Soft gluon resummation effects for processes with final state jets

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Outline

- QCD, Parton Model and Factrorization
- Precision electroweak Physics at Hadron Colliders
- Transverse Momentum Resummation framework
- Drell-Yan production
- Higgs +two jets production
- T-channel single-top quark production
- Summary

Quantum Chromodynamics

Fields: Quarks ψ^{color}_{flavor} and Gluon $G^{color}(A{\cdot}T, g)$.

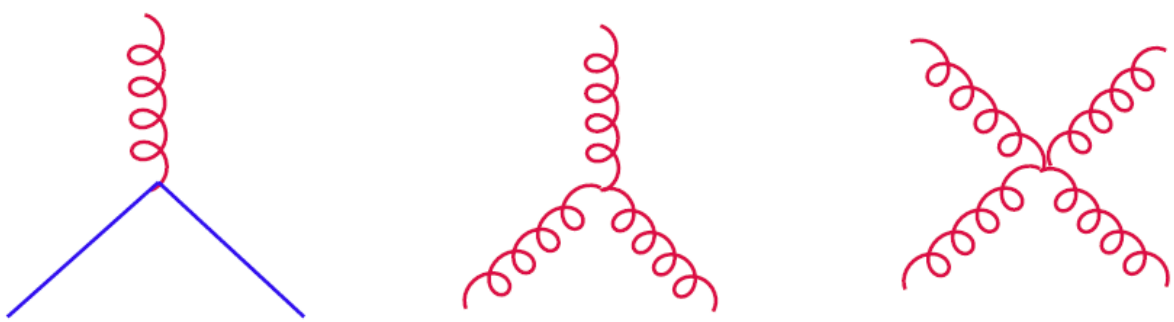
Basic Lagrangian:

$$\mathcal{L} = \bar{\psi}(i \not{\partial} - g \not{A} \cdot t - m)\psi - \frac{1}{4} G(A \cdot T, g) \cdot G(A \cdot T, g)$$

- g: gauge Coupling Strength
- m_i: quark masses
- t & T: color SU(3) matrices in the fundamental and adjoint representations.

  Group factors: $C_F = \frac{4}{3}$; $T_F = \frac{1}{2}$; $C_A = 3$

Interaction Vertices:



Why does QCD play such a crucial role in High Energy Phenomenology?

- The parton picture language provides the foundation on which all modern particle theories are formulated, and all experimental results are interpreted.
- The validity of the parton picture is based empirically on an overwhelming amount of experimental evidence collected in the last 40-50 years, and theoretically on the Factorization Theorems of PQCD.

How could the *simple* (almost non-interacting) *parton picture* possibly hold in QCD — a strongly interacting quantum gauge field theory?

Answer: 3 unique features of QCD:

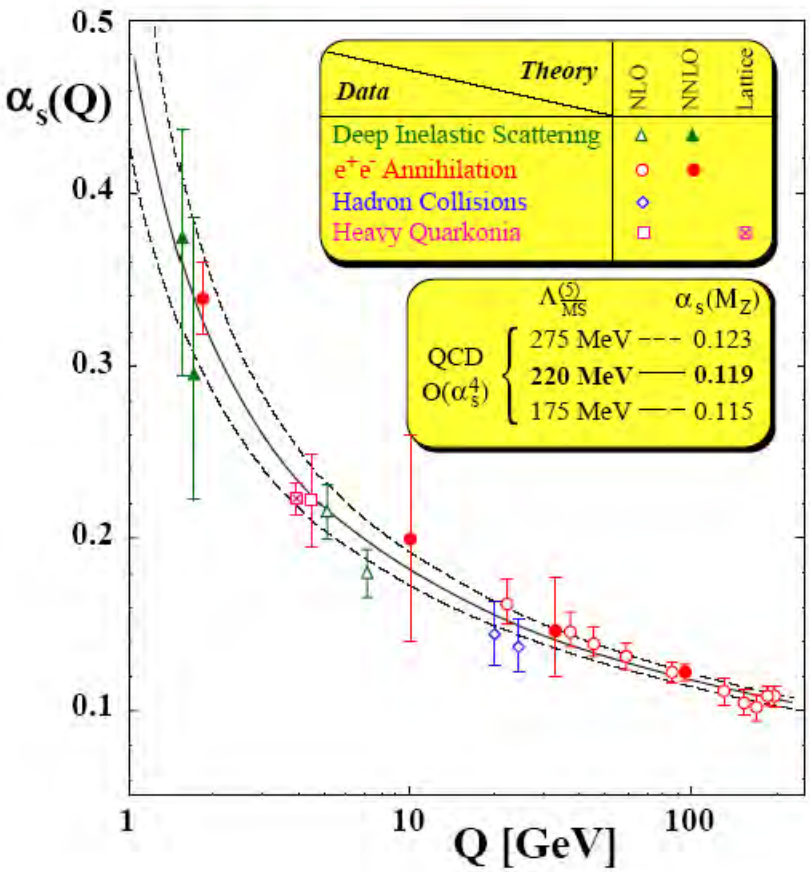
1. Asymptotic Freedom:
   A strongly interacting theory at long-distance can become weakly interacting at short-distance.
2. Infra-red Safety:
   There are classes of *infra-red safe* quantities which are independent of long-distance physics, hence are calculable in PQCD.
3. Factorization:
   There are an even wider class of physical quantities which can be *factorized* into a long-distance piece (not calculable, but *universal*) and short-distance piece (process-dependent, but infra-red safe, hence *calculable*).

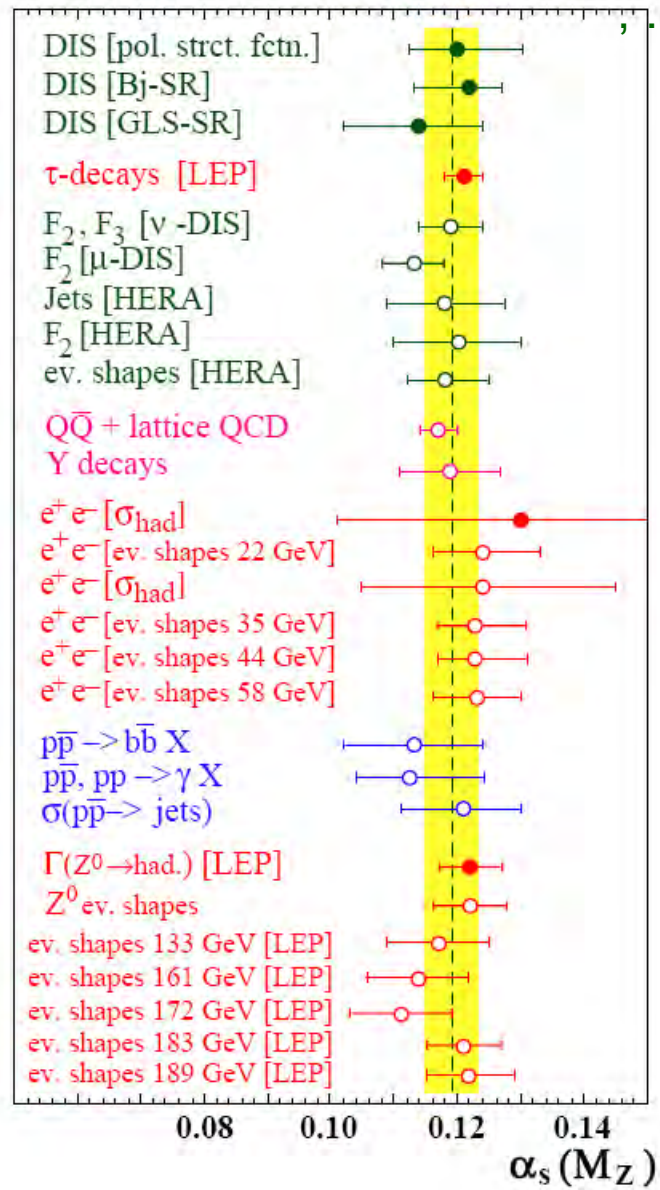
Key concepts: Ultra-violet divergences, bare Green fns, renormalization, RGE, anomalous dimensions, renormalized G.Fs , … etc.

Asymptotic Freedom

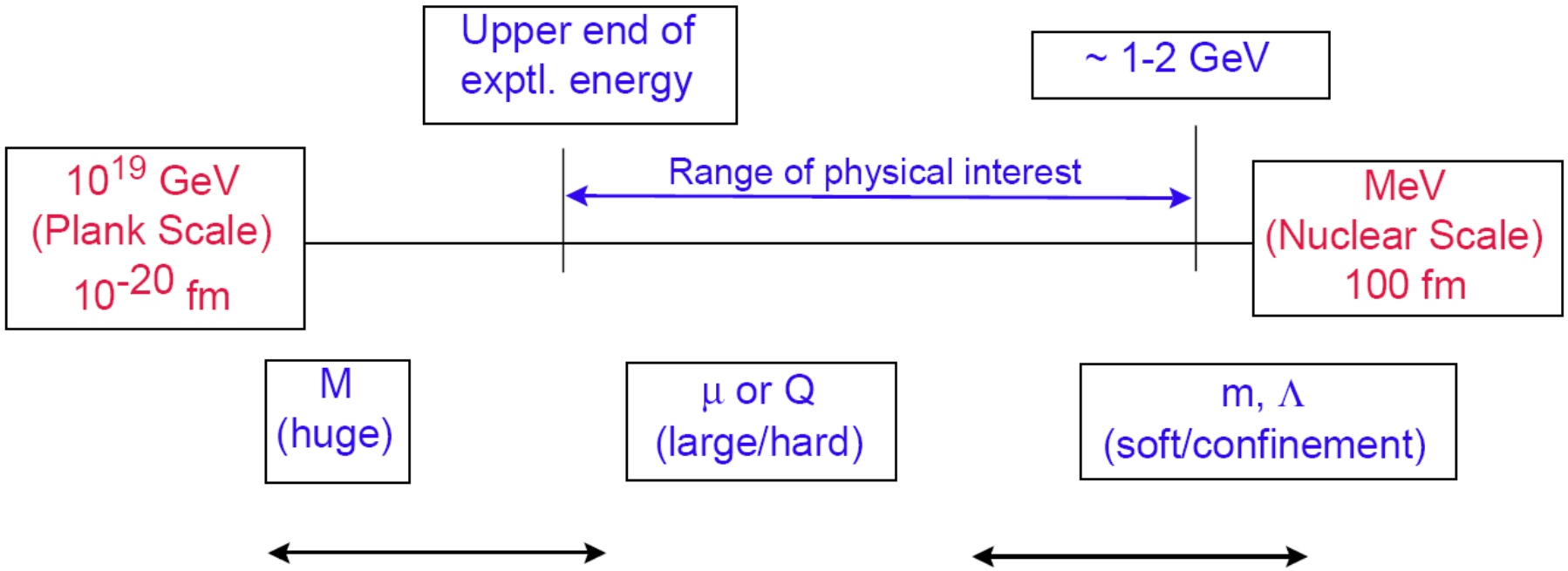
Universal (running) coupling:

$$\alpha_s(Q) = \frac{g^2}{4\pi} = \frac{b}{\ln(Q/\Lambda)}(1 + \ldots)$$





The importance of ***Scales*** -- Renormalization and Factorization



Renormalization Group Equations (RGE) relates physics at different scales

Ultra-violet Renormalization hides / summarizes our ignorance of physics at huge scale in $\alpha_s(\mu_R)$, $m_i(\mu_R)$, ...	Analogies and correspondences (see later) ↔	Infra-red / collinear Factorization hides / summarizes non-perturbative QCD physics at confinement scale in $f_a(x, \mu_F)$, $d_a(x, \mu_F)$, ...

"Renormalization" and "Factorization"

	UV renormalization		Collinear/soft factorization	
A:	Bare Green Func.	$G_0(\alpha_0, m_0, ..)$	Partonic X-sect	F_a
B:	Ren. constants	$Z_i(\mu)$	Pert. parton dist.	$f_a^b(\mu)$
C:	Ren. Green Fun.	$G_R = G_0/Z$	Hard X-sect	$\widehat{F} = F\,/\,f$
D:	Anomalous dim.	$\gamma = \frac{\mu}{Z}\frac{d}{d\mu}Z$	Splitting fun.	$P = \frac{\mu}{f}\frac{d}{d\mu}f$
E:	Phys. para. α, m	$\alpha_0 Z_i \ldots$	Had. parton dist. f_A	resummed
F:	Phys sc. amp.	$\alpha(\mu)\, G_R(m,\mu)$	Hadronic S.F.'s F_A	$f_A(\mu) \times \widehat{F}(\mu)$

Some common features:

A : divergent; but, independent of "scheme" and scale μ;

B : divergent; scale and scheme dependent;
universal; absorbs all ultra-violet/soft/collinear divergences;

C & D : finite; scheme-dependent;
D controls the μ dependence of E & F;

E : physical parameters to be obtained from experiment;

F : Theoretical "prediction"; μ-indep. to all orders,
but μ-dep. at finite order n; $\mu\frac{d}{d\mu} \sim \mathcal{O}(\alpha^{n+1})$

Note: "Renormalization" is factorization (of UV divergences);
"factorization" is renormalization (of soft/collinear div.)

What to do with the long-distance physics associated with colinear/soft singularities in PQCD?

1st strategy:

Identify physical observables which are insensitive to the singularities! (Infra-red-safe (IRS) quantities)

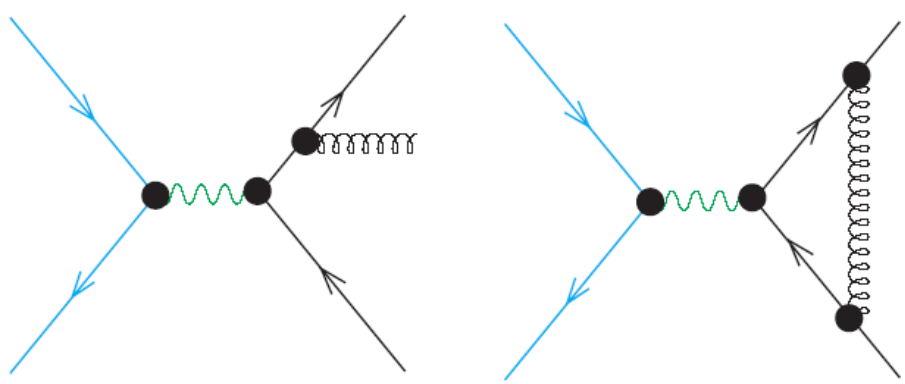
Total Hadronic Cross-section (*inclusive*):

$$\sigma_{tot}(s) = \sigma_0(s)[1 + \alpha_s(s)\, c_1 + \ldots]$$

Block – Nordsieck Thm $\rightarrow$ $c_{1,2,\ldots}$ *are finite, i.e. IRS (unitarity)*

Order α_s:

Cancellation of the colinear/soft singularities between real and virtual diagrams



Infra-Red-Safe observables:

Total hadronic Cross-section $\sigma_{tot}/\sigma_{\mu+\mu-}$

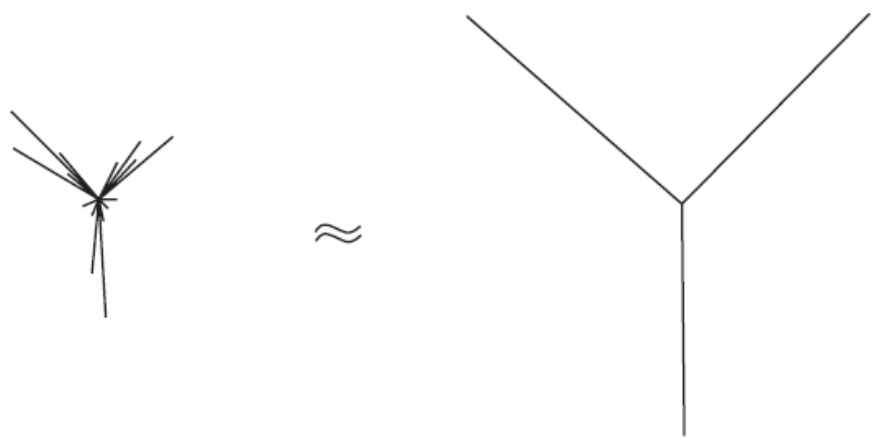
Sterman-Weinberg jet cross-sections and their modern variations (*Jade-, Durham-, ... algorithms*);

Jet shape observables: Thrust, ... ;

energy-energy correlation ; ....

