Soft gluon resummation effects for processes with final state jets

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Outline

• QCD, Parton Model and Factorization
• Precision electroweak Physics at Hadron Colliders
• Transverse Momentum Resummation framework
• Drell-Yan production
• Higgs +two jets production
• T-channel single-top quark production
• Summary
Quantum Chromodynamics

Fields: Quarks $\psi^{\text{color}}_{\text{flavor}}$ and Gluon $G^{\text{color}}(A \cdot T, g)$.

Basic Lagrangian:

$$\mathcal{L} = \overline{\psi}(i \not\!\! \partial - g \not\!\! A \cdot t - m)\psi - \frac{1}{4} G(A \cdot T, g) \cdot G(A \cdot T, g)$$

- $g$: gauge Coupling Strength
- $m_i$: quark masses
- $t$ & $T$: color SU(3) matrices in the fundamental and adjoint representations.

Group factors: $C_F = \frac{4}{3}$; $T_F = \frac{1}{2}$; $C_A = 3$

Interaction Vertices:
Why does QCD play such a crucial role in High Energy Phenomenology?

- The parton picture language provides the foundation on which all modern particle theories are formulated, and all experimental results are interpreted.
- The validity of the parton picture is based empirically on an overwhelming amount of experimental evidence collected in the last 40-50 years, and theoretically on the Factorization Theorems of PQCD.

How could the simple (almost non-interacting) parton picture possibly hold in QCD — a strongly interacting quantum gauge field theory?
Answer: 3 unique features of QCD:

1. Asymptotic Freedom:
   A strongly interacting theory at long-distance can become weakly interacting at short-distance.

2. Infra-red Safety:
   There are classes of infra-red safe quantities which are independent of long-distance physics, hence are calculable in PQCD.

3. Factorization:
   There are an even wider class of physical quantities which can be factorized into a long-distance piece (not calculable, but universal) and short-distance piece (process-dependent, but infra-red safe, hence calculable).
Key concepts: Ultra-violet divergences, bare Green fns, renormalization, RGE, anomalous dimensions, renormalized G.Fs, ... etc.

Asymptotic Freedom

Universal (running) coupling:

\[ \alpha_s(Q) = \frac{g^2}{4\pi} = \frac{b}{\ln(Q/\Lambda)} (1 + \ldots) \]
The importance of **Scales** -- Renormalization and Factorization

Renormalization Group Equations (RGE) relates physics at different scales

**Upper end of exptl. energy**

$10^{19}$ GeV (Plank Scale)
$10^{-20}$ fm

Range of physical interest

$\sim 1-2$ GeV

MeV (Nuclear Scale)
$100$ fm

**M** (huge)

$\mu$ or $Q$
(large/hard)

$m$, $\Lambda$
(soft/confinement)

Ultra-violet Renormalization hides / summarizes our ignorance of physics at huge scale in $\alpha_s(\mu_R)$, $m_i(\mu_R)$, ...

Analogaies and correspondences (see later)

Infra-red / collinear Factorization hides / summarizes non-perturbative QCD physics at confinement scale in $f_\alpha(x, \mu_F)$, $d_\alpha(x, \mu_F)$, ...
### “Renormalization” and “Factorization”

<table>
<thead>
<tr>
<th>UV renormalization</th>
<th>Collinear/soft factorization</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A:</strong> Bare Green Func.</td>
<td>Partonic X-sect</td>
</tr>
<tr>
<td>$G_0(\alpha_0, m_0, ..)$</td>
<td>$F_a$</td>
</tr>
<tr>
<td><strong>B:</strong> Ren. constants</td>
<td>Pert. parton dist.</td>
</tr>
<tr>
<td>$Z_i(\mu)$</td>
<td>$f_a^b(\mu)$</td>
</tr>
<tr>
<td><strong>C:</strong> Ren. Green Fun.</td>
<td>Hard X-sect</td>
</tr>
<tr>
<td>$G_R = G_0/Z$</td>
<td>$\hat{F} = F/f$</td>
</tr>
<tr>
<td><strong>D:</strong> Anomalous dim.</td>
<td>Splitting fun.</td>
</tr>
<tr>
<td>$\gamma = \frac{\mu}{Z} dZ/d\mu$</td>
<td>$P = \frac{\mu}{f} d_f f$</td>
</tr>
<tr>
<td><strong>E:</strong> Phys. para. $\alpha, m$</td>
<td>Had. parton dist.</td>
</tr>
<tr>
<td>$\alpha_0 Z_i \ldots$</td>
<td>$f_A$ resummed</td>
</tr>
<tr>
<td><strong>F:</strong> Phys sc. amp.</td>
<td>Hadronic S.F.'s</td>
</tr>
<tr>
<td>$\alpha(\mu) G_R(m, \mu)$</td>
<td>$F_A$</td>
</tr>
</tbody>
</table>

