Computer simulation of the $\mathcal{N} = 2$ Landau–Ginzburg model

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- O.M., PTEP **2019** (2019) 103B03 [arXiv:1906.00653 [hep-lat]].
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Motivation

- Superstring theory:
  4D spacetime + 6D space (Calabi–Yau (CY) manifold)
- CY $\to$ a 2D $\mathcal{N}=2$ Superconformal field theory (SCFT) on the 2D string world sheet
- In general, not a solvable minimal model of SCFT (or Gepner model = $\otimes$minimal models)
- SCFT corresponding to general CY??

- 2D SUSY Landau–Ginzburg (LG) model $\xrightarrow{\text{IR limit}}$ SCFT
- LG/CY correspondence [Green-Vafa-Warner '89, Witten '93]

$$\text{LG with potential } \underline{W(\Phi)} \overset{SCFT?}{\longleftrightarrow} \text{CY with def. } \underline{G(z)} = 0$$

▶ Numerical simulation of LG model:
new approach to investigate superstring theory via LG/CY

LG model: 2D $\mathcal{N}=2$ Wess–Zumino model

- QFT on RG fixed point $\to$ scale inv. $\approx$ conformal inv.
- LG description provides CFT in terms of Lagrangian
- An example of the LG description
  $\Rightarrow$ 2D $\mathcal{N}=2$ Wess–Zumino (WZ) model
  - Dimensional reduction of 4D $\mathcal{N}=1$ WZ model,

$$\int d^4x d^4\theta\, \bar{\Phi}_I \Phi_I + \int d^4x d^2\theta\, W(\Phi_I) + \int d^4x d^2\bar{\theta}\, W(\bar{\Phi}_I)$$

- The action is given by

$$S = \int d^2x \sum_I \Bigg[4\partial_z A_I^* \partial_{\bar{z}} A_I + \frac{\partial W(A)^*}{\partial A_I^*} \frac{\partial W(A)}{\partial A_I} + (\bar{\psi}_{\dot{1}}, \psi_2)_I \sum_J \begin{pmatrix} 2\delta_{IJ}\partial_z & \frac{\partial^2 W(A)^*}{\partial A_I^* \partial A_J^*} \\ \frac{\partial^2 W(A)}{\partial A_I \partial A_J} & 2\delta_{IJ}\partial_{\bar{z}} \end{pmatrix} \begin{pmatrix} \psi_1 \\ \bar{\psi}_{\dot{2}} \end{pmatrix}_J \Bigg]$$

where $z, \bar{z} = x_0 \pm i x_1$, and $\partial_z, \partial_{\bar{z}} = \frac{1}{2}(\partial_0 \mp i\partial_1)$

WZ/SCFT correspondence

- WZ with quasi-homogeneous superpotential $W(\Phi_I)$ such that $W(\Lambda^{\omega_I}\Phi_I) = \Lambda W(\Phi_I) \xrightarrow{\text{IR limit}} \mathcal{N} = 2$ SCFT
- $\mathcal{N} = 2$ minimal model (ADE classification [Vafa–Warner '89])

Algebra	Superpotential W	Central charge c
A_n	x^{n+1}, $n \geqq 1$	$3 - 6/(n+1)$
D_n	$x^{n-1} + xy^2$, $n \geqq 3$	$3 - 6/2(n-1)$
E_6	$x^3 + y^4$	$3 - 6/12$
E_7	$x^3 + xy^3$	$3 - 6/18$
E_8	$x^3 + y^5$	$3 - 6/30$

  - Finite number of primary fields: $1, x, \dots, x^{n-1}$ (A_n)

$$\Phi(z,\bar{z}) \xrightarrow{\text{conformal transf.}} (\partial_z z')^{-h} (\partial_{\bar{z}} \bar{z}')^{-\bar{h}} \Phi(z,\bar{z})$$

    Operator algebra $\Phi_{l,l}\Phi_{n,n} \sim \Phi_{l+n,l+n}$ $(x^l = \Phi_{l,l})$
  - Conformal weight $(h, \bar{h})_x = (\omega/2, \omega/2) = (1/2n, 1/2n)$
  - Central charge $\langle T_{zz}(z) T_{zz}(0) \rangle = c/2z^4$ with EMT $T_{\mu\nu}$

Lattice regularization vs SUSY

- Various evidences for LG/minimal-model [Vafa–Warner '88, Howe–West '89, Witten '93, ...]
- Strong coupling at IR and severe IR divergences
  $\rightarrow$ No complete proof of the correspondence
- An alternative approach:
  Non-perturbative numerical study such as lattice field theory

- Lattice regularization breaks SUSY...

  translational invariance: $\{Q, \bar{Q}\} \sim \cancel{\partial}$,

  fermion doubling, Leibniz rule, locality
  - Lattice SUSY [Cohen–Kaplan, Sugino, D'Adda et al., Catterall, ...]
  - Numerical study (e.g., $\mathcal{N} = 1$ SYM [Ali–Gerber–Montvay–Münster–Piemonte–Scior–Bergner '18])
  - Gradient flow representation of supercurrent [Hieda–Kasai–Makino–Suzuki '17, Kasai–O.M.–Suzuki '18]

