Duality and Weak Gravity

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refs: 2004.13732 w/S. Andriolo, T-C. Huang, H. Ooguri G. Shiu

1909.01352, 2006.06696 w/ G. Loges, G. Shiu

1810.13637 w/Y. Hamada, G. Shiu

July 20th 2020 @ Osaka

swampland：
apparently consistent, but not UV completable when coupled to gravity

Weak Gravity Conjecture (WGC):
conjectured condition defining boundary of landscape and swampland

landscape：
QFT models consistent w/quantum gravity

main results toward a proof of WGC:

1. positivity bounds imply WGC in many theories

2. but it is not the case once dilaton is turned on

$\rightarrow$ duality symmetries are useful for WGC

plan

1. Introduction

2. WGC vs. positivity bounds

3. Role of duality symmetries

4. Summary and prospects

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Swampland Program [Vafa '05, Ooguri-Vafa '06]

goal: identify consistency conditions

for a QFT model to be embedded into quantum gravity!

1. better understanding of quantum gravity and string theory

 - which stringy ingredients are crucial for quantum gravity?

2. toward phenomenological tests of quantum gravity

 - test swampland conditions via particle phys. & cosmology

various swampland conditions motivated by string compactification:

no global symmetry, weak gravity conjecture, distance conjecture, ⋯

main question in this talk:

- string theory accommodates a rich structure (perhaps too complete?):

  consistent amplitudes, ∞ gauge symmetries, dualities, holography, ⋯

- which ingredients are necessary for each swampland condition (if true)?

- specific to string theory or more robust in quantum gravity?

string theory = insurance w/full options

such a direction is better explored recently

in the context of Weak Gravity Conjecture

Weak Gravity Conjecture

[Arkani-Hamed et al '06]

claim: gravity is the weakest force [see next slide for motivation]

in graviton-photon system,

∃ a charged state w/ $g^2q^2 \geq \dfrac{m^2}{2M_{\rm Pl}^2}$ (gauge force ≧ gravity)

- never be satisfied if we decouple photon $g \to 0$

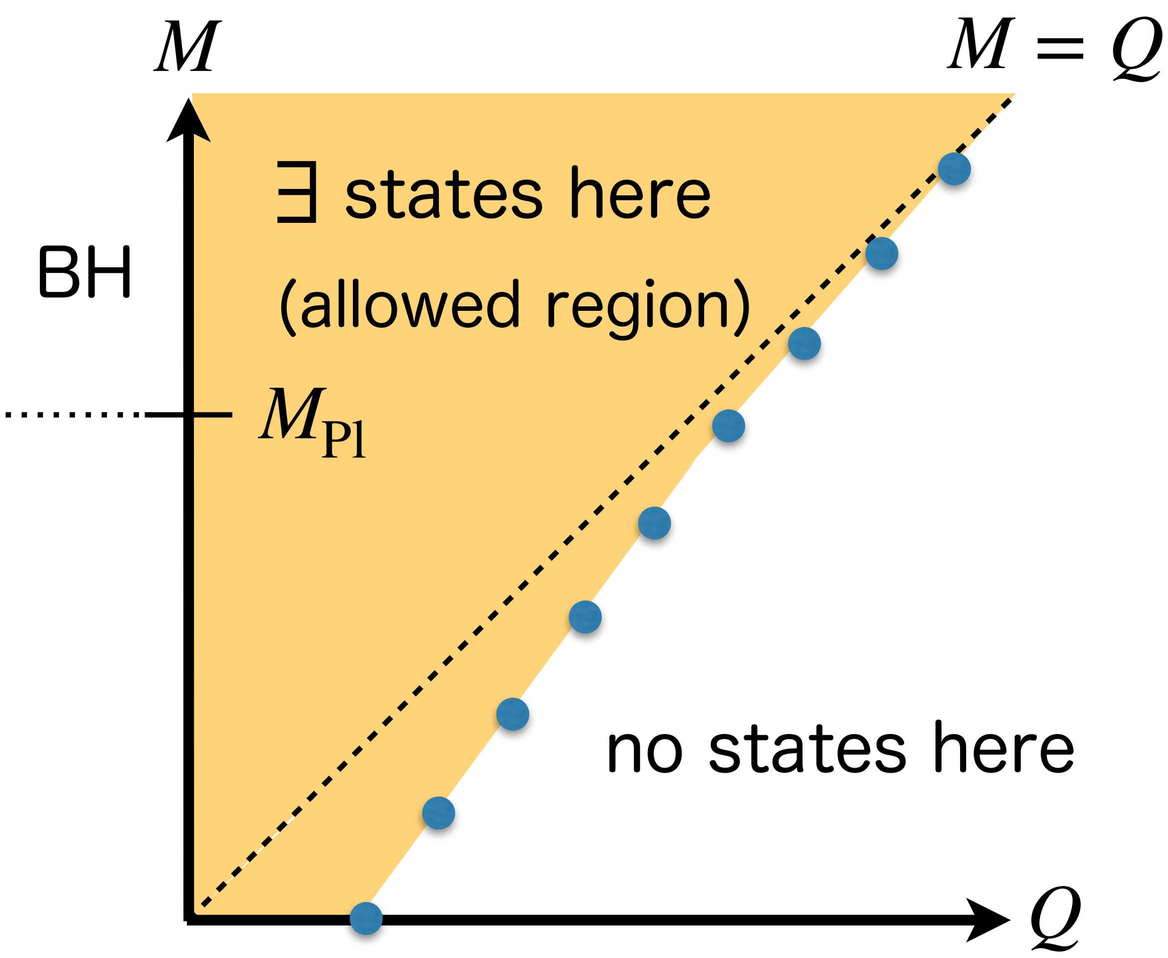
  → generalization of "no global symmetry in quantum gravity"