Some common features:

- **A**: divergent; but, independent of “scheme” and scale $\mu$;
- **B**: divergent; scale and scheme dependent;
  - universal; absorbs all ultra-violet/soft/collinear divergences;
- **C & D**: finite; scheme-dependent;
  - D controls the $\mu$ dependence of E & F;
- **E**: physical parameters to be obtained from experiment;
- **F**: Theoretical “prediction”; $\mu$-indep. to all orders,
  - but $\mu$-dep. at finite order $n$; $\mu d/du \sim \mathcal{O}(\alpha^{n+1})$

Note: “Renormalization” is factorization (of UV divergences);
“factorization” is renormalization (of soft/collinear div.)
What to do with the long-distance physics associated with colinear/soft singularities in PQCD?

1st strategy:
Identify physical observables which are insensitive to the singularities! (Infra-red-safe (IRS) quantities)

**Total Hadronic Cross-section (inclusive):**

\[ \sigma_{tot}(s) = \sigma_0(s)[1 + \alpha_s(s)c_1 + \ldots] \]

Block – Nordsieck Thm → \(c_{1,2,\ldots}\) are finite, i.e. IRS (unitarity)

**Order \(\alpha_s\):**
Cancelling of the colinear/soft singularities between real and virtual diagrams
Infra-Red-Safe observables:

Total hadronic Cross-section $\sigma_{\text{tot}} / \sigma_{\mu^+ \mu^-}$

Sterman-Weinberg jet cross-sections and their modern variations (Jade-, Durham-, ... algorithms);
Jet shape observables: Thrust, ... ;
energy-energy correlation ; ....

Essential feature of a general IRS physical quantity:

*the observable must be such that it is insensitive to whether $n$ or $n+1$ particles contributed -- if the $n+1$ particles has $n$-particle kinematics*  

\[ \approx \] 

*example of IRS "jet algorithm"*
Figure 40.6: World data on the total cross section of $\sigma(e^+e^- \rightarrow \text{hadrons})$ and the ratio $R(s) = \sigma(e^+e^- \rightarrow \text{hadrons}, s)/\sigma(e^+e^- \rightarrow \mu^+\mu^-, s)$, $\sigma(e^+e^- \rightarrow \text{hadrons}, s)$ is the experimental cross section corrected for initial state radiation and electron-positron vertex loops, $\sigma(e^+e^- \rightarrow \mu^+\mu^-, s) = 4\pi\alpha^2(s)/3s$. Data errors are total below 2 GeV and statistical above 2 GeV. The curves are an educative guide: the broken one (green) is a naive quark-parton model prediction and the solid one (red) is 3-loop pQCD prediction (see "Quantum Chromodynamics" section of this Review, Eq. (9.12) or, for more details, K. G. Chetyrkin et al., Nucl. Phys. B586, 56 (2000) (Erratum ibid. B634, 413 (2002)). Breit-Wigner parameterizations of $J/\psi, \psi(2S)$, and $Y(nS), n = 1, 2, 3, 4$ are also shown. The full list of references to the original data and the details of the $R$ ratio extraction from them can be found in [arXiv:hep-ph/0512114]. Corresponding computer-readable data files are available at http://pdg.lbl.gov/exact/contents.html. (Courtesy of the COMPAS/Protvino and HEPDATA/Durham) Groups, August 2005. Corrections by P. Janet (CERN) and M. Schmitt (Northwestern U.).) See full-color version on color pages at end of book.
The 2nd strategy:
Factorization $\rightarrow$ QCD Parton model

Factorize the physical observable into a calculable IRS part and a non-calculable but universal piece.

Example: One particle inclusive cross-section

\[
\sigma(s, z) = \int_1^z \frac{d\zeta}{\zeta} \hat{\sigma}^a \left( \frac{s}{\mu}, \frac{z}{\zeta}, \alpha_s(\mu) \right) \cdot D_a(\zeta, \mu)
\]
Lepton-hadron Sc.

