Partial deconfinement in gauge theories

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arXiv: 1911.11465 (proceedings)
JHEP 03 (2019) 145 (arXiv:1812.05494)

Hanada & Maltz (2016)
Hanada, Jevicki, Peng & Wintergerst, (2019)
Hanada & Robinson, (2019)

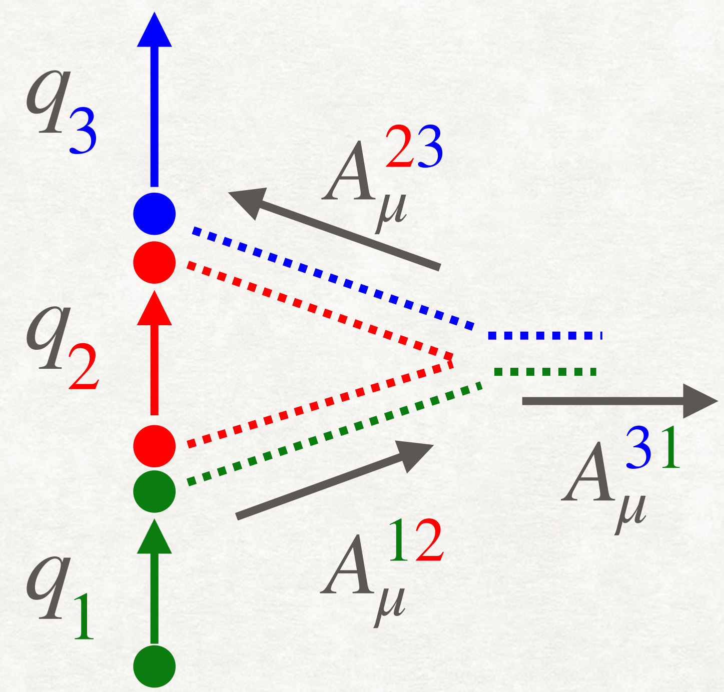
2019/12/10 Seminar@Osaka University

Introduction: large-Nc gauge theory

QCD : SU(3) gauge theory, 3 colors

$$\begin{pmatrix} A_\mu^{11} & A_\mu^{12} & A_\mu^{13} \\ A_\mu^{21} & A_\mu^{22} & A_\mu^{23} \\ A_\mu^{31} & A_\mu^{32} & A_\mu^{33} \end{pmatrix} \qquad \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}$$

gauge field (gluon) quark



SU(Nc) gauge theory, Nc colors

$$\begin{pmatrix} A_\mu^{11} & \cdots & A_\mu^{1N} \\ \vdots & \ddots & \vdots \\ A_\mu^{N1} & \cdots & A_\mu^{NN} \end{pmatrix} \qquad \begin{pmatrix} \Psi^{11} & \cdots & \Psi^{1N} \\ \vdots & \ddots & \vdots \\ \Psi^{N1} & \cdots & \Psi^{NN} \end{pmatrix} \qquad \begin{pmatrix} \Phi^{1} \\ \vdots \\ \Phi^{N} \end{pmatrix}$$

Large N limit

\`t Hooft large N limit;  [\`t Hooft, (1974)]

$$N \to \infty, \quad \lambda \equiv g_{\mathrm{YM}}^2 N : \text{fixed}$$

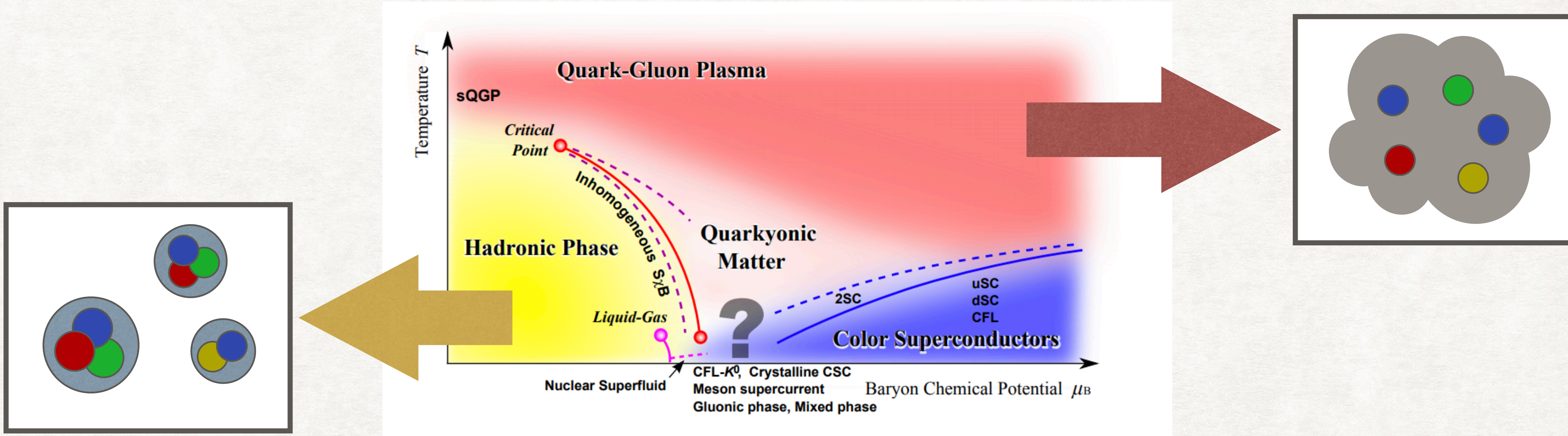
$\longrightarrow$ Saddle point approximation becomes exact

(up to 1/N corrections)

- This often gives good approximation to finite N, even N=3
- Energy and entropy can be estimated by counting color d.o.f.
- Another motivation; equivalence to string theory

Summing up vacuum diagrams $\Longleftrightarrow$ Genus expansion of string worldsheet

Deconfinement transition



From [Fukushima & Hatsuda, (2012)]

"deconfinement criterion" in large-N theory

Confined phase;

$$E \sim N^0$$

Phase transition takes place
even finite volume.

Deconfined phase;

$$E \sim N^a$$

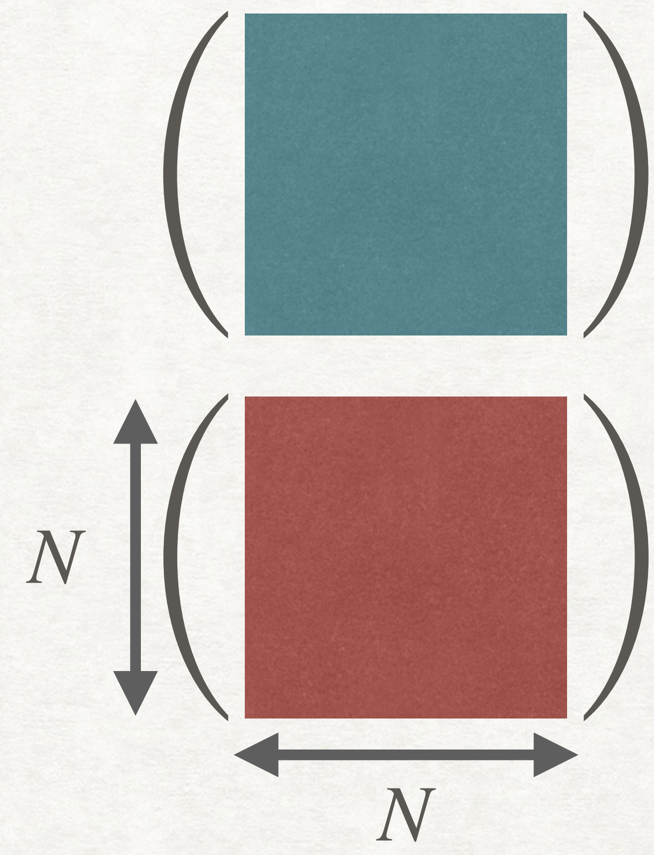
$$\begin{cases} a = 1 & \text{fundamental matter} \\ a = 2 & \text{adjoint matter} \end{cases}$$

“Partial” deconfinement?

- Confined phase; $E \sim N^0$

- Deconfined phase; $E \sim N^2$

    (thermal excitation of color d.o.f.)



What happens
if energy is not so large or small?

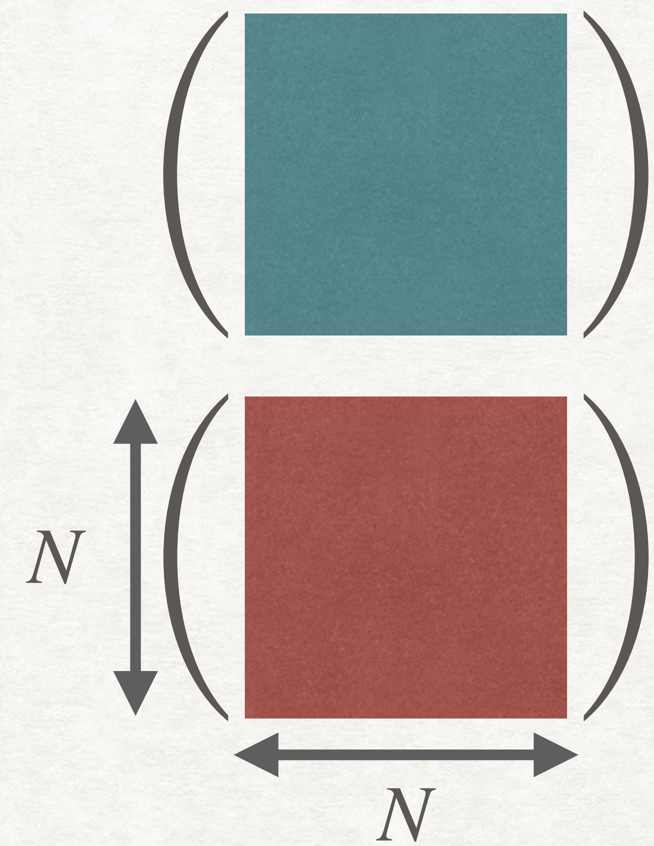
ex.) $E \sim N^2/100$

“Partial” deconfinement?

- Confined phase; $E \sim N^0$

- Deconfined phase; $E \sim N^2$

  (thermal excitation of color d.o.f.)

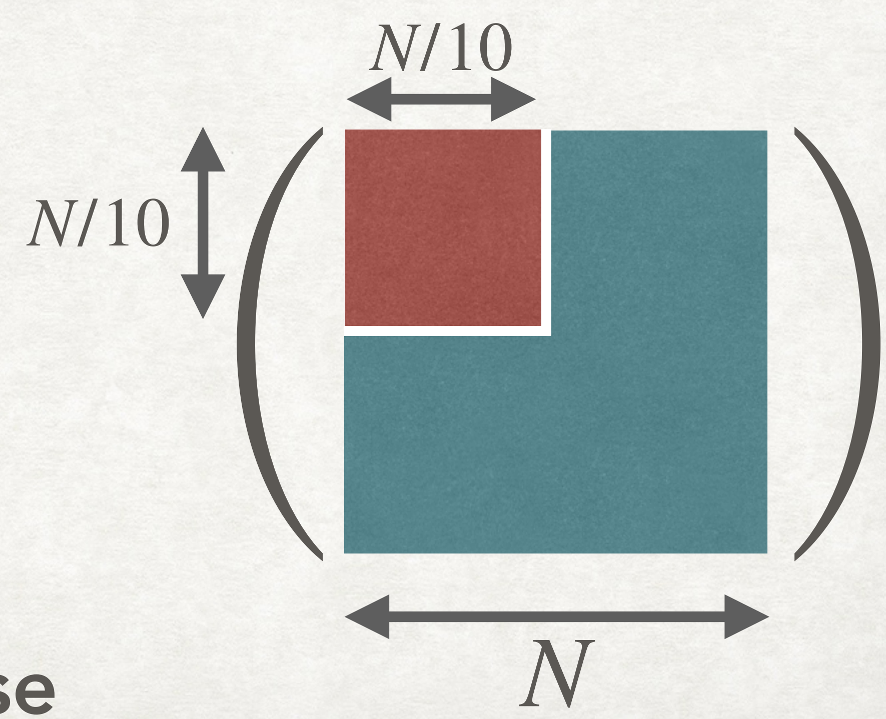


What happens
if **energy is not so large or small?**

ex.) $E \sim N^2/100$

only SU(N/10) sector deconfines.

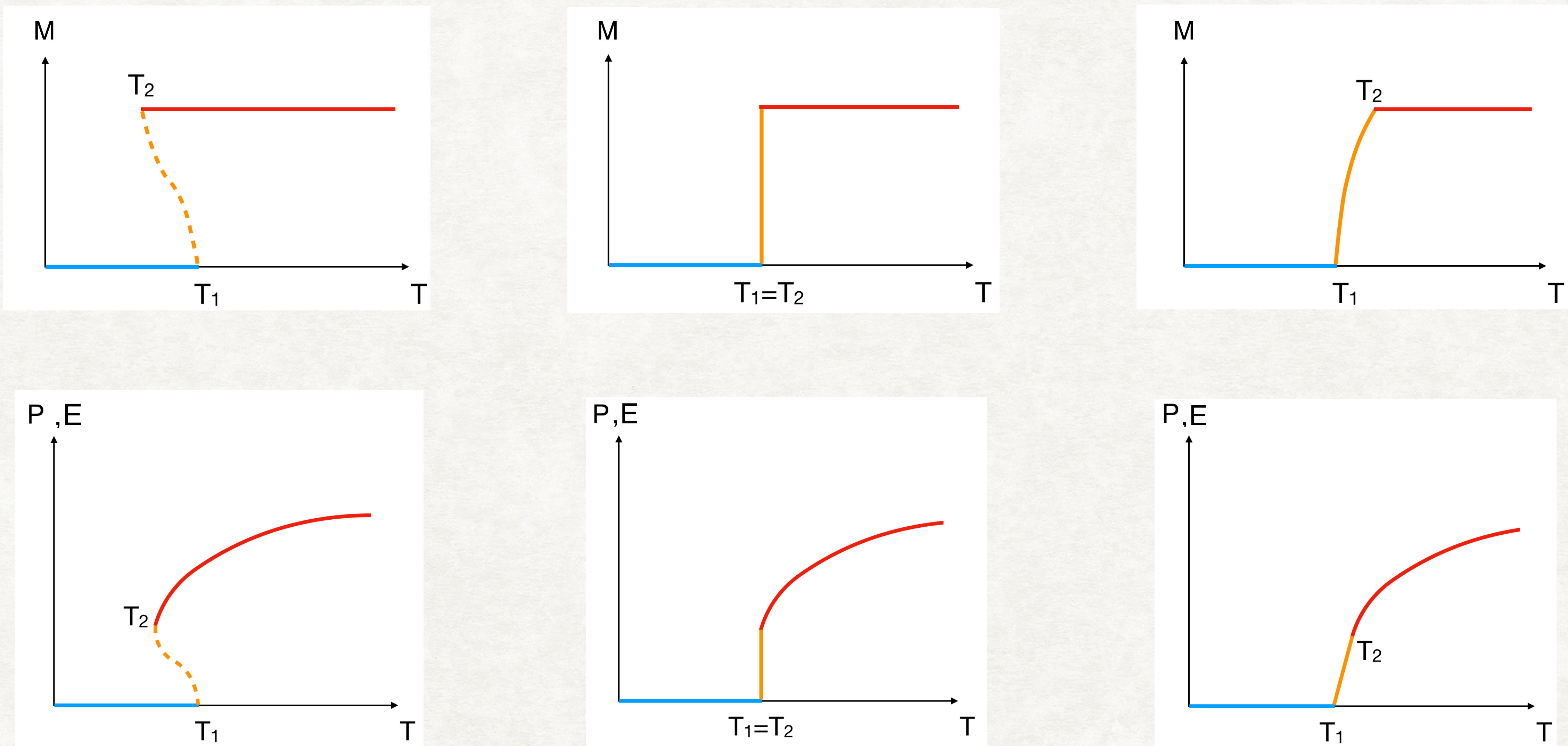
$E \sim M^2 = \varepsilon N^2$ with $M \sim \sqrt{\varepsilon} N = N/10$



