Spherical transverse M5-branes from the plane wave matrix model

Goro Ishiki
(University of Tsukuba)

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Collaborators:
Y. Asano (DIAS⇒KEK), S. Shimasaki (Keio U), S. Terashima (YITP)

Matrix Model = Second Quantization?

Does BFSS MM contain all states in M-theory?

$$|\text{a single M2}\rangle$$

$$|\text{multiple M2}\rangle$$

Relatively well understood in MM

$$|\text{a single M5}\rangle$$

$$|\text{multiple M5}\rangle$$

$$|\text{multiple M2/M5}\rangle$$

Longstanding problem
⇒ This talk

Why is M5 difficult in MM?

- The world volume theory of M5-branes is not known.
- M5-brane charge is missing in SUSY algebra of MM.

  [Banks-Seiberg-Shenker]

No guiding principle to find M5-branes in MM

My talk

- Mass deformation of BFSS matrix model is very useful to understand this problem

Outline

BFSS MM ⇒ M-theory on $R^{1,10}$

↓ Mass deformation

Plane wave (BMN) MM ⇒ M-theory on pp-wave

[Berenstein-Maldacena-Nastase]

- M-theory on 11D pp-wave background admits spherical M2/M5 branes with vanishing light cone energy.
- This M5-brane should be described in PWMM as a vacuum state.

In finding this M5-brane in PWMM,
the target is restricted to the vacuum sector

Conjecture [BMN, Maldacena-Sheiki-Jabbari-Raamsdonk] :

" The trivial vacuum (all fields=0) in PWMM corresponds to the spherical M5."

The conjecture for M5-branes in PWMM

Vacua of PWMM

- The potential terms vanish for

$$\mu X_i + \frac{i}{2}\epsilon_{ijk}[X_j, X_k] = 0, \quad X_a = 0$$

This is solved by any N-dimensional representation of SU(2) generator

$$X_i = \mu L_i \qquad [L_i, L_j] = i\epsilon_{ijk}L_k$$

Fuzzy sphere

- The fuzzy sphere is the regularization of S^2, not of S^5.
- We cannot see any spherical M5-brane like object…

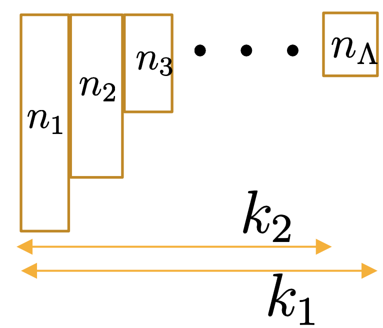
Conjecture

- "Trivial vacuum $(X_i = 0)$ corresponds to a single spherical M5". [BMN]
- More generally: Make an irreducible decomposition, [Maldacena-Sheikh-Jabbari-Raamsdonk]

$$X_i = \mu L_i = \mu \bigoplus_{s=1}^{\Lambda} L_i^{[n_s]}$$

n_s dim irrep

- Vacua are labelled by partitions of N
  ⇔ Young Tableau
- Let k_a be the length of a-th row



$n_s \longrightarrow \infty$ is commutative limit of fuzzy sphere (M2-brane limit)