Essential feature of a general IRS physical quantity:

the observable must be such that it is insensitive to whether n or n+1 particles contributed -- if the n+1 particles has n-particle kinematics



e.g. a IRS "jet algorithm"

$$R = \frac{\sigma(e^-e^+ \to hadrons)}{\sigma(e^-e^+ \to \mu^-\mu^+)}$$

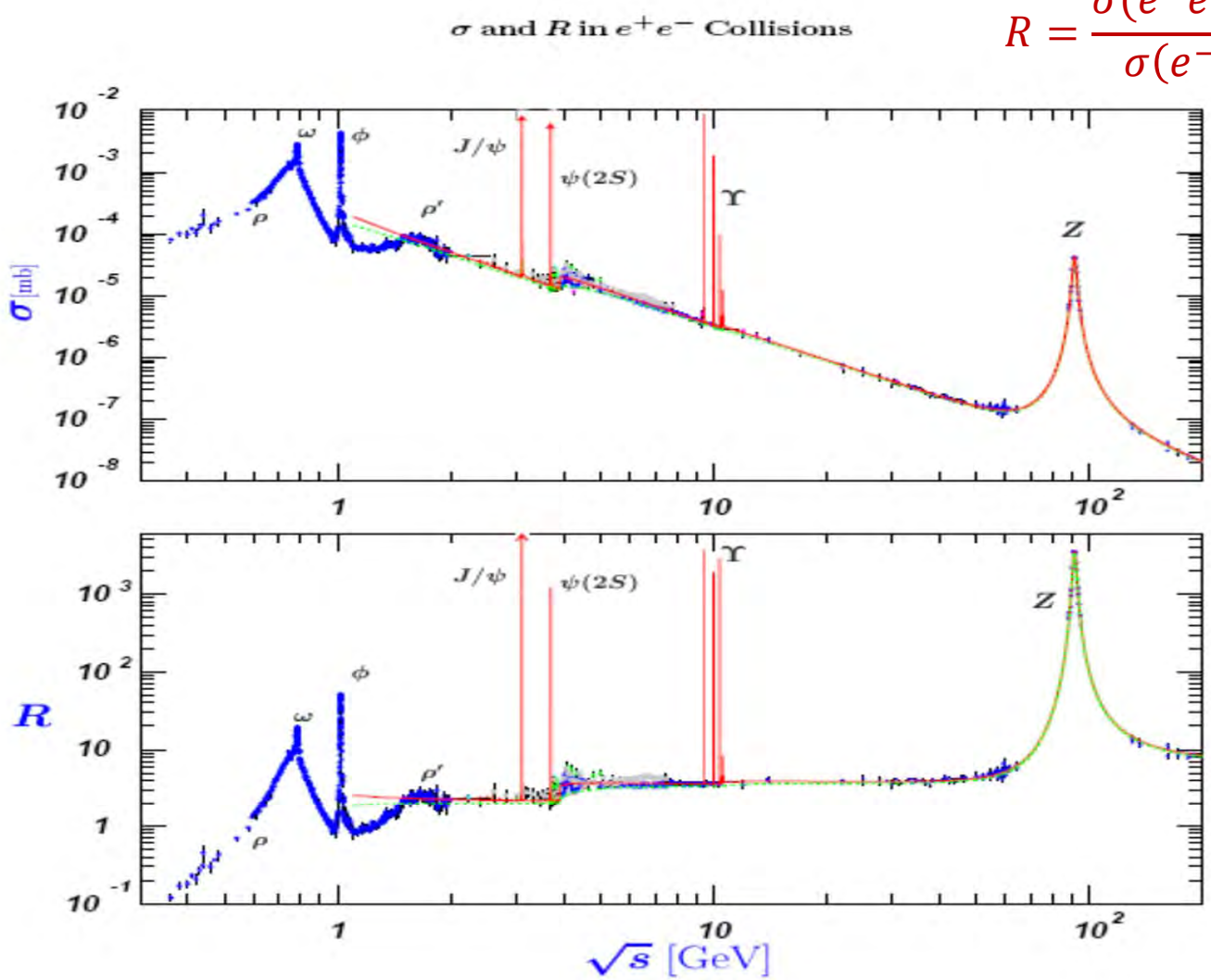


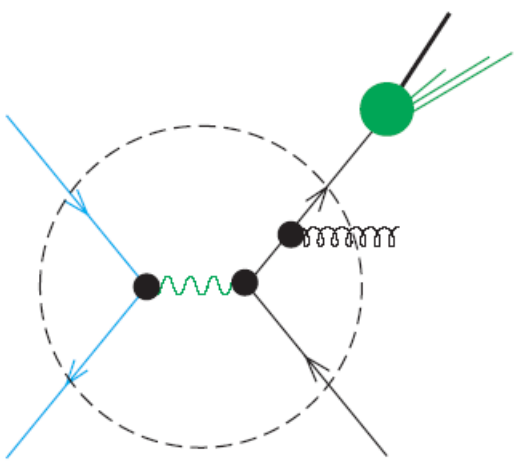
Figure 40.6: World data on the total cross section of $e^+e^- \to hadrons$ and the ratio $R(s) = \sigma(e^+e^- \to hadrons, s)/\sigma(e^+e^- \to \mu^+\mu^-, s)$. $\sigma(e^+e^- \to hadrons, s)$ is the experimental cross section corrected for initial state radiation and electron-positron vertex loops, $\sigma(e^+e^- \to \mu^+\mu^-, s) = 4\pi\alpha^2(s)/3s$. Data errors are total below 2 GeV and statistical above 2 GeV. The curves are an educative guide: the broken one (green) is a naive quark-parton model prediction and the solid one (red) is 3-loop pQCD prediction (see "Quantum Chromodynamics" section of this *Review*, Eq. (9.12) or, for more details, K. G. Chetyrkin *et al.*, Nucl. Phys. B586, 56 (2000) (Erratum *ibid.* B634, 413 (2002)). Breit-Wigner parameterizations of J/ψ, $\psi(2S)$, and $\Upsilon(nS)$, $n = 1, 2, 3, 4$ are also shown. The full list of references to the original data and the details of the R ratio extraction from them can be found in [arXiv:hep-ph/0312114]. Corresponding computer-readable data files are available at http://pdg.ihep.su/xsect/contents.html. (Courtesy of the COMPAS(Protvino) and HEPDATA(Durham) Groups, August 2005. Corrections by P. Janot (CERN) and M. Schmitt (Northwestern U.)) See full-color version on color pages at end of book.

The 2nd strategy:

Factorization $\longrightarrow$ QCD Parton model

Factorize the physical observable into a calculable ***IRS*** part and a non-calculable but ***universal*** piece.

Example: One particle inclusive cross-section



Fragmentation function:
Long-distance physics;
Universal.

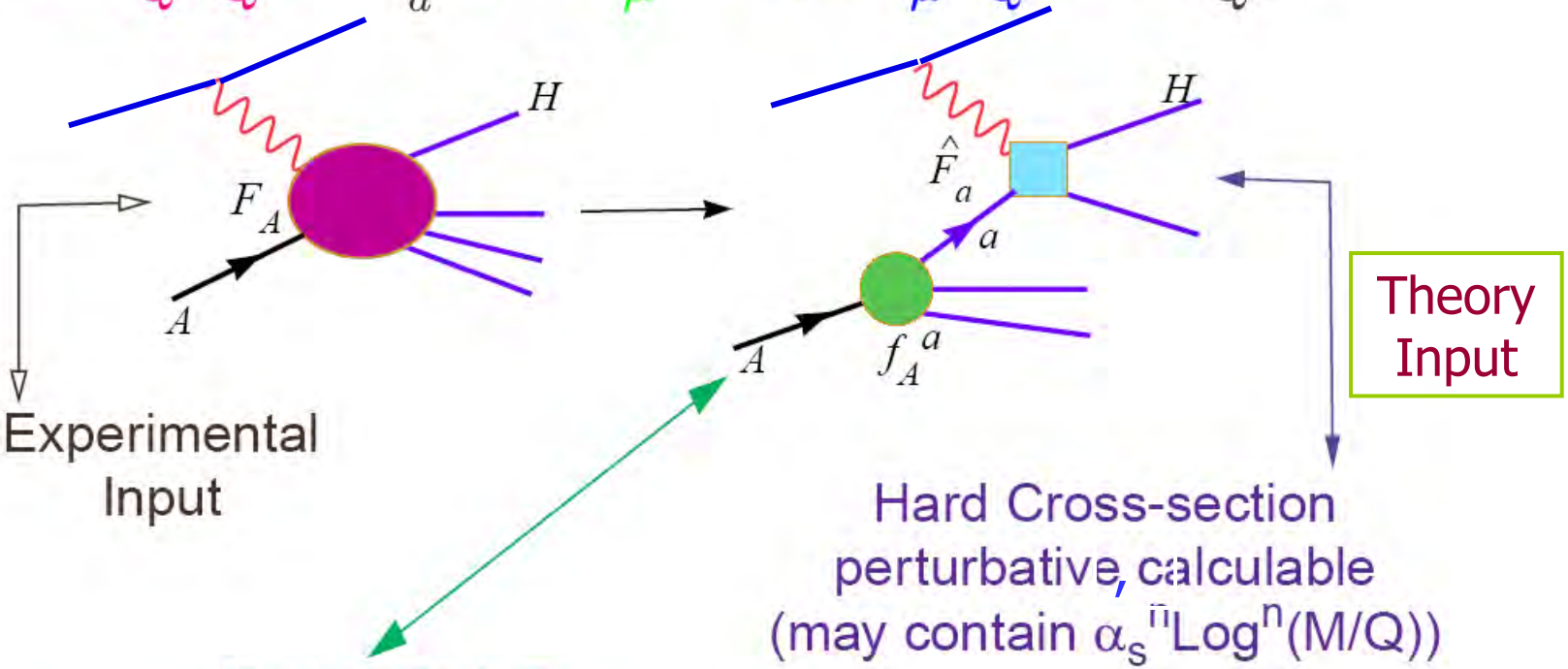
Hard scattering:
Short distance physics;
IRS, perturbatively cal.

$$\sigma(s,z) = \hat{\int}_z^1 \frac{d\varsigma}{\varsigma}\, \hat{\sigma}^a\left(\frac{s}{\mu}, \frac{z}{\varsigma}, \alpha_s(\mu)\right) \cdot D_a(\varsigma, \mu)$$

Lepton-hadron Sc.

Master Equation for QCD Parton Model – the Factorization Theorem

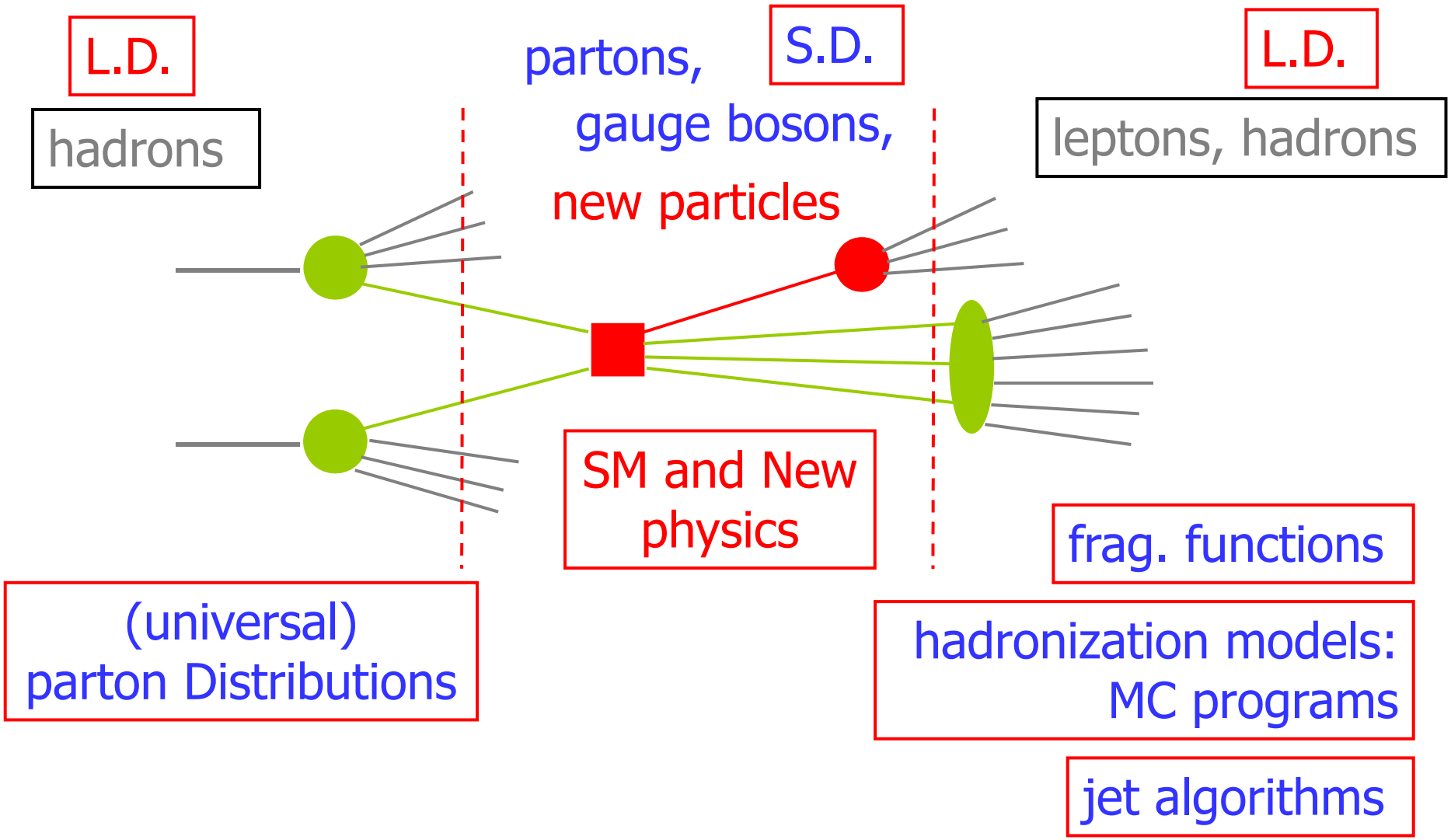
$$F_A^\lambda(x, \frac{m}{Q}, \frac{M}{Q}) = \sum_a f_A^a(x, \frac{m}{\mu}) \otimes \hat{F}_a^\lambda(x, \frac{Q}{\mu}, \frac{M}{Q}) + \mathcal{O}((\frac{\Lambda}{Q})^2)$$



universal Parton Dist. Fn.
Non-Perturbative Parametrization at Q_0
DGLAP Evolution to Q

Extracted by global analysis

Hadron Collider Physics



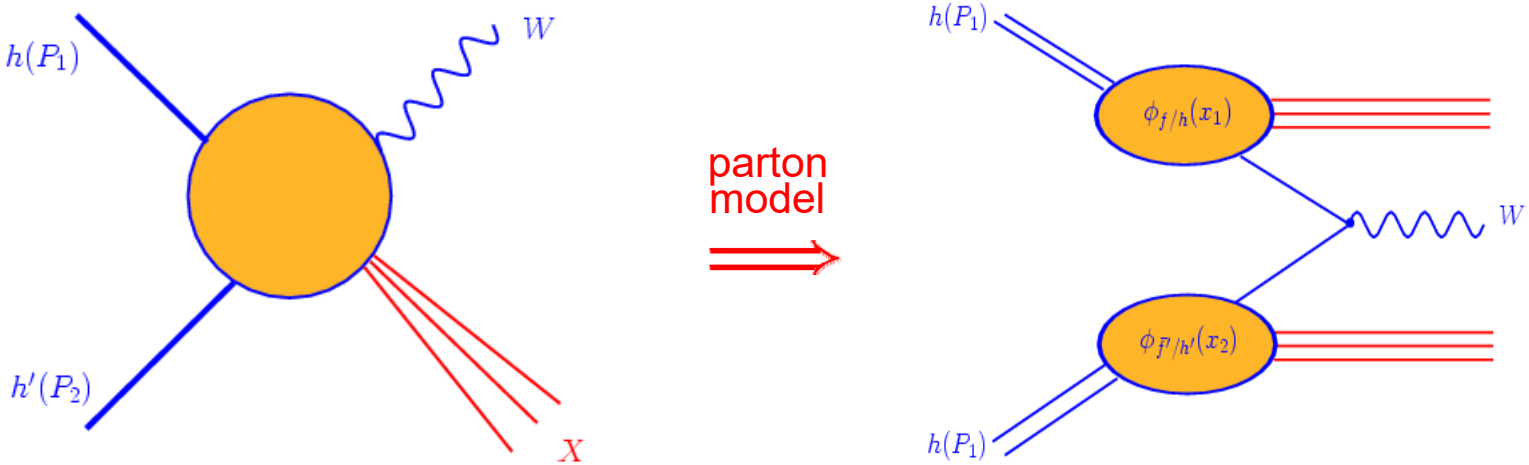
Precision Electroweak Physics at Hadron Colliders

Physics of Drell-Yan, W and Z Bosons

W-boson physics

1. W-boson production and decay at hadron collider
2. How to measure W-boson mass and width?
3. High order radiative corrections:
   - ☞ QCD (NLO, NNLO, Resummation)
   - ☞ EW (QED-like, NLO)
4. ResBos and ResBos-A

W-boson production at hadron colliders

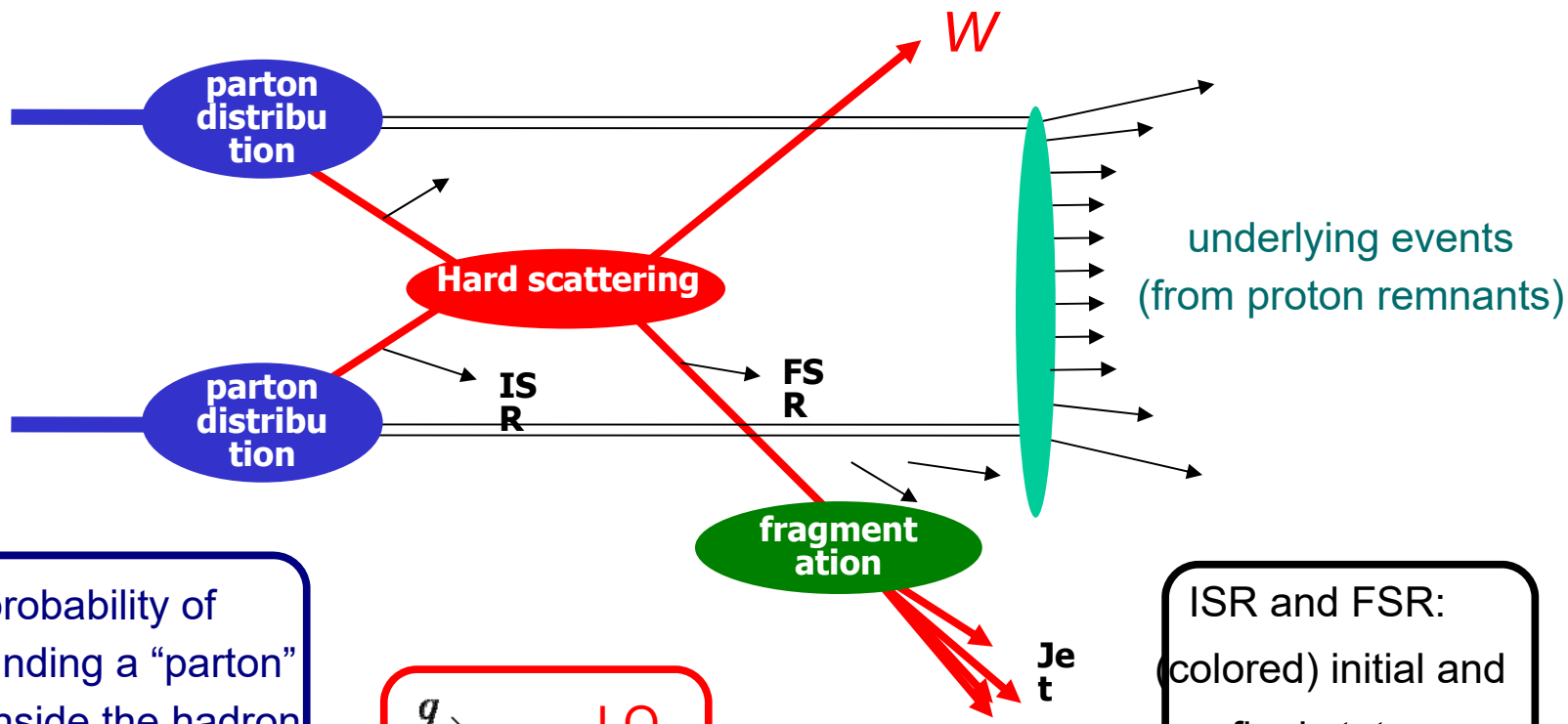


$$\sigma_{hh'\to W^+X} = \sum_{f,f'} \int_0^1 dx_1 dx_2 \left\{ \phi_{f/h}(x_1)\, \hat{\sigma}_{ff'} \phi_{\bar{f}'/h'}(x_2) + (x_1 \leftrightarrow x_2) \right\}$$

