Toward a Generalization of Higgs Effective Field Theory

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Based on 1904.07618

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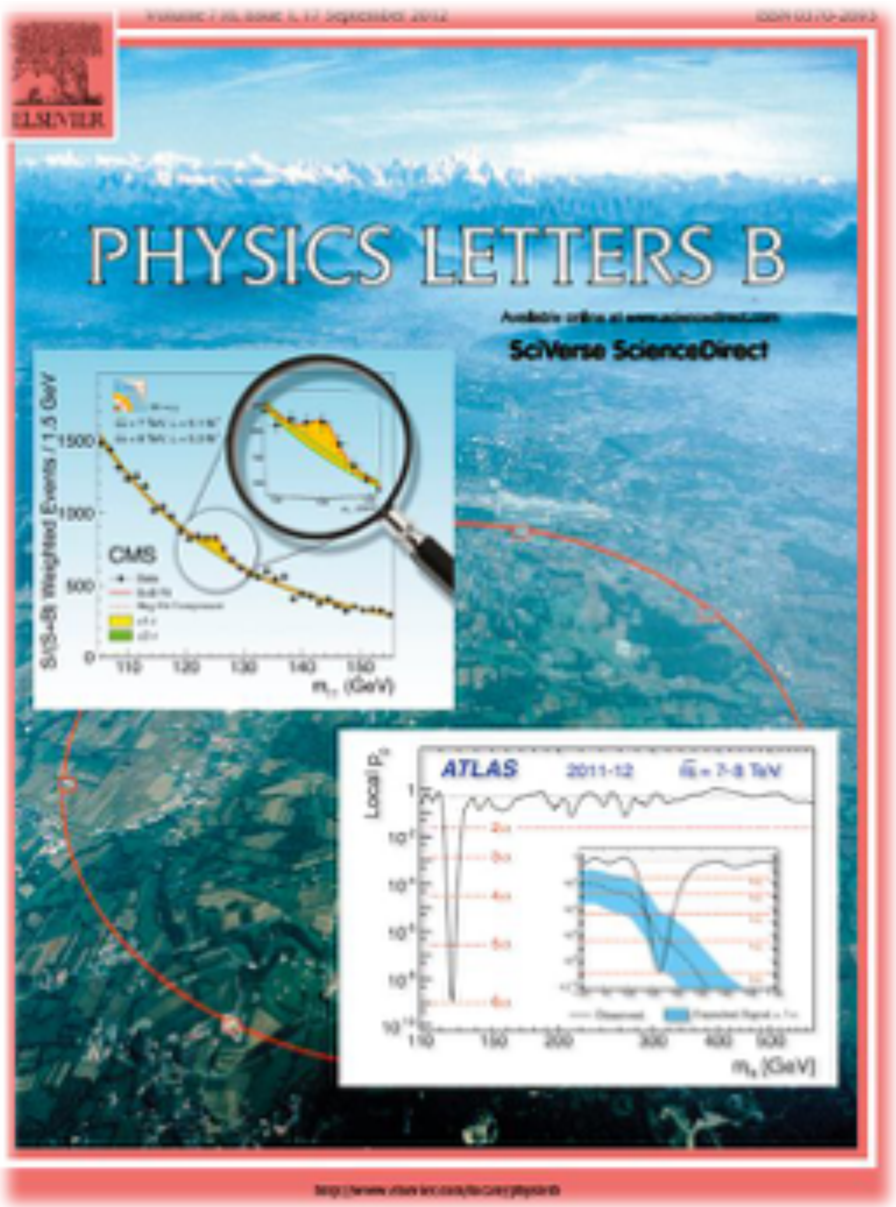
Osaka Univ. Particle Physics Theory Group Seminar

Plan

- Introduction
- CCWZ method
- HEFT
- GHEFT
- Summary

We found a h(125) !



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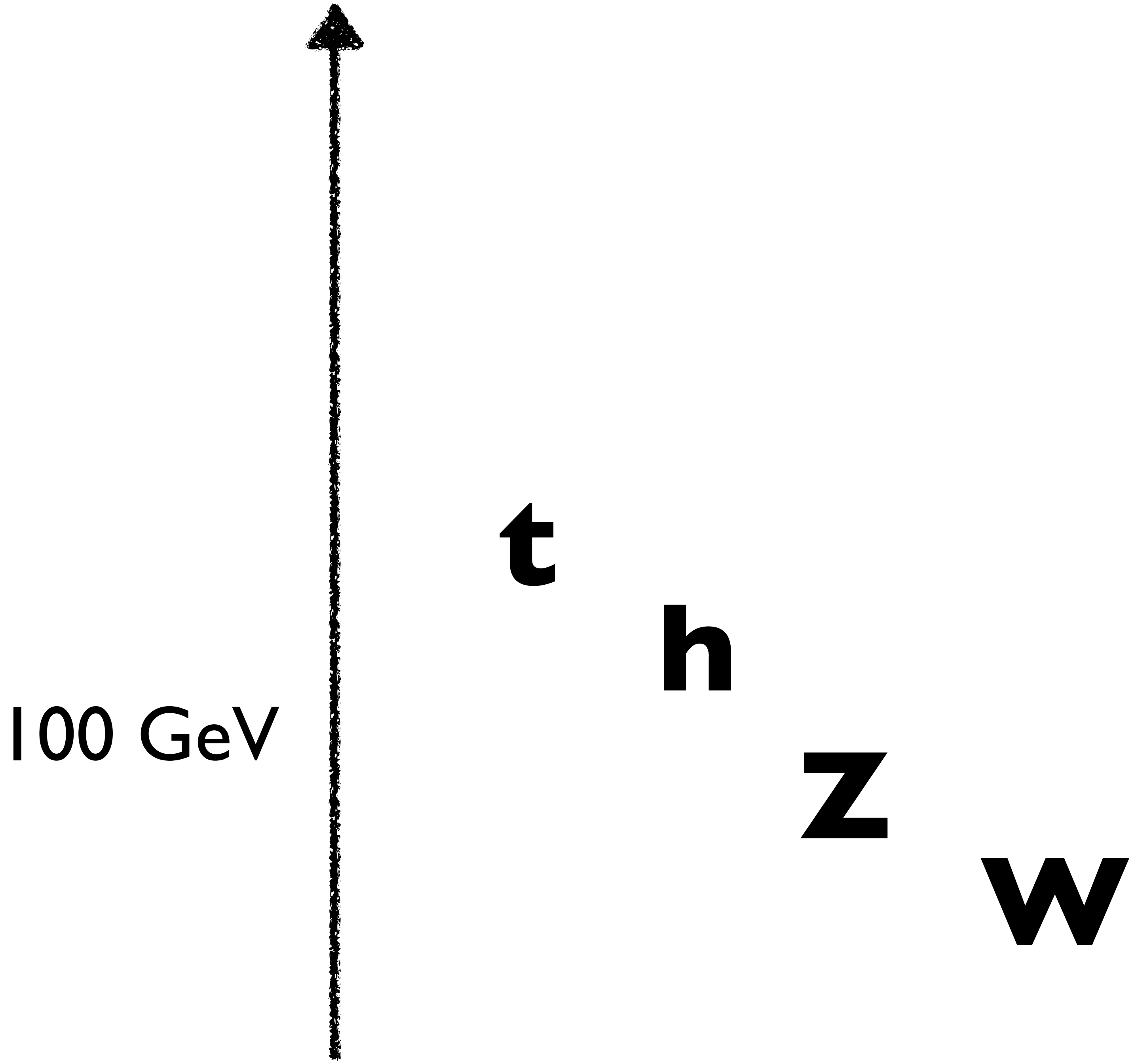
Particle Zoo