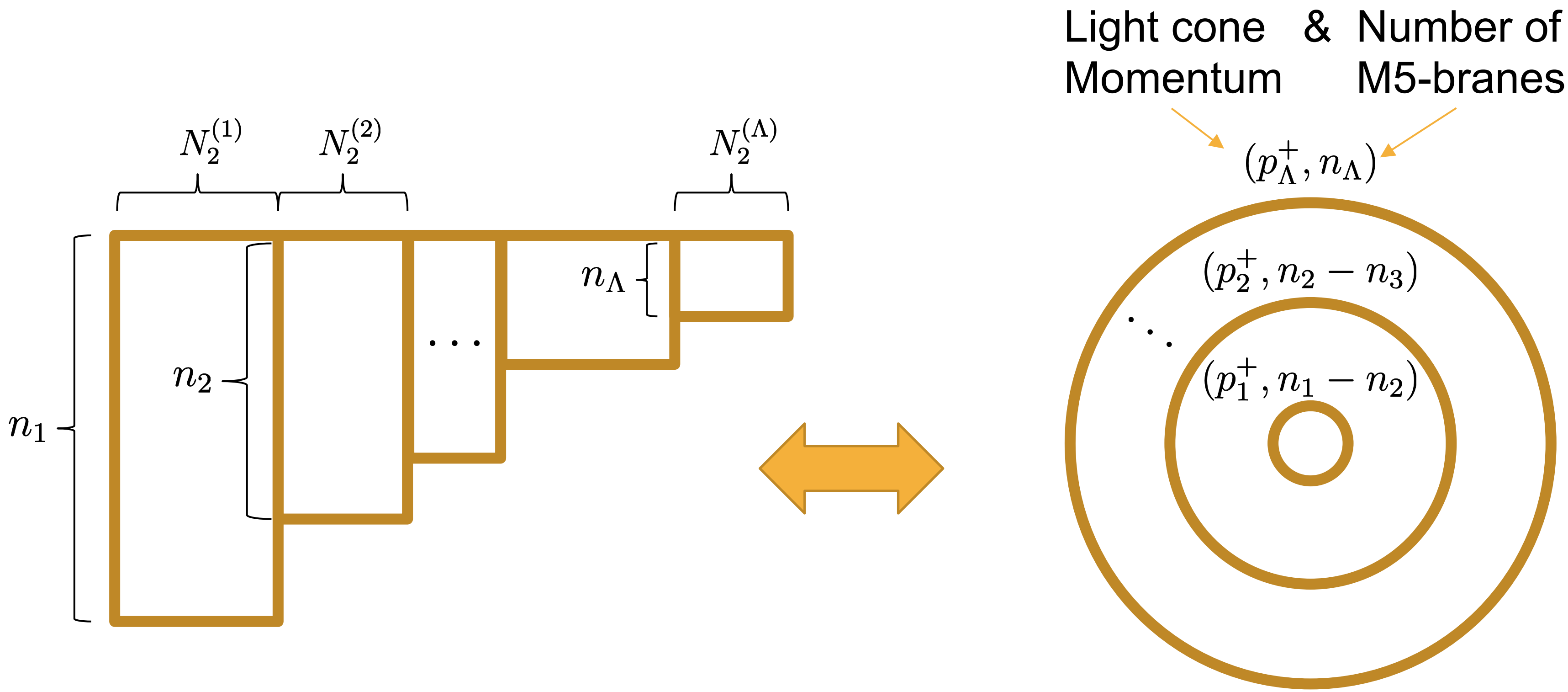
Dual limit $k_a \longrightarrow \infty$ is conjectured to be the M5-brane limit

For the trivial vacuum ($n_s = 1$), they checked the conjecture by showing that the mass spectra of PWMM and a single M5-brane agree with each other.

Partitions ⇔ M5-branes

$$X_i = \mu \bigoplus_{s=1}^{\Lambda} L_i^{[n_s]} \otimes \mathbf{1}_{N_2^{(s)}}$$

$$p_s^+ = \sum_{r=1}^{s} N_2^{(r)}/R$$



$N_2^{(s)} \to \infty, \quad n_s$: fixed

Radii $\sim (p^+)^{1/4}$

Decoupling limit

Decoupling limit of M5-brane

In order to isolate M5-branes form bulk modes, we can take the decoupling limit

Bulk newton constant $G_{11} \sim l_p^9$: positive length dimension
⇒ if we look at very large distance than G_{11} , the bulk mode decouples.

We take a limit where the radius of M5 becomes large (in the Planck unit)

$$r_{M5} \gg l_p$$

In this limit, we can fine tune other parameters in such a way that only DOF on M5-branes survive.

