Superconformal subcritical hybrid inflation and leptogenesis

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Kanazawa University

Based on

- PLB782 (2018) 367-371
- JHEP 1909 (2019) 065 with Y. Gunji

Osaka, June 30, 2020

1. Introduction

A rough current status of particle physics

- Standard Model (SM) is a good theory (at least) up to the electroweak (EW) scale
- No significant signal beyond the SM so far (except for neutrino masses)

On the other hand

- SM has several of inconsistencies in cosmology or astrophysics

Possible ways to go:

- Find out the BSM in precision measurements or further analysis

- Focus on the cosmological issues

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- Focus on the cosmological issues

The cosmological issues

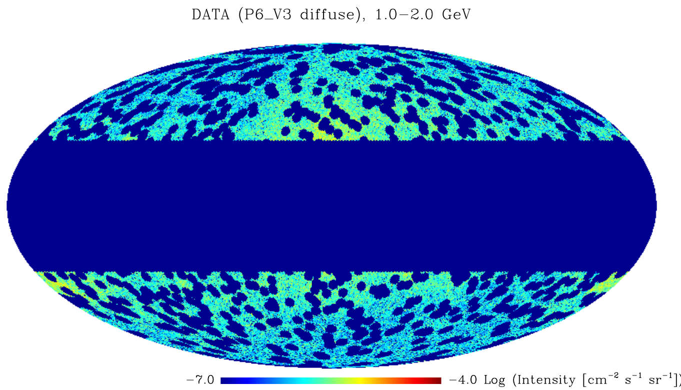
- Isotropic, homogeneous, flat universe
- Baryon asymmetry
- Dark matter
- Dark energy
- Missing satellite, cusp, Too-big-to-fail, etc.

The cosmological issues

- Isotropic, homogeneous, flat universe
- Baryon asymmetry
- Dark matter
  - Indirect detection
  - Direct detection
- Dark energy
- Missing satellite, cusp, Too-big-to-fail, etc.

Tomographic cross-correlation using local galaxy distribution

Ando, KI ’16



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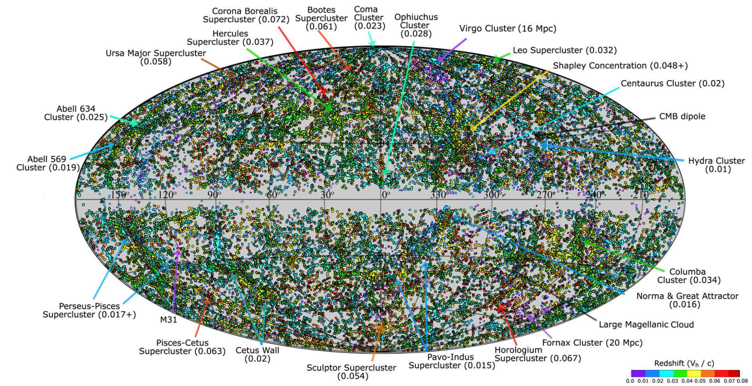
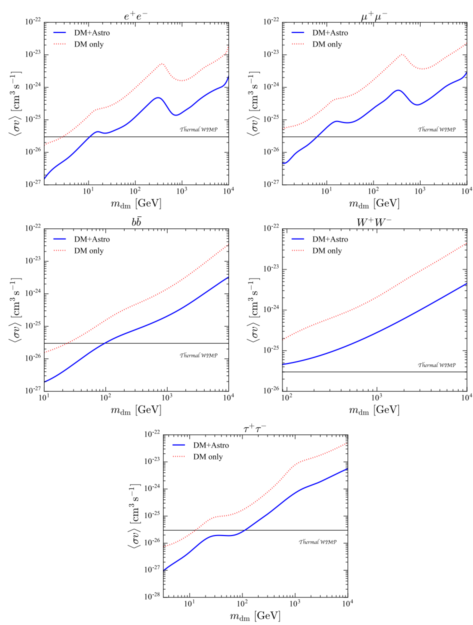
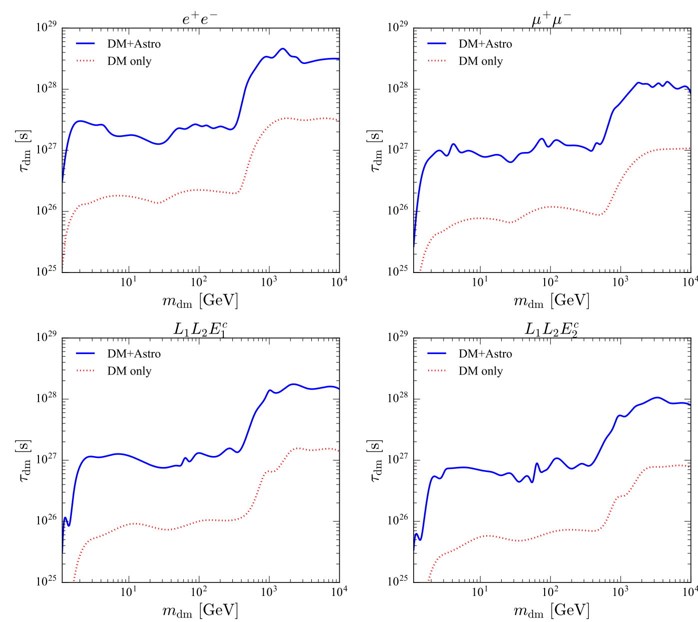
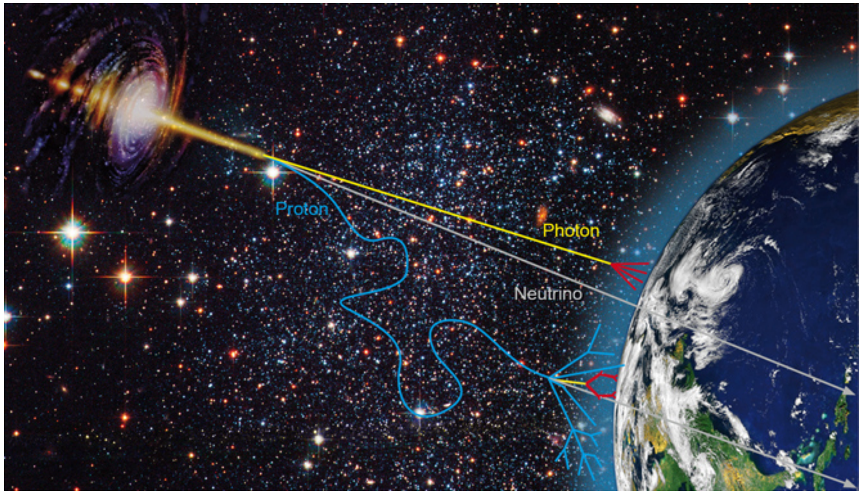


→ Constraints on DM annihilation cross-section or lifetime



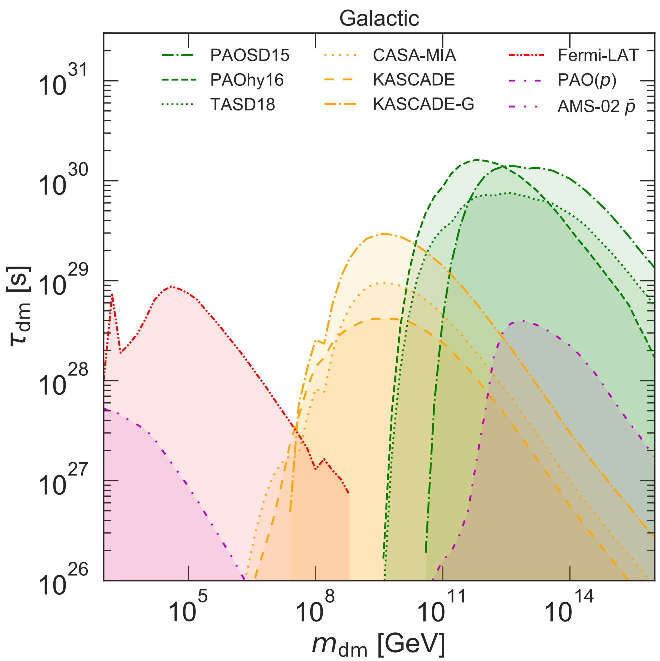


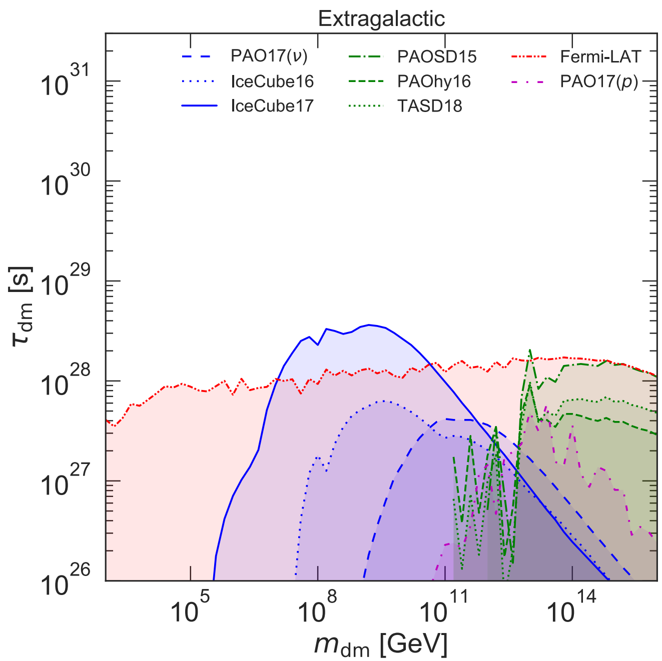
With multi-messenger astrophysical data



astro.desy.de

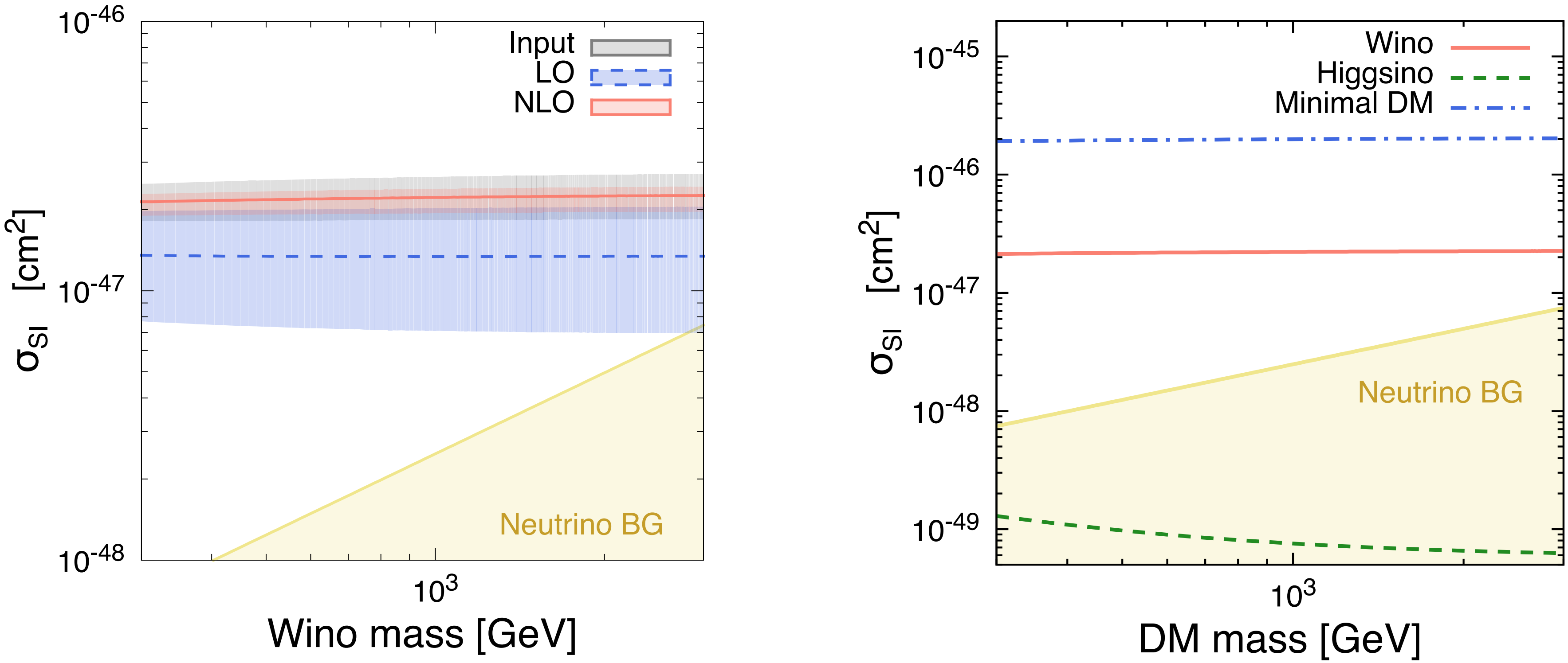
Ando, Arimoto, KI, Macias '19





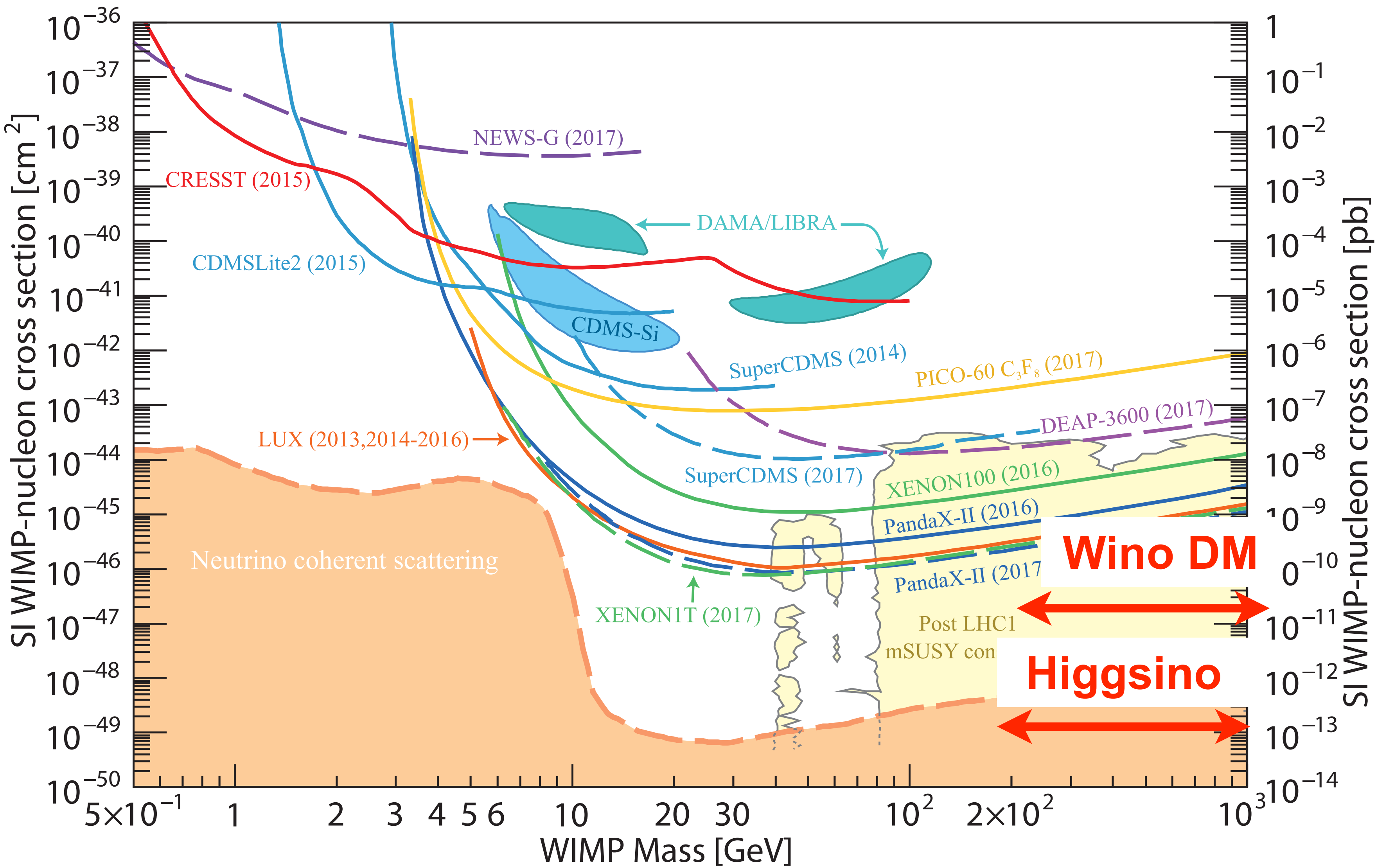
Direct detection of E-WIMP DM

Hisano, KI, Nagata '15



Neutrino BG from

Billard, Figueroa-Feliciano, Strigari '14 from PDG



Wino DM can be detected

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Outline

1. Introduction
2. Superconformal subcritical hybrid inflation
3. Leptogenesis after the inflation
4. Conclusions

2. Superconformal subcritical hybrid inflation

The cosmological issues

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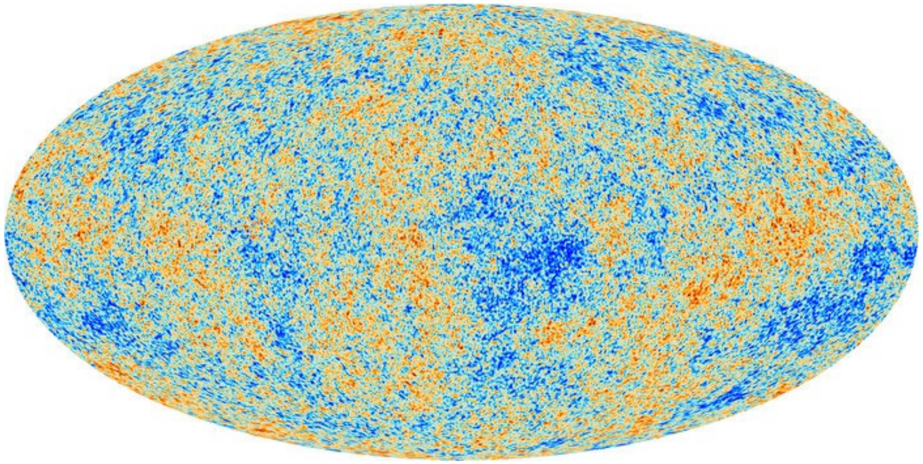
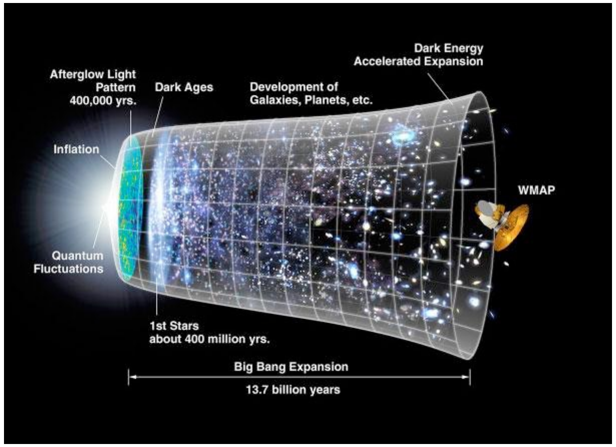
Cosmic Microwave Background (CMB)

- It tells the fundamental cosmological parameters
- It strongly supports inflation