➡ **“Partially deconfined” phase**

Phase structures

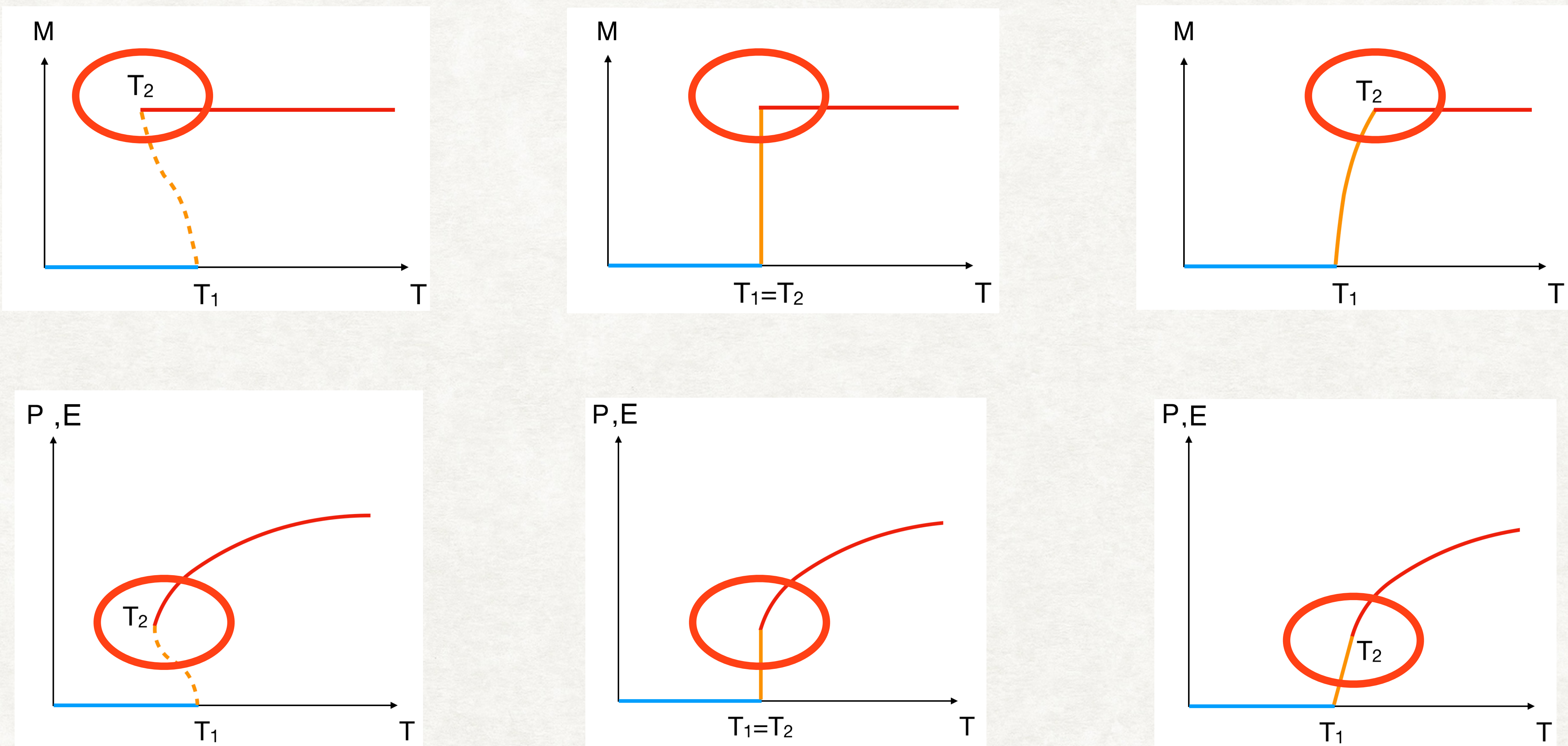
(Precise arguement is given later)



Phase structures

(Precise arguement is given later)

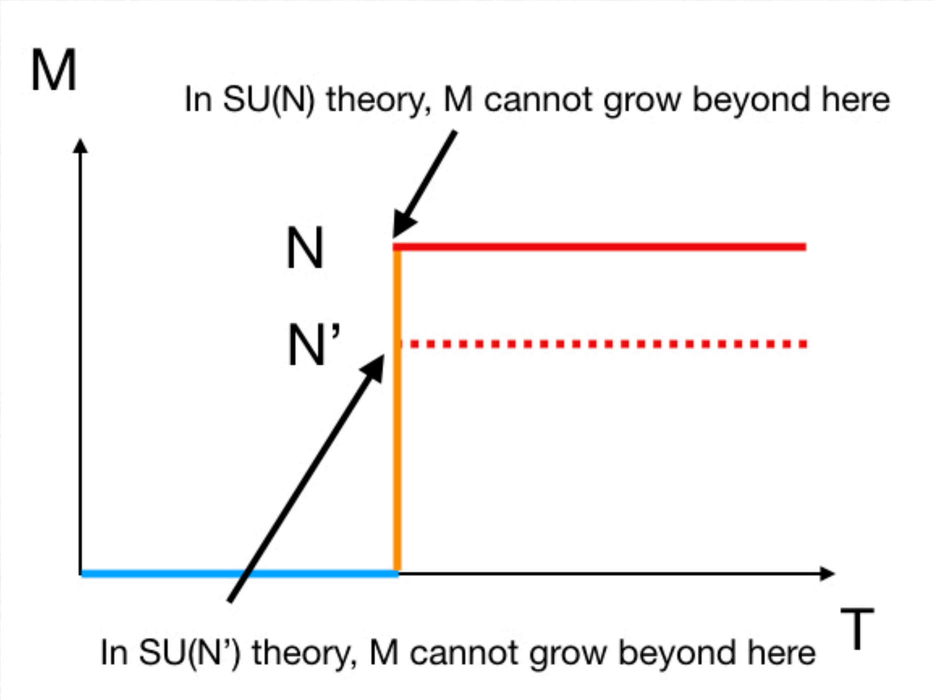
There are TWO transitions.



One of transitions is **Gross-Witten-Wadia(GWW) transition**

[Gross & Witten, (1980)/ Wadia, (1980)]

Relation to Gross-Witten-Wadia transition



1/N and 1/N′ corrections are both negligible.

At $M \leq N'$, SU(N) and SU(N′) theories behave in same manner.

SU(N′) sector in SU(N) theory ⟷ GWW-transition point of SU(N′) theory

In partially deconfined phase,

$$E = E_{\mathrm{GWW}}(M), \quad S = S_{\mathrm{GWW}}(M)$$

Point of view from Polyakov loop

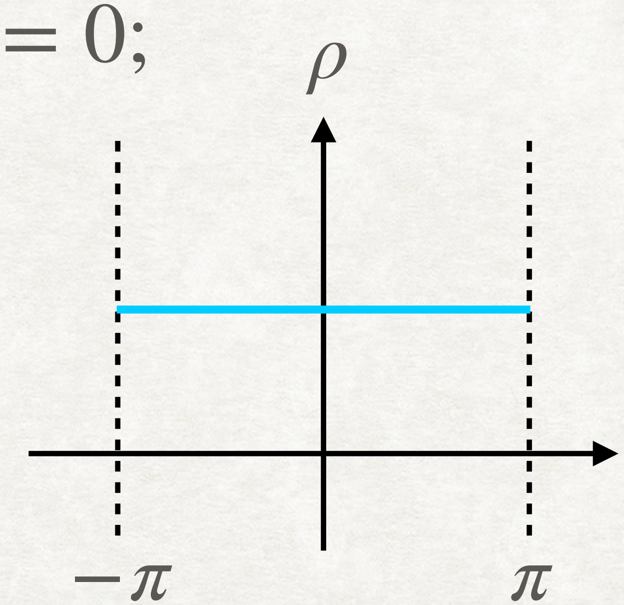
Polyakov loop : an order parameter of confine/deconfine transition

$$P = \frac{1}{N}\mathrm{Tr}\mathscr{P}\exp\left[-\oint_{\mathrm{temporal}} A_t\right] = \frac{1}{N}\sum_{j=1}^{N} \mathrm{e}^{\mathrm{i}\theta_j} = \int d\theta\, \rho(\theta)\mathrm{e}^{\mathrm{i}\theta}$$

$$\rho(\theta) = \frac{1}{N}\sum_j \delta(\theta - \theta_j)$$: phase distribution

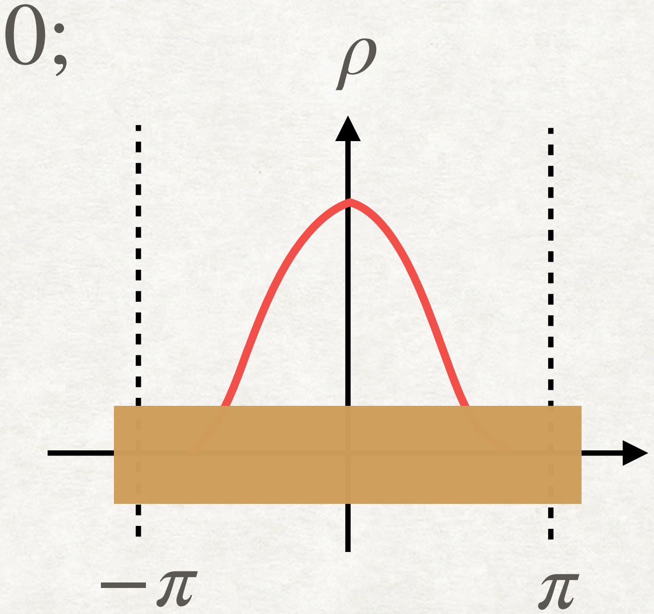
Can be regarded as continuous function in large N limit

$P = 0;$



Confined phase

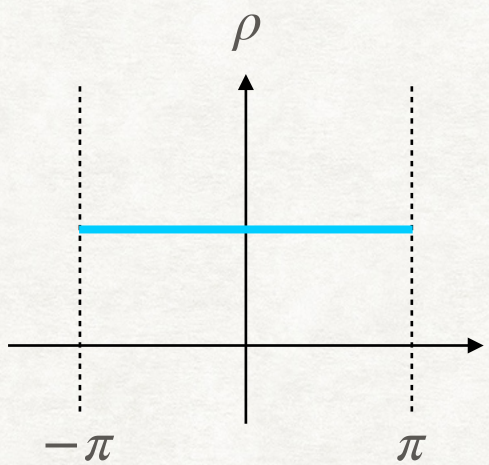
$P \neq 0;$



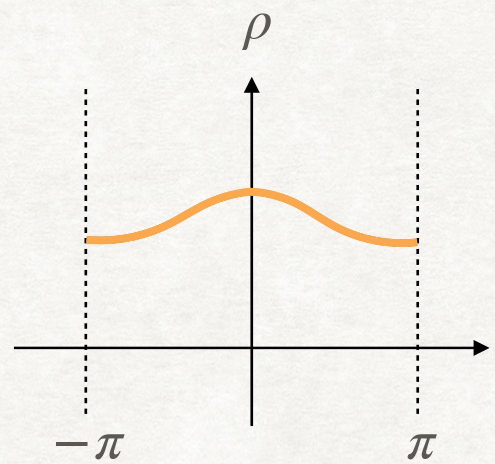
Deconfined phase

Phase distribution of Polyakov line phases

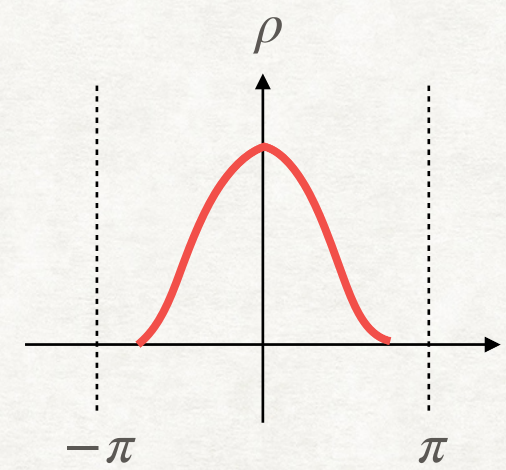
Polyakov loop : an order parameter of confine/deconfine transition



confined



"partially" deconfined



"completely" deconfined

Deconfinement transition
(Hagedorn transition)

GWW transition

$$\boxed{\rho(\theta) = \frac{N-M}{N}\rho_{\text{conf}}(\theta) + \frac{M}{N}\rho_{\text{GWW},M}(\theta) = \frac{N-M}{N}\cdot\frac{1}{2\pi} + \frac{M}{N}\rho_{\text{GWW},M}(\theta)}$$

Partial deconfinement is "the mixture."