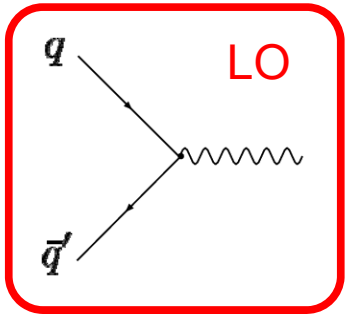
PDFs are known from deep inelastic scattering

partonic "Born" cross section of $f\bar{f}' \to W^+$

W-boson production at hadron colliders



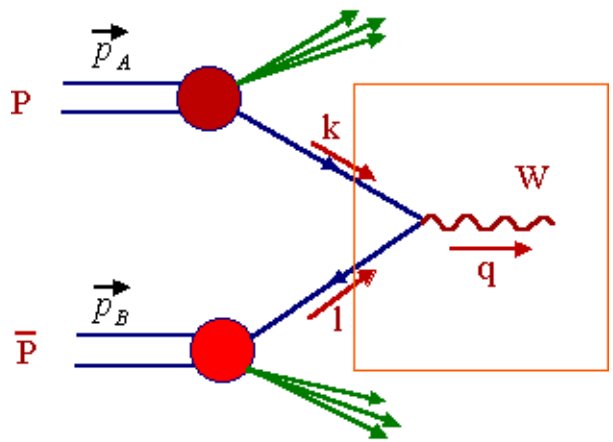
PDFs: probability of finding a "parton" inside the hadron



ISR and FSR: (colored) initial and final states can radiate gluons

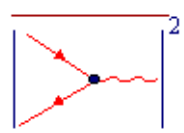
Fixed order pQCD prediction

$\alpha_S^{(0)}$



$$\sigma = \frac{1}{2S}\int \frac{d\xi_A}{\xi_A}\frac{d\xi_B}{\xi_B} f_{i/A}(\xi_A,\mu)\, f_{i/B}(\xi_B,\mu)\cdot d\hat{\sigma}$$

$$d\hat{\sigma} = \underbrace{\overline{|M|^2}}(2\pi)^4\,\delta^{(4)}(q-k-l)\frac{d^3q}{(2\pi)^3\,2q_0}$$



$$\frac{d\sigma}{dq_T^2\,dy\,dQ^2} = \frac{1}{S}\int \frac{d\xi_A}{\xi_A}\frac{d\xi_B}{\xi_B} f_{i/A}(\xi_A,\mu)\, f_{i/B}(\xi_B,\mu)$$

$$\cdot\left(\frac{\pi^2}{Q^2}\right)\cdot\overline{|M|^2}\cdot\delta\left(1-\frac{x_A}{\xi_A}\right)\cdot\delta\left(1-\frac{x_B}{\xi_B}\right)$$

$$\cdot\delta\left(q_T^2\right)\cdot\delta\left(Q^2-M_W^2\right)$$

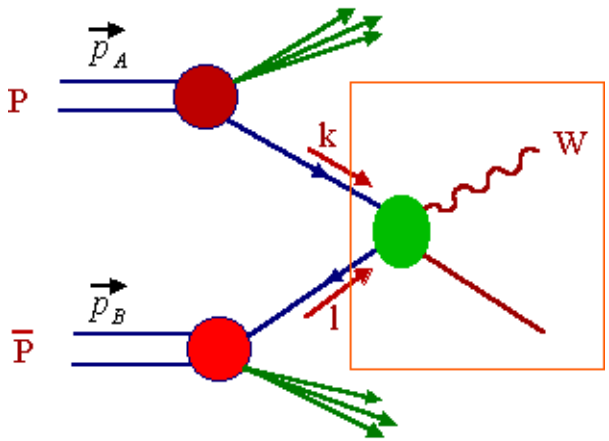
$$s = \left(p_A + p_B\right)^2$$

$$k = \xi_A p_A$$

$$l = \xi_B p_B$$

$$Q \equiv \sqrt{Q^2} = \sqrt{q^2},\ \mu = Q = M_W,\ x_A = \frac{Q}{\sqrt{S}}e^{y},\ x_B = \frac{Q}{\sqrt{S}}e^{-y}$$

$\alpha_S^{(1)}$



$$\frac{d\sigma}{dq_T^2 dy dQ^2} = \int \frac{d\xi_A}{\left(\xi_A S + U - Q^2\right)} \left(\frac{\hat{s} d\hat{\sigma}}{d\hat{t}}\right) \cdot f_{i/A}\left(\xi_A, \mu\right)$$

$$\cdot f_{j/B}\left(\xi_B = \frac{-Q^2 - \xi_A\left(T - Q^2\right)}{\xi_A S + U - Q^2}, \mu\right) \cdot \delta\left(Q^2 - M_W^2\right)$$

$$+ \int \frac{d\xi_B}{\left(\xi_B S + T - Q^2\right)} \left(\frac{\hat{s} d\hat{\sigma}}{d\hat{t}}\right) \cdot f_{j/B}\left(\xi_B, \mu\right)$$

$$\cdot f_{i/A}\left(\xi_A = \frac{-Q^2 - \xi_B\left(U - Q^2\right)}{\xi_B S + T - Q^2}, \mu\right) \cdot \delta\left(Q^2 - M_W^2\right)$$

$$T = Q^2 - \sqrt{q_T^2 + Q^2}\sqrt{S}\ e^{-y},$$

$$U = Q^2 - \sqrt{q_T^2 + Q^2}\sqrt{S}\ e^{y},$$

$$\hat{s} = \xi_A \xi_B S$$

$$\hat{t} = \xi_A\left(T - Q^2\right) + Q^2$$

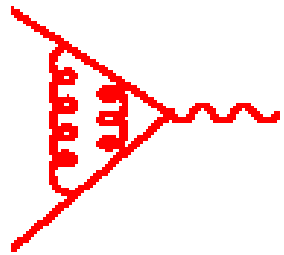
$$\frac{\hat{s}\, d\hat{\sigma}}{d\hat{t}} = \frac{1}{16\pi^2}\overline{\left|M\right|^2}$$

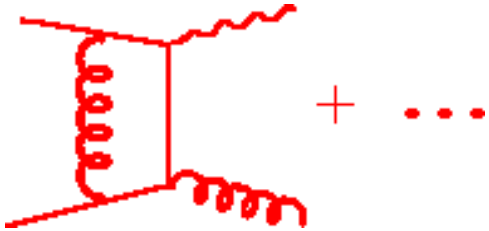
$M =$ 

(For simplicity, only consider qq→Wg)

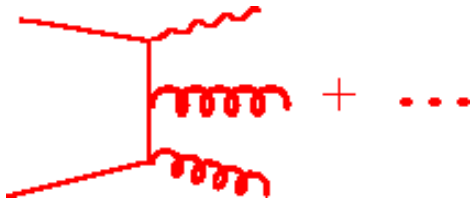
$\alpha_S^{(2)}$

- Virtual Corrections





- Real emission contributions



Theory Calculations

There are a variety of programs available for comparison of data to theory and/or predictions.

- Tree level (Alpgen, CompHEP, Grace, Madgraph…)

  ↓ Les Houches accord

- Parton shower Monte Carlos (Herwig, Pythia,…

  ↑ MC@NLO

- N^nLO (EKS, Jetrad, Dyrad, Wgrad, Zgrad,.Horace

  ↓ recover NLO (NNLO?) normalization

- Resummed (ResBos)

Important to know strengths/weaknesses of each.

Fixed order Perturbative calculations

- Higher order in $\alpha_S^{(n)}$
  ➡ Less sensitive to Factorization Scale μ

- High q_T and smaller y (i.e. more central)
  ➡ PDF (parton distribution function) better known

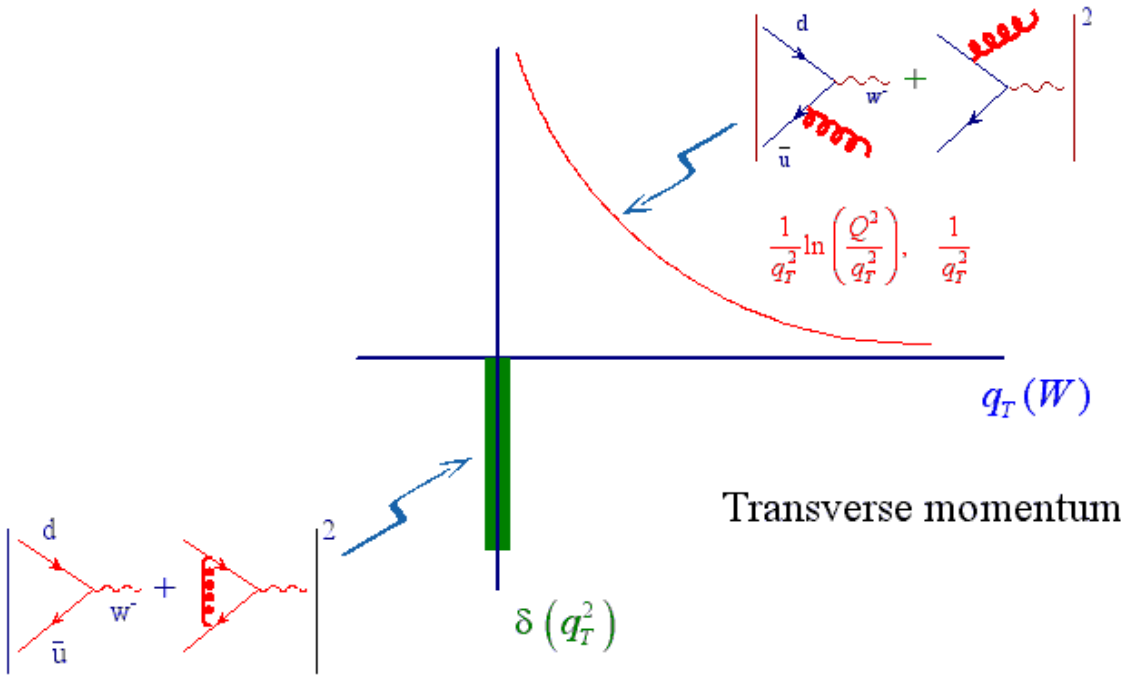
- With larger Luminosity
  ➡ Test QCD in one large scale problem (i.e. $q_T \sim Q$)

- Up to now, most of the Data used in Testing QCD were
  One large scale observables, e.g., Jet-P_T.

- Observables involving Multiple Scales, e.g., q_T of W-Boson with mass M_W, can only be accurately described in QCD after including effects of Resummation.

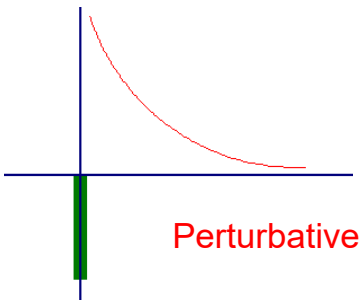
Shortcoming of fixed order calculation

- Cannot describe data with small q_T of W-boson.
- Cannot precisely determine m_W at hadron colliders without knowing the transverse momentum of W-boson. Most events fall in the small q_T region.

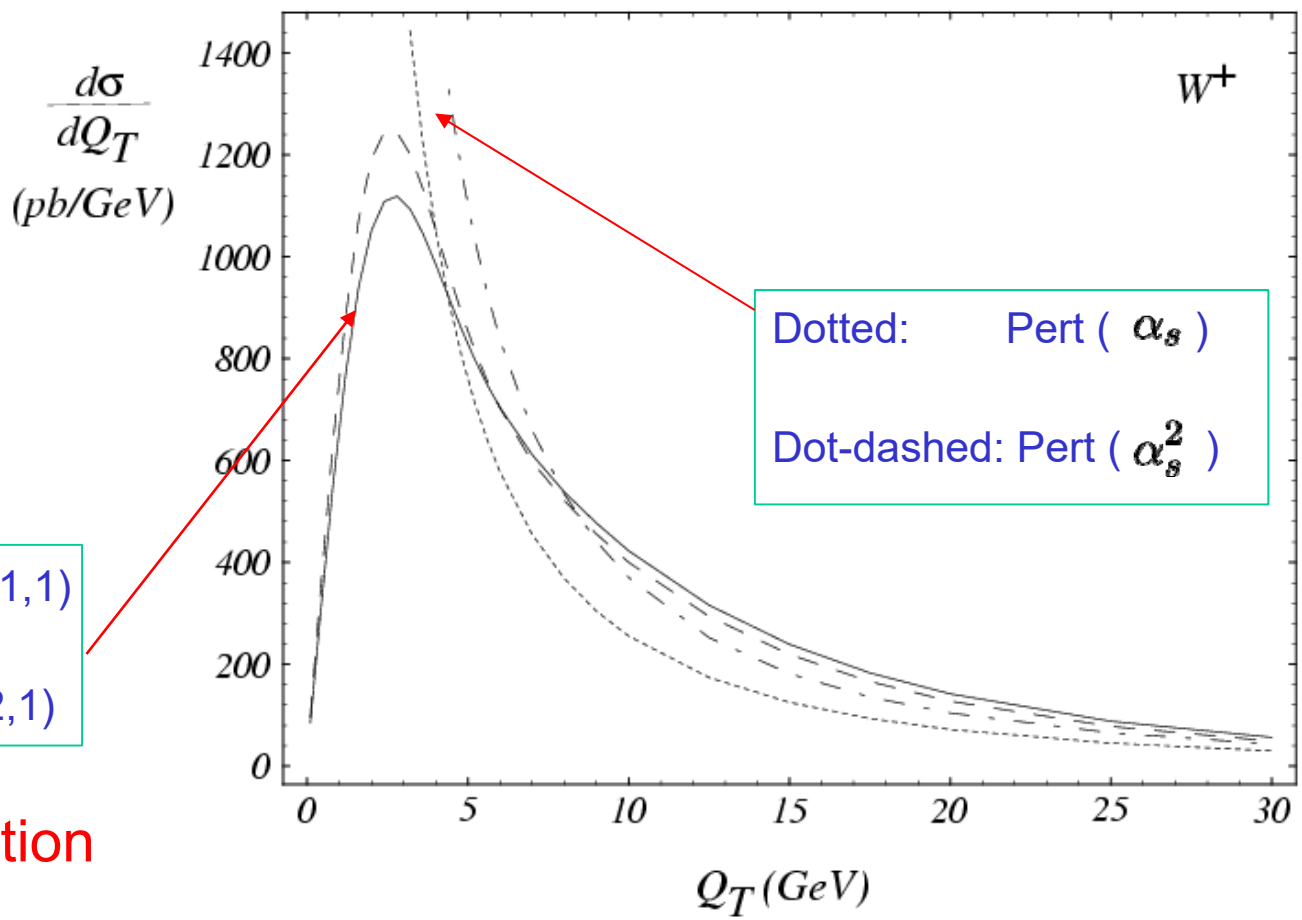


(at NLO)

QCD Resummation is needed



Perturbative



Dotted: Pert (α_s)

Dot-dashed: Pert (α_s^2)

Dashed: CSS (1,1,1)

Solid: CSS (2,2,1)

