THE UNIVERSITY OF TOKYO

ALIGNMENT LIMIT IN MULTI-HIGHS DOUBLET MODELS

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Based on:
- Phys.Rev. D100 (2019) no.3, 035021
- Phys.Rev. D91 (2015) no.9, 095024

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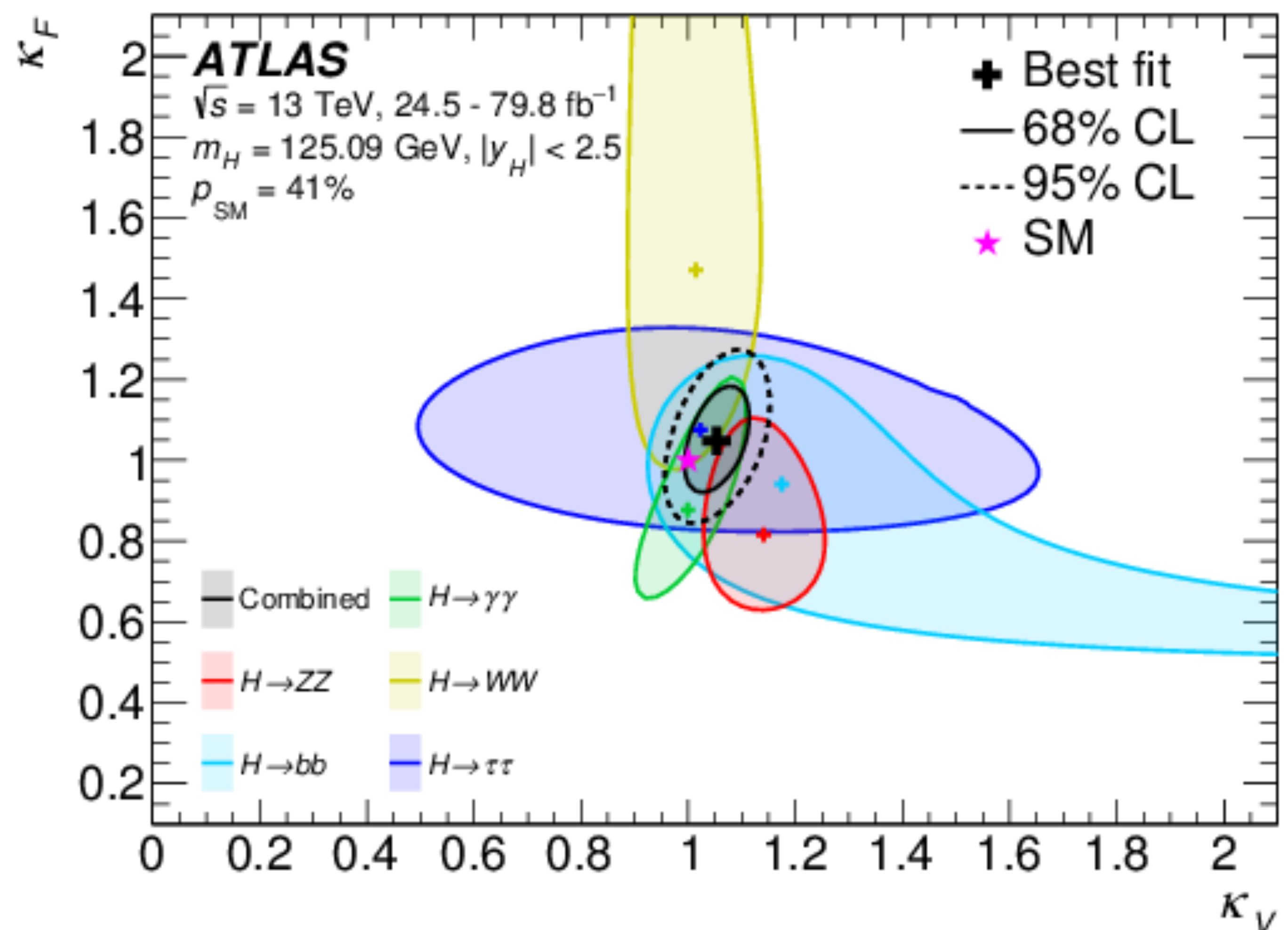
The 125 GeV HIGGS



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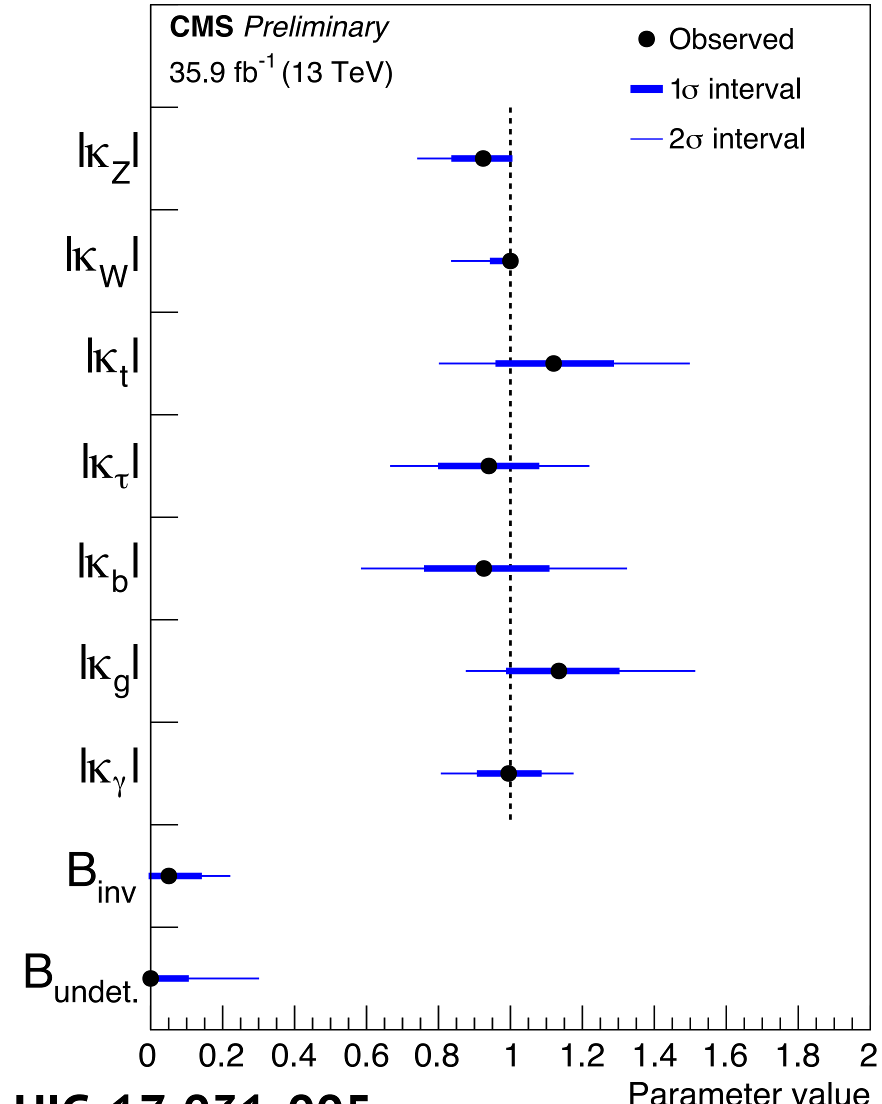

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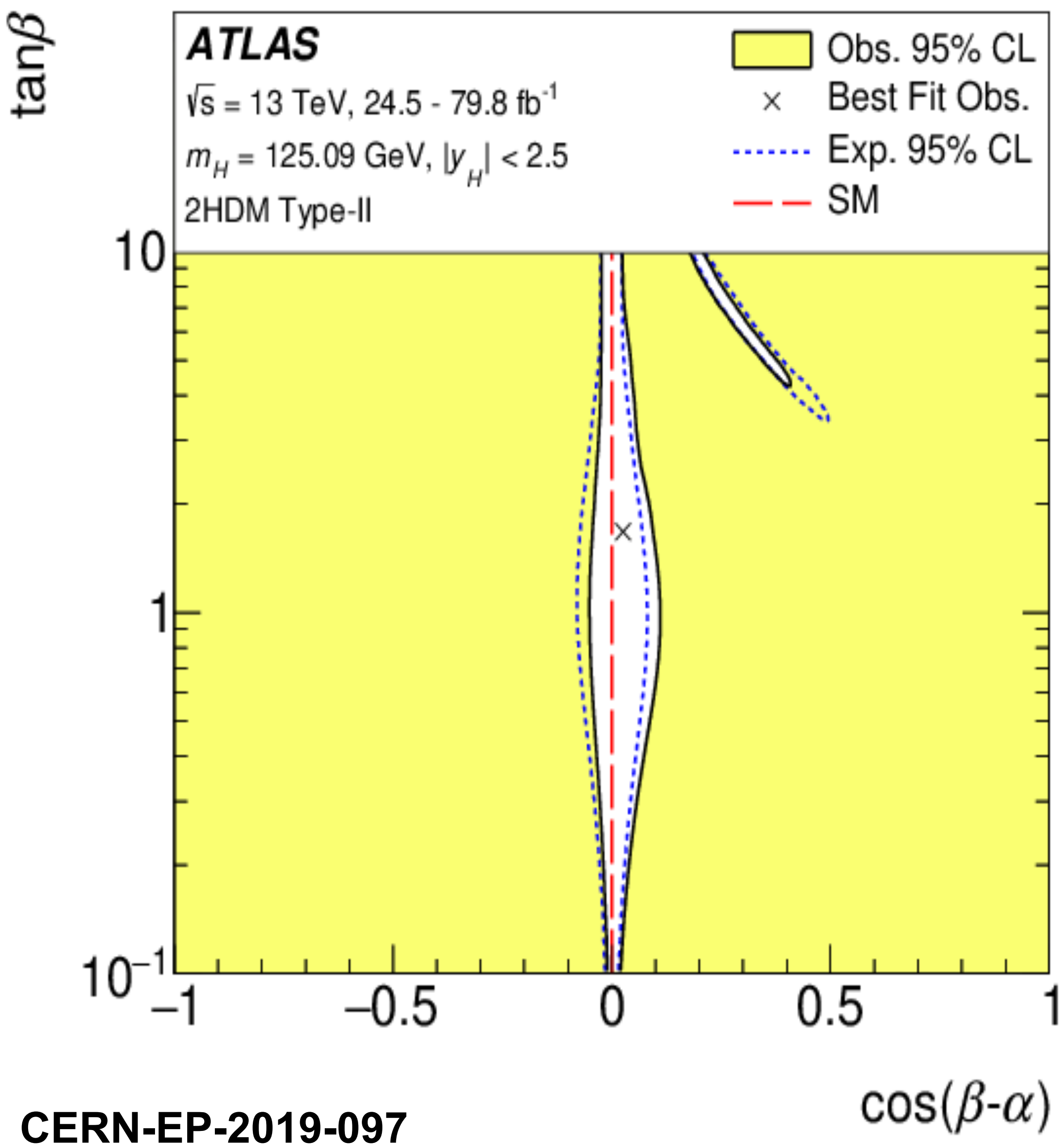
The 125 GeV HIGGS In 2HDM



