DM heating vs. rotochemical heating in old neutron stars

Keisuke Yanagi

University of Tokyo

Based on KY, Nagata Hamaguchi, accepted by MNRAS [arXiv:1905.02991]
Hamaguchi, Nagata, KY, Phys.Lett. B795 (2019) 484-489

Seminar @ Osaka University
Jan. 14, 2020

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This is astrophysics journal!

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Overview

- DM heating
  - DM accretion heats up old NS
  - NS surface temperature measurement can probe DM

- Rotochemical heating
  - it occurs w/o any new or exotic physics (induced by NS rotation)
  - it explains observed warm NSs

- (DM heating) > (rotochemical heating)?
  - $P_0 \lesssim 10 - 100\,\mathrm{ms}$: (DM heating) < (rotochemical heating)
  - $P_0 \gtrsim 10 - 100\,\mathrm{ms}$: (DM heating) > (rotochemical heating)
  - old ordinary pulsar is suitable target

Outline

1. DM heating of NS
2. Standard cooling of NS
3. Rotochemical heating
4. DM heating vs. Rotochemical heating

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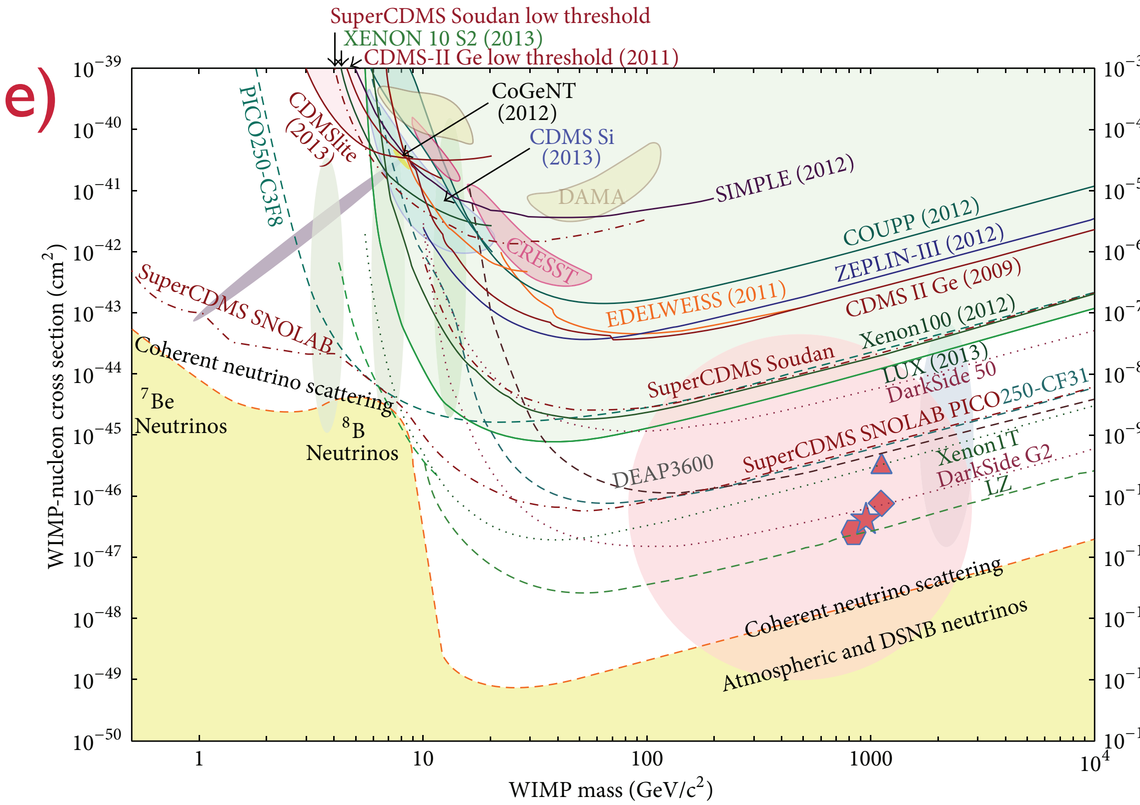
Dark matter

- Dark matter: 25 % of the energy density of the universe

- Particle DM candidate
    - WIMP (weakly interacting massive particle)
    - SIMP
    - FIMP
    - Axion
    - …



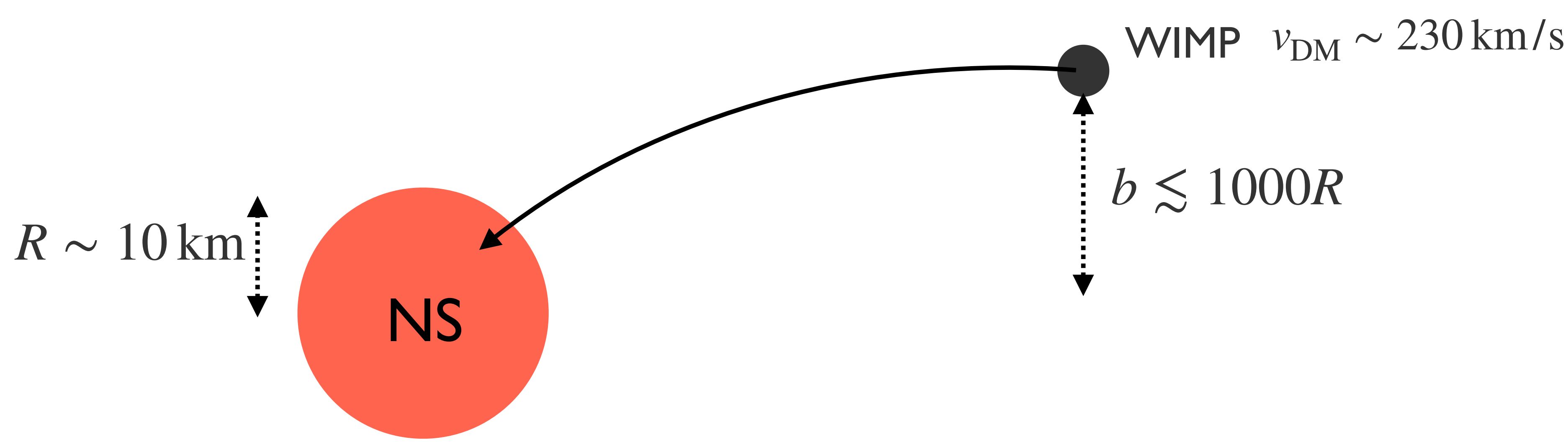
- Direct detection experiment
    - No signal
    - Stringent limit on DM-nucleon cross section
    - Recent trend: DM model which is difficult to probe in direct detection
    - How can we probe such DM models?

Dark matters accrete in neutron stars

We can probe the DM signature in Neutron stars [Kouvaris (2007)]

- DMs accrete in NS by gravity
- If $\sigma_n \gtrsim 10^{-45}\,\mathrm{cm}^2$, (DM mean free path) < (NS radius)

  → DM loses initial kinetic energy, trapped in NS gravitational potential



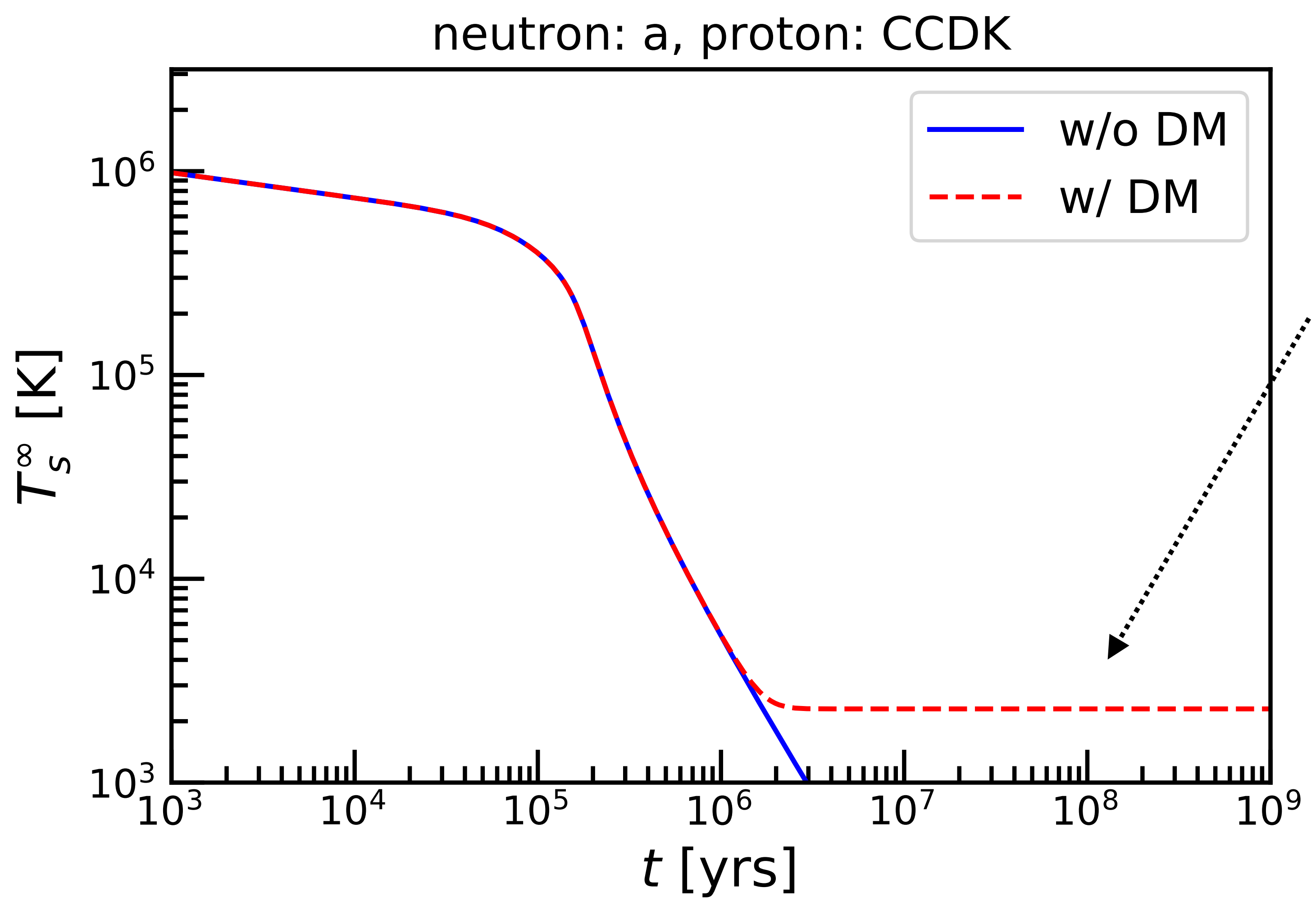
DM kinetic/mass energy → Heat in a NS

Heating rate: $L_{\rm DM} \sim \rho_{\rm DM} v_{\rm DM} \pi b_{\rm max}^2$ w/ $b_{\rm max} \sim \sqrt{2GMR/v_{\rm DM}^2}$

Prediction of dark matter heating

$T_s^\infty \sim 3000\,\mathrm{K}$ at $t \gtrsim 10\,\mathrm{Myr}$ is a signal of DM!

[Kouvaris (2007); Baryakhtar et al. (2017)]



Balance between photon emission and DM heating

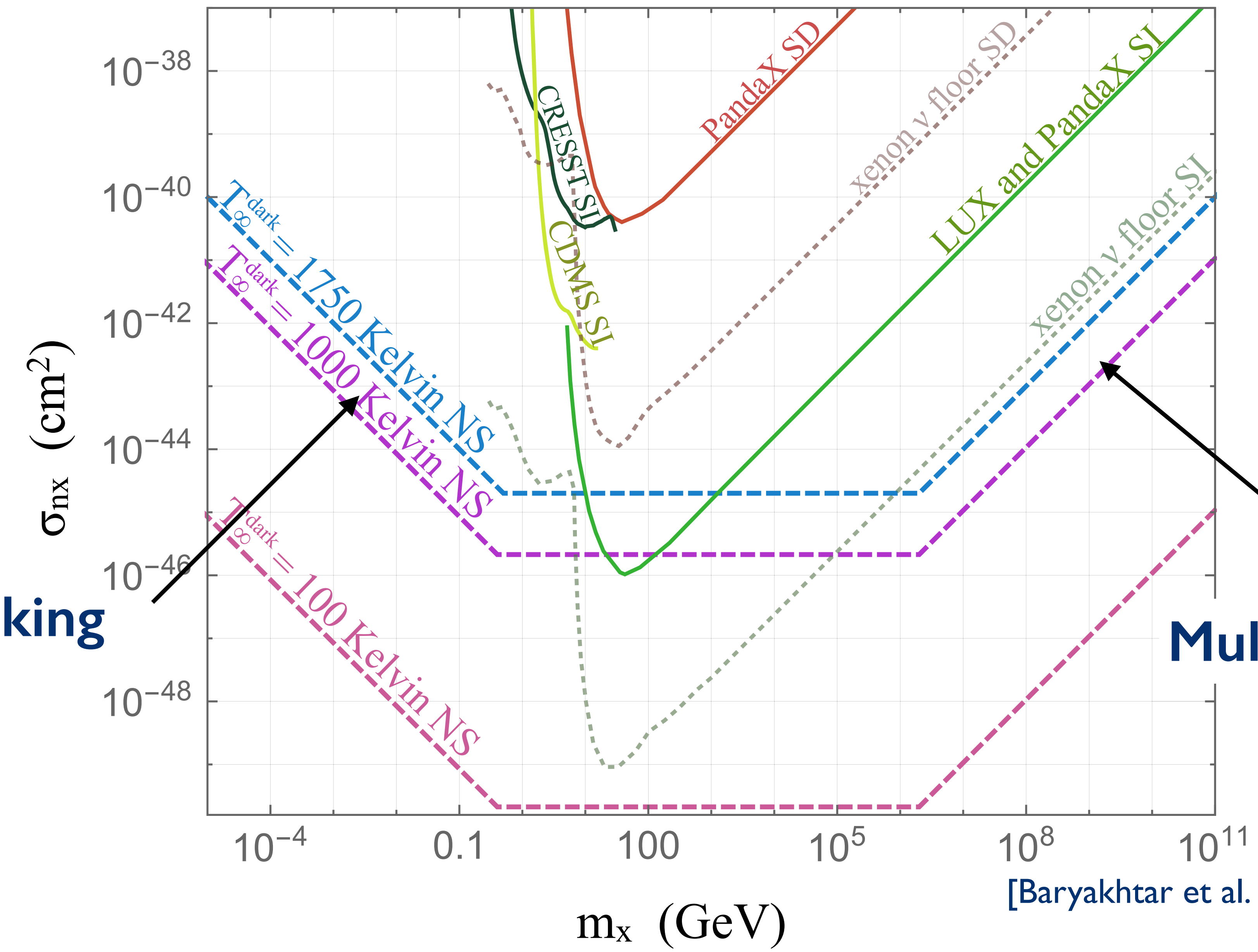
$$4\pi R^2 \sigma_B T_s^4 = 2\pi G M R \rho_{\mathrm{DM}} / v_{\mathrm{DM}}$$

Surface temperature of old NSs can probe/constrain DM models!

Prospects

Constraints on DM-neutron cross section

Particularly sensitive to m = 1 GeV - 1 PeV



Pauli blocking

Multiple scattering

[Baryakhtar et al. (2017)]

Advantages over the terrestrial experiments

- Large DM velocity on NS surface

$$v_{\rm esc} = \sqrt{\frac{2GM}{R}} \sim 0.6c$$ $(M = 1.4\,M_\odot$ and $R = 10\,\mathrm{km})$

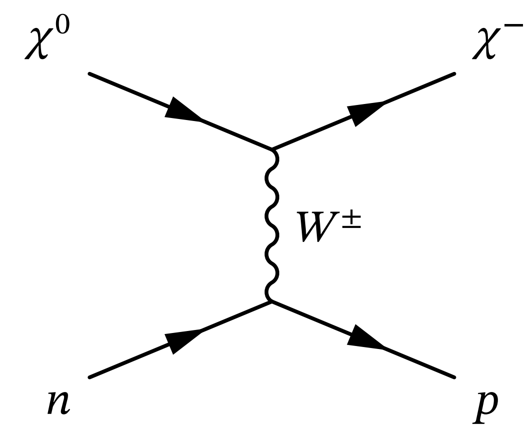
- Inelastic scattering of electroweak DM $(\tilde{H},\tilde{W}\,,\ldots)$ [Baryakhtar et al. (2017); Bell et al. (2018)]

$$\Delta E = \frac{m_n m_\chi^2 \gamma^2}{m_n^2 + m_\chi^2 + 2\gamma m_n m_\chi} v_{\rm esc}^2 (1 - \cos\theta_{\rm CM}) \sim \mathcal{O}(1)\,\mathrm{GeV}$$

c.f. $\Delta E \sim 100\,\mathrm{keV}$ on the earth

- Velocity suppressed scattering [Raj et al. (2018)]

- Spin-dependent scattering



- No detector threshold for light DM

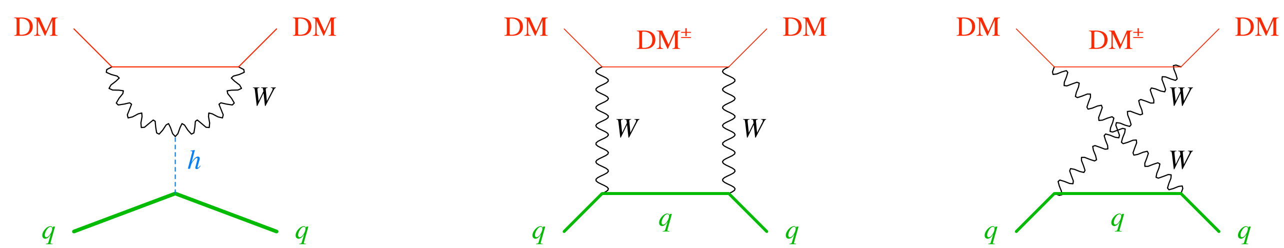
- No limitation from neutrino floor

Electroweak DM

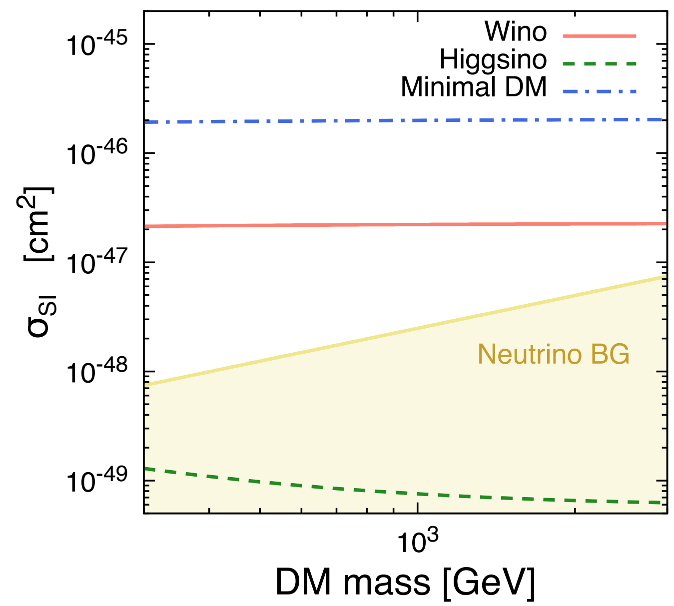
- DM originally in electroweak multiplet (e.g., Wino, Higgsino, minimal DM…)
- Mass splitting after EW symmetry breaking

$$\Delta M = m_{\chi^+} - m_{\chi^0} = \mathcal{O}(100)\,\mathrm{MeV}$$

- Elastic scattering is generally loop-suppressed

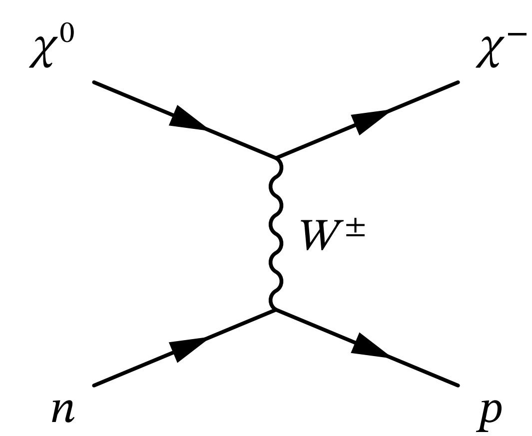


[Cirelli and Strumia (2009)]



[Hisano et al. (2015)]