Preceding numerical studies

- Numerical studies of A_2-type model ($W(\Phi) = \Phi^3$) based on Nicolai map (path integral to Gaussian form)
  - Scaling dimension $1 - h - \bar{h} = 2/3 = 0.666\ldots$
  - Central charge $c = 1$

1. $1 - h - \bar{h} = 0.660(11)$ [Kawai-Kikukawa '09]
   based on a lattice formulation [Kikukawa-Nakayama '02]
   - One nilpotent SUSY and discrete R symmetry preserved
   - Automatic restoration of full SUSY for *massive* WZ [Kadoh–Suzuki '10]
2. $1 - h - \bar{h} = 0.616(25)(13)$, $c = 1.09(14)(31)$ [Kamata-Suzuki '10]
   based on a numerical algorithm [Kadoh-Suzuki '09]
   - Manifest full SUSY, translational and R symmetries
   - Straightforward construction of supercurrent, EMT and $U(1)_R$ current

- Non-perturbative evidence of the A_2-type correspondence

Today's talk...

- Let us apply the SUSY-preserving formulation [Kadoh–Suzuki] to various ADE-type models

1. Review of the formulation
2. Simulation setup and SUSY Ward–Takahashi identity
3. Direct measurement of $h + \bar{h}$ and c [O.M.–Suzuki, O.M. '18]
   - Non-perturbative evidences of the conjectured correspondence for A_2, A_3, D_3, D_4, E_6 ($\cong A_2 \otimes A_3$), E_7 models
4. Precision measurement of $h + \bar{h}$ through the continuum limit and finite-size scaling [O.M. '19]

- These numerical studies support the validity of the formulation $\rightarrow$ possible application to more general SCFTs
- This numerical approach will be useful to investigate superstring theory.

Supersymmetric formulation [Kadoh–Suzuki 2009]

- Continuum physical box $L \times L$
- We work in the momentum space;

$$\varphi(x) = \frac{1}{L^2} \sum_p e^{ipx} \varphi(p), \qquad p_\mu \in \frac{2\pi}{L}\mathbb{Z}.$$

- Let us introduce a UV cutoff Λ as

$$-\Lambda \leq p_\mu \leq \Lambda$$

  - $\Lambda = \pi/a$ ("lattice spacing" a; "continuum limit" $a \to 0$)
- Non-locality because of the cutoff
  - $\sim$ (4D) lattice formulation [Bartels-Bronzan '83] based on SLAC derivative [Drell-Weinstein-Yankielowicz '76, Rabin '81]
  - Restoration in continuum limit?
    - ⋆ 4D: No [Karsten–Smit '79, Kato–Sakamoto–So '08]
    - ⋆ 2D, 3D: Yes for massive WZ [Kadoh-Suzuki '09]

Supersymmetric formulation [Kadoh–Suzuki 2009]

- SUSY (similarly for $\bar{Q}_{\dot{\alpha}}$),

$$
\begin{aligned}
Q_1\bar{\psi}_{\dot{1}}(p) &= -2ip_{\bar{z}}A^*(p), & Q_1A^*(p) &= 0,\\
Q_1F^*(p) &= 2ip_{\bar{z}}\bar{\psi}_{\dot{2}}(p), & Q_1\bar{\psi}_{\dot{2}}(p) &= 0,\\
Q_1A(p) &= \psi_1(p), & Q_1\psi_1(p) &= 0,\\
Q_1\psi_2(p) &= F(p), & Q_1F(p) &= 0,
\end{aligned}
$$

$$
\begin{aligned}
Q_2\bar{\psi}_{\dot{2}}(p) &= -2ip_zA^*(p), & Q_2A^*(p) &= 0,\\
Q_2F^*(p) &= -2ip_z\bar{\psi}_{\dot{1}}(p), & Q_2\bar{\psi}_{\dot{1}}(p) &= 0,\\
Q_2A(p) &= \psi_2(p), & Q_2\psi_2(p) &= 0,\\
Q_2\psi_1(p) &= -F(p), & Q_2F(p) &= 0,
\end{aligned}
$$

translational and R symmetries are conserved

$\rightarrow$ Straightforward construction of supercurrent, EMT, ...

(Symmetries under linear transformations are also conserved)

WZ model and Nicolai map [Nicolai 1980]

- Let's rewrite WZ action ($N_\Phi = 1$) as

$$S = S_B + \frac{1}{L^2}\sum_p \left[(\bar{\psi}_{\dot{1}}, \psi_2)\,(-p) \begin{pmatrix} \frac{\partial N(p)}{\partial A(p)} & \frac{\partial N(p)}{\partial A^*(p)} \\ \frac{\partial N^*(p)}{\partial A(p)} & \frac{\partial N^*(p)}{\partial A^*(p)} \end{pmatrix} \begin{pmatrix} \psi_1 \\ \bar{\psi}_{\dot{2}} \end{pmatrix} (p) \right],$$

where $p_z = \frac{1}{2}(p_0 - ip_1)$, $p_{\bar{z}} = \frac{1}{2}(p_0 + ip_1)$, $*$: convolution,

$$S_B = \frac{1}{L^2}\sum_p N(p)^* N(p), \quad N(p) \equiv 2ip_z A(p) + W'(A)^*(p). \quad (\star)$$