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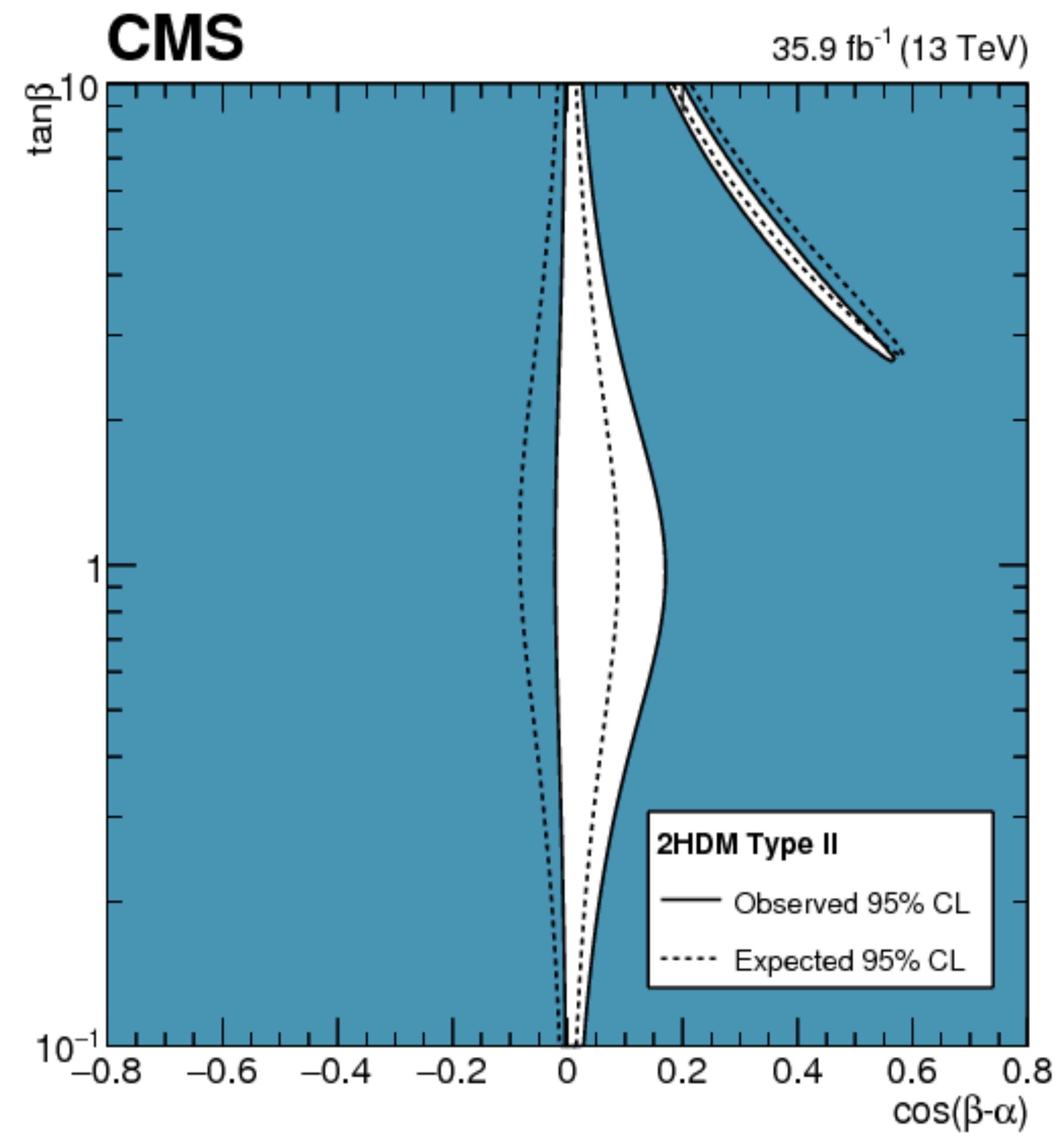


CMS-HIG-17-031-005

arXiv:1909.02845 [hep-ex]

ALIGNMENT LIMIT

The set of conditions under which the lightest CP-even scalar mimics the SM Higgs by possessing SM-like gauge and Yukawa couplings at the tree-level.

In two Higgs doublet models in usual convention

$$\cos(\alpha - \beta) = 0$$

Minimizing the number of independent parameters of scalar potential

ALIGNMENT LIMIT IN NHDM

$$\phi_k = \begin{pmatrix} w_k^+ \\ (h_k + i z_k)/\sqrt{2} \end{pmatrix}, \qquad (k = 1,2,\ldots,n)\,.$$

$$\langle \phi_k \rangle = v_k/\sqrt{2}; \qquad v^2 = \sum_{k=1}^{n} v_k^2 = (246\ \mathrm{GeV})^2$$

<u>The gauge Higgs trilinear coupling :</u>

$$\mathscr{L}_{\mathrm{kin}}^S = \sum_{k=1}^{n} |D_\mu \phi_k|^2 \quad \ni \frac{g^2}{2} W_\mu^+ W^{\mu -} \left(\sum_{k=1}^{n} v_k h_k \right)$$

$$\equiv \frac{g^2 v}{2} W_\mu^+ W^{\mu -} \left(\frac{1}{v} \sum_{k=1}^{n} v_k h_k \right)$$

ALIGNMENT LIMIT IN nHDM

$$\mathscr{L}^{S}_{\text{kin}} = \sum_{k=1}^{n} |D_\mu \phi_k|^2 \ni \frac{g^2 v}{2} W^+_\mu W^{\mu -} \left(\frac{1}{v} \sum_{k=1}^{n} v_k h_k \right)$$

This suggests that the SM-like Higgs stands for the combination

$$H_0 = \frac{1}{v} \sum_{k=1}^{n} v_k h_k$$

Therefore exact SM-like gauge and Yukawa coupling at tree-level.

This state however is not guaranteed to be the physical eigenstate.

RETRIEVING ALIGNMENT LIMIT IN 2HDM

$$H_0 = \frac{1}{v}(v_1 h_1 + v_2 h_2)$$

The orthogonal states can be obtained as,

$$\begin{pmatrix} H_0 \\ R \end{pmatrix} = \mathcal{O}_\beta \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$

where, $\tan\beta = v_2/v_1$

The physical mass eigenstates,

$$\begin{pmatrix} h \\ H \end{pmatrix} = \mathcal{O}_\alpha \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$

REPRODUCING ALIGNMENT LIMIT OF 2HDM

Hence, the combination yield

$$\begin{pmatrix} h \\ H \end{pmatrix} = \underline{\mathcal{O}_\alpha \mathcal{O}_\beta^T} \begin{pmatrix} H_0 \\ R \end{pmatrix} = \begin{pmatrix} \cos(\alpha-\beta) & \sin(\alpha-\beta) \\ -\sin(\alpha-\beta) & \cos(\alpha-\beta) \end{pmatrix} \begin{pmatrix} H_0 \\ R \end{pmatrix}$$

The alignment limit becomes

$$\cos(\alpha-\beta) = 1 \Rightarrow \alpha = \beta$$

ALIGNMENT LIMIT IN 3HDM

Three doublets with three non-zero vevs,

$$v_1 = v \cos\beta_1 \cos\beta_2\,, \qquad v_2 = v \sin\beta_1 \cos\beta_2\,, \qquad v_3 = v \sin\beta_2$$

The orthogonal combinations:

$$\begin{pmatrix} H_0 \\ R_1 \\ R_2 \end{pmatrix} = \mathcal{O}_\beta \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \begin{pmatrix} \cos\beta_2 \cos\beta_1 & \cos\beta_2 \sin\beta_1 & \sin\beta_2 \\ -\sin\beta_1 & \cos\beta_1 & 0 \\ -\cos\beta_1 \sin\beta_2 & -\sin\beta_1 \sin\beta_2 & \cos\beta_2 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}$$

The elements of first row are derived from previous analogy.

$$H_0 = \frac{v_1}{v} h_1 + \frac{v_2}{v} h_2 + \frac{v_3}{v} h_3$$

ALIGNMENT LIMIT IN 3HDM

The physical basis transformation requires a 3X3 orthogonal rotation

$$\mathcal{O}_\alpha = \mathcal{R}_3 \cdot \mathcal{R}_2 \cdot \mathcal{R}_1$$

$$\mathcal{R}_1 = \begin{pmatrix} \cos\alpha_1 & \sin\alpha_1 & 0 \\ -\sin\alpha_1 & \cos\alpha_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathcal{R}_2 = \begin{pmatrix} \cos\alpha_2 & 0 & \sin\alpha_2 \\ 0 & 1 & 0 \\ -\sin\alpha_2 & 0 & \cos\alpha_2 \end{pmatrix}$$

$$\mathcal{R}_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha_3 & \sin\alpha_3 \\ 0 & -\sin\alpha_3 & \cos\alpha_3 \end{pmatrix}$$

ALIGNMENT LIMIT IN 3HDM

$$\begin{pmatrix} h \\ H_1 \\ H_2 \end{pmatrix} = \underline{\mathcal{O}_\alpha \cdot \mathcal{O}_\beta^T} \begin{pmatrix} H_0 \\ R_1 \\ R_2 \end{pmatrix}$$

$$\mathcal{O} \equiv \mathcal{O}_\alpha \cdot \mathcal{O}_\beta^T$$

For h to overlap completely with H_0

$$\boxed{\mathcal{O}_{11} = 1}$$

Which also ensures,

$$\boxed{\mathcal{O}_{12} = \mathcal{O}_{21} = \mathcal{O}_{13} = \mathcal{O}_{31} = 0}$$

ALIGNMENT LIMIT IN 3HDM

$$\mathcal{O}_{11} = 1$$

$$\cos\alpha_2 \cos\beta_2 \cos(\alpha_1 - \beta_1) + \sin\alpha_2 \sin\beta_2 = 1$$

$$\rightarrow \left[\sin\left(\frac{\alpha_1 - \beta_1}{2}\right)\cos\left(\frac{\alpha_2 + \beta_2}{2}\right)\right]^2 + \left[\cos\left(\frac{\alpha_1 - \beta_1}{2}\right)\sin\left(\frac{\alpha_2 - \beta_2}{2}\right)\right]^2 = 0$$