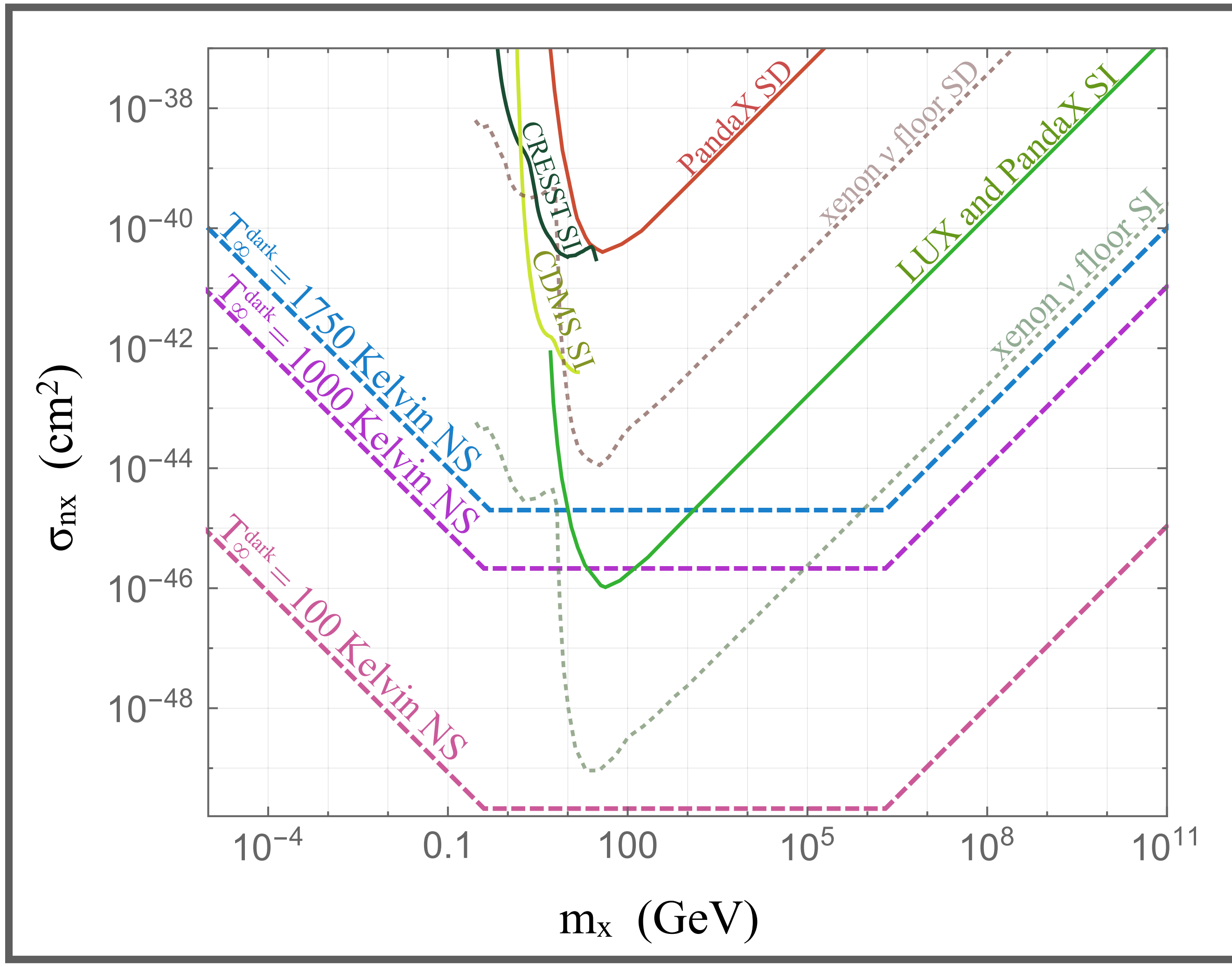
- Inelastic scattering cross section is tree-level
  - $\sigma \sim 10^{-39}\,\mathrm{cm}^2$
  - Highly suppressed on earth by kinematics

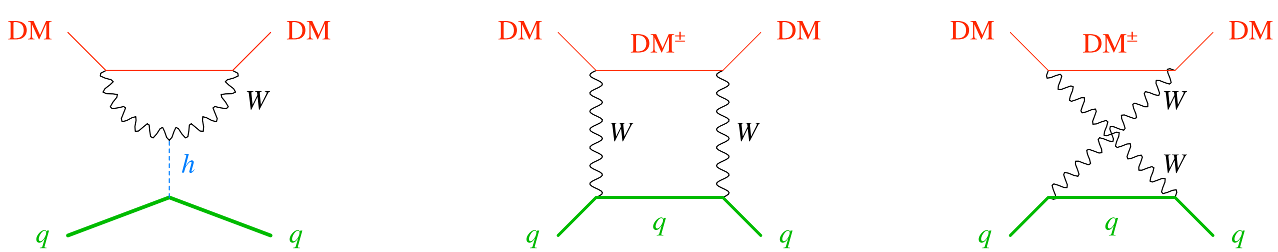


ak DM

no, Higgsino, minimal DM…)

00) MeV





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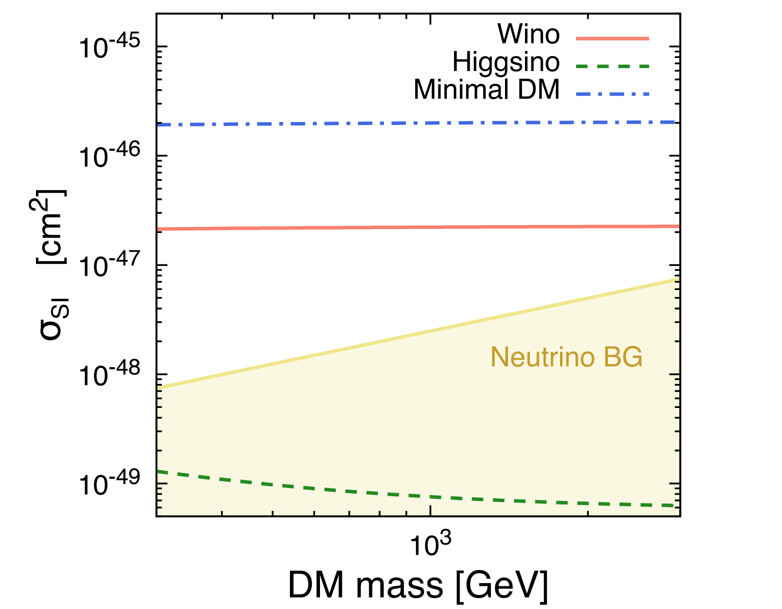

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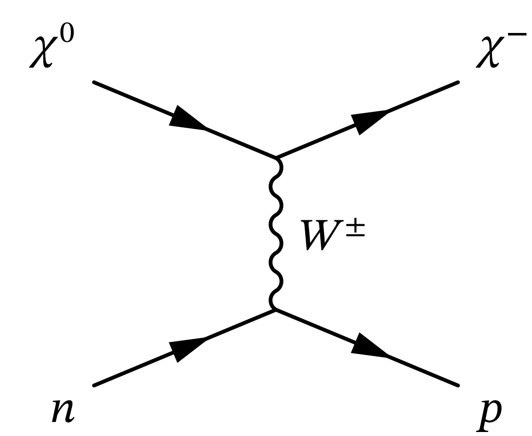

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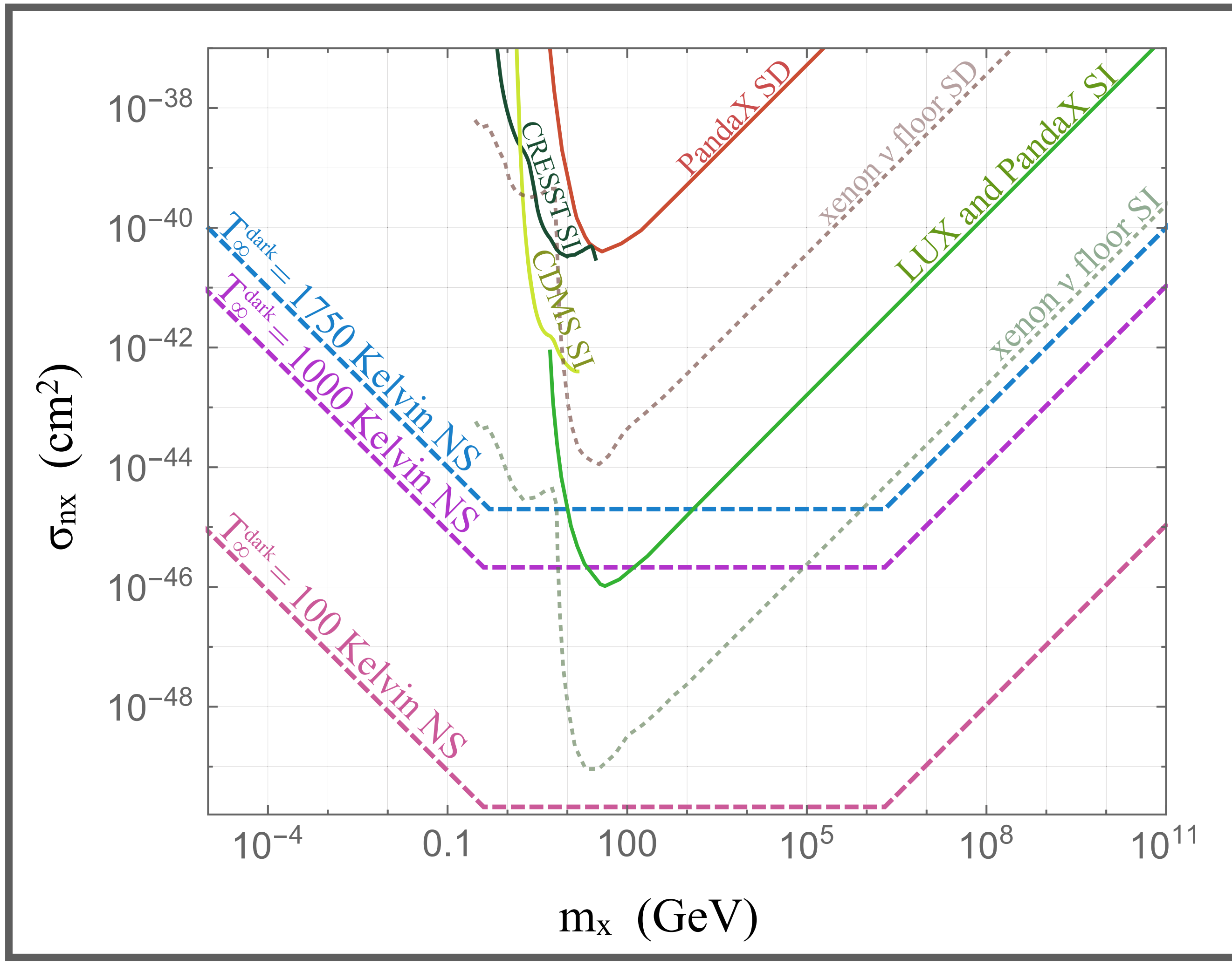

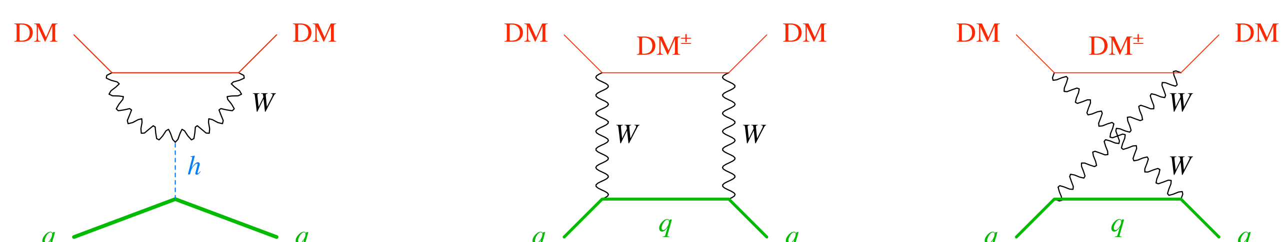

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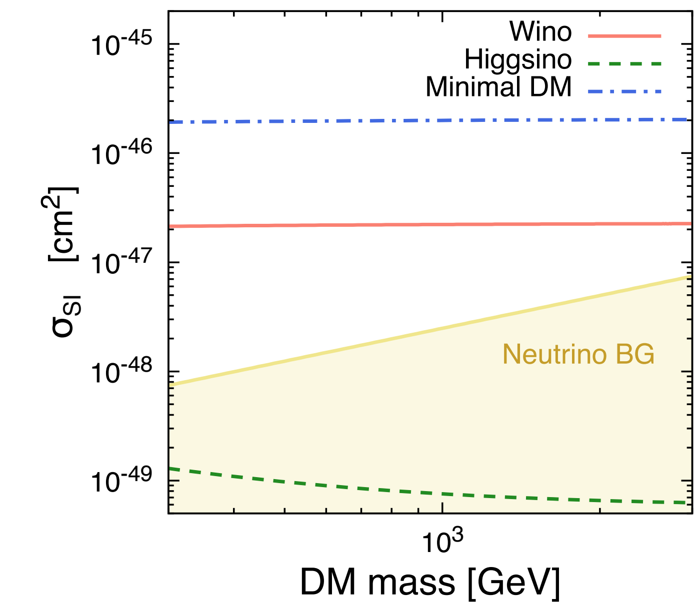

[Hisano et al. (2015)]

DM heating can probe!

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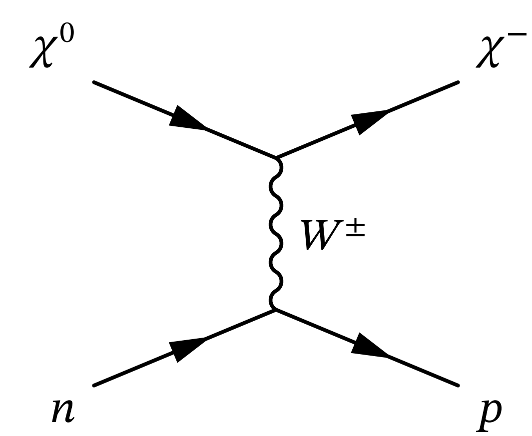
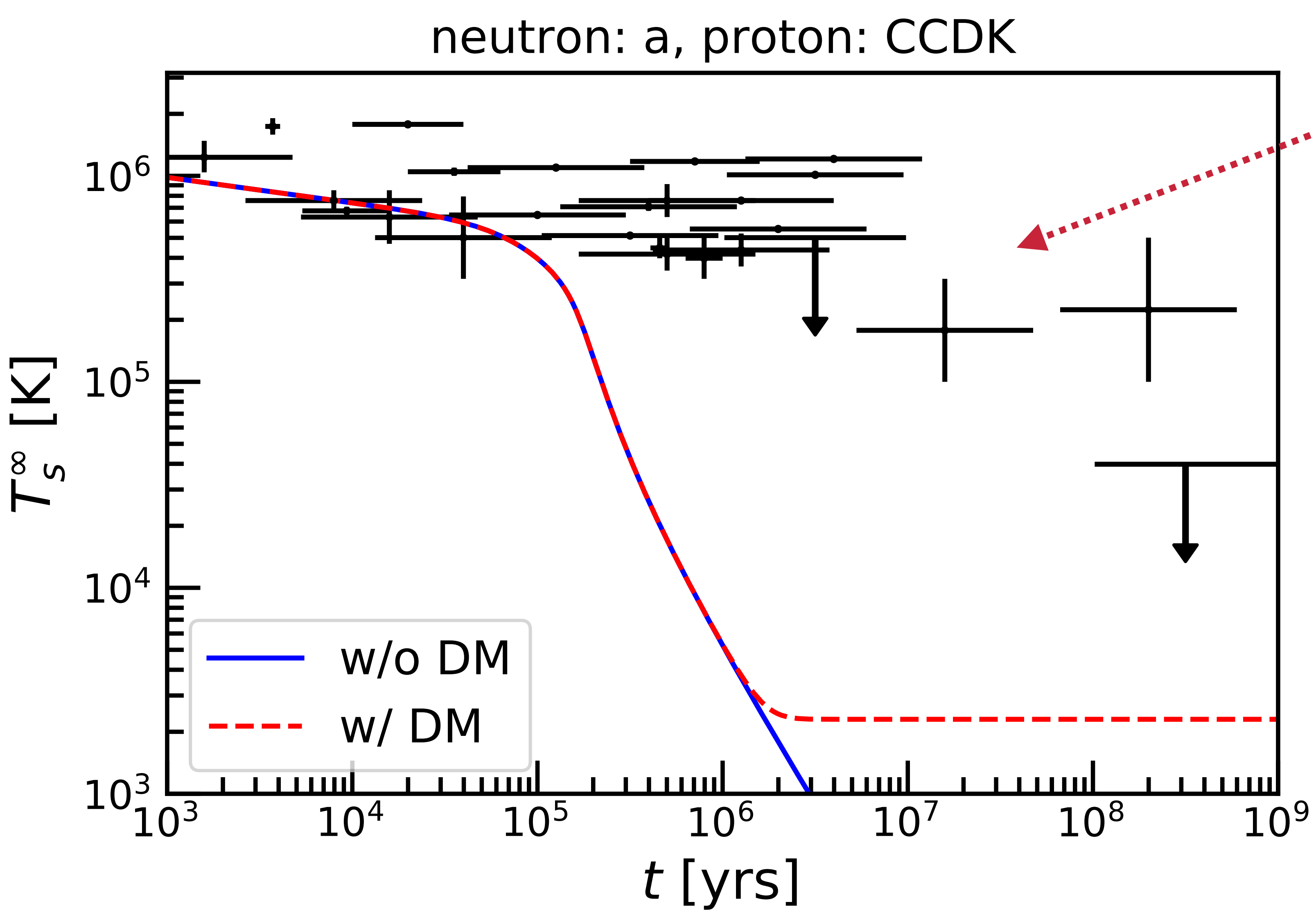


Can we really see DM heating?

The observation suggests presence of other heating mechanisms



Hotter than theory prediction even if DM is included

Question: (DM heating) > (other heating) really occurs?

Outline

1. DM heating of NS
2. Standard cooling of NS
3. Rotochemical heating
4. DM heating vs. Rotochemical heating

Standard theory of NS cooling

$$C\frac{dT}{dt} = -L_\nu - L_\gamma$$

Heat capacity (n, p, e, μ)

Surface photon luminosity:

$$L_\gamma = 4\pi R^2 \sigma_B T_s^4$$

- L_ν: Neutrino emission luminosity
  - (Direct Urca process)
  - Modified Urca process
  - Cooper pair breaking and formation
  - + minor processes

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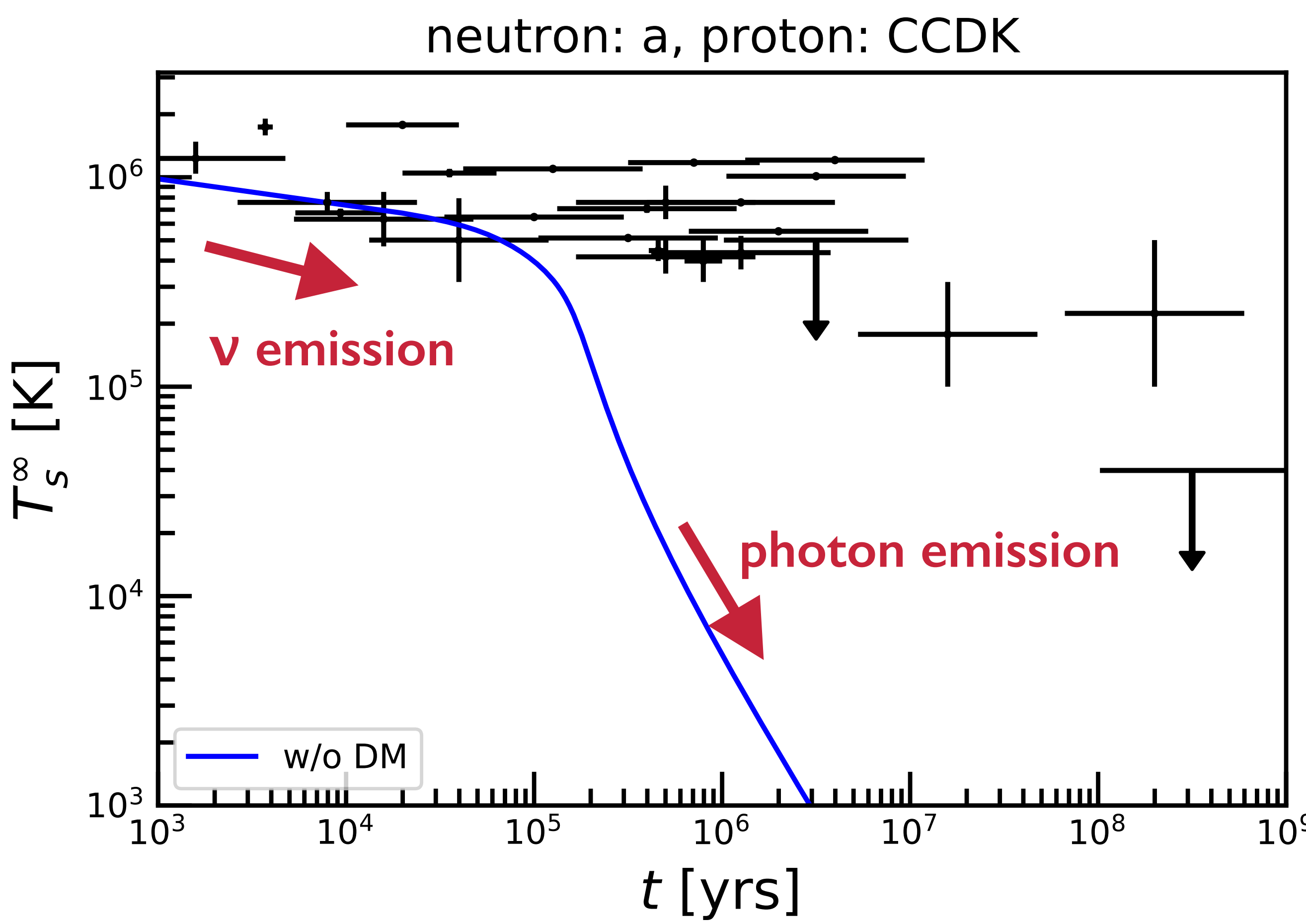