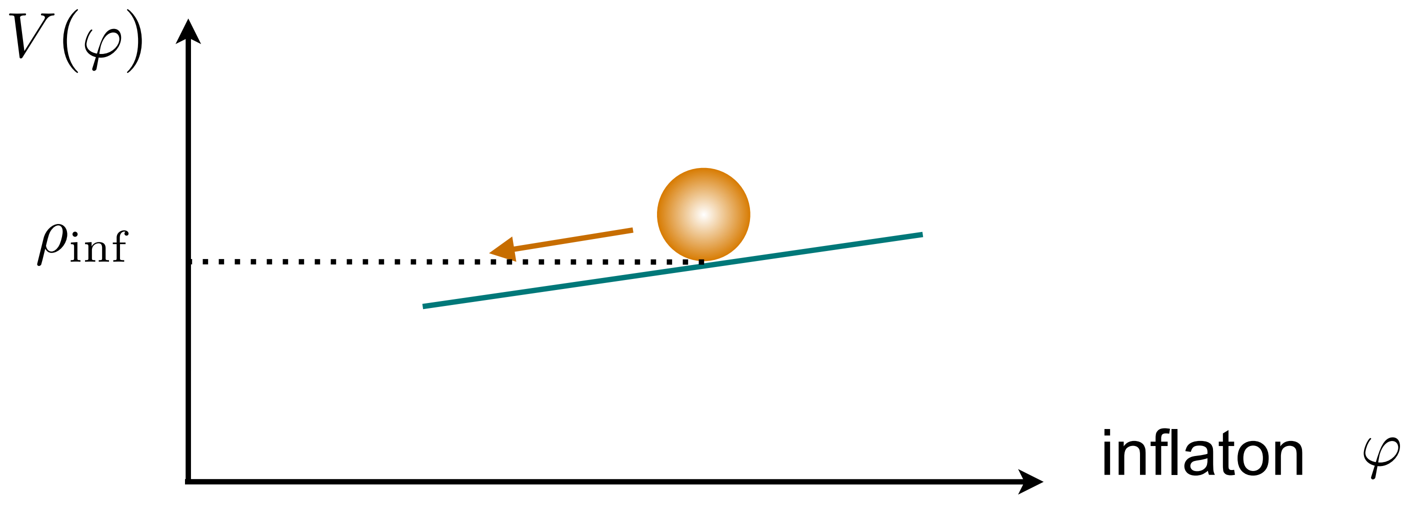




WMAP, Planck ’13

Inflation

- The paradigm of early universe, supported by the CMB observations
- It can be driven by a slow-roll scalar, inflaton
- It explains isotropic, homogenous and flat universe

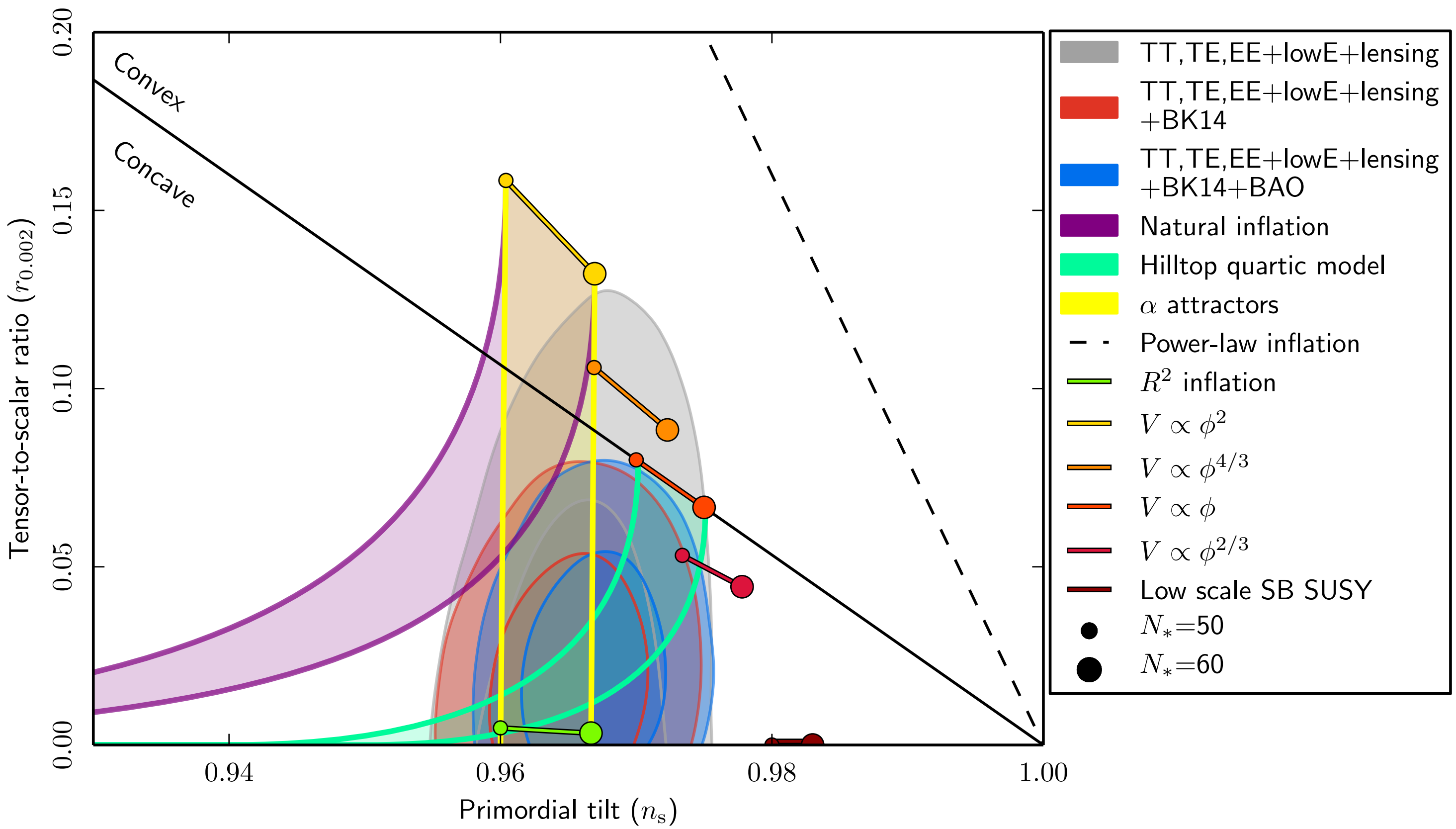


Inflation models

- Starobinsky model — Starobinsky '80
- Chaotic inflation — Linde '83
- Hybrid inflation — Linde '94
- Superconformal α attractor model — Kallosh, Linde '13; Kallosh, Linde, Roest '13
- etc.

The observations of the CMB constrain the inflation models

Planck '18



The CMB *constrains* the inflation models, which can be a hint for new physics

Supersymmetry is one of possible candidates for new physics

- Superstring theory requires SUSY
- It is compatible with the GUT
- There are flat directions $\longrightarrow$ Inflation

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Many hybrid inflation models have been proposed

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F/D-term hybrid inflation

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$$V_{\mathrm{SUSY}} = V_F + \boxed{V_D}$$

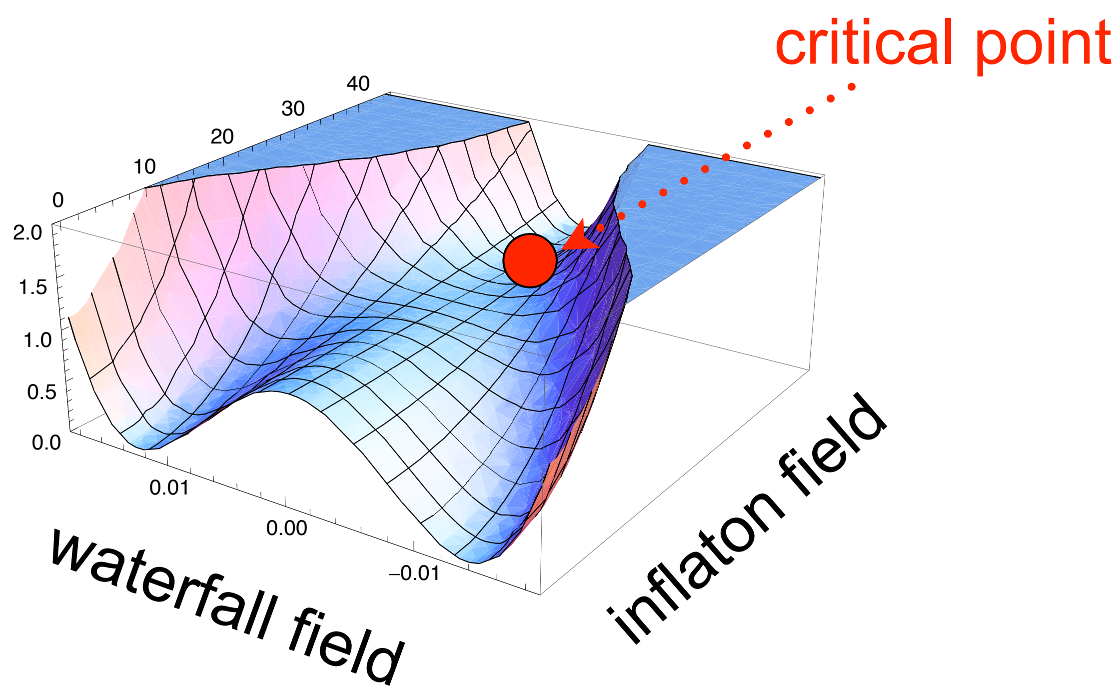
F/D-term hybrid inflation

D-term hybrid inflation (canonical case)

Binetruy, Dvali '96

Halyo '96

- Inflaton field slowly rolls down to the critical point
- At the critical point, "waterfall field" becomes tachyonic, and then inflation ends



Dynamics of inflaton-waterfall field system depends on the *Kähler potential*

- Superconformal → *Starobinsky model*

Buchmüller, Domcke, Schmitz '12
Buchmüller, Domcke, Kamada '13
Buchmüller, Domcke, Wieck '13

- Shift-symmetric → "*Chaotic regime*" below the critical point

Buchmüller, Domcke, Schmitz '14
Buchmüller, KI '15

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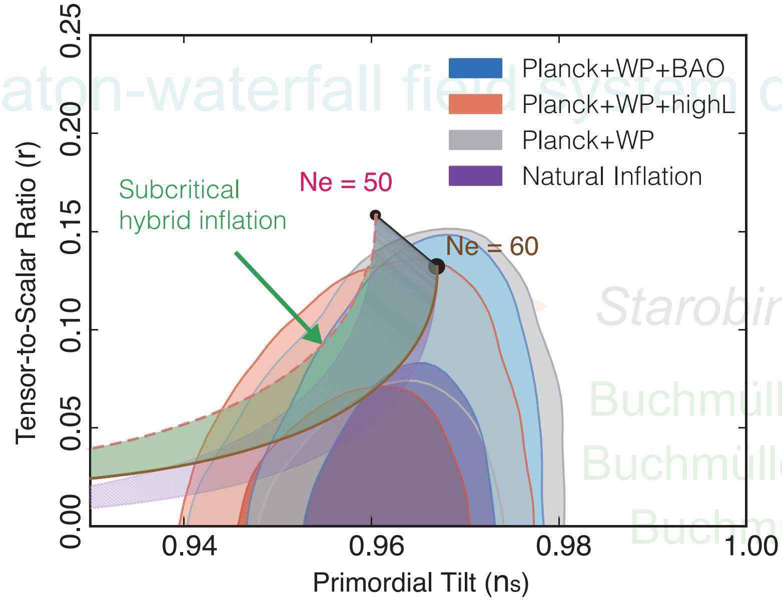

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"Subcritical hybrid inflation"

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- Superconformal + apprx. shift-symmetric → **?**

The model

Buchmüller, Domcke, Schmitz ’12
Buchmüller, Domcke, Kamada ’13

Superconformal D-term hybrid inflation model

	Φ	S_+	S_-
U(1)	0	q	$-q$

$q > 0$

- Superpotential: $W = \lambda \Phi S_+ S_-$

- Kähler potential: $\mathcal{K} = -3 \log \Omega^{-2}$

with $$\Omega^{-2} = 1 - \frac{1}{3}\left(|S_+|^2 + |S_-|^2 + |\Phi|^2\right) - \frac{\chi}{6}\left(\Phi^2 + \bar{\Phi}^2\right)$$

$M_{pl} = 1$

The model

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Superconformal D-term hybrid inflation model

(with an explicit ~~superconformal~~ term)

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~~Superconformal~~ term

$$\mathcal{K} = -3\log\Omega^{-2}$$

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Kähler potential has a shift symmetry

$$\mathrm{Re}\,\Phi \to \mathrm{Re}\,\Phi + \mathrm{const} \qquad \chi = -1$$

for

$$\mathrm{Im}\,\Phi \to \mathrm{Im}\,\Phi + \mathrm{const} \qquad \chi = 1$$

which is broken in superpotential by non-zero λ

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$\longrightarrow$

- $\mathrm{Re}\,\Phi\ (\mathrm{Im}\,\Phi)$ can be inflaton for $\chi \simeq -1\,(1)$
- $\lambda \ll 1$ is expected

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- $\lambda \ll 1$ is expected

In the following discussion, we focus on

$\chi \simeq -1$ (and $\lambda \ll 1$) and $\mathrm{Re}\,\Phi$ as inflaton

Scalar potential (in Einstein frame)

- F-term

$$V_F = \Omega^4 \lambda^2 \left[|\Phi|^2 \left(|S_+|^2 + |S_-|^2 \right) + |S_+ S_-|^2 - \frac{\chi^2 |S_+ S_- \Phi|^2}{3 + \frac{\chi}{2}\left(\Phi^2 + \bar{\Phi}^2\right) + \chi^2 |\Phi|^2} \right]$$

- D-term

$$V_D = \frac{1}{2} g^2 \left(q\Omega^2 (|S_+|^2 - |S_-|^2) - \xi \right)^2$$

$\xi (> 0)$: constant Fayet-Iliopoulos term

Buchmüller, Domcke, Schmitz ’12

see also

Binetruy, Dvali, Kallosh, Proeyen ’04
Dienes, Thomas ’09
Komargodski, Seiberg ’10
Catino, Villadoro, Zwirner ’11

$$
\begin{aligned}
\longrightarrow \quad V_{\rm tot}(\phi, s) &= V_F + V_D \\
&= \frac{\Omega^4(\phi, s)\lambda^2}{4} s^2\phi^2 + \frac{g^2}{8}\left(q\Omega^2(\phi, s)s^2 - 2\xi\right)^2 \\
\Omega^{-2}(\phi, s) &= 1 - \frac{1}{6}\left(s^2 + (1+\chi)\phi^2\right)
\end{aligned}
$$

$s \equiv \sqrt{2}|S_+| \longrightarrow$ waterfall field

$\phi \equiv \sqrt{2}\mathrm{Re}\,\Phi \longrightarrow$ inflaton field

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$$

$s \equiv \sqrt{2}|S_+|$ → waterfall field

$\phi \equiv \sqrt{2}{\rm Re}\,\Phi$ → inflaton field

Suppose the initial inflaton value is super-Planckian (due to the approximated shift symmetry)

We have found that the dynamics of the inflaton/waterfall fields are similar to *subcritical hybrid inflation*

Subcritical hybrid inflation

1) Inflaton rolls down to the critical point from super-Planckian value

2) At the critical point, waterfall field begins to grow

3) Inflaton is still slow-rolling and inflation continues

Clesse ’11, Clesse, Garbrecht ’12

Kodama, Kohri, Nakayama ’11

Lyth ’12

Buchmüller, Domcke, Schmitz ’14

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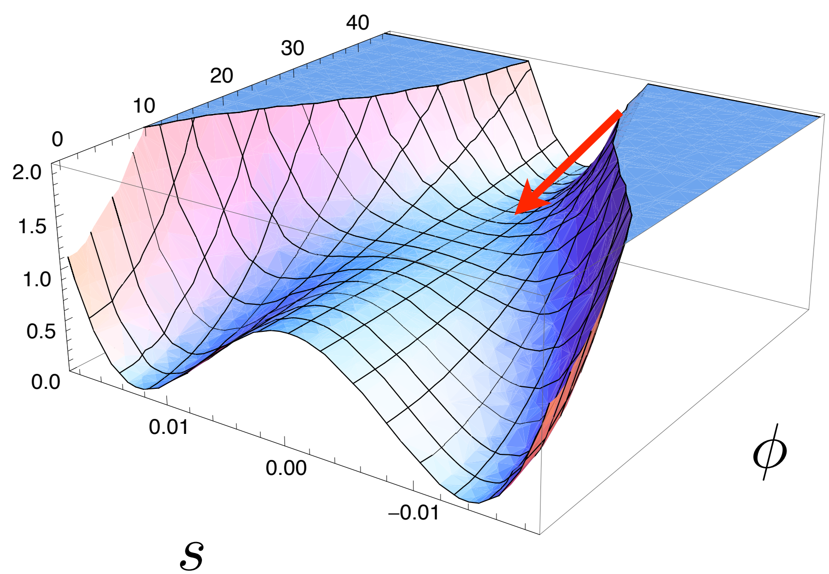

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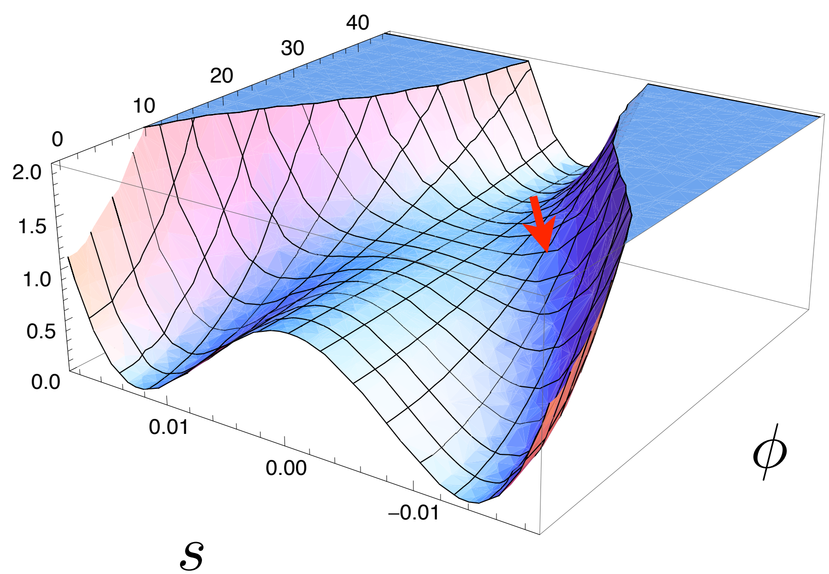

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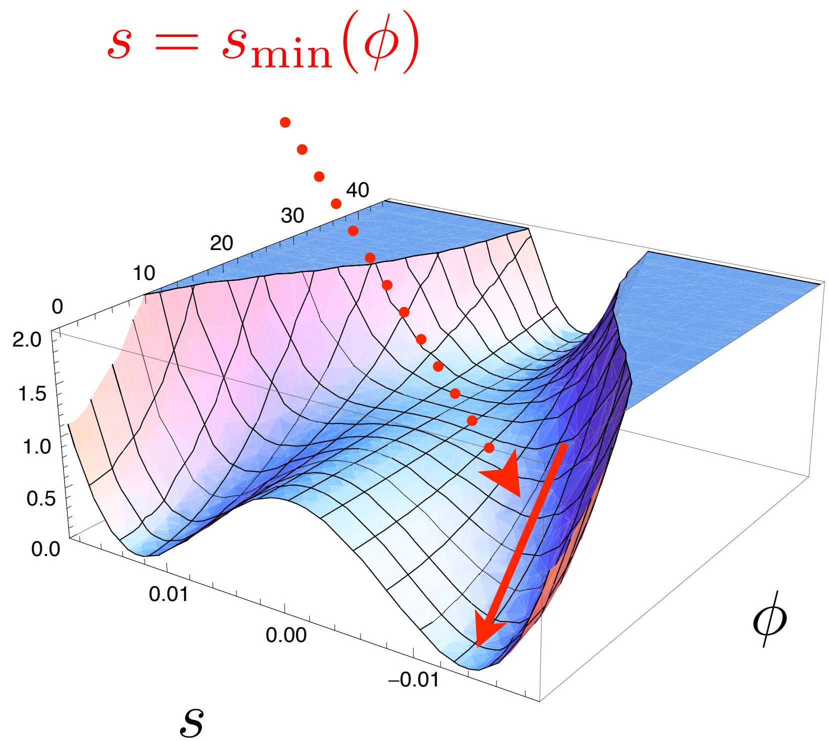