100 GeV

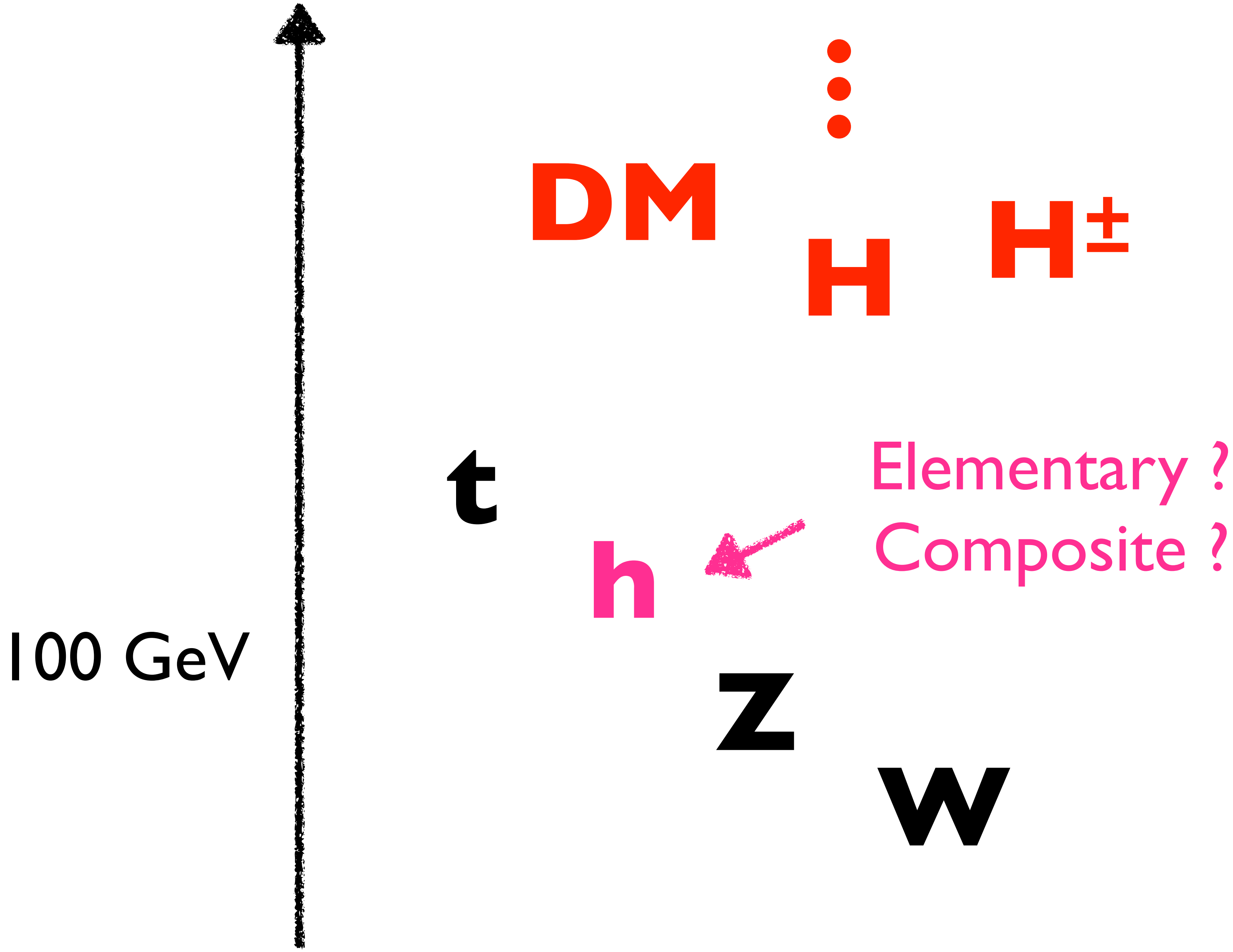
t

h

Z

W

Particle Zoo



⋮

DM

H

$H^{\pm}$

t

h

Elementary ?

Composite ?

100 GeV

Z

W

Question

Unified description ??

“Generalized Higgs EFT”

Difficulty

- Spontaneous symmetry breaking

$$\underset{SU(2)_L \times U(1)_Y}{G} \quad \rightarrow \quad \underset{U(1)_{EM}}{H}$$

- What does G look like at vacuum?

- Generally speaking, G is **non-linearly** realized.

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CCWZ method

Coleman-Callen-Wess-Zumino (1969)

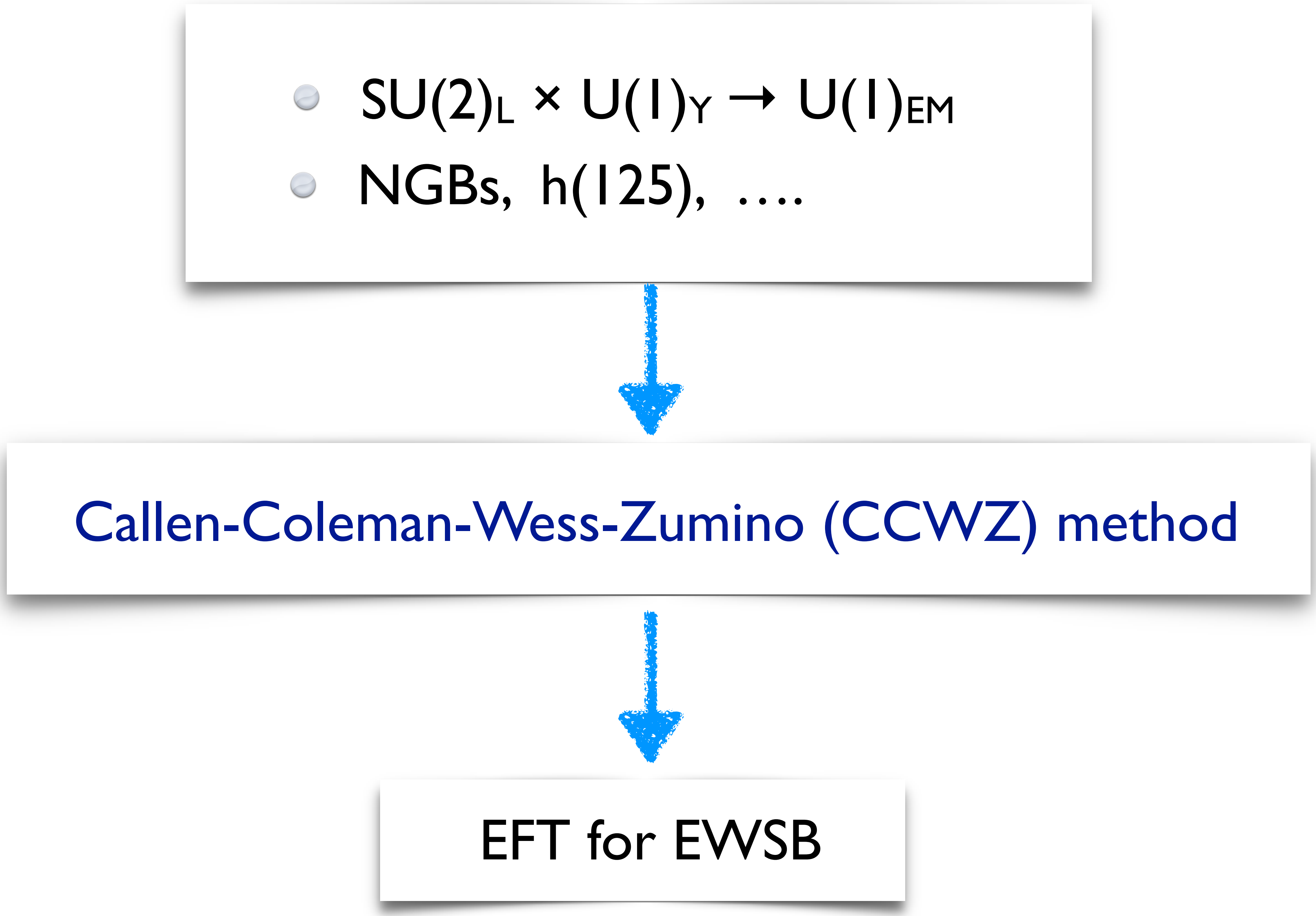
- G → H
- NGBs, Matters

Callen-Coleman-Wess-Zumino (CCWZ) method

EFT on G/H

CCWZ method

Coleman-Callen-Wess-Zumino (1969)



- $SU(2)_L \times U(1)_Y \to U(1)_{EM}$
- NGBs, h(125), ….