$$\alpha_1 = \beta_1 \,; \alpha_2 = \beta_2$$

Or,

$$\alpha_1 = \pi + \beta_1 \,; \alpha_2 = \pi - \beta_2$$

↓

Redefinition of fields as, $H_1 \rightarrow -H_1 \,, H_2 \rightarrow -H_2$

3HDM WITH Z3 SYMMETRY

Higgs coupling modifiers → $\kappa_x = \dfrac{g_{hxx}}{(g_{hxx})^{\rm SM}}$

Z3 charges assignment

$$\phi_1 \to \omega\,\phi_1\,, \qquad \phi_2 \to \omega^2\phi_2$$

$$d_R \to \omega\, d_R\,, \qquad \ell_R \to \omega^2\ell_R$$

ϕ_3 → Up type quark mass

ϕ_2 → Down type quark mass

ϕ_1 → Charged lepton mass

The fermionic coupling modifiers

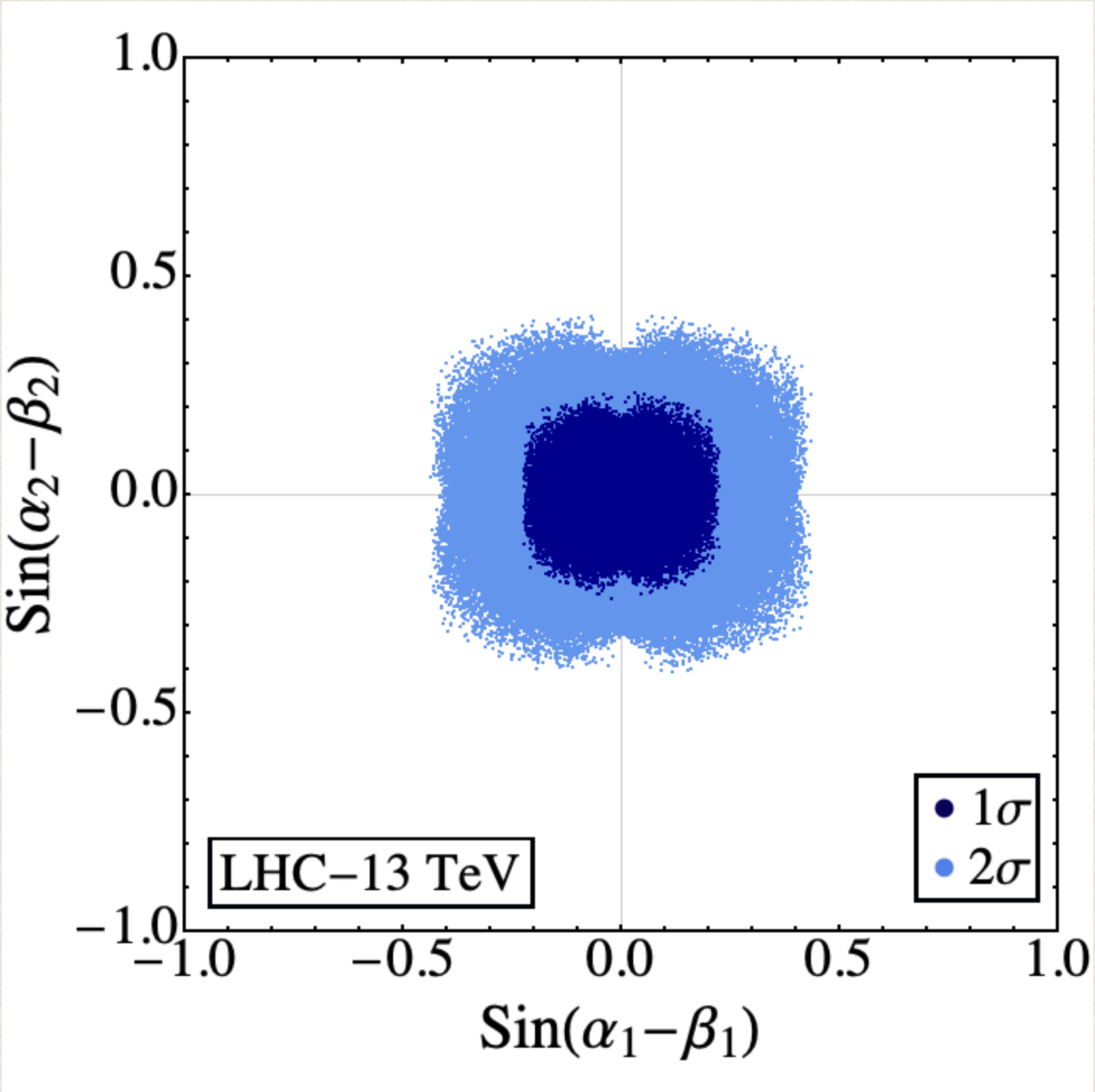
$$\kappa_u = \frac{\sin\alpha_2}{\sin\beta_2}$$

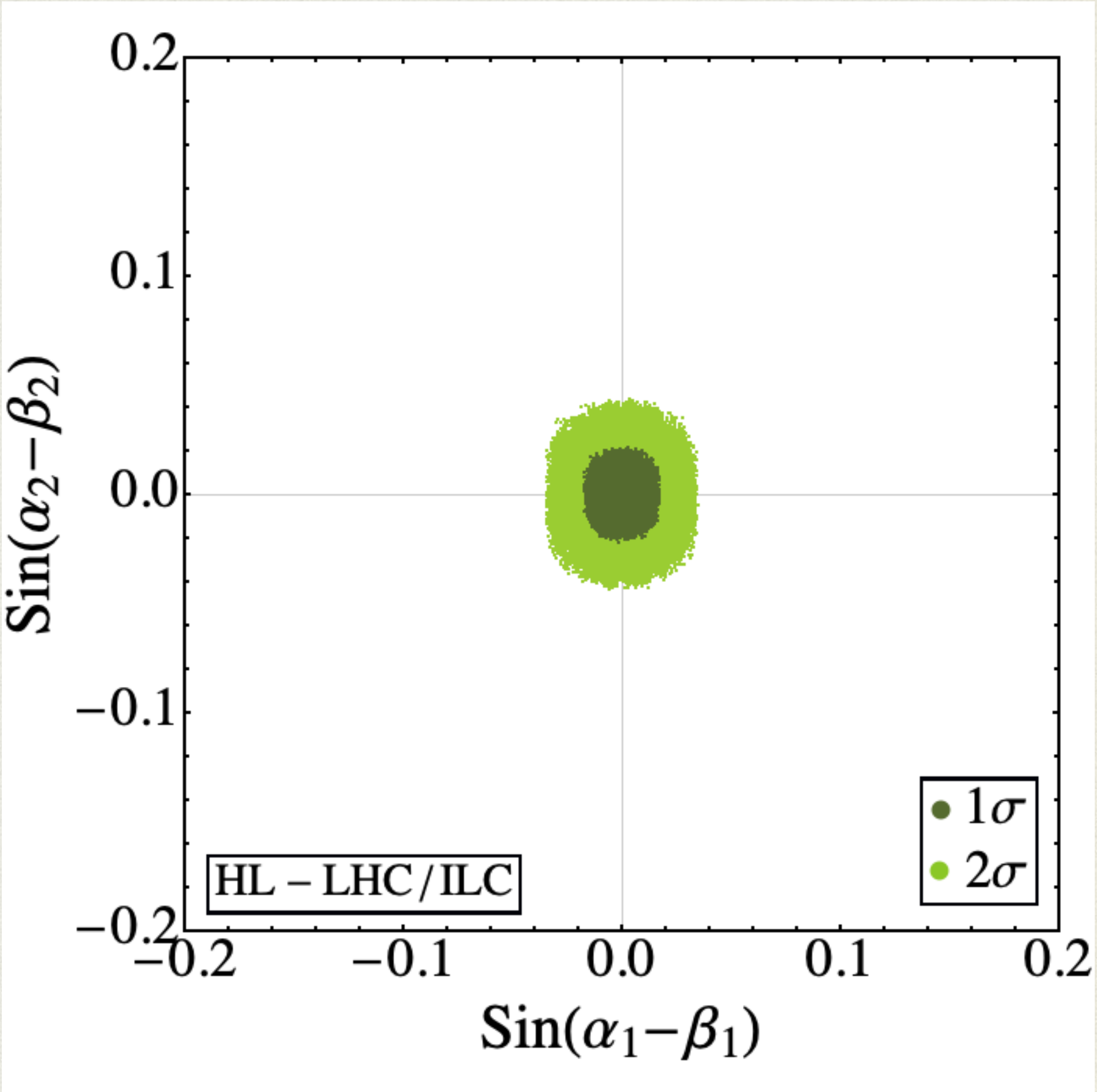
$$\kappa_d = \frac{\sin\alpha_1\cos\alpha_2}{\sin\beta_1\cos\beta_2}$$

$$\kappa_\ell = \frac{\cos\alpha_1\cos\alpha_2}{\cos\beta_1\cos\beta_2}$$

$$\kappa_V \equiv \mathcal{O}_{11} = \cos\alpha_2\cos\beta_2\cos(\alpha_1-\beta_1) + \sin\alpha_2\sin\beta_2$$

3HDM WITH Z3 SYMMETRY



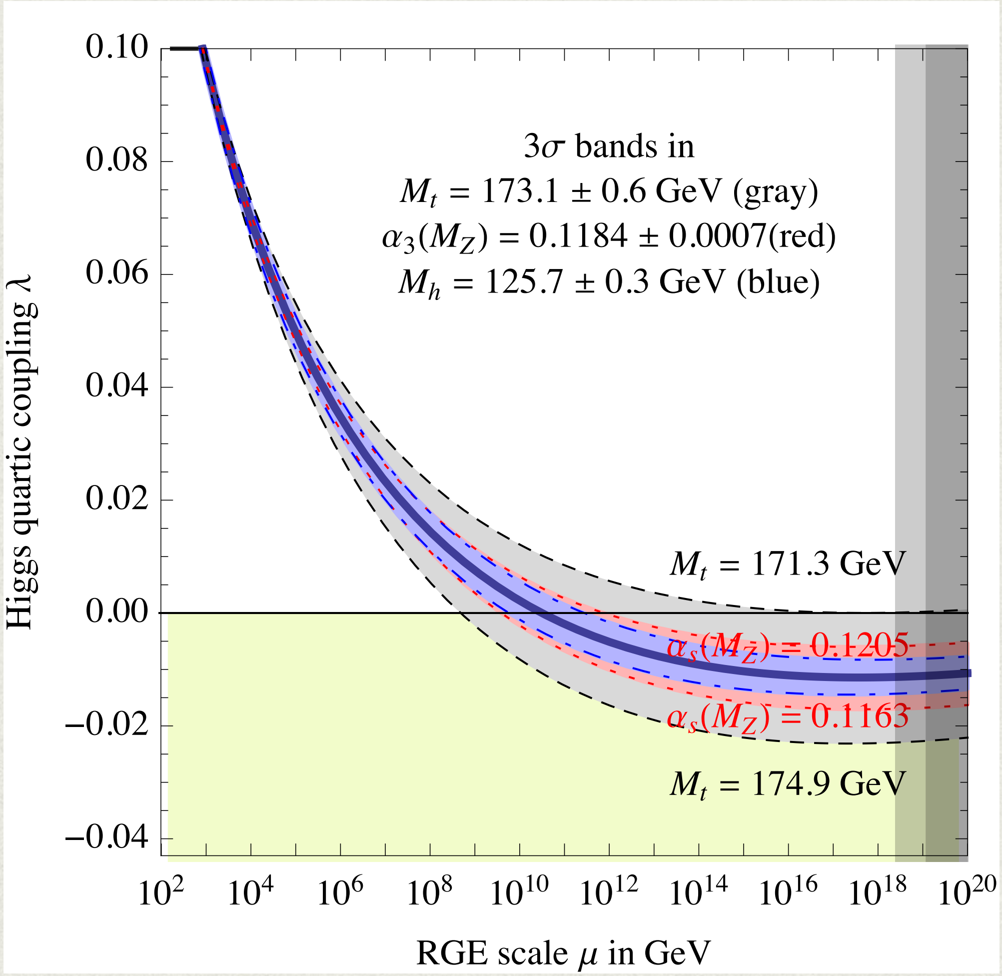


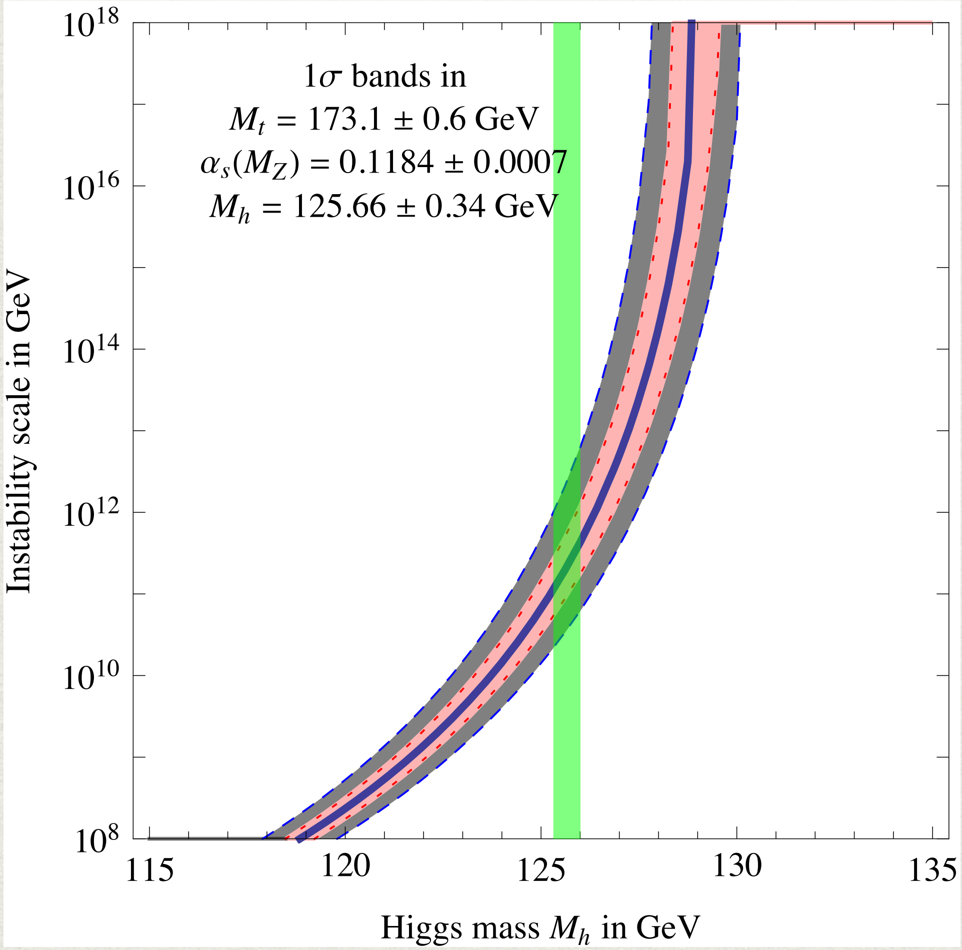
SUMMARY

- Alignment limit is a recipe to recover a SM-like Higgs in multi-Higgs doublet model.
- A suitable parametrization in the 3HDM leads to the alignment limit that looks very similar to 2HDM case.
- Our analysis provides a way to efficiently implement the alignment limit in case of a CP-conserving 3HDM

STABLE ALIGNMENT LIMIT IN 2HDM

VACUUM INSTABILITY PROBLEM





$$M_h\ [\mathrm{GeV}] > 129 + 2.0(\mathrm{M_t} - 173.34) - 0.5\left(\frac{\alpha_s(\mathrm{M_Z}) - 0.1184}{0.0007}\right) \pm 0.3$$

THE 2HDM

Introduce second Higgs doublet.