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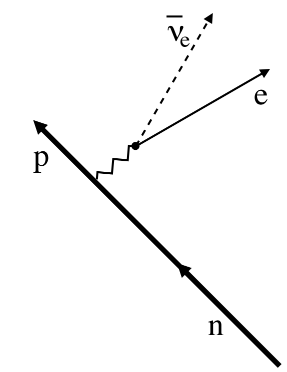
Beta decay and its inverse

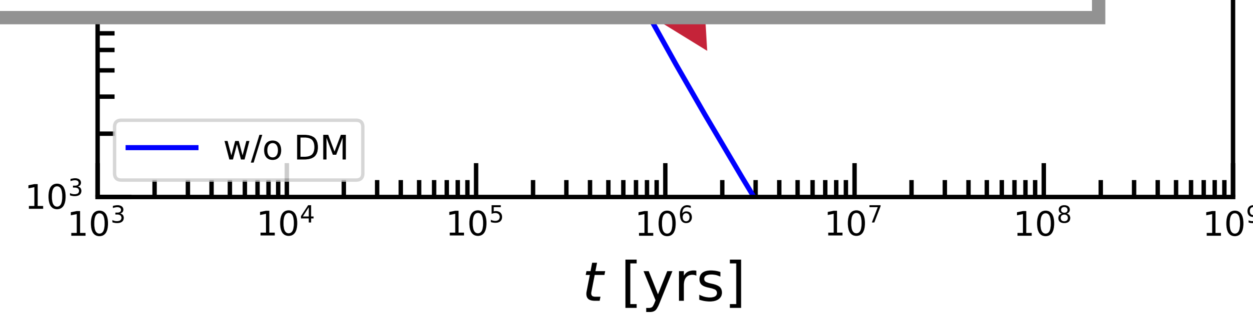
$$n \rightarrow p + e + \bar{\nu}_e$$

$$p + e \rightarrow n + \nu_e$$



- Occurs only when in a very heavy NS

e.g. $M \gtrsim 1.97\,M_\odot$ for APR EOS



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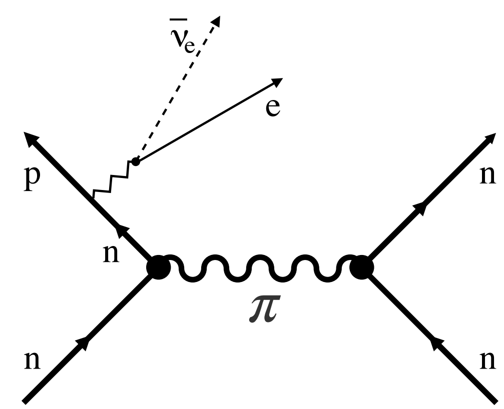
Modified Urca process

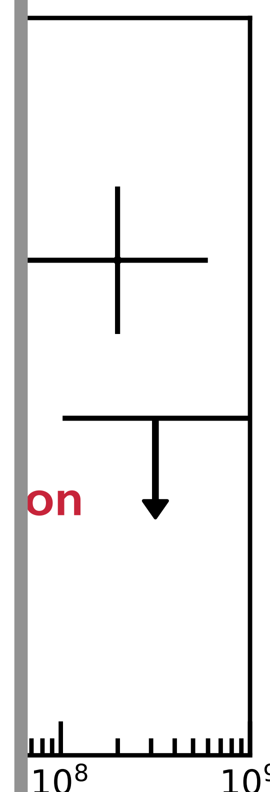
$$n + N \to p + N + e + \bar{\nu}_e$$

$$p + N + e \to n + N + \nu_e$$

$N = n, p$



- Dominant process (before Cooper pairing)
- $L_\nu \propto T^8$



Standard theory of NS cooling

$$C\frac{dT}{dt} = -L_\nu - L_\gamma$$

Heat capacity (n, p, e, μ)

Surface photon luminosity:

Suppression by $\sim e^{-\Delta/T}$

- L_ν: Neutrino emission luminosity
  - (Direct Urca process)
  - Modified Urca process
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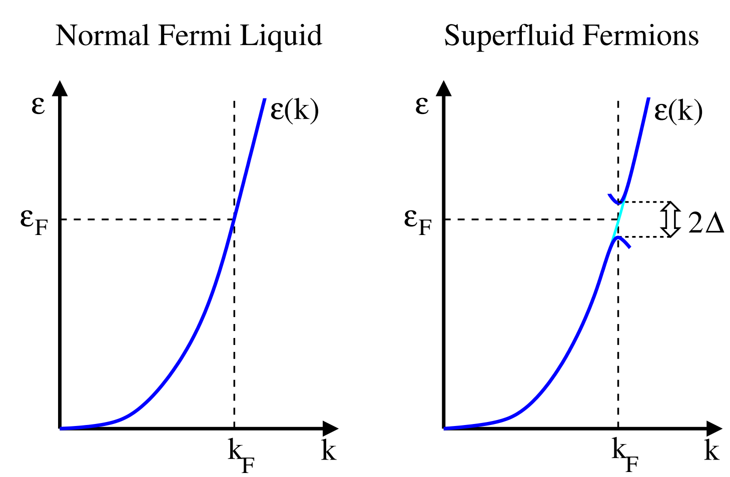
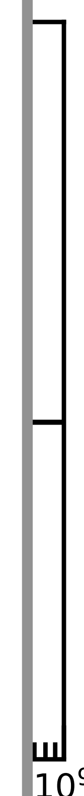
Nucleon Cooper pairing ($T < T_c$)

- Attractive nuclear force induces n-n and p-p pairing
- Energy gap Δ appears in the spectrum



$$\epsilon_N(\mathbf{p}) \simeq \mu_N + \mathrm{sign}(p - p_{F,N})\sqrt{\Delta_N^2 + v_{F,N}^2(p - p_{F,N})^2}$$

- Suppresses heat capacity and Urca process



Standard theory of NS cooling

$$C\frac{dT}{dt} = -L$$

Heat capacity (n, p, e, μ)

- L_ν: Neutrino emission luminosity
  - (Direct Urca process)
  - Modified Urca process
  - Cooper pair breaking and formation
  - + minor processes

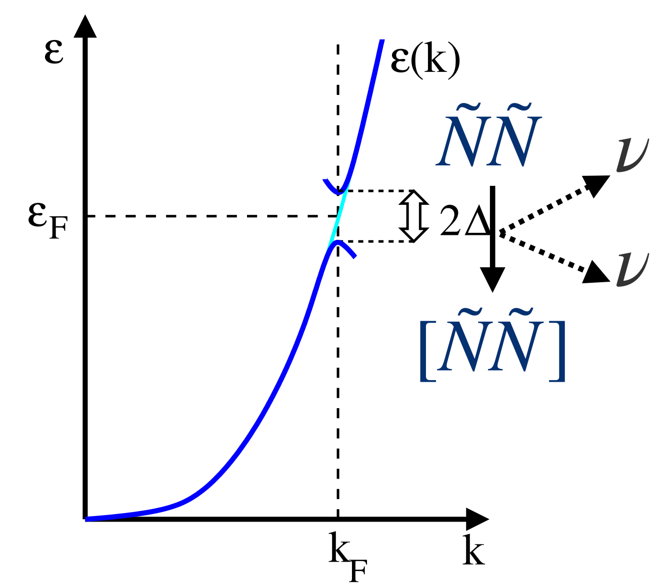
Cooper pairing triggers pair-breking and formation (PBF) process

Pair-breaking: $[\tilde{N}\tilde{N}] \to \tilde{N} + \tilde{N}$

Pair-formation: $\tilde{N} + \tilde{N} \to [\tilde{N}\tilde{N}] + \nu + \bar{\nu}$

[Flowers et al. (1976)]

Superfluid Fermions

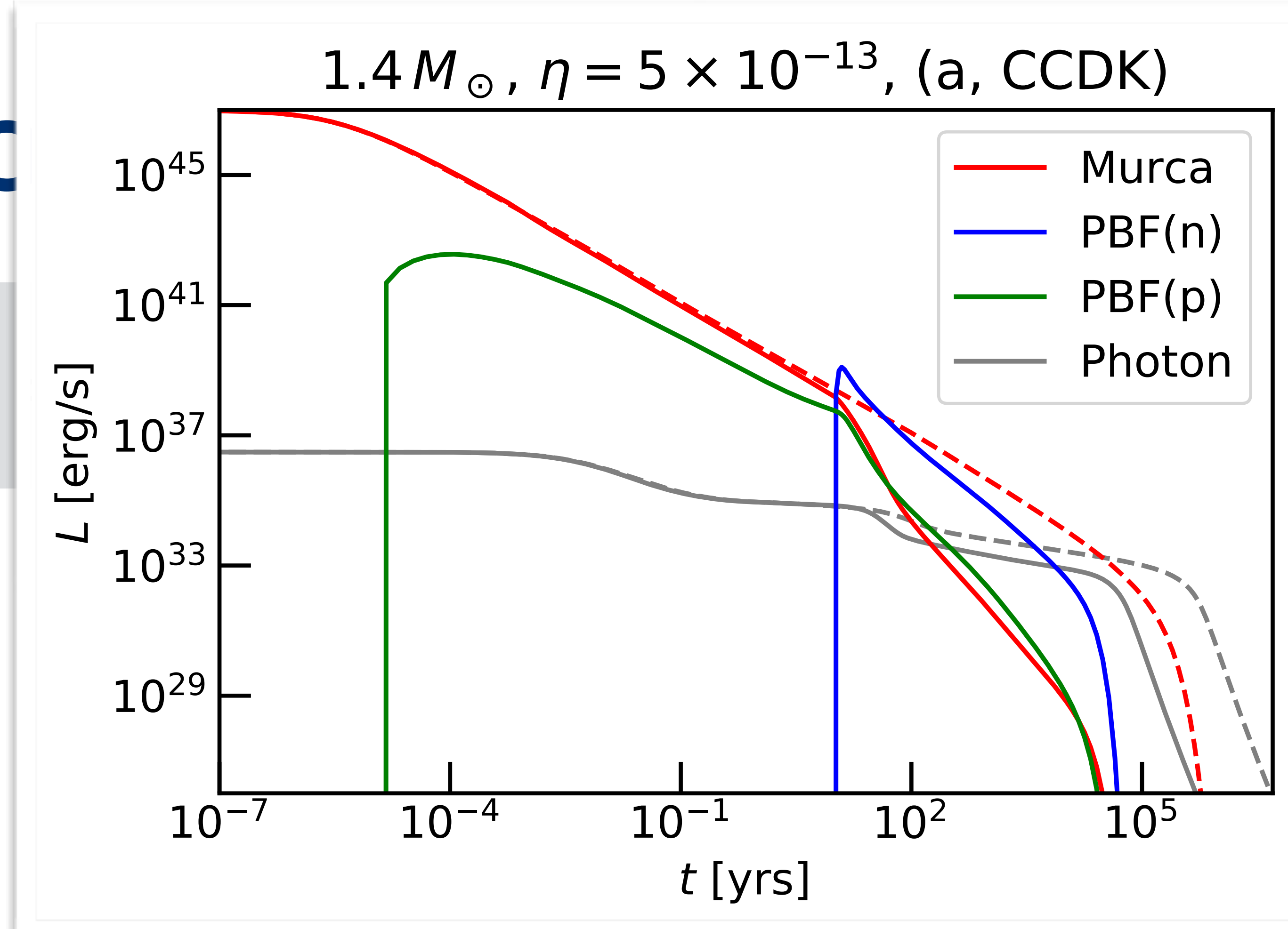


- Efficiently occurs for $T \lesssim T_c$
- Dominant neutrino emission process after pairing

Standard theo

$$C\frac{dT}{dt}$$

Heat capacity (n, p, e, μ)



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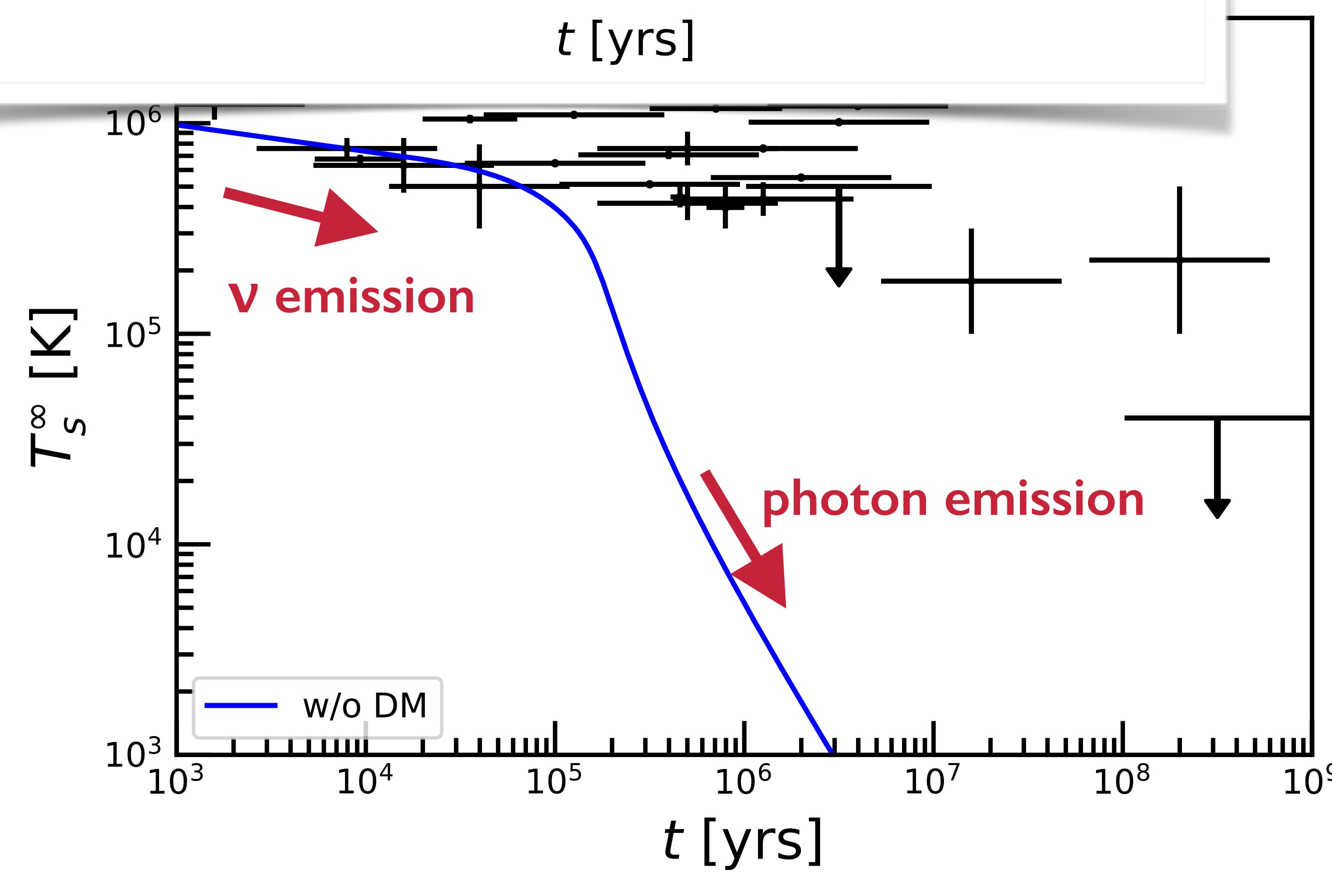

y of NS cooling

$L_\nu - L_\gamma$

Surface photon luminosity:

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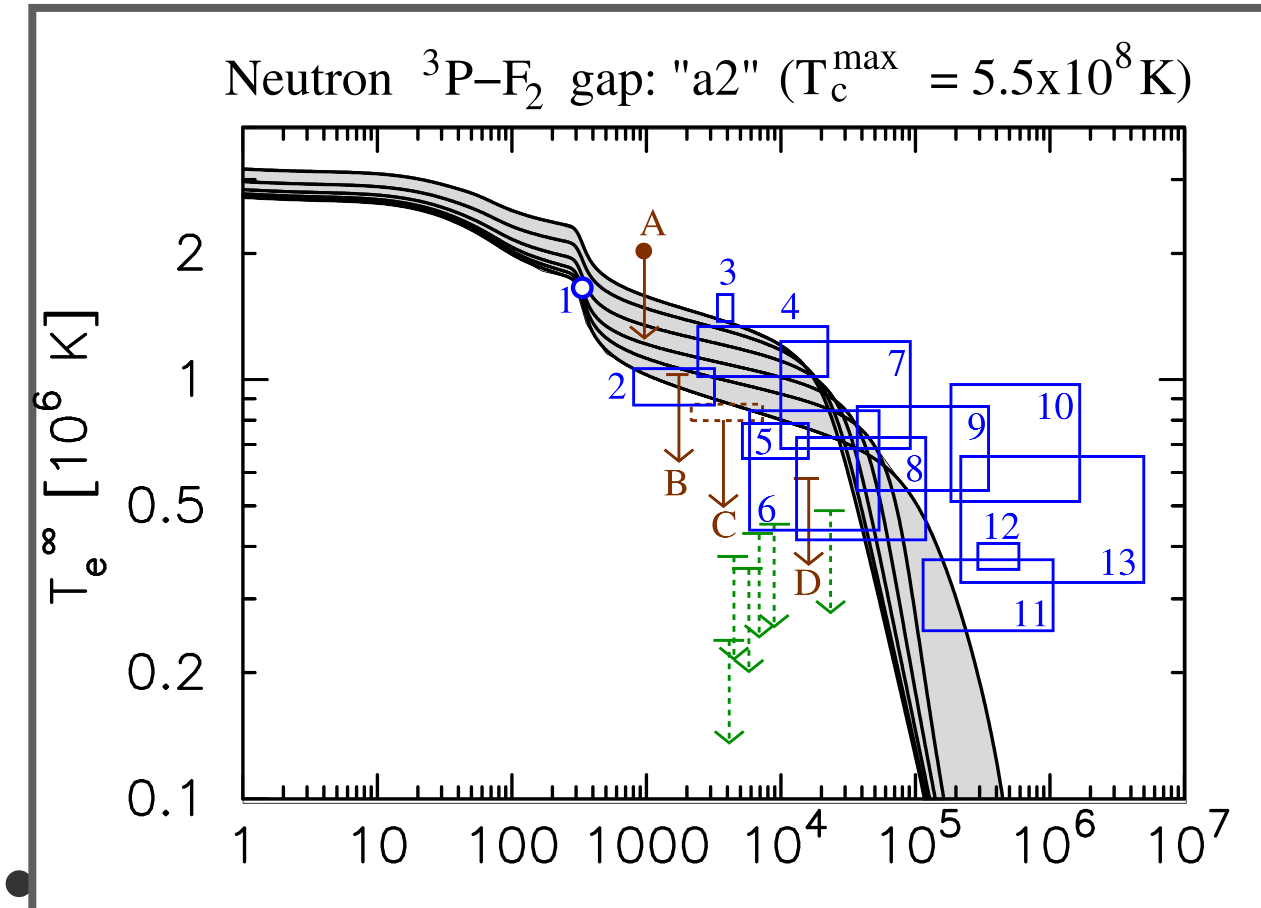
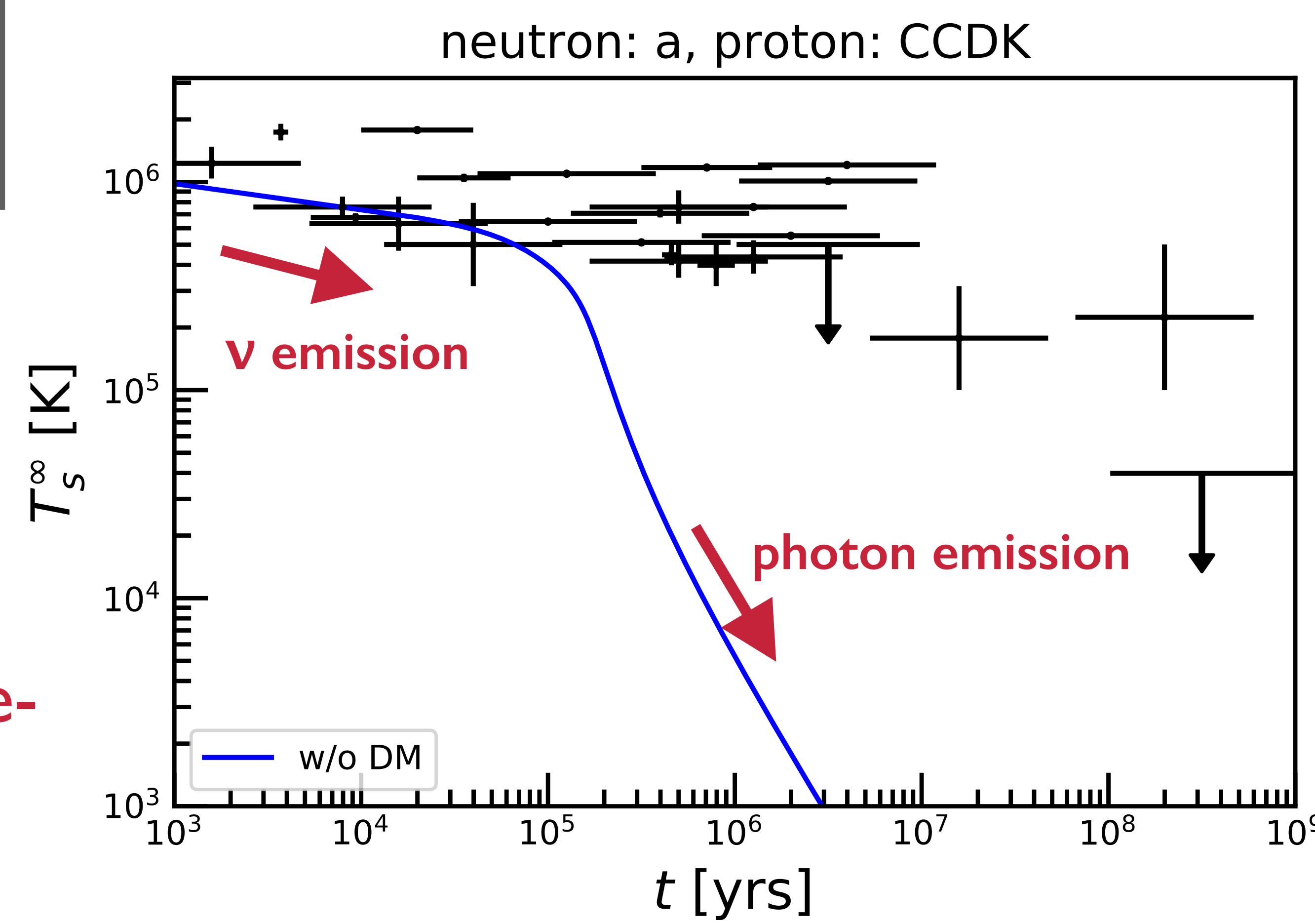