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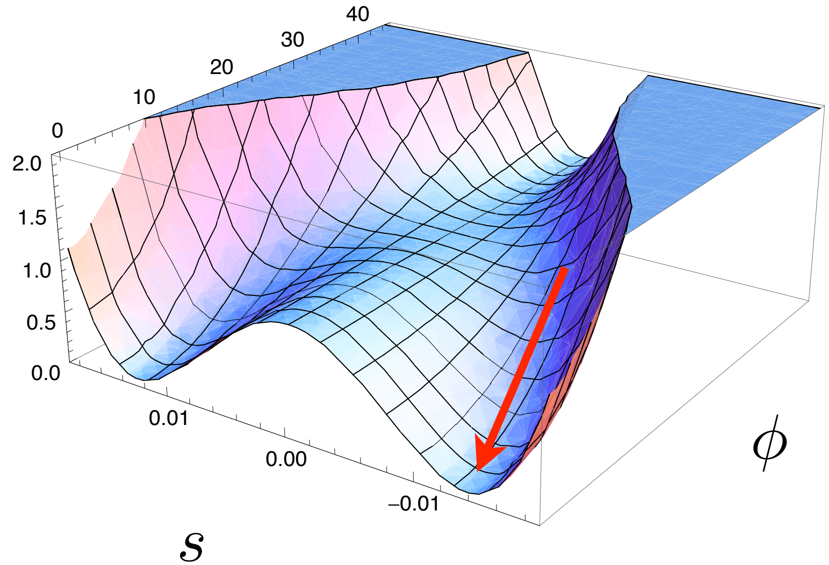
→ Cosmological consequences are determined by the subcritical regime



Potential in subcritical regime

$$V \equiv V_{\rm tot}(\phi, s_{\rm min})$$

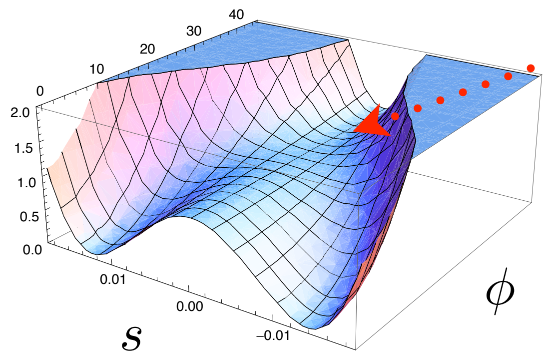
$$= g^2\xi^2(1+\tilde{\xi})\Psi^2 \frac{1 - \frac{\Psi^2}{2(1+\tilde{\xi})}}{1 + 2\tilde{\xi}\Psi^2}$$

$$\Psi \equiv \frac{\Omega(\phi, 0)\phi}{\Omega(\phi_c, 0)\phi_c}$$

$$\tilde{\xi} \equiv \xi/(3q)$$

$$\phi_c^2 = \frac{6qg^2\xi}{3\lambda^2 + (1+\chi)qg^2\xi}$$

critical point



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→ Remaining task:

Solve the dynamics of the inflaton to give cosmological parameters

Observables

- Scalar spectral index: n_s
- Tensor-to-scalar ratio: r
- Scalar amplitude: A_s

$\longleftarrow$ Slow-roll parameters ϵ, η for given number of e-folds N_*

$\longrightarrow$ n_s and r are determined for given N_*

Preparation for numerical study

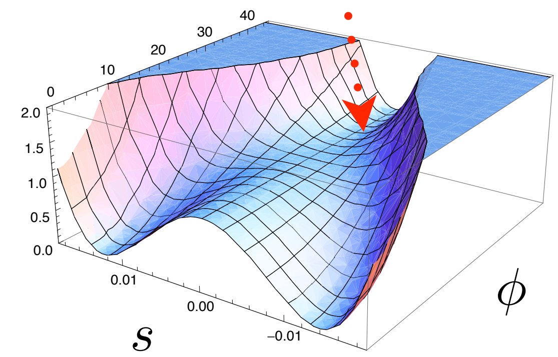
- We rewrite χ as

$$\chi = -1 - \frac{3\lambda^2}{qg^2\xi}\delta\chi \qquad \boxed{(0 < \delta\chi < 1)}$$

$$\phi_c^2 = \boxed{\frac{2qg^2\xi}{\lambda^2(1-\delta\chi)} > 0}$$

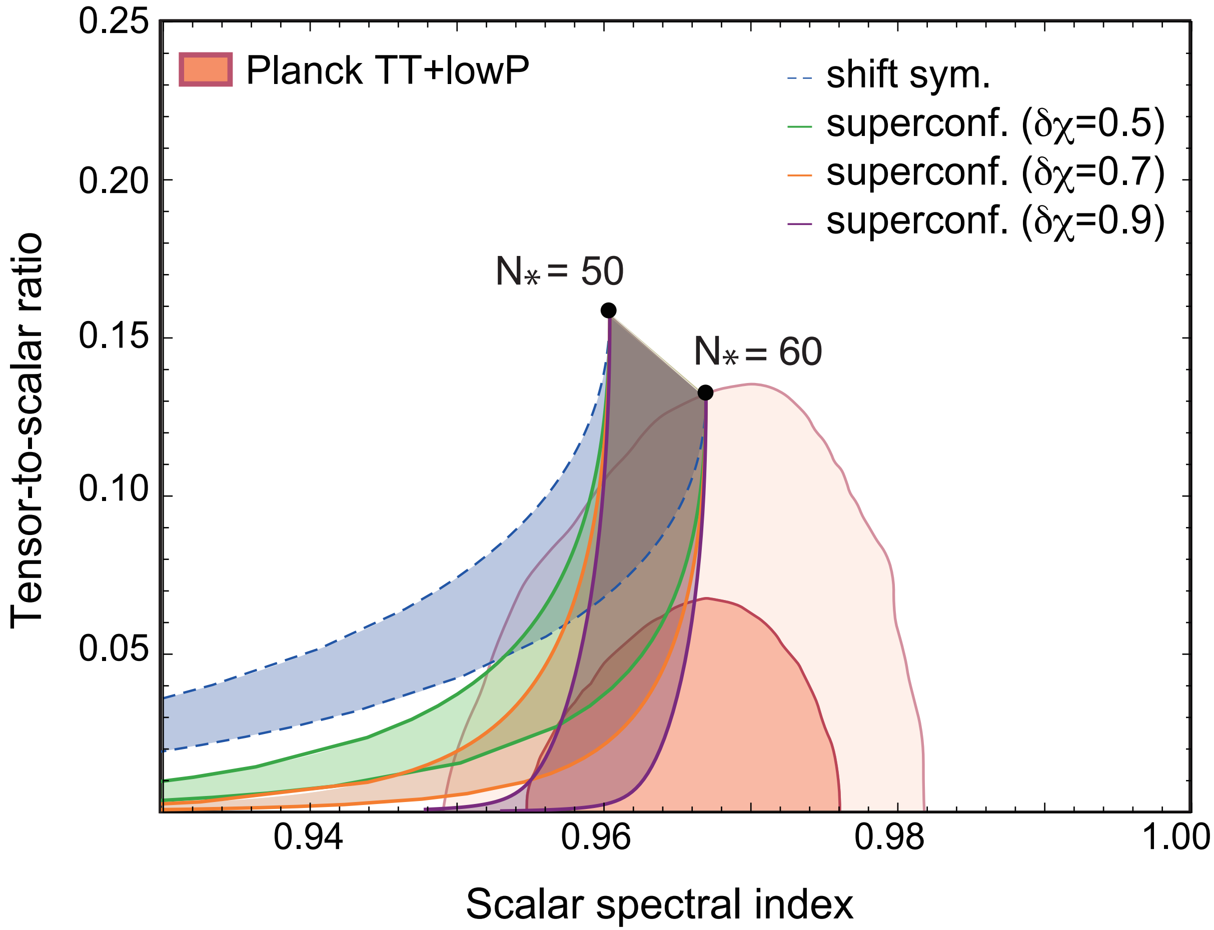
critical point value

critical point

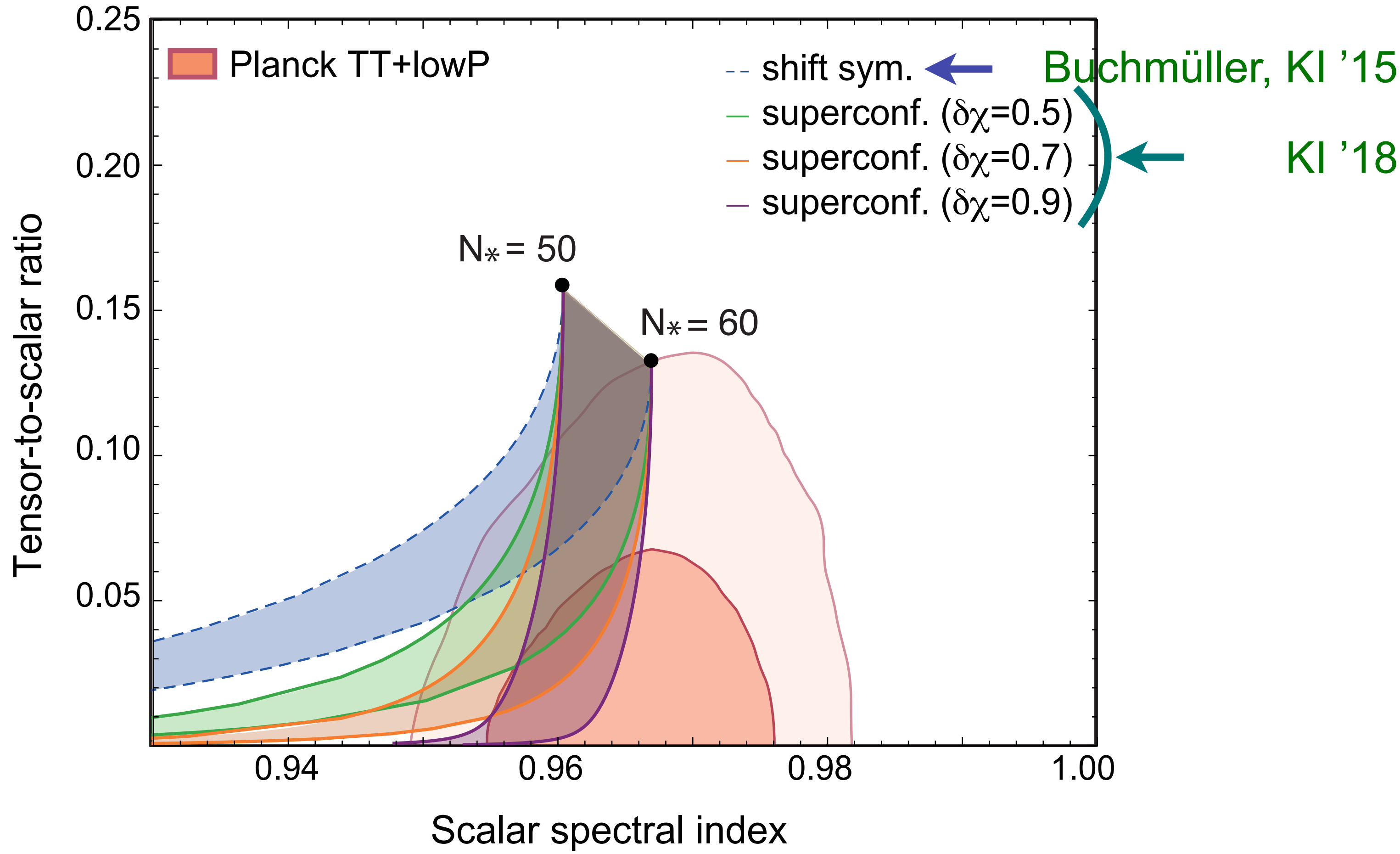


- $q,\ g$ can be absorbed in redefinition of $\lambda,\ \xi$

$$\longrightarrow \quad q = g = 1$$



- $\delta\chi \to 0$ approaches to the shift symmetric Kähler case
- $\delta\chi \to 1$ looks similar to α attractor models



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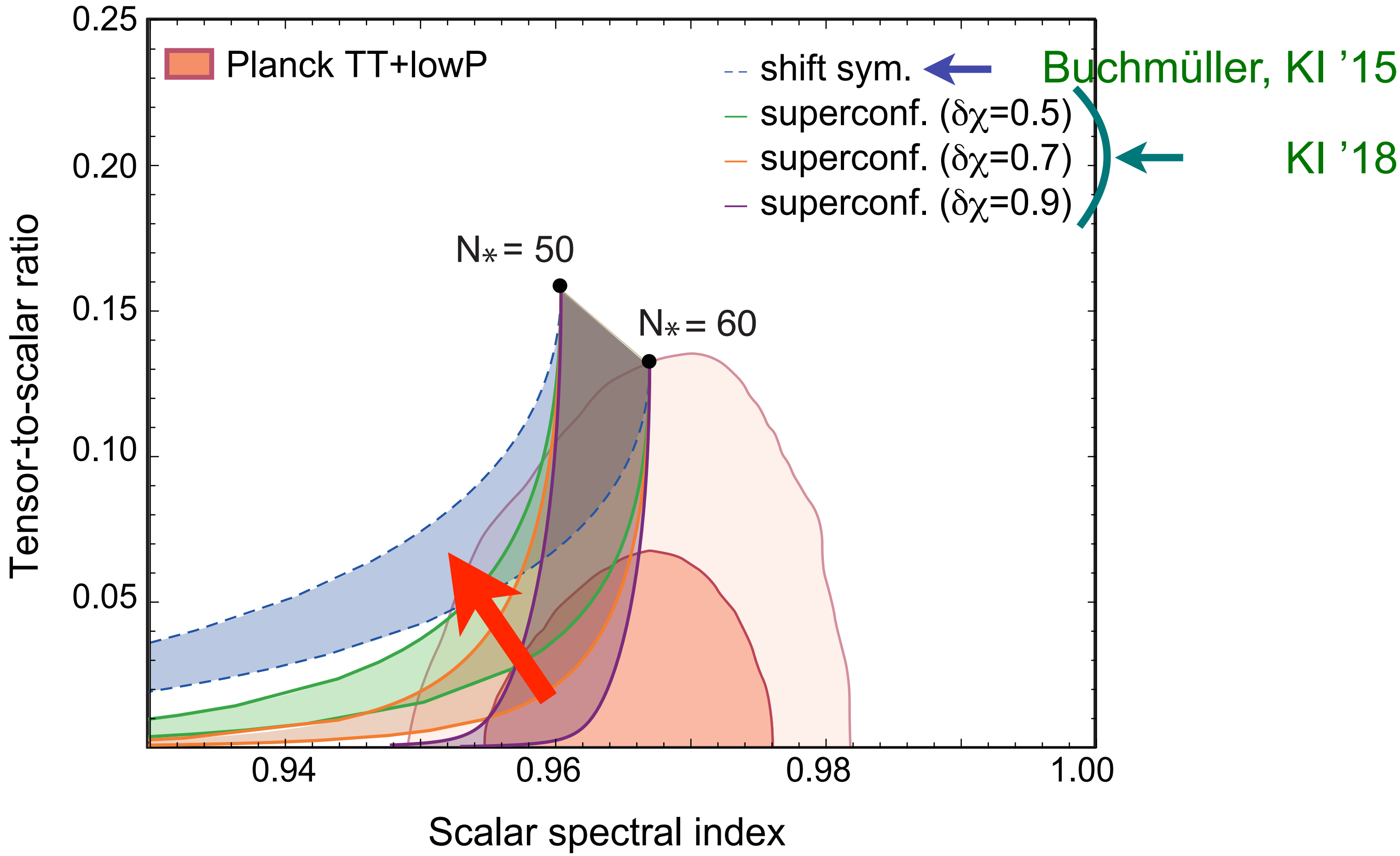

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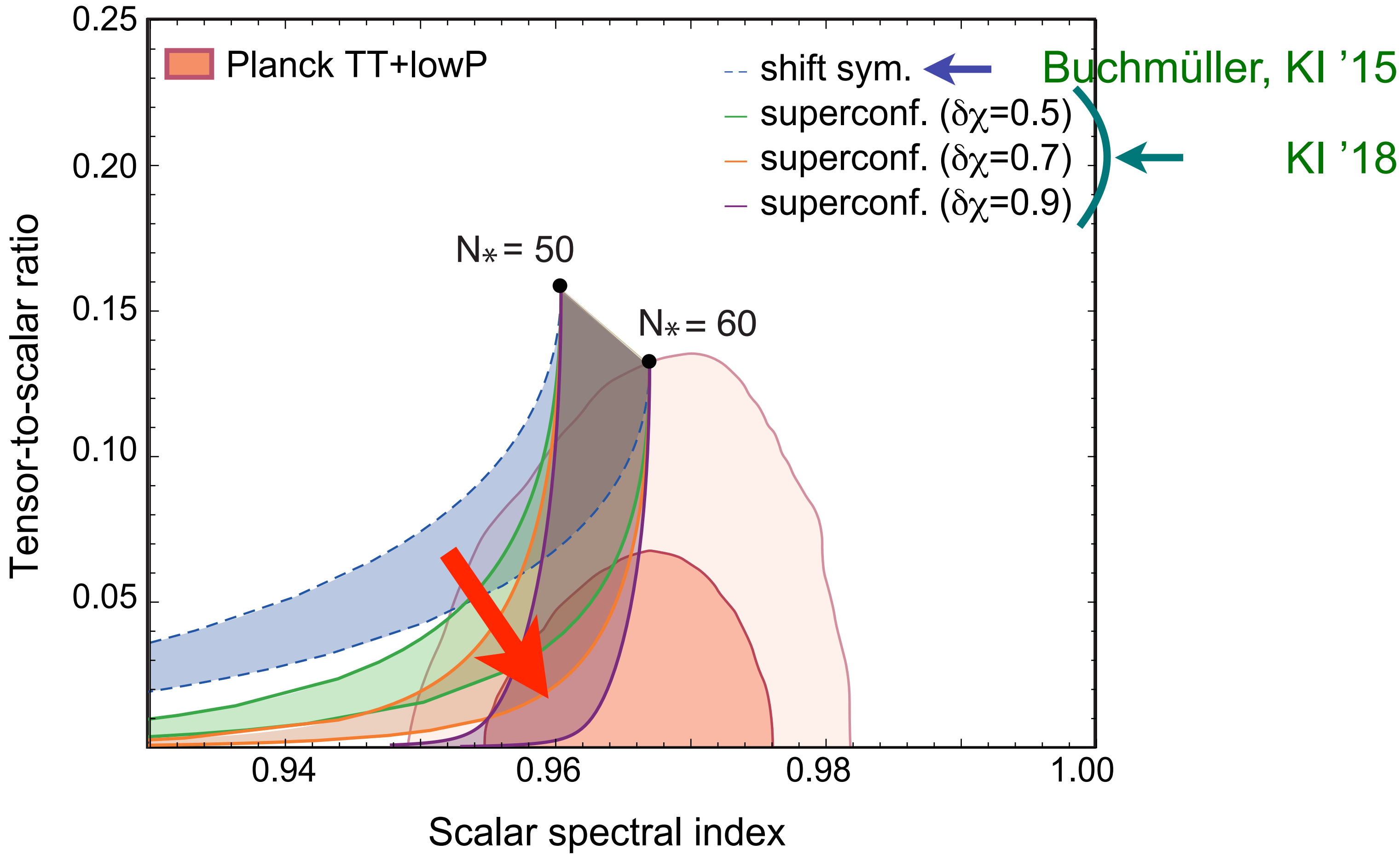