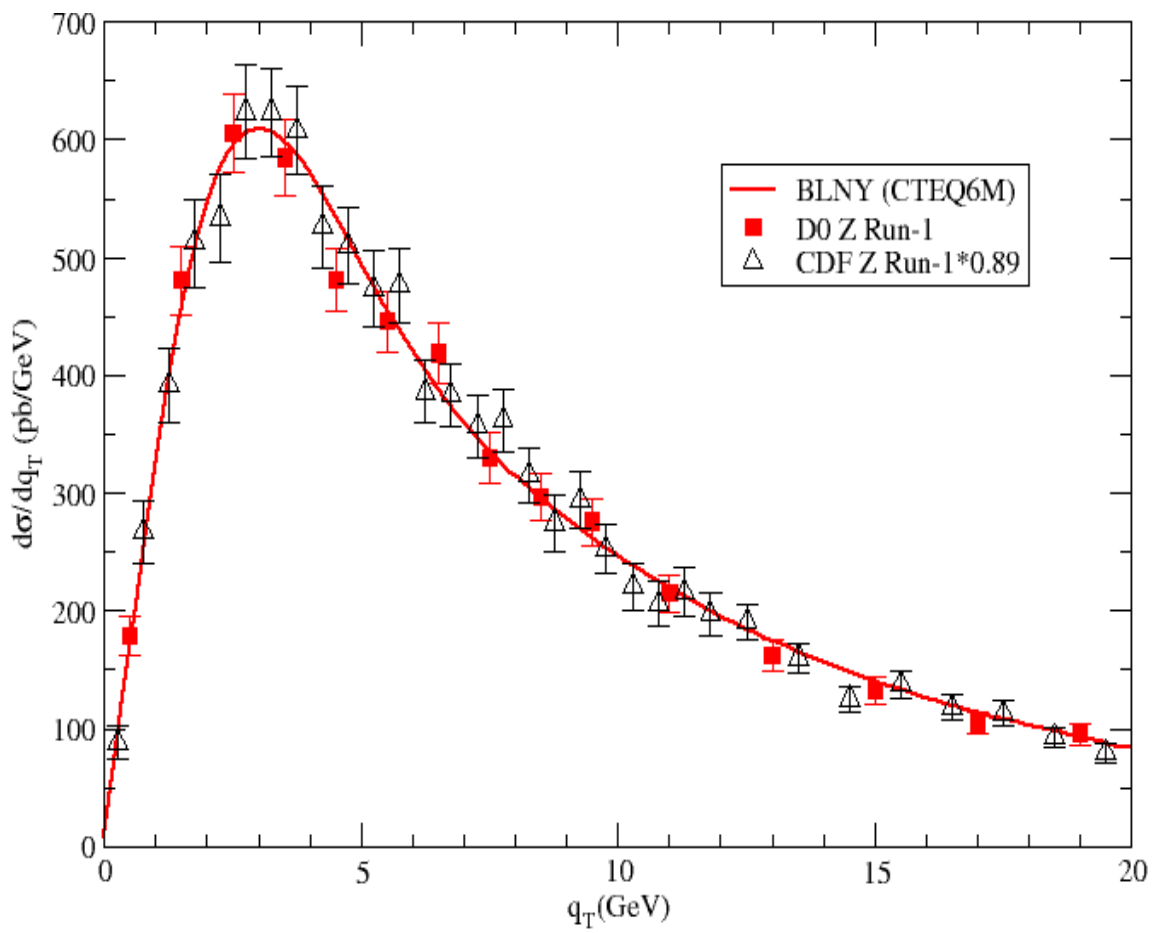
Resummation

Resummation calculations agree with data very well

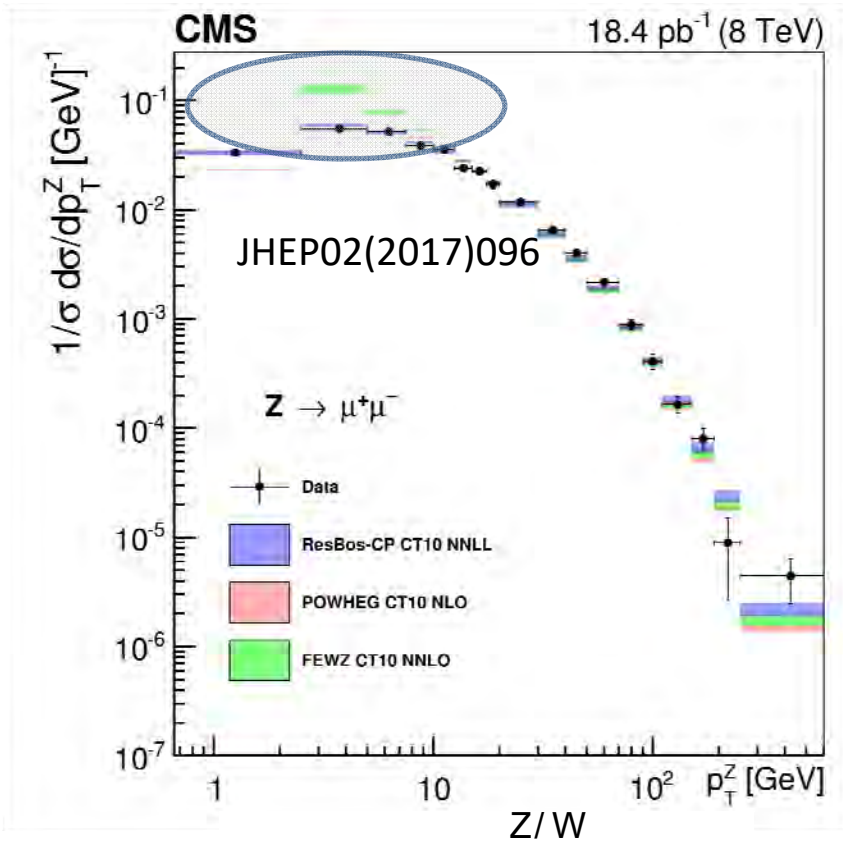
Predicted by ResBos:

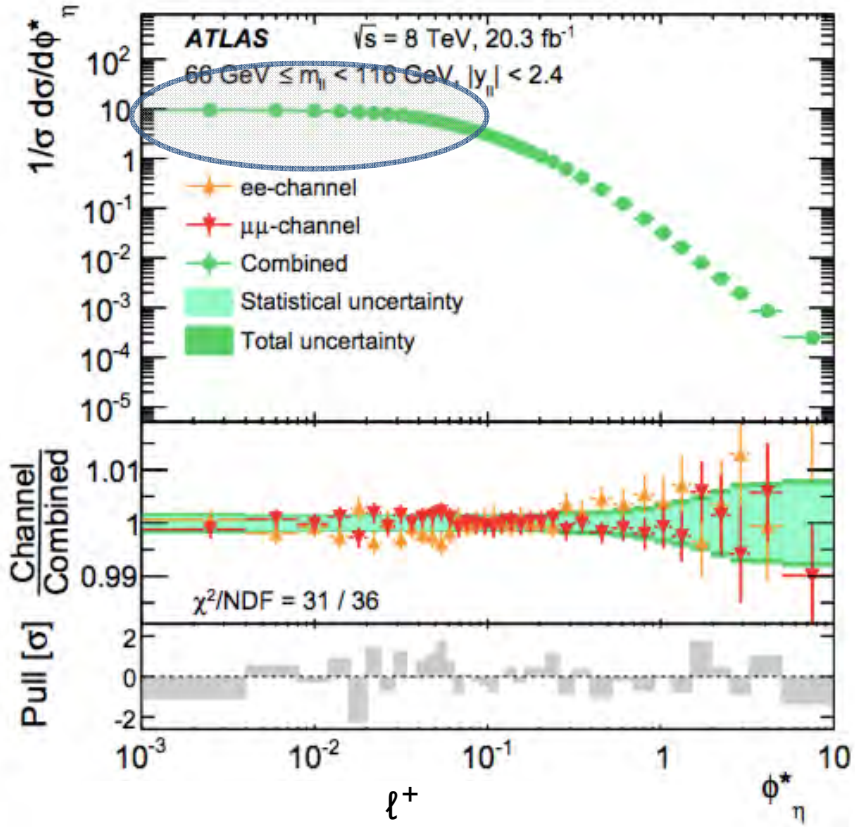
A program that
includes the effect of
multiple soft gluon emission
on the production of
W and Z bosons
in hadron collisions.

$P\bar{P} \rightarrow Z$ @ Tevatron

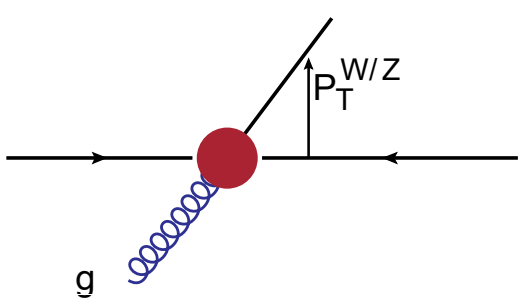


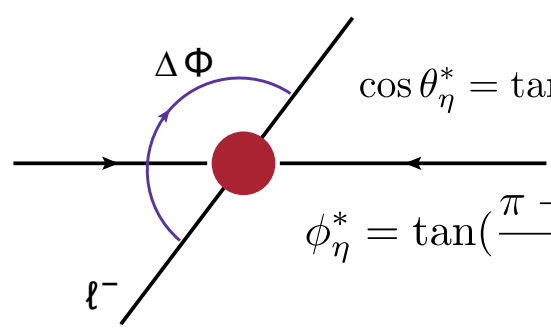
Transverse Momentum and ϕ^*_η distributions





Z/ W

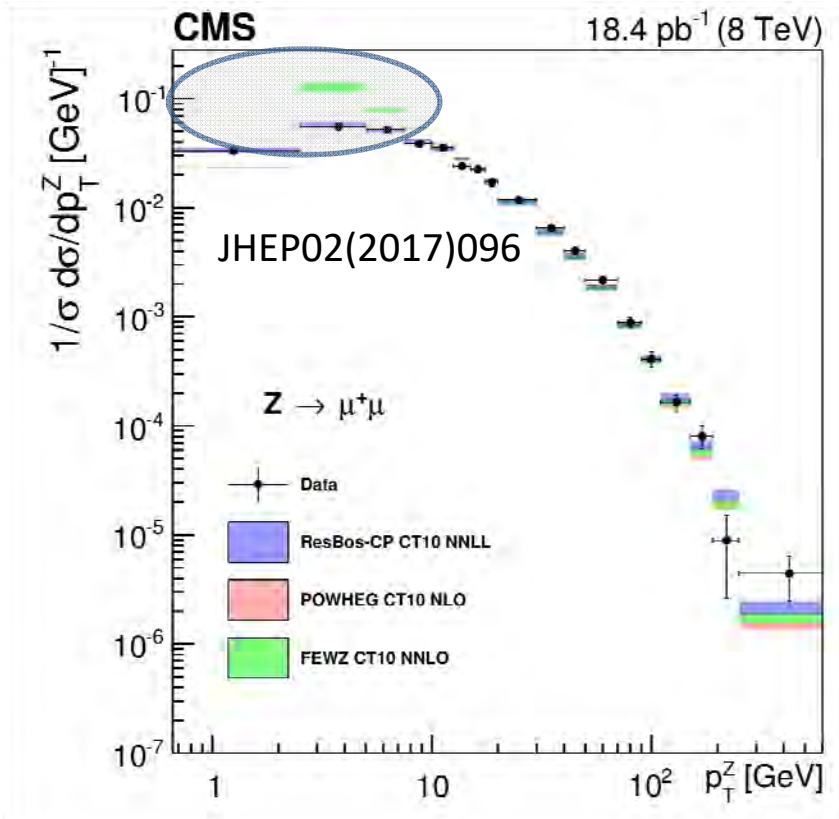


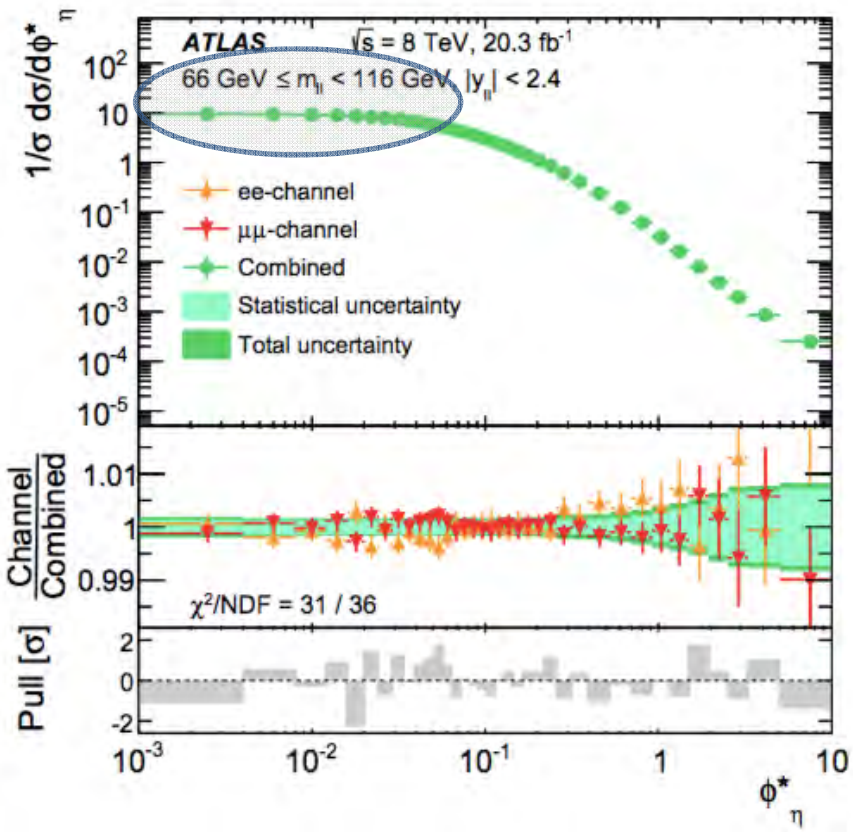


$$\cos\theta^*_\eta = \tanh\left[\frac{\eta_{\ell^+} - \eta_{\ell^-}}{2}\right]$$

$$\phi^*_\eta = \tan\left(\frac{\pi - \Delta\phi}{2}\right)\sin\theta^*_\eta$$

Transverse Momentum and ϕ^*_η distributions





1. Both distributions are important observables for the DY production and are extremely precise;
2. Both observables are sensitive to the soft gluon radiations $\ln \frac{Q^2}{P_T^{V\,2}}$
3. All order resummation is needed;

$$\phi^*_\eta = \tan\left(\frac{\pi - \Delta\phi}{2}\right) \sin\theta^*_\eta$$

Transverse Momentum Resummation

- Resummation for DY-like processes are well understand;

  Collins, Soper, Sterman ’85

- Resummation carried out mostly in impact parameter b space;

  Resbos [Balaze, Belyaev, Berger, Cao, Chen, Isaacson, Nadolsky, Yuan et al ‘93], DYRes[Catani, de Florian, Ferrera, Grazzini ‘15], HRes[de Florian, Ferrera, Grazzini, Tommasini ‘12] , Cute [Becher, Neubert, Wilhelm ‘12, ‘13], arTeMiDe [Scimemi, Vladimirov ‘17]

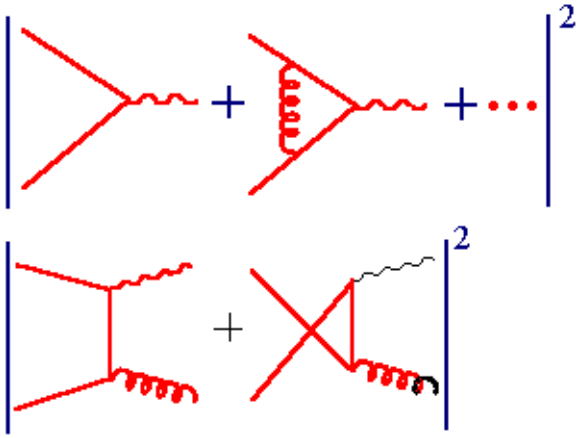
- Resummaion also carried out in momentum space;

  Frixione, Nason, Ridolf ‘97; Ellis, Veseli ’98; Kulesza, Stirling ’00; Monni, Re, Torrielli ’16; Bizon, Monni, Re, Rottoli, Rorrielli ‘17;

To recover the "K-factor" in NLO total rate

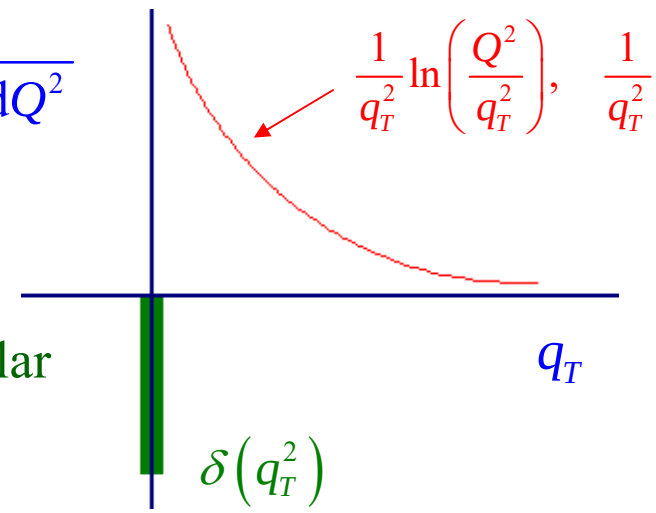
To include the C-Functions

$$\frac{d\sigma}{dQ^2 dy} \square$$

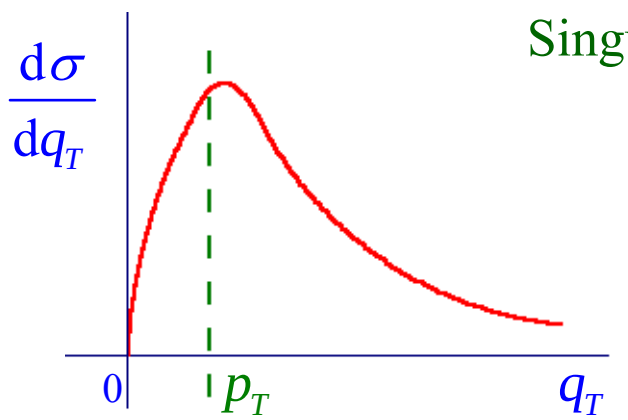


Finite

$$\frac{d\sigma}{dq_T\, d\, y dQ^2}$$



Singular



The area under the q_T – curve will reproduce the total rate at the order $\alpha_S^{(1)}$ if Y term is calculated to $\alpha_S^{(1)}$ as well.

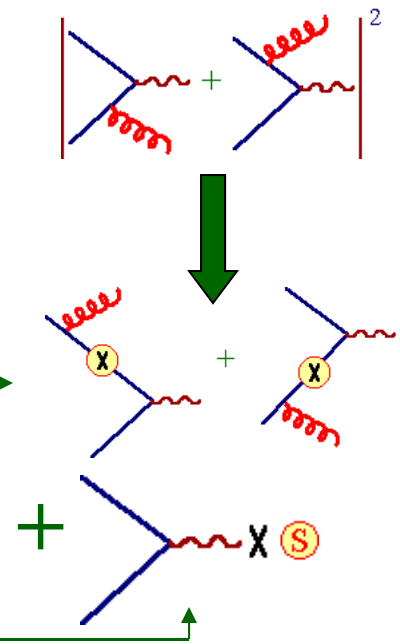
As $q_T \to 0$

$$\left.\frac{d\sigma}{dq_T^{2\backslash} dy dQ^2}\right|_{q_T\to 0}$$

$$= \left(\frac{\pi}{s}\sigma_0\right)\cdot\delta\left(Q^2 - M_W^2\right)\cdot\left(\frac{1}{2\pi q_T^2}\right)\left(\frac{\alpha_s(Q)}{\pi}\right)$$

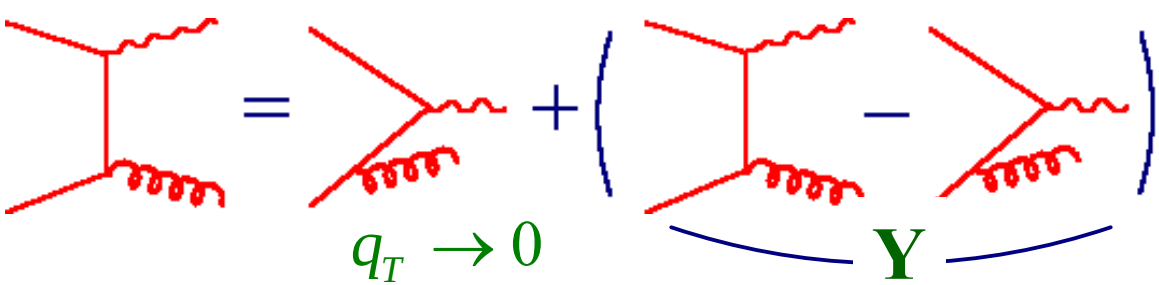
$$\cdot\left\{ f_{q/A}(x_A,Q)\left[P_{\bar{q}\leftarrow\bar{q}}\otimes f_{\bar{q}}\right]_{x_B,Q} + \left[P_{q\leftarrow q}\otimes f_q\right]_{x_A,Q} f_{\bar{q}/B}(x_B,Q) + f_{q/A}(x_A,Q) f_{\bar{q}/B}(x_B,Q)\cdot\left[2\left(\frac{4}{3}\right)\ln\left(\frac{Q^2}{q_T^2}\right) + 2(-2)\right]\right\}$$

Diagramatically,



Exponentiate

To preserve transverse momentum conservation, we have to go to the impact parameter space (b-space) to perform resummation.



