Duality and Weak Gravity

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refs: 2004.13732 w/S. Andriolo, T-C. Huang, H. Ooguri G. Shiu
1909.01352, 2006.06696 w/ G. Loges, G. Shiu
1810.13637 w/Y. Hamada, G. Shiu

July 20th 2020 @ Osaka
Swampland:
apparently consistent, but not UV completeable when coupled to gravity

Weak Gravity Conjecture (WGC):
conjectured condition defining boundary of landscape and swampland

Landscape:
QFT models consistent w/quantum gravity
main results toward a proof of WGC:

1. positivity bounds imply WGC in many theories
2. but it is not the case once dilaton is turned on
   → duality symmetries are useful for WGC
plan

1. Introduction
2. WGC vs. positivity bounds
3. Role of duality symmetries
4. Summary and prospects
plan

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Swampland Program [Vafa ’05, Ooguri-Vafa ’06]

goal: identify consistency conditions
for a QFT model to be embedded into quantum gravity!

1. better understanding of quantum gravity and string theory
   - which stringy ingredients are crucial for quantum gravity?

2. toward phenomenological tests of quantum gravity
   - test swampland conditions via particle phys. & cosmology

various swampland conditions motivated by string compactification:

no global symmetry, weak gravity conjecture, distance conjecture, …
main question in this talk:

- string theory accommodates a rich structure (perhaps too complete?): consistent amplitudes, ∞ gauge symmetries, dualities, holography, ...

- which ingredients are necessary for each swampland condition (if true)?

- specific to string theory or more robust in quantum gravity?

string theory = insurance w/full options
such a direction is better explored recently in the context of Weak Gravity Conjecture
Weak Gravity Conjecture
[Arkani-Hamed et al ’06]

# claim: gravity is the weakest force [see next slide for motivation]

# in graviton-photon system,

\[ \exists \text{ a charged state } w/ \ g^2 q^2 \geq \frac{m^2}{2M_{Pl}^2} \] (gauge force \( \geq \) gravity)

- never be satisfied if we decouple photon \( g \rightarrow 0 \)
  
  \rightarrow \text{generalization of “no global symmetry in quantum gravity”}

- trivially be satisfied if we decouple gravity \( M_{Pl} \rightarrow \infty \)
  
  \rightarrow \text{special in quantum gravity}
Motivation from string compactification

ex. heterotic string compactified on tori w/generic Wilson lines

existence of states w/ $M \leq Q$ is common in string theory

[ArkaniHamed-Motl-Nicolis-Vafa 06’, … ]
How generic this picture is?

- this asymptotic behavior (of BHs) follows from positivity bounds in graviton-photon systems [Hamada-TN-Shiu ’18]
  - existence proof of (mild) WGC

- if UV theory has a worldsheet structure, spectral flow may relate the two regions [Heidenreich et al ’16, Alasma et al ’19]

- combination of two observations
  - suggests a stronger condition called sublattice/tower WGC [Heidenreich et al ’16, Andriolo-Junghanns-TN-Shiu ’18]
Positivity bounds are not enough??

Recently, we collected more data on WGC vs. positivity bounds beyond graviton-photon systems

1. positivity bounds imply WGC
   in graviton-photon systems and graviton-axion systems

2. but it is not the case once dilaton is turned on:
   in these theories duality symmetries are useful for WGC

In the rest of my talk, I will explain details for axionic WGC
(which is technically simpler than the Maxwell case)
plan

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axionic WGC vs. Euclidean wormholes

[Andriolo-Huang-TN-Ooguri-Shiu ’20]
### axionic WGC

<table>
<thead>
<tr>
<th>form field</th>
<th>charged state</th>
<th>gravitational objects</th>
<th>coupling</th>
<th>size</th>
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<tr>
<td>photon</td>
<td>particle</td>
<td>charged BH</td>
<td>$qg$</td>
<td>mass</td>
</tr>
<tr>
<td>axion</td>
<td>instanton</td>
<td>Euclidean (semi)wormhole</td>
<td>$\frac{n}{f}$</td>
<td>action</td>
</tr>
</tbody>
</table>

(size) < (coupling) implies $\exists$ an instanton w/ $S < \mathcal{O}(1) \cdot \frac{|n| M_{\text{Pl}}}{f}$

cf. instanton generates axion potential

→ implications to axion cosmology (inflation, DM)

[see, e.g., Hebecker-Mikhali-Soler ’18 for a review ]
Giddings-Strominger wormhole

Euclidean wormhole can be regarded as an instanton anti-instanton pair

Euclidean (semi)wormhole in Einstein-axion theory:

\[ ds^2 = \frac{dr^2}{1 - (r_0/r)^4} + r^2 d\Omega_3^2, \quad r_0^4 = \frac{n^2 f^2}{24\pi^4 M_{Pl}^6} \]  
(n : axion charge, f : decay const.)