- Integrating over ψ, Nicolai mapping $A(p) \to N(p)$,

$$\mathcal{Z} = \int \prod_{|p_\mu| \le \Lambda} [dN(p)dN^*(p)]\, e^{-S_B} \sum_i \operatorname{sign}\det \left.\frac{\partial(N, N^*)}{\partial(A, A^*)}\right|_{A=A_i}.$$

  - Jacobian cancellation thanks to Nicolai map $\Rightarrow$ Gaussian weight
  - $A_i(p)$ ($i = 1, 2, \ldots$): solutions of ($\star$)

Algorithm

1. Generate Gaussian random numbers $(N(p), N^*(p))$
2. Solve the algebraic equation numerically
$$2ip_z A(p) + W'(A)^*(p) - N(p) = 0,$$
with respect to $A(p)$; find all solutions A_i $(i = 1, 2, \dots)$
3. Calculate the following sums:
$$\sum_i \text{sign det} \frac{\partial(N, N^*)}{\partial(A, A^*)} \mathcal{O}(A, A^*) \bigg|_{A=A_i}, \quad \sum_i \text{sign det} \frac{\partial(N, N^*)}{\partial(A, A^*)} \bigg|_{A=A_i}$$
4. Repeat steps (1)–(3), and average
$$\langle \mathcal{O} \rangle = \frac{1}{\Delta} \left\langle \sum_i \text{sign det} \frac{\partial(N, N^*)}{\partial(A, A^*)} \mathcal{O}(A, A^*) \bigg|_{A=A_i} \right\rangle$$
$$\Delta \equiv \left\langle \sum_i \text{sign det} \frac{\partial(N, N^*)}{\partial(A, A^*)} \bigg|_{A=A_i} \right\rangle$$

(Dis)Advantages of the algorithm

1. Advantages

- No auto-correlation among conf $N(p)$
- Correctly normalized partition function Δ

$$\Delta \equiv \left\langle \sum_i \text{sign det} \left. \frac{\partial(N, N^*)}{\partial(A, A^*)} \right|_{A=A_i} \right\rangle$$

This is equal to Witten index ($\Delta = n$ for A_n, D_n, E_n)

- No sign problem when $\Delta \neq 0$

2. Disadvantages

- It is not a priori clear how many solutions $A(p)_i$
  - Signs for $\{A(p)_i\}_{N(p)}$: $(+\cdots+-\cdots-) \to \Delta = \langle n_+ - n_- \rangle$
  - Newton–Raphson method: *many* initial trial configurations
- Supersymmetric boundary condition only (periodic)
  - Finite temperature (fermion: anti-periodic)

Simulation of *ADE*-type LG models

- Consider following *ADE*-type models

Algebra	Superpotential
A_n $(n = 2,\ 3)$	$\frac{\lambda}{n+1} x^{n+1}$
D_n $(n = 3,\ 4)$	$\frac{\lambda}{n-1} x^{n-1} + \frac{\lambda}{2} x y^2$
E_7	$\frac{\lambda}{3} x^3 + \frac{\lambda}{3} x y^3$

  - $a\lambda = 0.3$ (same as [Kawai–Kikukawa, Kamata–Suzuki])
  - L/a as various even integers; $L \gtrsim \lambda^{-1} \approx 3a$
    - ⋆ A_2: 8–36; A_3: 8–30 ($E_6 \cong A_2 \otimes A_3$) [O.M.–Suzuki '18]
    - ⋆ D_3: 8, 16, 24, 32, 40, 44; D_4: 8, 16, 24, 32, 40, 42; E_7: 8, 16, 24 [O.M. '18]
  - 640 conf of $N(p)$
- Solve algebraic eq. by Newton–Raphson method for each $N(p)$
  - 100 convergent trial conf of $A(p)$ (allowing repetition of identical solutions)
  - [KK, KS] 100 trial conf including divergent ones

Classification of configurations

A_2 ($W = \Phi^3$)

L/a	30	32	34	36
$(++)_2$	626	633	628	627
$(+++-)_2$	14	7	12	13
Δ	2	2	2	2

A_3 ($W = \Phi^4$)

L/a	24	26	28	30
$(+++)_3$	625	616	614	615
$(++++-)_3$	15	23	20	22
$(+++++--)_3$	0	0	2	0
$(++++)_4$	0	1	3	2
$(+++++-)_4$	0	0	1	0
$(++)_2$	0	0	0	1
Δ	3	3.002(2)	3.006(3)	3.002(3)

SUSY Ward–Takahashi identity

- Numerical confirmation of exact SUSY invariance
- One-point SUSY WT identity [Catterall–Karamov '01]

$$\delta = \langle S_B \rangle / (L/a + 1)^2 - 1 = 0$$

Algebra	L/a	δ
A_2	36	0.0007(11)
A_3	30	−0.0010(15)
D_3	44	−0.00092(63)
D_4	42	0.00019(64)
E_7	24	0.0009(17)

- Two-point SUSY WT identity

$$\left\langle Q_1 (A(p) \bar{\psi}_{\dot{1}}(-p)) \right\rangle = 0$$
$$\Rightarrow 2ip_{\bar{z}} \langle A(p) A^*(-p) \rangle = - \left\langle \psi_1(p) \bar{\psi}_{\dot{1}}(-p) \right\rangle , \qquad \text{etc.}$$