What's QCD Resummation?

- Perturbative expansion

$$\frac{d\hat{\sigma}}{dq_T^2} \sim \alpha_S \left\{ 1 + \alpha_S + \alpha_S^2 + \cdots \right\}$$

- The singular pieces, as $\frac{1}{q_T^2}$ (1 or log's)

$$\frac{d\hat{\sigma}}{dq_T^2} \sim \frac{1}{q_T^2} \sum_{n=1}^{\infty} \sum_{m=0}^{2n-1} \alpha_S^{(n)} \ln^{(m)} \left(\frac{Q^2}{q_T^2} \right)$$

$$\sim \frac{1}{q_T^2} \{ \quad \alpha_S \left(\underline{L+1} \right)$$

$$+ \alpha_S^2 \left(\underline{L^3 + L^2} + \underline{L+1} \right)$$

$$+ \alpha_S^3 \left(\underline{L^5 + L^4} + \underline{L^3 + L^2} + \underline{L+1} \right)$$

$$+ \cdots \quad \}$$

$$L \equiv \ln \left(\frac{Q^2}{q_T^2} \right)$$

Resummation is to reorganize the results in terms of the large Log's.

Resummed results:

Determined by $A^{(1)}$ and $B^{(1)}$

$$\frac{d\sigma}{dq_T^2} \sim \frac{1}{q_T^2}\Big\{ [\alpha_S(L+1) \quad +\alpha_S^2(L^3+L^2)+\alpha_S^3(L^5+L^4) \quad +\cdots \ ]$$

$$+[\quad +\alpha_S^2(L+1) \quad +\alpha_S^3(L^3+L^2) \quad +\cdots \ ]$$

Determined by $A^{(2)}$ and $B^{(2)}$

$$+[\quad +\alpha_S^3(L+1) \quad +\cdots \ ]$$

$$+\cdots \ \Big\}$$

Determined by $A^{(3)}$ and $B^{(3)}$

QCD Resummation

In the formalism by Collins-Soper-Sterman, in addition to these perturbative results, the effects from physics beyond the leading twist is also implemented as

[non-perturbative functions].

CSS Resummation Formalism

$$\frac{d\sigma}{dq_T^2\, dy\, dQ^2} = \frac{\pi}{S}\sigma_0 \delta\left(Q^2 - M_W^2\right)\cdot$$

$$\left\{\frac{1}{(2\pi)^2}\int d^2b \quad e^{i\vec{q}_T\cdot\vec{b}}\tilde{W}\left(b,Q,x_A,x_B\right)\cdot\text{[Non-perturbative functions]}\right.$$

$$\left. + Y\left(q_T,y,Q\right)\right\}$$

$$\tilde{W} = e^{-S(b)}\cdot C\otimes f\left(x_A\right)\cdot C\otimes f\left(x_B\right)$$

$$\sum_j \int_{x_A}^1 \frac{d\xi_A}{\xi_A} C_{qj}\left(\frac{x_A}{\xi_A},b,\mu\right)\cdot f_{j/A}\left(\xi_A,\mu\right)$$

$$\sum_k \int_{x_B}^1 \frac{d\xi_B}{\xi_B} C_{qk}\left(\frac{x_A}{\xi_A},b,\mu\right)\cdot f_{k/B}\left(\xi_B,\mu\right)$$

Sudakov form factor $S(b) = \int_{\left(\frac{b_0}{b}\right)^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[\ln\left(\frac{Q^2}{\bar{\mu}^2}\right)A(\bar{\mu}) + B(\bar{\mu})\right]$

[Non-perturbative functions] are functions of (b,Q,x_A,x_B) which include QCD effects beyond Leading Twist.

Example: for $W^{\pm}$

$$\sigma_0 = \left(\frac{4\pi^2 \alpha}{3} \sum_{jj'} Q_{jj'}^{(W)} \right), \qquad Q_{jj'}^{(W)} = \frac{1}{4\sin^2\theta_W} \left(kM \right)_{jj'}^2$$

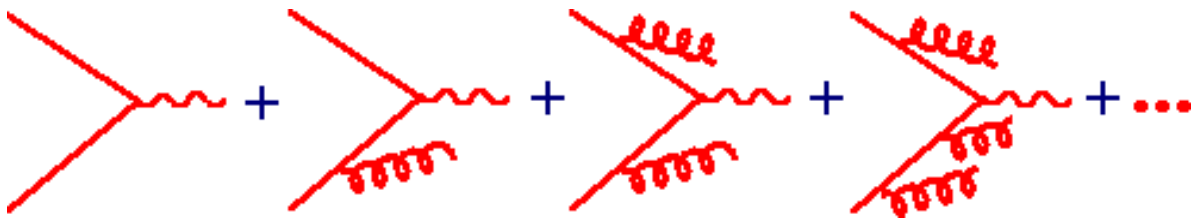
The couplings of gauge bosons to fermions are expressed in the way to include the dominant electroweak radiative corrections. The propagators of gauge bosons also contain energy-dependent width, as done in LEP precision data analysis.

Note:

$$A \equiv \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n \cdot A^{(n)}, \qquad B \equiv \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n \cdot B^{(n)},$$

$$C \equiv \sum_{n=0}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n \cdot C^{(n)}$$

Diagramatically, Resummation is doing



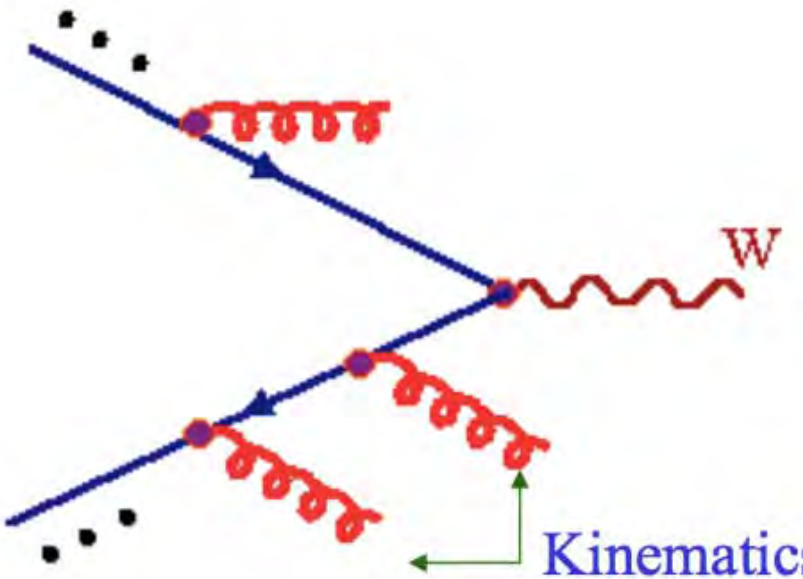
Resum large $\alpha_S^n \ln^m\left(\frac{Q^2}{q_T^2}\right)$ terms

$$\left.\frac{d\sigma}{dq_T^2 dy}\right|_{q_T \to 0} \sim \frac{1}{q_T^2}\sum_{n=1}^{\infty}\sum_{m=0}^{2n-1}\alpha_S^n \ln^m\left(\frac{Q^2}{q_T^2}\right)\cdot C_m^n$$

Monte-Carlo programs ISAJET, PYTHIA, HERWIG contain these physics.

(Note: Arbitrary cut-off scale in these programs to affect the amount of Backward radiation , i.e. Initial state radiation.)

Monte-Carlo Approach (Pythia8)
Parton showers



Backward Radiation
(Initial State Radiation)

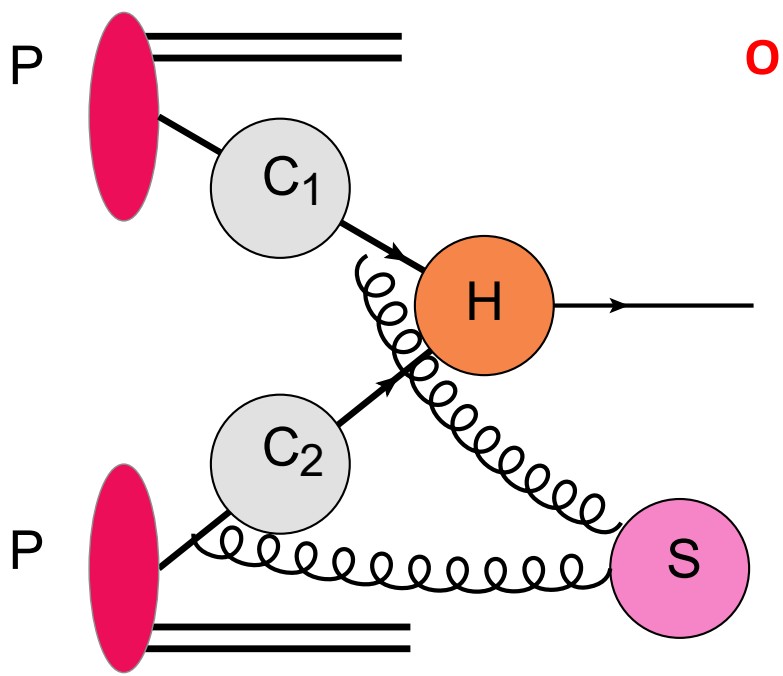
Kinematics of the radiated gluon, controlled by Sudakov form factor with some arbitrary cut-off. (In contrast to perform integration in impact parameter space, i.e., b space.)

Collins-Soper-Sterman (CSS) Formalism For Drell-Yan processes

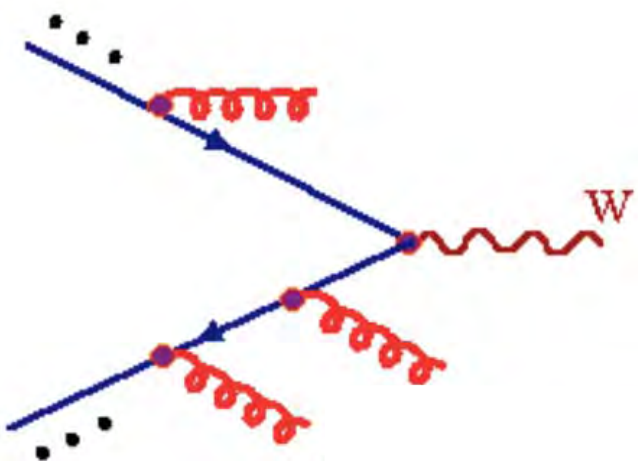
The large logs will be resummed into the exponential form factor:

$$W(b,Q) = exp\left\{-\int_{b_0/b^*}^{Q} \frac{d\mu}{\mu}(A\ln\frac{Q^2}{\mu^2}+B)\right\}(C\otimes f)_1\,(C\otimes f)_2$$

J. C. Collins, D. E. Soper and G. F. Sterman, Nucl. Phys. B 250,199(1985)



Only for initial state soft gluon resummation

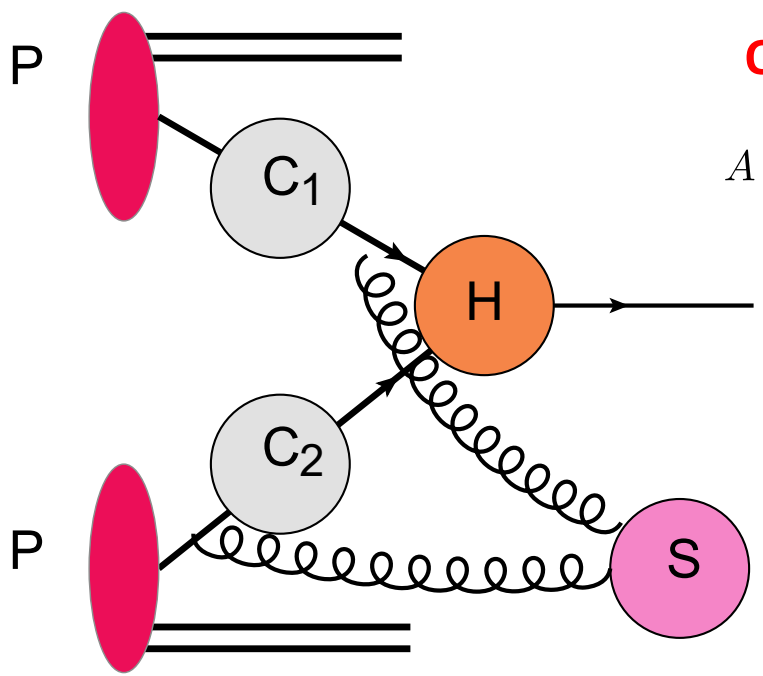


CSS Formalism
(For Drell-Yan-like processes)

The large logs will be resummed into the exponential form factor:

$$W(b,Q) = exp\left\{-\int_{b_0/b^*}^{Q}\frac{d\mu}{\mu}(A\ln\frac{Q^2}{\mu^2}+B)\right\}(C\otimes f)_1\,(C\otimes f)_2$$

J. C. Collins, D. E. Soper and G. F. Sterman, Nucl. Phys. B 250,199(1985)



Only for initial state soft gluon resummation

$$A=\sum_n A^n(\frac{\alpha_s}{\pi})^n, B=\sum_n B^n(\frac{\alpha_s}{\pi})^n, C=\sum_n C^n(\frac{\alpha_s}{\pi})^n$$

A, B, C functions are perturbatively calculable

A: Sudakov double logs

B: anomalous dimension

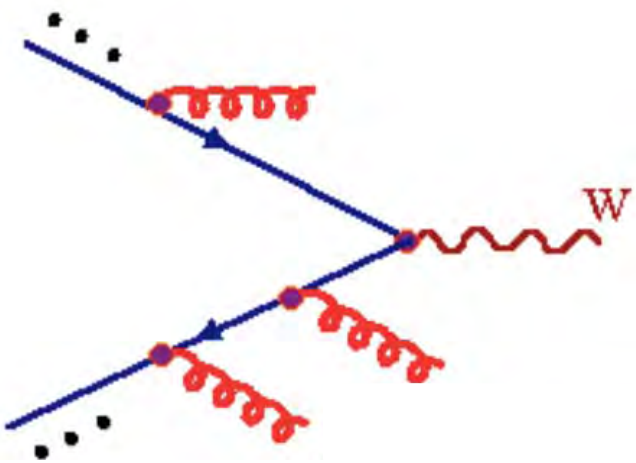
C: hard coefficients

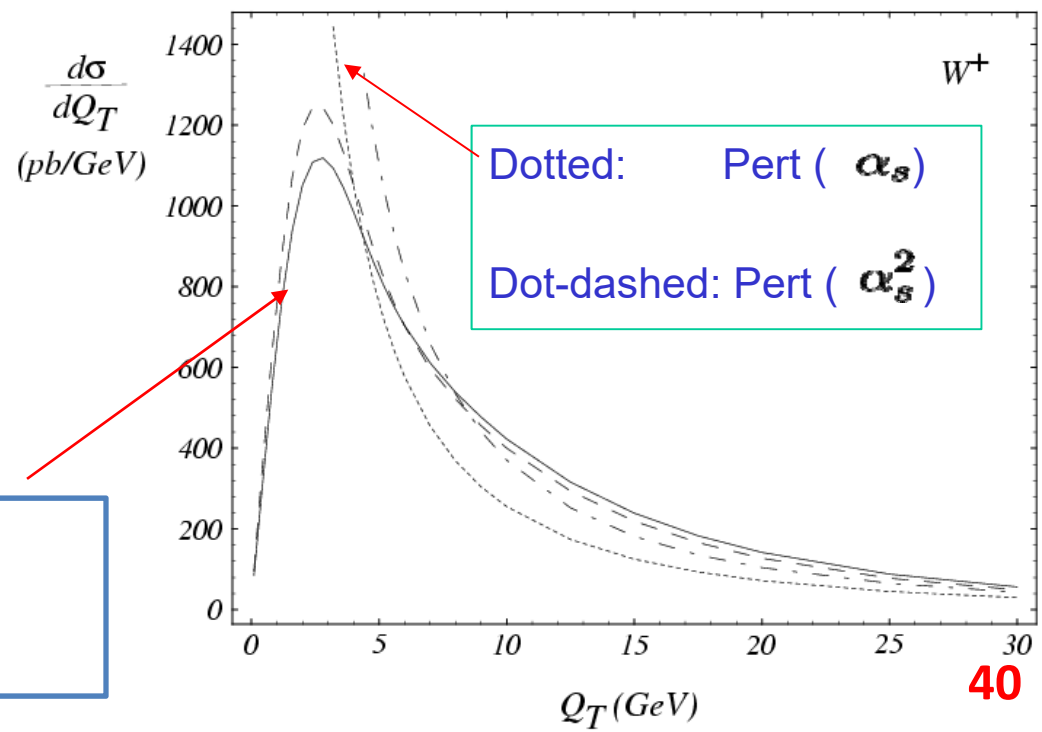
CSS Formalism
(For Drell-Yan-like processes)

The large logs will be resummed into the exponential form factor:

$$W(b,Q) = exp\left\{-\int_{b_0/b^*}^{Q} \frac{d\mu}{\mu}(A\ln\frac{Q^2}{\mu^2}+B)\right\}(C\otimes f)_1\,(C\otimes f)_2$$

$$A=\sum_n A^n(\frac{\alpha_s}{\pi})^n, B=\sum_n B^n(\frac{\alpha_s}{\pi})^n, C=\sum_n C^n(\frac{\alpha_s}{\pi})^n$$