[Page et al. (2013)]

- (Direct Urca process)
  - Modified Urca process
  - Cooper pair breaking and formation
  - + minor processes

- Standard cooling explains young/middle-aged NSs



Standard theory of NS cooling

$$C\frac{dT}{dt} = -L_\nu - L_\gamma$$

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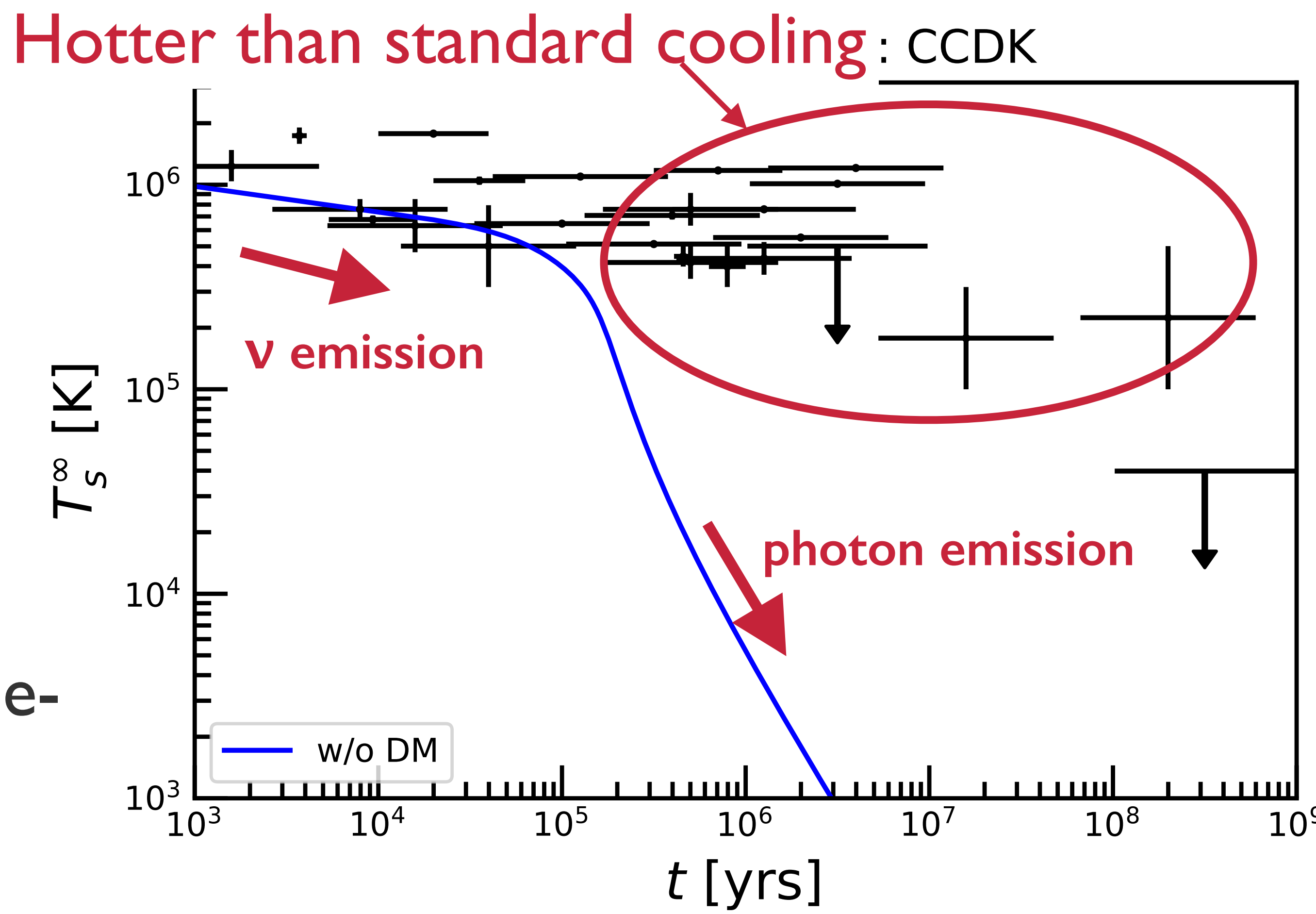

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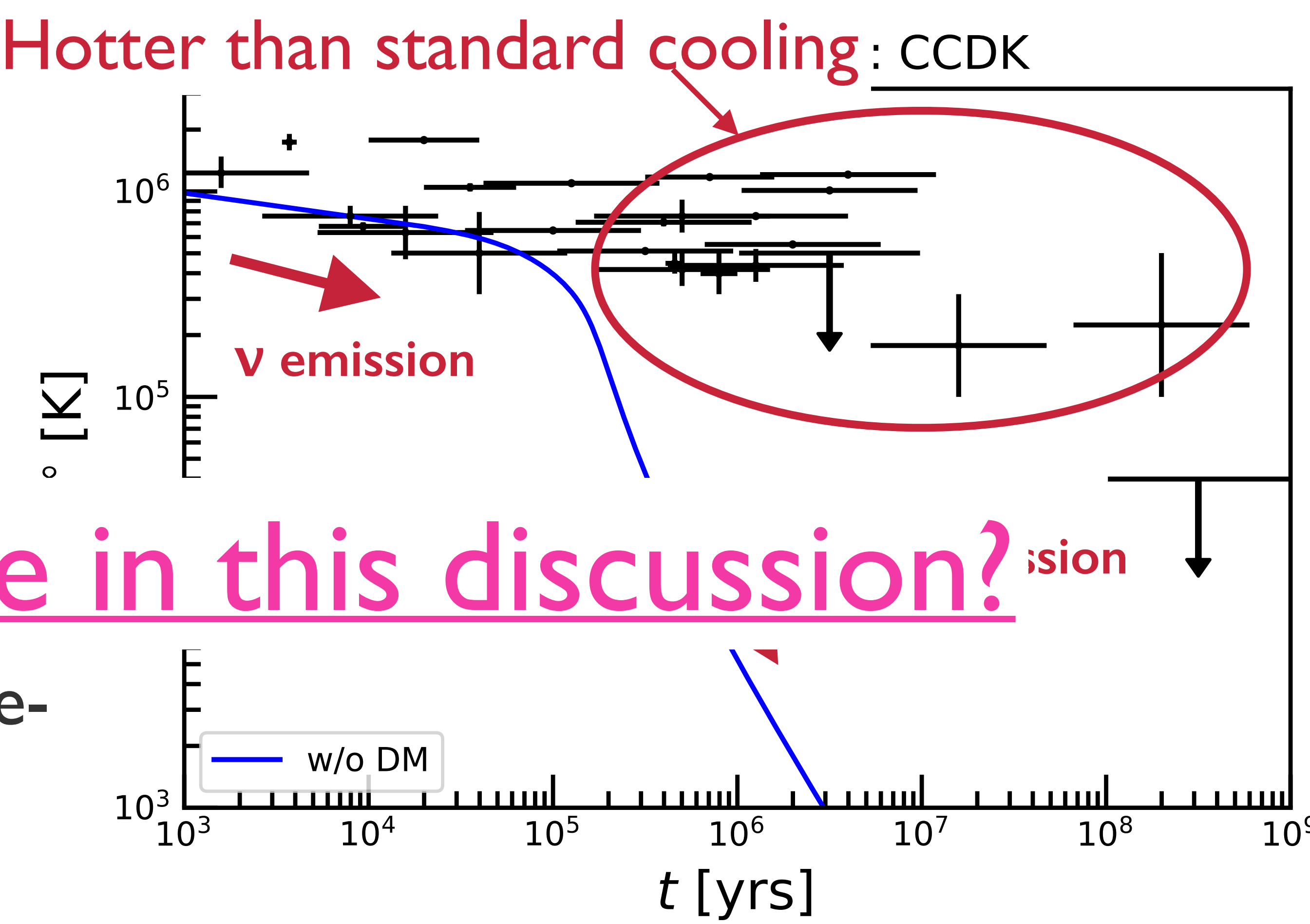

Is there any loophole in this discussion?

- Standard cooling explains young/middle-aged NSs

Outline

1. DM heating of NS
2. Standard cooling of NS
3. Rotochemical heating
4. DM heating vs. Rotochemical heating

Assumption of β-equilibrium

Conventional assumption: matters are in β-equilibrium by Urca processes

$$\Gamma_{n\to p+e} = \Gamma_{p+e\to n} \quad \mu_n = \mu_p + \mu_e$$

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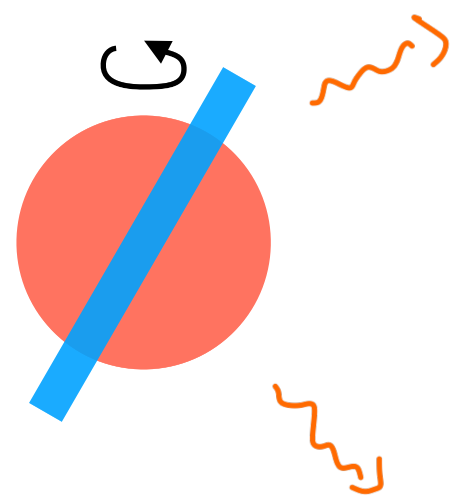
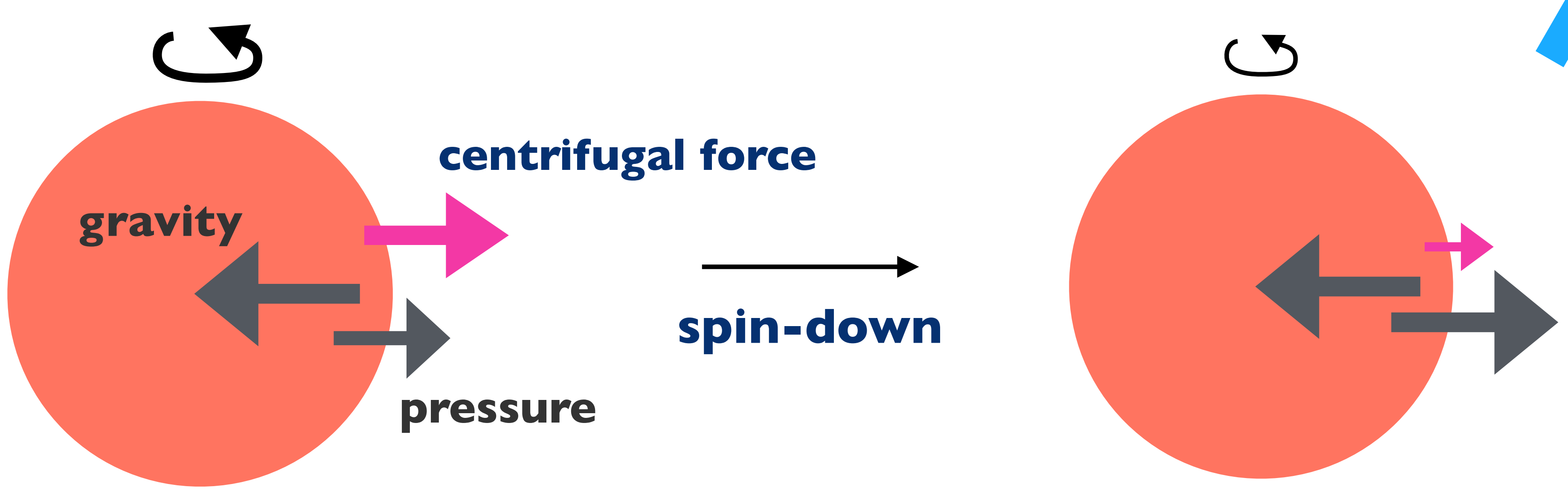


Spin-down of NS violates β-equilibrium [Reisenegger (1994)]

- NS rotation is slowing down by magnetic dipole radiation



- Continuous change of equilibrium condition (local pressure)

  → Urca processes is not fast enough to catch up this change

Evolution of chemical imbalance

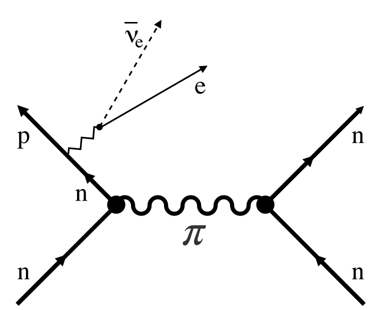
Evolution of imbalance $\eta_e = \mu_n - \mu_p - \mu_e$

[Fernández and Reisenegger (2005)]

$$\frac{d\eta_e^\infty}{dt} = -\sum_{N=n,p}\int dV\,\underbrace{\left(Z_{npe}\Delta\Gamma_{M,Ne} + Z_{np}\Delta\Gamma_{M,N\mu}\right)}_{\text{equilibration by modified Urca}} e^{\Phi(r)} + \underbrace{2W_{npe}\Omega\dot{\Omega}}_{\text{spin-down drives NS out of equilibrium}}$$

equilibration by modified Urca

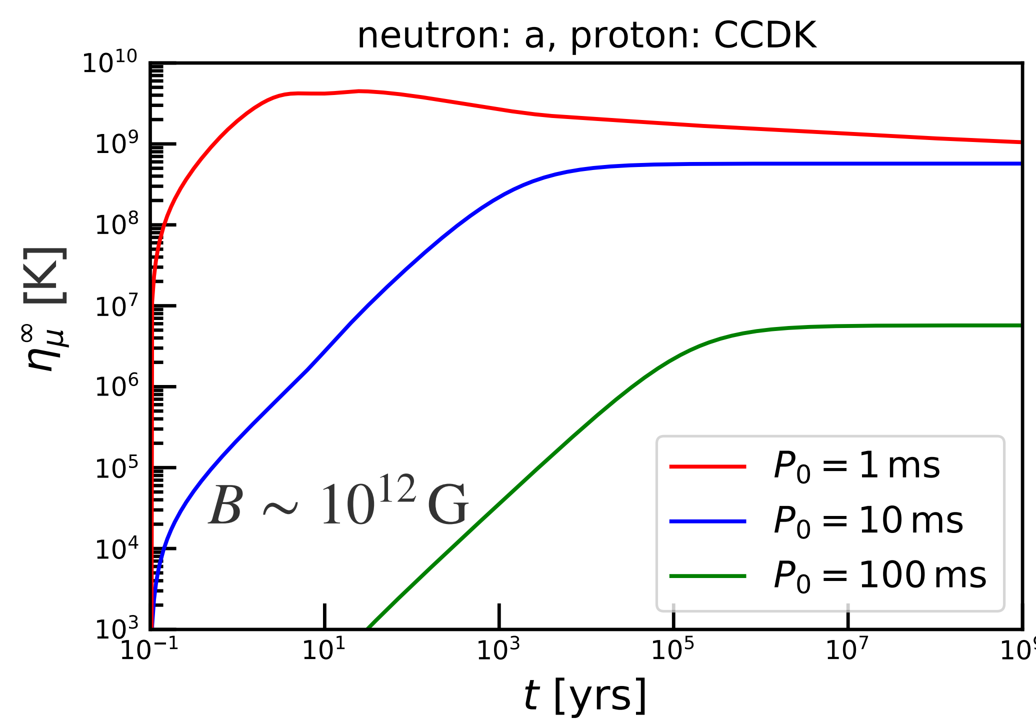
spin-down drives NS out of equilibrium



$$\dot{\Omega} = -k\Omega^3 \qquad k = \frac{2B_s^2R^6\sin^2\alpha}{3I}$$

(Z>0 and W<0 are constants which depends on NS structure)

- If modified Urca is slow, chemical imbalance grows



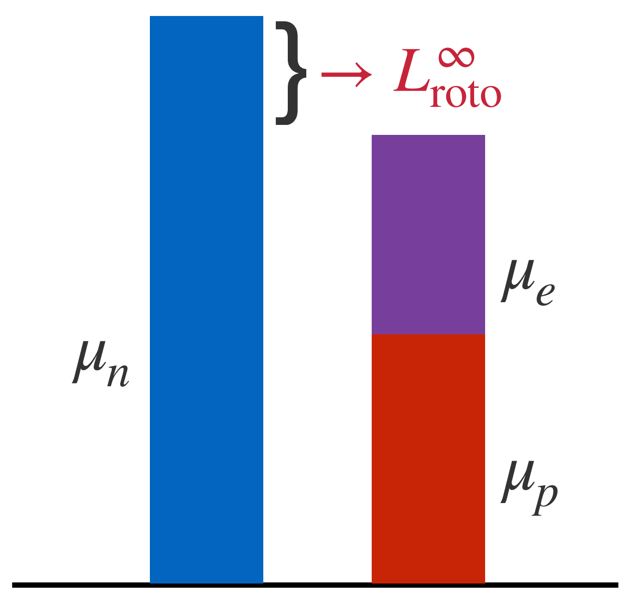
Rotochemical heating

In non-equilibrium modified Urca process, imbalance between chemical potentials is converted to heat [Reisenegger (1994)]

$$C\frac{dT^\infty}{dt} = -L_\nu^\infty - L_\gamma^\infty + L_{\rm roto}^\infty$$

$$L_{\rm roto}^\infty = \sum_{\ell=e,\mu} \sum_{N=n,p} \int dV\, e^{2\Phi(r)} \left(\mu_n - \mu_p - \mu_\ell\right) \cdot \left(\Gamma_{n\to p+\ell} - \Gamma_{p+\ell\to n}\right)$$

[Fernández and Reisenegger (2005)]