Derivation of the decoupling limit

$$ds^2 = -2dx^+dx^- - \frac{\mu^2 r^2}{36}dx^+dx^+ + r^2 d\Omega_5^2 + \cdots \qquad r^2 = \sum_{a=4}^{9} x_a^2$$

$$\tilde{x}^+ = x^+ + \frac{36}{\mu^2 r^2}dx^-, \quad \tilde{x}^- = x^-$$

$$ds^2 = r^2\left(-\frac{\mu^2}{36}d\tilde{x}^+d\tilde{x}^+ + d\Omega_5^2\right) + \frac{36}{\mu^2 r^2}d\tilde{x}^-d\tilde{x}^- + \cdots$$

$$(\tilde{x}^+, \tilde{x}^-) \sim (\tilde{x}^+ + 36R/(\mu r)^2, \tilde{x}^- + R) \Leftarrow (x^+, x^-) \sim (x^+, x^- + R)$$

We look at $r \sim r_{M5} \gg 1$ $\Rightarrow$ spatial compactification with radius $R/(\mu r_{M5})$
$\Rightarrow$ NS5-brane in the type IIA superstring
(bulk modes decouple)

We look at $r \sim r_{M5} \gg 1$ ⇒ spatial compactification with radius $R/(\mu r_{M5})$
⇒ NS5-brane in the type IIA superstring
(bulk modes decouple)
⇒ The theory on NS5-brane (Little string theory)
is parametrized by the tension $\sim 1/l_s^2$

▪Decoupling limit of NS5-brane

$$\frac{r_{M5}}{l_p} \sim \left(l_p^2 \mu p^+\right)^{1/4} \to \infty \qquad \frac{r_{M5}^2}{l_s^2} \sim \left(\frac{R^4 p^+}{\mu^3 l_p^6}\right) : \ \text{fixed} \qquad (\mu, l_p : \text{fixed})$$

Bulk modes decouple
for large radius

DOF on NS5 survive

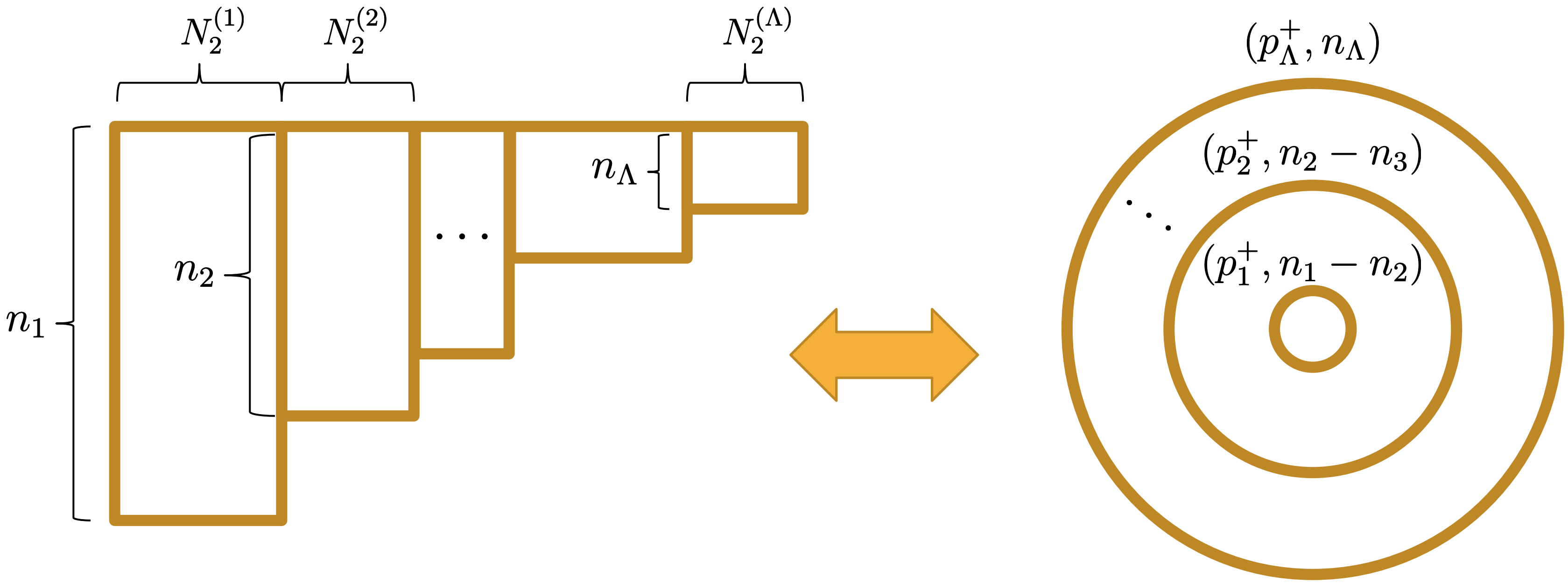
$$\Longleftrightarrow \qquad N \to \infty, \quad \lambda = g^2 N : \ \text{fixed}$$