Electroweak ρ- parameter remains unity at tree level.

FCNC is possible at tree level → Need some additional discrete Z_2 or continuous U(1) symmetry to prevent.

Four variants following Z2 assignments.

Z2 CHARGE ASSIGNMENT

	Φ_1	Φ_2	U_R	d_R	E_R	Q_L, L_L
Type I	+	-	-	-	-	+
Type II	+	-	-	+	+	+
Type X	+	-	-	-	+	+
Type Y	+	-	-	+	-	+

2HDM POTENTIAL

Notation 1

$$V = m_{11}^2\phi_1^\dagger\phi_1 + m_{22}^2\phi_2^\dagger\phi_2 - \left(m_{12}^2\phi_1^\dagger\phi_2 + \mathrm{h.c.}\right) + \frac{\lambda_1}{2}\left(\phi_1^\dagger\phi_1\right)^2 + \frac{\lambda_2}{2}\left(\phi_2^\dagger\phi_2\right)^2$$
$$+\lambda_3\left(\phi_1^\dagger\phi_1\right)\left(\phi_2^\dagger\phi_2\right) + \lambda_4\left(\phi_1^\dagger\phi_2\right)\left(\phi_2^\dagger\phi_1\right) + \left\{\frac{\lambda_5}{2}\left(\phi_1^\dagger\phi_2\right)^2 + \mathrm{h.c.}\right\}$$

Notation 2

$$V = \beta_1\left(\phi_1^\dagger\phi_1 - \frac{v_1^2}{2}\right)^2 + \beta_2\left(\phi_2^\dagger\phi_2 - \frac{v_2^2}{2}\right)^2 + \beta_3\left(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2 - \frac{v_1^2+v_2^2}{2}\right)^2$$
$$+\beta_4\left\{(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) - (\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1)\right\} + \beta_5\left(\mathrm{Re}\ \phi_1^\dagger\phi_2 - \frac{v_1v_2}{2}\right)^2 + \beta_6\left(\mathrm{Im}\ \phi_1^\dagger\phi_2\right)^2$$

- The additional symmetry is softly broken by the term proportional to β_5 or m_{12}^2
- Nonzero $\tan\beta$ implies two parametrizations are equivalent.

THE 2HDM

- Notation-II is useful for tracking breaking parameter effect and defining

scalar masses in terms of couplings.

- The symmetry enhanced from Z2 to a U(1) for $\beta_5 = \beta_6$.
- The equivalence of the two sets of parameters are given by the following relations:

$$m_{11}^2 = -(\beta_1 v_1^2 + \beta_3 v^2)\ ;$$

$$m_{22}^2 = -(\beta_2 v_2^2 + \beta_3 v^2)\ ;$$

$$m_{12}^2 = \frac{\beta_5}{2} v_1 v_2\ ;$$

$$\lambda_4 = \frac{\beta_5 + \beta_6}{2} - \beta_4\ ;$$

$$\lambda_1 = 2(\beta_1 + \beta_3)\ ;$$

$$\lambda_2 = 2(\beta_2 + \beta_3)\ ;$$

$$\lambda_3 = (2\beta_3 + \beta_4)\ ;$$

$$\lambda_5 = \frac{\beta_5 - \beta_6}{2}\ .$$

COUPLINGS TO MASSES RELATION

Five independent physical parameters : $(m_H, m_A, m_{H^\pm}, \tan\beta, m_0)$

$$\beta_1 = \frac{1}{2v^2 c_\beta^2}\left[m_H^2 c_\alpha^2 + m_h^2 s_\alpha^2 - \frac{s_\alpha c_\alpha}{\tan\beta}\left(m_H^2 - m_h^2\right)\right] - \frac{\beta_5}{4}\left(\tan^2\beta - 1\right)$$

$$\beta_2 = \frac{1}{2v^2 s_\beta^2}\left[m_h^2 c_\alpha^2 + m_H^2 s_\alpha^2 - s_\alpha c_\alpha \tan\beta\left(m_H^2 - m_h^2\right)\right] - \frac{\beta_5}{4}\left(\cot^2\beta - 1\right)$$

$$\beta_3 = \frac{1}{2v^2}\frac{s_\alpha c_\alpha}{s_\beta c_\beta}\left(m_H^2 - m_h^2\right) - \frac{\beta_5}{4}$$

$$\beta_4 = \frac{2}{v^2} m_{H^+}^2$$

$\beta_6 = \dfrac{2}{v^2} m_A^2\,;$ $\quad m_0 = \dfrac{1}{2}\beta_5 v^2$ is the soft-symmetry breaking parameter.

THEORETICAL CONSTRAINTS

Vacuum stability :

$$\lambda_1 > 0\,,$$

$$\lambda_2 > 0\,,$$

$$\lambda_3 + \sqrt{\lambda_1\lambda_2} > 0\,, \quad \lambda_3 + \lambda_4 - |\lambda_5| + \sqrt{\lambda_1\lambda_2} > 0\,.$$

Perturbative unitarity:

$$a_1^\pm = \frac{3}{2}(\lambda_1 + \lambda_2) \pm \sqrt{\frac{9}{4}(\lambda_1 - \lambda_2)^2 + \left(2\lambda_3 + \lambda_4\right)^2}\,, \qquad b_1^\pm = \lambda_3 + 2\lambda_4 \pm 3\lambda_5\,,$$

$$a_2^\pm = \frac{1}{2}(\lambda_1 + \lambda_2) \pm \frac{1}{2}\sqrt{(\lambda_1 - \lambda_2)^2 + 4\lambda_4^2}\,, \qquad b_2^\pm = \lambda_3 \pm \lambda_5\,,$$

$$a_3^\pm = \frac{1}{2}(\lambda_1 + \lambda_2) \pm \frac{1}{2}\sqrt{(\lambda_1 - \lambda_2)^2 + 4\lambda_5^2}\,, \qquad b_3^\pm = \lambda_3 \pm \lambda_4\,.$$

The requirement of tree unitarity then restricts the above eigenvalues as
$|a_i^\pm|, |b_i^\pm| \leq 16\pi$

EXPERIMENTAL CONSTRAINTS

- Oblique T-parameter constraint to restrict the splitting between the nonstandard masses.

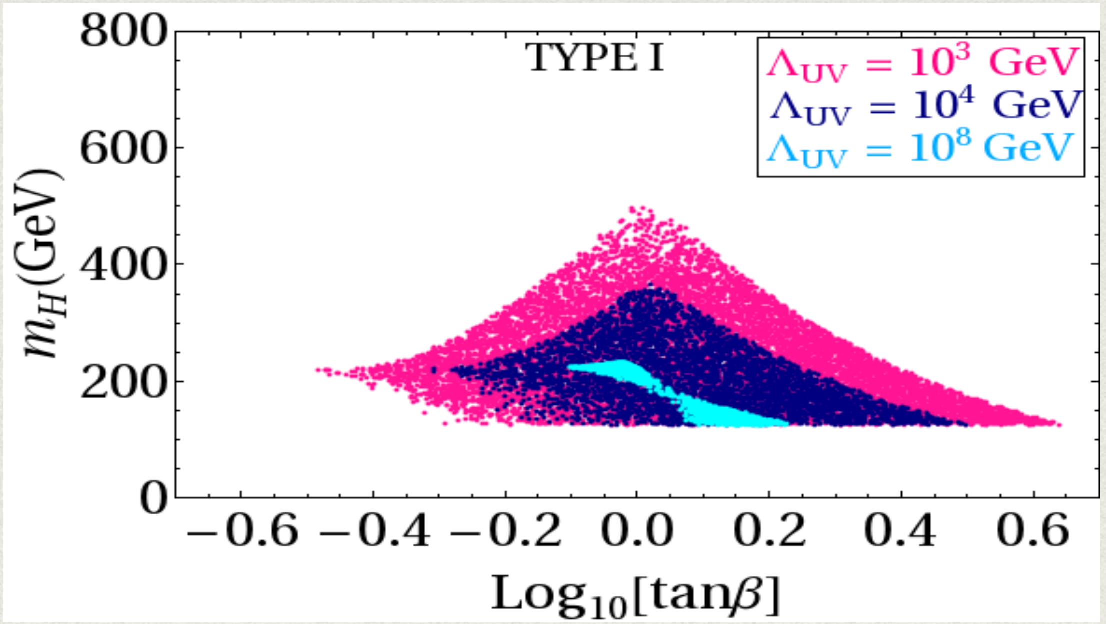
$$\Delta T = \frac{1}{16\pi \sin^2\theta_w M_W^2}\left[F(m_{H^+}^2, m_H^2) + F(m_{H^+}^2, m_A^2) - F(m_H^2, m_A^2)\right] ,$$

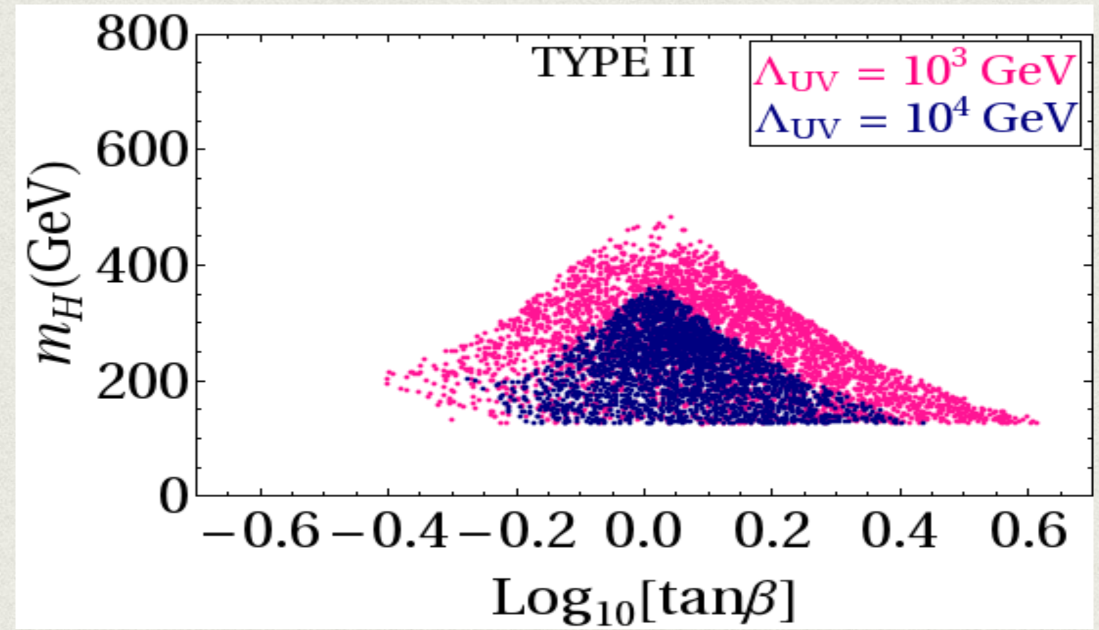
$$F(x,y) = \begin{cases} \frac{x+y}{2} - \frac{xy}{x-y}\ln\left(\frac{x}{y}\right) & , x \neq y \\ 0 & , x = y \end{cases}$$