- trivially be satisfied if we decouple gravity $M_{\rm Pl} \to \infty$

  → special in quantum gravity

Motivation from string compactification

ex. heterotic string compactified on tori w/generic Wilson lines



typical spectrum of charged state

1. a tower of states w/ $M \leq Q$
2. monotonic “boundary”
3. it approaches to $M = Q$ for $M \gg M_{\rm Pl}$

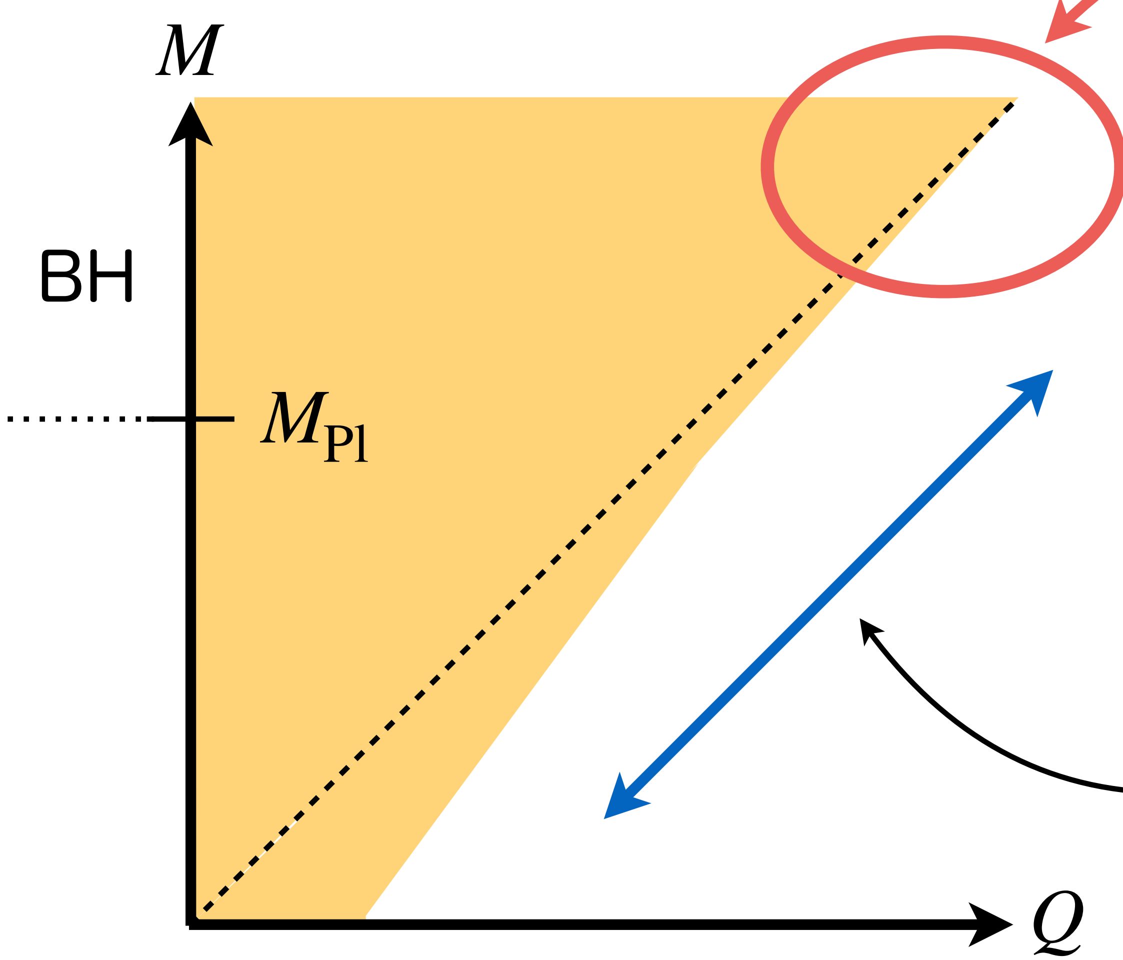
   ※ extremal bounds on large BHs

in the unit $M = Q$ for large extremal BHs

existence of states w/ $M \leq Q$ is common in string theory

[ArkaniHamed-Motl-Nicolis-Vafa 06’, ⋯]