't Hooft limit

▪Decoupling limit of M5-branes

Take $\lambda \to \infty$ in the 't Hooft limit

Decoupling limit of multiple M5



We first consider the ‘t Hooft limit such that the large-N limit is taken as

$$N_2^{(s)} \to \infty, \quad n_s : \text{fixed} \qquad \sum_{s=1}^{\Lambda} n_s N_2^{(s)} = N \to \infty$$

and then, take the strong coupling limit. $\lambda \to \infty$

Assumption

Assumption

- We first assumed that in the strong coupling limit, the matrices (dominant configurations) become mutually commuting

$$[X^A, X^B] \to 0$$

We thought this is an reasonable assumption but…

- Maldacena gave us an email saying that this assumption should be wrong!

- But for the moment, let us assume this (though this may be wrong) and in the last part of my talk, I will introduce Maldacena's suggestion.

(Actually, Maldacena's comment does not change our main claim. it only changes some interpretation of the result.)

Our result

Localization in PWMM

$$S_{PWMM}=\frac{1}{g^2}\int dt\mathrm{Tr}\left[\frac{1}{2}DX_M^2-\frac{1}{4}[X_a,X_b]^2-\frac{1}{2}[X_a,X_i]^2\right.$$
$$\left.-\frac{\mu^2}{8}X_a^2-\frac{1}{2}\left(\mu X_i+\frac{i}{2}\epsilon_{ijk}[X_j,X_k]\right)^2+\text{fermions}\right]$$

- For the moment, let us consider the simplest partition,

$$X_i=\mu L_i=\mu L_i^{[N_5]}\otimes\mathbf{1}_{N_2}$$ (N_5=1 ⇔ the trivial vacuum)

- After Wick rotation, we set the boundary condition as

All fields → The vacuum configuration $(\tau\to\pm\infty)$

- We consider a complex scalar field,

$$\phi(t) = X_3(t) + i(X_8(t)\sin t + X_9(t)\cos t)$$

There exist supersymmetries such that $Q\phi = 0$ (4 SUSYs, ¼ BPS)

At $t = 0$, $\phi = X_3 + iX_9$

Eigenvalue distribution of ϕ gives
the moduli distribution on the (X_3, X_9)-plane

The eigenvalue distribution of ϕ is computable by the localization

- Any correlation function made of ϕ can be computed by localization

$$\langle \mathrm{Tr}\phi^{n_1}(t_1)\mathrm{Tr}\phi^{n_2}(t_2)\cdots\rangle$$

- Localization

We add a nice Q-exact term tQV to the action

⇒ The partition function is shown to be independent of t

⇒ Taking the large-t limit, the saddle points of the Q-exact term dominate

⇒ Saddle point $\phi = X_3 + iX_9 = \mu L_3 + iM$

M : constant matrix commuting with L_i

- Our result of the localization

$$\langle \mathrm{Tr}\phi^{n_1}(t_1)\mathrm{Tr}\phi^{n_2}(t_2)\cdots\rangle = \langle \mathrm{Tr}(\mu L_3 + iM)^{n_1}\mathrm{Tr}(\mu L_3 + M)^{n_2}\cdots\rangle_{eff}$$

Effective action

$$S = \frac{1}{g^2}\sum_{i=1}^{N_2} m_i^2 + \sum_{J=0}^{N_5-1}\sum_{i\neq j}\log\left(\frac{((2J)^2+(m_i-m_j)^2)((2J+2)^2+(m_i-m_j)^2)}{((2J+1)^2+(m_i-m_j)^2)^2}\right)$$

Classical part

1-loop determinant

This also means that the spectrum of ϕ is determined by the effective action;

$$\langle \mathrm{Tr}(z-\phi)^{-1} \rangle = \langle \mathrm{Tr}(z-\mu L_3 + iM)^{-1} \rangle_{eff}$$

Under the assumption of the commutativity of matrices,
eigenvalue distribution of M = eigenvalue distribution of X^9

◆ In the large-N limit, the saddle points of S_{eff} dominate.