Master Equation for QCD Parton Model – the Factorization Theorem

\[ F_A^λ(x, \frac{m}{Q}, \frac{M}{Q}) = \sum_a f_A^a(x, \frac{m}{μ}) \otimes \tilde{F}_a^λ(x, \frac{Q}{μ}, \frac{M}{Q}) + \mathcal{O}\left(\left(\frac{λ}{Q}\right)^2\right) \]

Experimental Input

- Universal Parton Dist. Fn.
- Non-Perturbative Parametrization at \( Q_0 \)
- DGLAP Evolution to \( Q \)

Hard Cross-section, calculable (may contain \( α_s^{\text{pert}} \text{Log}^n(M/Q) \))

Theory Input

Extracted by global analysis
Hadron Collider Physics

hadrons, leptons, hadrons

partons, gauge bosons, new particles

SM and New physics

frag. functions
hadronization models: MC programs
jet algorithms

(universal) parton Distributions

L.D. hadrons
S.D. partons, gauge bosons, new particles
L.D. leptons, hadrons
Precision Electroweak Physics at Hadron Colliders

Physics of Drell-Yan, $W$ and $Z$ Bosons
W-boson physics

1. W-boson production and decay at hadron collider
2. How to measure W-boson mass and width?
3. High order radiative corrections:
   - QCD (NLO, NNLO, Resummation)
   - EW (QED-like, NLO)
4. ResBos and ResBos-A
W-boson production at hadron colliders

PDFs are known from deep inelastic scattering

\[ \sigma_{hh' \rightarrow W+X} = \sum_{f,f'} \int_0^1 dx_1 dx_2 \left\{ \phi_{f/h}(x_1) \hat{\sigma}_{ff'} \phi_{f'/h'}(x_2) + (x_1 \leftrightarrow x_2) \right\} \]

partonic "Born" cross section of $f f' \rightarrow W^+$
W-boson production at hadron colliders

PDFs: probability of finding a “parton” inside the hadron

parton distribution

Hard scattering

ISR and FSR: (colored) initial and final states can radiate gluons

underlying events (from proton remnants)

Jet
Fixed order pQCD prediction

\[ \sigma = \frac{1}{2S} \int \frac{d\xi_A}{\xi_A} \frac{d\xi_B}{\xi_B} f_{i/A}(\xi_A, \mu) f_{i/B}(\xi_B, \mu) \cdot d\hat{\sigma} \]

\[ d\hat{\sigma} = \left| M \right|^2 (2\pi)^4 \delta^{(4)}(q - k - l) \frac{d^3q}{(2\pi)^3} 2q_0 \]

\[ \frac{d\sigma}{dq_T^2 dy dQ^2} = \frac{1}{S} \int \frac{d\xi_A}{\xi_A} \frac{d\xi_B}{\xi_B} f_{i/A}(\xi_A, \mu) f_{i/B}(\xi_B, \mu) \]

\[ \cdot \left( \frac{\pi^2}{Q^2} \right) \cdot \left| M \right|^2 \cdot \delta \left( 1 - \frac{x_A}{\xi_A} \right) \cdot \delta \left( 1 - \frac{x_B}{\xi_B} \right) \]

\[ \cdot \delta(q_T^2) \cdot \delta(Q^2 - M_W^2) \]

\[ Q = \sqrt{Q^2} = \sqrt{q^2}, \ \mu = Q = M_W, \ x_A = \frac{Q}{\sqrt{S}} e^y, \ x_B = \frac{Q}{\sqrt{S}} e^{-y} \]
\[
\frac{d\sigma}{dq^2_T dy dQ^2} = \int \frac{d\xi_A}{(\xi_A S + U - Q^2)} \left( \frac{\hat{s} d\hat{\sigma}}{d\hat{t}} \right) \cdot f_{j/A}(\xi_A, \mu) 
\]
\[
\cdot f_{j/B} \left( \xi_B = \frac{-Q^2 - \xi_A (T - Q^2)}{\xi_A S + U - Q^2}, \mu \right) \cdot \delta (Q^2 - M_W^2) 
\]
\[
+ \int \frac{d\xi_B}{(\xi_B S + T - Q^2)} \left( \frac{\hat{s} d\hat{\sigma}}{d\hat{t}} \right) \cdot f_{j/B}(\xi_B, \mu) 
\]
\[
\cdot f_{j/A} \left( \xi_A = \frac{-Q^2 - \xi_B (U - Q^2)}{\xi_B S + T - Q^2}, \mu \right) \cdot \delta (Q^2 - M_W^2) 
\]