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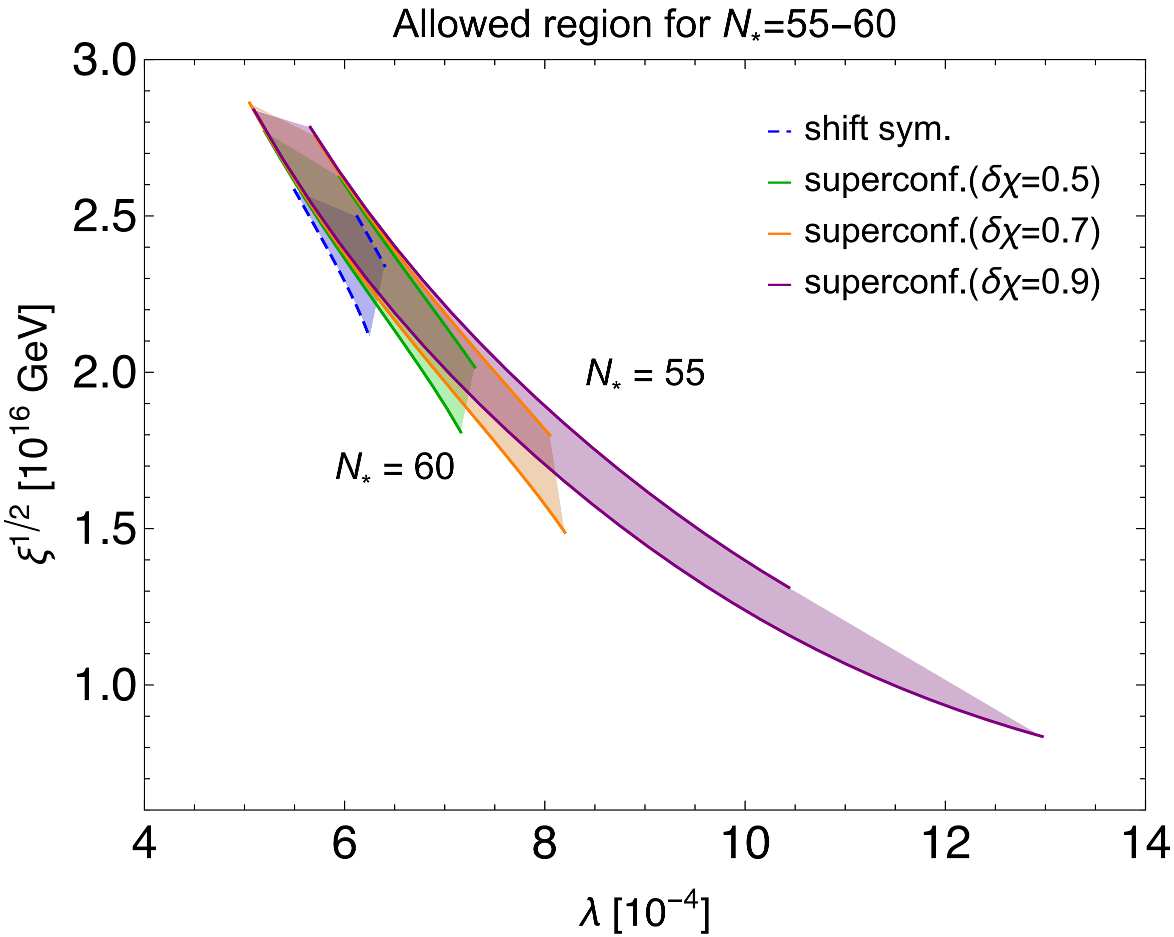
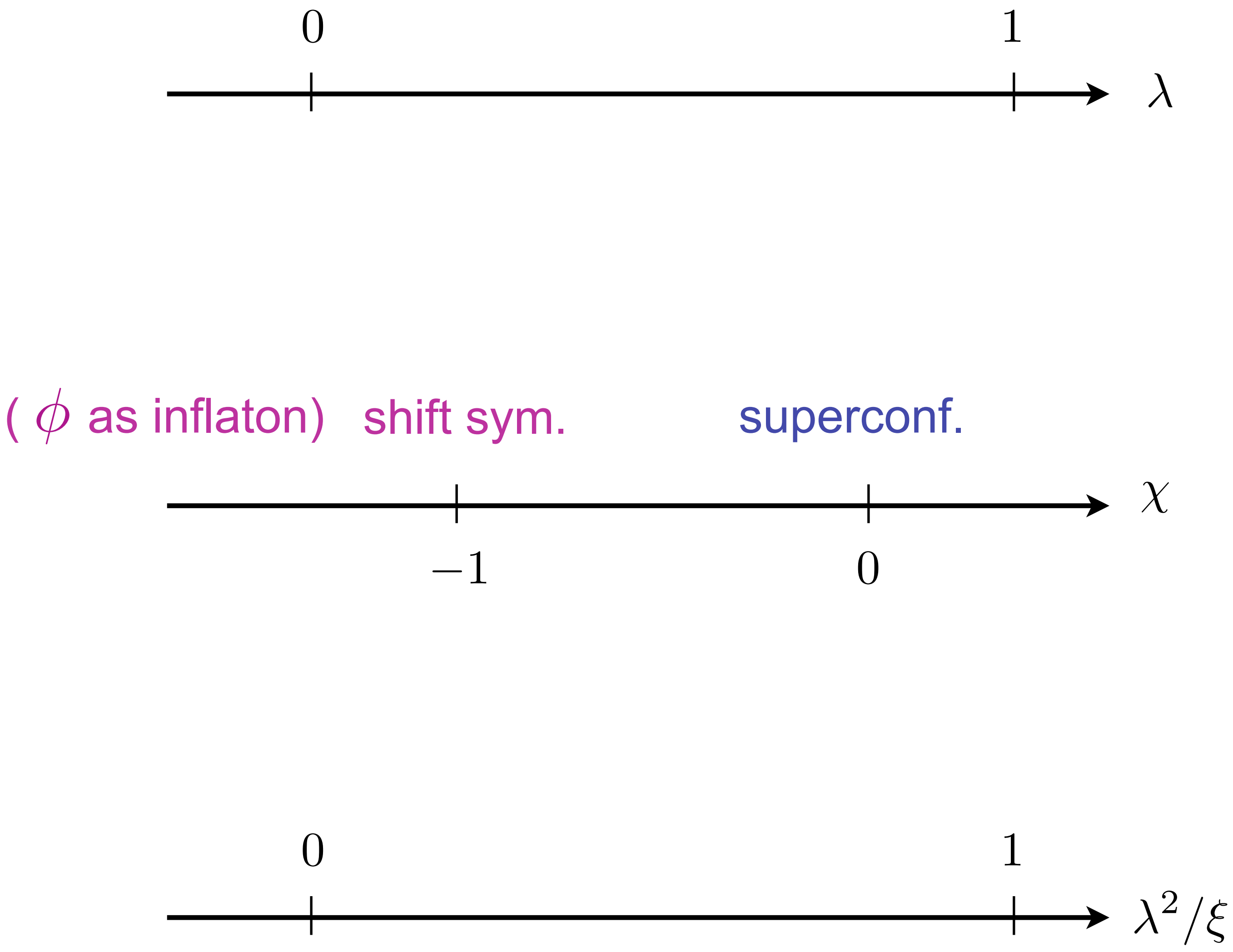
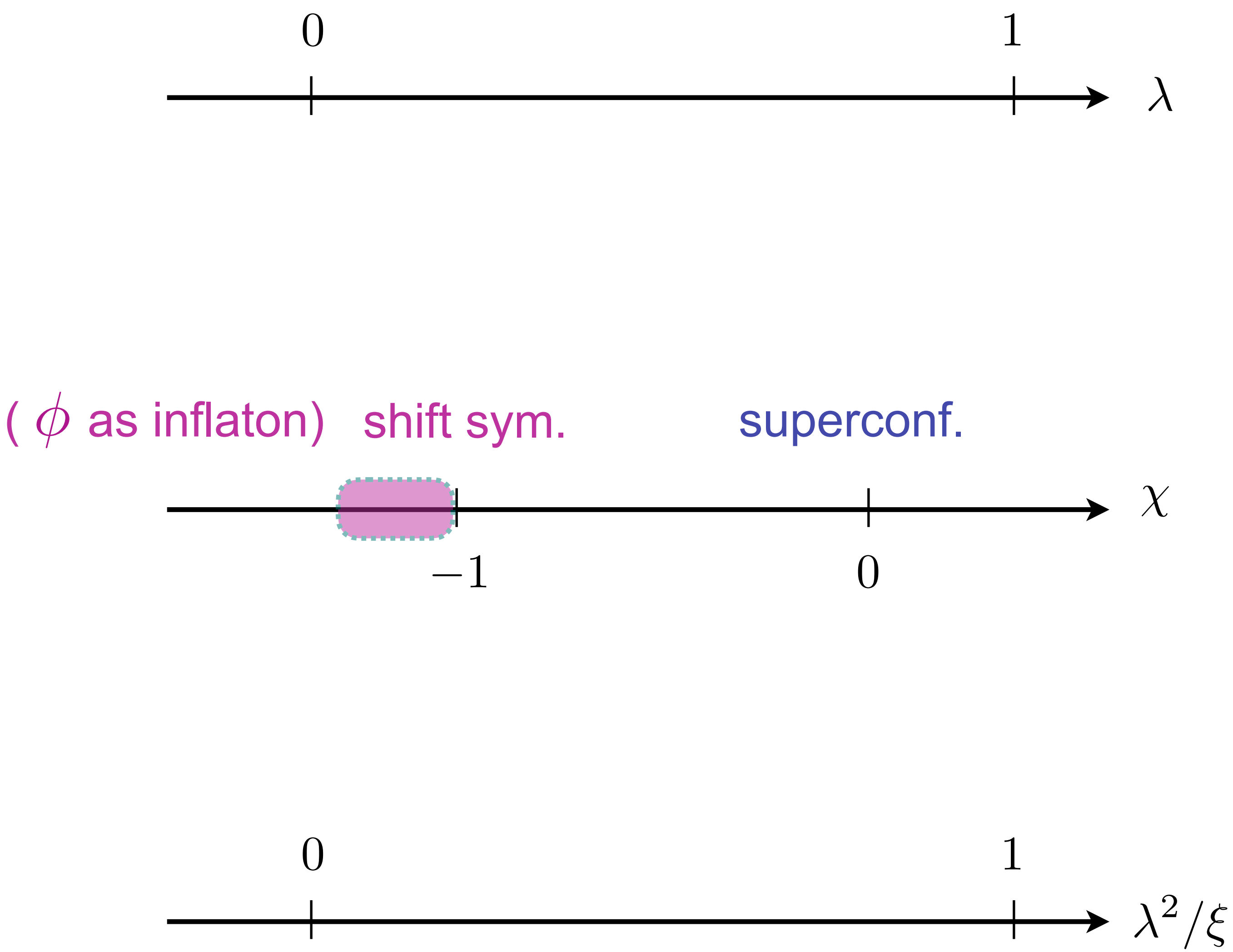


$\lambda \sim 10^{-4}$-10^{-3} and $\sqrt{\xi} \sim 10^{16}\,\mathrm{GeV}$ are consistent with the data

$\longrightarrow$ $m_\phi \simeq \lambda\sqrt{\xi} \sim 10^{13}\,\mathrm{GeV}$

Inflaton mass


0
1
λ
(ϕ as inflaton)
shift sym.
superconf.
χ
-1
0
0
1
λ^2/ξ



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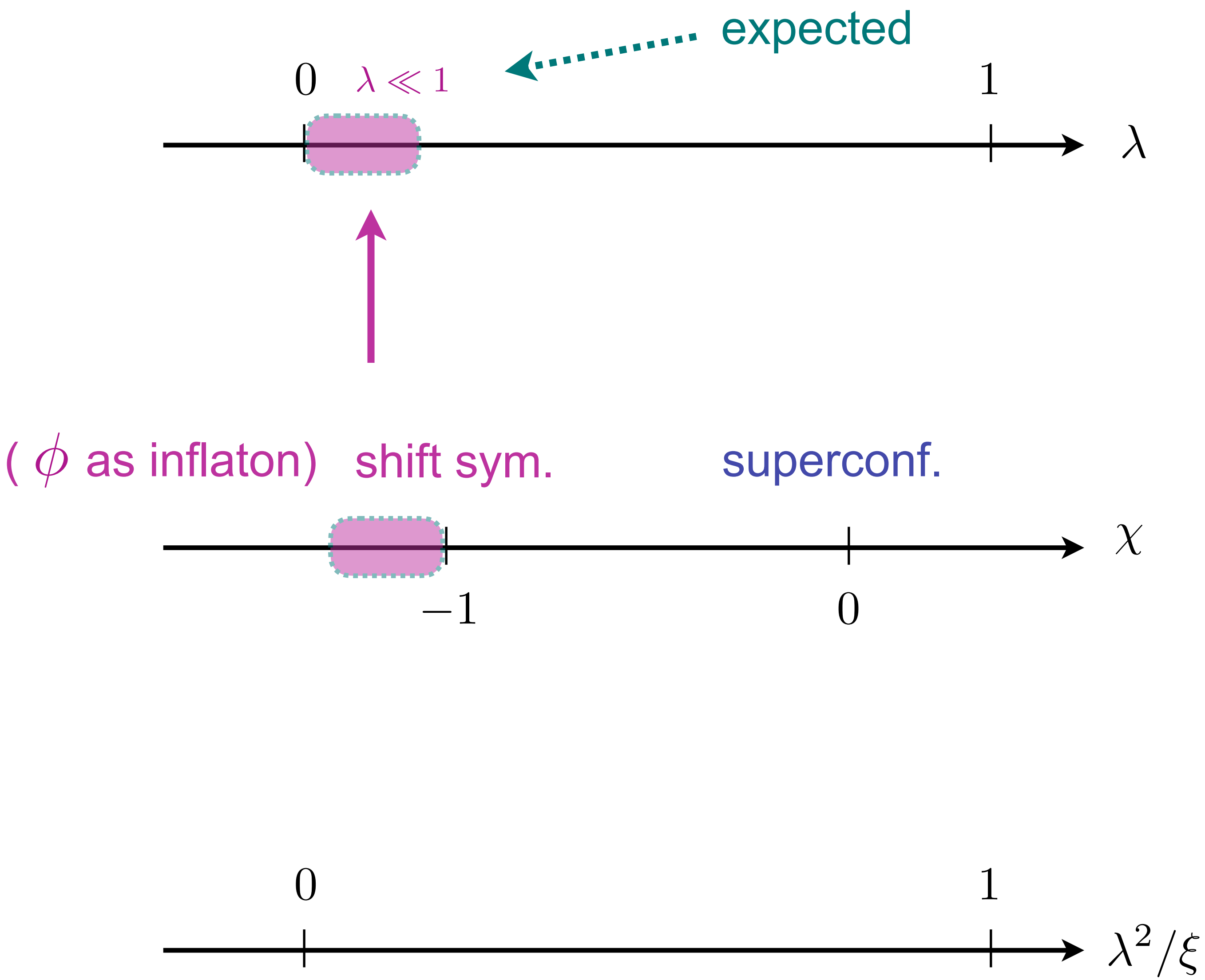

expected
0
$\lambda \ll 1$
1
λ
(ϕ as inflaton)
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superconf.
χ
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0
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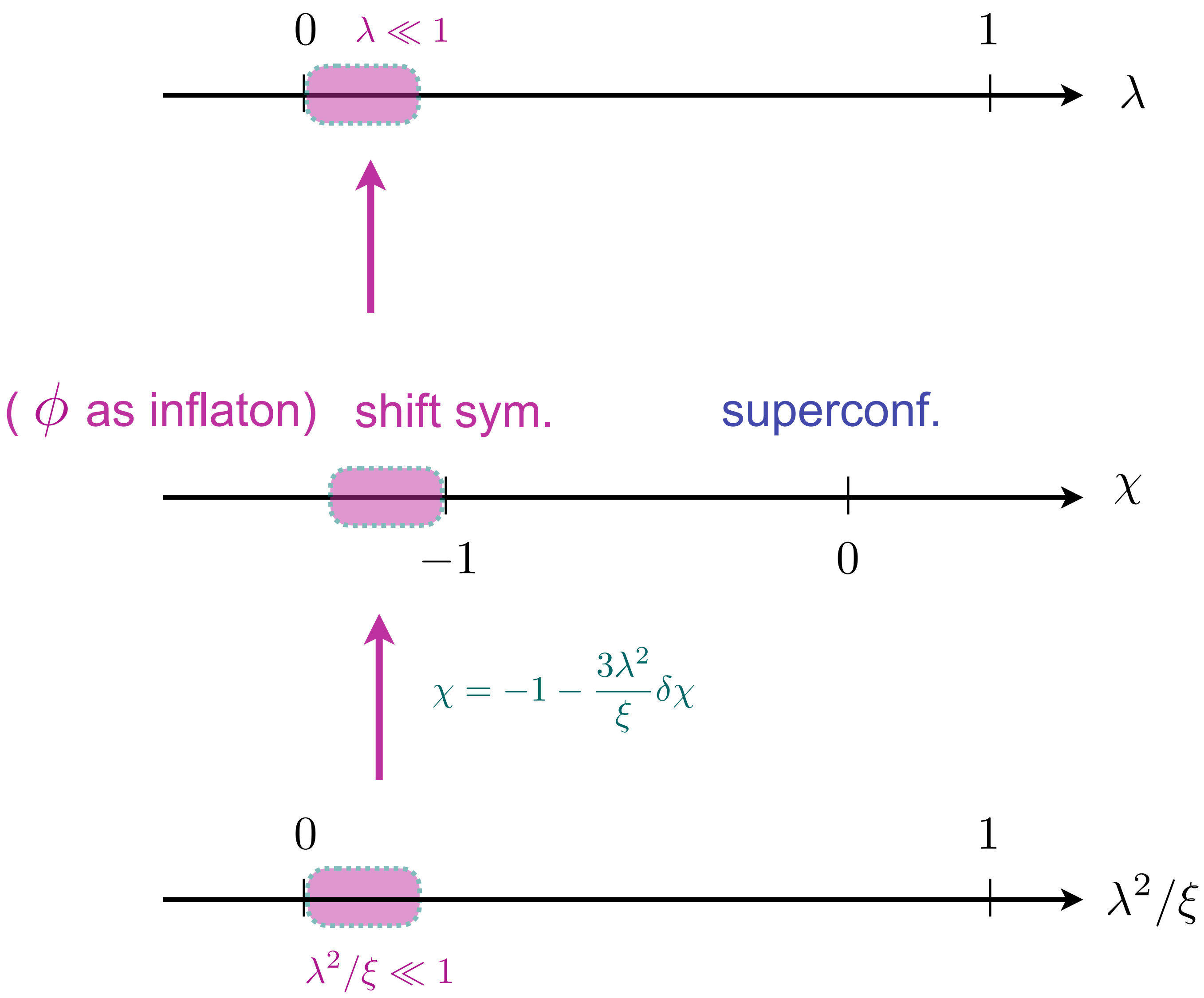