SUSY Ward–Takahashi identity

$$\mathrm{Re}\left[2ip_{\bar{z}}\left\langle A(p)A^{*}(-p)\right\rangle\right] = -\,\mathrm{Re}\left[\left\langle \psi_1(p)\bar{\psi}_{\dot{1}}(-p)\right\rangle\right]$$

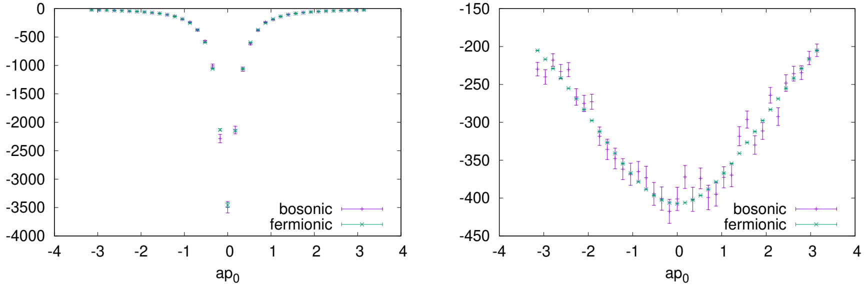


Figure: A_2, $L/a = 36$, $ap_1 = 2\pi/L = \pi/18$ (left); $ap_1 = \pi$ (right)

SUSY Ward–Takahashi identity

$$\mathrm{Im}\left[2ip_{\bar{z}}\left\langle A(p)A^*(-p)\right\rangle\right] = -\,\mathrm{Im}\left[\left\langle \psi_1(p)\bar{\psi}_{\dot{1}}(-p)\right\rangle\right]$$

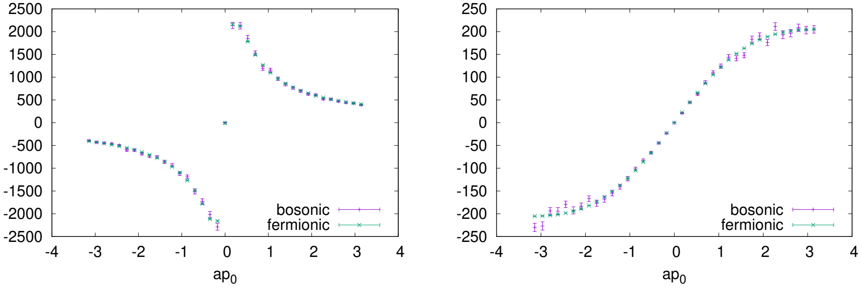


Figure: A_2, $L/a = 36$, $ap_1 = 2\pi/L = \pi/18$ (left); $ap_1 = \pi$ (right)

- Statistical error for A > statistical error for ψ
  - Too large error in, e.g., four-point function $\langle A^* A^* A A\rangle$
  - SUSY WT: bosonic $\to$ fermionic ones (less noisy)

IR behavior of correlation functions

- Simulation of scaling dimension and central charge
- Two-point function

$$\underbrace{\langle \varphi_1(p)\varphi_2(-p)\rangle}_{\text{Numerical simulation}} = L^2 \int d^2x\, e^{-ipx}\, \underbrace{\langle \varphi_1(x)\varphi_2(0)\rangle}_{\text{Relations in SCFT}}$$

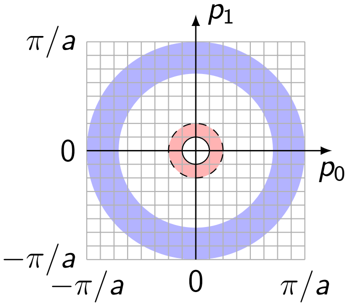
- E.g., scalar field $A(x)$ ($\to$ primary field with $(h, \bar{h})$)

$$\langle A(x)A^*(0)\rangle \propto \frac{1}{z^{2h}\bar{z}^{2\bar{h}}} \quad \Rightarrow \langle A(p)A^*(-p)\rangle \propto 1/(p^2)^{1-h-\bar{h}}$$

IR behavior of $\langle \varphi_1(p)\varphi_2(-p)\rangle$
$\Longrightarrow$ Scaling dimension $h+\bar{h}$
Central charge c ($\leftarrow$ EMT, ...)

- Fitting $\langle \varphi_1(p)\varphi_2(-p)\rangle$
  - IR: $2\pi/L \le |p| < 2\pi/L \times 2$

  (UV: free SCFT)