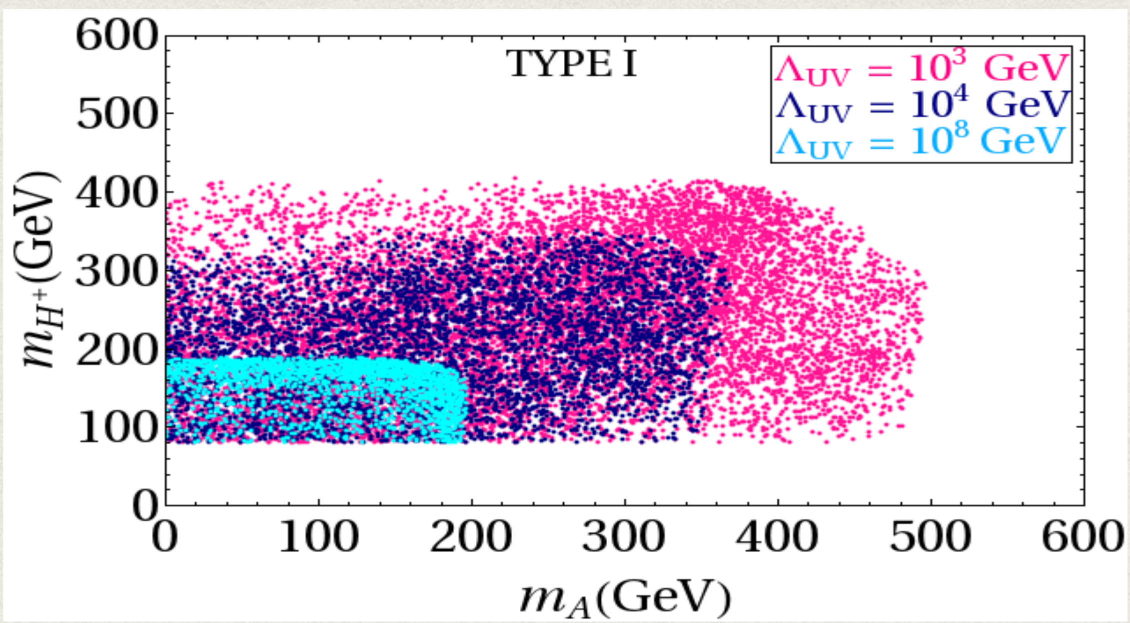
$$\Delta T = 0.05 \pm 0.12$$

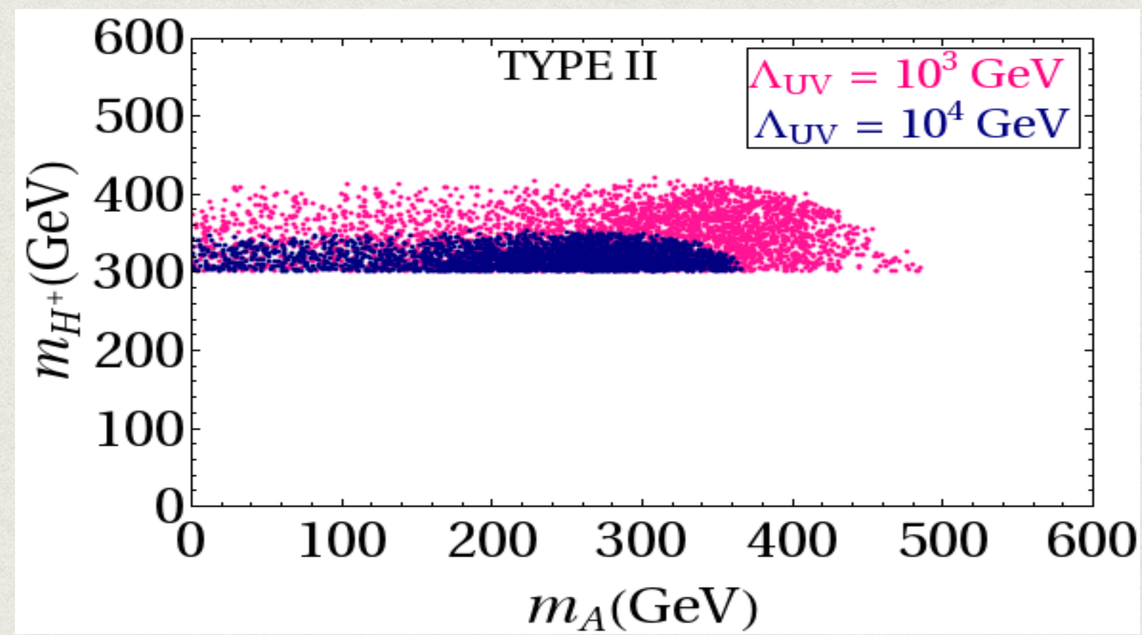
  - If, $m_H \approx m_A$ then ΔT severely restricts the splitting between charged and neutral scalar masses.
  - For Type II models $m_H^+ > 300$ GeV due to $b \to s\gamma$.
  - For Type I models, $m_H^+ > 80$ GeV from direct search limit.
  - Alignment limit, $\cos(\beta - \alpha) = 0$

EXACT Z2 SYMMETRIC CASE





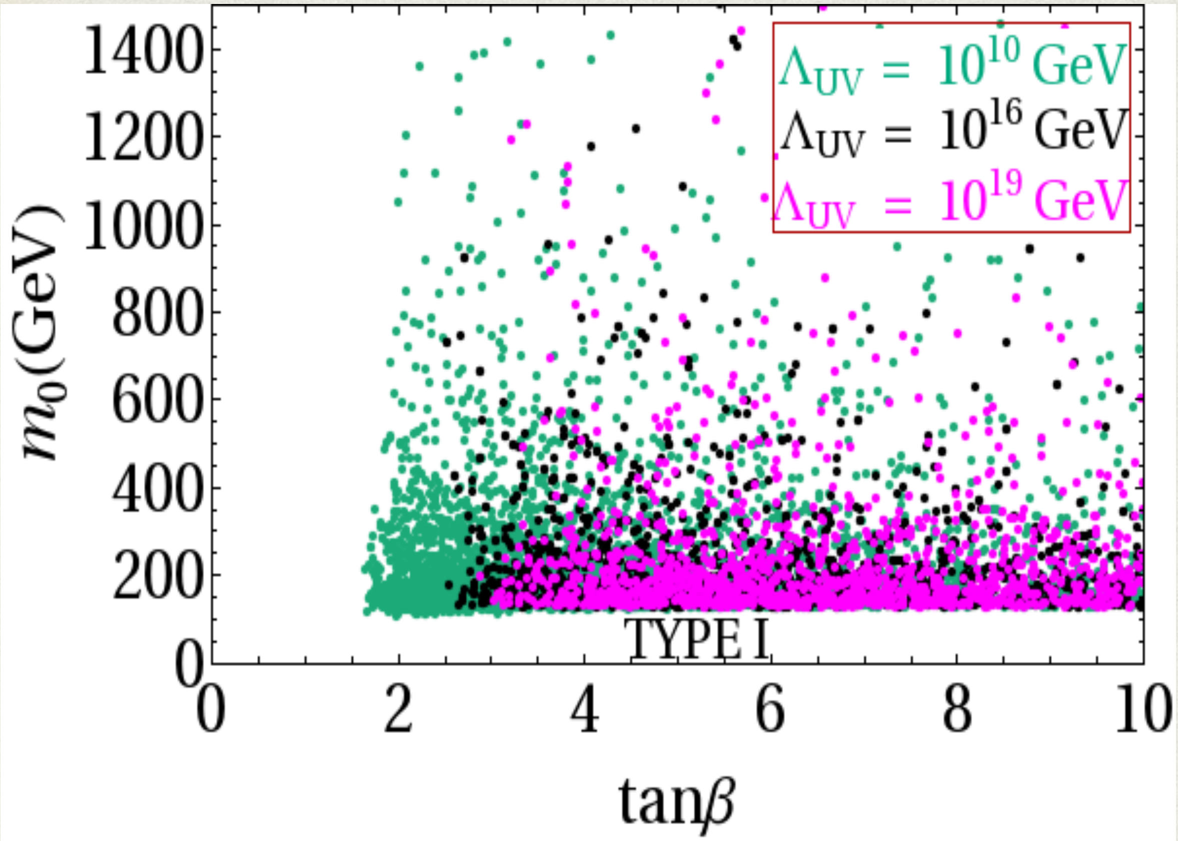


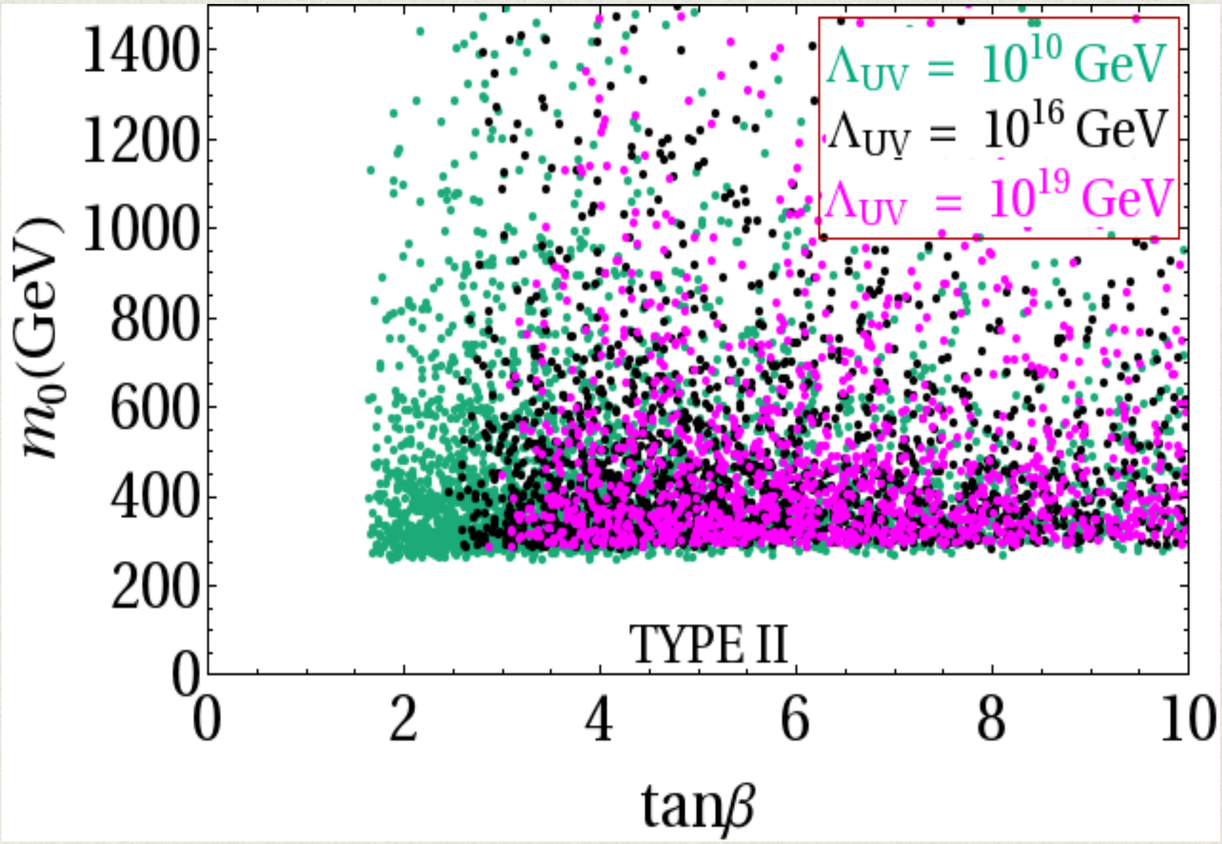


- Potential is not stable until Planck scale.
- Type I models remain stable up to a maximum of 10^8 GeV whereas Type II models can only be stable up to 10^4 GeV due to charged Higgs mass bound.
- $\tan\beta$ is bounded depending upon the energy scale Λ_{UV} up to which stability is demanded.