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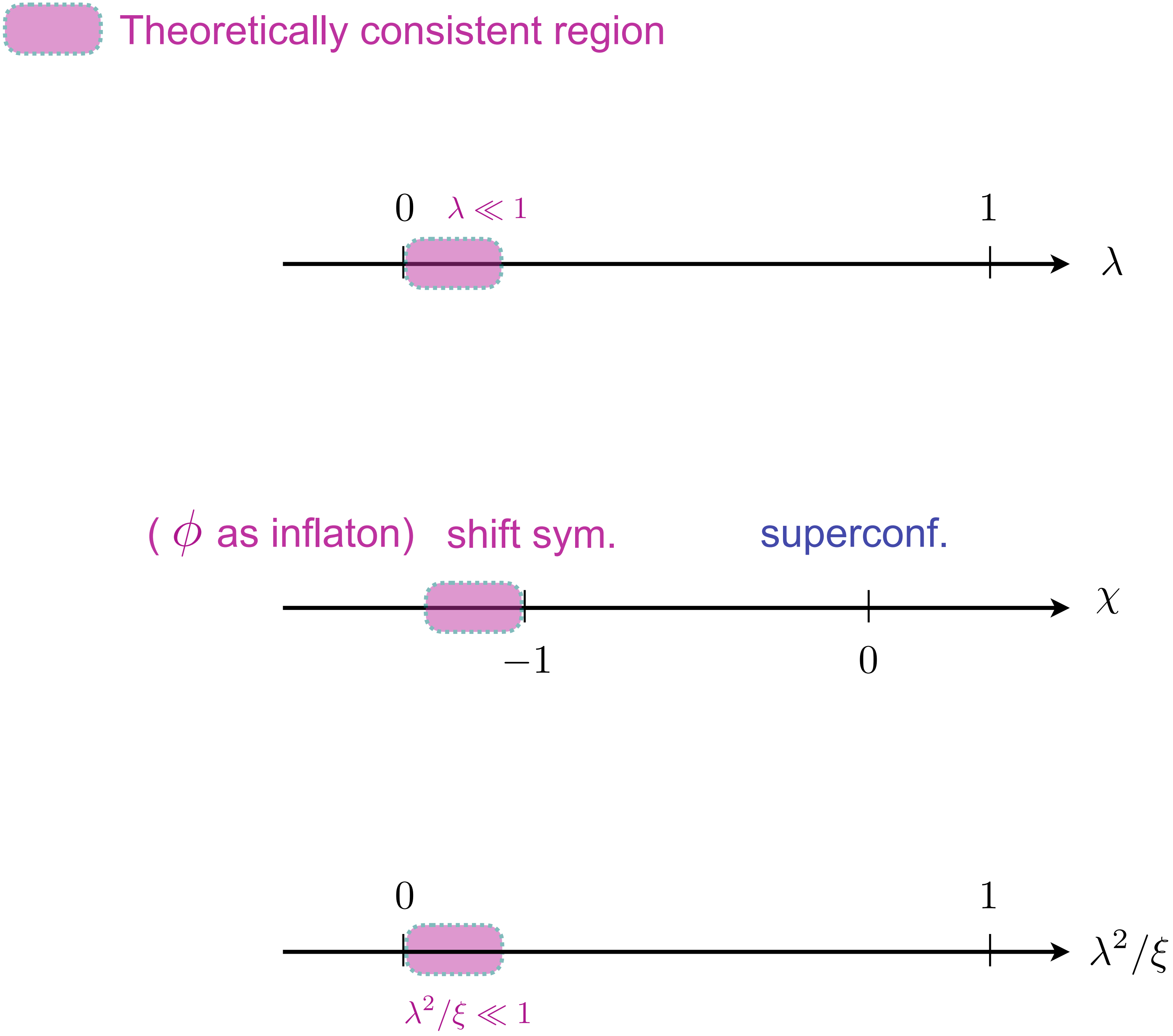
shift sym.
superconf.
χ
-1
0
$\chi = -1 - \frac{3\lambda^2}{\xi}\delta\chi$
0
1
λ^2/ξ
$\lambda^2/\xi \ll 1$


Theoretically consistent region
0
$\lambda \ll 1$
1
λ
(ϕ as inflaton) shift sym.
superconf.
χ
-1
0
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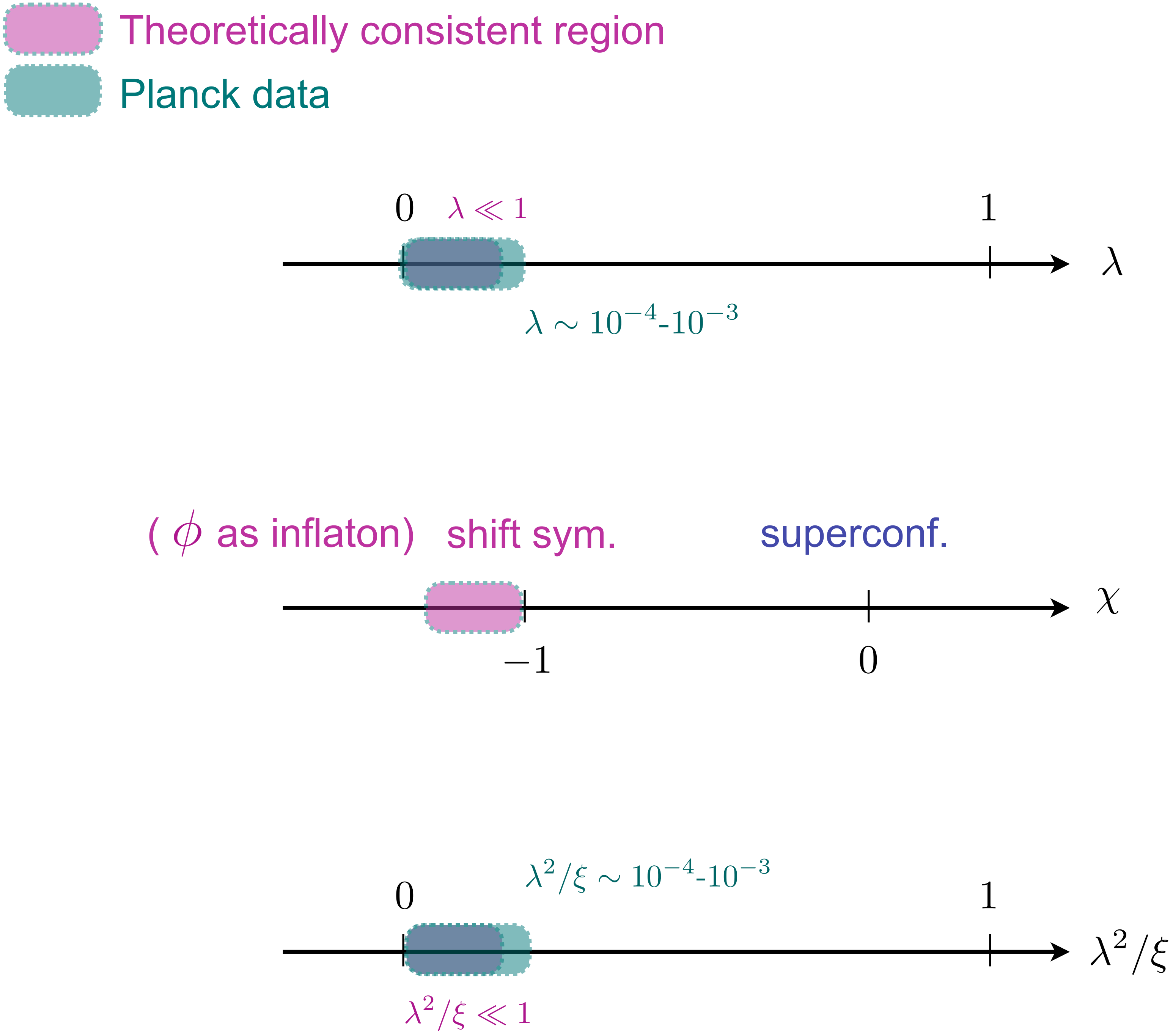
Theoretically consistent region
Planck data
0
$\lambda \ll 1$
1
λ
$\lambda \sim 10^{-4}$-10^{-3}
(ϕ as inflaton)
shift sym.
superconf.
χ
-1
0
$\lambda^2/\xi \sim 10^{-4}$-$10^{-3}$
0
1
λ^2/ξ
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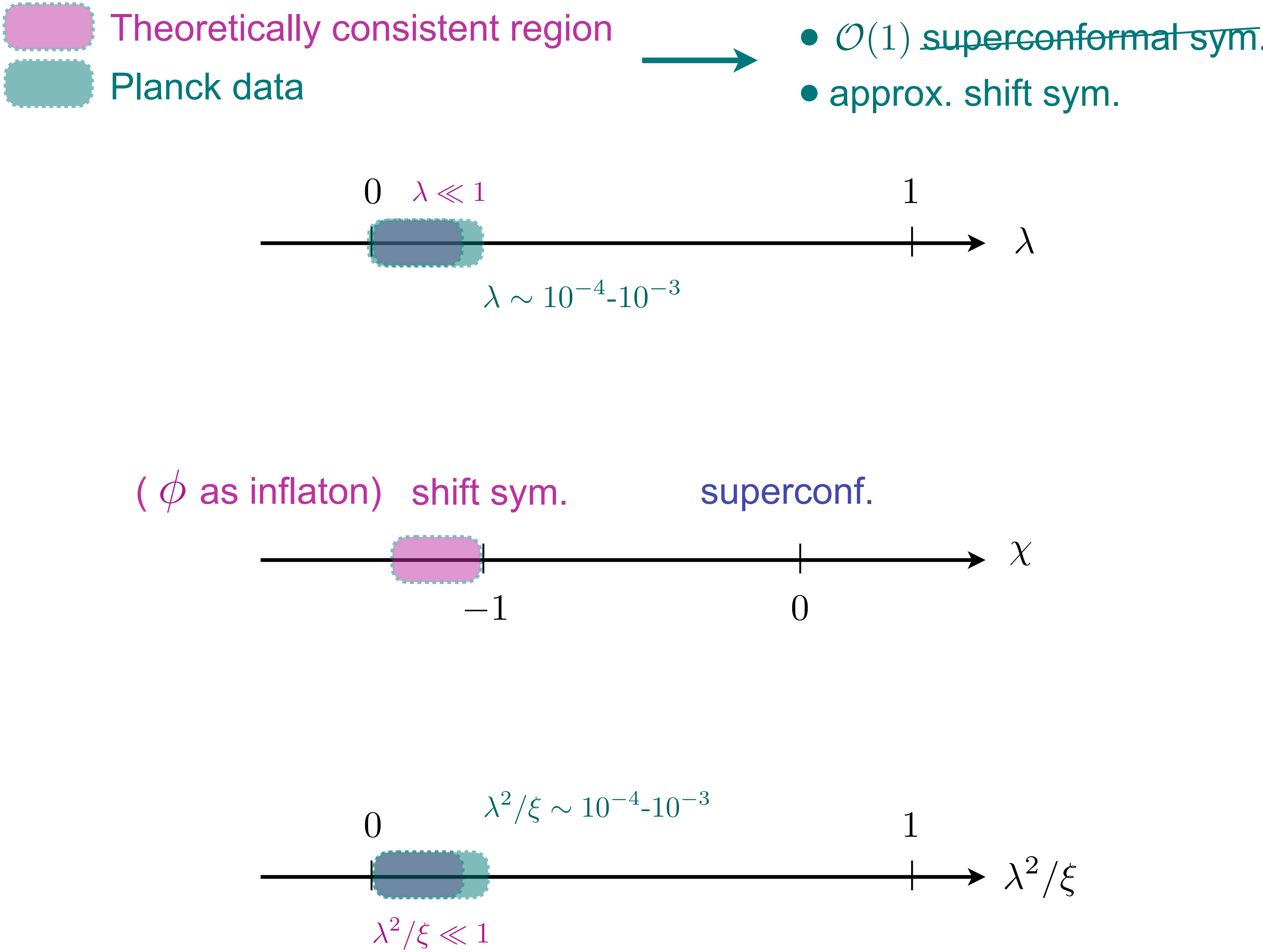

Theoretically consistent region
Planck data
$\mathcal{O}(1)$ ~~superconformal sym.~~
approx. shift sym.
0
$\lambda \ll 1$
1
λ
$\lambda \sim 10^{-4}$-10^{-3}
(ϕ as inflaton) shift sym.
superconf.
χ
-1
0
$\lambda^2/\xi \sim 10^{-4}$-$10^{-3}$
0
1
λ^2/ξ
$\lambda^2/\xi \ll 1$

What we've done so far

Inflation → okay

Reheating → ?

Baryon asymmetry → ?

Dark matter → ?

What we’ve done so far

Inflation	→	okay
Reheating	→	?
Baryon asymmetry	→	?
Dark matter	→	?

3. Leptogenesis after the inflation

Buchmüller, Domcke, Schmitz '12
Buchmüller, Domcke, Kamada '13
KI '18

The original model

- Superpotential: $W = \lambda \Phi S_+ S_-$

	Φ	S_+	S_-
U(1)	0	q	$-q$

$q > 0$

- Kähler potential: $\mathcal{K} = -3 \log \Omega^{-2}$

with $$\Omega^{-2} = 1 - \frac{1}{3}\left(|S_+|^2 + |S_-|^2 + |\Phi|^2\right) - \frac{\chi}{6}\left(\Phi^2 + \bar{\Phi}^2\right)$$

- Fayet-Iliopoulos (FI) term: $\xi \ (> 0)$

The extended model

We introduce three right-handed neutrinos N_i^c that have Majorana mass terms:

- Superpotential:

$$W_{\rm neu} = \lambda_i N_i^c S_+ S_- + \frac{1}{2} M_{ij} N_i^c N_j^c + y_{\nu\, ij} N_i^c L_j H_u$$

- Kähler potential: $\mathcal{K} = -3 \log \Omega^{-2}$

with $$\Omega^{-2} = 1 - \frac{1}{3}\left(|S_+|^2 + |S_-|^2 + |N_i^c|^2\right) - \frac{\chi_i}{6}\left(N_i^{c\,2} + \bar{N}_i^{c\,2}\right)$$

- Fayet-Iliopoulos (FI) term: $\xi \ (> 0)$

For further study, let us take

- $\begin{cases} \lambda_3 \neq 0 \\ \chi_3 \neq 0 \end{cases}$, the rests = 0

Minimal extension

- $M_{ij} = \mathrm{diag}(M_1, M_2, M_3)$

For further study, let us take

- $\begin{cases} \lambda_3 \neq 0 \\ \chi_3 \neq 0 \end{cases}$, the rests = 0

Minimal extension

- $M_{ij} = \mathrm{diag}(M_1, M_2, M_3)$

$$\begin{aligned} W_{\rm neu} =& \lambda_3 N_3^c S_+ S_- + \frac{1}{2} M_i N_i^c N_i^c + y_{\nu\,ij} N_i^c L_j H_u \\ \Omega^{-2} =& 1 - \frac{1}{3}\left(|S_+|^2 + |S_-|^2 + |N_i^c|^2\right) - \frac{\chi_3}{6}\left({N_3^c}^2 + {\bar{N}_3^c}^2\right) \end{aligned}$$

For further study, let us take

- $\begin{cases} \lambda_3 \neq 0 \\ \chi_3 \neq 0 \end{cases}$, the rests = 0
- $M_{ij} = \mathrm{diag}(M_1, M_2, M_3)$

Minimal extension

$$\longrightarrow \begin{aligned} W_{\mathrm{neu}} =& \lambda_3 N_3^c S_+ S_- + \frac{1}{2} M_i N_i^c N_i^c + y_{\nu\, ij} N_i^c L_j H_u \\ \Omega^{-2} =& 1 - \frac{1}{3}\left(|S_+|^2 + |S_-|^2 + |N_i^c|^2\right) - \frac{\chi_3}{6}\left({N_3^c}^2 + {\bar{N}_3^c}{}^2\right) \end{aligned}$$

Our goal is to see the impact on the thermal history

$\phi \equiv \sqrt{2}\mathrm{Re} N_3^c$ as Inflaton

Impact of M_i on inflation

During inflation, M_3 gives additional gradient for inflaton:

$$V_{\rm inf} = V + \Delta V(M_3)$$

Inflationary trajectory is not affected if

$$\frac{\Delta V(M_3)}{V} \ll 1$$

$$\longrightarrow \quad M_3 \ll 2\times 10^{11}\,{\rm GeV}$$

On the other hand, M_1 and M_2 are not constrained

We consider two representative cases:

(I). $M_1,\, M_2 < m_\phi$

(II). $M_1,\, M_2 > m_\phi$

c.f., inflaton mass

$m_\phi \simeq 10^{13}\,\mathrm{GeV}$

Preparation: neutrino sector

Light neutrino mass matrix is given by seesaw mechanism:

$$M_\nu = -\tilde{m}_\nu^T \tilde{M}^{-1} \tilde{m}_\nu$$

$$\tilde{m}_\nu = \begin{pmatrix} & m_\nu & \\ 0 & 0 & 0 \end{pmatrix}$$

← 4×3 matrix

$$m_\nu = y_\nu \langle H_u^0 \rangle$$

$$\tilde{M} = \left(\begin{array}{ccc|c} M_1 & & & 0 \\ & M_2 & & 0 \\ & & M_3 & m_\phi \\ \hline 0 & 0 & m_\phi & 0 \end{array}\right)$$

← 4×4 matrix

Preparation: neutrino sector

Features of light neutrino mass matrix

- One neutrino is massless

$$(\because \mathrm{rank}\,(M_\nu) = 2)$$

- The mass matrix becomes

$$M_{\nu ij} = -\langle H_u^0 \rangle^2 \sum_{k=1}^{2} \frac{y_{\nu ki} y_{\nu kj}}{M_k}$$

(independent of $m_\phi,\, M_3,\, y_{\nu 3i}$)

⟶ $m_\phi,\, M_3,\, y_{\nu 3i}$ are not constrained by the neutrino observations

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$\longrightarrow$ $m_\phi,\ M_3,\ \boxed{y_{\nu 3i}}$ are not constrained by the neutrino observations

$\uparrow$

determines the reheating temperature

Reheating

Inflaton decays to Higgses and leptons to reheat the universe

$$T_R \simeq 1.4 \times 10^{10}\,\mathrm{GeV} \left(\frac{(y_\nu y_\nu^\dagger)_{33}}{10^{-9}} \right)^{1/2}$$

$$\Gamma_\phi \simeq \frac{(y_\nu y_\nu^\dagger)_{33}}{8\pi} m_\phi$$

$$\propto \sum_i |y_{\nu 3i}|^2$$

$$T_R \sim \sqrt{\Gamma_\phi M_{pl}}$$

Reheating

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$$\propto \sum_i |y_{\nu 3i}|^2$$

$$T_R \sim \sqrt{\Gamma_\phi M_{pl}}$$

$\longrightarrow$ T_R is a free parameter

On the other hand, M_1 and M_2 are not constrained

We consider two representative cases:

(I). $M_1,\, M_2 < m_\phi$

(II). $M_1,\, M_2 > m_\phi$

c.f., inflaton mass

$$m_\phi \simeq 10^{13}\,\mathrm{GeV}$$

Leptogenesis: case (I) $M_1,\, M_2 < m_\phi$

Thermal leptogenesis takes place when $T_R \gtrsim M_1,\, M_2$, which is always possible

see, e.g.,

Buchmüller, Peccei, Yanagida '05
Davidson, Nardi, Nir '08

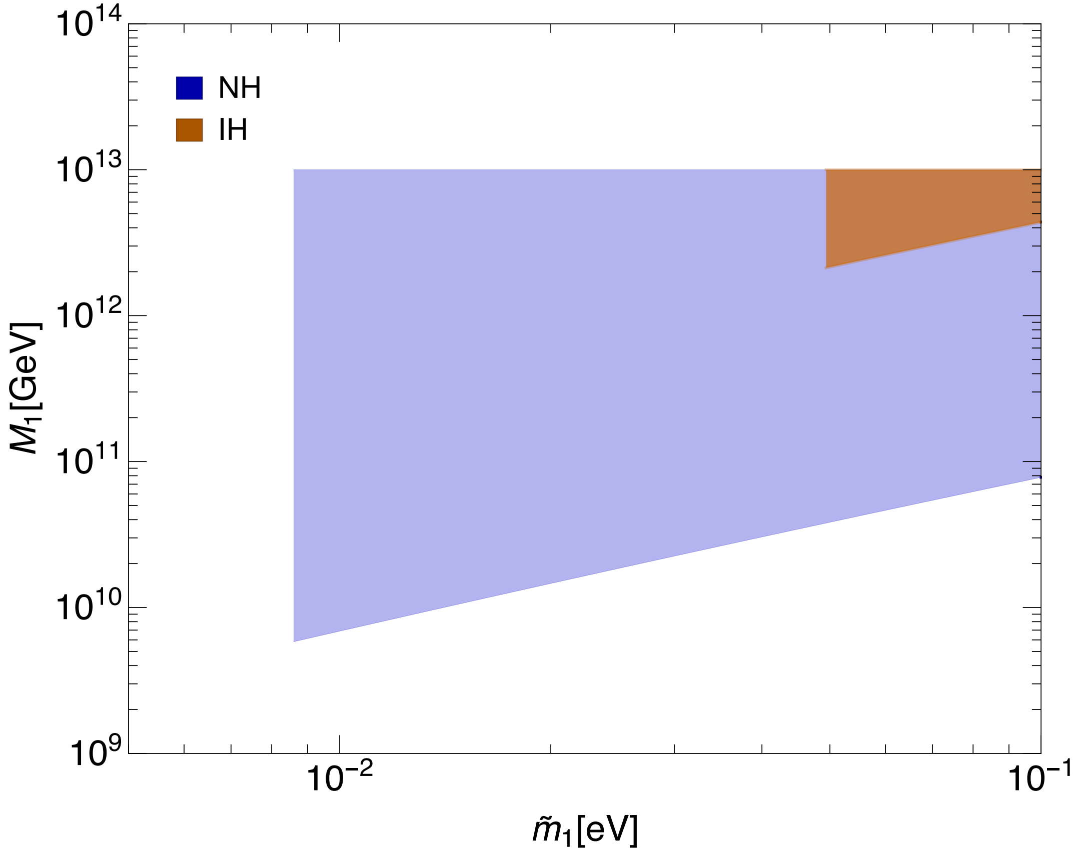
→ Baryon number:

$$\eta_B \simeq 2.7 \times 10^{-10} \left(\frac{\epsilon_1}{10^{-6}}\right) \left(\frac{\kappa_f}{2 \times 10^{-2}}\right)$$