M θ_j s are in deconfined phase and N-M θ_j s are in confined phase

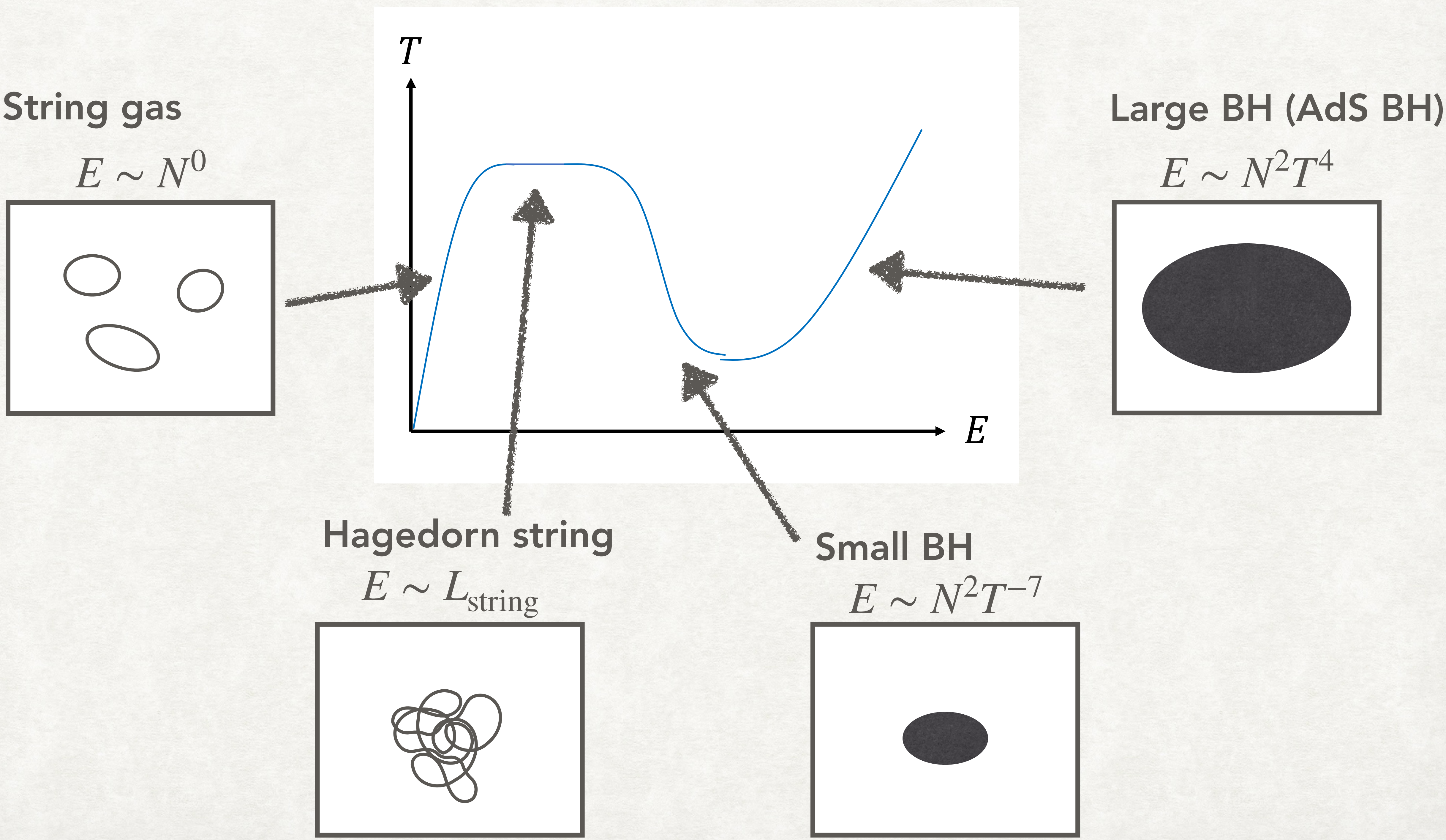
(For string theorists)

Connection to String Theory

Berkowitz, Hanada & Maltz, (2016)
Hanada & Maltz, (2016)
Berenstein, (2018)

Black hole in $\mathrm{AdS}_5\times S^5$ <=> 4d N=4 $\mathrm{SU}(N)$ SYM

Strongly coupled 4d SYM / dual string theory (E: fix)



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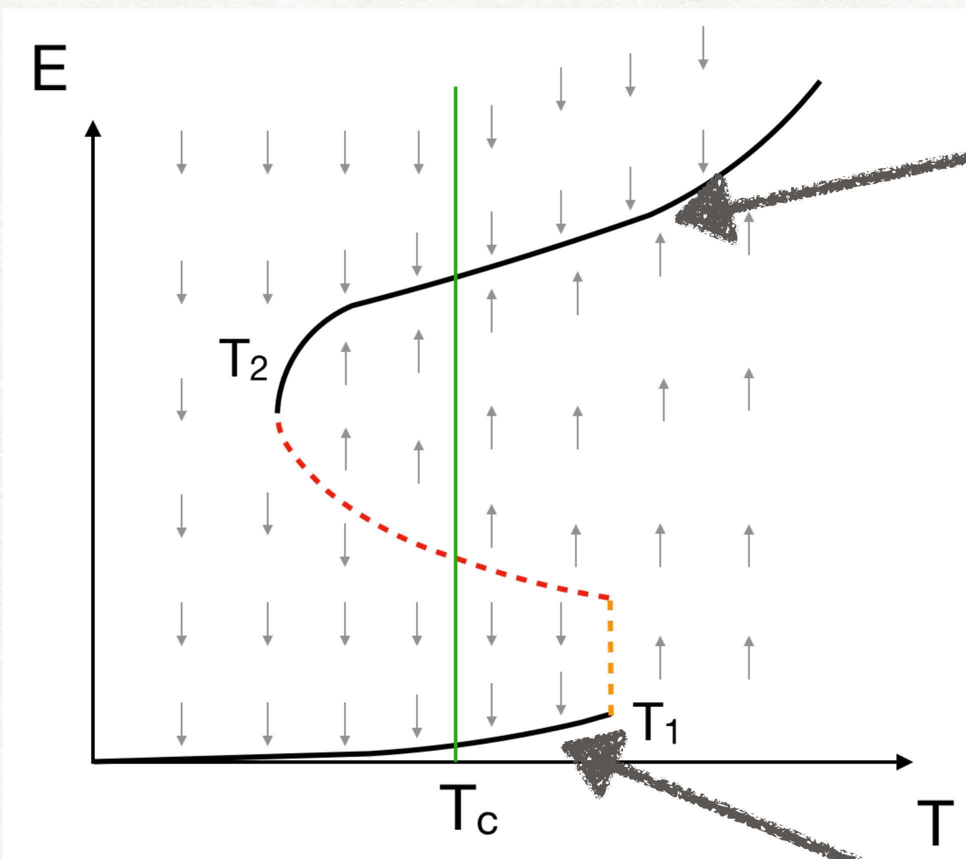
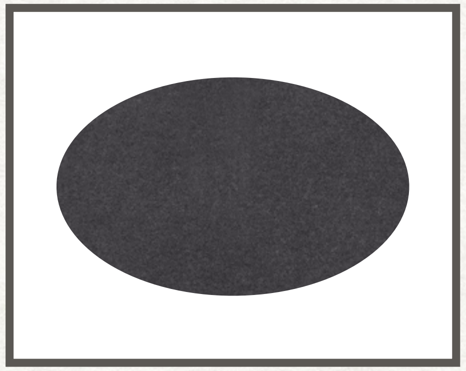


(T: fix)

How about
small BH or Hagedorn string?

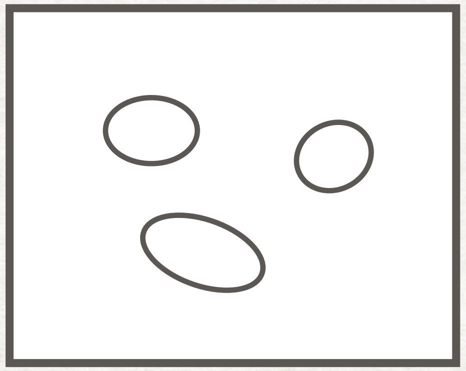
Large BH (AdS BH)

$E \sim N^2 T^4$



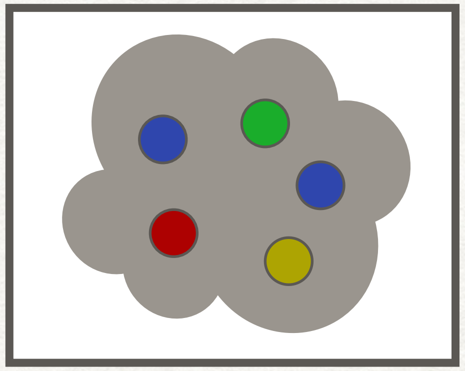
String gas

$E \sim N^0$



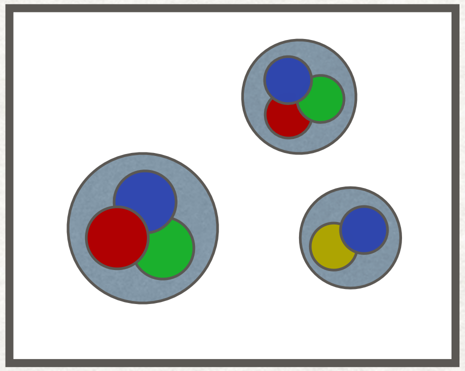
Gauge theory side;

Deconfined phase



phase transition

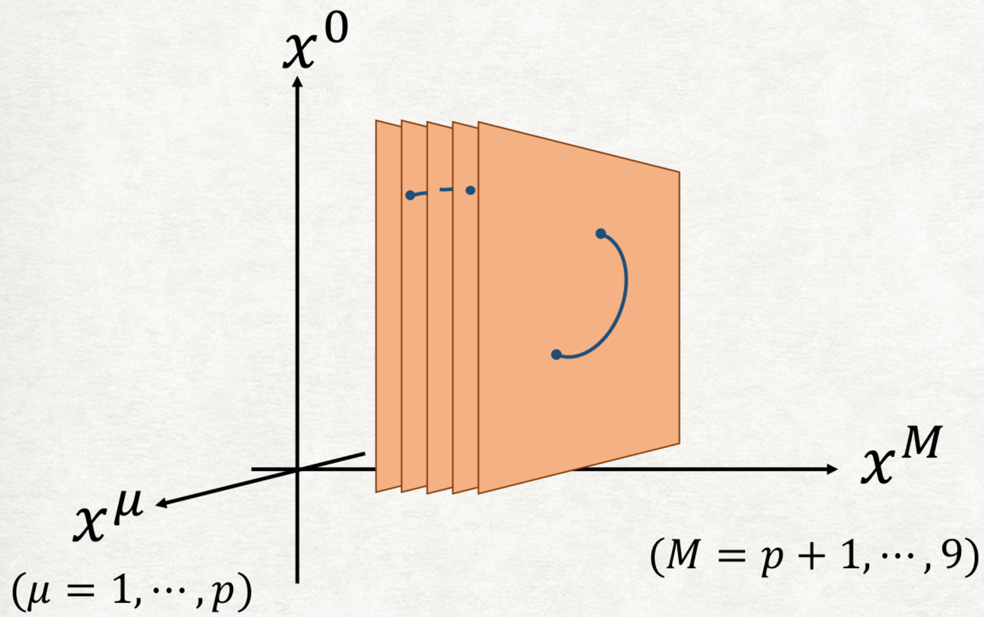
Confined phase



[Witten, (1998)]

D-branes with open strings & BH

Dp-brane : the objects that open strings can put their endpoints.



($p = 3$ for $\mathcal{N} = 4$ SYM)

$$S_{\mathrm{DBI}} = -T_{D_p}\,\mathrm{Tr}\int \mathrm{d}^{p+1}\sigma\sqrt{-\det\left(G_{ab} + 2\pi l_s^2 \mathscr{F}_{ab} + \cdots\right)}$$

$$\Big\downarrow$$

$$S_{\mathrm{eff}} = \frac{1}{g_{\mathrm{YM}}^2}\int d^{p+1}x\,\mathrm{Tr}\left\{\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}(D_\mu X_M)^2 + \frac{1}{4}[X_M, X'_M]^2 + \cdots\right\}$$

$$D_\mu = \partial_\mu - i[A_\mu, \cdot\,] \qquad \begin{cases} A_\mu(x^\mu) & (\mu = 0,\ 1,\ \cdots,\ p) \\ X_M(x^\mu) & (M = p+1,\ \cdots,\ 9) \end{cases}$$

Classical vacua (:minima of potential)

$$X_M = \mathrm{diag}\left(x_M^1, x_M^2, \cdots, x_M^N\right)$$

$(X_M)_{ii} = x_M^i$: Position of i th Dp-brane

$(X_M)_{ij}$'s fluctuation

: Open strings between i th and j th Dp-brane

D-branes with open strings & BH

Classical vacua (:minima of potential)

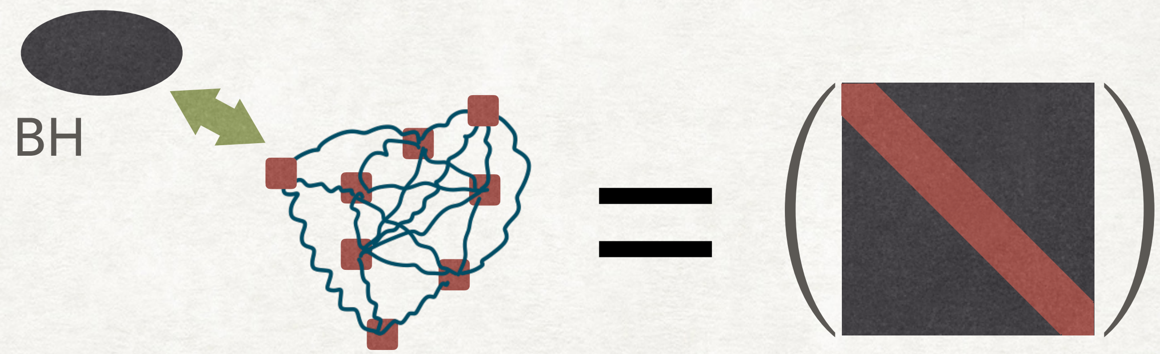
$$X_M = \mathrm{diag}\left(x_M^1, x_M^2, \cdots, x_M^N\right)$$

$(X_M)_{ii} = x_M^i$: Position of i th Dp-brane

$(X_M)_{ij}$'s fluctuation

: Open strings between i th and j th Dp-brane

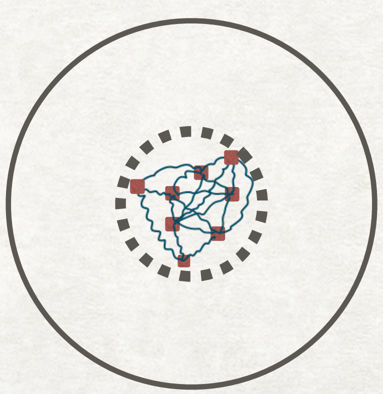
High energy region,

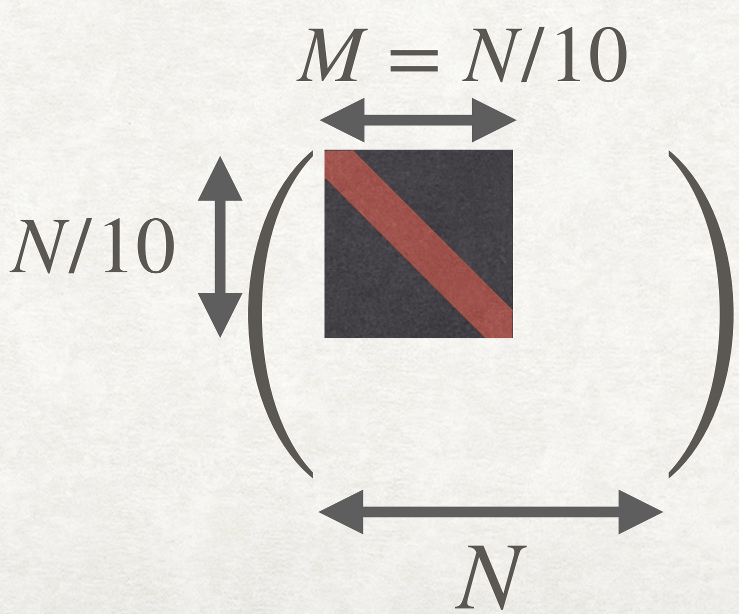


The bound state of D-branes & open strings

Configuration of scalar fields X_M

When E is not so large/small, [Hanada & Maltz, (2016)/Berkowitz, Hanada & Maltz, (2016)]

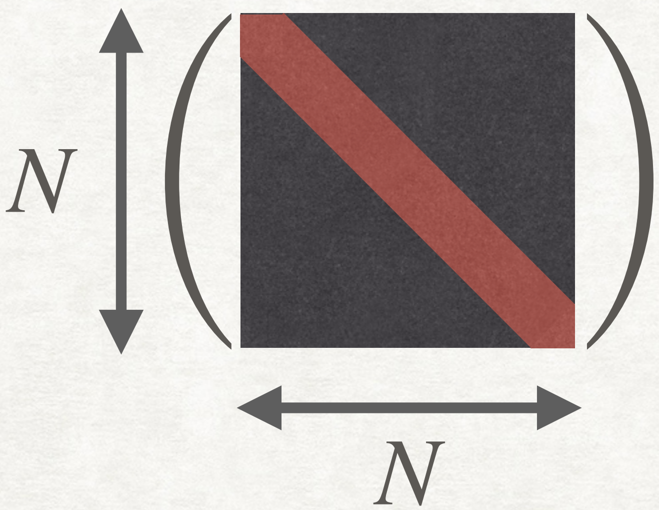


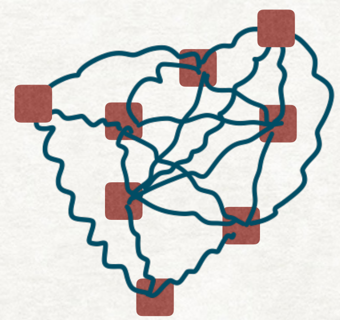


gauge theory counterpart of small black hole

Negative specific heat can appear

Roughly, temperature ~ energy per d.o.f.