SOFTLY BROKEN Z2 SYMMETRIC 2HDM

- A stable alignment limit can only be achieved with a softly broken 2HDM potential.
- Stability of the 2HDM potential up to a cut-off scale Λ_{UV} yield a lower bound on $\tan\beta$ and eventually on m_0 (or equivalently on β_5).
- $\tan\beta \geq 3$ and $m_0 \geq 120(280)$ GeV for Type I(II).

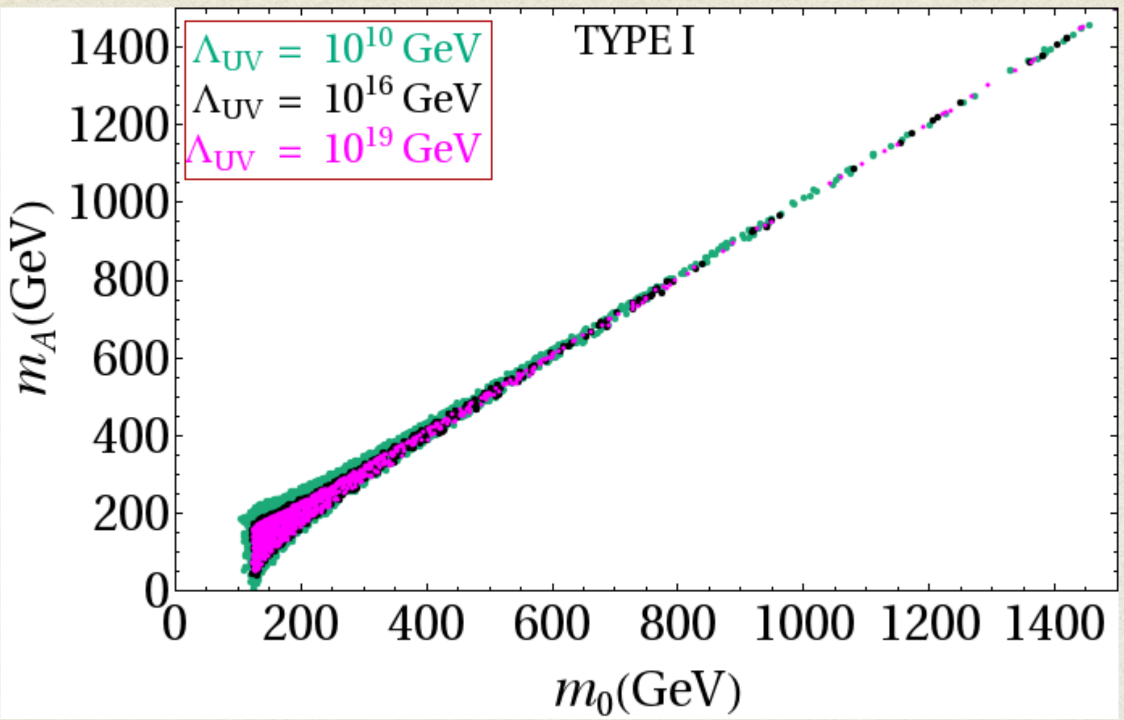


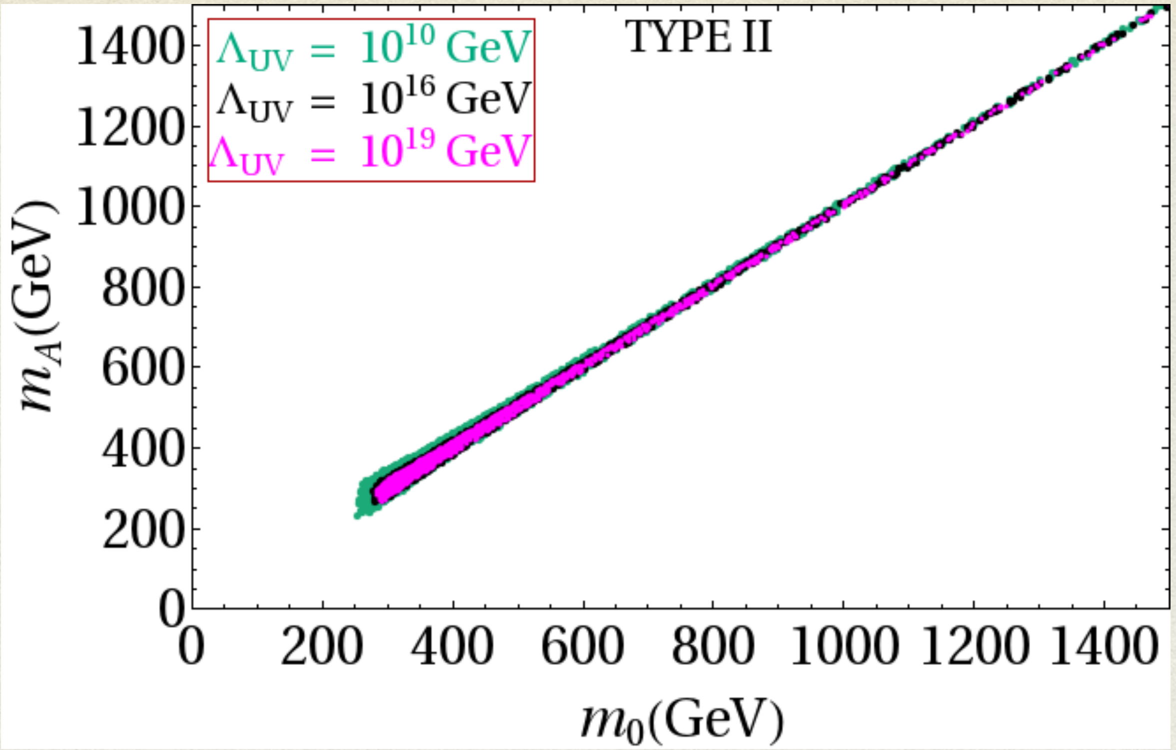


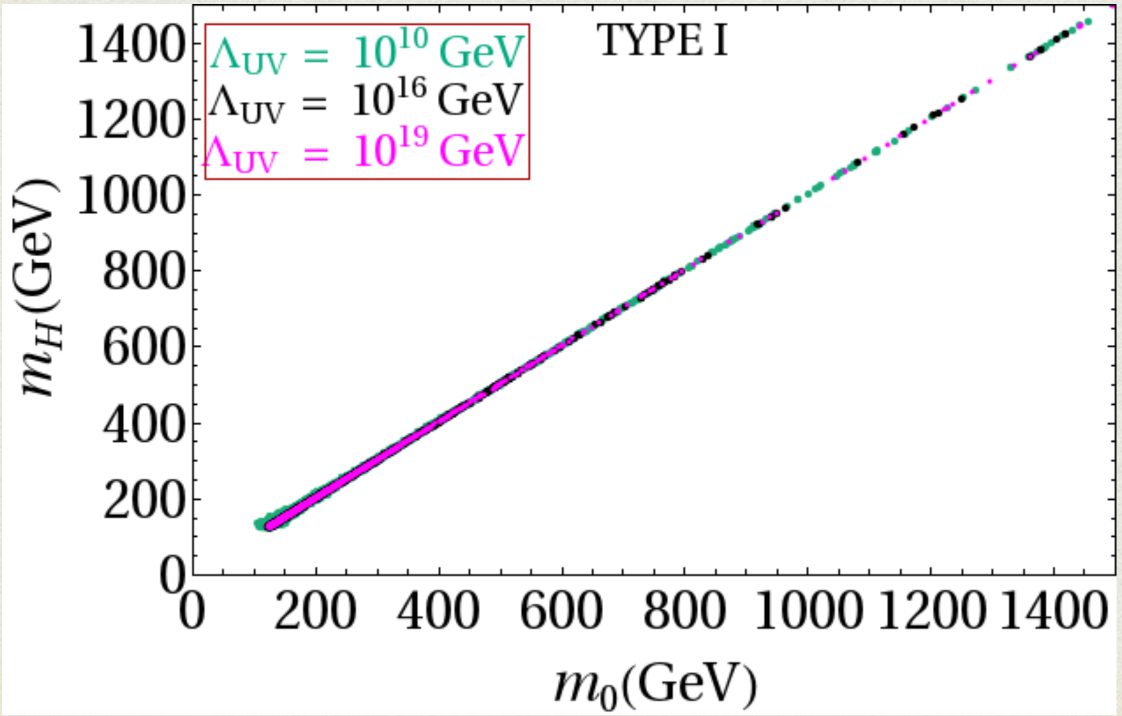
For $\tan\beta$ lower than the required limit → top Yukawa coupling higher than the SM value which worsen the stability of the potential.

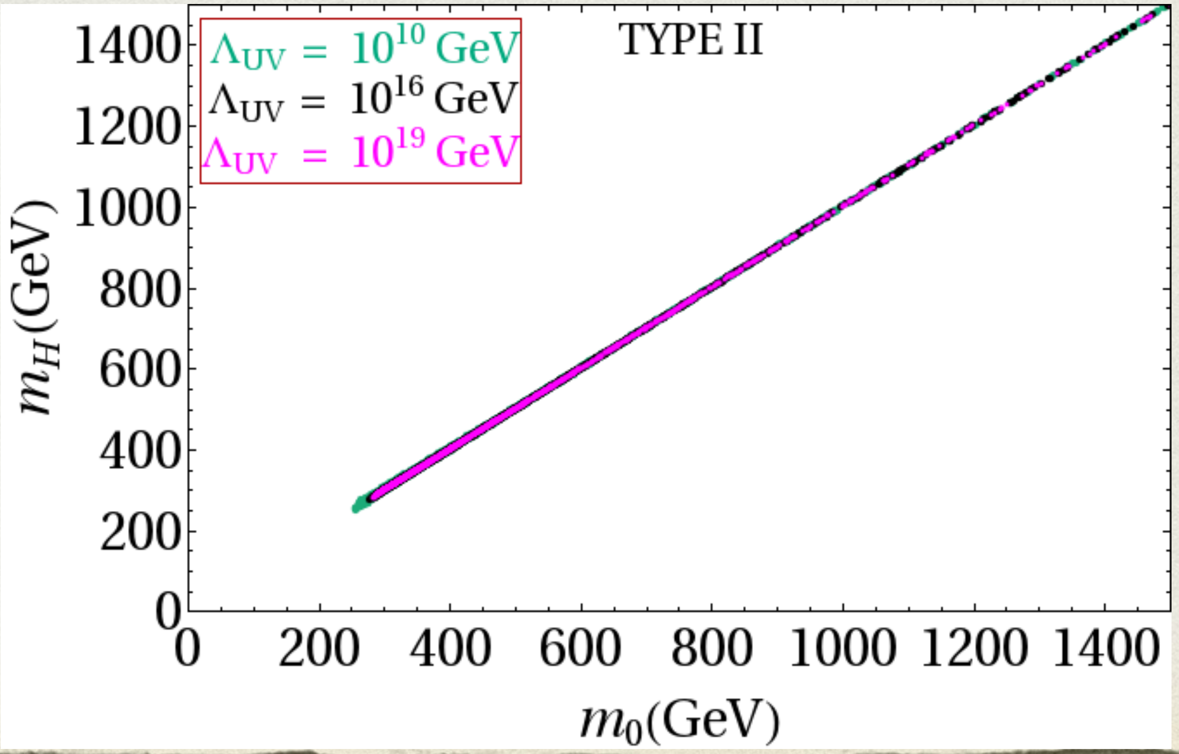
Significantly large $\tan\beta$ → enough positive contribution to Higgs self-coupling.