- ϵ_1 : Asymmetric parameter
- κ_f : Efficiency factor

Covi, Roulet, Vissani '96
Giudice, Notari, Raidal, Riotto, Strumia '04
Buchmüller, Di Bari, Plümacher '04
Davidson, Nardi, Nir '08

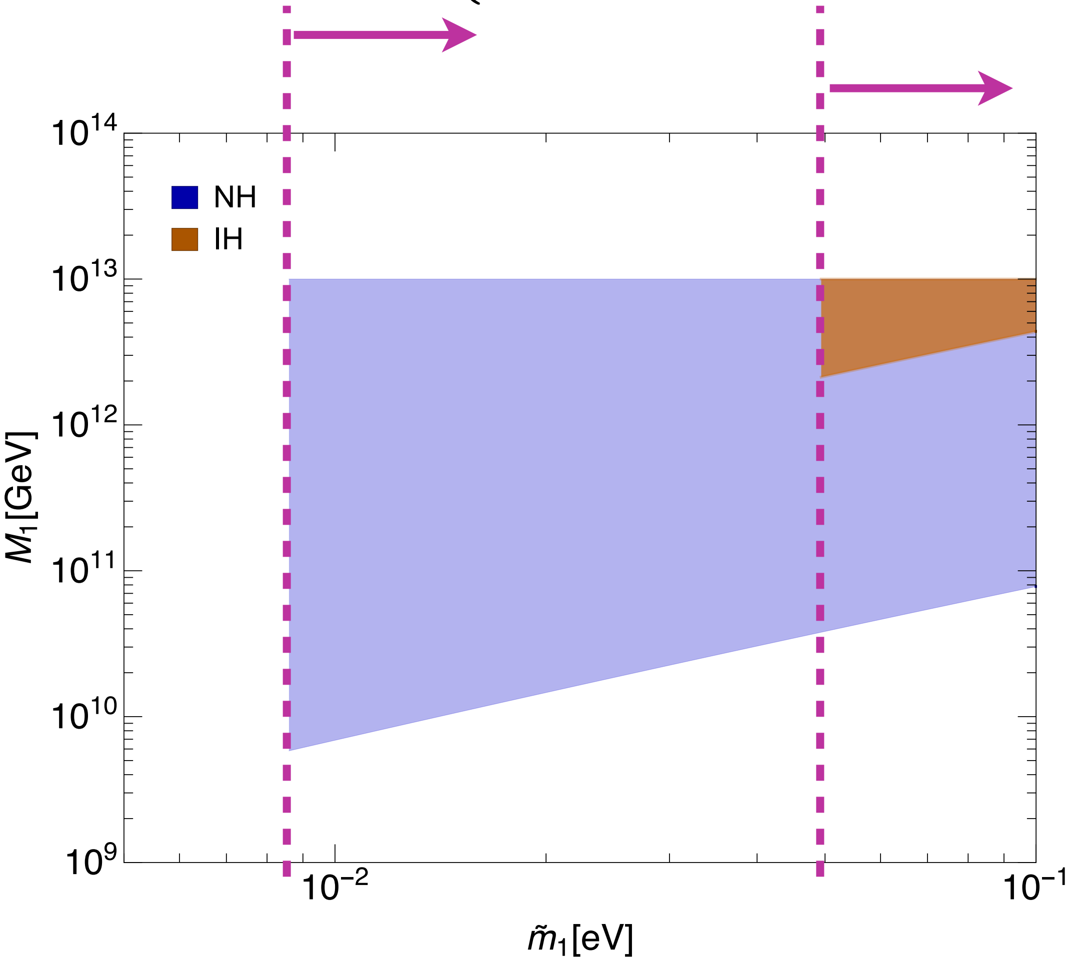
Leptogenesis: case (I)



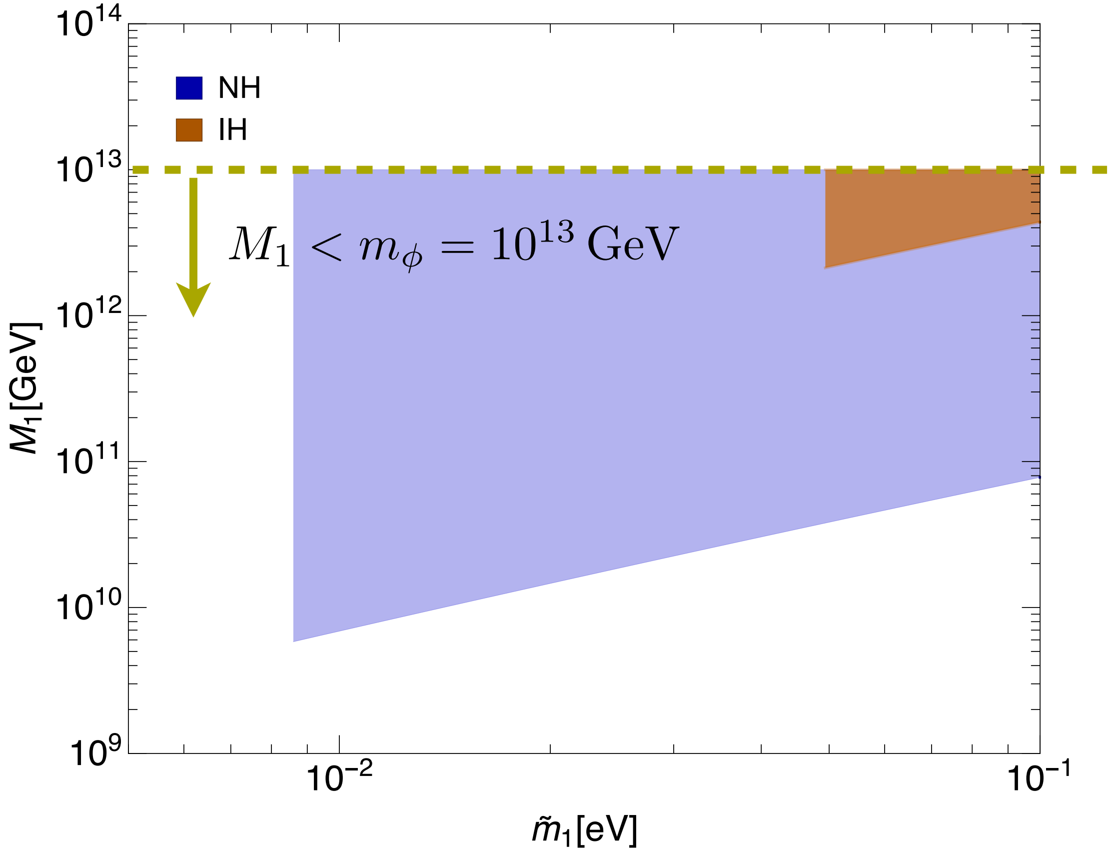
$$\tilde{m}_1 \equiv \frac{(m_\nu m_\nu^\dagger)_{11}}{M_1}$$

Leptogenesis: case (I)

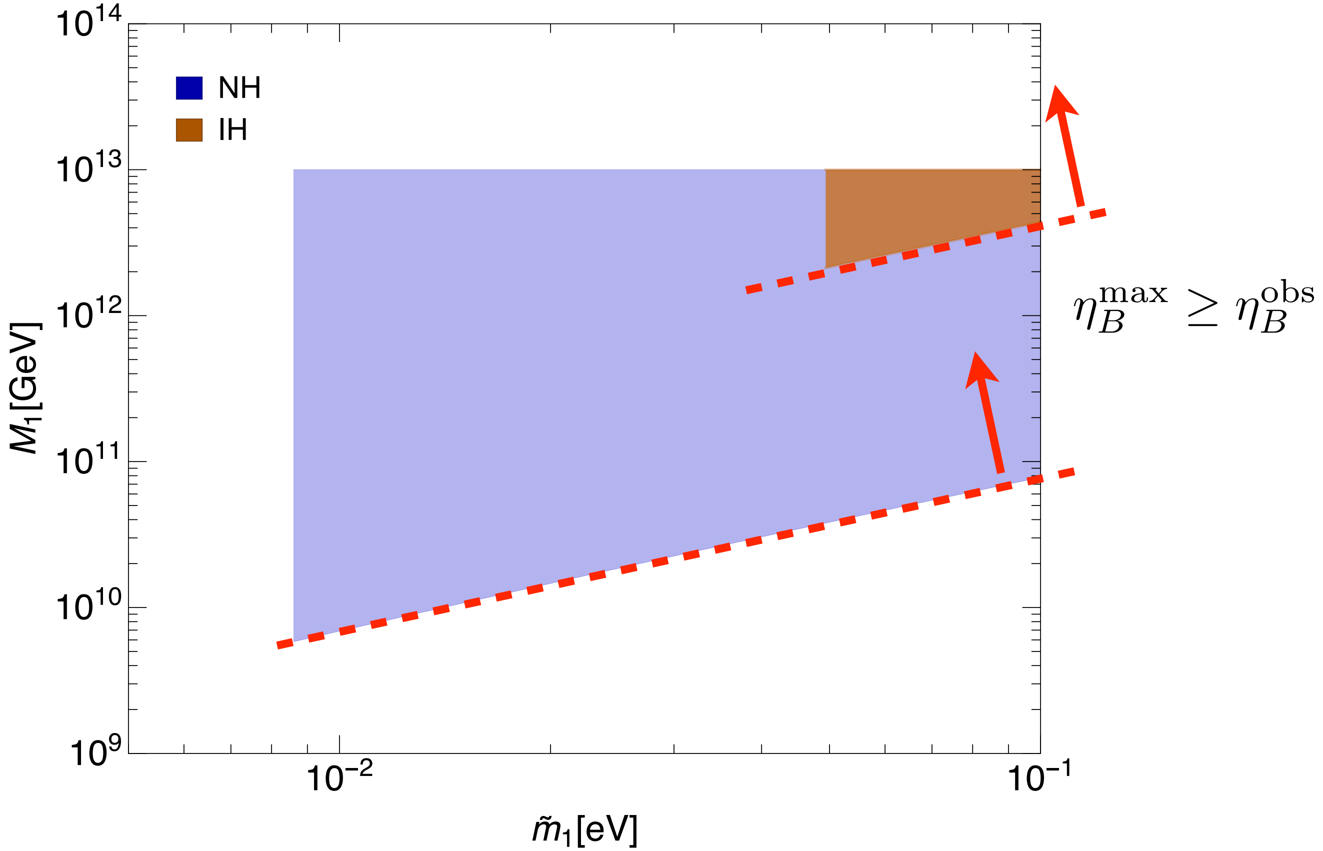
$$\tilde{m}_1 \geq \begin{cases} m_2 \simeq 8.6 \times 10^{-3}\,\mathrm{eV} & (\mathrm{NH}) \\ m_1 \simeq 4.9 \times 10^{-2}\,\mathrm{eV} & (\mathrm{IH}) \end{cases}$$



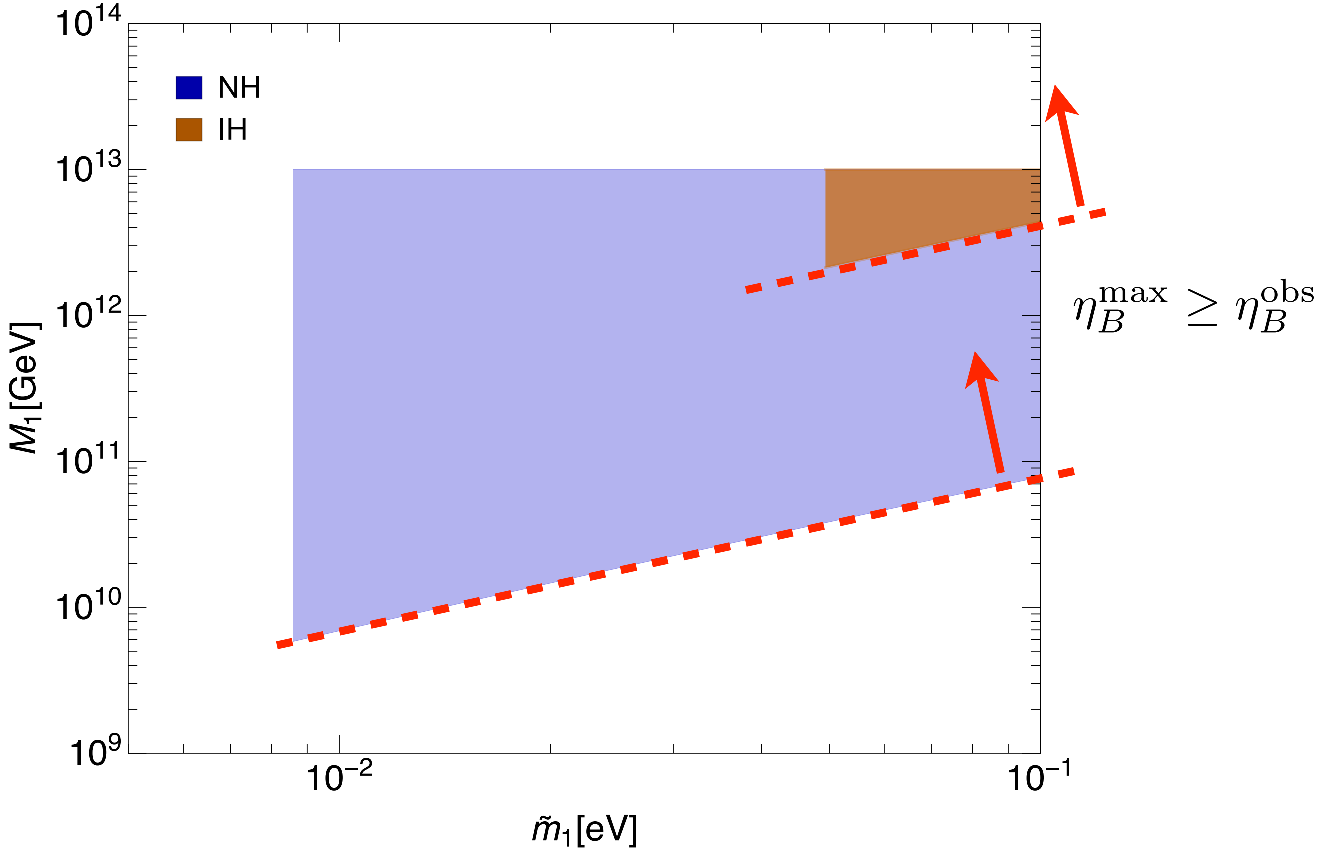
Leptogenesis: case (I)



Leptogenesis: case (I)



Leptogenesis: case (I)



Successful leptogenesis in wide parameter space

Leptogenesis: case (II) $M_1,\, M_2 > m_\phi$

Inflaton decay produces baryon number
(similar to sneutrino leptogenesis)

see, e.g.,

Murayama, Suzuki, Yanagida, Yokoyama '93
Hamaguchi, Murayama, Yanagida '02
Ellis, Raidal, Yanagida '04
Nakayama, Takahashi, Yanagida '16

→ Baryon number:

$$\eta_B = \frac{3}{4}\frac{T_R}{m_\phi} a_{\rm sph} d \epsilon_\phi$$

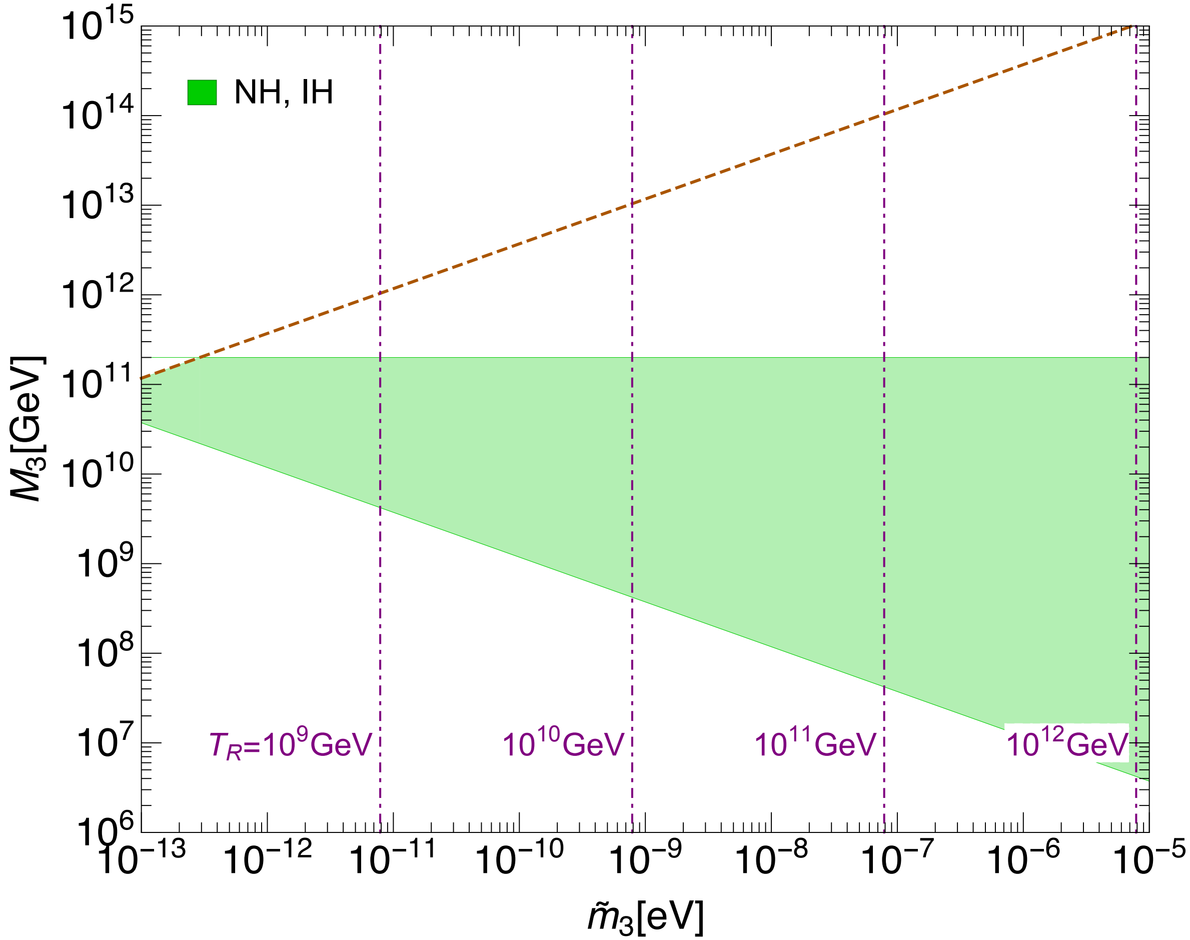
(Independent of m_ϕ)