Scaling dimension from $\langle A(p)A^*(-p)\rangle \sim 1/(p^2)^{1-h-\bar{h}}$

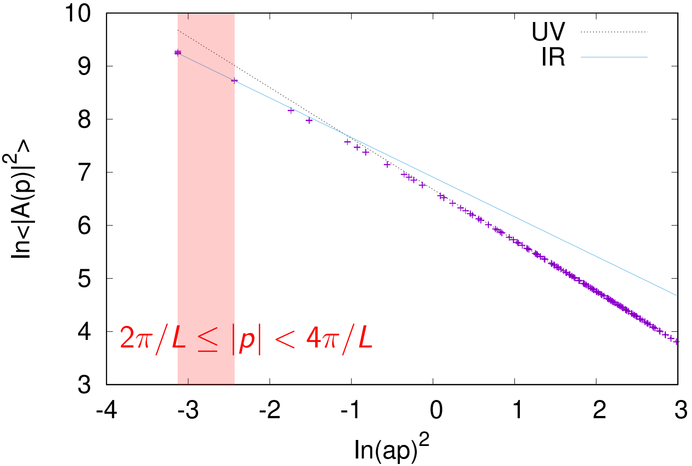
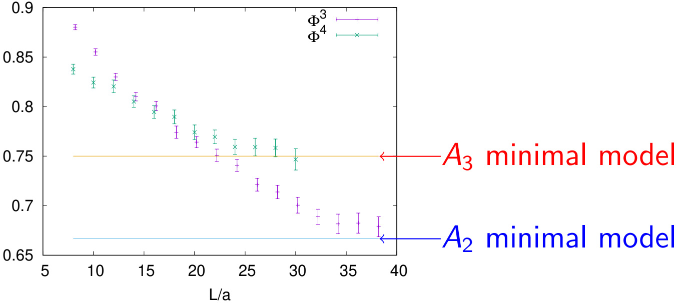


Figure: A_3 ($W = \Phi^4$), $L/a = 30$

Scaling dimension $h + \bar{h}$



- Scaling dimension (largest L) [O.M.–Suzuki '18]

	L/a	$1 - h - \bar{h}$	Expected value
A_2	36	0.682(10)(7)	0.666...
A_3	30	0.747(11)(12)	0.75

  - A_2: Kawai–Kikukawa 0.660(11), Kamata–Suzuki 0.616(25)(13)

- Continuum limit of $h + \bar{h}$ [O.M. '19] (later)

"Effective scaling dimension"

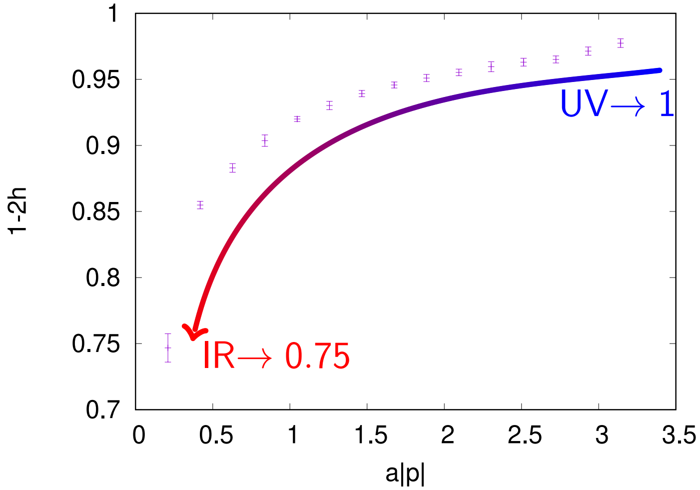


Figure: A_3 ($W = \Phi^4$), $L/a = 30$. Fitting in various momentum regions, $\frac{2\pi}{L} n \leq a|p| < \frac{2\pi}{L}(n+1)$

Central charge

- Central charge appears in

$$\left\langle S_z^+(z) S_z^-(0) \right\rangle = \frac{2c}{3z^3}, \quad \langle T_{zz}(z) T_{zz}(0) \rangle = \frac{c}{2z^4}, \quad \langle J_z(z) J_z(0) \rangle = \frac{c}{3z^2}.$$

- Construction of $(S^\pm,\ T_{zz},\ J_z)$ is straightforward
  - Free WZ ($W = 0$) $\to \mathcal{N} = 2$ super-Virasoro algebra
- Noting that

$$T_{zz} = \frac{1}{4} Q_2 S_z^+ - \frac{1}{4} \bar{Q}_{\dot{2}} S_z^-, \qquad S_z^+ = \bar{Q}_{\dot{2}} J_z, \qquad S_z^- = Q J_z,$$

we can obtain a less noisy form,

$$\langle T_{zz}(p) T_{zz}(-p) \rangle = -\frac{2ip_z}{16} \left\langle S_z^+(p) S_z^-(-p) + S_z^-(p) S_z^+(-p) \right\rangle$$

$$\to L^2 \frac{\pi c}{12} \frac{p_z^3}{p_{\bar{z}}}$$

  - (Anti-)Symmetry under $p \to -p$; better χ^2-fit than $\langle SS \rangle$

Central charge from $\langle T_{zz}(p) T_{zz}(-p)\rangle = L^2\pi c p_z^3/12 p_{\bar{z}}$

(Real part)