\[
T = Q^2 - \sqrt{q^2_T + Q^2} \sqrt{S} e^{-y}, \\
U = Q^2 - \sqrt{q^2_T + Q^2} \sqrt{S} e^{y}, \\
\hat{s} = \xi_A \xi_B S, \\
\hat{t} = \xi_A (T - Q^2) + Q^2 \\
M = \left( \begin{array}{c}
\vdots \\
\text{For simplicity, only consider } qq \rightarrow Wg \\
\end{array} \right)
\]
• Virtual Corrections

• Real emission contributions
Theory Calculations

There are a variety of programs available for comparison of data to theory and/or predictions.

- Tree level (Alpgen, CompHEP, Grace, Madgraph…)
  Les Houches accord
- Parton shower Monte Carlos (Herwig, Pythia,…
  MC@NLO
- $N^n$LO (EKS, Jetrad, Dyrad, Wgrad, Zgrad, Horace
  recover NLO (NNLO?) normalization
- Resummed (ResBos)

Important to know strengths/weaknesses of each.
Fixed order Perturbative calculations

- Higher order in $\alpha_s^{(n)}$
  - Less sensitive to Factorization Scale $\mu$

- High $q_T$ and smaller $y$ (i.e. more central)
  - PDF (parton distribution function) better known

- With larger Luminosity
  - Test QCD in one large scale problem (i.e. $q_T \sim Q$)

- Up to now, most of the Data used in Testing QCD were One large scale observables, e.g., Jet-$P_T$.

- Observables involving Multiple Scales, e.g., $q_T$ of W-Boson with mass $M_W$, can only be accurately described in QCD after including effects of Resummation.
Shortcoming of fixed order calculation

- Cannot describe data with small $q_T$ of $W$-boson.
- Cannot precisely determine $m_W$ at hadron colliders without knowing the transverse momentum of $W$-boson. Most events fall in the small $q_T$ region.

\[ \delta(q_T^2) \]

Transverse momentum

(at NLO)
QCD Resummation is needed

\[ \frac{d\sigma}{dQ_T} \text{ (pb/GeV)} \]

- Dashed: CSS (1,1,1)
- Solid: CSS (2,2,1)
- Dotted: Pert (\(\alpha_s\))
- Dot-dashed: Pert (\(\alpha_s^2\))
Resummation calculations agree with data very well

Predicted by ResBos:

A program that includes the effect of multiple soft gluon emission on the production of W and Z bosons in hadron collisions.
Transverse Momentum and $\phi_{\eta}^*$ distributions

\[ \Delta \Phi \]

\[ \cos \theta_{\eta}^* = \tanh\left[ \frac{\eta_{\ell^+} - \eta_{\ell^-}}{2} \right] \]

\[ \phi_{\eta}^* = \tan\left( \frac{\pi - \Delta \phi}{2} \right) \sin \theta_{\eta}^* \]
1. Both distributions are important observables for the DY production and are extremely precise;
2. Both observables are sensitive to the soft gluon radiations $\ln \frac{Q^2}{P_T V^2}$
3. All order resummation is needed;

$$\phi^*_\eta = \tan\left(\frac{\pi - \Delta \phi}{2}\right) \sin \theta^*_\eta$$
Transverse Momentum Resummation

• Resummation for DY-like processes are well understand;
  Collins, Soper, Sterman ’85

• Resummation carried out mostly in impact parameter b space;
  Resbos [Balaze, Belyaev, Berger, Cao, Chen, Isaacson, Nadolsky, Yuan et al ‘93],
  DYRes[Catani, de Florian, Ferrera, Grazzini ’15],
  HRes[de Florian, Ferrera, Grazzini, Tommasini ‘12],
  Cute [Becher, Neubert, Wilhelm ‘12, ‘13],
  arTeMiDe [Scimemi, Vladimirov ’17]

