Realization of impossible anomalies

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Anomaly 101

- Classical conservation law may be broken by quantum effects e.g.

$$\partial^\mu j_\mu = \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$$

- More generally operator relations that cannot be obtained from (local) action principle

- Some anomalies are experimentally observed (others are never observed because it would be inconsistent)

Anomaly Quiz

Rule 1: Multiple choice question (yes or no)

Rule 2: Don’t trust anyone over thirty (Don’t raise your hand if you know QFT more than ten years)

Anomaly Quiz 1

$$\partial^\mu j_\mu = \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$$

is usual chiral anomaly.
But can we have the anomaly such as

$$\partial^\mu j_\mu = F_{\mu\nu} F^{\mu\nu}$$

① Yes we can!

② No you cannot…

Anomaly Quiz 2

Is theta parameter in YM theory renormalized?

$$S = \int d^4x \left(\frac{1}{g^2} F_{\mu\nu} F^{\mu\nu} + \theta \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} \right)$$

$$T^\mu_\mu = \beta_g F_{\mu\nu} F^{\mu\nu} + \beta_\theta \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$$

① Of course yes…

② No. You don't know QFTs!

These two questions are related by SUSY

$$T^{\mu}_{\mu} = \beta_g F_{\mu\nu} F^{\mu\nu} + \beta_\theta \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$$

$$\partial^\mu J^R_\mu = \beta_g \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} + \beta_\theta F_{\mu\nu} F^{\mu\nu}$$

So if one is yes, the other will be yes as well.

Note that CP must be broken to have non zero beta functions for theta parameter (and so does for the unconventional current anomaly)

In textbooks?

- 1-loop computation only gives conventional chiral anomaly
- All loop argument was given by Adler-Bardeen (but I'm not sure if this is proved with CP violation, let alone with non-perturbative effects)

- Wess-Zumino consistency condition? (→ they are allowed)
- Descent formulation (anomaly polynomial in d+2 dimensions) → sufficient but not necessary(??)
- You may want to recheck Kugo-san's textbook after this talk

Theta parameter is most probably renormalized

- I haven't found good reference on it, but…
- 2D NLSM suggests β_g depends (non-perturbatively) on θ
- $\theta = 0$ → confining $\theta = \pi$ → conformal fixed point

- If RG flows are gradient flow, $\beta^a = g^{ab}\partial_b V$

  β_θ must be non-zero

- In holography, it is very likely when dilaton (dual to gauge coupling) has potential, then axion (dual to theta parameter) also has potential (with discrete shift symmetry).

- If we want to gauge it, did we have to cancel this part (which I don't know)?

Impossible Anomalies

Nonperturbative go-go argument in CFT

- General QFTs are too difficult. Focus on CFTs.
- Anomaly is constrained by Wess-Zumino consistency condition
- Although non-local, anomaly is variation of the effective action and variation satisfies an algebra:

$$[\delta_{\lambda(x)}, \delta_{\tilde{\lambda}(y)}] = 0$$

- Non-abelian case is non-trivial, but for Abelian anomaly, the both are trivially consistent (because the second variation is obviously zero)

$$\delta_{\lambda(x)} Z = \lambda(x)(c_1 F^{\mu\nu} F_{\mu\nu} + c_2 \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma})$$

Nonperturbative no-go argument in CFT

- General QFTs are too difficult. Focus on CFTs
- Consider (conserved) current three-point functions

$$\partial^\mu \langle J^1_\mu J^2_\nu J^3_\rho \rangle = ?$$

- Must be zero except at $x_i = x_j$ (this is anomaly!)
- Conformal symmetry dictates the form of $\langle J_\mu J_\nu J_\rho \rangle$ up to regularizations (I'll discuss this later)

- Unregularized three-point functions do not contain parity violating form

→ $\partial^\mu j_\mu = F_{\mu\nu}F^{\mu\nu}$ is impossible!

Nonperturbative argument(?) in CFT

- I will critically discuss if this argument is really anomaly free (pun intended)

- To avoid the confusion, even if true, this argument did NOT say theta parameter is not renormalized. It only says there is no one-loop renormalization for the theta parameter near the conformal fixed point.

Simpler d=2 example: $\partial^\mu J_\mu = c_J R$

- Go-go argument
  - Obviously the RHS is gauge invariant, so Wess-Zumino consistency condition is satisfied
- No-go argument
  - In CFT, this anomaly would correspond to the OPE

$$\langle T(z) J(0) \rangle = \frac{c_J}{z^3}$$

  - $\bar{\partial}$ gives $\partial^2 \delta(z)$, meaning the possible anomaly

$$\bar{\partial} J \propto c_J R$$

  - However, in CFT two-point functions of primary operators must be diagonal w.r.t. conformal weight
    → $c_J = 0$

d=2 example $\partial^\mu J_\mu = c_J R$

- But actually we know this anomaly does exist: ghost number anomaly in world-sheet string theory

- This is based on the (topological) twist

$$T \to \tilde{T} = T + \partial J$$
$$\langle \tilde{T}(z) J(0) \rangle = \frac{c_J}{z^3}$$

- In other words, J is no-longer primary
- The price we had to pay is it is no longer unitary

- This also means theta parameter is remormalized

$$\tilde{T}^\mu_\mu = \epsilon_{\mu\nu} F^{\mu\nu}$$

- Lesson: impossible anomaly can be realized and in this case it has corresponding non-local correlation functions

d=4 example $\partial^\mu j_\mu = F_{\mu\nu}F^{\mu\nu}$

- Go-go argument
  - Obviously the RHS is gauge invariant, so Wess-Zumino consistency is satisfied
- No-go argument
  - In CFT, the (unregulated) three-point function

$$\langle J_\mu J_\rho J_\sigma \rangle = (\mathrm{CP\ odd}) + (\mathrm{CP\ even})$$