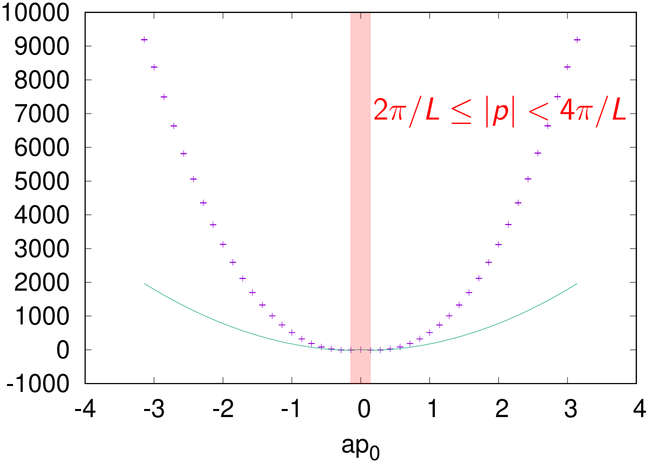


Figure: D_3 ($W = x^2 + xy^2$), $L/a = 44$, $ap_1 = \pi/22$

Central charge from $\langle T_{zz}(p) T_{zz}(-p) \rangle = L^2 \pi c p_z^3 / 12 p_{\bar{z}}$

(Imaginary part)

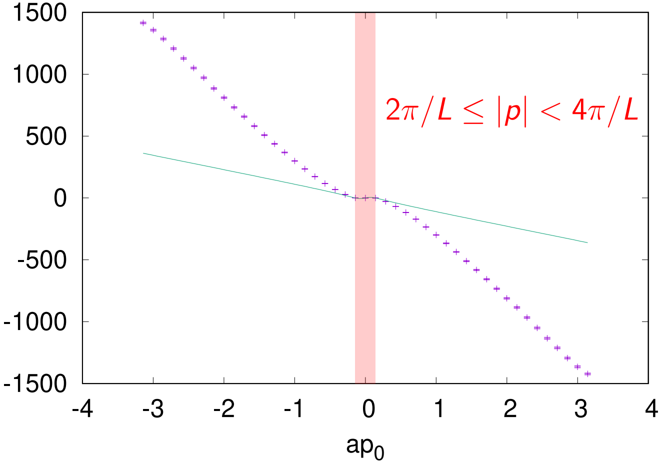


Figure: D_3 ($W = x^2 + xy^2$), $L/a = 44$, $ap_1 = \pi/22$

Backup: Central charge from $\langle S_z^+(p) S_z^-(-p) \rangle = L^2 i\pi c p_z^2 / 3p_{\bar{z}}$

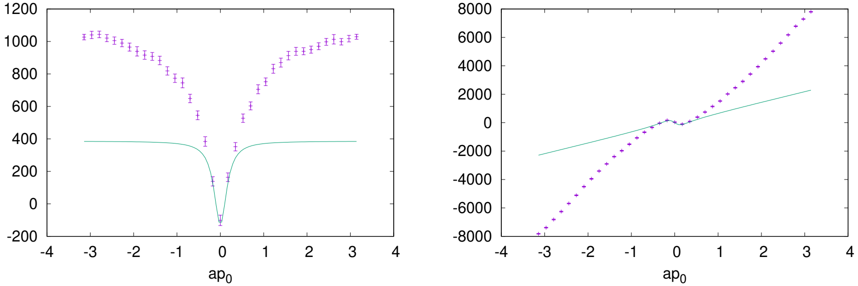
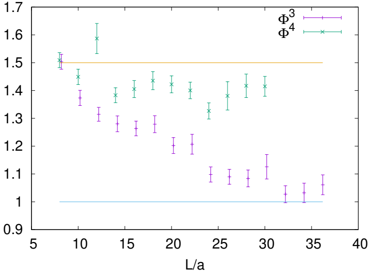
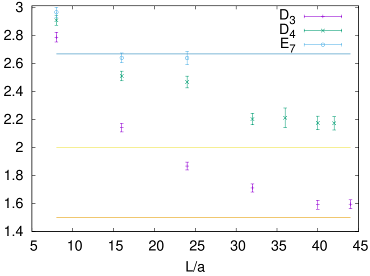


Figure: A_2, $L/a = 36$, $ap_1 = \pi/18$, real part (left); imaginary part (right)

- (Anti-)symmetry under $p \to -p$ within statistical error
- $TT \sim S^+S^- + S^-S^+$: better χ^2-fit and smaller statistical error

Central charge c





	L/a	Central charge	Expected value
A_2	36	1.061(36)(34)	1
A_3	30	1.415(36)(36)	1.5
D_3	44	1.595(31)(41)	1.5
D_4	42	2.172(48)(39)	2
E_7	24	2.638(47)(59)	2.66...

- c from $\langle SS \rangle$: Kamata-Suzuki 1.09(14)(31) for A_2

"Effective central charge"

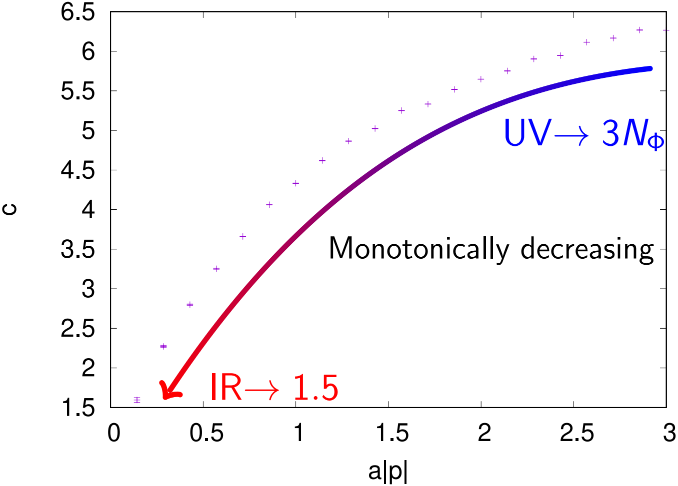


Figure: D_3, $L/a = 44$ (Analogous to Zamolodchikov's C-function ['86])