※ each semiwormhole (instanton) has an action \( S = |n| \frac{\sqrt{6\pi}}{4} \cdot \frac{M_{Pl}}{f} \)

※ this fixes the \( \mathcal{O}(1) \) constant in the WGC bound: \( S \leq \frac{\sqrt{6\pi}}{4} \cdot \frac{|n| M_{Pl}}{f} \)
higher derivative corrections

\[ S = |n| \frac{\sqrt{6\pi}}{4} \cdot \frac{M_{\text{Pl}}}{f} \]

\[ \Delta S = -24\pi^2 M_{\text{Pl}}^4 \alpha + \mathcal{O}(1/n) \]

# graviton-axion EFT up to four-derivatives

\[ S = \int d^4x \sqrt{-g} \left[ \frac{M_{\text{Pl}}^2}{2} R - \frac{1}{2} \partial_\mu a \partial^\mu a + \alpha (\partial_\mu a \partial^\mu a)^2 + \beta_1 W_{\mu\nu\rho\sigma}^2 + \beta_2 a W_{\mu\nu\rho\sigma} \tilde{W}^{\mu\nu\rho\sigma} \right] + \text{appropriate boundary terms} \]

\[ \text{※ modify wormhole solutions and so their action: } \Delta S = -24\pi^2 M_{\text{Pl}}^4 \alpha + \mathcal{O}(1/n) \]
if the $\alpha$ operator has a positive coefficient $\alpha > 0$, macroscopic (semi)wormholes satisfy the WGC bound. Indeed, $\alpha > 0$ follows from analyticity, unitarity and locality of UV scattering amplitudes (positivity bounds) [Adams et al ’06] → an existence proof of (the mild form of) WGC

caveat: applicable only when gravitational Regge states are negligible [see Hamada-TN-Shiu ’18 for details]
plan

1. Introduction ✔
2. WGC vs. positivity bounds ✔
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Generalization to graviton-axion-dilaton system

[Andriolo-Huang-TN-Ooguri-Shiu ’20]
graviton-axon-dilaton EFT

# Einstein-axon-dilaton action

\[ S = \int d^4x \sqrt{-g} \left[ \frac{M_{Pl}^2}{2} R - \frac{1}{2} e^{\lambda \phi} \partial_\mu a \partial^\mu a - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi \right] \]

- we focus on \(|\lambda| < 4/\sqrt{6}\), otherwise no regular wormholes

# four-derivative terms relevant to our problem

\[ \Delta \mathcal{L} = \alpha_1 e^{2\lambda \phi} (\partial_\mu a \partial^\mu a)^2 + \alpha_2 (\partial_\mu \phi \partial^\mu \phi) \]

\[ + \alpha_3 e^{\lambda \phi} (\partial_\mu a \partial^\mu a)(\partial_\nu \phi \partial^\nu \phi) + \alpha_4 e^{\lambda \phi} (\partial_\mu a \partial^\mu \phi)^2 \]

- suppressed terms w/Weyl tensor, which do not correct the action

- also see our paper for more general dilaton couplings
Corrections to (semi)wormhole action

# four-derivative corrections to the (semi)wormhole action

\[ \Delta S = 36\pi^2 M_{Pl}^4 \int_0^{\pi/2} dt \cos^3 t \left[ -\alpha_1 \sec^4 \left( \frac{\sqrt{6}}{4} \lambda \cdot t \right) - \alpha_2 \tan^4 \left( \frac{\sqrt{6}}{4} \lambda \cdot t \right) \right. \\
\left. + \left( \alpha_3 + \alpha_4 \right) \sec^2 \left( \frac{\sqrt{6}}{4} \lambda \cdot t \right) \tan^2 \left( \frac{\sqrt{6}}{4} \lambda \cdot t \right) \right] + \mathcal{O}(1/n) \]

- the condition for $\Delta S < 0$ and so WGC reads

\[ \alpha_3 + \alpha_4 < A_1(\lambda) \alpha_1 + A_2(\lambda) \alpha_2 \quad (A_{1,2} : \lambda \text{-dep. positive coefficients}) \]
Implications of positivity bounds