• Resummaion also carried out in momentum space;
  Frixione, Nason, Ridolf ‘97;
  Ellis, Veseli ’98;
  Kulesza, Stirling ’00;
  Monni, Re, Torrielli ’16;
  Bizon, Monni, Re, Rottoli, Rorrielli ‘17;
To recover the “K-factor” in NLO total rate

To include the C-Functions

\[ \frac{d\sigma}{dQ^2 dy} \]

Finite +

Singular

The area under the \( q_T \) curve will reproduce the total rate at the order \( \alpha_s^{(1)} \) if \( Y \) term is calculated to \( \alpha_s^{(1)} \) as well.
As $q_T \to 0$

$$\left. \frac{d\sigma}{dq_T^2 \, dydQ^2} \right|_{q_T \to 0} = \left( \pi \sigma_0 \right) \cdot \delta(Q^2 - M_w^2) \cdot \left( \frac{1}{2\pi q_T^2} \right) \left( \frac{\alpha_s(Q)}{\pi} \right)$$

$$\cdot \left\{ f_{q_A}(x_A, Q) \left[ \frac{P_{q_A} \otimes f_{\bar{q}}}{x_A, Q} \right] x_B, Q + \left[ P_{q_B} \otimes f_{\bar{q}} \right] x_A, Q f_{\bar{q}_B}(x_B, Q) + f_{q_A}(x_A, Q) f_{\bar{q}_B}(x_B, Q) \cdot \left[ 2 \left( \frac{4}{3} \right) \ln \left( \frac{Q^2}{q_T^2} \right) + 2(-2) \right] \right\}$$

Exponentiate

Diagrammatically, to preserve transverse momentum conservation, we have to go to the impact parameter space (b-space) to perform resummation.
What’s QCD Resummation?

• Perturbative expansion

\[
\frac{d\hat{\sigma}}{dq_T^2} \sim \alpha_s \left\{ 1 + \alpha_s + \alpha_s^2 + \cdots \right\}
\]

• The singular pieces, as \( \frac{1}{q_T^2} \) (1 or log’s)

\[
\frac{d\hat{\sigma}}{dq_T^2} \sim \frac{1}{q_T^2} \sum_{n=1}^{\infty} \sum_{m=0}^{2^{n-1}} \alpha_s^{(n)} \ln^{(m)} \left( \frac{Q^2}{q_T^2} \right)
\]

\[
\sim \frac{1}{q_T^2} \left\{ \alpha_s \left( L + 1 \right) + \alpha_s^2 \left( L^3 + L^2 + L + 1 \right) + \alpha_s^3 \left( L^5 + L^4 + L^3 + L^2 + L + 1 \right) + \cdots \right\}
\]

Resummation is to reorganize the results in terms of the large Log’s.
Resummed results:

\[ \frac{d\sigma}{dq_T^2} \sim \frac{1}{q_T^2} \left\{ \left[ \alpha_s (L+1) + \alpha_s^2 (L^3 + L^2) + \alpha_s^3 (L^5 + L^4) + \cdots \right] + \left[ + \alpha_s^2 (L+1) + \alpha_s^3 (L^3 + L^2) + \cdots \right] + \left[ + \alpha_s^3 (L+1) + \cdots \right] \right\} \]

- Determined by \( A^{(1)} \) and \( B^{(1)} \)
- Determined by \( A^{(2)} \) and \( B^{(2)} \)
- Determined by \( A^{(3)} \) and \( B^{(3)} \)

QCD Resummation

In the formalism by Collins-Soper-Sterman, in addition to these perturbative results, the effects from physics beyond the leading twist is also implemented as

[non-perturbative functions].
CSS Resummation Formalism

\[
\frac{d\sigma}{dq_T^2 \, dy \, dQ^2} = \frac{\pi}{s} \sigma_0 \delta \left( Q^2 - M_W^2 \right). \\
\left\{ \frac{1}{(2\pi)^2} \int d^2 b \, e^{i q_T \cdot b} \tilde{W} (b, Q, x_A, x_B) \cdot [\text{Non-perturbative functions}] \right\}
\]

\[
+ Y(q_T, y, Q) \rightleftharpoons \sum_j \int_{x_A}^1 \frac{d\xi_A}{\xi_A} C_q \left( \frac{x_A}{\xi_A}, b, \mu \right) f_j / (\xi_A, \mu)
\]

\[
\tilde{W} = e^{-s(b)} \cdot C \otimes f(x_A) \cdot C \otimes f(x_B)
\]

\[
= \sum_k \int_{x_B}^1 \frac{d\xi_B}{\xi_B} C_{qk} \left( \frac{x_A}{\xi_A}, b, \mu \right) f_k / (\xi_B, \mu)
\]