- Weak coupling limit → the saddle is trivial $(X_9 = M = 0)$
- Strong coupling limit → There is a non-trivial equilibrium

→ Moduli distribution of X_9 in the decoupling limit of M5-brane

$\rho(x) = c(a-x^2)^{3/2}$ $\quad a, c$: constants

($M \gg L_3$ in this limit ⇒ only SO(6) scalars expand)

Uplift to SO(6) symmetric distribution

- Assuming SO(6) symmetry and the commutativity of matrices we introduce the six dimensional distribution $\tilde{\rho}$ by [Filev-O'Connor]

$$\int dx_9 x_9^n \rho(x_9) =: \int dx_4 dx_5 \cdots dx_9 x_9^n \tilde{\rho}(r) \qquad \text{for any } n$$

$$r = \sqrt{x_4^2 + \cdots + x_9^2}$$

- For $\rho(x) = c(a - x^2)^{3/2}$, we find $\tilde{\rho}(r) = \tilde{c}\,\delta(r - r_0)$

SO(6) scalars form a spherical shell in R^6

- Furthermore, the radius of the distribution agrees with the radius of M5,

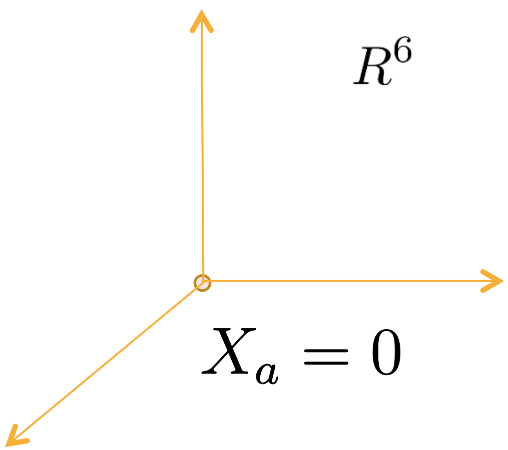
$$r_0 = \left(\frac{\mu p^+}{6\pi^3}\right)^{1/4} = r_{M5}$$

In the M5-brane limit, SO(6) scalars form a spherical M5-brane!

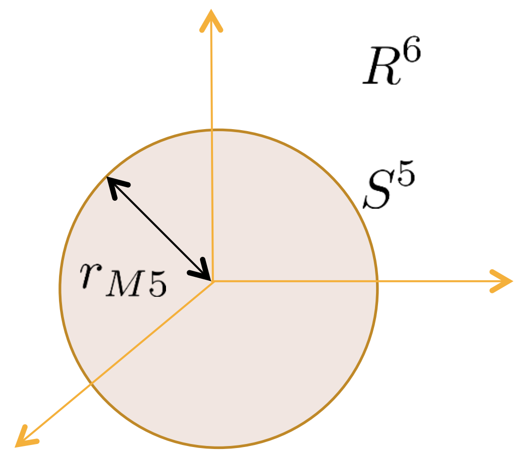
Emergent M5-branes

At weak coupling, vacuum configuration was trivial

At strong coupling, however, typical moduli configuration is a spherical shell



Trivial



Spherical M5-brane

The most general partition

For the most general partition. the eigenvalue integral is given by a multi-matrix model.

$$Z = \int \prod_{s=1}^{\Lambda} \prod_{i=1}^{N_2^{(s)}} dm_{si} Z_{1-loop} e^{-\frac{2n_s}{g^2} m_{si}^2}$$

$$Z_{1-loop} = \prod_{s,t=1}^{\Lambda} \prod_{J=|n_s-n_t|/2}^{(n_s+n_t)/2-1} \prod_{i=1}^{N_2^{(s)}} \prod_{j=1}^{N_2^{(t)}} \left[\frac{\{(2J+2)^2 + (m_{si}-m_{tj})^2\}\{(2J)^2 + (m_{si}-m_{tj})^2\}}{\{(2J+2)^2 + (m_{si}-m_{tj})^2\}^2} \right]^{1/2}$$