Callen-Coleman-Wess-Zumino (CCWZ) method

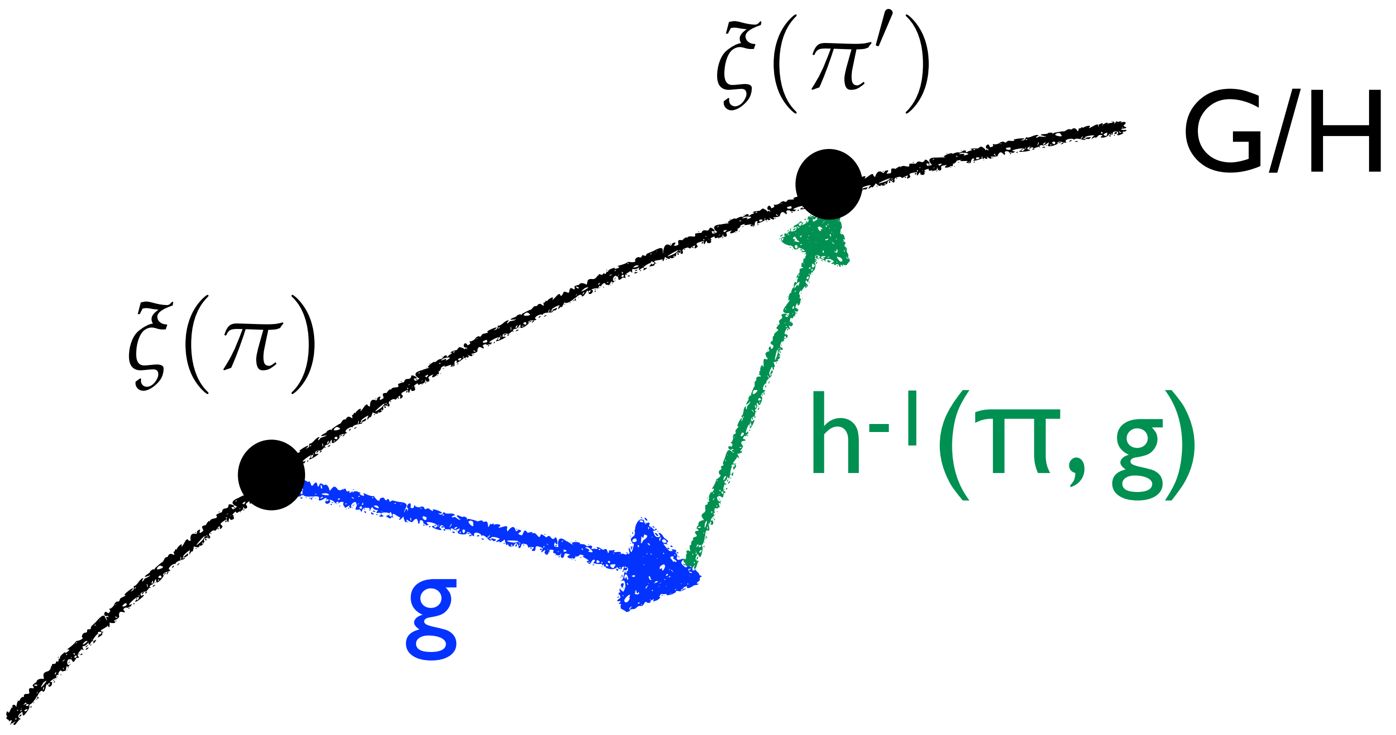
EFT for EWSB

NGBs on G/H

- NGB fields $\xi(\pi) = \exp(i\pi^a X^a) \quad \in G/H$

X^a : generators for G/H

$$\boxed{\xi(\pi) \xrightarrow{G} \xi(\pi') = \mathfrak{g}\,\xi(\pi)\,\mathfrak{h}^{-1}(\pi, \mathfrak{g})}$$



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- G **nonlinearly** acts on NGBs !

$$\pi' = \pi + \mathcal{O}(\pi^3)$$

NGBs on G/H

- NGB fields $\xi(\pi) = \exp(i\pi^a X^a) \quad \in G/H$

X^a : generators for G/H

$$\boxed{\xi(\pi) \xrightarrow{G} \xi(\pi') = \mathfrak{g}\,\xi(\pi)\,\mathfrak{h}^{-1}(\pi, \mathfrak{g})}$$

- G **nonlinearly** acts on NGBs !
- Kinetic term? $(\partial_\mu \xi)^2$ is NOT invariant under G.

$$\partial_\mu \xi \xrightarrow{G} \mathfrak{g}\,(\partial_\mu \xi)\,\mathfrak{h}^{-1} + \mathfrak{g}\,\xi\underline{(\partial_\mu \mathfrak{h}^{-1})}$$

NGBs on G/H

- NGB fields $\xi(\pi) = \exp(i\pi^a X^a) \quad \in G/H$

X^a : generators for G/H

$$\boxed{\xi(\pi) \xrightarrow{G} \xi(\pi') = \mathfrak{g}\,\xi(\pi)\,\mathfrak{h}^{-1}(\pi, \mathfrak{g})}$$

- Maurer-Cartan 1-form $\alpha_\mu = -i\xi^{-1}(\partial_\mu \xi)$

- Irreducible decomp

G/H : $\alpha_{\perp\mu} = \mathrm{tr}[\alpha_\mu X^a] X^a$

H : $\alpha_{\parallel\mu} = \mathrm{tr}[\alpha_\mu S^A] S^A$

S^A : generators for H

NGBs on G/H

- G-transformation :

$$\alpha_{\perp\mu} \xrightarrow{G} \mathfrak{h}\,\alpha_{\perp\mu}\,\mathfrak{h}^{-1}$$

$$\alpha_{\parallel\mu} \xrightarrow{G} \mathfrak{h}\,\alpha_{\parallel\mu}\,\mathfrak{h}^{-1} - i(\partial\mathfrak{h})\mathfrak{h}^{-1}$$

- G-invariant building block :

O(p²) $\mathrm{tr}[\alpha_{\perp\mu}\alpha_{\perp}^{\mu}]$

O(p⁴) $\mathrm{tr}[\alpha_{\perp\mu}\alpha_{\perp}^{\mu}\alpha_{\perp\nu}\alpha_{\perp}^{\nu}]$