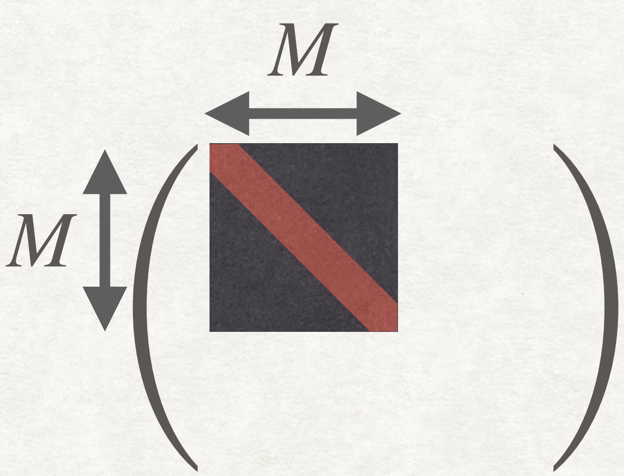


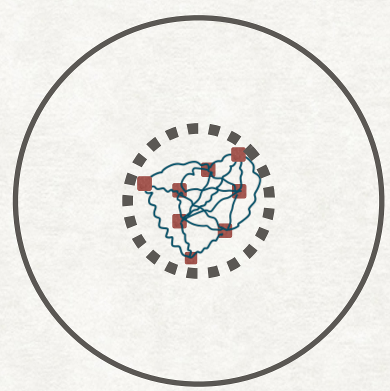


$$T \sim E/N^2$$

$$T' \sim E'/N^2$$

N^2:fixed, $\rightarrow$ $T' > T$, if $E' > E$





$$T \sim \frac{E}{M^2(E)}$$

It's possible that T decreases when E increases.

Contents

(For all)

Explicit demonstrations of partial deconfinement

Hanada, Ishiki & HW, (2018)

Hanada, Jevicki, Peng & Wintergerst, (2019)

Hanada & Robinson, (2019)

Weakly-coupled 4D Yang-Mills theories

- 4d $\mathrm{SU}(N)$ Yang-Mills theory with adjoint matters on S^3 ;

[Sundborg, (2000)/Aharony, Marsano, Minwalla, Papadodimas & Raamsdonk, (2003)]

At zero 't Hooft coupling;

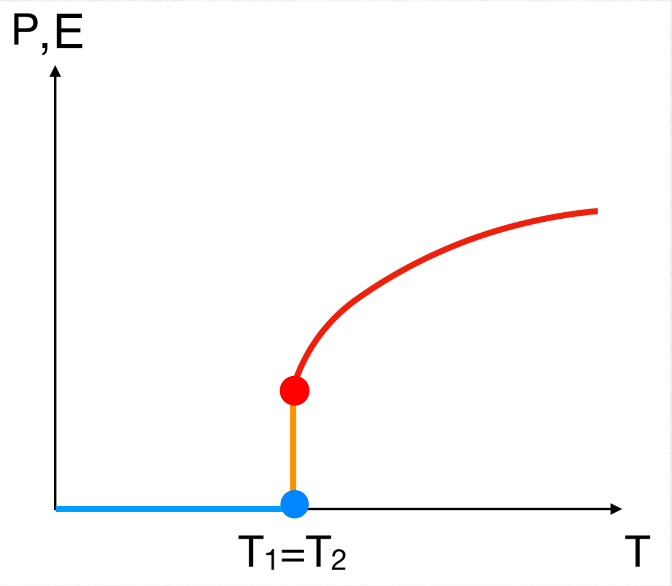
$$Z(x) = \int [dU] \exp\left\{ \sum_{m=1}^{\infty} \frac{1}{m} \big(z_B(x^m) + (-1)^{m+1} z_F(x^m) \big) \mathrm{tr}(U^m) \mathrm{tr}((U^\dagger)^m) \right\}$$

$x \equiv e^{-\beta}, \quad z(x) = \sum_i x^{E_i}$:Single particle partition function

$$\int [dU] \to \prod_i \int_{-\pi}^{\pi} [d\theta_i] \prod_{i<j} \sin^2\left(\frac{\theta_i - \theta_j}{2}\right), \quad \mathrm{tr}(U^n) \to \sum_j e^{in\theta_j}$$

$$Z(x) = \int [d\theta_i] \ \exp\Big(- \sum_{i \neq j} V(\theta_i - \theta_j) \Big)$$

$$V(\theta) = \ln(2) + \sum_{n=1}^{\infty} \frac{1}{n} \big(1 - z_B(x^n) - (-1)^{n+1} z_F(x^n)\big) \cos(n\theta)$$



At ● ; GWW transition

At small nonzero 't Hooft coupling;

$$Z(\beta) = \int [dU] \ \exp\left[- \Big(|\mathrm{tr}(U)|^2 \big(m_1^2 - 1\big) + b\, |\mathrm{tr}(U)|^4 / N^2 \Big) \right]$$

[Aharony, Marsano, Minwalla, Papadodimas & Raamsdonk, (2005)]

Depending on sign of b, phase structure changes.

Weakly-coupled 4D Yang-Mills theories

- 4d $\mathrm{SU}(N)$ Yang-Mills theory with adjoint matters on S^3 ;

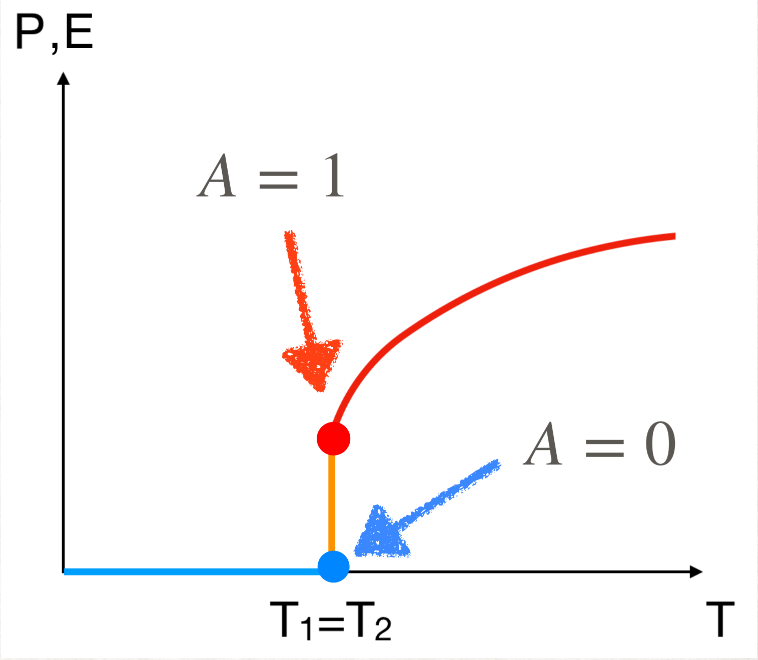
[Sundborg, (2000)/Aharony, Marsano, Minwalla, Papadodimas & Raamsdonk, (2003)]

$$\rho(\theta) = \begin{cases} \frac{1}{2\pi} & \left(T \le T_1 = T_2\right) \\ \frac{1}{2\pi}(1 + A\cos\theta) & \left(T = T_1 = T_2\right) \\ \frac{A}{\pi}\cos\frac{\theta}{2}\sqrt{\frac{1}{A} - \sin^2\frac{\theta}{2}} & \left(T \ge T_1 = T_2, |\theta| < 2\arcsin\sqrt{A^{-1}}\right) \end{cases}$$

At $T = T_1 = T_2$,

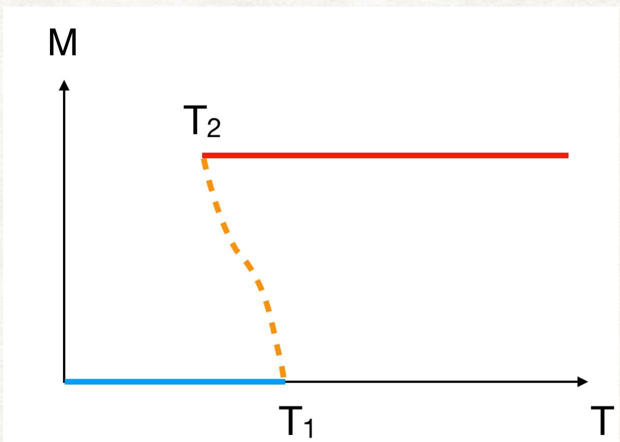
$$\begin{aligned} \rho(\theta) &= \frac{1}{2\pi}(1 + A\cos\theta) \\ &= (1 - A)\cdot\frac{1}{2\pi} + A\cdot\frac{1}{2\pi}(1 + \cos\theta) \\ &= (1 - A)\rho_{\mathrm{conf}} + A\cdot\rho_{\mathrm{GWW}}(\theta) \end{aligned}$$

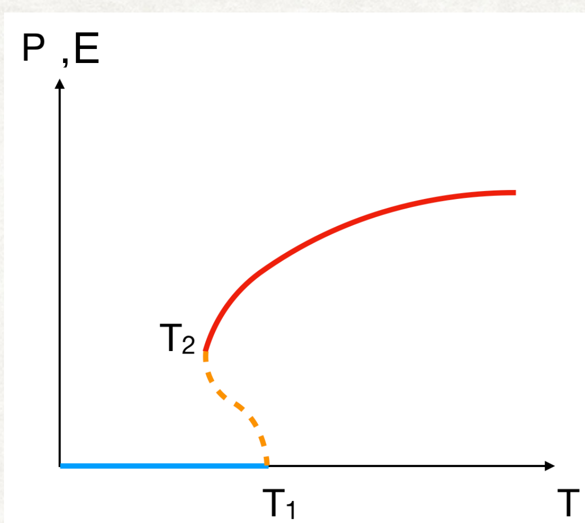
$$\boxed{\rho(\theta) = \frac{N - M}{N}\cdot\rho_{\mathrm{conf}} + \frac{M}{N}\rho_{\mathrm{GWW},M}(\theta)}$$ with $\frac{M}{N} = A$



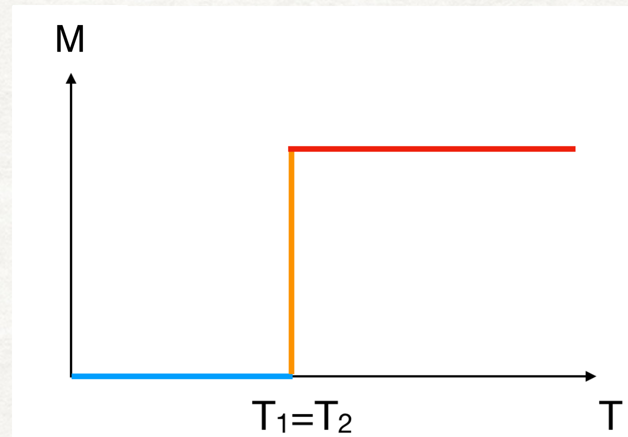
At ● ; GWW transition

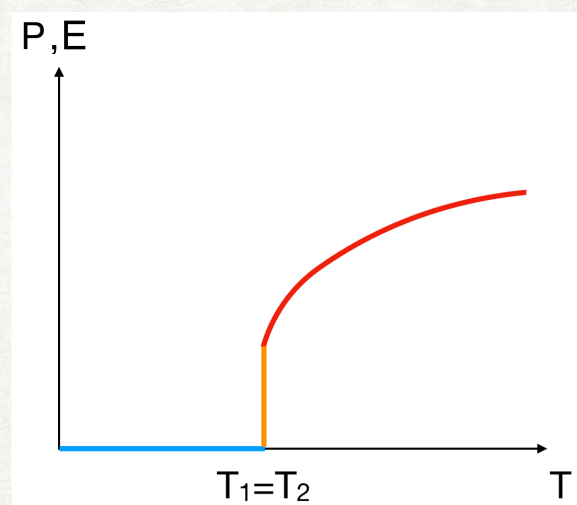
Phase structures revisited



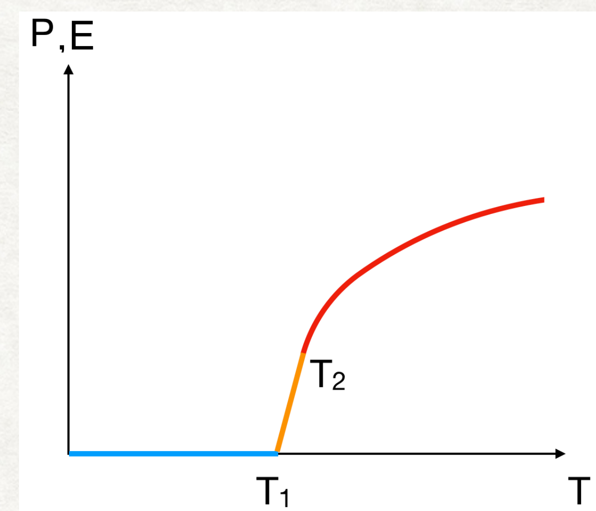


Strongly coupled 4d SYM





Weakly coupled 4d SYM



real-world QCD?