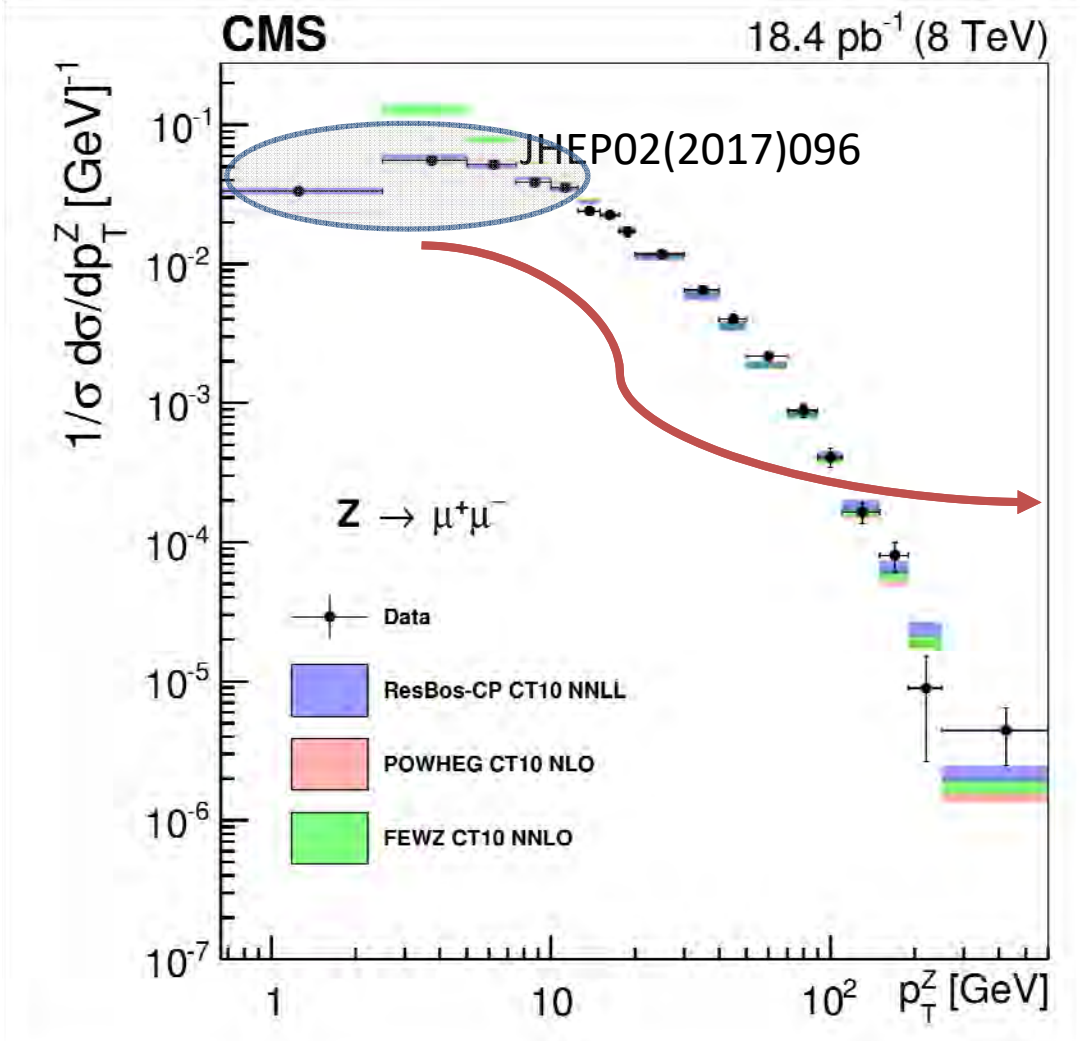




Dashed: CSS (1,1,1)

Solid: CSS (2,2,1)

CSS Formalism
(For Drell-Yan-like processes)

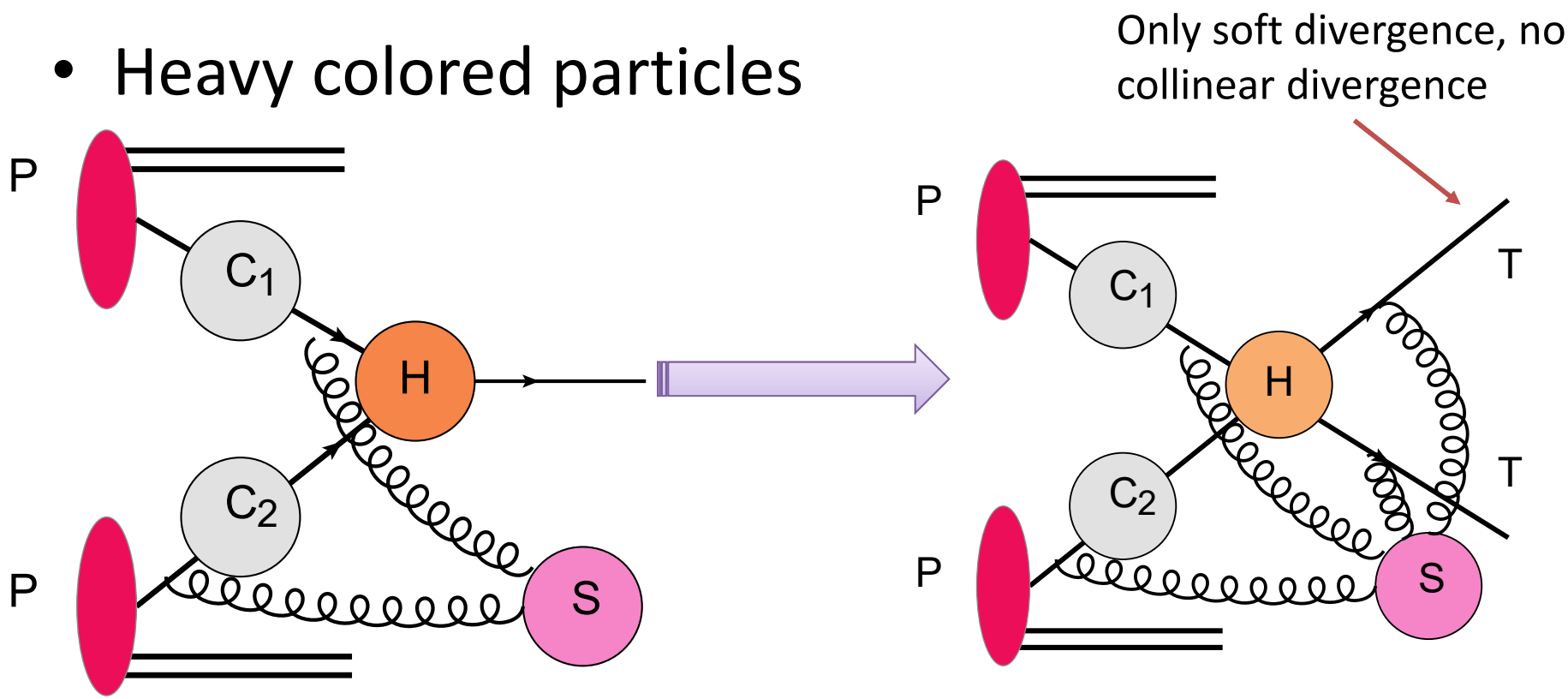


ResBos prediction agrees with data very well

Resummation effects are important for the Z boson transverse momentum distribution

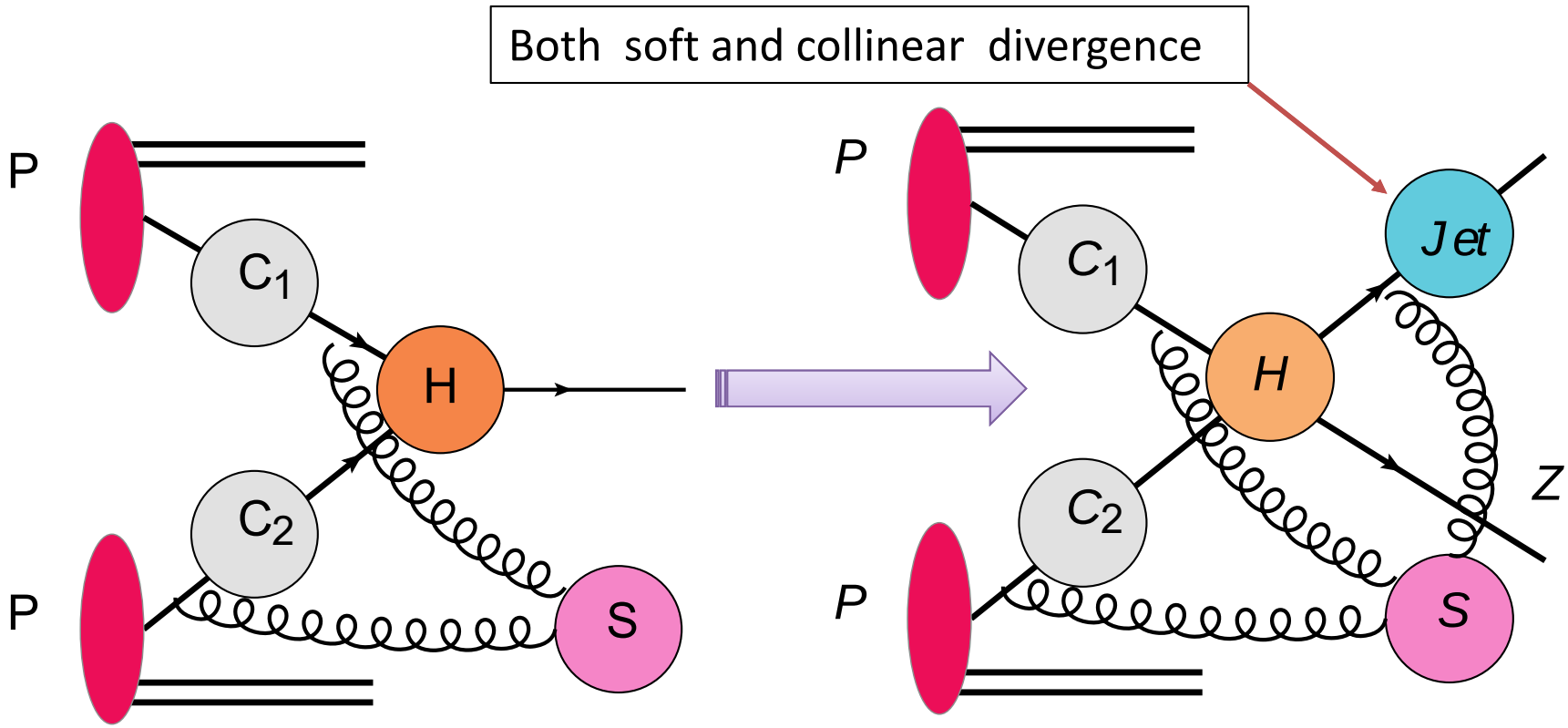
How to extend CSS formula to general processes?

- Heavy colored particles



Final states carry color, so we should also consider complete color coherence radiation between initial and final states, and final state radiation;

Final state Jet Production



A. Final states carry color, so we should also consider complete color coherence radiation between initial and final states;

B. Jet algorithm will enter into the calculations as well;

Only out of the cone radiation contributions to transverse momentum of the system

C. Jet function from collinear gluon radiation

Recent progress of modified q_T resummation

- Heavy quark production

  A. top quark pair production

  H. X. Zhu, C. S. Li, H. T. Li, D. Y. Shao and L. L. Yang, PRL110(2013)8,082001; PRD88(2013)074004

  B. Heavy quarkonium production

  P. Sun, C.-P. Yuan and F. Yuan, PRD88(2013)054008

- Processes involving multijets in the final state

  A. Dijet production

  P. Sun, C.-P. Yuan and F. Yuan, PRL113(2014)23,232001; PRD92(2015)9,094007

  B. Higgs+jet production

  P. Sun, C.-P. Yuan and F. Yuan, PRL114(2015)20,202001;
  P. Sun, J. Isaacson, C.-P. Yuan and F. Yuan, PLB769(2017)57-61

  C. Higgs+2jet production

  P. Sun, C.-P. Yuan and F. Yuan, PLB762(2016)47-51;arxiv:1802.02980

  D. Single top quark production

  Q. H. Cao, P. Sun, Bin Yan, C.-P. Yuan and F. Yuan, PRD98(2018)5,054032;1902.09336
  P. Sun, Bin Yan and C.-P. Yuan, arxiv:1811.01428

  E. Z boson+jet production

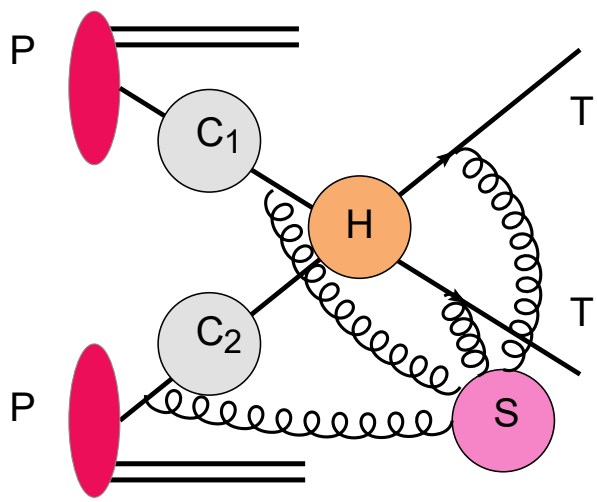
  P. Sun, Bin Yan, C.-P. Yuan and F. Yuan, arxiv:1810.03804

Transverse Momentum Dependent (TMD) Factorization

Collinear PDFs

$F(x, Q)$

1. DGLAP evolution
2. Resum $\ln(Q^2/\mu^2)$
3. Kernel: purely perturbative



TMD PDFs

$F(x, k_\perp, Q)$

1. Collins-Soper evolution
2. Resum $\ln(Q^2/k_\perp^2)$
3. Kernel: can be non-perturbative

$k_\perp \sim \Lambda_{\rm QCD}$

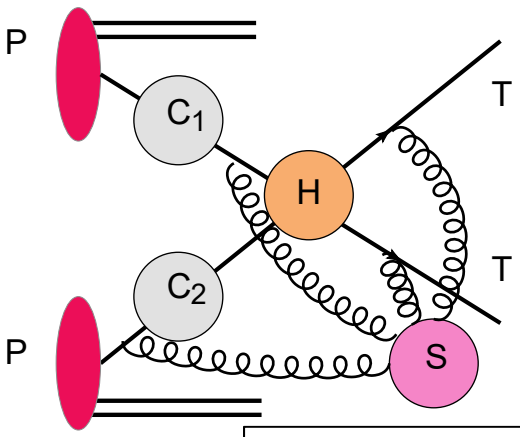
Collinear part

Sudakov Factor

Non-perturbative: fitted from data

$$F(x, b, Q) = C \otimes F(x, b_0/b^*) \times exp\left\{-\int_{b_0/b^*}^{Q} \frac{d\mu}{\mu}\left(A \ln \frac{Q^2}{\mu^2} + B\right)\right\} \times exp\left(-\mathcal{F}_{\rm NP}(b, Q)\right)$$

Transverse Momentum Dependent (TMD) Factorization



$$\frac{d\sigma}{q_\perp} \sim \sum_{ab} \left[\int \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp \cdot \vec{b}_\perp} W_{ab\to cd}(x_1, x_2, b_\perp) + Y_{ab\to cd} \right]$$

TMD factorization, all order resummation for multiple gluon radiation effects.

Y = Pert - Asy

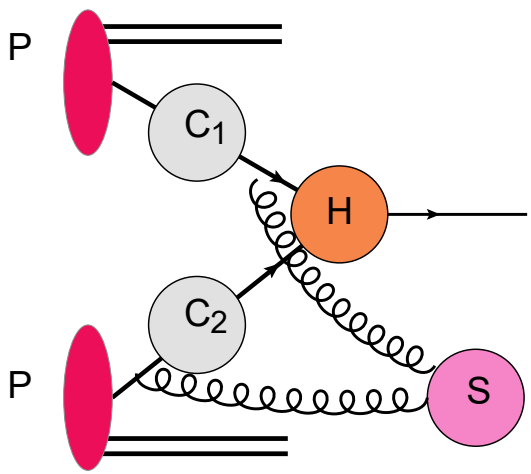
The Matrix Form is defined in the color space:

$$x_1 f_a(x_1, \mu = b_0/b_\perp) x_2 f_b(x_2, \mu = b_0/b_\perp) e^{-S_{\rm Sud}(Q^2, b_\perp)}$$

$$\mathrm{Tr}\left[\mathbf{H}_{ab\to cd} \exp[-\int_{b_0/b_\perp}^{Q} \frac{d\mu}{\mu} \gamma^{s\dagger}] \mathbf{S}_{ab\to cd} \exp[-\int_{b_0/b_\perp}^{Q} \frac{d\mu}{\mu} \gamma^{s}] \right]$$

P. Sun, C.-P. Yuan, F. Yuan, PRL 2014

Drell-Yan production



$$\frac{d\sigma}{q_\perp} \sim \sum_{ab} \left[\int \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp \cdot \vec{b}_\perp} W_{ab\to cd}(x_1, x_2, b_\perp) + Y_{ab\to cd} \right]$$