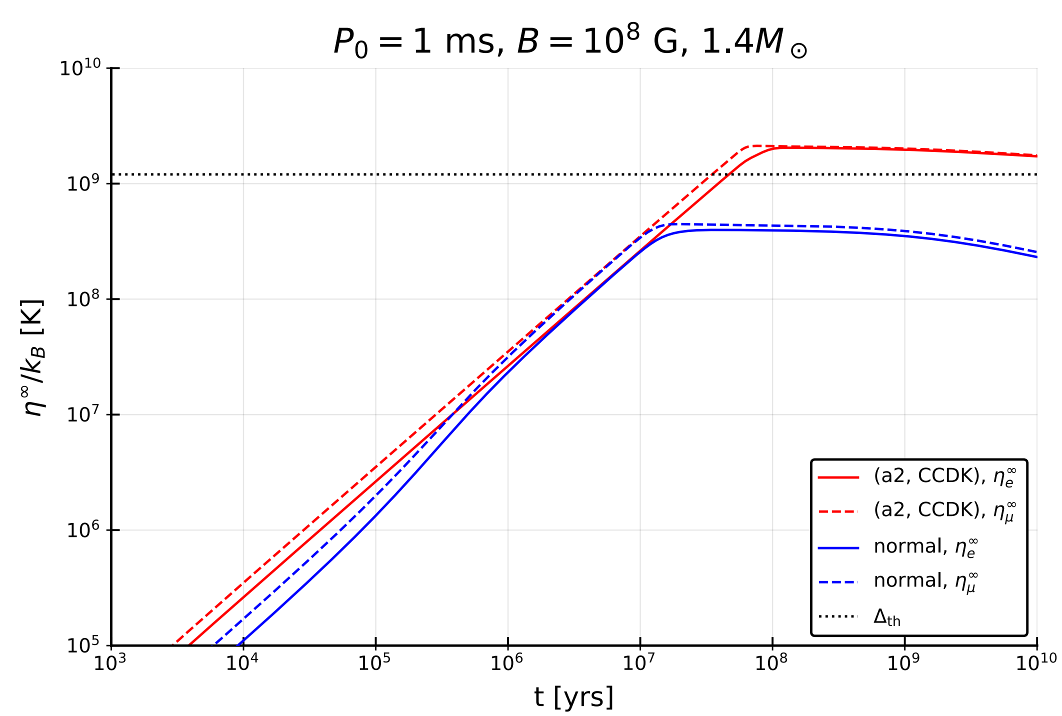


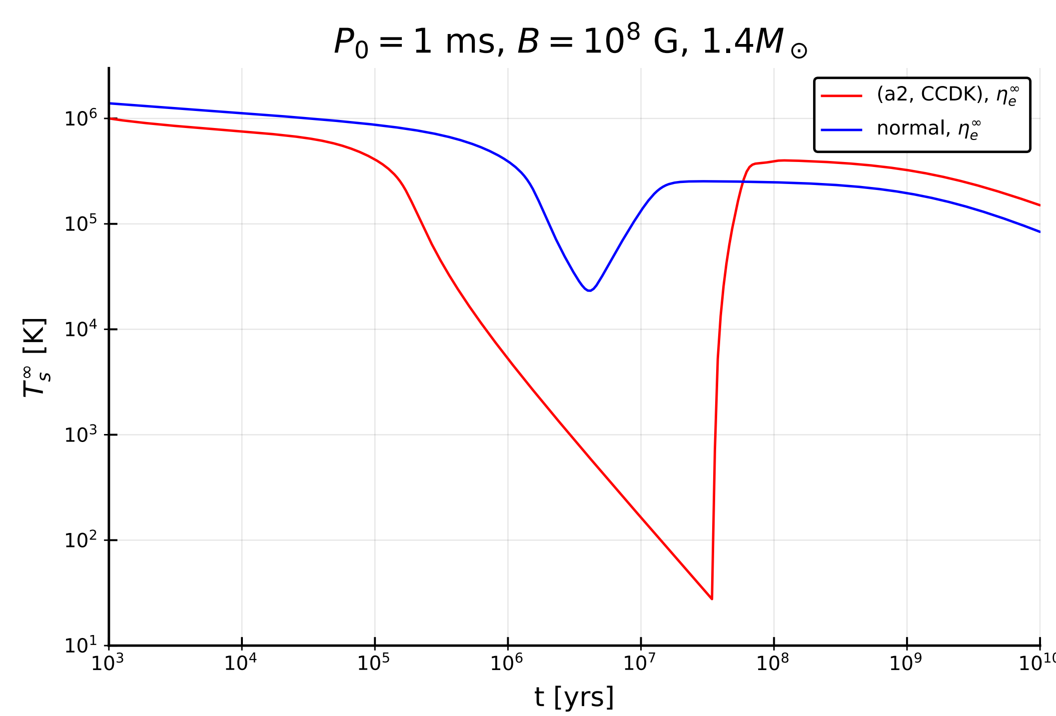
Heating occurs w/o any exotic physics

Details: Effect of Cooper pairing

- Nucleon superfluidity generates threshold $\Delta_{\rm th} = \min\{3\Delta_n + \Delta_p, \Delta_n + 3\Delta_p\}$
- Once η_ℓ exceeds $\Delta_{\rm th}$, rotochemical heating begins
- Larger gap → larger η_ℓ → hotter NS

[Petrovich & Reisenegger (2009)]





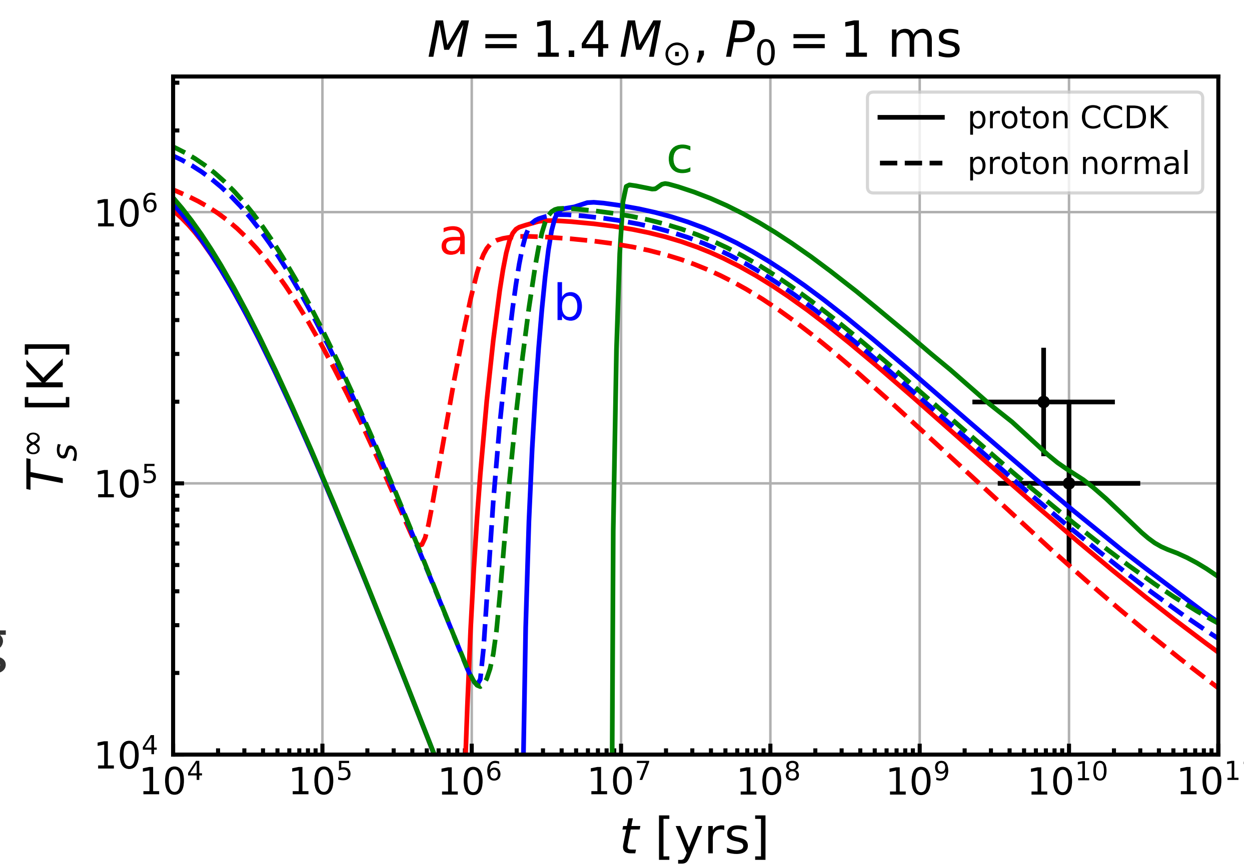
We improve previous works (e.g. González-Jiménez et al. (2014)) by including both neutron and proton pairing in numerical calculation

[KY, Nagata, Hamaguchi (2019)]

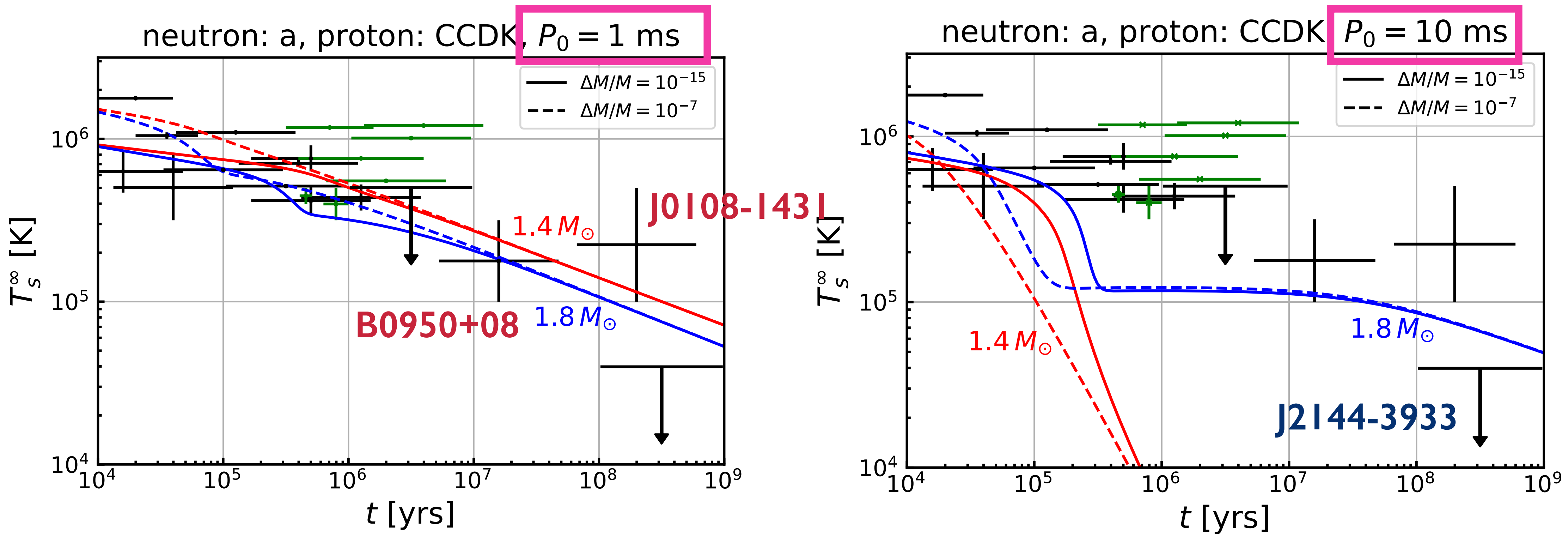
Result: Millisecond pulsars

- Millisecond pulsars (MSPs): short period (P~1ms) and small magnetic field (B~10^8 G)
- Two very old MSPs (PSR J2124-3358 & PSR J0437-4715) are much hotter than standard cooling prediction
- These are explained by rotochemical heating
- Including both neutron and proton pairing is advantageous for the explanation

[KY, Nagata, Hamaguchi (2019)]



Result: Ordinary pulsars and XDINSs



- Ordinary pulsars : P~1s and B~10^{12} G; XDINSs: larger magnetic field
- Old hot NSs (PSR J0108-1431& PSR B0950+08): P_0 = 1ms is necessary
- Old cold NS (PSR J2144-3933): P_0 > 10 ms is necessary
- Some XDINSs are even hotter. Maybe due to the magnetic field decay

[KY, Nagata, Hamaguchi (2019)]

Outline

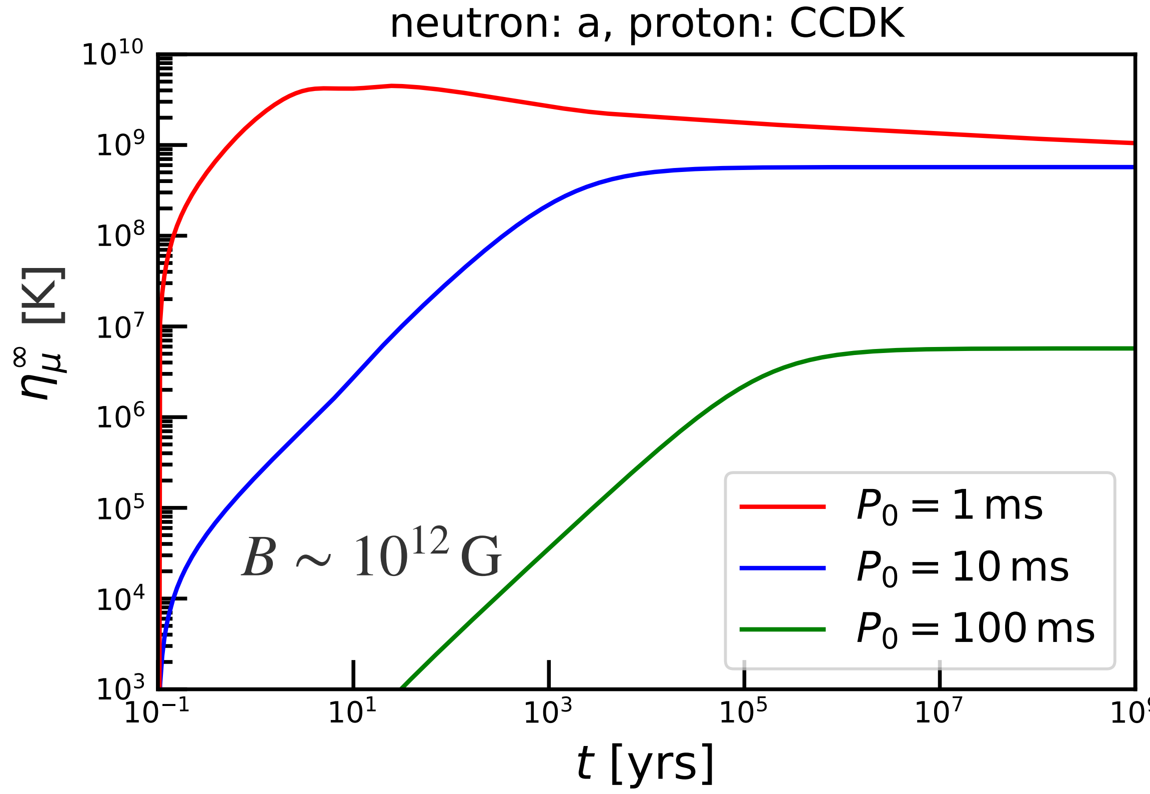
1. DM heating of NS
2. Standard cooling of NS
3. Rotochemical heating
4. DM heating vs. Rotochemical heating

DM heating vs. rotochemical heating

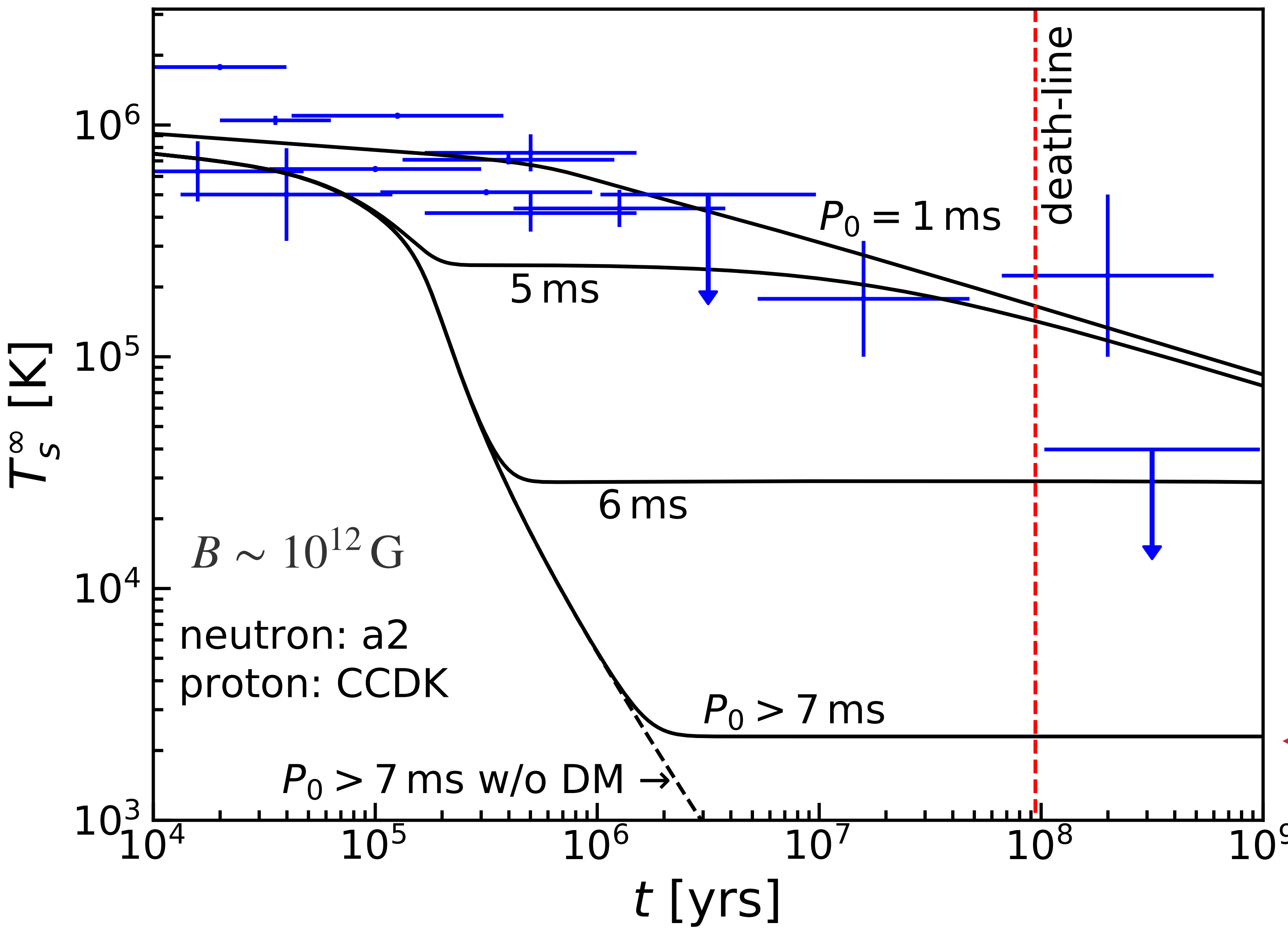
- DM heating
  - $T_s \sim 3000\,\mathrm{K}$
  - For nearby NSs, this prediction cannot change by order

- Rotochemical heating
  - If it operates, typically $T_s \sim 10^{5-6}\,\mathrm{K}$
  - Heating rate is strongly dependent on the initial rotation period P_0
  - Heating is more efficient for smaller P_0

$$\eta_\ell = \mu_n - \mu_p - \mu_\ell$$



DM heating vs. rotochemical heating



$$C\frac{dT^\infty}{dt} = -L_\nu^\infty - L_\gamma^\infty + L_{\rm roto}^\infty + L_{\rm DM}^\infty$$

$L_{\rm DM} < L_{\rm roto}$

$L_{\rm DM} > L_{\rm roto}$ for $P_0 \gtrsim 7$ ms

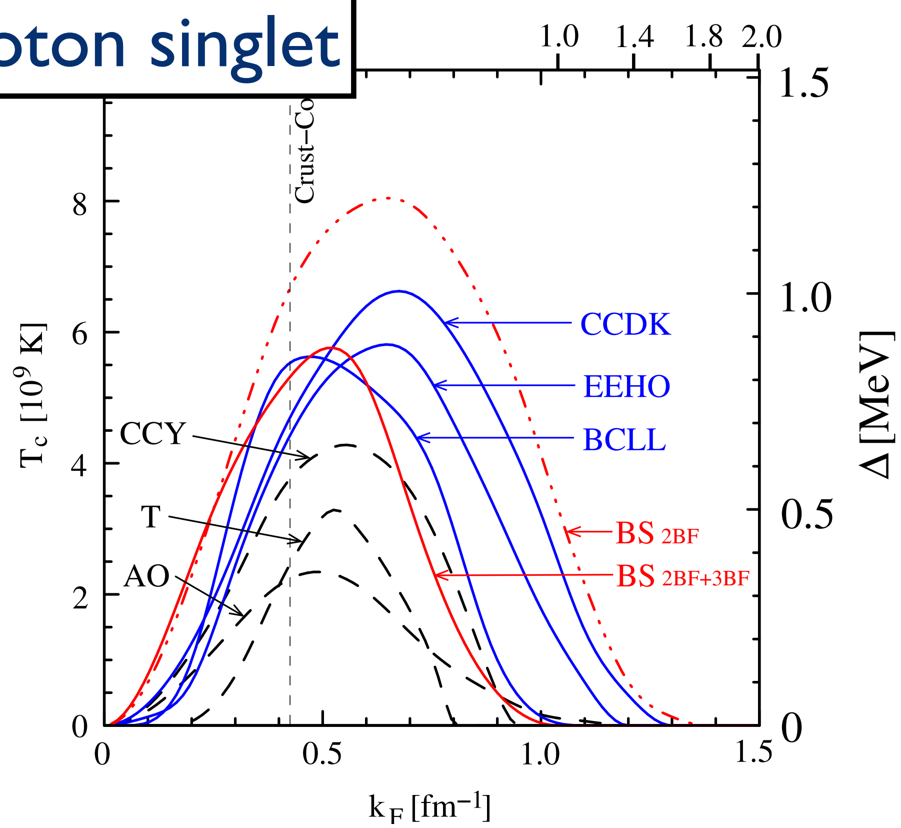
[Hamaguchi, Nagata, KY (2019)]

DM heating effect is visible if the initial period is sufficiently large!