$$C = 2F - G - \frac{3}{8}H \quad \to \quad \frac{d}{d(\ln x^2)}C = -\frac{3}{4}H \leq 0 \;\&\; C \xrightarrow{\mathrm{IR}} c$$

where $F(x^2) = z^4\langle T_{zz}(x)T_{zz}(0)\rangle$, $G(x^2) = 4z^3\bar{z}\langle T_{zz}(x)T_{z\bar{z}}(0)\rangle$, $H(x^2) = z^2\bar{z}^2\langle T_{z\bar{z}}(x)T_{z\bar{z}}(0)\rangle$

Continuum limit and finite-size scaling

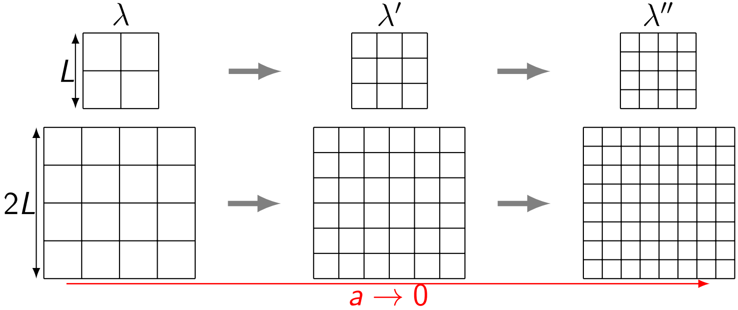
- We have obtained $h + \bar{h}$ and c at largest volume L^2 with fixed a
- In this analysis, L dependence is not quite clear
  $\overset{??}{\Rightarrow} L/a \to \infty$ (continuum limit, infinite volume limit)
- Susceptibility of the scalar field [Kawai–Kikukawa '09]

$$\chi(L) = \frac{1}{a^2} \int_{L^2} d^2x \, \langle A(x) A^*(0) \rangle \propto (L^2)^{1-h-\bar{h}}$$

- Measurement for various sizes of L
  $\to$ we can read $1 - h - \bar{h}$ from L dependence (finite-size scaling)
- Sensitivity to UV details for finite L/a in Kadoh–Suzuki formulation [O.M.-Suzuki]
  - $0.666\ldots$ vs $0.616(25)(13)$ by $\chi(L)$ with $L \leq 36$ [Kamata-Suzuki]
  - Precise and *reliable* result requires continuum limit
- Let us try to extrapolate $\chi(L)$ to $a \to 0$

Continuum limit extrapolation

- Similar to the continuum limit of the step scaling function [Lüscher–Weisz–Wolff '91]



- ${}^{\forall}a$, λ is tuned such that $\ln\chi(L) = u$ is fixed
  $\to \Sigma(u, a/L) = \ln\chi(2L)$ for (a, λ)
- From def. of the susceptibility, we have

$$1 - h - \bar{h} = \frac{1}{\ln 4}\left[\lim_{a\to 0}\Sigma(u, a/L) - u\right]$$

Tuning λ

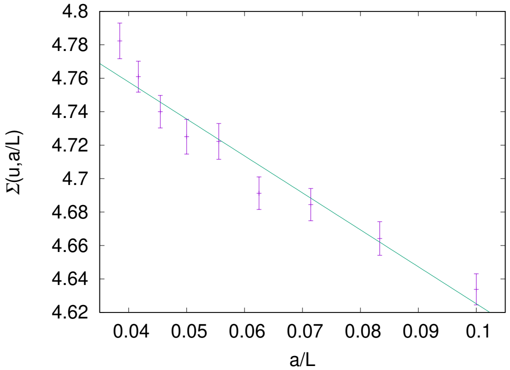
- Simulation of the simplest case: A_2 model ($W = \Phi^3$)

L/a	L'/a	$a\lambda$	#conf(L)	#conf(L')	$\ln\chi(L)$
10	20	0.1780	7680	5120	3.9174(59)
12	24	0.2135	5120	5120	3.9175(73)
14	28	0.2538	5120	5120	3.9193(70)
16	32	0.3000	5120	4320	3.9171(69)
18	36	0.3420	5120	2592	3.9166(68)
20	40	0.3888	5120	2592	3.9215(65)
22	44	0.4500	5120	2592	3.9162(62)
24	48	0.5100	5120	2592	3.9186(60)
26	52	0.5705	5120	1512	3.9175(56)

$\cdots$

- $\ln\chi(L) = u = 3.9175$; then, $\Sigma(u, a/L) = \ln\chi(L' = 2L)$?

(a/L)-dependence of $\Sigma(u, a/L)$



From a linear fit,

$$\lim_{a/L\to 0} \Sigma(3.9175, a/L) = 4.8461(107)$$

- Scaling dimension (A_2)

$1 - h - \bar{h}$	
0.6699(77)(87)	0.6666...