$\vdots$

NGBs on $SU(2)_L \times U(1)_Y / U(1)_{EM}$

- NGB fields
- MC 1-form
- Irreducible decomp
- Use G/H components

NGBs on $SU(2)_L \times U(1)_Y / U(1)_{EM}$

- NGB fields $$(\xi_W, \xi_Y) = (e^{i\pi^i \frac{\tau^i}{2}}, e^{i\pi^3 \frac{\tau^3}{2}})$$

$$\boxed{\begin{aligned} \xi_W &\xrightarrow{G} \mathfrak{g}_W \, \xi_W \, \mathfrak{h}^{-1}(\pi, \mathfrak{g}_W, \mathfrak{g}_Y) \\ \xi_Y &\xrightarrow{G} \mathfrak{g}_Y \, \xi_Y \, \mathfrak{h}^{-1}(\pi, \mathfrak{g}_W, \mathfrak{g}_Y) \end{aligned}}$$

- MC 1-form $$\alpha_{W\mu} = -i\xi_W^\dagger(\partial_\mu \xi_W) \quad \alpha_{Y\mu} = -i\xi_Y^\dagger(\partial_\mu \xi_Y)$$

- Irreducible decomp

G/H : $$\alpha_\perp^3 = \mathrm{tr}\left[\left(\frac{\alpha_W - \alpha_Y}{2}\right)\tau^3\right] \qquad \alpha_\perp^i = \mathrm{tr}\left[\alpha_W \tau^i\right]$$

(i=1,2)

H : $$\alpha_\parallel^3 = \mathrm{tr}\left[\left(\frac{\alpha_W + \alpha_Y}{2}\right)\tau^3\right]$$

NGBs on SU(2)$_L$×U(1)$_Y$ /U(1)$_{EM}$

- SU(2)$_W$ × U(1)$_Y$ transformation

$$\alpha^i_{\perp\mu}\frac{\tau^i}{2} \xrightarrow{G} \mathfrak{h}\left(\alpha^i_{\perp\mu}\frac{\tau^i}{2}\right)\mathfrak{h}^{-1} \quad \text{(i=1,2)}$$

$$\alpha^3_{\perp\mu}\frac{\tau^3}{2} \xrightarrow{G} \mathfrak{h}\left(\alpha^3_{\perp\mu}\frac{\tau^3}{2}\right)\mathfrak{h}^{-1} = \alpha^3_{\perp\mu}\frac{\tau^3}{2}$$

$$\mathfrak{h} = \exp\left(i\theta_h(\pi,\mathfrak{g})\frac{\tau^3}{2}\right)$$

- Building blocks:

$$\alpha^i_{\perp\,\text{(i=1,2)}} \quad , \quad \alpha^3_\perp$$

NGBs on $SU(2)_L \times U(1)_Y / U(1)_{EM}$

- $SU(2)_W \times U(1)_Y$ invariant Lagrangian up to $O(p^2)$

$$\boxed{\mathcal{L}_{\rm NGB} = \frac{1}{2} G^{(0)}_{ab} \alpha^a_{\perp\mu} \alpha^{b\mu}_{\perp}}$$

$$G^{(0)}_{ab} := \frac{1}{4} \begin{pmatrix} v^2 & & \\ & v^2 & \\ & & v_Z^2 \end{pmatrix}$$

- If we introduce $U = \xi_W \xi_Y^\dagger$,

$$\mathcal{L} = \frac{v^2}{4} {\rm tr}[\partial_\mu U^\dagger \partial^\mu U] + \frac{v^2 - v_Z^2}{8} {\rm tr}[U \partial_\mu U^\dagger \tau^3] {\rm tr}[U \partial^\mu U^\dagger \tau^3]$$

"Electroweak Chiral Lagrangian"

EW gauge sector

- SU(2)$_W$ × U(1)$_Y$ invariant Lagrangian up to O(p^2)

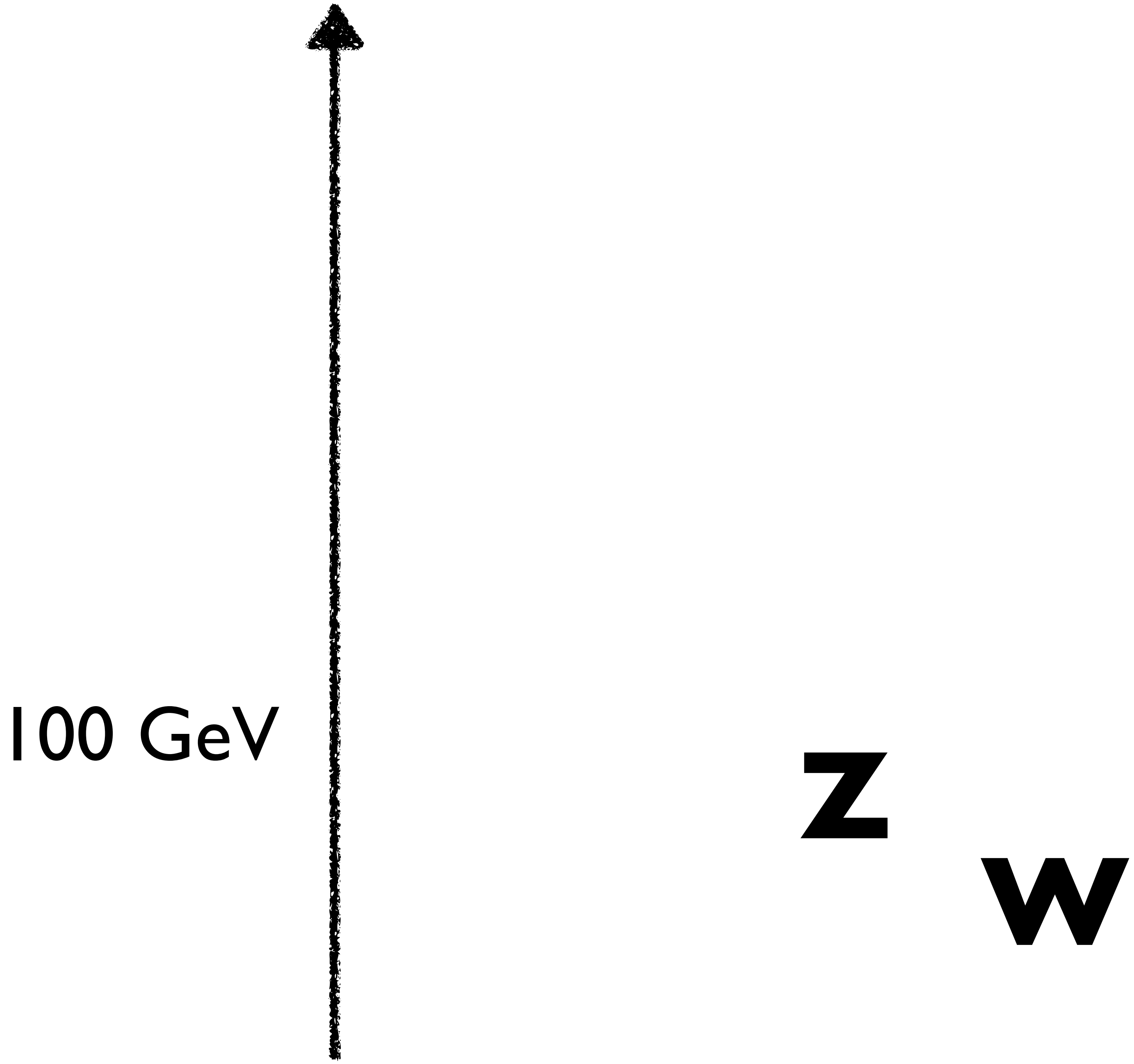
$$\boxed{\mathcal{L}_{\mathrm{NGB}} = \frac{1}{2} G^{(0)}_{ab} \alpha^a_{\perp\mu} \alpha^{b\mu}_\perp}$$

$$G^{(0)}_{ab} := \frac{1}{4} \begin{pmatrix} v^2 & & \\ & v^2 & \\ & & v_Z^2 \end{pmatrix}$$