Y=

$q_T \to 0$

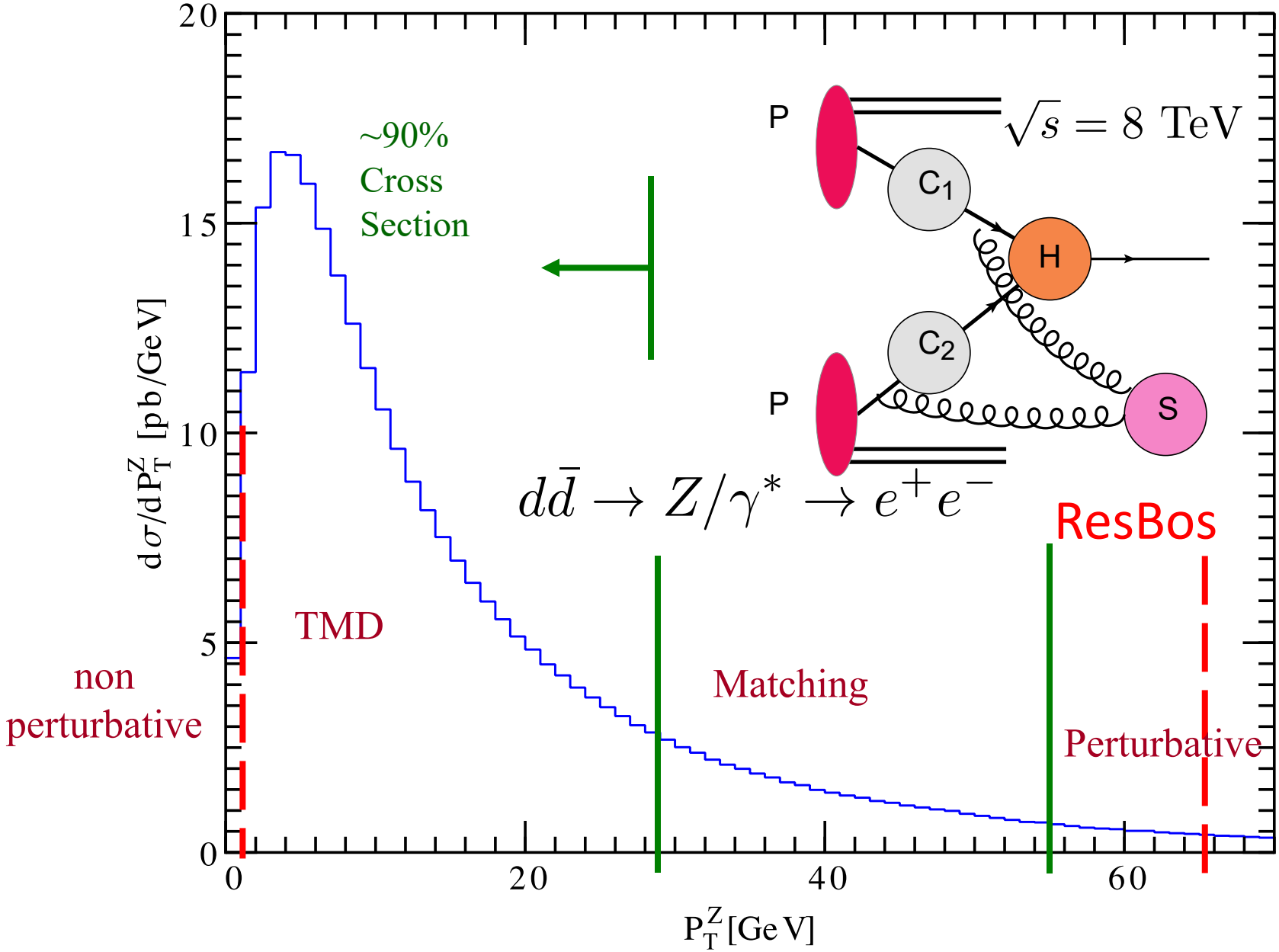
$$W(b,Q) = exp\left\{ -\int_{b_0/b^*}^{Q} \frac{d\mu}{\mu} (A \ln \frac{Q^2}{\mu^2} + B) \right\} \times exp(-\mathcal{F}_{\rm NP}(b,Q)) \times (C \otimes f)_1 \, (C \otimes f)_2$$

$$\mathcal{F}_{\rm NP}(b,Q) = g_1 b^2 + g_2 \ln \frac{Q}{Q_0} \ln \frac{b}{b_*}$$

P. Sun, J. Isaacson, C.-P. Yuan and F. Yuan, Int.J.Mod.Phys. A33 (2018) no.11, 1841006

$$g_1 = 0.21, \quad g_2 = 0.84, \quad Q_0^2 = 2.4 \text{ GeV}^2, \quad b_* = \frac{b}{\sqrt{1 + b^2/b_{\rm max}^2}}$$

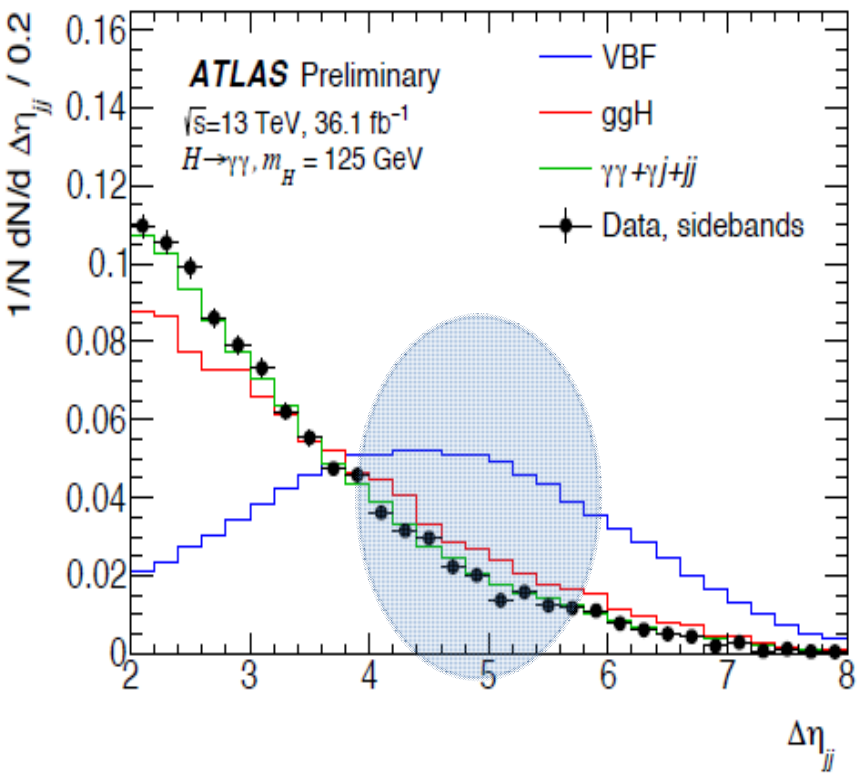
Z boson production

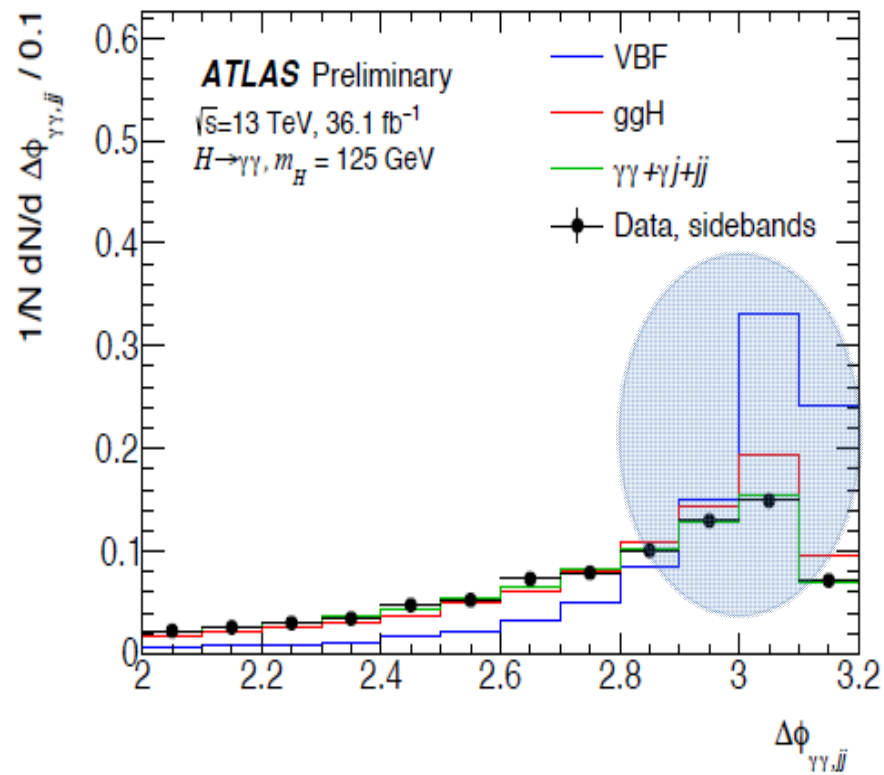


Higgs plus two jets production at LHC

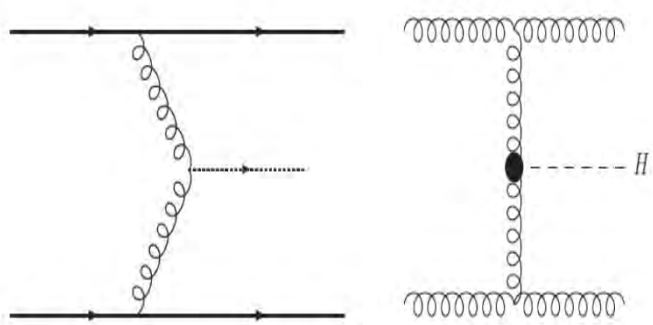
Peng Sun, C.-P. Yuan and Feng Yuan, arxiv:1802.02980, PLB762(2016)47-51

Higgs plus two jets production at LHC

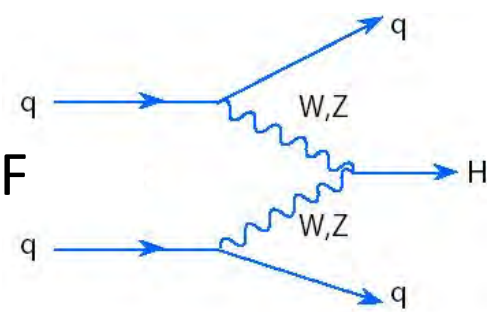




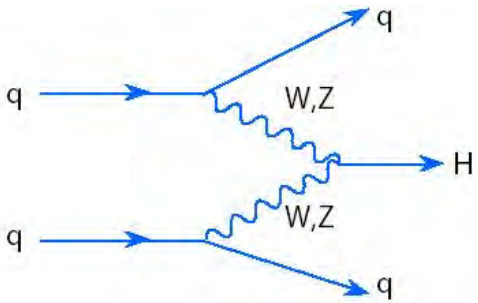
ggH

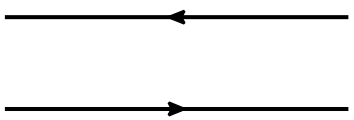


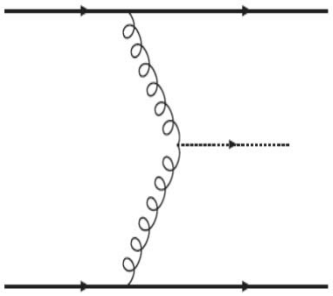
VBF

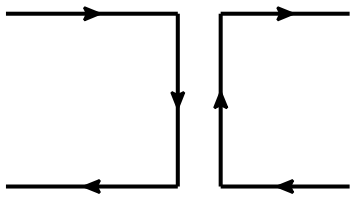


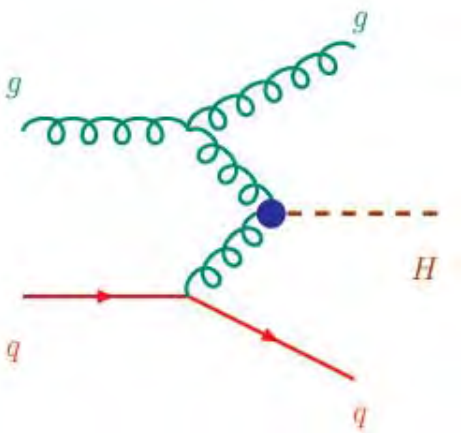
Color structure of Higg+2jet

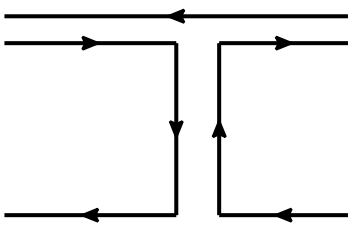


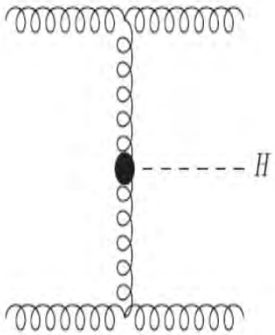


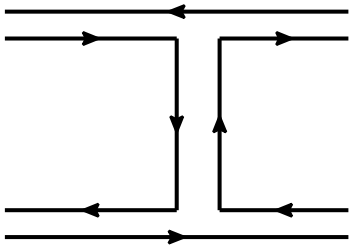




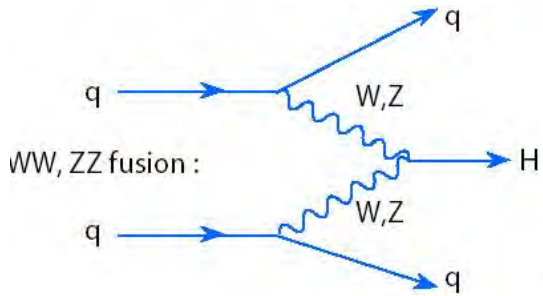






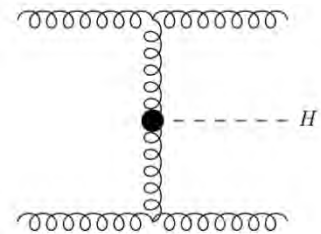


qT Distributions



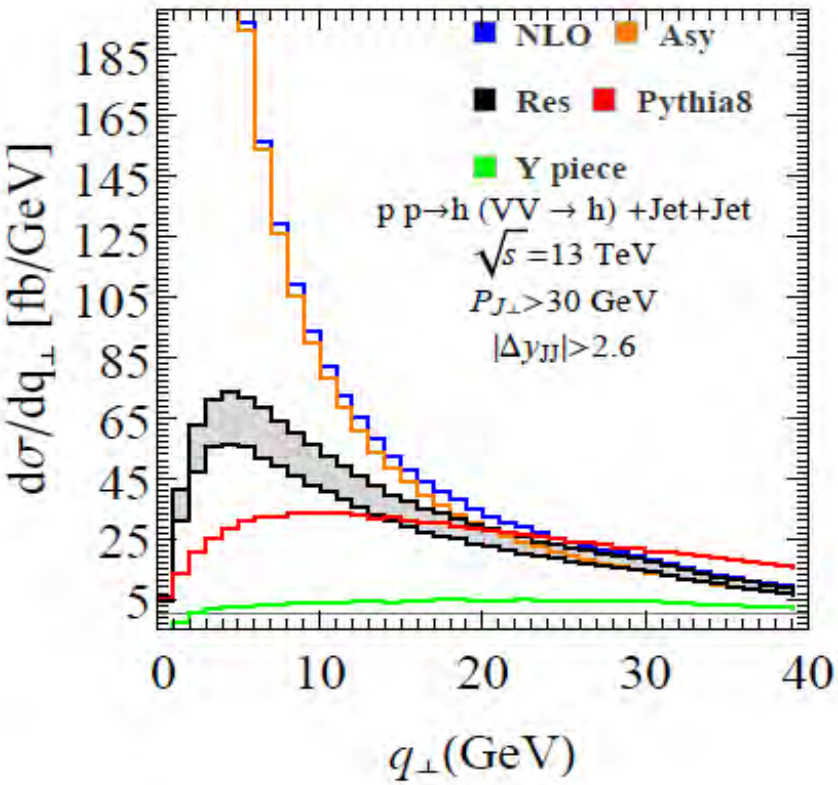
Black: resummation calculation

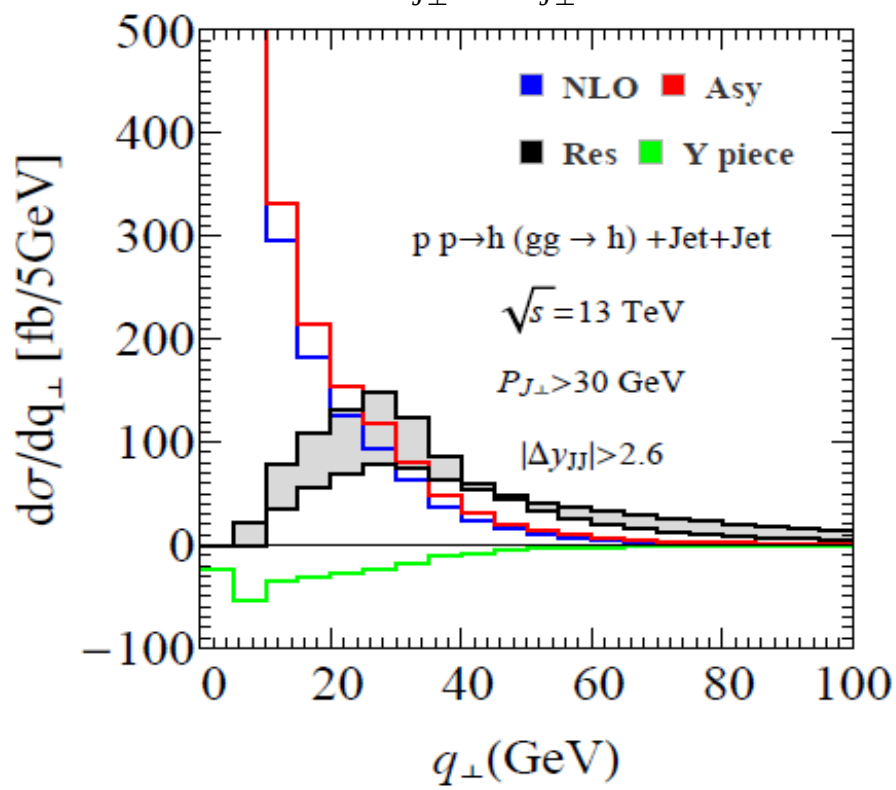
Red: leading order matrix element with parton showers by Pythia