[Sudakov form factor] \( S(b) = \int_{(\frac{b}{\mu})^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \ln \left( \frac{Q^2}{\bar{\mu}^2} \right) A(\bar{\mu}) + B(\bar{\mu}) \)]

[Non-perturbative functions] are functions of \((b, Q, x_A, x_B)\) which include QCD effects beyond Leading Twist.
Example: for $W^\pm$

$$
\sigma_0 = \left( \frac{4\pi^2 \alpha}{3} \sum_{jj'} Q_{jj'}^{(w)} \right), \quad Q_{jj'}^{(w)} = \frac{1}{4\sin^2 \theta_W} (kM)^2_{jj'}
$$

The couplings of gauge bosons to fermions are expressed in the way to include the dominant electroweak radiative corrections. The propagators of gauge bosons also contain energy-dependent width, as done in LEP precision data analysis.

Note:

$$
A \equiv \sum_{n=1}^{\infty} \left( \frac{\alpha_s}{\pi} \right)^n \cdot A^{(n)}, \quad B \equiv \sum_{n=1}^{\infty} \left( \frac{\alpha_s}{\pi} \right)^n \cdot B^{(n)}
$$

$$
C \equiv \sum_{n=0}^{\infty} \left( \frac{\alpha_s}{\pi} \right)^n \cdot C^{(n)}
$$
Diagramatically, **Resummation** is doing

\[ \alpha_s^n \ln^m \left( \frac{Q^2}{q_T^2} \right) \]

Monte-Carlo programs **ISAJET, PYTHIA, HERWIG** contain these physics.

( Note: Arbitrary cut-off scale in these programs to affect the amount of Backward radiation, i.e. Initial state radiation. )
Monte-Carlo Approach (Pythia8)
Parton showers

Backward Radiation
(Initial State Radiation)

Kinematics of the radiated gluon, controlled by Sudakov form factor with some arbitrary cut-off. (In contrast to perform integration in impact parameter space, i.e., b space.)
Collins-Soper-Sterman (CSS) Formalism
For Drell-Yan processes

The large logs will be resummed into the exponential form factor:

\[ W(b, Q) = \exp \left\{ - \int_{b_0/b^*}^{Q} \frac{d\mu}{\mu} \left( A \ln \frac{Q^2}{\mu^2} + B \right) \right\} (C \otimes f)_1 (C \otimes f)_2 \]


Only for initial state soft gluon resummation
CSS Formalism
(For Drell-Yan-like processes)

The large logs will be resummed into the exponential form factor:

\[
W(b, Q) = \exp \left\{- \int_{b_0/b^*}^{Q} \frac{d\mu}{\mu} \left( A \ln \frac{Q^2}{\mu^2} + B \right) \right\} (C \otimes f)_1 (C \otimes f)_2
\]


Only for initial state soft gluon resummation

\[
A = \sum_n A^n \left( \frac{\alpha_s}{\pi} \right)^n, B = \sum_n B^n \left( \frac{\alpha_s}{\pi} \right)^n, C = \sum_n C^n \left( \frac{\alpha_s}{\pi} \right)^n
\]

A, B, C functions are perturbatively calculable
A: Sudakov double logs
B: anomalous dimension
C: hard coefficients
CSS Formalism  
(For Drell-Yan-like processes)

The large logs will be resummed into the exponential form factor:

$$W(b, Q) = \exp \left\{ - \int_{b_0/b^*}^{Q} \frac{d\mu}{\mu} \left( A \ln \frac{Q^2}{\mu^2} + B \right) \right\} (C \otimes f)_1 (C \otimes f)_2$$

$$A = \sum_n A^n \left( \frac{\alpha_s}{\pi} \right)^n, \quad B = \sum_n B^n \left( \frac{\alpha_s}{\pi} \right)^n, \quad C = \sum_n C^n \left( \frac{\alpha_s}{\pi} \right)^n$$
Resummation effects are important for the $Z$ boson transverse momentum distribution.

ResBos prediction agrees with data very well.
How to extend CSS formula to general processes?