We found an exact solution to this model

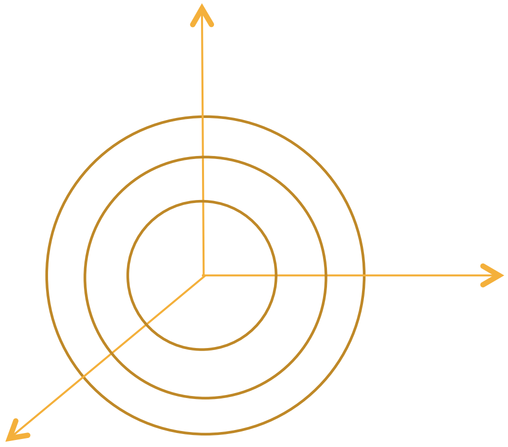
Multiple M5-branes

$$\hat{\rho}_s = \sum_{r=1}^{s} \rho_r$$

Solution in the decoupling limit

$$\hat{\rho}_s(x) = \frac{8^{3/4}\sum_{r=1}^{s} N_2^{(r)}}{3\pi\lambda_s^{1/4}}\left[1-\left(\frac{x}{x_s}\right)^2\right]^{3/2}, \quad x_s = (8\lambda_s)^{1/4}, \quad \lambda_s = g^2\sum_{r=1}^{s} N_2^{(r)}$$

The SO(6) symmetric uplift is Λ-stacks of spherical shells



Agrees with the claim of the conjecture!

$$r_s = \left(\frac{\mu p_s^+}{6\pi^3}\right)^{1/4} \quad p_s^+ = \sum_{r=1}^{s} N_2^{(r)}/R$$

Maldacena's comments

- We computed eigenvalue distribution of $\phi = X_3 + iX_9$ Correct, Nice!

- Assuming the commutativity, $[X_A, X_B] \sim 0$, we identified the real and imaginary parts of the distribution of $\phi = X_3 + iX_9$ as the distributions of X_3, X_9, respectively.

Maybe wrong! Some modification needed.

- Assuming the commutativity again, we defined uplifted distribution function in 6D. And then we found that the uplifted distribution agrees with the shape of M5-branes.

With a suitable modification of the interpretation, this result gives a nice evidence for the emergence of the M5-branes!

Maldacena's suggestion

- Assumption of the commutativity contradicts with Polchinski's paper [hep-th/9903165]

- From some basic equations like the uncertainty inequality and the Ward identity, Polchinki showed that

$$\mathrm{Tr}(X^A)^2 \gg \mathcal{O}(r_{M5}^2)$$

Contradicts with our result ⇒ Our commuting assumption may be wrong

- Supergravity objects corresponds to low energy modes of the matrices

$$\mathrm{Tr}(X^A)^2 = \mathrm{Tr}(X^A)^2|_{\text{high eng}} + \mathrm{Tr}(X^A)^2|_{\text{low eng}}$$

High energy modes will be irrelevant in low energy physics

This part can be $\mathcal{O}(r_{M5}^2)$

Resolution by Maldacena

- Our correlators are independent of time coordinates

$$\langle \mathrm{Tr}\phi^{n_1}(t_1)\mathrm{Tr}\phi^{n_2}(t_2)\cdots\rangle = \langle \mathrm{Tr}(\mu L_3 + iM)^{n_1}\mathrm{Tr}(\mu L_3 + M)^{n_2}\cdots\rangle_{eff}$$

- They are invariant under taking time averages ⇒ contain only low energy modes

- Then, we can just change our assumption as

X^A are commuting ⇒ Low energy modes of X^A are commuting

- The conclusion is that the M5-branes are formed by the eigenvalues of the low energy modes of the matrices.

Summary

- By applying localization, we derived a spherical distribution for low energy moduli of SO(6) scalars in the M5-brane limit of PWMM.
- The radius agrees with that of the spherical M5-brane in M-theory
- This result can be generalized to multiple concentric M5-branes
- (multiple) M5-brane states are indeed contained in PWMM

Nice evidence for PWMM to be the second quantization of M-theory!

Outlook

- Excited states? (Rotating M5 etc)
- Theory for multiple M5-branes?
- Numerical work?