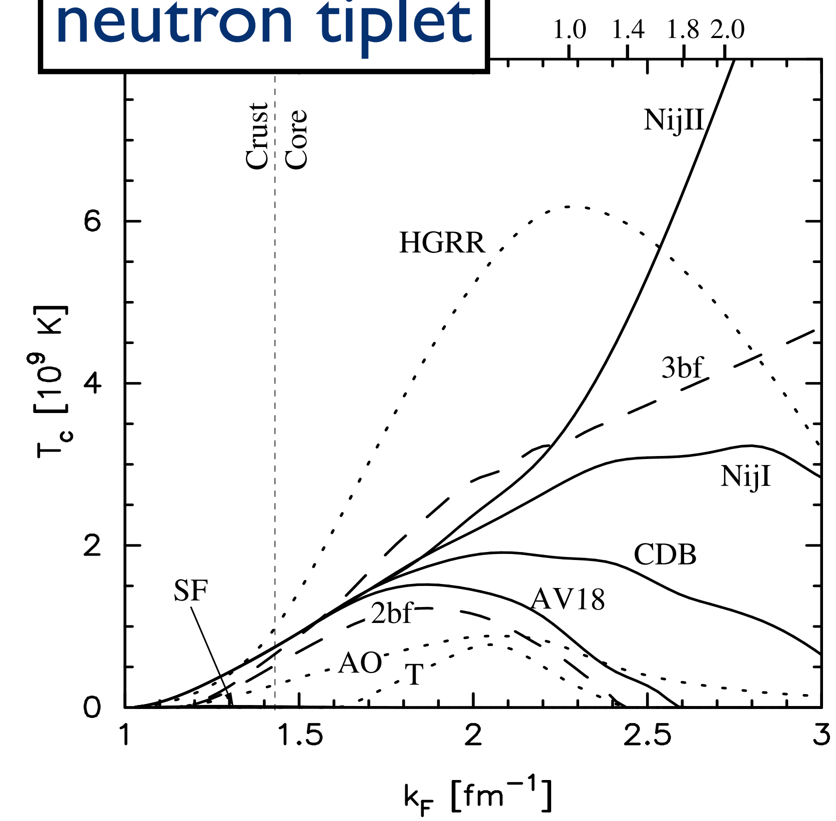
Uncertainty from pairing gap

- So far we have fixed Cooper pairing gap model
- But the gap amplitude has uncertainties due to nuclear force modeling

proton singlet



neutron tiplet



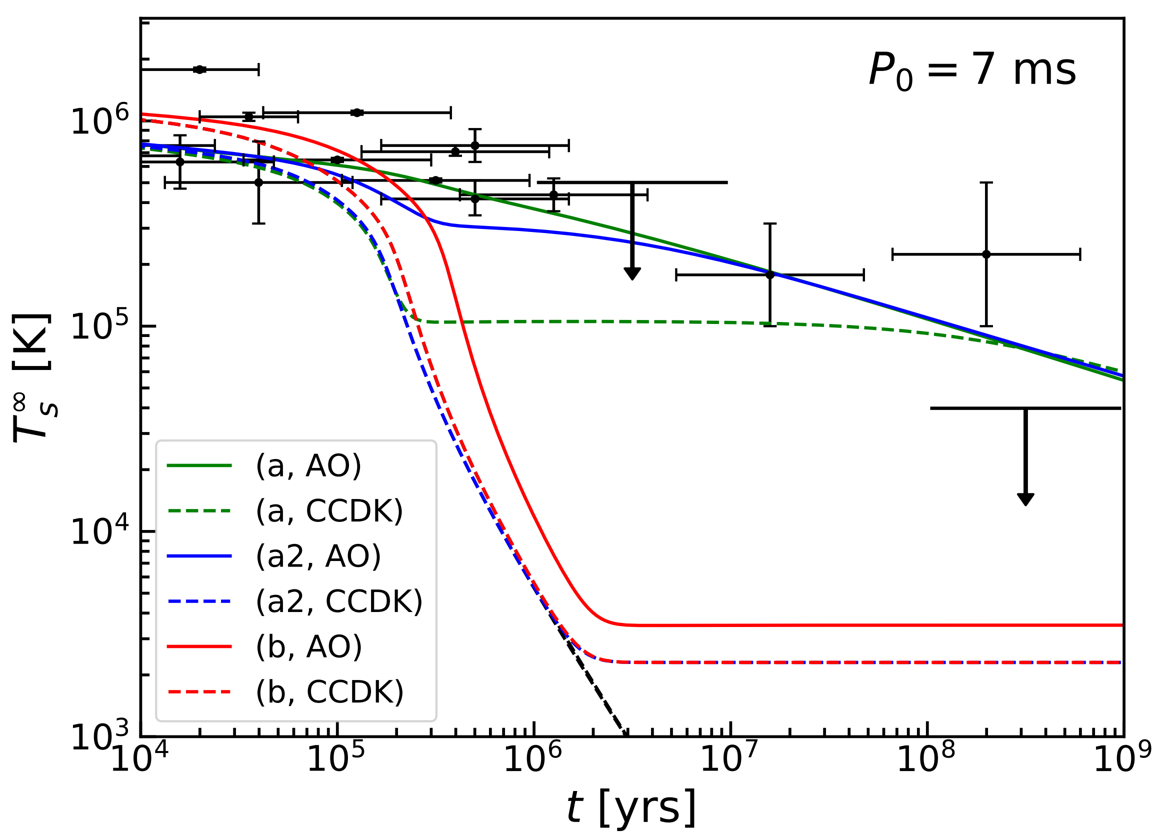
$$\epsilon_N(\mathbf{p}) \simeq \mu_N + \mathrm{sign}(p - p_{F,N})\sqrt{\Delta_N^2 + v_{F,N}^2(p - p_{F,N})^2}$$

- Proton singlet gap is rather well constrained
- Neutron triplet gap is highly uncertain. It is often taken as a free parameter

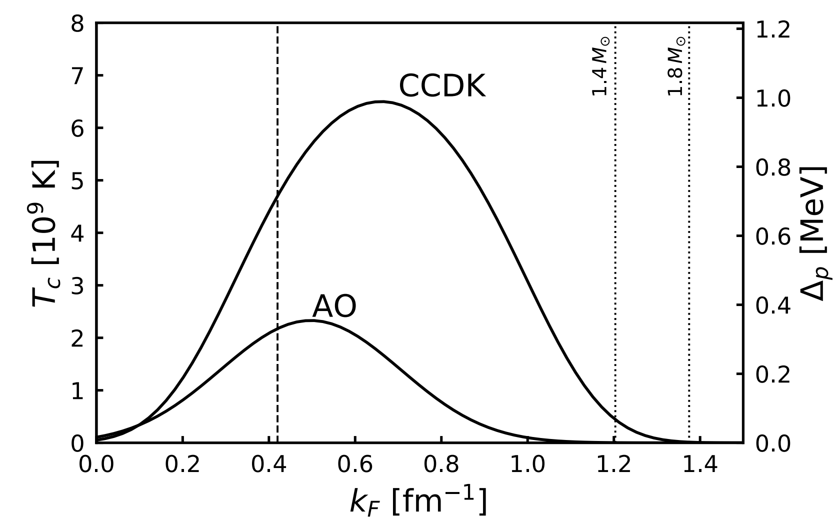
[Figures from Page et al. (2013)]

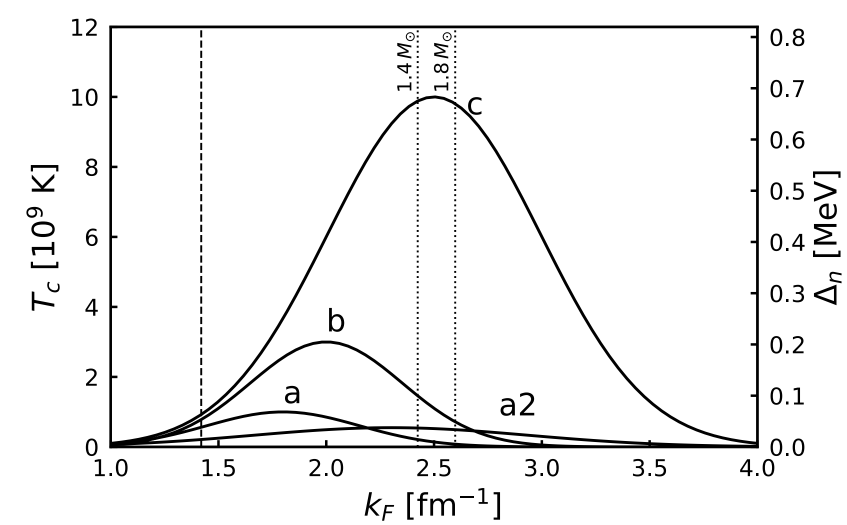
Uncertainty from pairing gap

- Strength of rotochemical heating depends on gap amplitude
- Critical P_0 depends on the choice of gap models
- If $P_0 \gtrsim 100\,\mathrm{ms}$, (DM heating) >> (rotochemical heating)



(neutron gap, proton gap)





Initial period

Several studies suggest the typical initial period of $P_0 \sim \mathcal{O}(100)\,\mathrm{ms}$

- Observed kinematic age
  [Popov & Turolla, 1204.0632; Noutsos et.al., 1301.1265; Igoshev & Popov, 1303.5258]

- Population synthesis
  [Faucher-Giguere & Kaspi, astro-ph/0512585; Popov et al., 0910.2190, Gullo´n et al., 1406.6794, 1507.05452]

- Supernova simulation for proto-NSs
  [Mu¨ller et al., 1811.05483]

Thus we expect

- For many NSs, DM heating > Rotochemical heating
- Some NSs accidentally have $P_0 \sim 1\mathrm{ms}$ → observed high $T_s \sim 10^{5-6}\,\mathrm{K}$

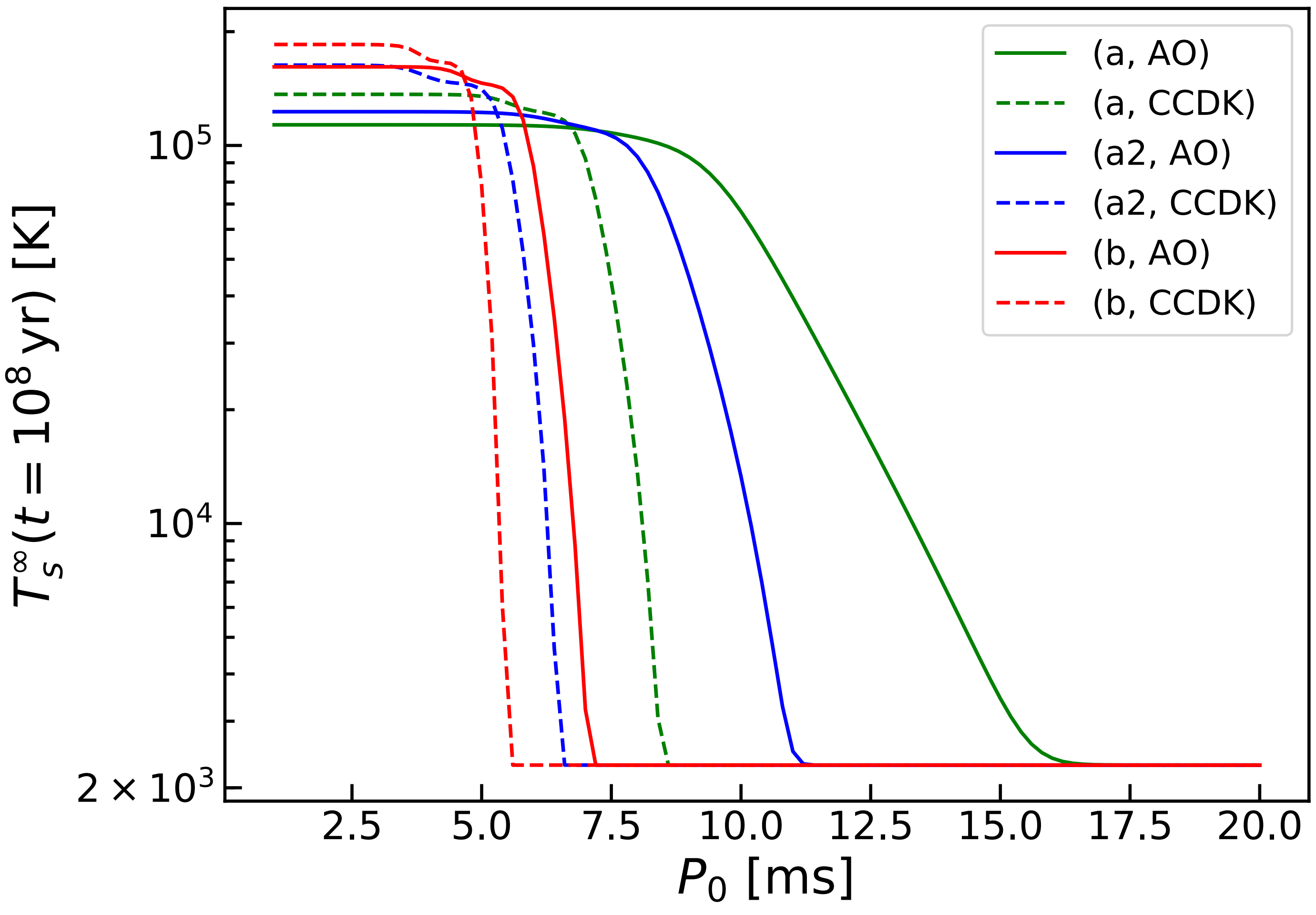
Summary

- DM heating
  - DM accretion heats up old NS
  - NS surface temperature measurement can probe DM
- Rotochemical heating
  - it occurs w/o any new or exotic physics (induced by NS rotation)
  - it explains observed warm NSs
- (DM heating) > (rotochemical heating)?
  - $P_0 \lesssim 10 - 100\,\mathrm{ms}$: (DM heating) < (rotochemical heating)
  - $P_0 \gtrsim 10 - 100\,\mathrm{ms}$: (DM heating) > (rotochemical heating)
  - old ordinary pulsar is suitable target

Backup

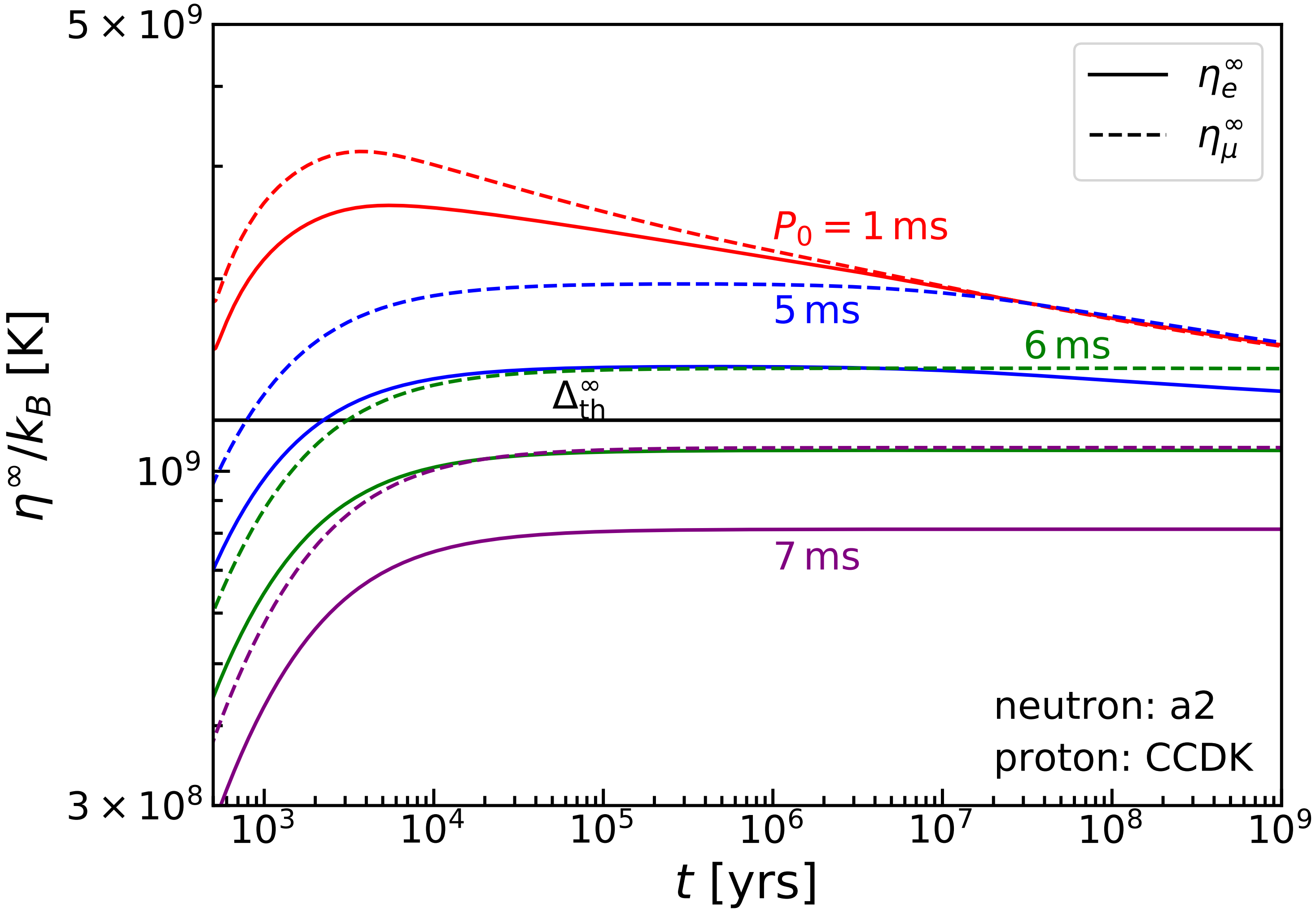
Uncertainty from pairing gap

- Strength of rotochemical heating depends on gap amplitude
- Critical P_0 depends on the choice of gap models
- $P_0 \gtrsim 100\,\mathrm{ms}$ is enough

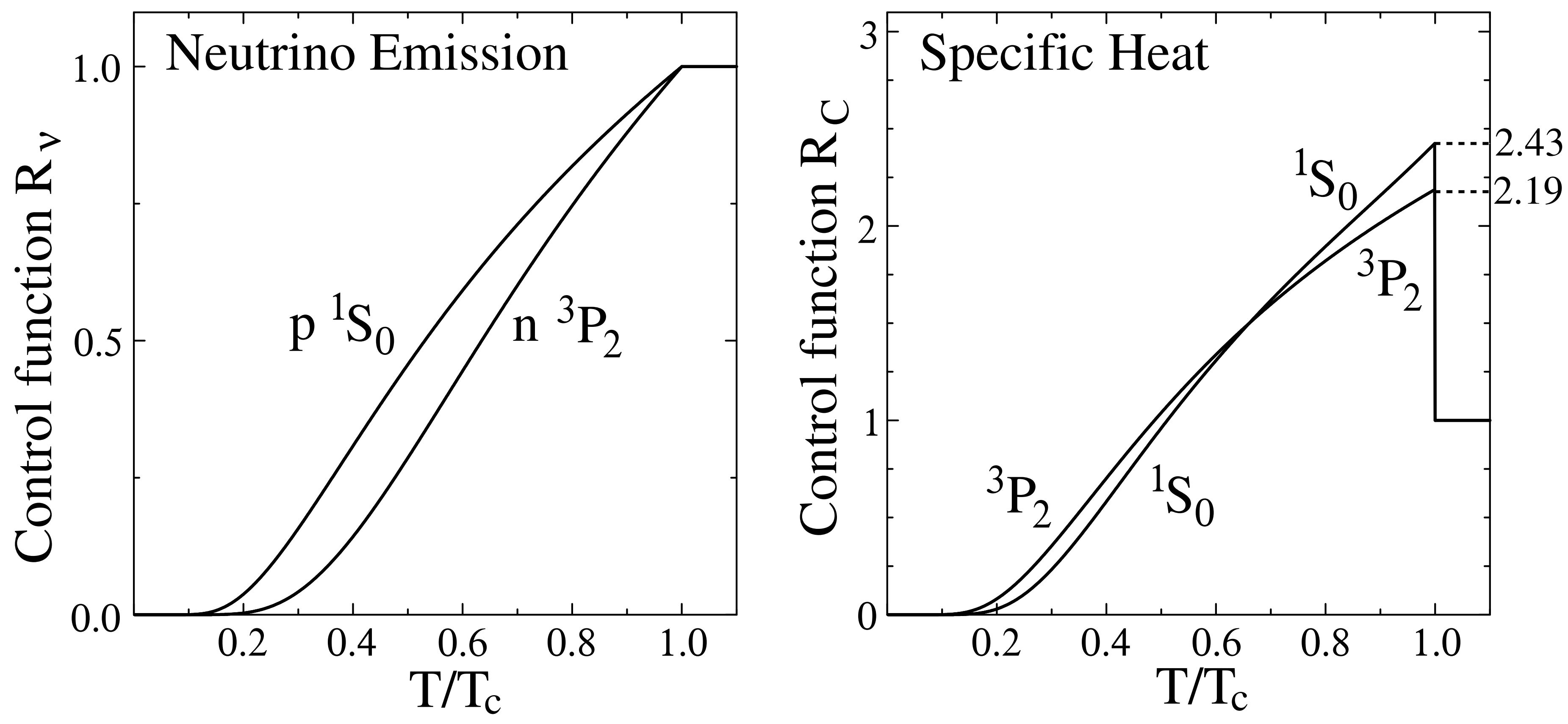


(neutron gap, proton gap)