- **Heavy colored particles**

Final states carry color, so we should also consider complete color coherence radiation between initial and final states, and final state radiation;
A. Final states carry color, so we should also consider complete color coherence radiation between initial and final states;
B. Jet algorithm will enter into the calculations as well;
Only out of the cone radiation contributions to transverse momentum of the system
C. Jet function from collinear gluon radiation
Recent progress of modified $q_T$ resummation

- Heavy quark production
  A. top quark pair production
  B. Heavy quarkonium production
    P. Sun, C.-P. Yuan and F. Yuan, PRD88(2013)054008

- Processes involving multijets in the final state
  A. Dijet production
    P. Sun, C.-P. Yuan and F. Yuan, PRL113(2014)23,232001; PRD92(2015)9,094007
  B. Higgs+jet production
    P. Sun, C.-P. Yuan and F. Yuan, PRL114(2015)20,202001;
    P. Sun, J. Isaacson, C.-P. Yuan and F. Yuan, PLB769(2017)57-61
  C. Higgs+2jet production
    P. Sun, C.-P. Yuan and F. Yuan, PLB762(2016)47-51; arxiv:1802.02980
  D. Single top quark production
    Q. H. Cao, P. Sun, Bin Yan, C.-P. Yuan and F. Yuan, PRD98(2018)5,054032; 1902.09336
    P. Sun, Bin Yan and C.-P. Yuan, arxiv:1811.01428
  E. Z boson+jet production
    P. Sun, Bin Yan, C.-P. Yuan and F. Yuan, arxiv:1810.03804
Transverse Momentum Dependent (TMD) Factorization

Collinear PDFs

\( F(x, Q) \)

1. DGLAP evolution
2. Resum \( \ln(Q^2/\mu^2) \)
3. Kernel: purely perturbative

TMD PDFs

\( F(x, k_{\perp}, Q) \)

1. Collins-Soper evolution
2. Resum \( \ln(Q^2/k_{\perp}^2) \)
3. Kernel: can be non-perturbative

\( k_{\perp} \sim \Lambda_{QCD} \)

Collinear part

\[ F(x, b, Q) = C \otimes F(x, b_0/b^*) \times exp \left\{ - \int_{b_0/b^*}^Q \frac{d\mu}{\mu} \left( A \ln \frac{Q^2}{\mu^2} + B \right) \right\} \times exp (-F_{NP}(b, Q)) \]
Transverse Momentum Dependent (TMD) Factorization

\[ \frac{d\sigma}{q_\perp} \sim \sum_{ab} \left[ \int \frac{d^2 \vec{b}_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp \cdot \vec{b}_\perp} W_{ab \rightarrow cd}(x_1, x_2, b_\perp) + Y_{ab \rightarrow cd} \right] \]

TMD factorization, all order resummation for multiple gluon radiation effects.

The Matrix Form is defined in the color space:

\[ x_1 f_a(x_1, \mu = b_0/b_\perp) x_2 f_b(x_2, \mu = b_0/b_\perp) e^{-S_{\text{Sud}}(Q^2, b_\perp)} \]

\[ \text{Tr} \left[ H_{ab \rightarrow cd} \exp\left[ - \int_{b_0/b_\perp}^{Q} \frac{d\mu}{\mu} \gamma^s \right] S_{ab \rightarrow cd} \exp\left[ - \int_{b_0/b_\perp}^{Q} \frac{d\mu}{\mu} \gamma^s \right] \right] \]

Y = Pert - Asy

P. Sun, C.-P. Yuan, F. Yuan, PRL 2014
Drell-Yan production

\[ \frac{d\sigma}{q_{\perp}} \sim \sum_{ab} \left[ \int \frac{d^2 \vec{b}_\perp}{(2\pi)^2} e^{-i \vec{q}_\perp \cdot \vec{b}_\perp} W_{ab \to cd}(x_1, x_2, b_{\perp}) + Y_{ab \to cd} \right] \]

\[ Y = \begin{pmatrix} \text{graph} \end{pmatrix} \quad q_T \to 0 \]

\[ W(b, Q) = \exp \left\{ - \int_{b_0/b_*}^{Q} \frac{d\mu}{\mu} (A \ln \frac{Q^2}{\mu^2} + B) \right\} \times \exp(-F_{\text{NP}}(b, Q)) \times (C \otimes f_1) (C \otimes f_2) \]

\[ F_{\text{NP}}(b, Q) = g_1 b^2 + g_2 \ln \frac{Q}{Q_0} \ln \frac{b}{b_*} \]

\[ g_1 = 0.21, \quad g_2 = 0.84, \quad Q_0^2 = 2.4 \text{ GeV}^2, \quad b_* = \frac{b}{\sqrt{1 + b^2/b_{\text{max}}^2}} \]