SOFTLY BROKEN Z2 SYMMETRIC CASE



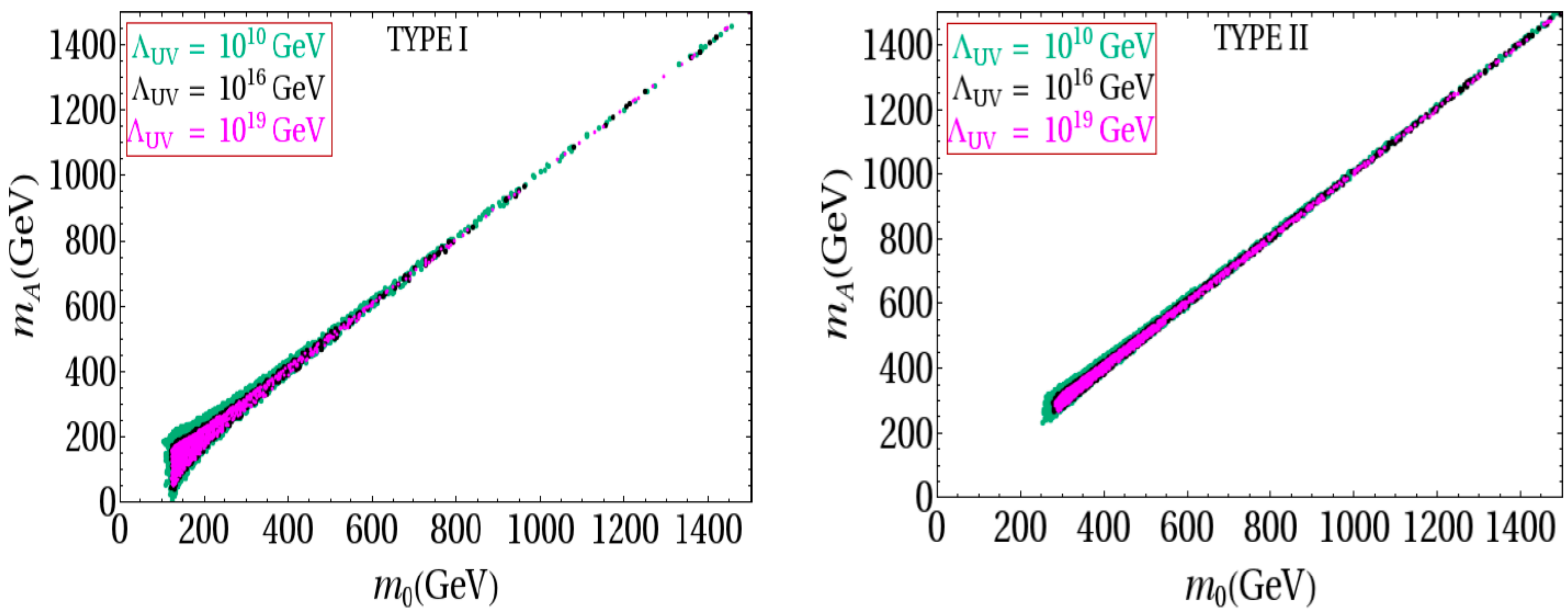






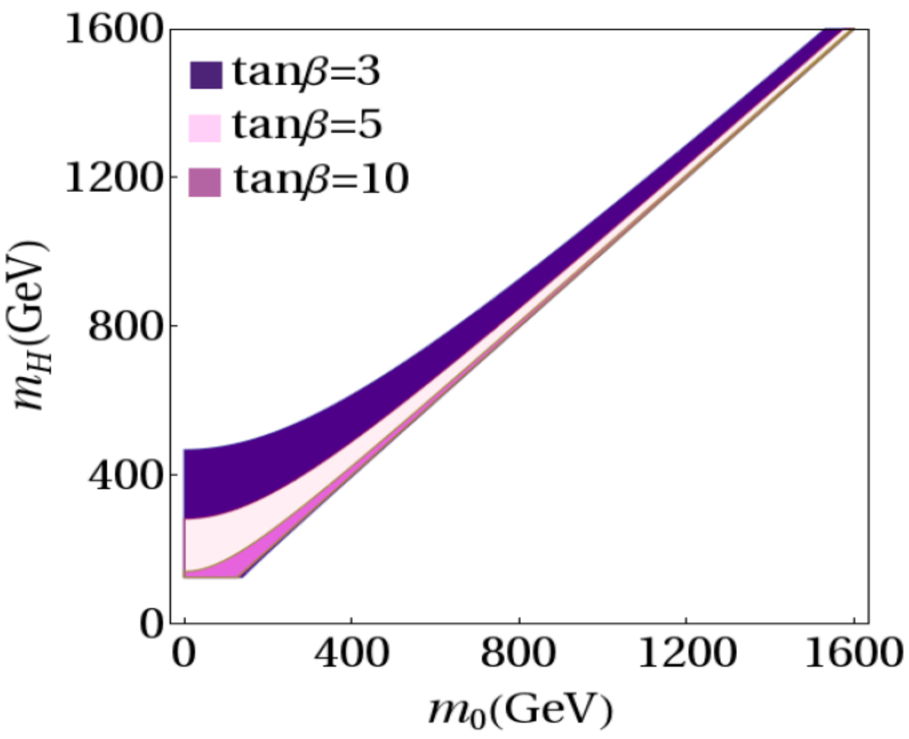
Theoretical Explanation for mass correlation

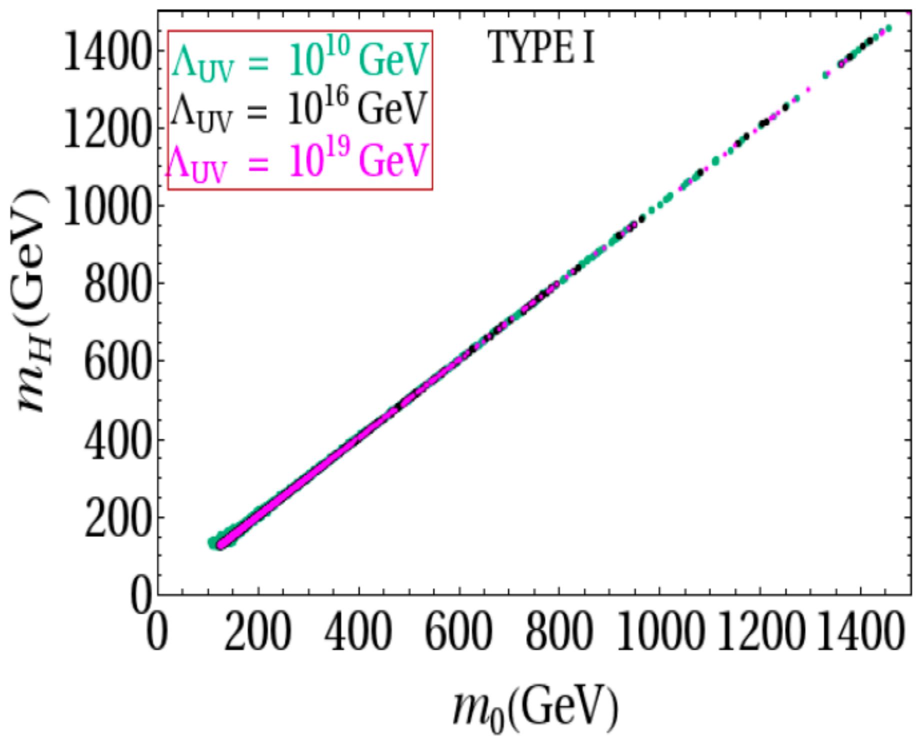
- The evolution of λ_5 is proportional to itself and any initial nonzero value of λ_5 will cause it to grow with energy. RGE
- $\lambda_5 \approx 0$ leads to $U(1)$ limit.
- $\lambda_5 \approx 0$ implies $\beta_5 \approx \beta_6$ and hence, $m_A^2 \approx m_0^2$.



Theoretical Explanation for mass correlation

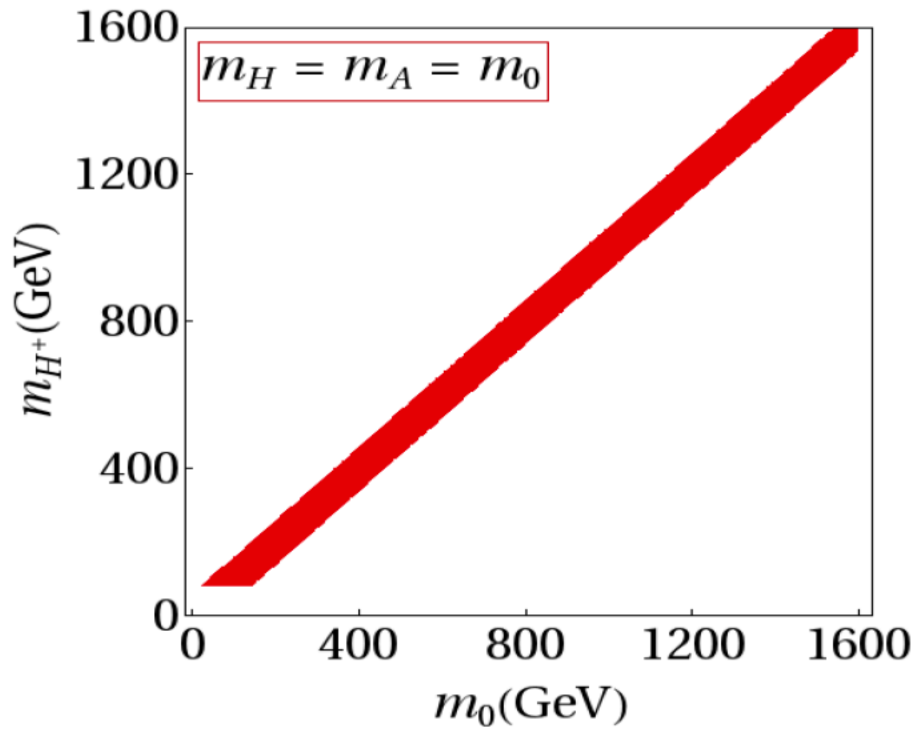
- The unitarity and stability conditions at the electroweak scale imply, [D. Das '15]
  $0 < (m_H^2 - m_0^2)(\tan^2\beta + \cot^2\beta) + 2m_h^2 < \frac{32\pi v^2}{3}$.
- For $\tan\beta$ away from unity, the inequality renders a degeneracy between m_H and m_0.

