\,\mathrm{tr}\left[\frac{1}{2}(D_tX_M)^2 - \frac{1}{4}[X_M,X_N]^2 + \frac{1}{2}\Psi^\dagger D_t\Psi - \frac{1}{2}\Psi^\dagger\gamma_M[X_M,\Psi]\right.$$
$$\left. + \frac{\mu^2}{2}(X_i)^2 + \frac{\mu^2}{8}(X_a)^2 + i\mu\epsilon_{ijk}X_iX_jX_k + i\frac{3\mu}{8}\Psi^\dagger\gamma_{123}\Psi\right] ,$$

- We consider the theory around

$$X_i = \mu L_i = \mu L_i^{[N_5]} \otimes \mathbf{1}_{N_2}$$

(N_5=1 ⇔ the trivial vacuum)

- We set the boundary condition as

All fields → The vacuum configuration. $(t \to \pm\infty)$

- We use a SUSY which keeps invariant the following combination

$$\phi = X_3 + iX_9 \qquad \left(\phi(t) = X_3(t) + i(X_8(t)\sin t + X_9(t)\cos t)\right)$$

After a standard computation of localizaiton, we obtain the following

- saddle point

$$\phi = X_3 + iX_9 = \mu L_3 + iM$$

M : Moduli for SO(6) scalar (constant matrix)

- Effective action

Classical part

Comes from 1-loop determinant

$$S = \frac{1}{g^2}\sum_{i=1}^{N_2} m_i^2 + \sum_{J=0}^{N_5-1}\sum_{i\neq j}\log\left(\frac{((2J)^2+(m_i-m_j)^2)((2J+2)^2+(m_i-m_j)^2)}{((2J+1)^2+(m_i-m_j)^2)^2}\right)$$

- Vacuum configuration can be evaluated by the saddle point analysis.

  - Weak coupling limit → eigenvalue distribution is trivial $(X_9 = M = 0)$
  - Strong coupling limit → There is a non-trivial equilibrium

→ Eigenvalue distribution of X_9 in the M5-brane limit

$$\rho(x) = c(a - x^2)^{3/2} \qquad a, c \text{ : constants}$$

Uplift to SO(6) symmetric distribution

Assuming SO(6) symmetry, we consider the six dimensional eigenvalue distribution $\tilde{\rho}$ defined by

$$\frac{1}{N}\langle \mathrm{tr} X_9^n \rangle \to \int dx_9\, x_9^n \rho(x_1) =: \int d^6x\, x_9^n\, \tilde{\rho}(r) \quad \text{for any } n$$

$$r = \sqrt{x_4^2 + \cdots + x_9^2}$$

For $\rho(x) = c(a - x^2)^{3/2}$, we found that

$$\tilde{\rho}(r) = \tilde{c}\,\delta(r - r_0)$$

SO(6) scalars form a spherical shell in R^6

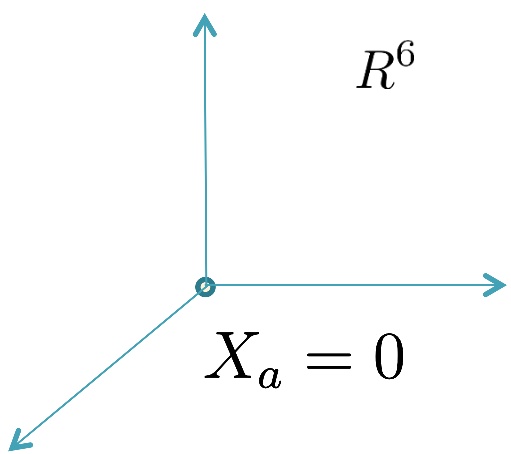
Furthermore, the radius of the distribution agrees with the radius of M5,

$$r_0 = \left(\frac{\mu p^+}{6\pi^3}\right)^{1/4} = r_{M5}$$

In the M5-brane limit, SO(6) scalars indeed form a spherical M5-brane

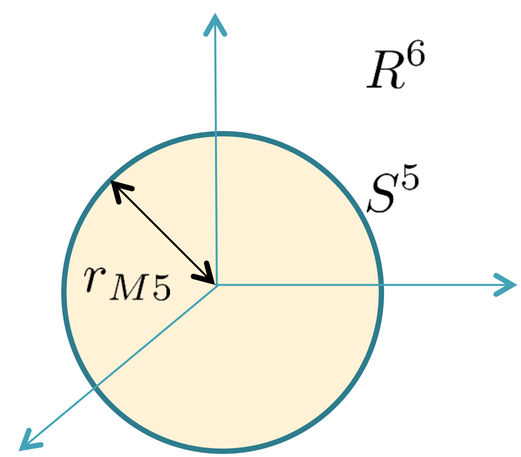
Emergent M5-branes

At weak coupling, vacuum configuration was trivial



Trivial

At strong coupling, however, typical moduli configuration is a spherical shell



Spherical M5-brane

Summary

- By applying localization, we derived a spherical distribution for SO(6) scalar fields in the M5-brane limit of PWMM.

- The radius agree with that of M5-brane

Evidence for PWMM to be M-theory !

- We also showed that multiple M5-branes can also be described in the similar way.

Outlook

- Excited states? (Rotating M5 etc)

- Theory for multiple M5-branes?