Imbalance evolution



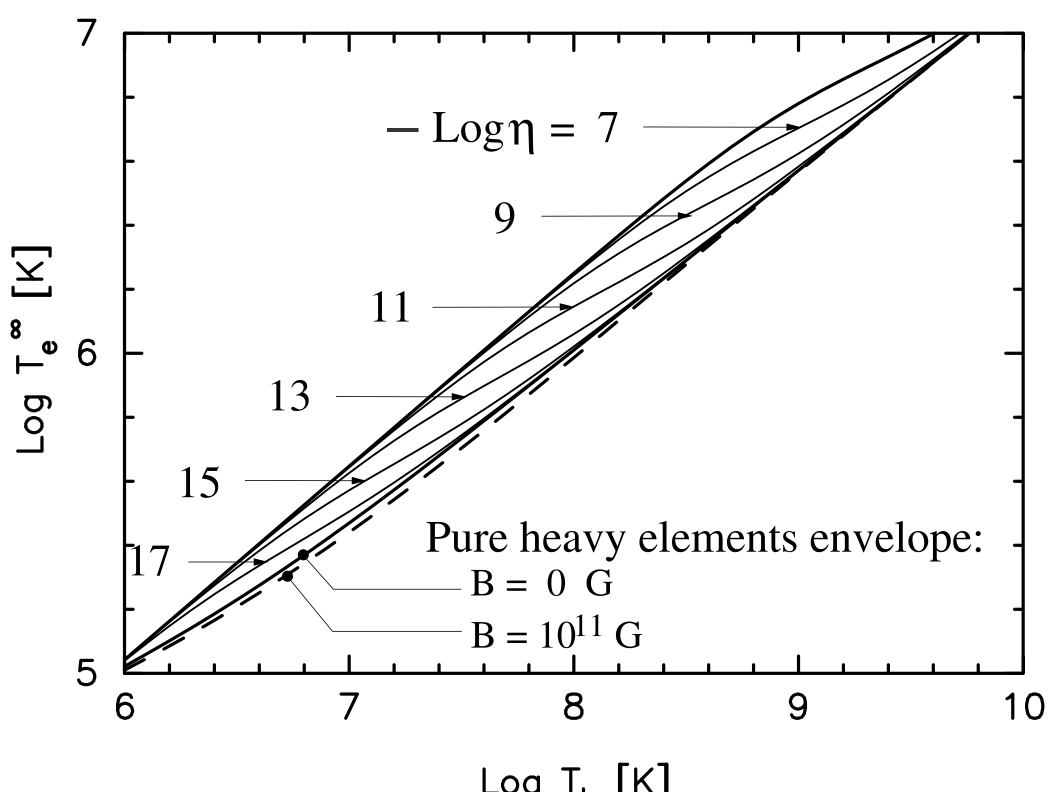
Superfluid suppression

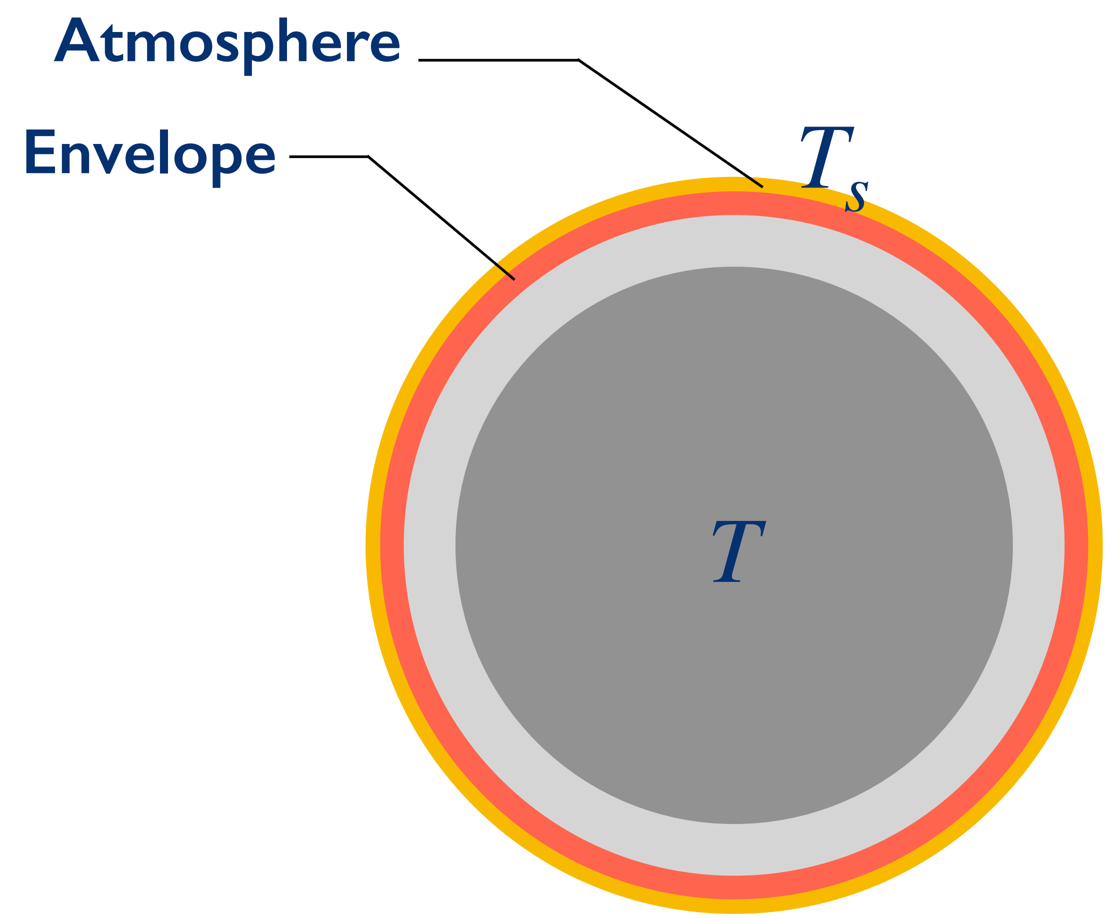


[Page et al. (2013)]

Details 2: Envelope

- Envelope shields atmosphere from core and crust
- Surface T and internal T are different
- $T - T_s$ relation depends on amount of light element in envelope [Potekhin et al. (1997)]





Parametrized by $\eta = g_{14}^2 \Delta M / M$

(surface gravity)/$(10^{14}\ \mathrm{cm/s^2})$ mass of light elements

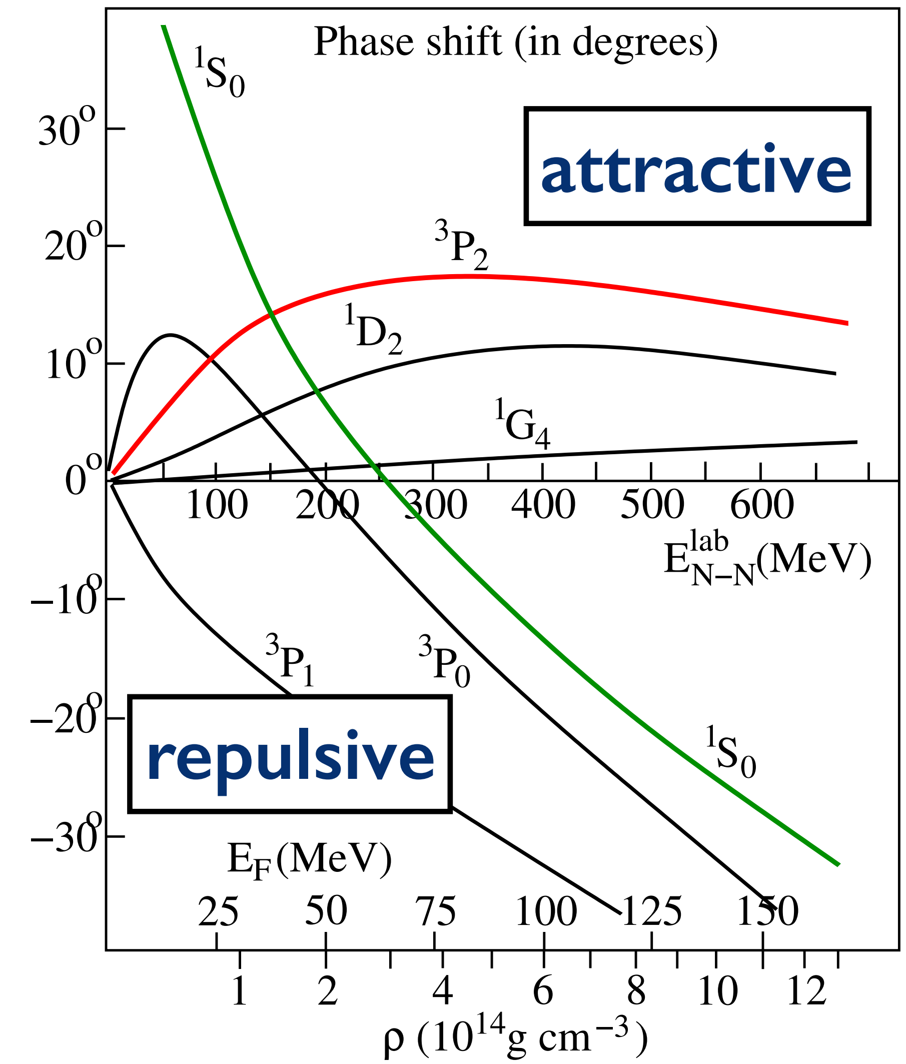
Nucleon Cooper pairing

- Attractive nuclear force induces the Cooper pairing of n-n and/or p-p

- **In the core**
  - n: spin-triplet ($^3P_2$) pairing
  - p: spin-singlet ($^1S_0$) pairing

    this difference is due to the difference of Fermi energy

- In the crust:
  - n: spin-singlet ($^1S_0$) pairing

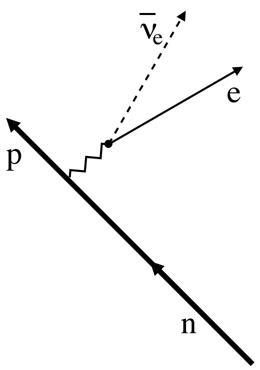


[Calculation by Tamagaki (1970), figure from Page et al. (2013)]

Direct Urca process

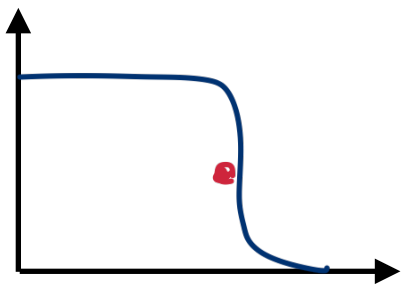
$$n \to p + \ell + \bar{\nu}_\ell \qquad p + \ell \to n + \nu_\ell$$

$\ell = e, \mu$



- Beta decay and its inverse

- Occurs around Fermi surface



- Direct Urca process does not operate unless NS is very heavy

[e.g., Lattimer et al. (1991)]

  - due to the energy and momentum conservations around Fermi surface
  - E.g., for APR EOS, $M \gtrsim 1.97\, M_\odot$ is required
  - We can neglect direct Urca in Cas A NS ($M \simeq 1.4\, M_\odot$)

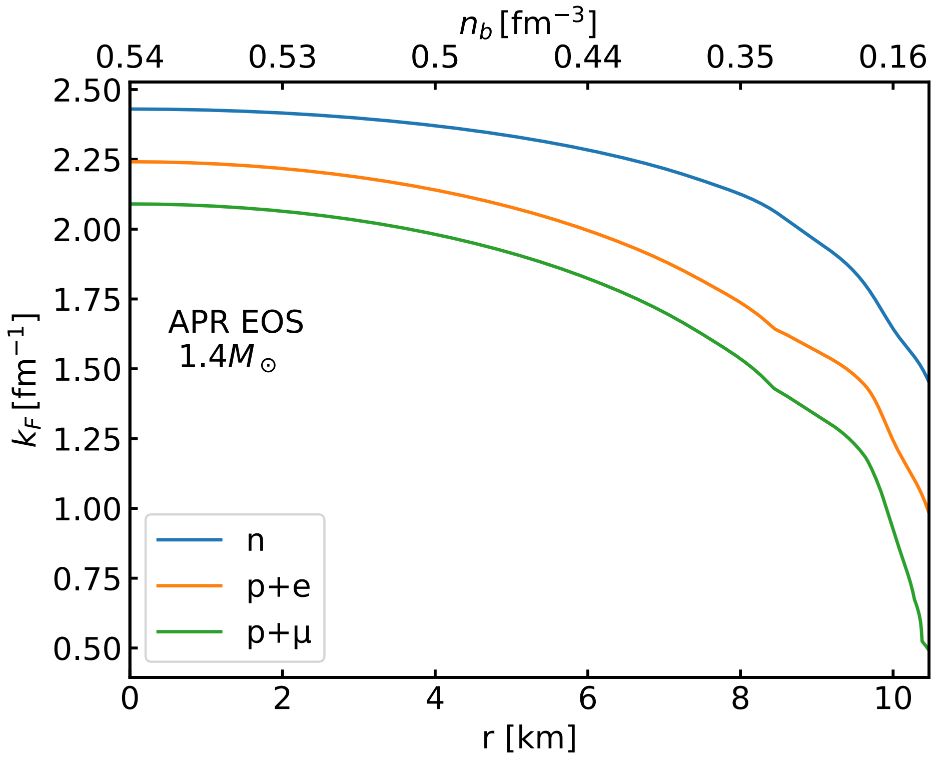
Threshold of direct Urca

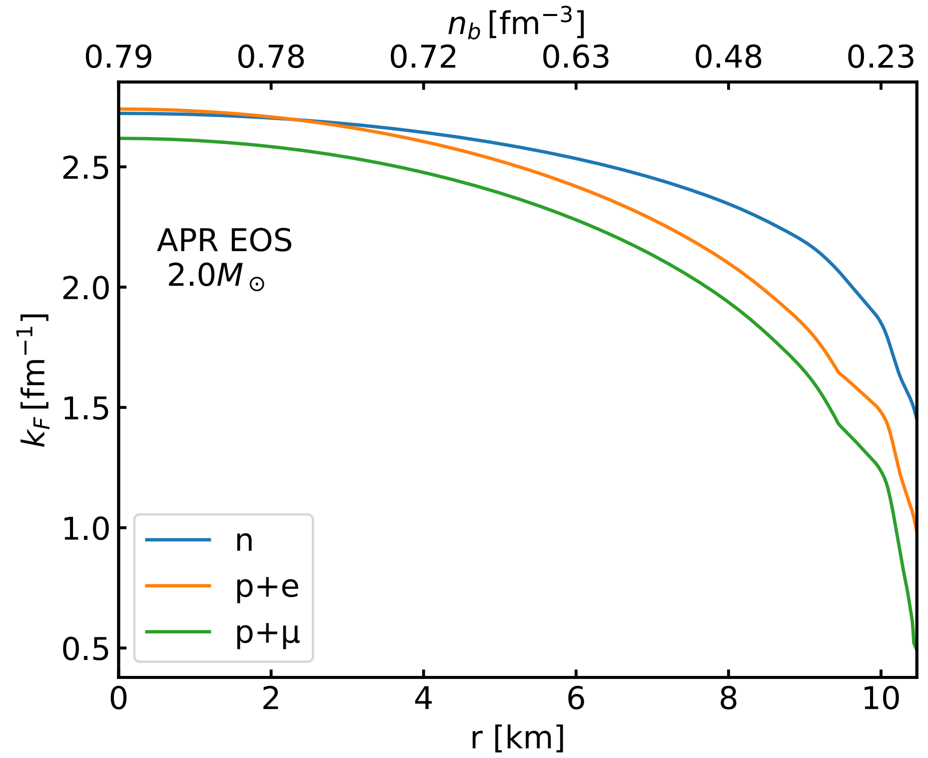
- Energy conservation $\varepsilon_n = \varepsilon_p + \varepsilon_\ell \pm \varepsilon_\nu$ and beta equilibrium $\mu_{F,n} = \mu_{F,p} + \mu_{F,\ell}$

  → Emitted neutrino momentum: $p_\nu \sim T \ll p_F$

- Momentum conservation: $\overrightarrow{p}_n \simeq \overrightarrow{p}_p + \overrightarrow{p}_\ell$

  → $p_{F,n} < p_{F,p} + p_{F,\ell}$, hence large proton fraction, is necessary

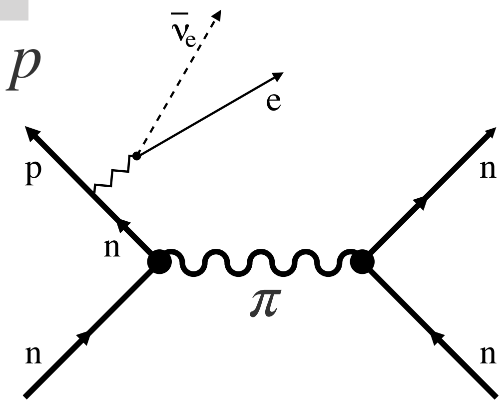




Modified Urca process

$$n + N \to p + N + \ell + \bar{\nu}_\ell \qquad p + N + \ell \to n + N + \nu_\ell$$

$N = n \,\mathrm{or}\, p$



- Threshold is relaxed

- Emissivity

$$Q_{M,N\ell} = \int \left[\prod_{j=1}^{4} \frac{d^3 p_j}{(2\pi)^3} \right] \frac{d^3 p_\ell}{(2\pi)^3} \frac{d^3 p_\nu}{(2\pi)^3} (2\pi)^4 \delta^4(P_f - P_i) \cdot \epsilon_\nu \cdot \frac{1}{2} \sum_{\text{spin}} |\mathcal{M}_{M,N\ell}|^2$$
$$\times \left[f_1 f_2 (1 - f_3)(1 - f_4)(1 - f_\ell) + (1 - f_1)(1 - f_2) f_3 f_4 f_\ell \right],$$

- Before pairing: $L_\nu \propto T^8$

- After pairing: exponentially suppressed by the gap: $f \sim \exp(-\Delta/T) \ll 1$

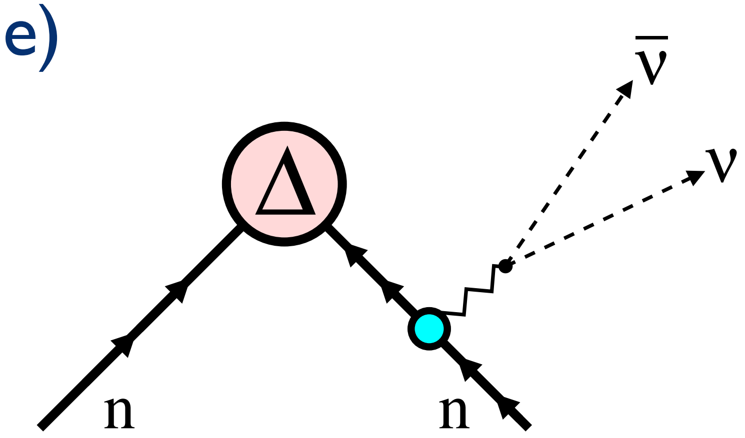
Cooper pair breaking and formation (PBF)

- Cooper pairing triggers another neutrino emission [Flowers et al. (1976)]

  - Pair-breaking: $[\tilde{N}\tilde{N}] \to \tilde{N} + \tilde{N}$ (thermal disturbance)

    Cooper pair **Single (quasi-)nucleon**

  - Pair-formation: $\tilde{N} + \tilde{N} \to [\tilde{N}\tilde{N}] + \nu + \bar{\nu}$