- EW gauge bosons are introduced as:

$$\begin{aligned} \partial_\mu \xi_W &\to \partial_\mu \xi_W - i g_W W^a_\mu \frac{\tau^a}{2} \xi_W \\ \partial_\mu \xi_Y &\to \partial_\mu \xi_Y - i g_Y B_\mu \frac{\tau^3}{2} \xi_Y \end{aligned}$$

EW gauge sector



Higgs EFT (HEFT)

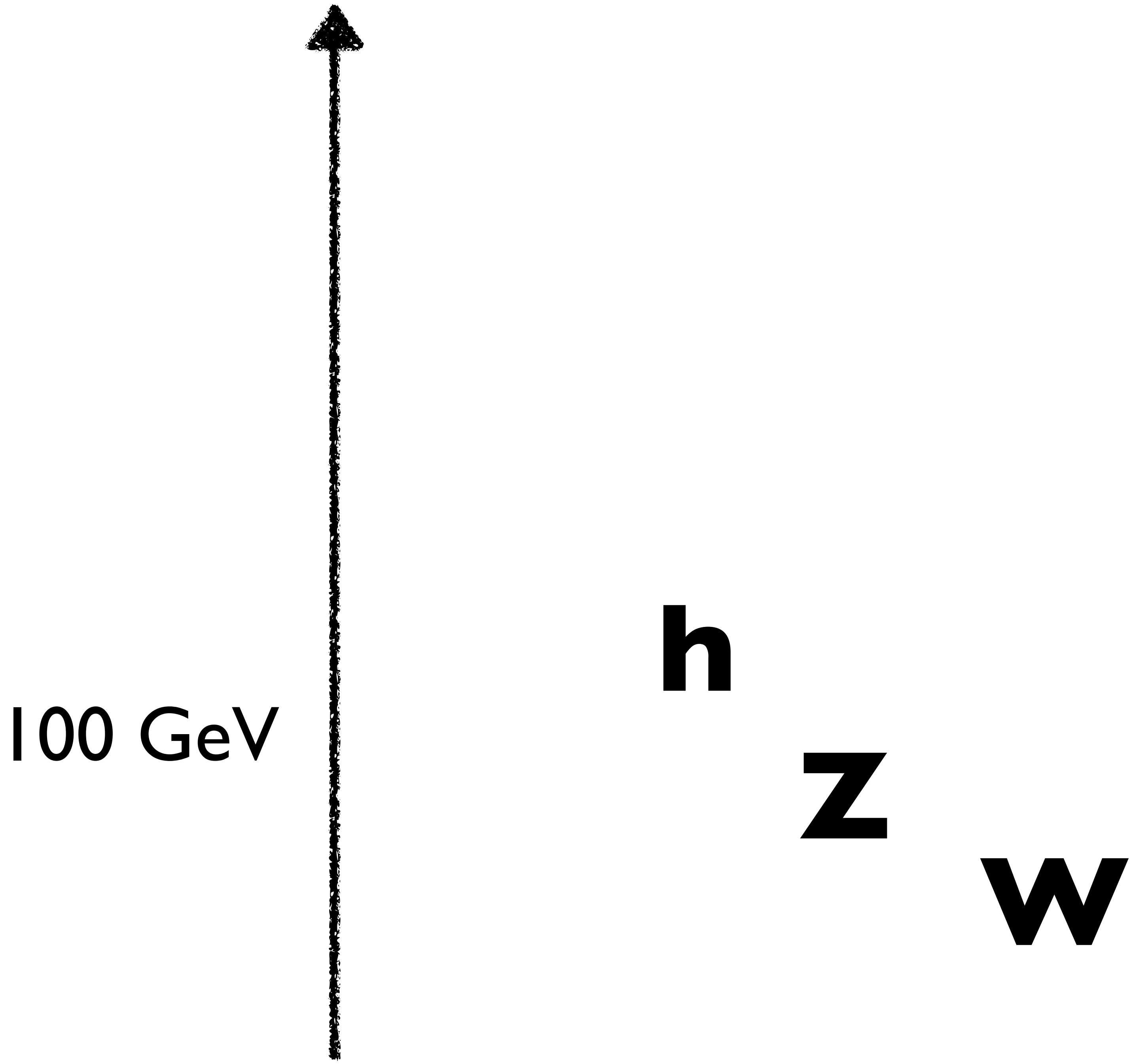
- It is straightforward to add h(125).

$$\mathcal{L}_{\rm HEFT} = \frac{1}{2} G_{ab}(h) \alpha_{\perp\mu}^{a} \alpha_{\perp}^{b\mu} + \frac{1}{2}(\partial_\mu h)(\partial^\mu h) - V(h)$$

$$G_{ab} = \frac{1}{4}\begin{pmatrix} v^2\mathcal{F}(h) & & \\ & v^2\mathcal{F}(h) & \\ & & v_Z^2\mathcal{F}_Z(h) \end{pmatrix}$$

- $\mathcal{F}(h)$ and $\mathcal{F}_Z(h)$ are arbitrary functions.
  - hVV coupling $\left.\frac{\partial}{\partial h}\mathcal{F}_Z(h)\right|_{h\to 0}$

Higgs EFT (HEFT)