$$a_{\rm sph} = 28/79$$

$$d = (s/n_\gamma)_0$$

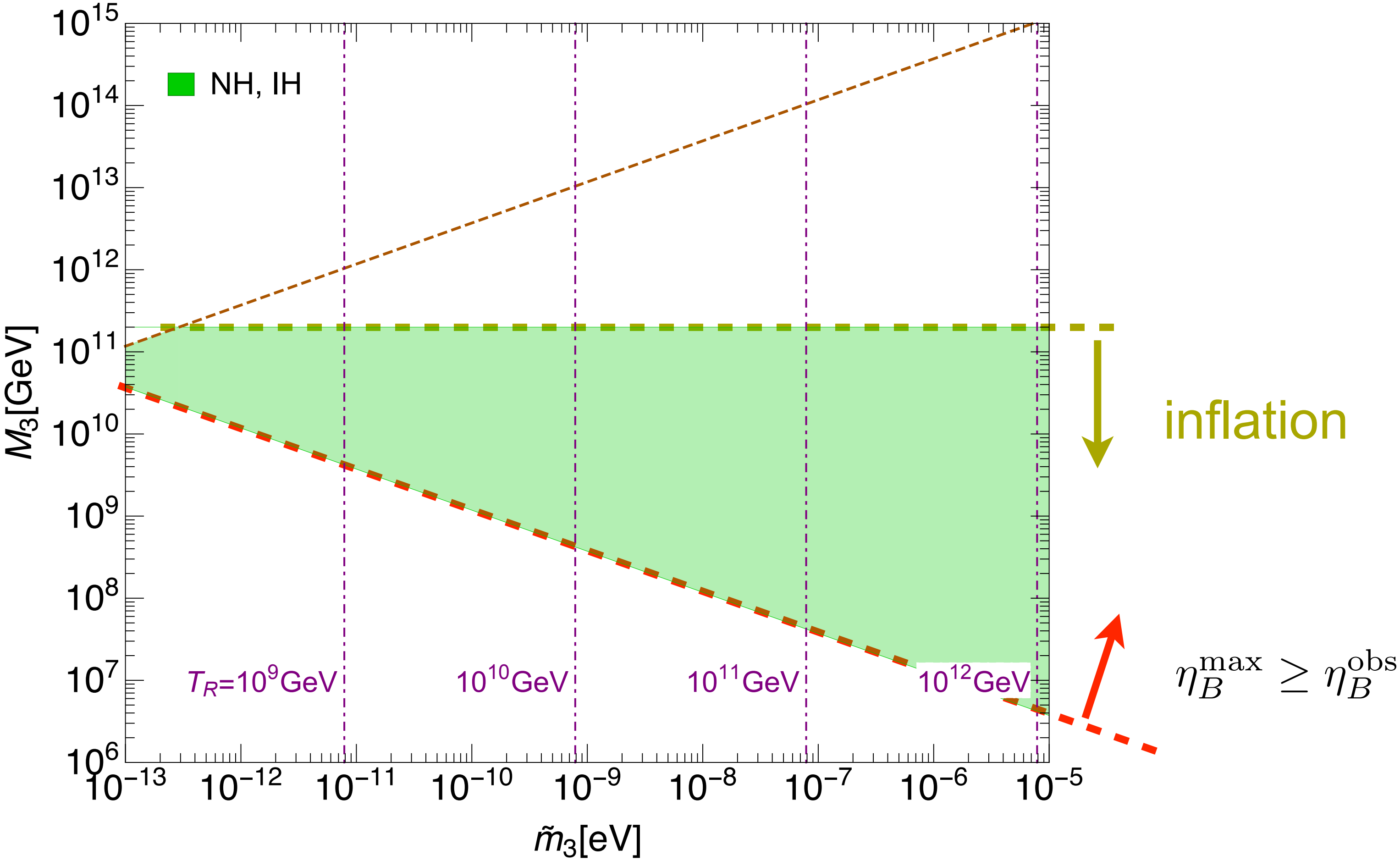
- ϵ_ϕ : Asymmetric parameter

Leptogenesis: case (II)



$$\tilde{m}_3 \equiv \frac{(m_\nu m_\nu^\dagger)_{33}}{m_\phi}$$

Leptogenesis: case (II)



Successful leptogenesis in both the NH and IH cases

4. Conclusions

We have studied phenomenology of superconformal subcritical hybrid inflation:

- Three right-handed (s)neutrinos are introduced, one of which becomes inflaton
- Predicted cosmological parameters fit with the CMB data
- Light neutrino mass matrix is given by the seesaw mechanism, but it has an unconventional structure
- (sneutrino) Inflaton decay reheats the universe, and T_R is not constrained by the neutrino data
- Thermal or sneutrino leptogenesis can be realized

Backups

Canonical superconformal supergravity models

Canonical superconformal supergravity models

Ferrara, Kallosh, Linde, Marrani, Van Proeyen '10, '11

- Lagrangian in Jordan frame becomes very simple form

$$\frac{1}{\sqrt{-g}}\mathcal{L}^{\text{scalar-grav}}_{\text{s.c.}} = -\frac{1}{6}\mathcal{N}(X,\bar{X})R - \eta_{I\bar{J}}D^\mu X^I D_\mu \bar{X}^{\bar{J}} - V_{\text{s.c.}}$$

with

$$V_{\text{s.c.}} = \eta^{I\bar{J}}\mathcal{W}_I\bar{\mathcal{W}}_{\bar{J}} + \frac{1}{2}(\text{Re}f)^{-1AB}\mathcal{P}_A\mathcal{P}_B$$

$$\eta_{0\bar{0}} = -1,\ \eta_{\alpha\bar{\beta}} = \delta_{\alpha\bar{\beta}}$$

X^0 : scalar compensator

X^α : matter scalar fields

Canonical superconformal supergravity models

Ferrara, Kallosh, Linde, Marrani, Van Proeyen '10, '11

- Lagrangian in Jordan frame becomes very simple form

$$\frac{1}{\sqrt{-g}}\mathcal{L}^{\text{scalar-grav}}_{\text{s.c.}} = -\frac{1}{6}\mathcal{N}(X,\bar{X})R - \boxed{\eta_{I\bar{J}}D^{\mu}X^{I}D_{\mu}\bar{X}^{\bar{J}}} - V_{\text{s.c.}}$$

canonical kinetic term

with

$$V_{\text{s.c.}} = \boxed{\eta^{I\bar{J}}\mathcal{W}_I\bar{\mathcal{W}}_{\bar{J}}} + \frac{1}{2}(\text{Re}f)^{-1AB}\mathcal{P}_A\mathcal{P}_B$$

simple F term

$$\boxed{\eta_{0\bar{0}} = -1,\ \eta_{\alpha\bar{\beta}} = \delta_{\alpha\bar{\beta}}}$$

X^0 : scalar compensator

X^α : matter scalar fields

Canonical superconformal supergravity models

Ferrara, Kallosh, Linde, Marrani, Van Proeyen '10, '11

- Compensator fields are decoupled from matter sector

$$\frac{1}{\sqrt{-g_J}}\mathcal{L}^0_{\rm sugra} = \frac{M_{pl}^2}{2}(R_J + 6A_\mu A^\mu)$$ ⟵ compensator part

$$\frac{1}{\sqrt{-g_J}}\mathcal{L}^{\rm m}_{\rm s.c.} = -\frac{1}{6}|X^\alpha|^2 R_J - \delta_{\alpha\bar\beta}D_\mu X^\alpha D^\mu \bar X^{\bar\beta} - V_J$$ ⟵ matter part

with $V_J = \delta^{\alpha\bar\beta}\mathcal{W}_\alpha \bar{\mathcal{W}}_{\bar\beta} + \frac{1}{2}({\rm Re} f)^{-1AB}\mathcal{P}_A\mathcal{P}_B$

Canonical superconformal supergravity models

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compensator part

$$\boxed{\begin{aligned}&\frac{1}{\sqrt{-g_J}}\mathcal{L}^{\rm m}_{\rm s.c.} = -\frac{1}{6}|X^\alpha|^2 R_J - \delta_{\alpha\bar\beta} D_\mu X^\alpha D^\mu \bar X^{\bar\beta} - V_J \\ &\text{with} \quad V_J = \delta^{\alpha\bar\beta}\mathcal{W}_\alpha \bar{\mathcal{W}}_{\bar\beta} + \frac{1}{2}({\rm Re}f)^{-1AB}\mathcal{P}_A\mathcal{P}_B\end{aligned}}$$

matter part

superconformal

Canonical superconformal supergravity models

Ferrara, Kallosh, Linde, Marrani, Van Proeyen '10, '11

Jordan frame $\longrightarrow$ Einstein frame

$$g_{J\mu\nu} = \Omega^2 g_{E\mu\nu} \quad \text{with} \quad \Omega^{-2} = 1 - \frac{|X^\alpha|^2}{3M_{pl}^2}$$

$$\frac{1}{\sqrt{-g_E}}\mathcal{L}^E_{\text{sugra}} = \frac{M_{pl}^2}{2}R_E - K_{\alpha\bar{\beta}}D_\mu X^\alpha D^\mu \bar{X}^{\bar{\beta}} - V_E$$

$$K_{\alpha\bar{\beta}} = \frac{\partial^2 K}{\partial X^\alpha \partial \bar{X}^{\bar{\beta}}}$$

with

$$K = -3M_{pl}^2 \log \Omega^{-2}$$

$$V_E = \Omega^4 V_J$$

Canonical superconformal supergravity models

Ferrara, Kallosh, Linde, Marrani, Van Proeyen '10, '11

Jordan frame ⟶ Einstein frame

$$\boxed{g_{J\mu\nu} = \Omega^2 g_{E\mu\nu} \quad \text{with} \quad \Omega^{-2} = 1 - \frac{|X^\alpha|^2}{3M_{pl}^2}}$$

kinetic term is not canonical

$$\frac{1}{\sqrt{-g_E}}\mathcal{L}^E_{\rm sugra} = \frac{M_{pl}^2}{2}R_E - \boxed{K_{\alpha\bar\beta}D_\mu X^\alpha D^\mu \bar X^{\bar\beta}} - V_E$$

$$K_{\alpha\bar\beta} = \frac{\partial^2 K}{\partial X^\alpha \partial \bar X^{\bar\beta}}$$

$$\boxed{K = -3M_{pl}^2 \log \Omega^{-2}}$$

with

"no-scale" Kähler potential

Canonical superconformal supergravity models

Ferrara, Kallosh, Linde, Marrani, Van Proeyen '10, '11

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$$g_{J\mu\nu} = \Omega^2 g_{E\mu\nu} \quad \text{with} \quad \Omega^{-2} = 1 - \frac{|X^\alpha|^2}{3M_{pl}^2}$$

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scalar potential has a simple form

Canonical superconformal supergravity models

Ferrara, Kallosh, Linde, Marrani, Van Proeyen '10, '11

Einstein frame

- No-scale Kähler potential $$K = -3M_{pl}^2 \log \Omega^{-2}$$
- Simple scalar potential $$V_E = \Omega^4 V_J$$
- Matter part is superconformal $$\Omega^{-2} = 1 - \frac{|X^\alpha|^2}{3M_{pl}^2}$$

Canonical superconformal supergravity models

Ferrara, Kallosh, Linde, Marrani, Van Proeyen '10, '11

Einstein frame

- No-scale Kähler potential $K = -3M_{pl}^2 \log \Omega^{-2}$
- Simple scalar potential $V_E = \Omega^4 V_J$ ← Model
- Matter part is superconformal $\Omega^{-2} = 1 - \dfrac{|X^\alpha|^2}{3M_{pl}^2}$

Canonical superconformal supergravity models

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Einstein frame

- No-scale Kähler potential $$K = -3M_{pl}^2 \log \Omega^{-2}$$
- Simple scalar potential $$V_E = \Omega^4 V_J$$ ← Model
- Matter part is superconformal $$\Omega^{-2} = 1 - \frac{|X^\alpha|^2}{3M_{pl}^2} + \ldots$$

~~Superconformal~~ term

(since superconformality is broken)

Potential in canonically-normalized inflation field

To understand the behavior,
let's derive the potential in canonically-normalized inflaton field $\hat{\phi}$

$$\frac{d\phi}{d\hat{\phi}} = K_{\Phi\bar{\Phi}}^{-1/2} \simeq \sqrt{1 - \frac{1}{6}(1+\chi)\phi^2}$$

$$\longrightarrow \quad \phi = \frac{1}{\sqrt{\beta}} \sinh \sqrt{\beta}\hat{\phi}$$

$$\beta = \frac{\lambda^2}{2qg^2\xi}\delta\chi$$

$$\longrightarrow \quad V \simeq g^2\xi^2\delta\chi^{-1} \tanh^2 \sqrt{\beta}\hat{\phi}\left[1 - \frac{\delta\chi^{-1}}{2}\tanh^2 \sqrt{\beta}\hat{\phi}\right]$$

- $\delta\chi \to 0$

$$V \simeq g^2\xi^2 \frac{\hat{\phi}^2}{\phi_c^2}\left(1 - \frac{\hat{\phi}^2}{2\phi_c^2}\right)$$

Buchmüller, Domcke, Schmitz '14
Buchmüller, KI '15

⟶ Same potential in shift sym. Kähler: $K = \frac{1}{2}(\Phi + \bar{\Phi})^2 + |S_+|^2 + |S_-|^2$

- $\delta\chi \to 1$

$$V \simeq \frac{1}{2}g^2\xi^2\left[1 - 16e^{-4\sqrt{\beta}\hat{\phi}}\right]$$ (for large $\hat{\phi}$)

⟶ Same asymptotic form with Starobinsky model or R^2 inflation or simplest class of superconformal α attractor model

Whitt '84
Kallosh, Linde '13
Kallosh, Linde, Roest '13

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Buchmüller, Domcke, Schmitz '14
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$$V \simeq \frac{1}{2} g^2\xi^2 \left[1 - 16 e^{-4\sqrt{\beta}\hat{\phi}}\right]$$ (for large $\hat{\phi}$)

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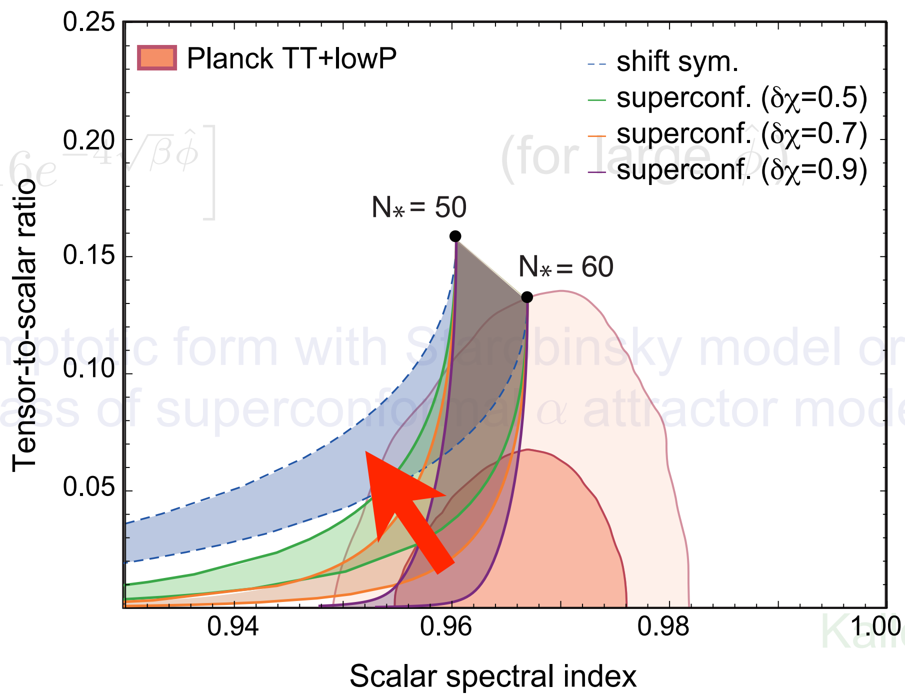
Whitt '84
Kallosh, Linde '13
Kallosh, Linde, Roest '13



- $\delta\chi \to 0$

$$V \simeq g^2\xi^2 \frac{\hat{\phi}^2}{\phi_c^2}\left(1 - \frac{\hat{\phi}^2}{2\phi_c^2}\right)$$

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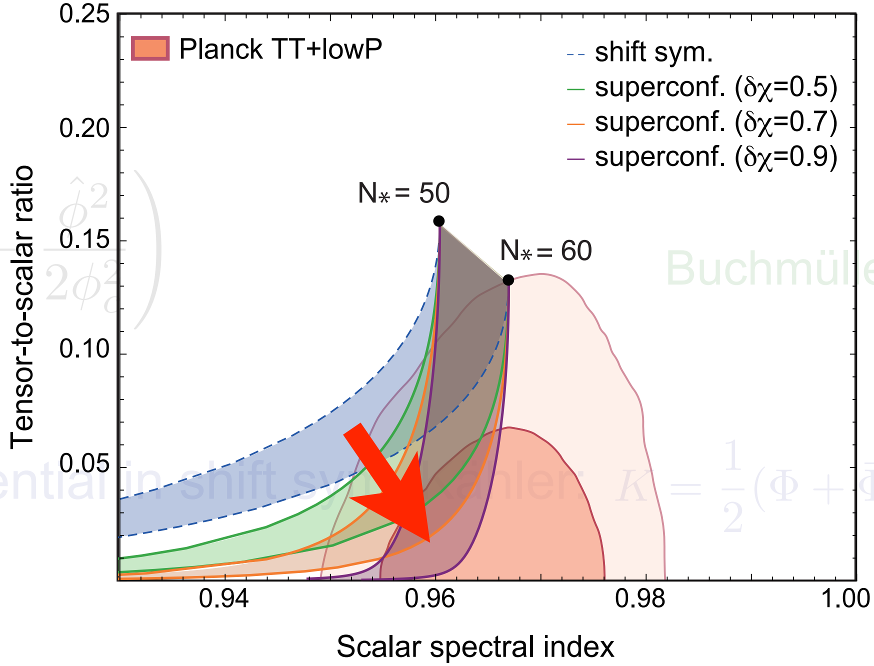