- Kawai-Kikukawa 0.660(11) ($\chi(L)$ with $L \leq 32$)
- Kamata-Suzuki 0.616(25)(13) ($\chi(L)$ with $L \leq 36$)
- cf. O.M.-Suzuki 0.682(10)(7)

Summary

- Numerical study of IR behavior of 2D $\mathcal{N}=2$ WZ model

  $\xrightarrow{\text{IR}}$ Non-perturbative evidences of WZ/SCFT correspondence
  - Scaling dimension & Central charge [O.M.–Suzuki, O.M. '18]

    $\Longrightarrow$ (First) Direct computation for typical ADE models
  - Finite-size effect & Continuum limit [O.M. '19]

    $\Longrightarrow$ Precise and reliable result of the scaling dimension
- Validity of the formulation (implication of locality restoration)

- E_8 ($\cong A_2 \otimes A_4$), A_4: $W = \Phi^5$
- Continuum limit for c, numerical measurement of C-function
- *Application to superstring theory*
  - Not ADE-type (Gepner) models; deformation of target space

$$W(\Phi) = \sum_{I=1}^{5} \Phi_I^5 \leftrightarrow G(z) = \sum_{i=1}^{5} z_i^5 = 0$$

  - Dynamics of superstring compactification?

Backup: Central charge from supercurrent

- The supercurrent is given by

$$
\begin{aligned}
S_z^+(x) &= 4\pi\bar{\psi}_{\dot{2}}(x)\partial_z A(x), \\
S_z^-(x) &= -4\pi\psi_2(x)\partial_z A^*(x),
\end{aligned}
$$

with a requirement (traceless)

$$
S_{\bar{z}}^+(x), S_{\bar{z}}^-(x) \to 0 \quad \text{in the UV limit (free SCFT).}
$$

- Two-point function of supercurrent $S^\pm$ in SCFT

$$
\left\langle S_z^+(z) S_z^-(0) \right\rangle = \frac{2c}{3z^3} \quad \Rightarrow \left\langle S_z^+(p) S_z^-(-p) \right\rangle = L^2 \frac{i\pi c}{3} \frac{p_z^2}{p_{\bar{z}}}
$$

Backup: Central charge from EMT

- Energy-momentum tensor is given by

$$\begin{aligned} T_{zz}(x) &= -4\pi\partial A^*(x)\partial A(x) \\ &\quad - \pi\psi_2(x)\bar{\psi}_{\dot{2}}(x) + \pi\partial\psi_2(x)\bar{\psi}_{\dot{2}}(x), \\ T_{\bar{z}\bar{z}}(x) &= -4\pi\bar{\partial}A^*(x)\bar{\partial}A(x) \\ &\quad - \pi\bar{\psi}_{\dot{1}}(x)\bar{\partial}\psi_1(x) + \pi\bar{\partial}\bar{\psi}_{\dot{1}}\psi_1(x), \end{aligned}$$

  with a requirement (traceless)

$$T_{z\bar{z}}(x) = T_{\bar{z}z}(x) \to 0 \quad \text{in the UV limit (free SCFT).}$$

- Two-point function of EMT in SCFT

$$\langle T_{zz}(x)T_{zz}(0)\rangle = \frac{c}{2z^4} \quad \Rightarrow \langle T_{zz}(p)T_{zz}(-p)\rangle = L^2\frac{\pi c}{12}\frac{p_z^3}{p_{\bar{z}}}.$$

Backup: Central charge from $U(1)$ current

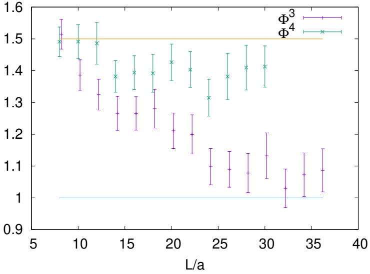
- $U(1)$ current is given by

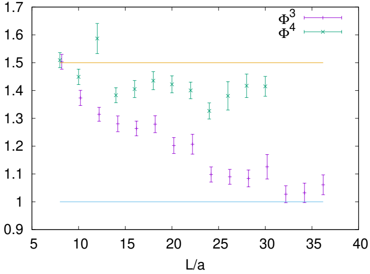
$$J_z(x) = 2\pi\bar{\psi}_{\dot{2}}\psi_2,$$
$$J_{\bar{z}}(x) = 2\pi\psi_1\bar{\psi}_{\dot{1}}.$$

- Two-point function of EMT in SCFT

$$\langle J_z(x)J_z(0)\rangle = \frac{c}{3z^2} \qquad \Rightarrow \langle J_z(p)J_z(-p)\rangle = -L^2\frac{\pi c}{3}\frac{p_z}{p_{\bar{z}}}.$$

Backup: Central charge c ($\langle SS \rangle$ vs $\langle TT \rangle$)





		L/a	Central charge
$\langle SS \rangle$ in [Kamata-Suzuki]	A_2	36	1.09(14)(31)
$\langle SS \rangle$ in [O.M.-Suzuki]	A_2	36	1.087(68)
	A_3	30	1.413(65)
$\langle TT \rangle$ in [O.M.-Suzuki]	A_2	36	1.061(36)(34)
	A_3	30	1.415(36)(36)