Weakly-coupled 4D Yang-Mills theories

- 4d $\mathrm{SU}(N_c)$ Yang-Mills theory with N_f fundamental matters on S^3 ;

[Schnitzer, (2004) / Hollowood & Myers, (2012)]

$N_f = 0$ [Hanada, Jevicki, Peng & Wintergerst, (2019) / Hanada & Robinson, (2019)]

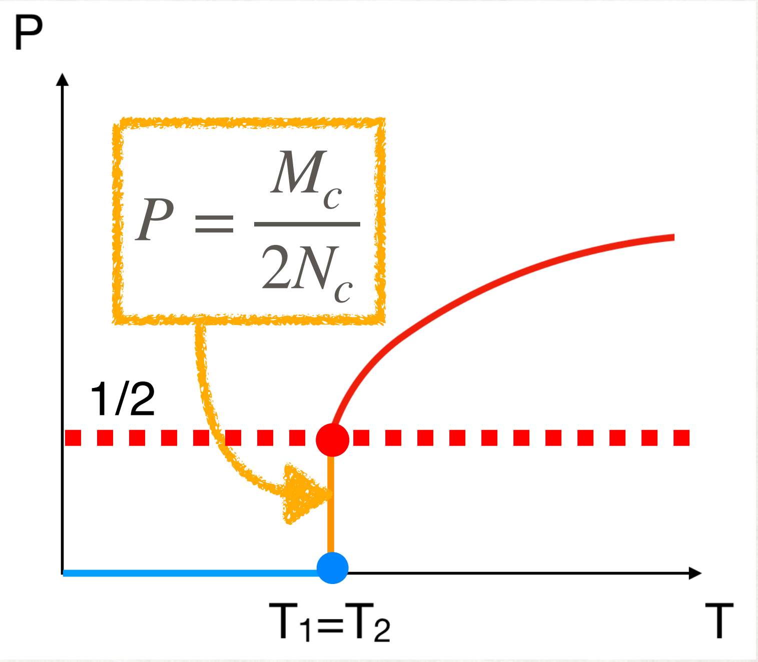
$$\beta F = N_c^2 \sum_n a_n(T) u_n^2 \qquad a_n(T) = \frac{1}{n}\left(1 - 2\sum_{l=1}^{\infty} l(l+2)\mathrm{e}^{-n\beta\frac{(l+1)}{R}}\right), \quad u_n = \frac{1}{N_c}\mathrm{tr}(U^n)$$

Searching the minimum of effective action;

- At low temperature, $a_n > 0, \quad \therefore u_n = 0$
- Phase transition happens at

$$a_1(T_c = T_1 = T_2) = 0$$

$$u_1 = P = \frac{1}{N_c}\,\mathrm{tr}\,U, \quad 0 \le P \le \frac{1}{2}$$



$$E = -\left.\frac{\partial(\beta F)}{\partial\beta}\right|_{T=T_c} = N_c^2 P^2 \times \left.\frac{\partial a_n}{\partial \beta}\right|_{T=T_c} = E_{\mathrm{GWW}}(M_c), \qquad S = \beta(E-F) = S_{\mathrm{GWW}}(M_c)$$

Weakly-coupled 4D Yang-Mills theories

- 4d $\mathrm{SU}(N_c)$ Yang-Mills theory with N_f fundamental matters on S^3 ;

[Schnitzer, (2004) / Hollowood & Myers, (2012)]

$N_f > 0$ [Hanada, Jevicki, Peng & Wintergerst, (2019) / Hanada & Robinson, (2019)]

$$\beta F = \sum_n \left(N_c^2 a_n(T) u_n^2 + N_c N_f b_n(T) u_n \right)$$

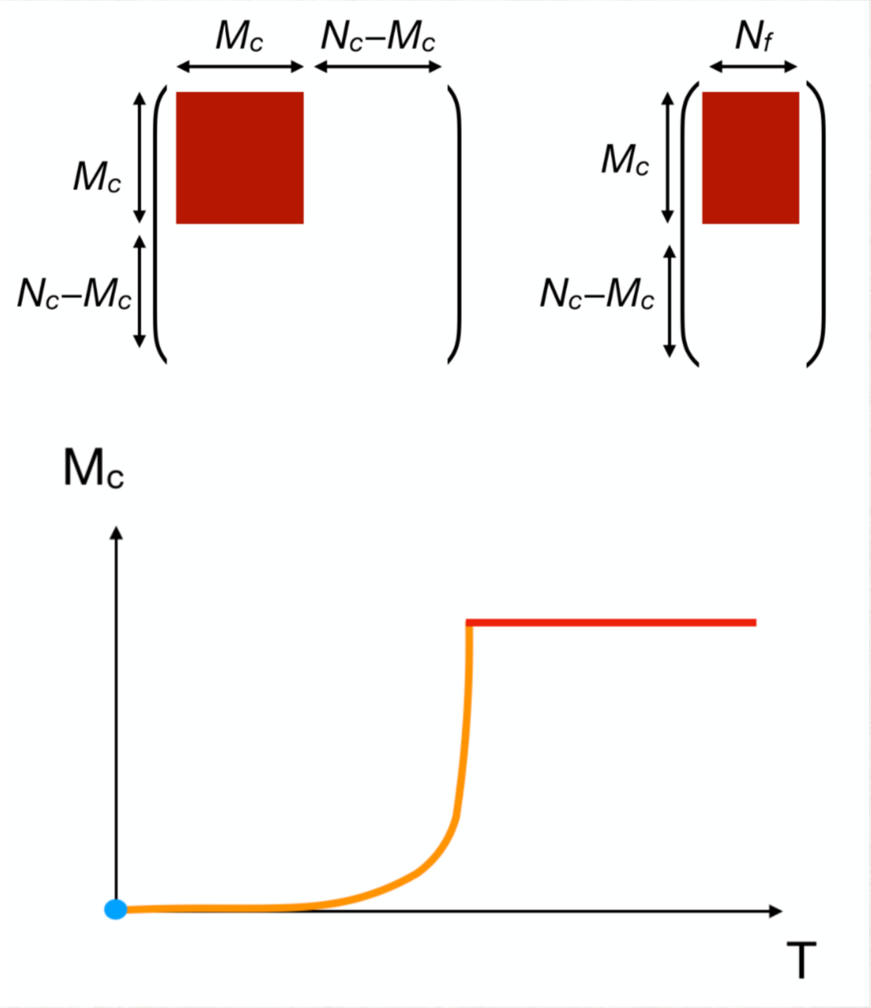
$$b_n(T) = \frac{(-1)^n}{n} \cdot 4 \sum_{l=1}^{\infty} l(l+1) \mathrm{e}^{-n\frac{\beta}{R}\sqrt{\left(l+\frac{1}{2}\right)^2 + m^2R^2}}$$

At $T = T_c = T_{\mathrm{GWW}}(M_c, N_f)$,

$$E = \sum_n \left(N_c^2 u_n^2 \frac{\partial a_n}{\partial \beta} + N_c N_f u_n \frac{\partial b_n}{\partial \beta} \right)$$

$$= \sum_n \left(M_c^2 \tilde{u}_n^2 \frac{\partial a_n}{\partial \beta} + M_c N_f \tilde{u}_n \frac{\partial b_n}{\partial \beta} \right) = E_{\mathrm{GWW}} \left(M_c, N_f \right)$$

$$S = S_{\mathrm{GWW}} \left(M_c, N_f \right) \qquad \text{with} \quad \tilde{u}_n = \frac{N_c}{M_c} u_n$$



3d free O(N) vector model (Nf=1 for simplicity)

Holographic dual to Vasiliev higher spin gravity

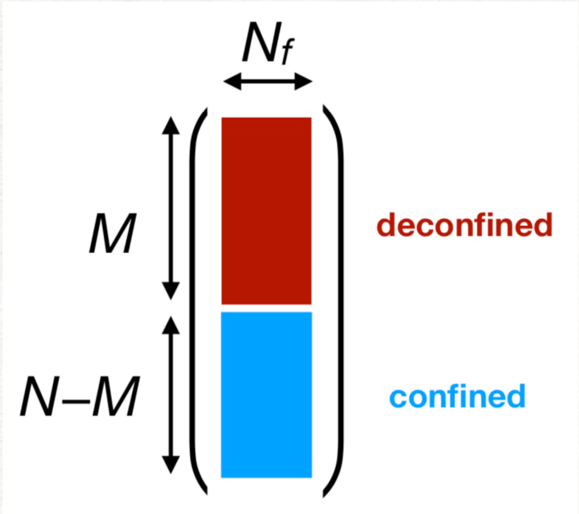
$$\rho(\theta) = \frac{1}{2\pi} - \frac{\pi b^2}{6} + \frac{b^2(|\theta| - \pi)^2}{2\pi}$$ [Shenker & Yin, (2011)]

$$b = \frac{T}{\sqrt{N}} \leq b_{\mathrm{GWW}} = \frac{\sqrt{3}}{\pi}$$

$$= \frac{1}{2\pi}\left(1 - \frac{b^2}{b_{\mathrm{GWW}}^2}\right) + \frac{b^2}{b_{\mathrm{GWW}}^2} \cdot \underline{\frac{b_{\mathrm{GWW}}^2(|\theta| - \pi)^2}{2\pi}}$$

$$= \rho(\theta; b_{\mathrm{GWW}}) = \rho_{\mathrm{GWW},M}(\theta)$$

with $\frac{M}{N} = \frac{b^2}{b_{\mathrm{GWW}}^2}$, $T = b\sqrt{N} = b_{\mathrm{GWW}}\sqrt{M}$



$$\boxed{\rho(\theta) = \frac{N-M}{N} \cdot \frac{1}{2\pi} + \frac{M}{N}\rho_{\mathrm{GWW},M}(\theta)}$$

At $1 \ll T \leq T_{\mathrm{GWW}}$,

$$E \approx 16\zeta(5)T^5 \sim M^{5/2}, \quad S \approx 20\zeta(5)T^4 \sim M^2$$

[Hanada, Jevicki, Peng & Wintergerst, (2019)]

Numerical check of partial deconfinement

- The bosonic part of **plane wave matrix model** (PWMM or BMN matrix model)
  = the mass deform. of (0+1)d SYM / Matrix quantum mechanics.

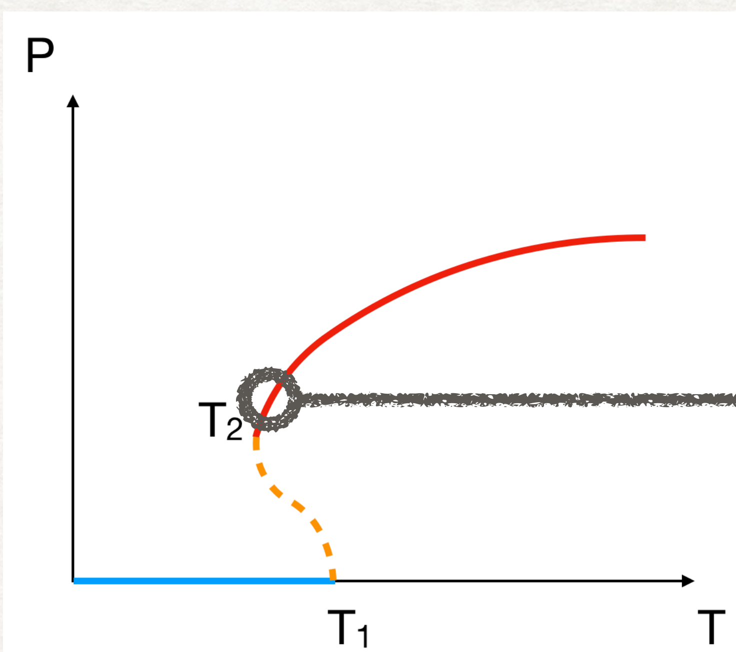
$$L = N\,\mathrm{Tr}\left(\frac{1}{2}\sum_{I=1}^{9}\left(D_t X_I\right)^2 + \frac{1}{4}\sum_{I,J=1}^{9}\left[X_I, X_J\right]^2 - \frac{\mu^2}{2}\sum_{i=1}^{3} X_i^2 - \frac{\mu^2}{8}\sum_{a=4}^{9} X_a^2 - \mathrm{i}\sum_{i,j,k=1}^{3}\mu\epsilon^{ijk}X_iX_jX_k\right)$$

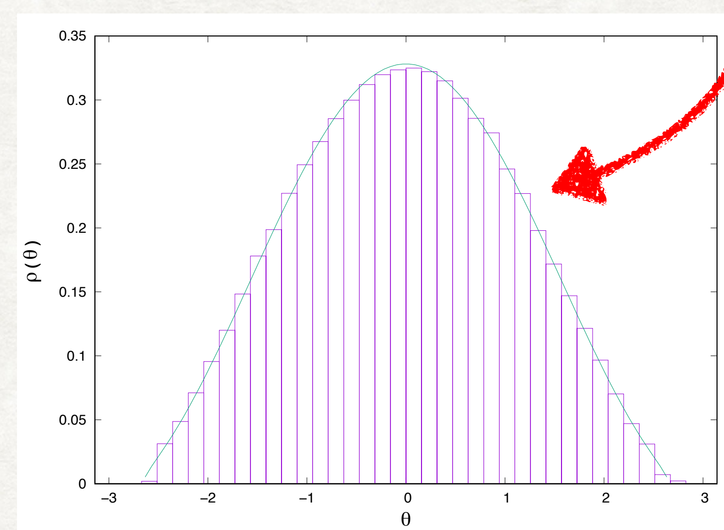
Check by Monte Carlo simulation;

- Hysteresis $(T_2 \leq T_1)$
- Phase distribution

GWW ansatz

$$\rho(\theta) = \begin{cases} \dfrac{1}{2\pi} \\ \dfrac{1}{2\pi}(1 + A\cos\theta) \\ \boxed{\dfrac{A}{\pi}\cos\dfrac{\theta}{2}\sqrt{\dfrac{1}{A} - \sin^2\dfrac{\theta}{2}}} \end{cases}$$





$$\begin{pmatrix} N = 128 \\ \mu = \ \ 5 \end{pmatrix}$$

- The bosonic part of **BFSS matrix model** (μ = 0) had been studied recently by [Bergner, Bodendorfer, Hanada, Rinaldi, Schafer & Vranas, (2019)]

gauged Gaussian matrix model

[Hanada, Jevicki, Peng & Wintergerst, (2019)]

[Bergner, Bodendorfer, Hanada, Rinaldi & HW, (in progress)]

$$S = N \sum_{I=1}^{D} \int_0^\beta \mathrm{d}t\, \mathrm{Tr}\left(\frac{1}{2}(D_t X_I)^2 + \frac{1}{2} X_I^2 \right)$$

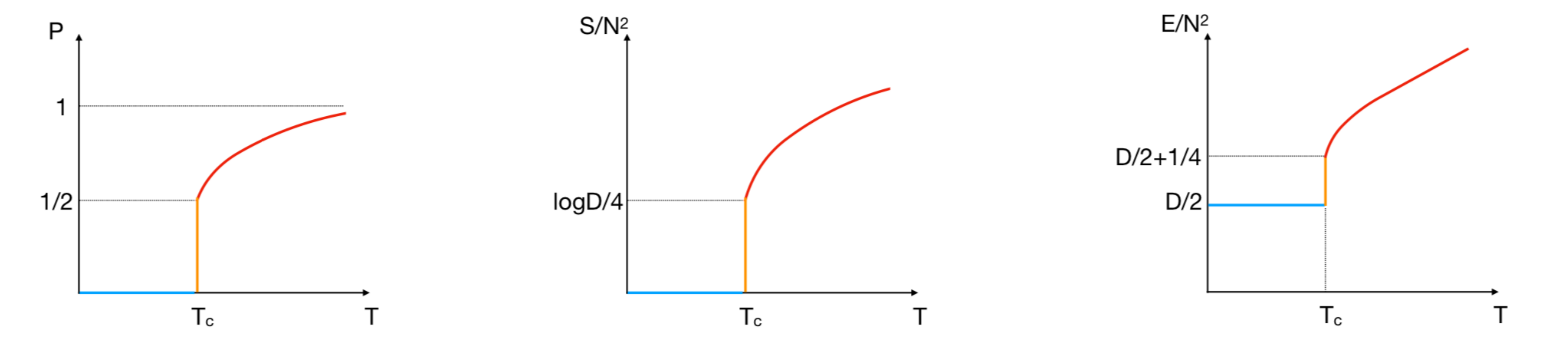
$$D_t X_I = \partial_t X_I - \mathrm{i}\left[A_t, X_I\right]$$