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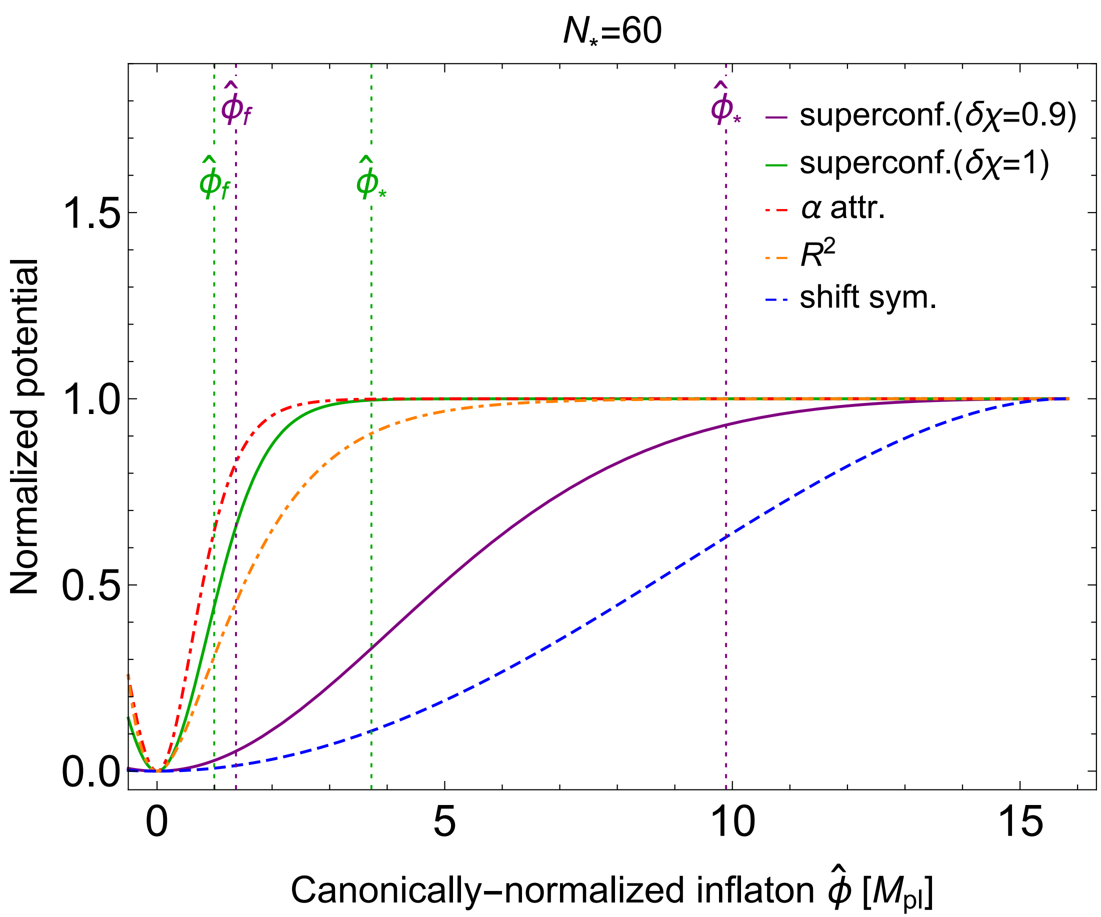
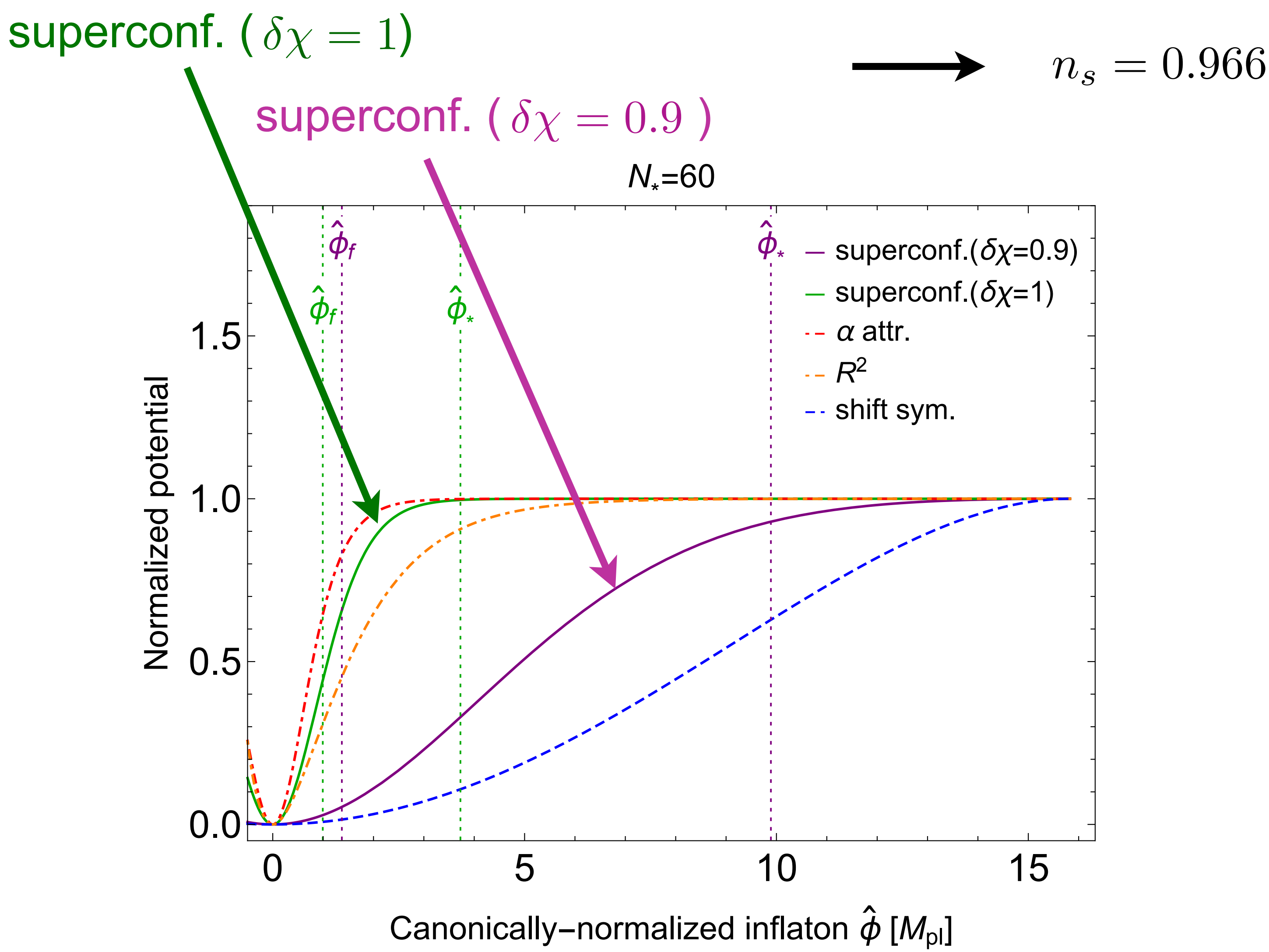
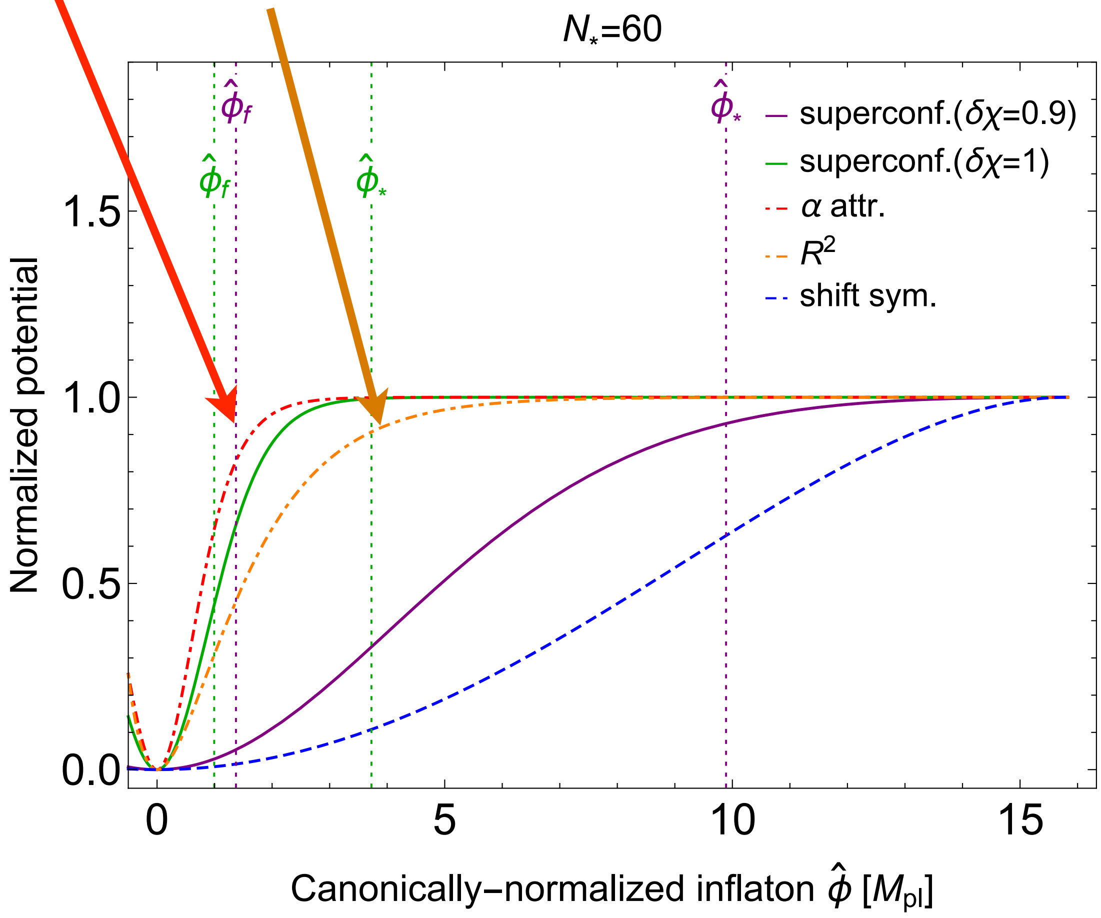

N_*=60
$\hat{\phi}_f$
$\hat{\phi}_*$
$\hat{\phi}_f$
$\hat{\phi}_*$
superconf.($\delta\chi$=0.9)
superconf.($\delta\chi$=1)
α attr.
R^2
shift sym.
Normalized potential
0.0
0.5
1.0
1.5
0
5
10
15
Canonically–normalized inflaton $\hat{\phi}$ [$M_{\rm pl}$]



superconf. ($\delta\chi = 1$)
$n_s = 0.966$
superconf. ($\delta\chi = 0.9$)
N_*=60
$\hat{\phi}_f$
$\hat{\phi}_*$
superconf.($\delta\chi$=0.9)
superconf.($\delta\chi$=1)
α attr.
R^2
shift sym.
Normalized potential
Canonically–normalized inflaton $\hat{\phi}$ [M_{pl}]

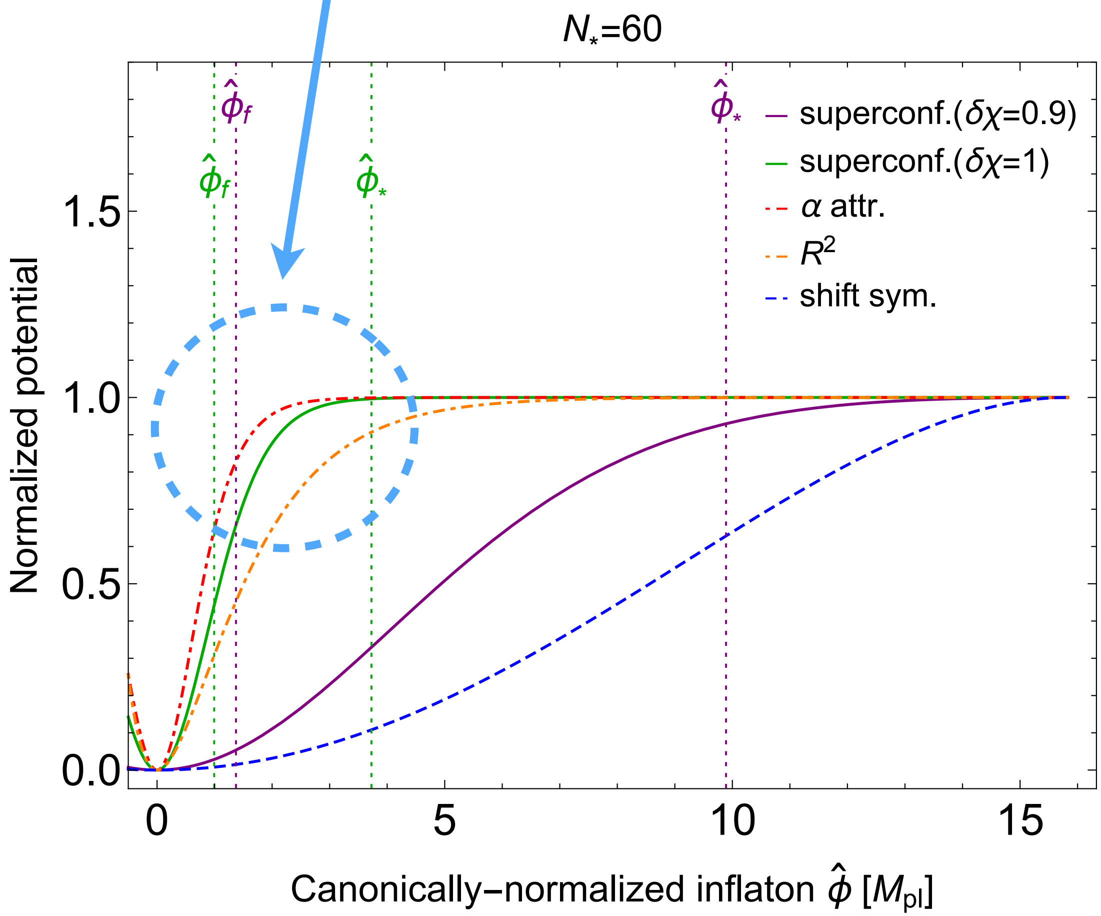
α attractor

R^2 the same asymptotic form as $\delta\chi = 1$



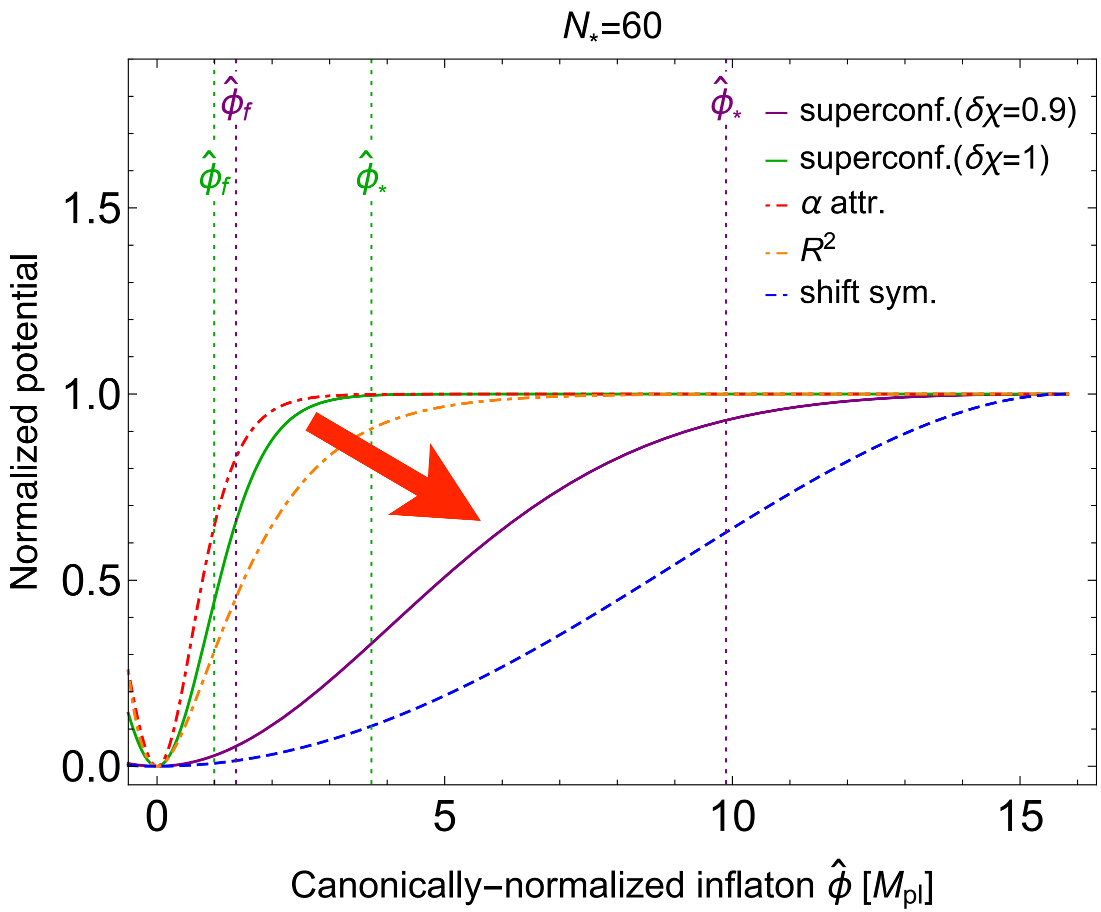
$\delta\chi = 1$ behaves similar to α attractors and R^2 model

But not exactly the same



$r = 0.00052$ for superconf. ($\delta\chi = 1$)

$r = 0.00044$ for α attractor , R^2



The deviation from the α attractors and R^2 model gets larger even for $\delta\chi = 0.9$

$\longrightarrow \quad r = 0.051$

Consistency in the parameters

Consistency in the parameters

- A shift symmetry for $\chi = -1$
- $\phi_c^2 > 0$

$$K = -3 \log \Omega^{-2}$$

$$\Omega^{-2} \simeq 1 - \frac{1}{6}(1+\chi)\phi^2$$

$$W = \lambda \Phi S_+ S_-$$

$$\chi = -1 - \frac{3\lambda^2}{\xi}\delta\chi \qquad (0 < \delta\chi < 1)$$

$$\phi_c^2 > 0$$

$\chi \simeq -1$ (ϕ as inflaton) $\longrightarrow$ $\lambda \ll 1$ (expectation)

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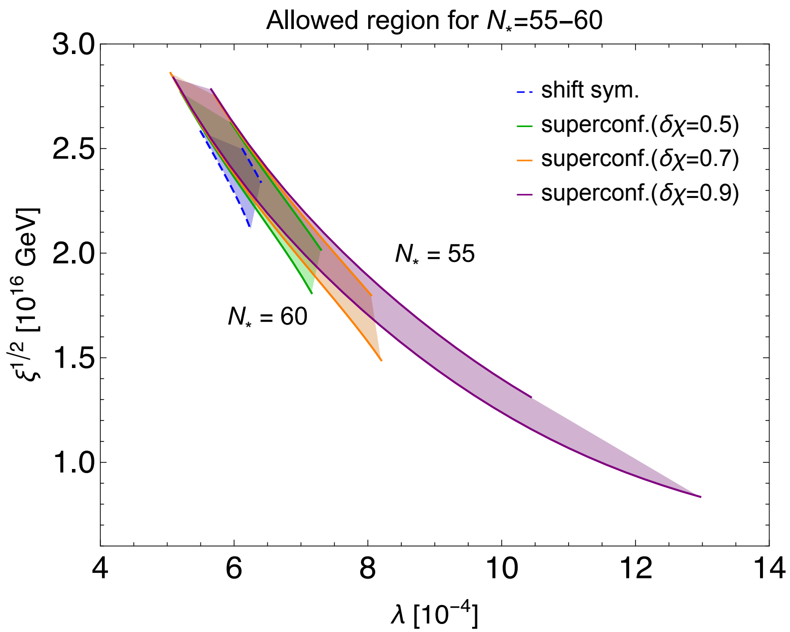
$\longleftarrow$ $\lambda^2/\xi \ll 1$

Targeted parameter space



$$\begin{cases} \lambda \sim 10^{-4}\text{–}10^{-3} \\ \sqrt{\xi} \sim 10^{16}\,\mathrm{GeV} \end{cases}$$

$$(\lambda \equiv \lambda_3\,, \chi \equiv \chi_3)$$

$$\longrightarrow \quad m_\phi \simeq \lambda\sqrt{\xi} \sim 10^{13}\,\mathrm{GeV}$$

Inflaton mass

$\phi \equiv \mathrm{Re} N_3^c$: Inflaton

Leptogenesis: case (I)

Casas, Ibarra '01
Davidson, Ibarra '02

- Asymmetric parameter

$$\epsilon_1 \simeq \begin{Bmatrix} 8.2 \times 10^{-7} & (\mathrm{NH}) \\ 1.5 \times 10^{-8} & (\mathrm{IH}) \end{Bmatrix} \times \left(\frac{M_1}{10^{10}\,\mathrm{GeV}}\right)\left(\frac{\sin\delta}{0.5}\right)$$

δ : CP phase

($M_1 \ll M_2$ for simplicity)

- Efficiency factor

$$\tilde{m}_1 \geq \begin{cases} m_2 \simeq 8.6 \times 10^{-3}\,\mathrm{eV} & (\mathrm{NH}) \\ m_1 \simeq 4.9 \times 10^{-2}\,\mathrm{eV} & (\mathrm{IH}) \end{cases}$$

$$\tilde{m}_1 \equiv \frac{(m_\nu m_\nu^\dagger)_{11}}{M_1}$$

$$> m_*$$

m_* : equilibrium neutrino mass

⟶ Strong washout regime

⟶ $$\kappa_f = (2 \pm 1) \times 10^{-2} \left(\frac{0.01\ \mathrm{eV}}{\tilde{m}_1}\right)^{1.1 \pm 0.1}$$

Buchmüller, Di Bari, Plümacher '04

Leptogenesis: case (II)

- T_R/m_ϕ

$$\tilde{m}_3 \equiv \frac{(m_\nu m_\nu^\dagger)_{33}}{m_\phi} \simeq 1.5 \times 10^{-9}\,\mathrm{eV}\left(\frac{(y_\nu y_\nu^\dagger)_{33}}{10^{-9}}\right)$$

$$\longrightarrow \quad T_R/m_\phi \simeq 0.71 \times \left(\frac{\tilde{m}_3}{m_*}\right)^{1/2}$$

- Asymmetric parameter

$$\epsilon_\phi \simeq 3.9 \times 10^{-9} \times \left(\frac{M_3}{10^7\,\mathrm{GeV}}\right)\left(\frac{\sin\delta'}{0.5}\right)$$

Both quantities are independent of m_ϕ