  - CP odd term gives usual chiral anomaly
  - CP even terms are

$$D_{\mu\nu\rho}(x,y,z) = \frac{1}{(x-y)^2(z-y)^2(x-z)^2}\frac{\partial}{\partial x_\mu}\frac{\partial}{\partial y_\nu}\log(x-y)^2\frac{\partial}{\partial z_\rho}\log\left(\frac{(x-z)^2}{(y-z)^2}\right)$$

$$C_{\mu\nu\rho}(x,y,z) = \frac{1}{(x-y)^4}\frac{\partial}{\partial x_\mu}\frac{\partial}{\partial z_\alpha}\log(x-z)^2\frac{\partial}{\partial y_\nu}\frac{\partial}{\partial z_\alpha}\log(y-z)^2\frac{\partial}{\partial z_\rho}\log\left(\frac{(x-z)^2}{(y-z)^2}\right)$$

  - The first term breaks conservation with delta function but this is expected violation in non-Abelian current
  - The second term is interesting but does not lead to our term
  - No unregulated three-point functions for impossible anomaly

d=4 example $\partial^\mu j_\mu = F_{\mu\nu}F^{\mu\nu}$

- If symmetry is non-linearly realized, we can construct the three-point functions with conformal invariance and the impossible anomaly

$$S = \int d^4x \left(\frac{1}{2}\phi\Box^2\phi + B^\mu \Box \partial_\mu \phi - \phi F_{\mu\nu}F^{\mu\nu} \right)$$

- The construction is possible because the impossible anomaly is consistent

$$\langle J^B_\mu J_\nu J_\rho \rangle = \int d^4w \langle \Box \partial_\mu \phi(x) \phi(w) \rangle \left(\delta_{\nu\rho} \partial^\alpha \delta^4(w-y) \partial_\alpha \delta^4(w-z) - \partial_\nu \delta^4(w-y) \partial_\rho \delta^4(w-z) \right) \ .$$

$$\langle \partial^\mu J^B_\mu J_\nu J_\rho \rangle = \delta_{\nu\rho} \partial^\alpha \delta^4(x-y) \partial_\alpha \delta^4(x-z) - \partial_\nu \delta^4(x-y) \partial_\rho \delta^4(x-z)$$

- The anomaly is supported by semi-local three-point functions.

Pontryagin trace anomaly?

(see also **arXiv:1201.3428**)

Anomaly Quiz 3

Consider a CFT in curved space-time, which terms can appear in the trace anomaly?

$$T^{\mu}_{\mu} = c\mathrm{Weyl}^2 - a\mathrm{Euler} + bR^2 + d\Box R + e\mathrm{Pontryagin}$$

$$\mathrm{Weyl}^2 = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 2R_{\mu\nu}R^{\mu\nu} + \frac{1}{3}R^2$$

$$\mathrm{Euler} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

$$\mathrm{Pontryagin} = \epsilon^{\alpha\beta}_{\rho\sigma}R_{\mu\nu\alpha\beta}R^{\mu\nu\rho\sigma}$$

Trace anomaly in d=4

$$T^{\mu}_{\mu} = c\mathrm{Weyl}^2 - a\mathrm{Euler} + bR^2 + d\Box R + e\mathrm{Pontryagin}$$

- a, c, d and e are all consistent
- b is not consistent
- d is trivial (i.e. can be removed by local counterterms)
- While we were chatting when we were students, Yuji Tachikawa suggested the possibility of e.

- After some computations, we can show the Pontryagin term is impossible in the sense there is no conformal invariant non-local three-point functions of CP violating <TTT>

Debates in Pontryagin trace anomaly

- Bonora et al claimed that a Lorentzian Weyl fermion gives the Pontryagin trace anomaly (which I don't believe at its face value)
- Duff et al showed a Euclidean Majorana Weyl fermion gives the Pontryagin "trace anomaly" (which I do believe but the definition is very subtle)
- These should be given by semi-local three-point functions of <TTT> rather than non-local ones
- As in the chiral anomaly case, we can construct the effective action

$$S = \int d^4x \sqrt{g} \left(\frac{1}{2} \phi \Delta_4 \phi - Q \mathcal{Q} \phi - \phi \epsilon^{\alpha\beta\gamma\delta} R_{\alpha\beta\mu\nu} R^{\mu\nu}{}_{\gamma\delta} \right) \ ,$$

$$\langle T^{\mu}_{\ \mu}(x) T_{\sigma\rho}(y) T_{\alpha\beta}(z) \rangle = Q \epsilon_{\sigma\alpha\epsilon\kappa} [(\partial_\beta \partial_\rho - \partial^2 \delta_{\beta\rho})(\partial^\epsilon \delta(x-y) \partial^\kappa \delta(x-z))] + \mathrm{sym}$$

- The full semi-local three-point function can be found in master thesis of Nakagawa
- It remains to see if the computation by Bonora et al gives THIS (or the other) semi-local term in full 3pt functions

Summary

- Impossible anomaly may be possible
- But effective field theory may (must?) violate the unitarity
- Studies of semi-local correlation functions in CFT sound interesting
- It remains open if the realization of (impossible) anomaly from semi-local correlation function is physically meaningful (certainly we do use them to cancel anomalies in non-CFTs)
- Don’t trust over anyone thirty (or QFT textbooks)