Physical quantities are solvable exactly;

$$E(T = T_c) = \frac{DN^2}{2} + N^2 P^2, \quad S(T = T_c) = N^2 P^2 \ln D$$

At $T = T_c = 1/\ln D$,

$$P = \frac{M}{2N} \qquad S_{\mathrm{GWW}}(M) = \frac{M^2}{4} \ln D \qquad E_{\mathrm{GWW}}(M) = \frac{M^2}{4}$$



$$\rho_{\mathrm{GWW},N}(\theta) = \frac{1}{2\pi}(1 + \cos\theta)$$

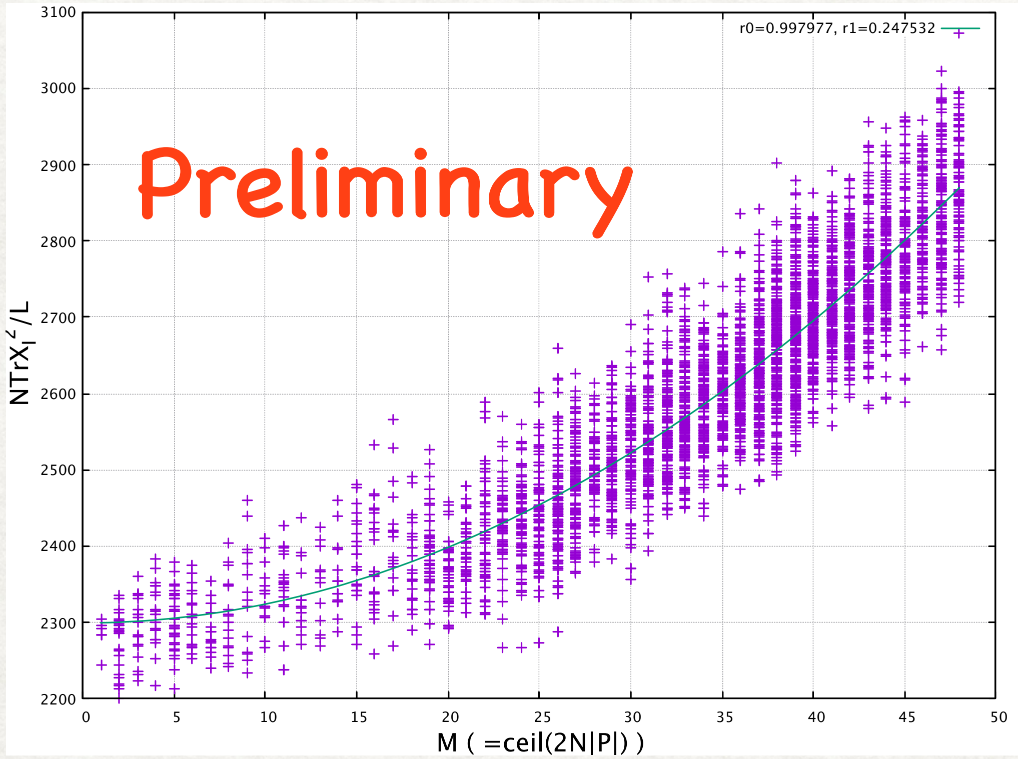
up to 1/N corrections.

Partial deconfinement and gauge fixing

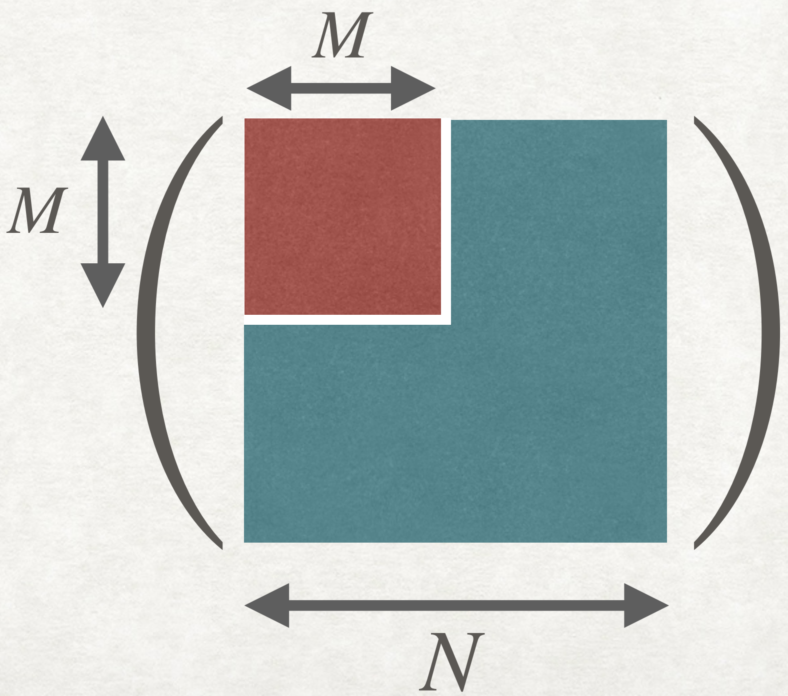
[Hanada, Jevicki, Peng & Wintergerst, (2019)]
[Bergner, Bodendorfer, Hanada, Rinaldi & HW, (in progress)]

$$L = N\,\mathrm{Tr}\left(\frac{1}{2}\sum_{I=1}^{9}(D_tX_I)^2 + \frac{1}{4}\sum_{I,J=1}^{9}[X_I, X_J]^2\right) \Big/ \; S = N\sum_{I=1}^{D}\int_0^{\beta} \mathrm{d}t\,\mathrm{Tr}\left(\frac{1}{2}(D_tX_I)^2 + \frac{1}{2}X_I^2\right)$$

- Determine M as $M = \lfloor 2NP \rfloor$ config. by confing.
- Plot and fit $\dfrac{N}{\beta}\displaystyle\int \mathrm{d}t\,\mathrm{Tr}\,X_I^2$ as $r_0N^2 + r_1M^2$,etc. to use as input parameter



Result of gauged Gaussian MM
with N=48, D=2, L=8 @ T=Tc

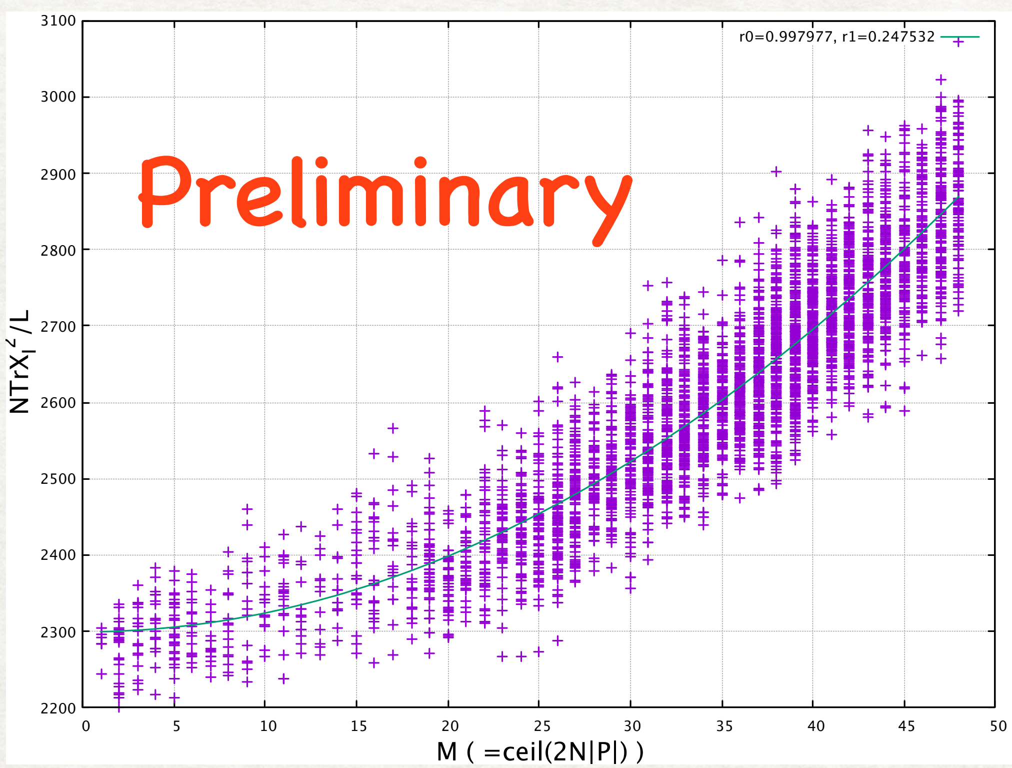


Partial deconfinement and gauge fixing

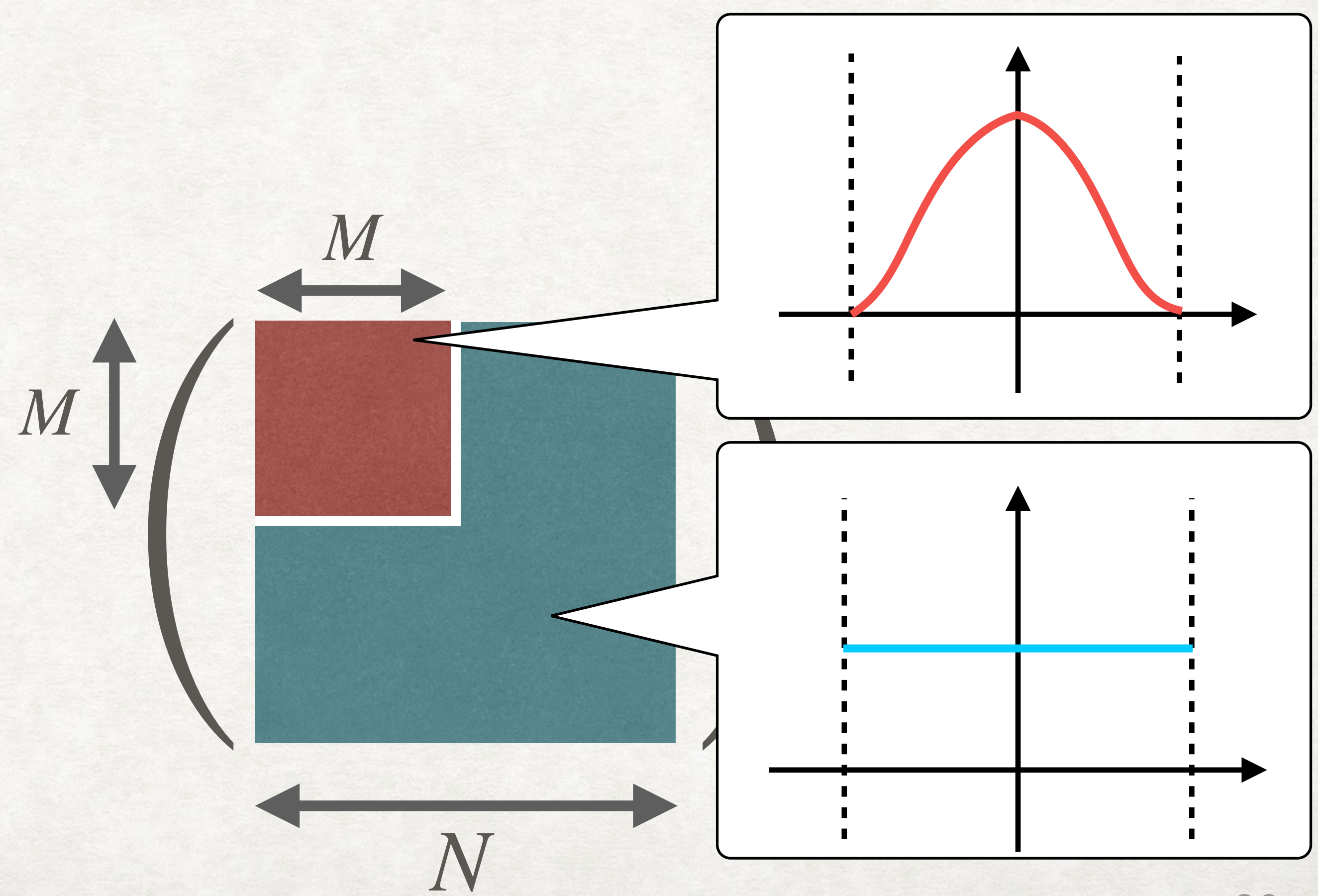
[Hanada, Jevicki, Peng & Wintergerst, (2019)]
[Bergner, Bodendorfer, Hanada, Rinaldi & HW, (in progress)]

$$L = N\,\mathrm{Tr}\left(\frac{1}{2}\sum_{I=1}^{9}(D_t X_I)^2 + \frac{1}{4}\sum_{I,J=1}^{9}[X_I, X_J]^2\right) \Big/ \; S = N\sum_{I=1}^{D}\int_0^{\beta} dt\,\mathrm{Tr}\left(\frac{1}{2}(D_t X_I)^2 + \frac{1}{2}X_I^2\right)$$

- Determine M as $M = \lfloor 2NP \rfloor$ config. by confing.
- Plot and fit $\frac{N}{\beta}\int dt\,\mathrm{Tr}\,X_I^2$ as $r_0 N^2 + r_1 M^2$,etc. to use as input parameter



Result of gauged Gaussian MM
with N=48, D=2, L=8 @ T=Tc



Other applications

- **Applying the same techniques to Wilson loop of spacial direction**
    - cf. black hole/black string topology change <=> 2D SYM
- **Prospects to Real-world QCD**
    - Neither center symmetry nor chiral symmetry is exact
    - How much contributes finite Nc correction?
    - Consequences of partial deconfinement?
        - Hadron/QGP phase transition is “crossover”.
        - SU(4) enhanced symmetry reported by Glozman et al.