Resummation scale dependence:

$$\hat{\mu} = P_{J\perp}^{sub} \Rightarrow P_{J\perp}^{lead}$$

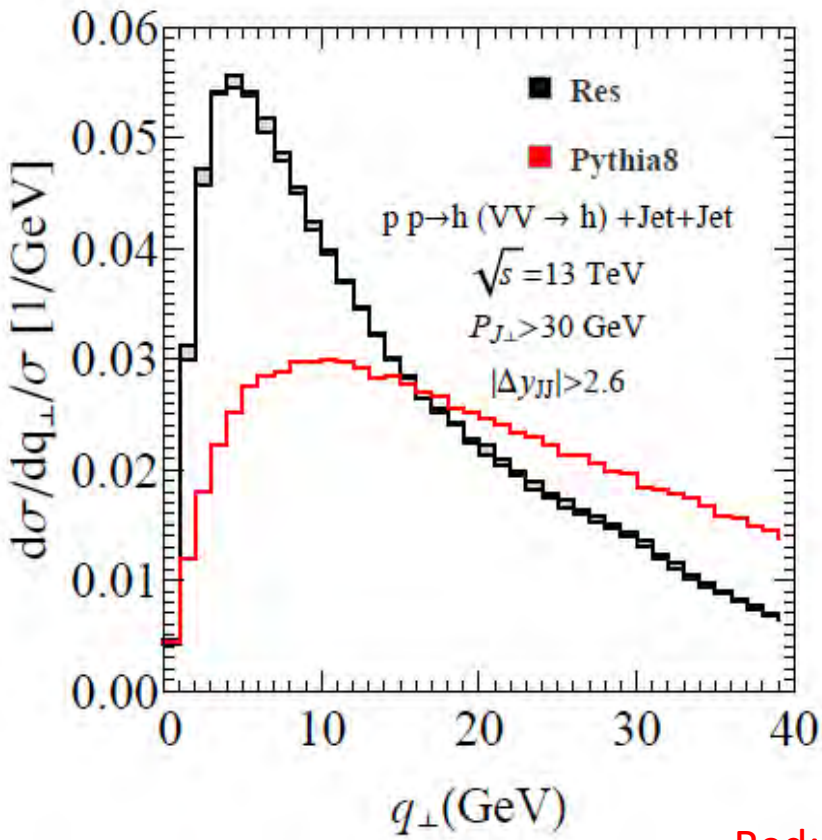


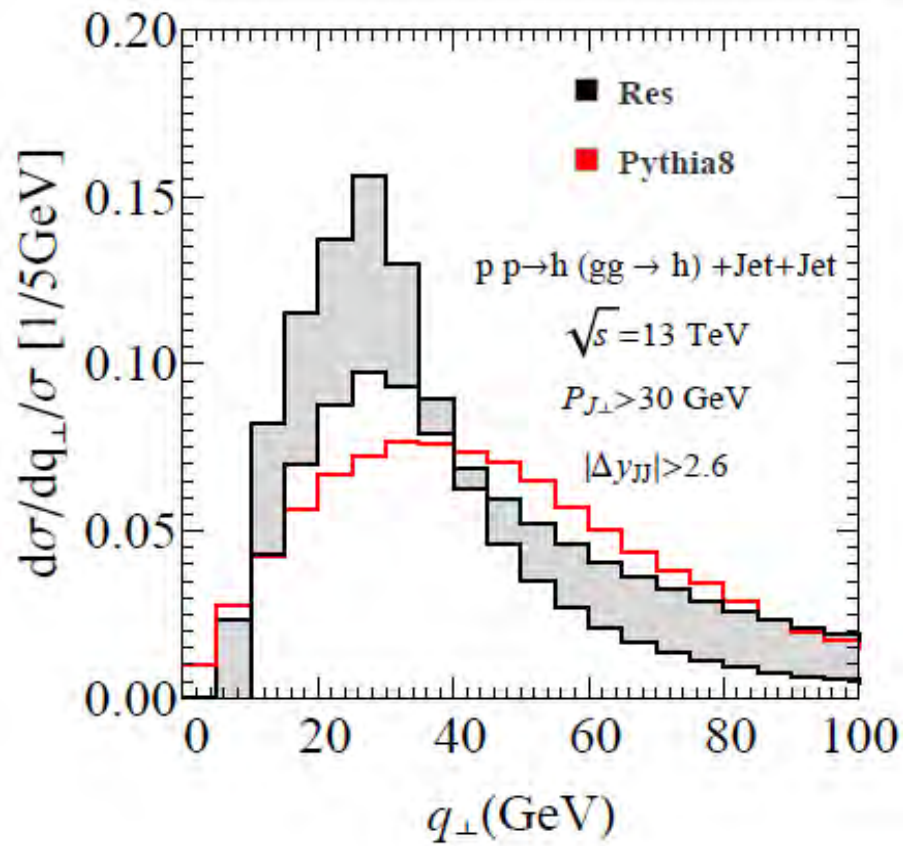


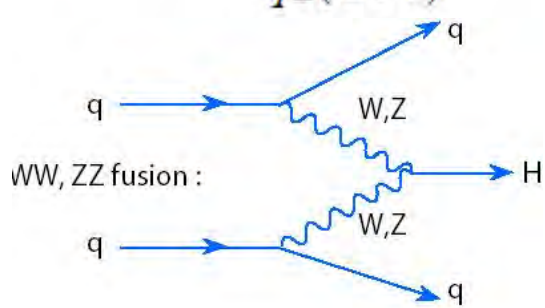
Blue: NLO calculation; Orange: Asymptotic part ; Green: Y-piece

Normalized qT Distributions

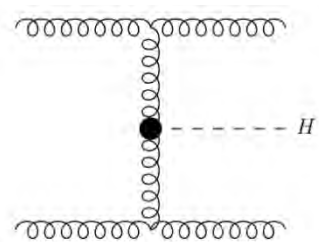
Black: resummation calculation



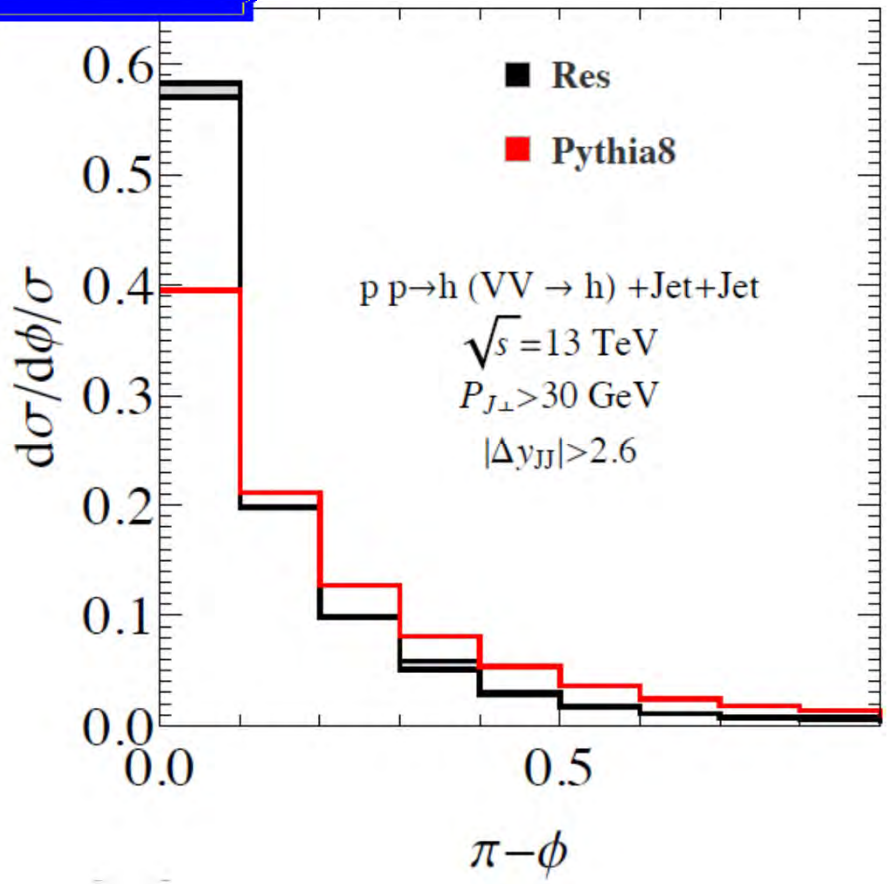


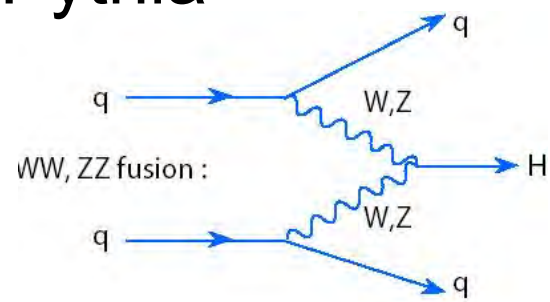


Red: leading order matrix element with parton showers by Pythia



Compared to Pythia





Experimental analysis:

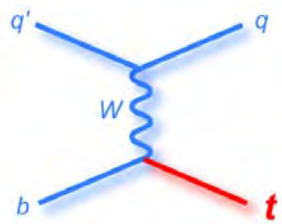
$$\Delta\phi_{\gamma\gamma,jj} > 2.6$$

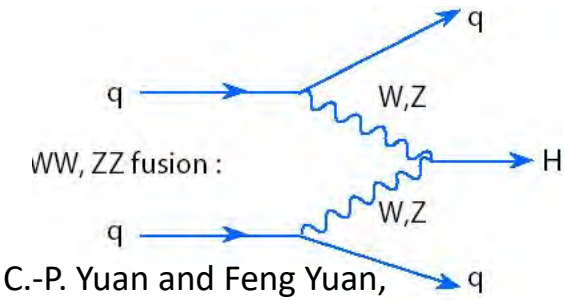
The difference between Res and Pythia: 8%

HL-LHC, VBF Higgs production, 10%

cut-off($\pi-\phi$)	<0.2	<0.3	<0.4	<0.5	<0.54
Res VBF	78.15~76.77%	88.00~86.59%	93.09~92.43%	95.85~95.49%	96.53~96.50%
Pythia8 VBF	60.64%	73.35%	81.45%	86.80%	88.44%

Single top quark vs. VBF Higgs+2jet





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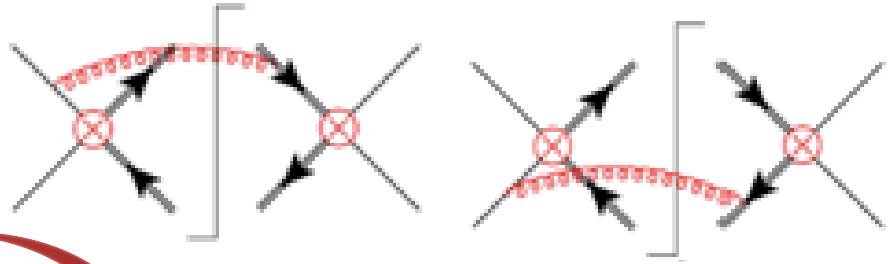
A. Final states carry color;

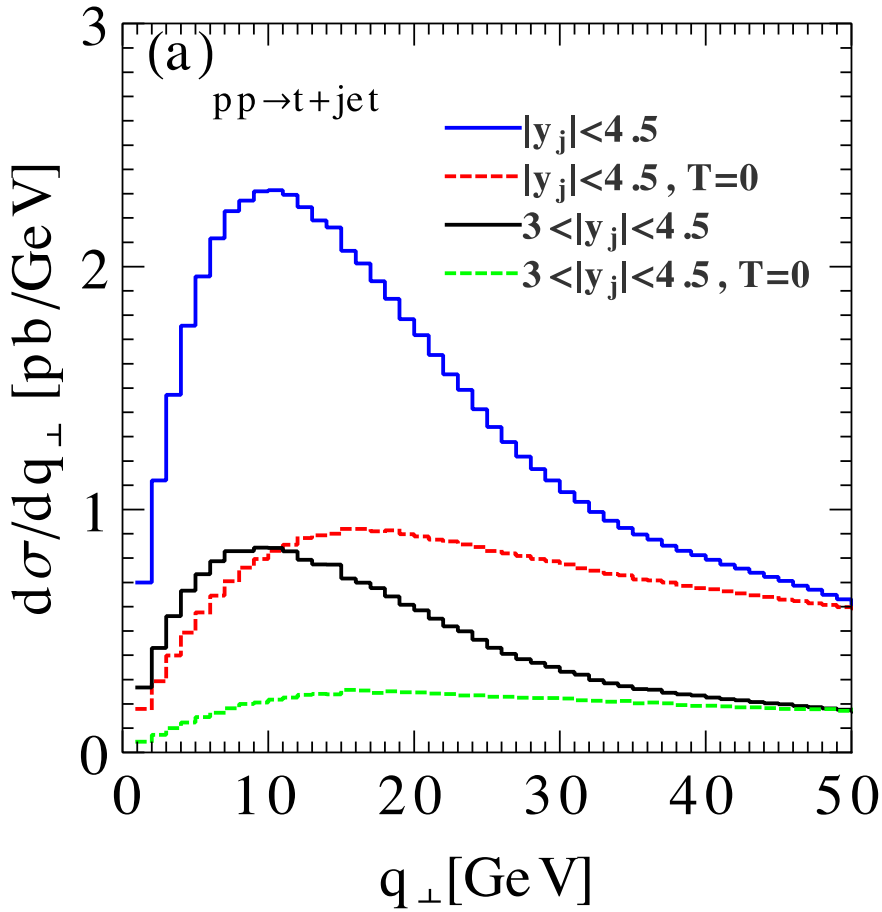
B. Jet algorithm will enter into the calculations as well

C. Color coherence effects between the initial and final states are important

To help VBF Higgs measurement and probe hVV couplings

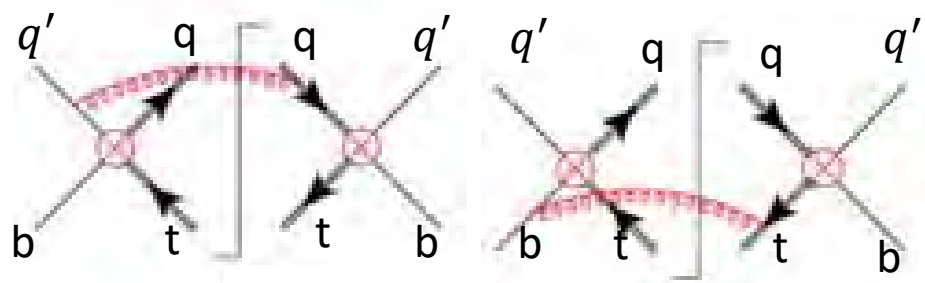


Color coherence effects



$$\hat{t} = \left(p_{q'} - p_q\right)^2$$

color coherence effects

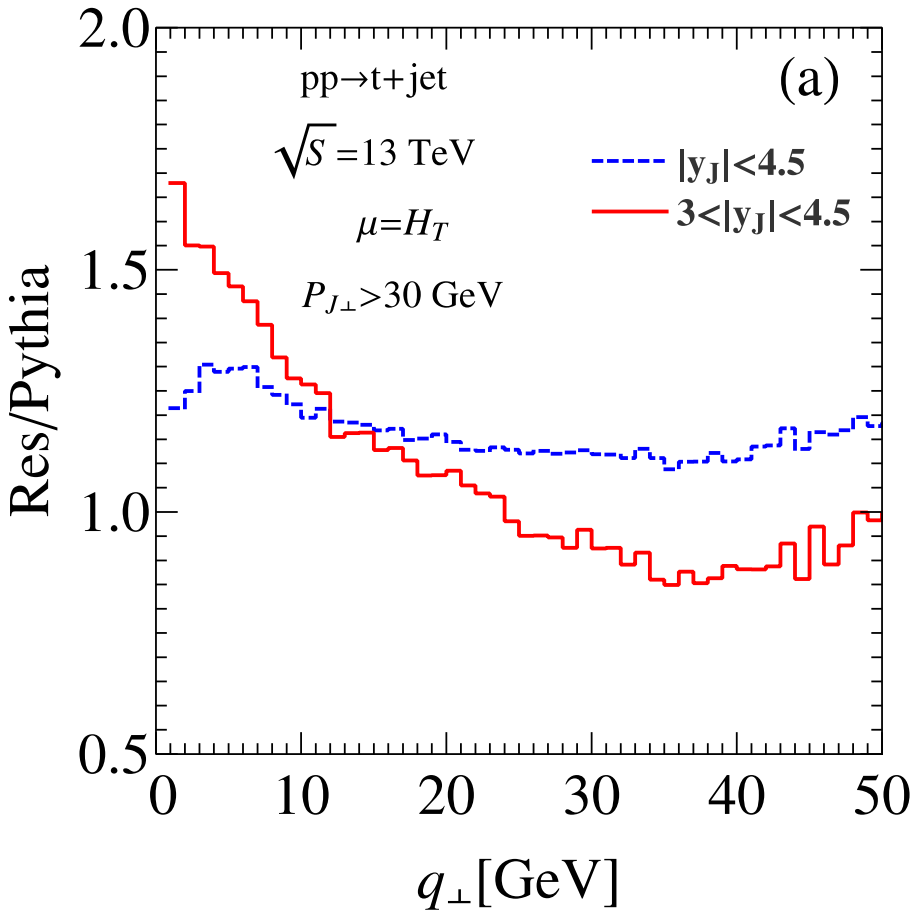


$$T = \ln\frac{-\hat{t}}{\hat{s}} + \ln\frac{-(\hat{t} - m_t^2)}{\hat{s} - m_t^2}$$

Blue: $|y_J| < 4.5$
Red: $|y_J| < 4.5$, turn off the color coherence factor T

Black: $3 < |y_J| < 4.5$
Green: $3 < |y_J| < 4.5$, turn off the color coherence factor T

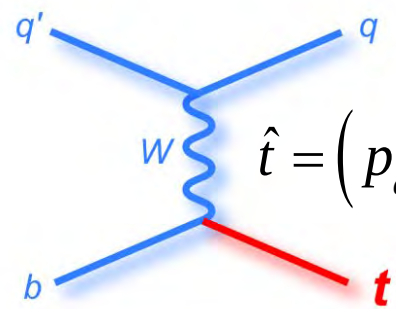
Compared to Pythia



Blue: $|y_J| < 4.5$

Red: $3 < |y_J| < 4.5$

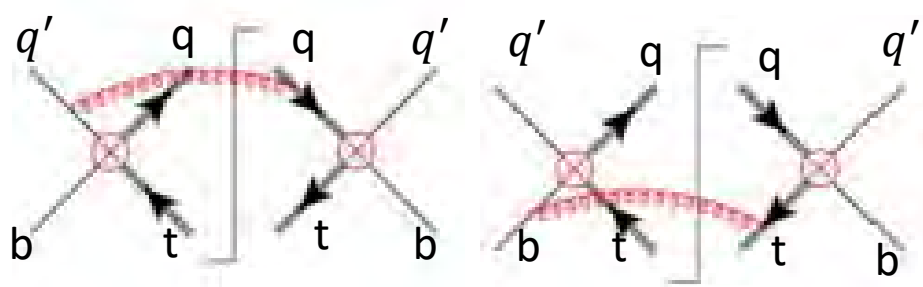
$$\hat{s} = \left(p_{q'} + p_b\right)^2$$



$$\hat{t} = \left(p_{q'} - p_q\right)^2$$

The resummation prediction is different from Pythia when $3 < |y_J| < 4.5$

color coherence factor plays an important role



$$T = \ln\frac{-\hat{t}}{\hat{s}} + \ln\frac{-(\hat{t} - m_t^2)}{\hat{s} - m_t^2}$$

Summary

A. Transverse momentum resummation framework for processes with final state jets

B. The resummation effects for Di-jet, Z/Higgs/top +jet and H+2jet productions

C. For Di-jet and Z+jet processes, our resummation calculation agree with data very well.

D. Color coherence effects in t-channel single top quark and H+2jet production are important.