# positivity of $aa \rightarrow aa$, $\phi\phi \rightarrow \phi\phi$, $a\phi \rightarrow a\phi$: $\alpha_1, \alpha_2, \alpha_4 > 0$

# scattering of superpositions of $a, \phi$:

2-para. family of bounds $\rightarrow$ its envelop gives $-\alpha_4 - 2\sqrt{\alpha_1\alpha_2} < \alpha_3 < 2\sqrt{\alpha_1\alpha_2}$

$\rightarrow$ large positive $\alpha_4$ violates WGC bound $\alpha_3 + \alpha_4 < A_1\alpha_1 + A_2\alpha_2$

# projection onto $\alpha_2 = \alpha_1$ plane for illustration

$\alpha_3 + \alpha_4 = (A_1 + A_2) \alpha_1 > 2\alpha_1$

prohibited by positivity

allowed by positivity, but WGC is not satisfied

satisfy positivity & WGC
positivity is not enough to demonstrate WGC
Q. any additional UV input which implies WGC?
$SL(2, R)$ duality symmetry of axion & dilaton

is an example for such UV information!
Implications of duality constraints

# $SL(2,R)$ transformation in our convention:

$$\tau \to \frac{a\tau + b}{c\tau + d} \quad (a, b, c, d \in R, \ ad - bc = 1) \quad \text{w/ } \tau = \frac{\lambda}{2}a + ie^{-\frac{i}{2}\phi}$$

only two $SL(2, R)$ invariant operators:

$$\frac{(\partial_\tau \partial^\mu \bar{\tau})^2}{(\text{Im } \tau)^4}, \quad \frac{|\partial_\tau \partial^\mu \tau|^2}{(\text{Im } \tau)^4}$$

in our language it means $\alpha_2 = \alpha_1, \ \alpha_3 + \alpha_4 = 2\alpha_1$

# under these conditions, we have $\Delta S = -24\pi^2 M_{\text{Pl}}^4 \alpha_1$

\rightarrow positivity $\alpha_1 > 0$ implies $\Delta S < 0$ and so WGC!

we have found that positivity alone is not enough, but positivity + $SL(2,R)$ duality invariance does imply WGC
Implications of duality constraints

# \( SL(2,R) \) transformation in our convention:

\[
\tau \rightarrow \frac{a\tau + b}{c\tau + d} \quad (a, b, c, d \in R, \ ad - bc = 1) \quad \text{w/} \ \tau = \frac{\lambda}{2}a + ie^{-\frac{i}{2}\phi}
\]

only two \( SL(2, R) \) invariant operators:

\[
\frac{(\partial_\mu \tau \partial^\mu \tau)^2}{(\text{Im} \tau)^4}, \quad \frac{|\partial_\mu \tau \partial^\mu \tau|^2}{(\text{Im} \tau)^4}
\]

in our language it means \( \alpha_2 = \alpha_1, \alpha_3 + \alpha_4 = 2\alpha_1 \)

\[\begin{align*}
\alpha_2 &= \alpha_1 \\
\alpha_4 &= 
\end{align*}\]

allowed by positivity, but WGC is not satisfied

\[\begin{align*}
\alpha_3 + \alpha_4 &= (A_1 + A_2) \alpha_1 > 2\alpha_1 \\
\text{SL}(2,R) \text{ invariant:} \quad \alpha_3 + \alpha_4 &= 2\alpha_1
\end{align*}\]
Related results for BHs

WGC in Einstein-Maxwell-dilaton-axion:

[Loges-TN-Shiu ’19]

there exists a parameter space

which is allowed by positivity, but does not satisfy WGC

[Loges-TN-Shiu ’20]

duality symmetries are again useful to demonstrate WGC

1. positivity + SL(2,R) → WGC

2. null energy condition + O(d,d;R) → WGC

※ positivity bounds are not applicable for O(d,d;R) case

because gravitational Regge states are not negligible
4. summary and prospects
summary and prospects

positivity implies WGC in graviton-photon and graviton-axion systems under the assumption that gravitational Regge states are negligible
※ can we incorporate gravitational Regge states in positivity?

but it is not the case once dilaton is turned on:
duality symmetries such as SL(2,R) and O(d,d;R) are useful for WGC
※ are there other UV inputs useful for demonstrating WGC?

we provided evidences for axionic WGC, which constrains axion potential
※ can we generalize our results to potential of other moduli fields?
    (cf. scalar WGC, non-SUSY AdS, dS, ...)
Thank you!