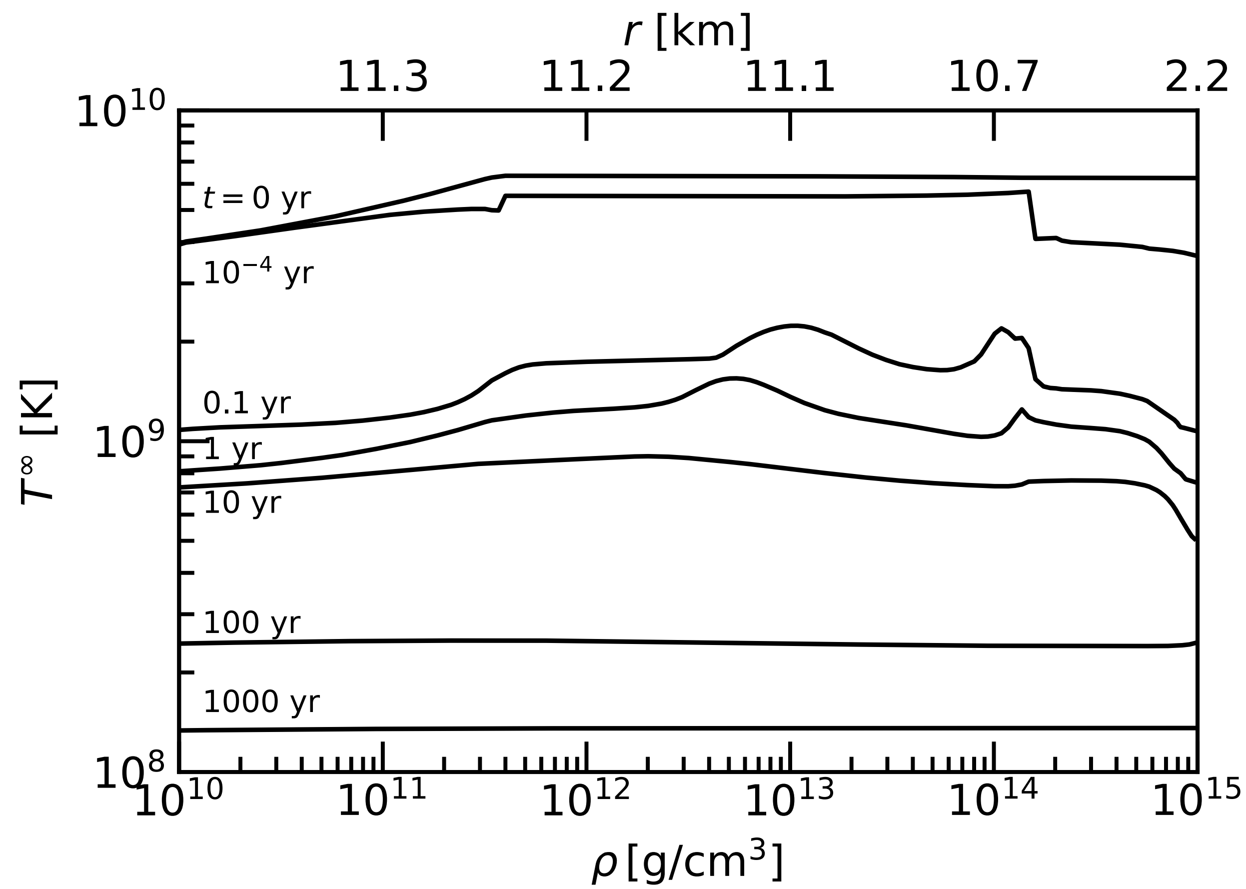


- Does not occur for $T > T_c$

- Efficiently occurs for $T \lesssim T_c$

- Suppressed for $T \ll T_c$ because excitation of quasi-nucleon is suppressed

Thermal relaxation



(a, CCDK) gap

- Relaxation time scale is $t \sim 10 - 100\,\mathrm{yr}$

Neutron singlet gap

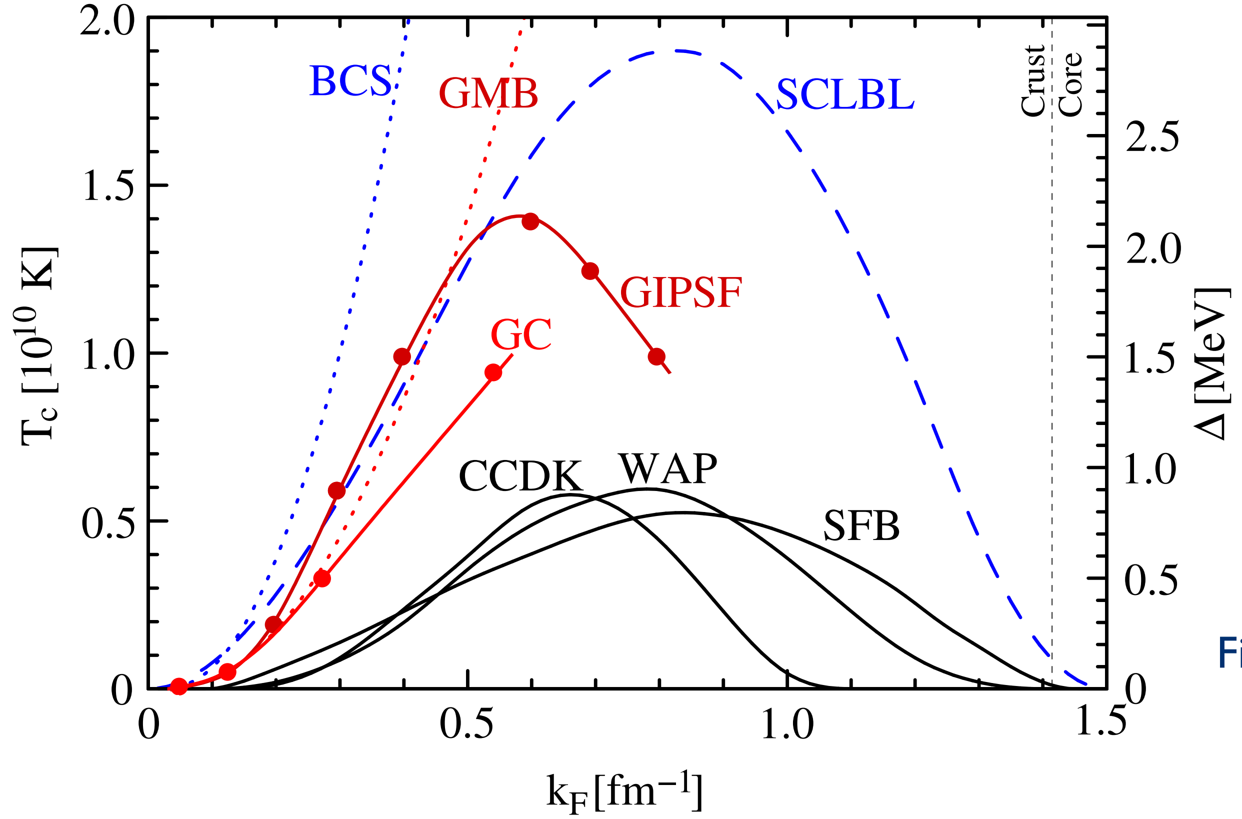
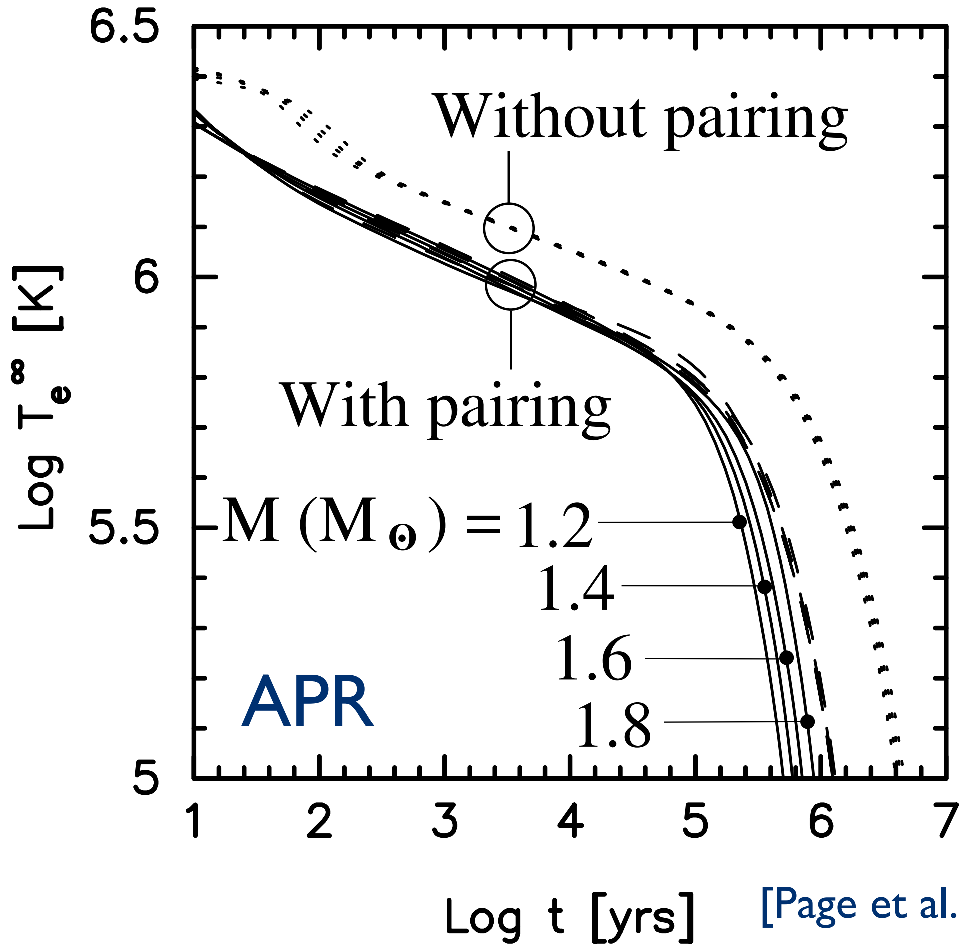


Figure from Page et al. (2013)]

- $T_c \sim (0.5 - 2) \times 10^{10}\,\mathrm{K}$
- Singlet pairing occurs only in the crust

Mass for thermal evolution

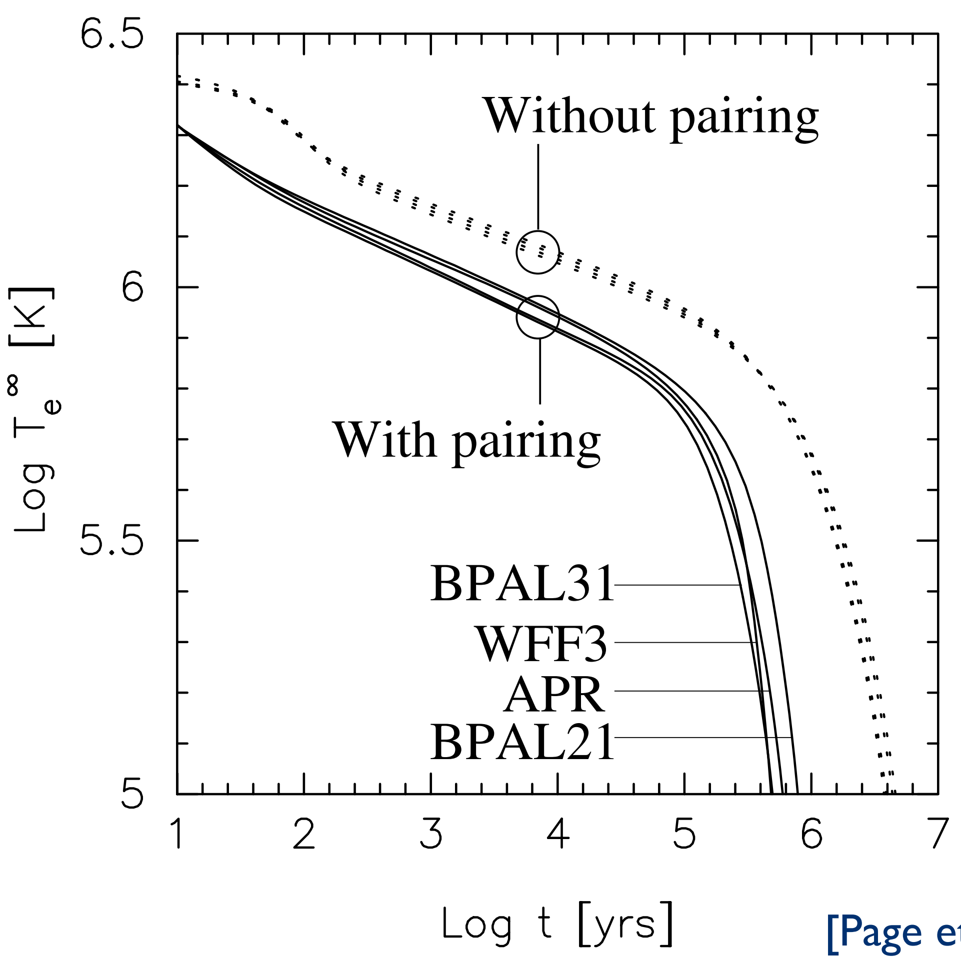


[Page et al. (2004)]

- Dashed: no proton singlet pairing
- Heavy element envelope
- (a, AO) gap

- Difference is due to the density dependence of pairing gap
- Heat capacity and neutrino luminosity slightly change

EOS for thermal evolution



- Heavy element envelope
- (a, AO) gap
- APR and WFF3 use different potential
- BPAL is phenomenological

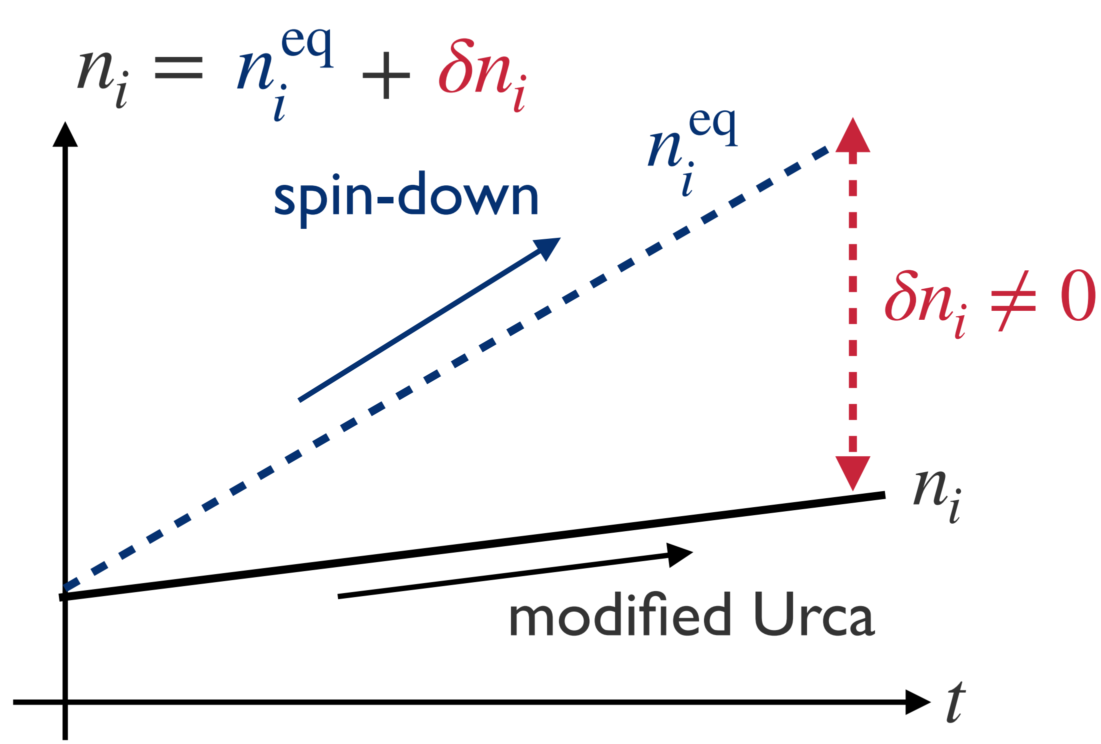
[Page et al. (2004)]

- Difference is also due to the density dependence of pairing gap

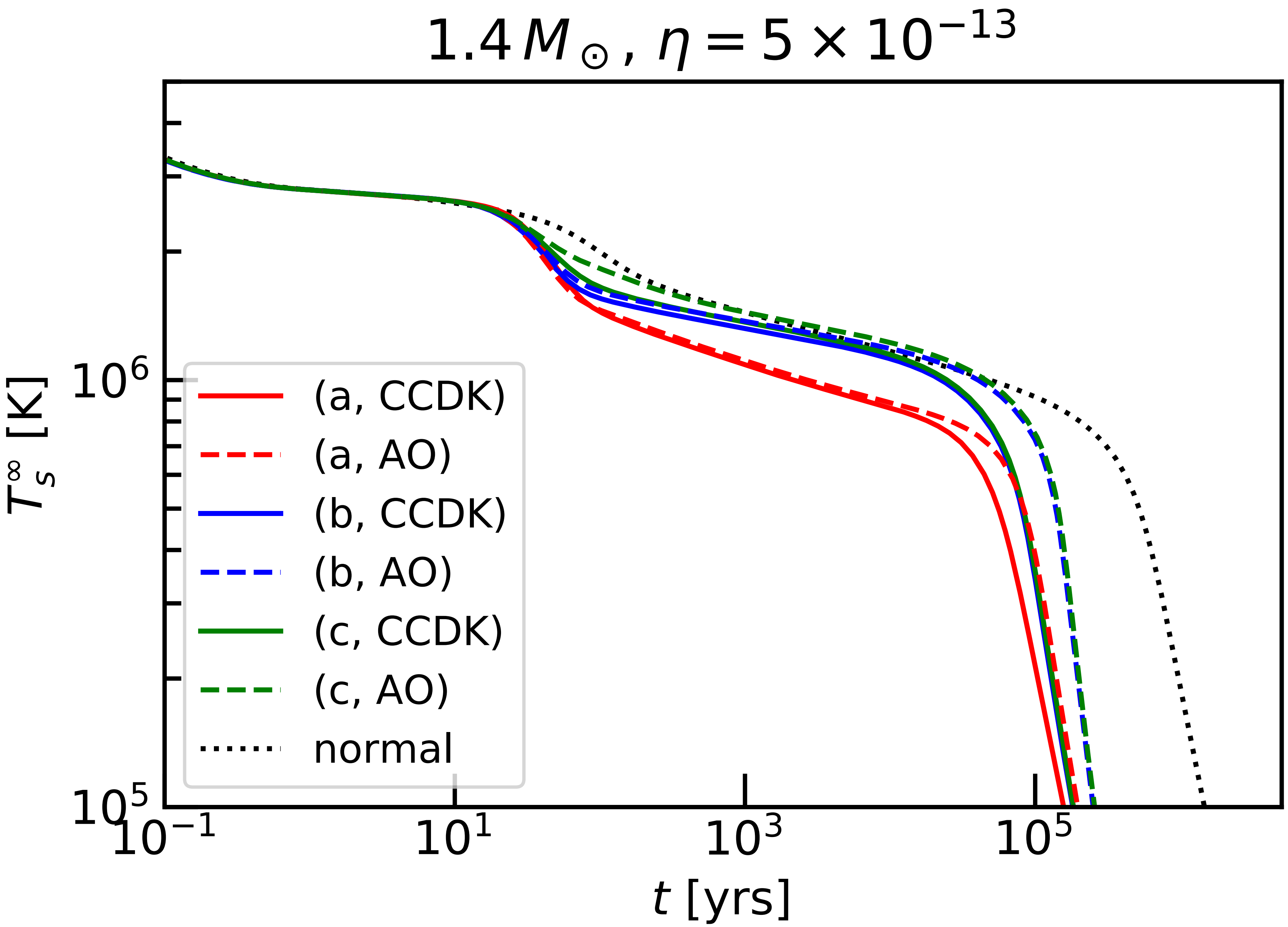
Pulsar spin-down violates β-equilibrium

- Each particle goes to new equilibrium $n_i^{\mathrm{eq}}(t)$ by modified Urca process
- If (modified) Urca is too slow, it cannot catch up with change of $n_i^{\mathrm{eq}}(t)$

(Schematic picture)



Gap dependence of cooling



Neutron star age

- Spin-down age
  - Assume rotational energy loss purely from magnetic dipole radiation

$$\dot{\Omega} = -k\Omega^3 \qquad k = \frac{2B_s^2 R^6 \sin^2\alpha}{3I}$$

$$\Rightarrow P(t) = \frac{2\pi}{\sqrt{P_0^2 + 2P_{\rm now}\dot{P}_{\rm now}t}}$$

if $P_{\rm now} \gg P_0$

$$\Rightarrow \boxed{t_{\rm sd} = \frac{P}{2\dot{P}}}$$

- Kinematic age
  - Estimate age from associated supernova remnant velocity