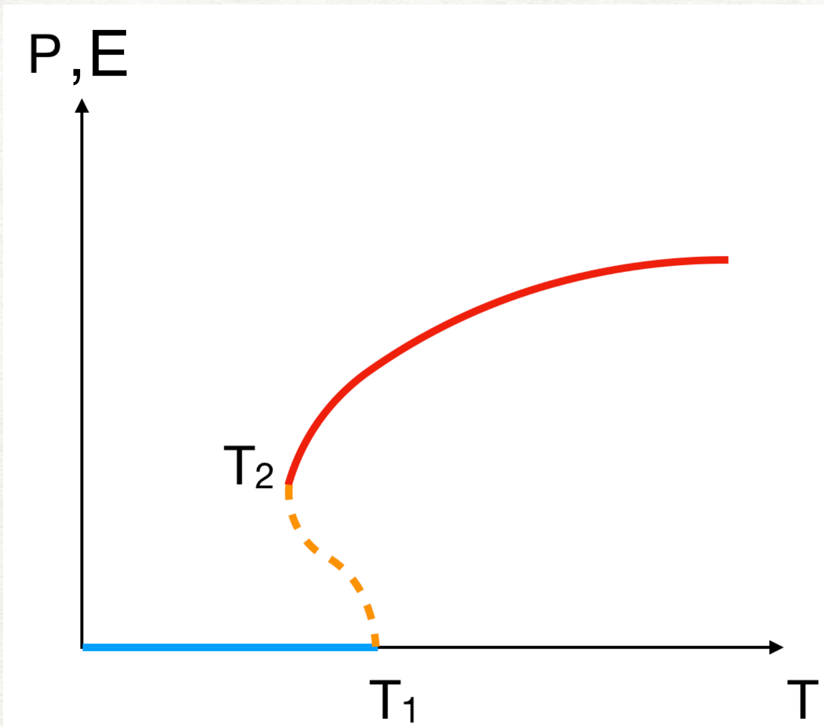
Summary & Discussion

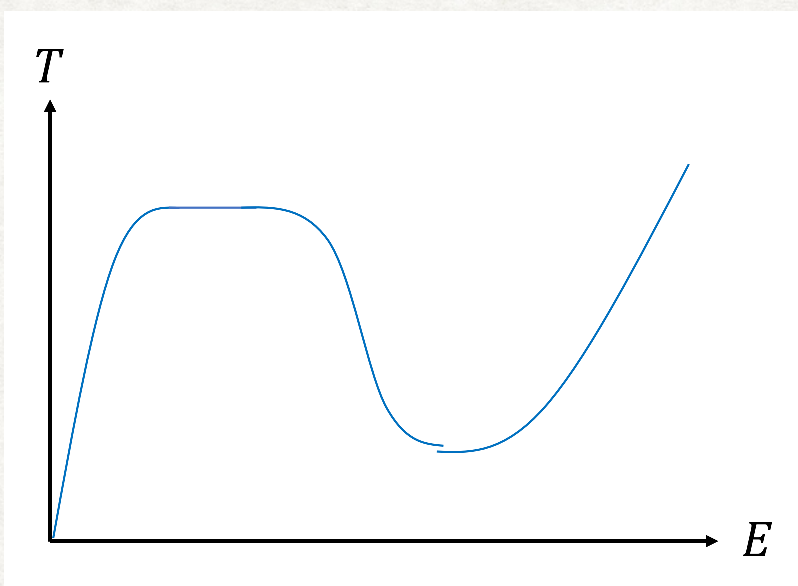
- We proposed the partial deconfinement in which only SU(M) sector of SU(N) theory deconfines.
  - It’s relating to small BH in dual gravity via holography
- Several examples of partial deconfinement are found in large-Nc gauge theories.
  - Needs to more analysis to strong-coupling theories.
  - Is this applicable to real QCD?

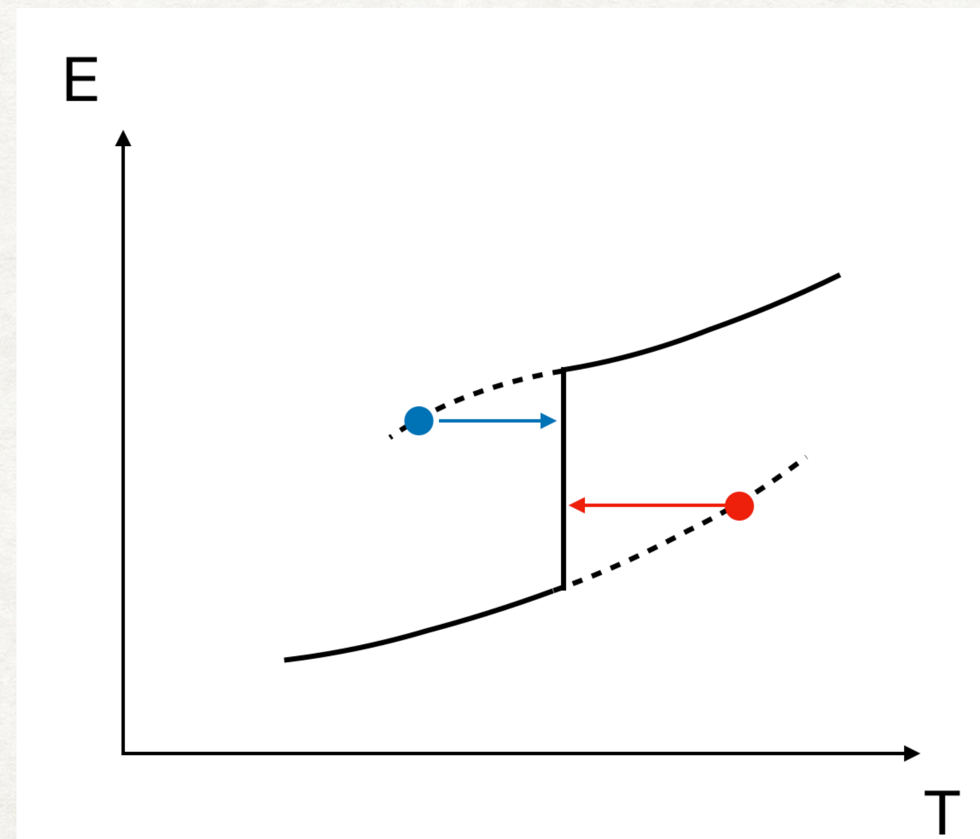
Backup Slides

Yang-Mills and water

[Hanada, Ishiki & HW, (2018)/Hanada & Robinson, (2019)]







Weakly-coupled 4D Yang-Mills theories

- 4d $\mathrm{SU}(N)$ Yang-Mills theory with adjoint matters on S^3 ;

[Sundborg, (2000)/Aharony, Marsano, Minwalla, Papadodimas & Raamsdonk, (2003)]

At zero 't Hooft coupling;

$$Z(x) = \int [d\theta_i] \ \exp\Big(-\sum_{i\neq j} V(\theta_i - \theta_j)\Big) \qquad V(\theta) = \ln(2) + \sum_{n=1}^{\infty} \frac{1}{n}\big(1 - z_B(x^n) - (-1)^{n+1} z_F(x^n)\big)\cos(n\theta)$$

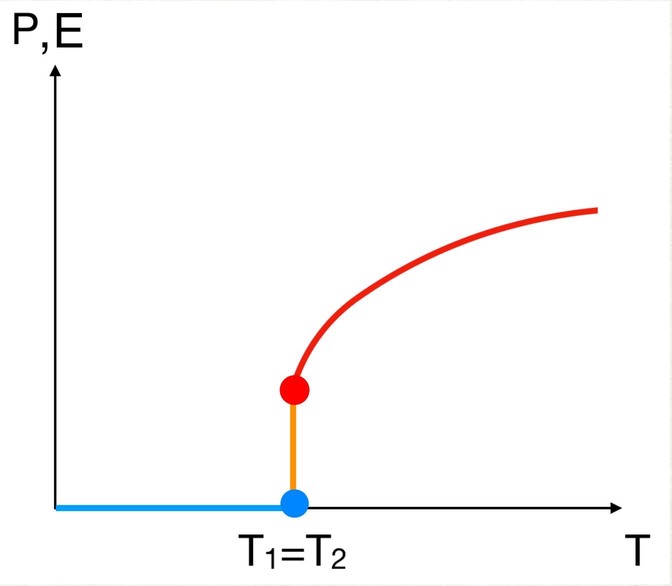
$$x \equiv e^{-\beta}$$

At low temperature;

$$S_{\mathrm{eff}}\left[\rho(\theta)\right] = N^2 \int d\theta_1 \int d\theta_2 \rho(\theta_1)\rho(\theta_2)V(\theta_1 - \theta_2) = \frac{N^2}{2\pi}\sum_{n=1} |\rho_n|^2 V_n(T)$$

$$\rho_n \equiv \int d\theta \rho(\theta)\cos(n\theta)$$

$$V_n \equiv \int d\theta V(\theta)\cos(n\theta) = \frac{2\pi}{n}\left(1 - z_B(x^n) - (-1)^{n+1} z_F(x^n)\right)$$



At ● ; GWW transition

$$T < T_H = -1/\ln x_H, \quad z_B(x^n) + (-1)^{n+1} z_F(x^n) < 1, \quad \rightarrow \rho_n = 0$$

$$T = T_H = -1/\ln x_H, \quad z(x_H) \equiv z_B(x_H) + z_F(x_H) = 1, \quad \rightarrow \rho_1 \neq 0$$

$$\Omega(E) \propto E^0 e^{\beta_H E}$$

; Hagedorn growth

gauged Gaussian matrix model

[Hanada, Jevicki, Peng & Wintergerst, (2019)]
[Bergner, Bodendorfer, Hanada, Rinaldi & HW, (in progress)]

$$S = N \sum_{I=1}^{D} \int_0^\beta \mathrm{d}t \, \mathrm{Tr} \left(\frac{1}{2}(D_t X_I)^2 + \frac{1}{2} X_I^2 \right)$$

$$D_t X_I = \partial_t X_I - \mathrm{i} \left[A_t, X_I \right]$$

Physical quantities are solvable exactly;

$$\begin{aligned}
\beta F &= -\ln Z(\beta) \\
&= \frac{N^2 D}{2} \ln \left(\det(-D_0^2 + 1) \right) - \underline{\frac{N^2}{2} \ln \left(\det(-D_0^2) \right)} \\
&= \frac{N^2 D \beta}{2} + N^2 \sum_{n=1}^{\infty} \frac{1 - D x^n}{n} |u_n|^2
\end{aligned}$$

Faddeev-Popov term
(static diagonal gauge)

$$x \equiv e^{-\beta}$$

At $T = T_c, \quad 0 \le |u_1| \le \frac{1}{2}, \quad |u_n| = 0 \ (n \ge 2)$

$$1 - D x_c = 1 - D \mathrm{e}^{-1/T_c} = 0, \qquad \therefore \ T_c = \frac{1}{\ln D}$$

Relation to gauge symmetry breaking

[Hanada, Jevicki, Peng & Wintergerst, (2019)]

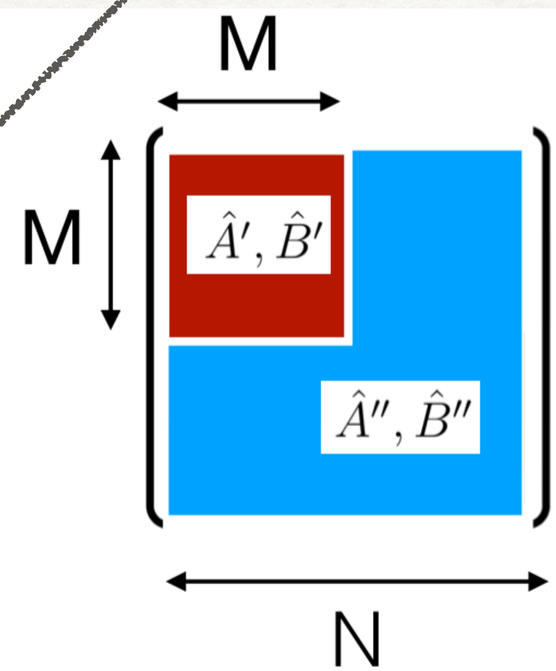
In Hamiltonian formalism of gaugeed Gaussian matrix model, ($D = 2$ for simplicity)

$$\hat{H} = \frac{1}{2}\,\mathrm{Tr}\left(\underline{\hat{P}_X^2 + \hat{X}^2} + \underline{\hat{P}_Y^2 + \hat{Y}^2}\right)$$

(From Hanada-san's talk)

$\hat{A}, \hat{A}^\dagger$

$\hat{B}, \hat{B}^\dagger$



not SU(N)-invariant

$$|E; \mathrm{SU}(M)\rangle = \mathrm{Tr}\left(\hat{A}'^\dagger \hat{A}'^\dagger \hat{B}'^\dagger \hat{A}'^\dagger \cdots\right)|0\rangle$$

At weak coupling, this is an energy eigenstate.

$$S = S_{\mathrm{GWW}}(M)$$

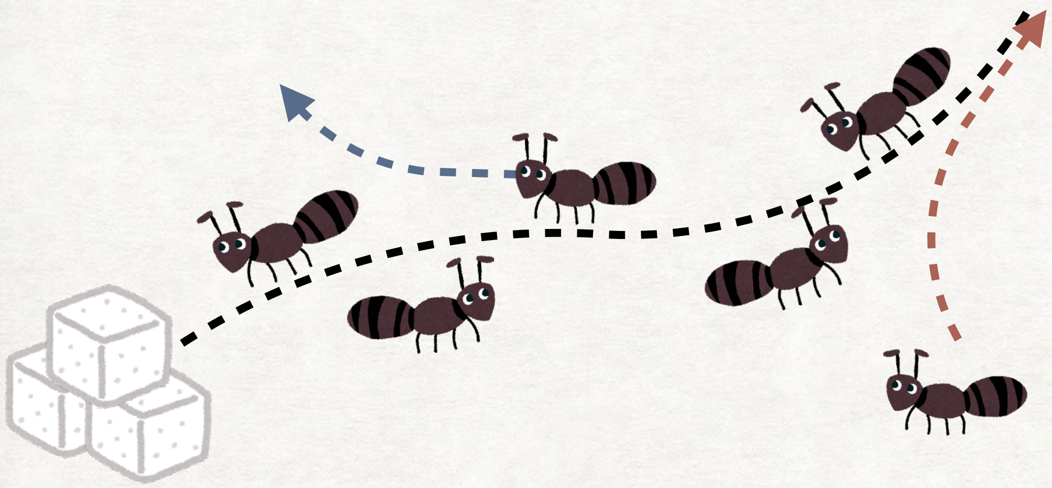
one-to-one correspondence

SU(N)-invariant

This is also an energy eigenstate.

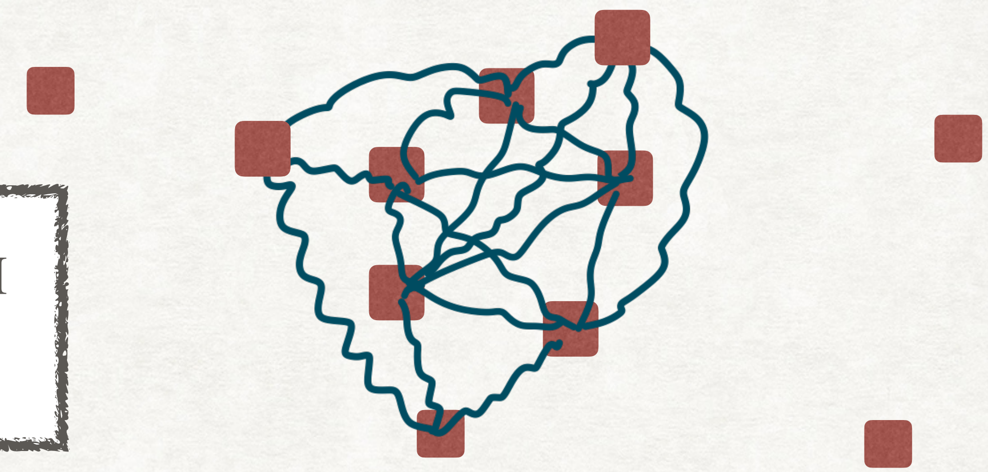
$$|E\rangle_{\mathrm{inv}} \equiv \mathcal{N}^{-1/2}\int dU\,\mathcal{U}\left(|E; \mathrm{SU}(M)\rangle\right)$$