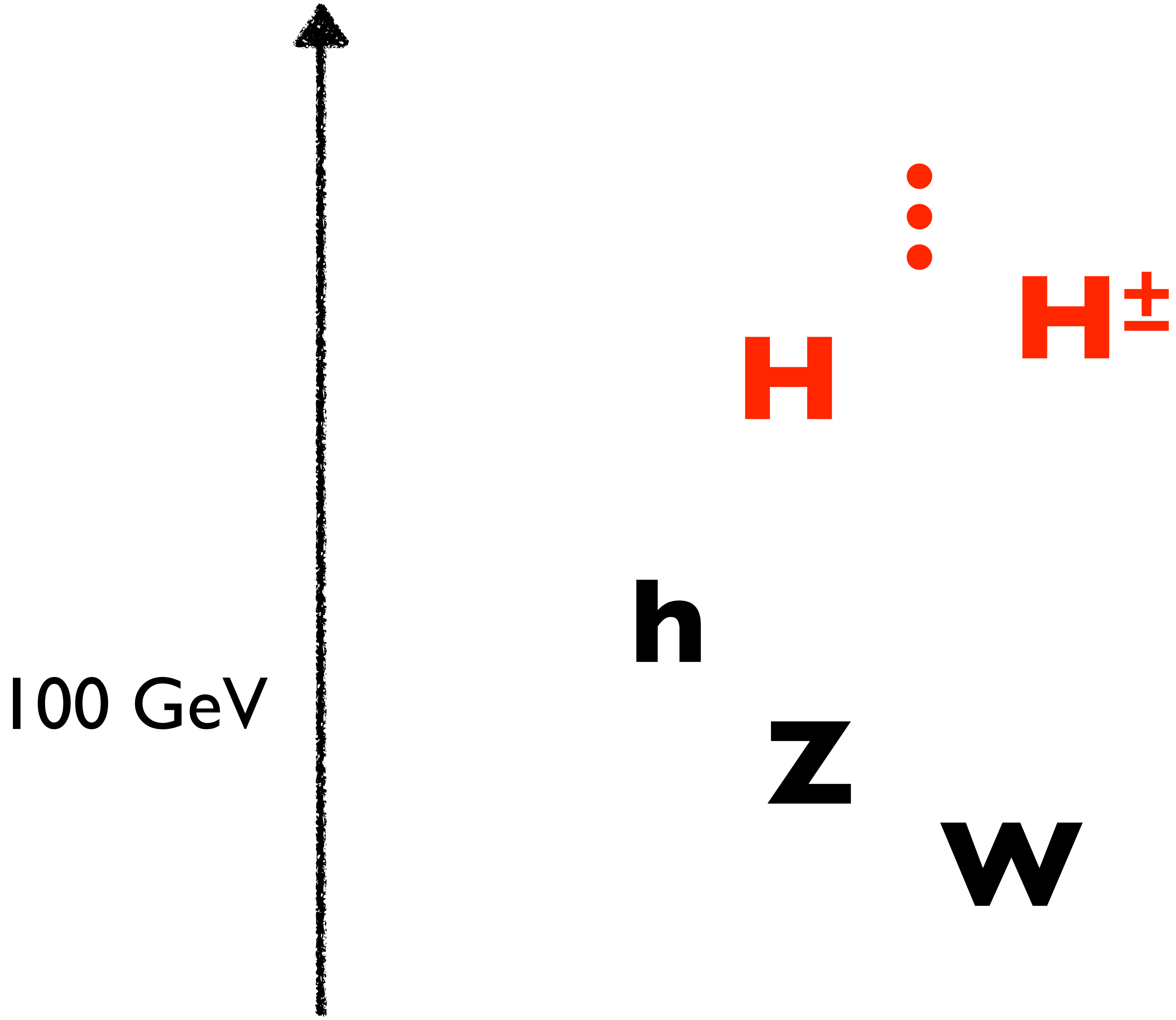
100 GeV

h

z

w

Beyond HEFT ?



⋮

$H^{\pm}$

H

100 GeV

h

Z

W

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Generalized HEFT

- Let us introduce

$$\phi = \Bigg(\underline{\overset{\text{Neutral}}{h, H, \cdots}}\,,\, \underline{\overset{\text{Charged}}{H^{+}, H^{-}, \cdots, H^{++}, H^{--}, \cdots}} \Bigg)$$

h(125) is here!

- Φ is a **linear** rep. of H = U(1)EM $\phi \xrightarrow{H} \rho_\phi(\mathfrak{h})\,\phi$
- G acts on it **non-linearly**: $\phi \xrightarrow{G} \rho_\phi(\mathfrak{h}(\pi, \mathfrak{g}_W, \mathfrak{g}_Y))\,\phi$

Generalized HEFT

- Note that

$$\partial_\mu \phi \xrightarrow{G} \rho_\phi(\mathfrak{h})(\partial_\mu \phi) + [\underline{\partial_\mu \rho_\phi(\mathfrak{h})}]\phi$$

- $(\partial_\mu \phi)^2$ is NOT G-invariant ….

Generalized HEFT

- Covariant derivative

$$\boxed{\mathcal{D}_\mu\phi = \partial_\mu\phi - i\rho_\phi(\alpha_{\|\mu})\,\phi}$$

$$\alpha_{\|\mu} \xrightarrow{G} \mathfrak{h}\,\alpha_{\|\mu}\,\mathfrak{h}^\dagger - i\mathfrak{h}(\partial_\mu\mathfrak{h}^\dagger)$$

- $\mathcal{D}_\mu\phi \xrightarrow{G} \rho_\phi(\mathfrak{h})(\mathcal{D}_\mu\phi)$

- G-inv. building blocks: $\boxed{\alpha^i_{\perp\mu}, \quad \alpha^3_{\perp\mu}, \quad \phi^I, \quad \mathcal{D}_\mu\phi^I}$

Generalized HEFT

- GHEFT up to O(p^2)

RN-Tanabashi-Tsumura-Uchida (2019)

NEW!

$$\mathcal{L}_{\rm GEFT} = \frac{1}{2} G_{ab}(\phi)\, \alpha_{\perp\mu}^{a} \alpha_{\perp}^{b\mu} + G_{aI}(\phi)\, \alpha_{\perp\mu}^{a} (\mathcal{D}^\mu \phi)^I + \frac{1}{2} G_{IJ}(\phi)\, (\mathcal{D}_\mu \phi)^I (\mathcal{D}^\mu \phi)^J - V(\phi)$$

- G's transform homogeneously as multiplets of corresponding representations.

- G's and V depend on the UV completion.

Generalized HEFT

- GHEFT up to O(p^2)

RN-Tanabashi-Tsumura-Uchida (2019)
Alonso-Jenkins-Manohar (2016)

$$\mathcal{L}_{\mathrm{GHEFT}} = \frac{1}{2} g_{ij}(\phi) \partial_\mu \phi^i \partial^\mu \phi^j - V(\phi)$$

- $\Phi = (\underbrace{\pi^1,\ \pi^2,\ \pi^3}_{\text{NGB}},\ \underbrace{h,\ H, \ldots,\ H^+,\ H^-,\ \ldots}_{\text{Higgs(es)}})$

- The scalar manifold and V depend on the UV completion.

Examples

The Standard Model

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} i(\phi^2 + i\phi^3) \\ \phi^1 - i\phi^4 \end{pmatrix}$$

$$\mathcal{L}_{\mathrm{SM}} = \partial_\mu \Phi^\dagger \partial^\mu \Phi \ = \frac{1}{2}(\partial_\mu \phi^i)(\partial^\mu \phi^i)$$

$$R_{ijkl} = 0$$

R⁴

Examples

MSSM (Two Higgs Doublet Model)

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} i(\phi^2 + i\phi^3) \\ \phi^1 - i\phi^4 \end{pmatrix}$$

$$\mathcal{L}_{\text{MSSM}} = \partial_\mu \Phi_1^\dagger \partial^\mu \Phi_1 + \partial_\mu \Phi_2^\dagger \partial^\mu \Phi_2$$

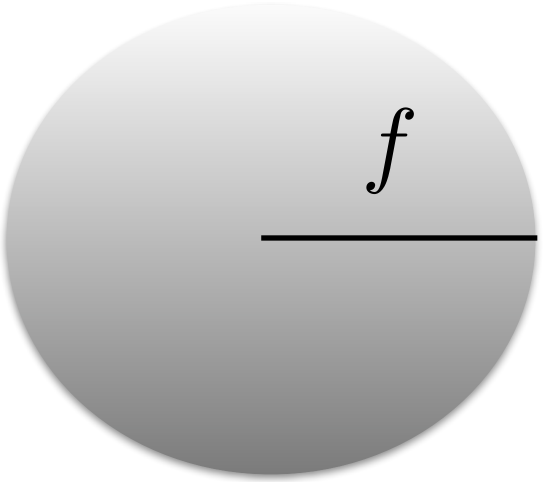
$$R_{ijkl} = 0 \qquad \mathbf{R^8}$$

Examples

Minimal Composite Higgs Model (SO(5)/SO(4))

Agache-Contino-Pomarol (2004)

$$\mathcal{L}_{SO(5)/SO(4)} = \frac{1}{2}\left(\frac{4f^2}{4f^2+\phi^j\phi^j}\right)^2 (\partial_\mu\phi^i)(\partial^\mu\phi^i)$$



$$R_{ijkl} = \left(\frac{4f^2}{4f^2+\phi^j\phi^j}\right)^2 \frac{1}{f^2}(\delta^{ik}\delta^{jl} - \delta^{il}\delta^{jk})$$

S^4

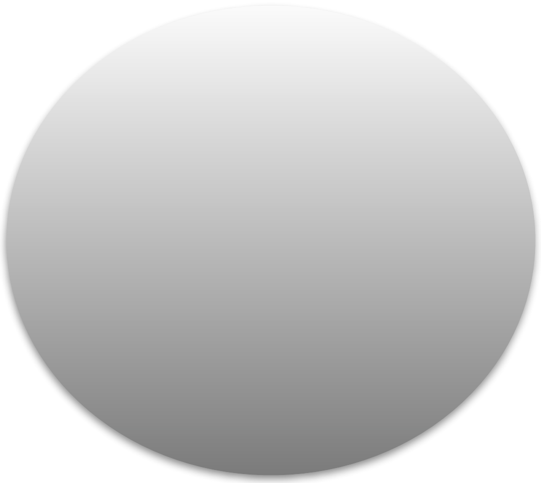
Examples

MSSM

Flat

MCSM

Curved



Scattering amplitude

- Let us look at Φ$_i$ Φ$_j$ → Φ$_k$ Φ$_l$ scattering.