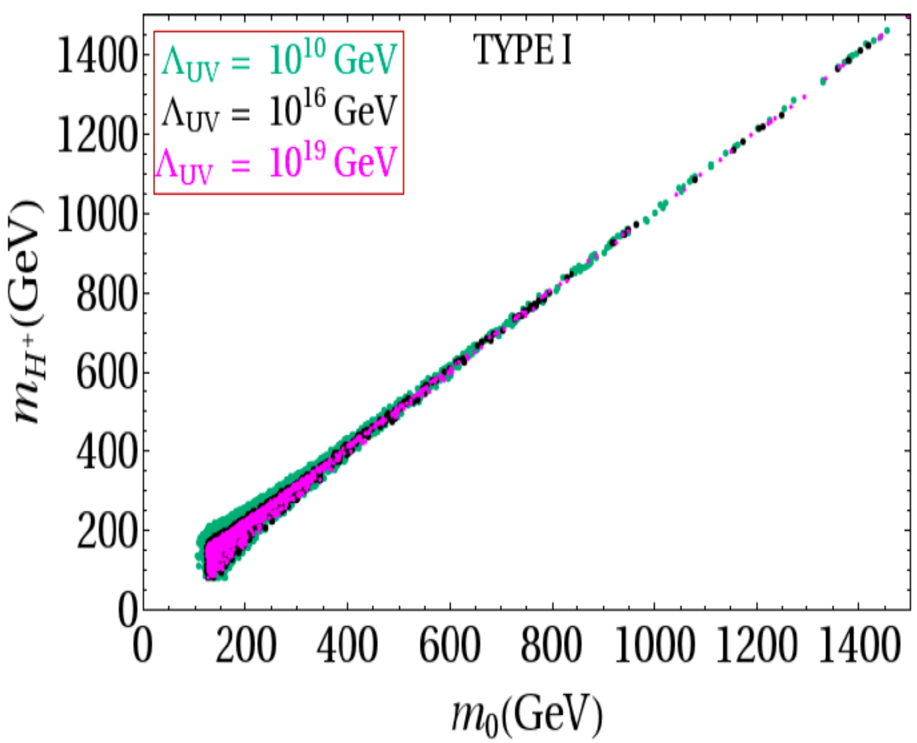




Theoretical Explanation for mass correlation

- With $m_H^2 \approx m_A^2 \approx m_0^2$, $\Delta T = \frac{1}{8\pi \sin^2\theta_w M_W^2} F\left(m_{H+}^2, m_0^2\right)$.
- $F\left(m_{H+}^2, m_0^2\right)$ restricts the splitting $|m_{H+}^2 - m_0^2|$, the experimental limit on ΔT imparts the degeneracy between m_{H+} and m_0.





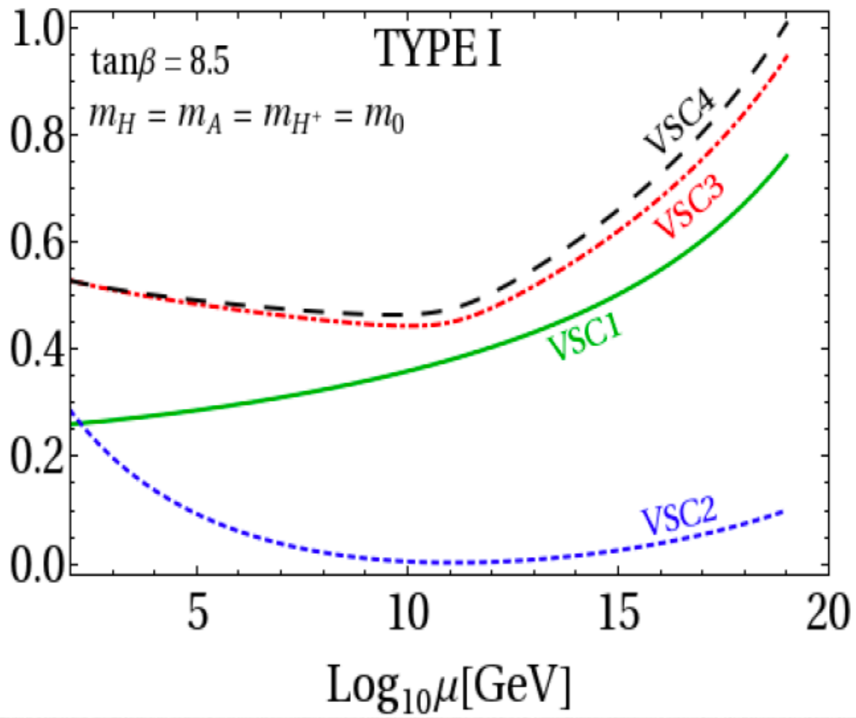
- Thus, $m_0^2 \approx m_A^2 \approx m_H^2 \approx m_{H+}^2$.
- The lower limit of charged Higgs mass for Type II (300 GeV) and limit on CP-even Higgs ($> m_h$) for Type I sets the limit on m_0.

Theoretical Explanation for $\tan\beta$ limit

- ϕ_2 gives masses to up-type quarks $\Rightarrow$ λ_2 will face the negative pull of top Yukawa copling $\to h_t = \frac{\sqrt{2}m_t}{(v\sin\beta)}$.
- In the limit of *exact degeneracy*, RG running of λ_2 is similar to SM Higgs self-coupling λ except some extra scalar contribution.

$$\begin{aligned}\mathcal{D}\lambda_2 &= 16\lambda_2^2 - 3\left(3g^2 + g'^2\right)\lambda_2 + \\ &\quad \frac{3}{4}\left(3g^4 + g'^4 + 2g^2g'^2\right) + 12h_t^2\lambda_2 - 12h_t^4 .\end{aligned}$$

- Not enough to overcome the negative pull of top Yukawa $\Rightarrow$ lower limit in $\tan\beta$.



Theoretical Explanation for $\tan\beta$ limit

Deviation in $m_H^{\pm}$ from degenerate value relax the $\tan\beta$ limit.

$$m_0 = m_A = m_H\,, \quad \text{and,} \;\; m_{H^+}^2 = m_0^2 + \Delta\,.$$

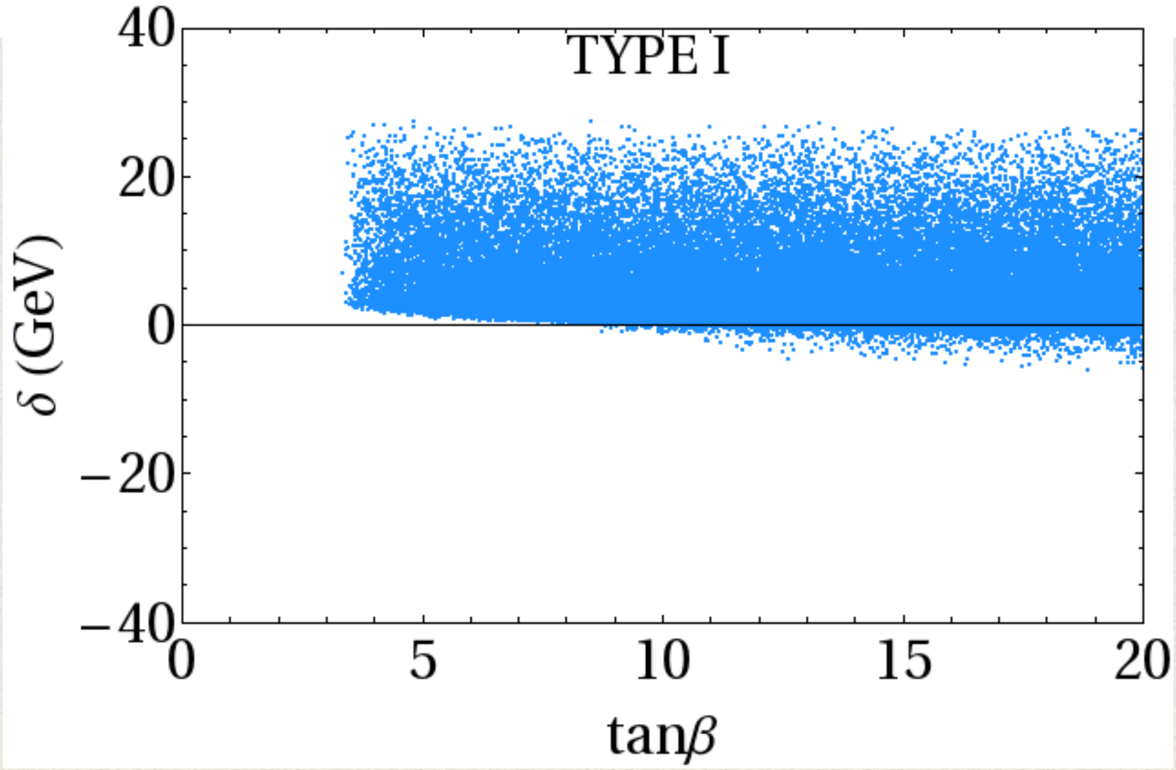
In this limit,

$$\lambda_1 = \lambda_2 = \frac{m_h^2}{v^2}\,, \quad \lambda_3 = \lambda_2 + \frac{2\Delta}{v^2}\,, \quad \lambda_4 = -\frac{2\Delta}{v^2}\,, \quad \lambda_5 = 0\,.$$

Using these, we can rearrange the terms that appear on the RHS in the RG equation for λ_2, to obtain

$$\mathcal{D}\lambda_2 = 14\lambda_2^2 + 2\left(\lambda_2 + \frac{2\Delta}{v^2}\right)^2 - 3\left(3g^2 + g'^2\right)\lambda_2 + \frac{3}{4}\left(3g^4 + g'^4 + 2g^2g'^2\right) + 12h_t^2\lambda_2 - 12h_t^4\,.$$

Extra scalar contribution aids to lower value of $\tan\beta$.



SUMMARY

- 2HDM scalar potential with an exact Z_2 symmetry is unable to maintain stability after 10^8 GeV (10^4 GeV) in the Type I (II) case. To ensure stability up to the Planck scale, Z_2 symmetry needs to be broken softly.

- By demanding high scale stability in the presence of a soft breaking, we are led to a situation where the symmetry of the potential is enhanced from softly broken Z_2 to softly broken U(1).

- To have stability up to very high energies ($\gtrsim 10^{10}$ GeV), all the nonstandard masses need to be nearly degenerate: $m_0 \approx m_A \approx m_H \approx m_{H^+}$. Thus, there is only one nonstandard mass parameter that governs the 2HDM in the *stable alignment limit*.

- The value of $\tan\beta$ is bounded from below.

THANK YOU

BACKUP

3HDM WITH Z3 SYMMETRY

The scalar potential

$$V = m_{11}^2(\phi_1^\dagger\phi_1) + m_{22}^2(\phi_2^\dagger\phi_2) + m_{33}^2(\phi_3^\dagger\phi_3) + \lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_3^\dagger\phi_3)^2$$

$$+\lambda_4(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_5(\phi_1^\dagger\phi_1)(\phi_3^\dagger\phi_3) + \lambda_6(\phi_2^\dagger\phi_2)(\phi_3^\dagger\phi_3)$$

$$+\lambda_7(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \lambda_8(\phi_1^\dagger\phi_3)(\phi_3^\dagger\phi_1) + \lambda_9(\phi_2^\dagger\phi_3)(\phi_3^\dagger\phi_2)$$

$$+\left[\lambda_{10}(\phi_1^\dagger\phi_2)(\phi_1^\dagger\phi_3) + \lambda_{11}(\phi_1^\dagger\phi_2)(\phi_3^\dagger\phi_2) + \lambda_{12}(\phi_1^\dagger\phi_3)(\phi_2^\dagger\phi_3) + \mathrm{h.c.}\right]$$

ADDITIONAL USAGE

Deviations from the exact alignment limit can also be parametrized rather conveniently.

$$\sin(\alpha_1 - \beta_1) = \delta_1 \qquad \sin(\alpha_2 - \beta_2) = \delta_2$$

$$\alpha_1 = \sin^{-1}(\delta_1) + \beta_1\,; \qquad \alpha_2 = \sin^{-1}(\delta_2) + \beta_2$$

Write the couplings in terms of physical masses and mixings.

$\delta_1 = \delta_2 = 0$ characterizes the exact alignment limit.

$$\mathscr{L}_{H_i^+ H_i^- h} = g_{H_i^+ H_i^- h}\, H_i^+ H_i^- h\,, \qquad (i = 1,2)$$

In the exact alignment limit,

$$g_{H_i^+ H_i^- h} = -\frac{1}{v}\left(m_h^2 + 2m_{Ci}^2\right) = -\frac{g m_{Ci}^2}{M_W}\left(1 + \frac{m_h^2}{2m_{Ci}^2}\right)$$

Introduce soft symmetry breaking term to avoid non-decoupling.