Ant model and partial deconfinement



$$N_{\mathrm{trail}} \leftrightarrow N_{\mathrm{BH}}$$

$$p \leftrightarrow T$$



Ant trail : ants bound by pheromone

p : pheromone from each ant

BH : D-branes bound by open strings

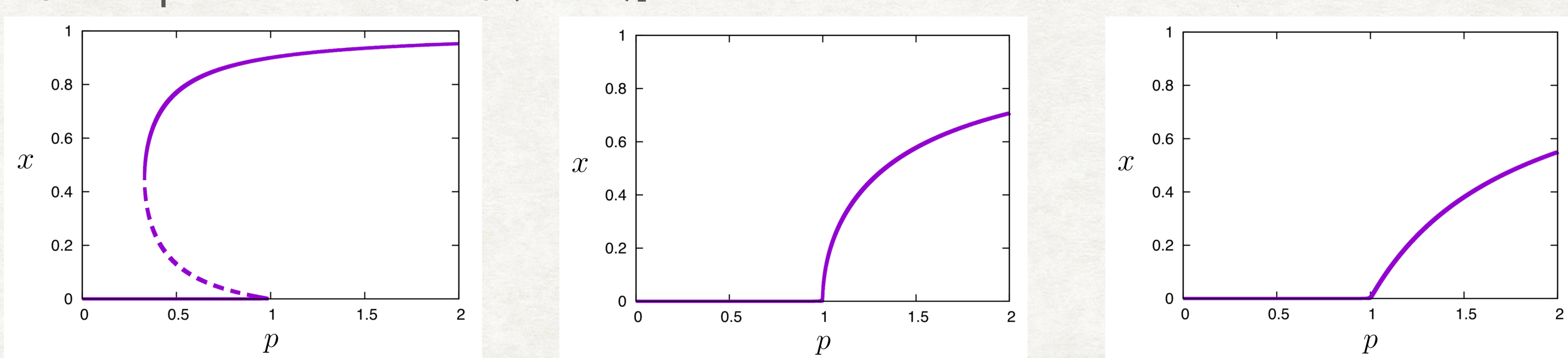
T : index for excitation of open string

$$\frac{\mathrm{d}N_{\mathrm{trail}}}{\mathrm{d}t} = \underbrace{(\alpha + pN_{\mathrm{trail}})(N - N_{\mathrm{trail}})}_{\text{Inflow effect}} - \underbrace{\frac{sN_{\mathrm{trail}}}{s + N_{\mathrm{trail}}}}_{\text{Outflow effect}}$$

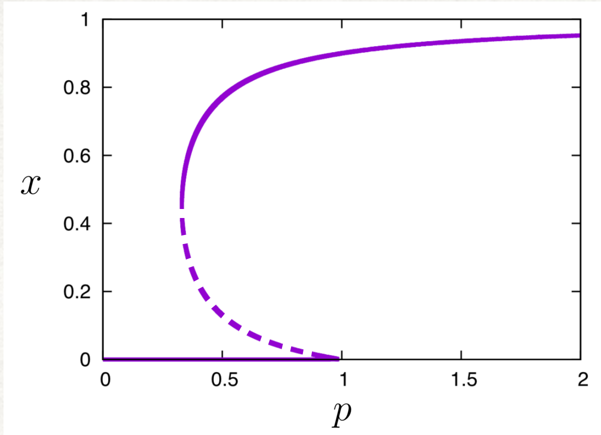
[Beekman, Sumpter & Ratnieks, (2001)]

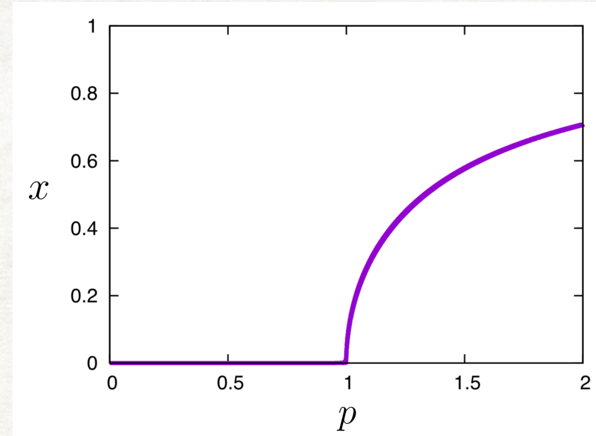
saddle point

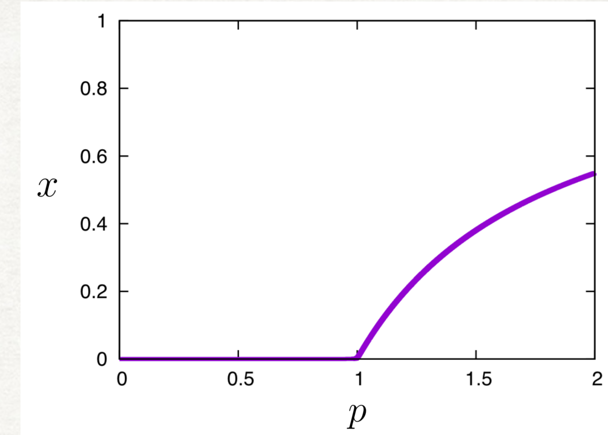
$$\frac{\mathrm{d}N_{\mathrm{trail}}}{\mathrm{d}t} = 0, \quad x \equiv \frac{N_{\mathrm{trail}}}{N}$$



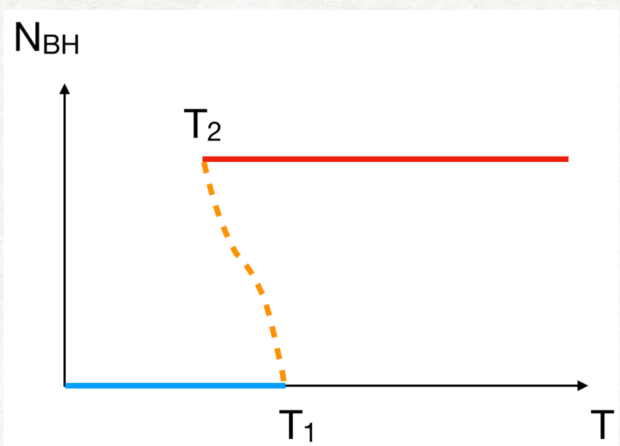
small s ⟷ large s

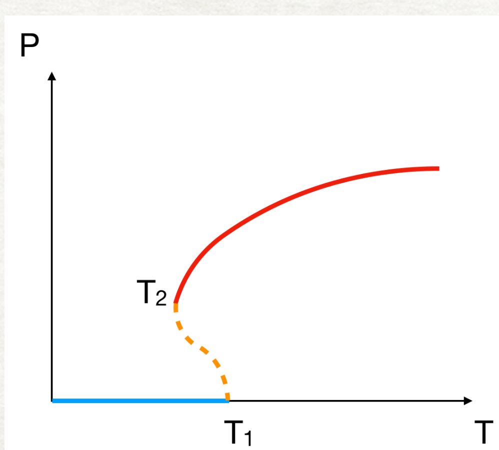




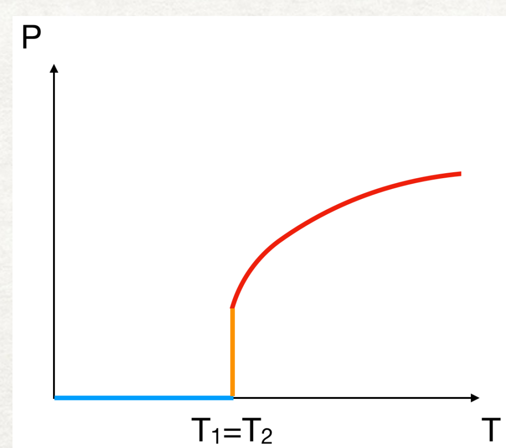


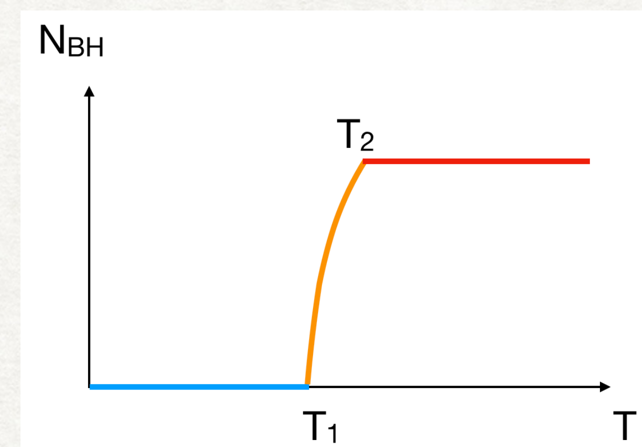
Strongly coupled 4d SYM

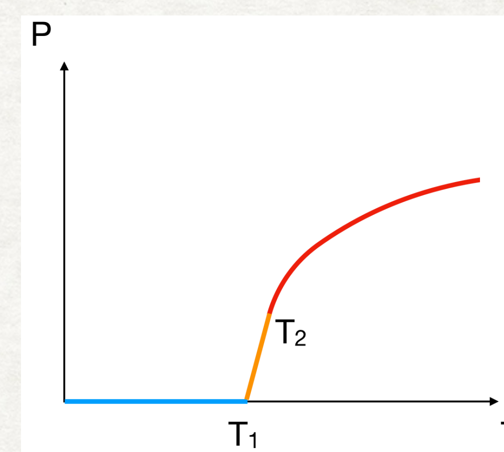




Weakly coupled 4d SYM



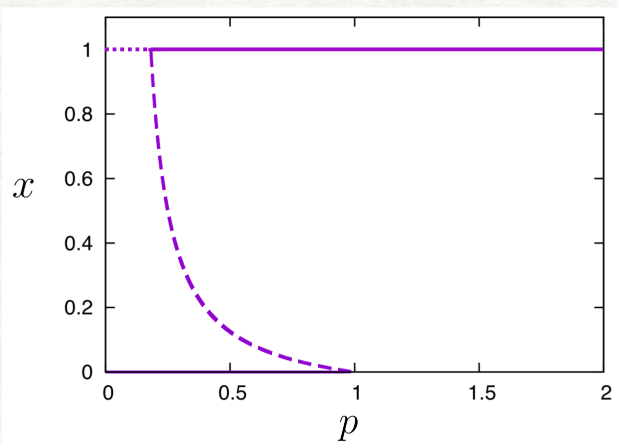


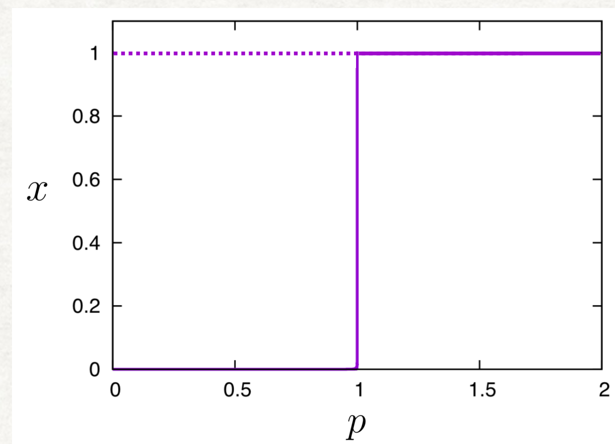


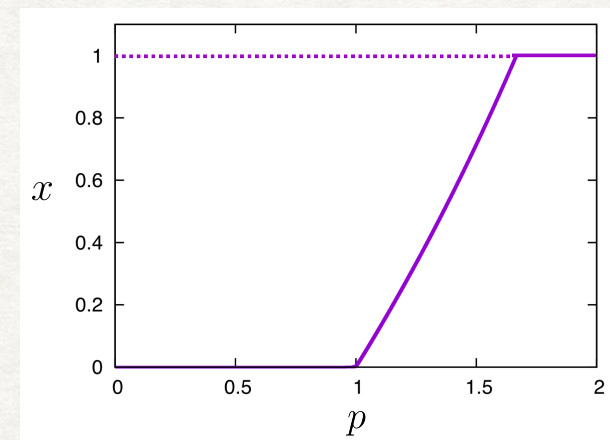
Modified ant model (Ant-Man model)

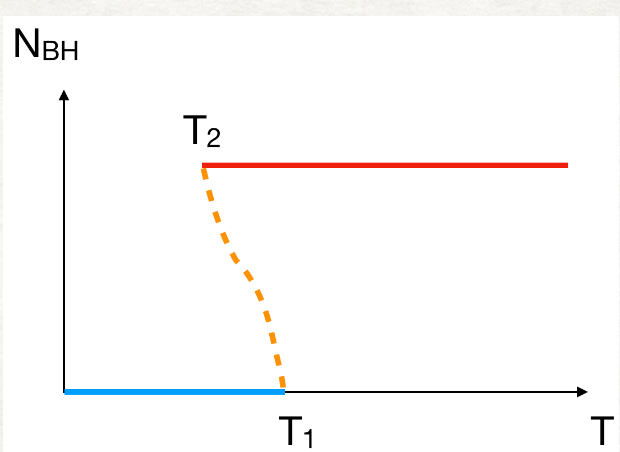
$$\frac{dx}{dt} = (\tilde{\alpha} + px)(1 - x) - \frac{\tilde{s}x}{\tilde{s} + x} \cdot \underline{(1 - x^2)} \qquad x \equiv \frac{N_{\mathrm{trail}}}{N}, \quad \tilde{s} = \frac{s}{N}, \quad \tilde{\alpha} = \frac{\alpha}{N}$$

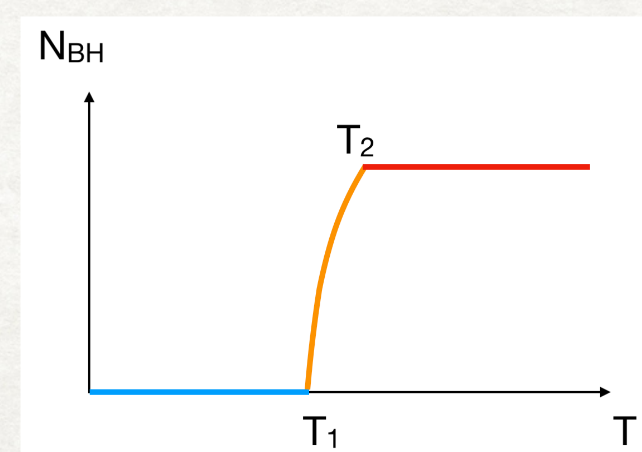
saddle point











Black hole thermodynamics in AdS space

$$S_{\rm BH} = \frac{k_{\rm B}}{4}\frac{A_{\rm BH}}{l_{\rm pl}^2} = \frac{A_{\rm BH}}{4G}\ , \qquad dM = \frac{T_{\rm BH}}{4G}dA_{\rm BH}\ , \quad T_{\rm BH} = \frac{f'(r_0)}{4\pi}\ \left(ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + \cdots\right)$$

AdS-Schwarzschild BH with horizon at $(r = r_+)$ (dual of super Yang-Mills theory)

$$ds^2 = -f(r, R_{\rm AdS})dt^2 + \frac{dr^2}{f(r, R_{\rm AdS})} + r^2 d\Omega_3^2 + R_{\rm AdS}^2 d\Omega_5^2 \qquad f(r, R_{\rm AdS}) = \frac{r^2}{R_{\rm AdS}^2} + 1 - \frac{r_0^4}{R_{\rm AdS}^2 r^2}$$

$$S_{\rm AdS-BH} = \frac{r_+^3 \pi^5 R_{\rm AdS}^5}{2G_{10,\rm N}}$$

$$T_{\rm AdS-BH} = \frac{2r_+^2 + R_{\rm AdS}^2}{2\pi R_{\rm AdS}^2 r_+}$$

$$E_{\rm AdS-BH} = \frac{3\pi^4 R_{\rm AdS}^3}{8G_{10,\rm N}}\left(r_+^4 + R_{\rm AdS}^2 r_+^2\right)$$