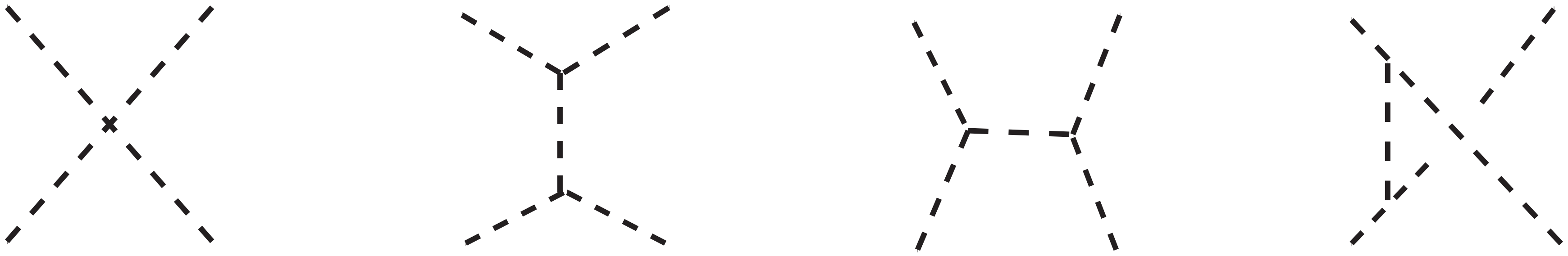
$$\mathcal{L} = \frac{1}{2} g_{ij}(\phi) \partial_\mu \phi^i \partial^\mu \phi^j - V(\phi)$$

- Expand around vacuum $\phi = \bar{\phi} + \varphi$

$$g_{ij}(\phi) = \bar{g}_{ij} + \bar{g}_{ij,k} \varphi^k + \frac{1}{2!} \bar{g}_{ij,kl} \varphi^k \varphi^l + \cdots$$

Scattering amplitude

- Let us look at $\Phi_i\ \Phi_j \rightarrow \Phi_k\ \Phi_l$ scattering.



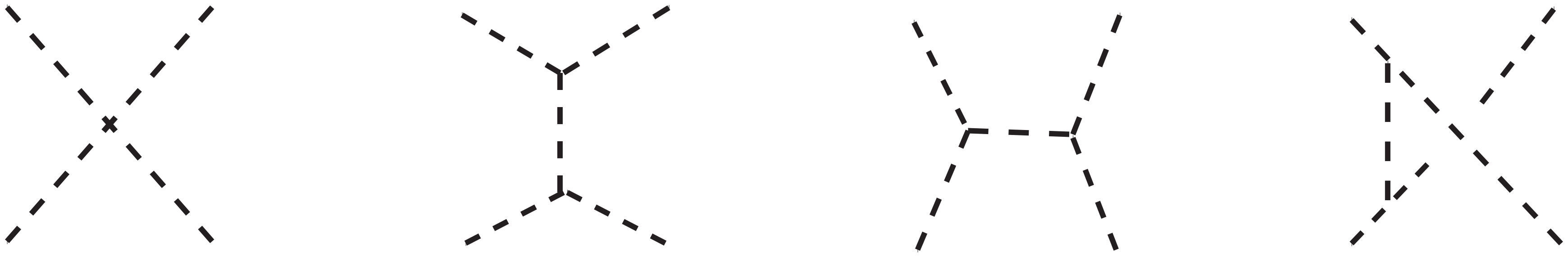
- (Scattering amp) ~ (CM energy)^2

$$\mathcal{M}_{\phi_i\phi_j\to\phi_k\phi_l} \sim \frac{s}{3}\left(\overline{R}_{iklj}+\overline{R}_{ilkj}\right)+\frac{t}{3}\left(\overline{R}_{ijlk}+\overline{R}_{iljk}\right)+\frac{u}{3}\left(\overline{R}_{ijkl}+\overline{R}_{ikjl}\right)+V_{;(ijkl)}$$

- Perturbative unitarity is violated at certain energy scale.

Scattering amplitude

- Let us look at $\Phi_i\ \Phi_j \rightarrow \Phi_k\ \Phi_l$ scattering.



- (Scattering amp) ~ (CM energy)^2

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Riemann curvature tensor

(evaluated at vacuum)

Scattering amplitude

$$\mathcal{M}_{\phi_i\phi_j\to\phi_k\phi_l} \sim \frac{s}{3}(\bar{R}_{iklj}+\bar{R}_{ilkj}) + \frac{t}{3}(\bar{R}_{ijlk}+\bar{R}_{iljk}) + \frac{u}{3}(\bar{R}_{ijkl}+\bar{R}_{ikjl})$$

- On-shell condition: $s+t+u = m_i^2+m_j^2+m_k^2+m_l^2$

$$= \frac{s}{3}(\bar{R}_{iklj}+\bar{R}_{ilkj}-\bar{R}_{ijlk}-\bar{R}_{iljk}) + \frac{u}{3}(\bar{R}_{ijkl}+\bar{R}_{ikjl}-\bar{R}_{ijlk}-\bar{R}_{iljk})$$

- Properties of R: $R_{ijkl} = -R_{ijlk}$

  $R_{ijkl}+R_{iklj}+R_{iljk} = 0$

$$= s(\bar{R}_{iklj}) + u(\bar{R}_{ijkl})$$

Scattering amplitude

- Let us look at Φ_i Φ_j → Φ_k Φ_l scattering.

- (Scattering amp) ~ (CM energy)^2

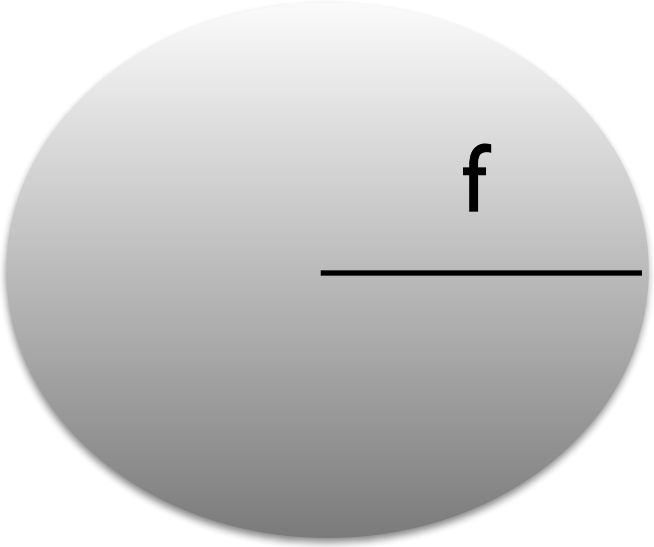
$$\mathcal{M}_{\phi_i\phi_j\to\phi_k\phi_l} \sim s(\bar{R}_{iklj}) + u(\bar{R}_{ijkl}) + \bar{V}_{;(ijkl)}$$

If M is curved, perturbative expansion fails.