Z boson production

\[ \sqrt{s} = 8 \text{ TeV} \]

\[ d\bar{d} \rightarrow Z/\gamma^* \rightarrow e^+e^- \]

\[ \sim 90\% \text{ Cross Section} \]

\[ \text{d}\sigma/\text{d}P_T^Z \text{ [pb/GeV]} \]

Perturbative

Matching

ResBos

TMD

non perturbative
Higgs plus two jets production at LHC

Peng Sun, C.-P. Yuan and Feng Yuan, arxiv:1802.02980, PLB762(2016)47-51
Higgs plus two jets production at LHC

ATLAS Preliminary
\( \sqrt{s} = 13 \, \text{TeV}, 36.1 \, \text{fb}^{-1} \)
\( H \rightarrow YY, m_H = 125 \, \text{GeV} \)

\( 1/N \frac{dN}{d\Delta \eta_{jj}} \) / 0.2

\( 1/N \frac{dN}{d\Delta \phi_{YY,jj}} \) / 0.1

Data, sidebands

\( ggH \)

VBF

ATLAS Preliminary
\( \sqrt{s} = 13 \, \text{TeV}, 36.1 \, \text{fb}^{-1} \)
\( H \rightarrow YY, m_H = 125 \, \text{GeV} \)

\( \gamma\gamma + jj \)

VBF
Color structure of Higg+2jet
qT Distributions

Black: resummation calculation
Red: leading order matrix element with parton showers by Pythia

Resummation scale dependence:

\[ \hat{\mu} = P_{J \perp}^{\text{sub}} \rightarrow P_{J \perp}^{\text{lead}} \]

Blue: NLO calculation; Orange: Asymptotic part; Green: Y-piece
Normalized qT Distributions

Red: leading order matrix element with parton showers by Pythia

Black: resummation calculation

\[ p p \rightarrow h (V^V \rightarrow h) + \text{Jet+Jet} \]
\[ \sqrt{s} = 13 \text{ TeV} \]
\[ P_{TJ} > 30 \text{ GeV} \]
\[ |\Delta y_{jj}| > 2.6 \]
Compared to Pythia

Experimental analysis:
\[ \Delta \phi_{\gamma\gamma, jj} > 2.6 \]

The difference between Res and Pythia: 8%

HL-LHC, VBF Higgs production, 10%
Single top quark vs. VBF Higgs+2jet

A. Final states carry color;

B. Jet algorithm will enter into the calculations as well

C. Color coherence effects between the initial and final states are important

To help VBF Higgs measurement and probe hVV couplings

Peng Sun, C.-P. Yuan and Feng Yuan, PLB762(2016)47-51, 1802.02980
Color coherence effects

\[ T = \ln \frac{-\hat{t}}{\hat{s}} + \ln \frac{-\hat{t} - m_t^2}{\hat{s} - m_t^2} \]

Blue: \( |y_J| < 4.5 \)
Red: \( |y_J| < 4.5 \), turn off the color coherence factor \( T \)
Black: \( 3 < |y_J| < 4.5 \)
Green: \( 3 < |y_J| < 4.5 \), turn off the color coherence factor \( T \)

\[
\hat{t} = \left( p_{q'} - p_q \right)^2
\]
Compared to Pythia

The resummation prediction is different from Pythia when $3 < |y_J| < 4.5$

The color coherence factor plays an important role in the comparison.

$T = \ln \frac{-\hat{t}}{\hat{s}} + \ln \frac{- (\hat{t} - m_t^2)}{\hat{s} - m_t^2}$
Summary

A. Transverse momentum resummation framework for processes with final state jets

B. The resummation effects for Di-jet, Z/Higgs/top +jet and H+2jet productions

C. For Di-jet and Z+jet processes, our resummation calculation agree with data very well.

D. Color coherence effects in t-channel single top quark and H+2jet production are important.