Breaking PU

- Perturbative unitarity breaking scale ~ **(Curvature)$^{-1/2}$**

e.g. SO(5)/SO(4) ~ S^4 Composite Higgs Model



Non perturbative effect at E ~ 4πf

Scalar manifold and UV

Perturbative

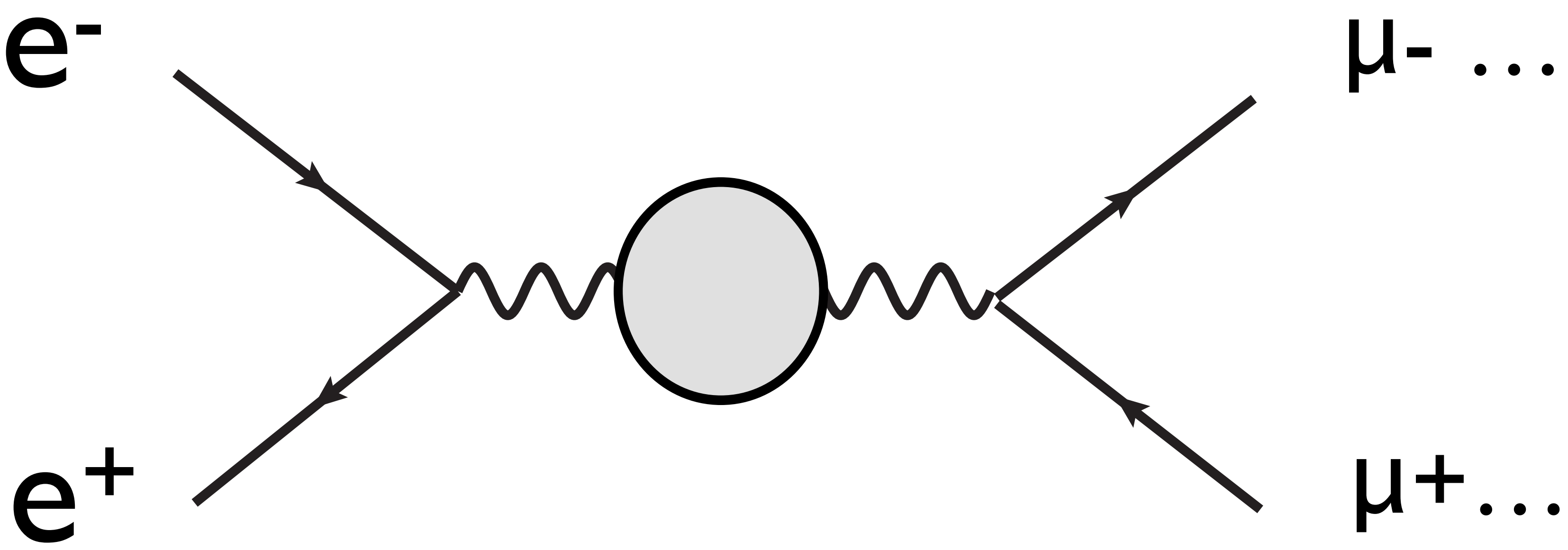
Flat

Non-Perturbative

Curved

- What does the experimental results tell us on the manifold?

Electroweak Precision Tests



- BSM particles can contribute radiatively.
- The BSM corrections should be tiny.
- **Constraint on the scalar manifold ?**

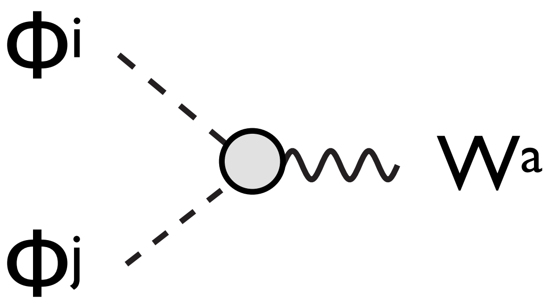
EW corrections

- EW corrections relate with the **symmetry** of the scalar manifold.

$$\mathcal{L} = \frac{1}{2} g_{ij}(\phi) D_\mu \phi^i D^\mu \phi^j - V(\phi)$$

$$D_\mu \phi^i = \partial_\mu \phi^i + W^a_\mu \underbrace{w^i_a(\phi)}_{\text{SU(2) Killing vector}} + B_\mu \underbrace{y^i(\phi)}_{\text{U(1) Killing vector}}$$

- WΦΦ interaction:



$$\propto (w^i_a)_{;j}$$

EW oblique corrections

- S-parameter at one-loop

$$S \sim (w_3^i)_{;j} (y^j)_{;i} \ln \frac{\Lambda^2}{m_h^2}$$

Alonso-Jenkins-Manohar (2016)

- Reminder: $S \sim \Pi'_{3Y} - \Pi'_{3Q} = \Pi'_{3Y}$

EW oblique corrections

- S-parameter at one-loop

$$S \sim (w_3^i)_{;j}(y^j)_{;i} \ln \frac{\Lambda^2}{m_h^2}$$

Alonso-Jenkins-Manohar (2016)

<u>SU(2)W×U(1)Y sym.</u>

$$[w_a, w_b] = -\epsilon_{abc} w_c$$

$$[w_a, y] = 0$$

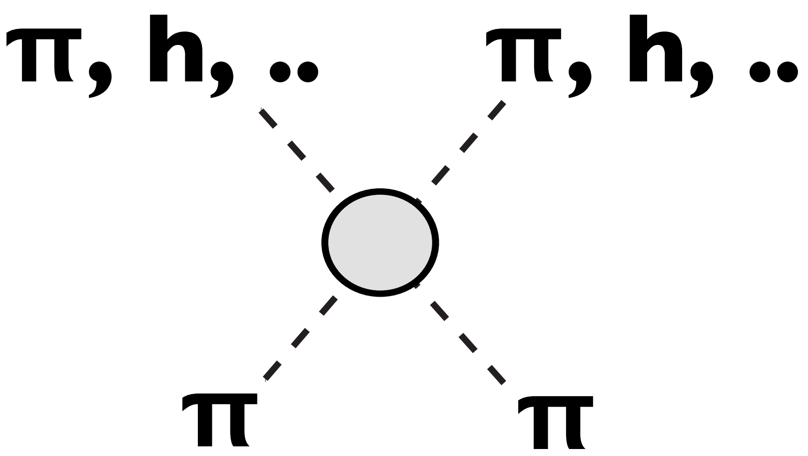
$$v_{i;j;k} = R^l{}_{kji} v_l \quad (v = w_a, y)$$

NEW!

$$S_{\log} \propto \epsilon_{3bc} \bar{R}^i_{jkl} \bar{w}^k_c \bar{w}^l_3 (\bar{w}^j_b)_{;i} + \epsilon_{3bc} \bar{R}^i_{jkl} \bar{w}^k_c \bar{w}^l_c (\bar{y}^j)_{;i}$$

EW oblique corrections

- S-parameter at one-loop



Non-zero only for NGB direction

$$\boxed{S_{\log} \propto \epsilon_{3bc}\bar{R}^i_{jkl}\underline{\bar{w}^k_c\bar{w}^l_3}(\bar{w}^j_b)_{;i} + \epsilon_{3bc}\bar{R}^i_{jkl}\bar{w}^k_c\bar{w}^l_c(\bar{y}^j)_{;i}}$$

EW oblique corrections

Perturbative **Non-Perturbative**

Flat **Curved**

S ~ 0 is possible in a curved theory ????

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Summary

- Gauge-scalar sector can be parametrized by
  - Geometry of the scalar manifold $g_{ij}(\phi)$
  - Higgs potential $V(\phi)$

Perturbative

Non-Perturbative

- Consistency with the EWPTs may not imply the complete flatness of the scalar manifold.