How generic this picture is?



this asymptotic behavior (of BHs) follows from positivity bounds in graviton-photon systems

[Hamada-TN-Shiu '18]

※ existence proof of (mild) WGC

if UV theory has a worldsheet structure, spectral flow may relate the two regions

[Heidenreich et al '16, Alasma et al '19]

combination of two observations

→ suggests a stronger condition called sublattice/tower WGC

[Heidenreich et al '16, Andriolo-Junghan-TN-Shiu '18]

Positivity bounds are not enough??

Recently, we collected more data on WGC vs. positivity bounds

beyond graviton-photon systems

[Loges-TN-Shiu '19, '20, Andriolo-Huang-TN-Ooguri-Shiu '20]

1. positivity bounds imply WGC

   in graviton-photon systems and graviton-axion systems

2. but it is not the case once dilaton is turned on:

   in these theories duality symmetries are useful for WGC

In the rest of my talk, I will explain details for axionic WGC

(which is technically simpler than the Maxwell case)

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axionic WGC vs. Euclidean wormholes

[Andriolo-Huang-TN-Ooguri-Shiu ’20]

axionic WGC

form field	charged state	gravitational objects	coupling	size
photon	particle	charged BH	qg	mass
axion	instanton	Euclidean (semi)wormhole	$\frac{n}{f}$	action

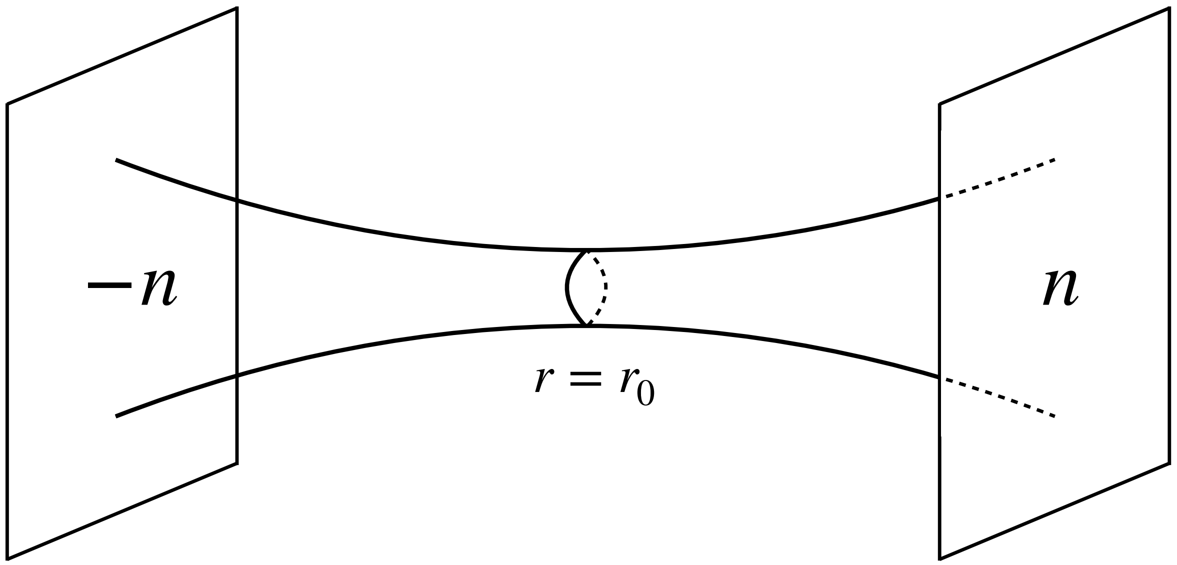
(size) < (coupling) implies $\exists$ an instanton w/ $S < \mathcal{O}(1) \cdot \frac{|n| M_{\rm Pl}}{f}$

cf. instanton generates axion potential

$\rightarrow$ implications to axion cosmology (inflation, DM)

[see, e.g., Hebecker-Mikhali-Soler '18 for a review]

Giddings-Strominger wormhole



Euclidean wormhole can be regarded as an instanton anti-instanton pair

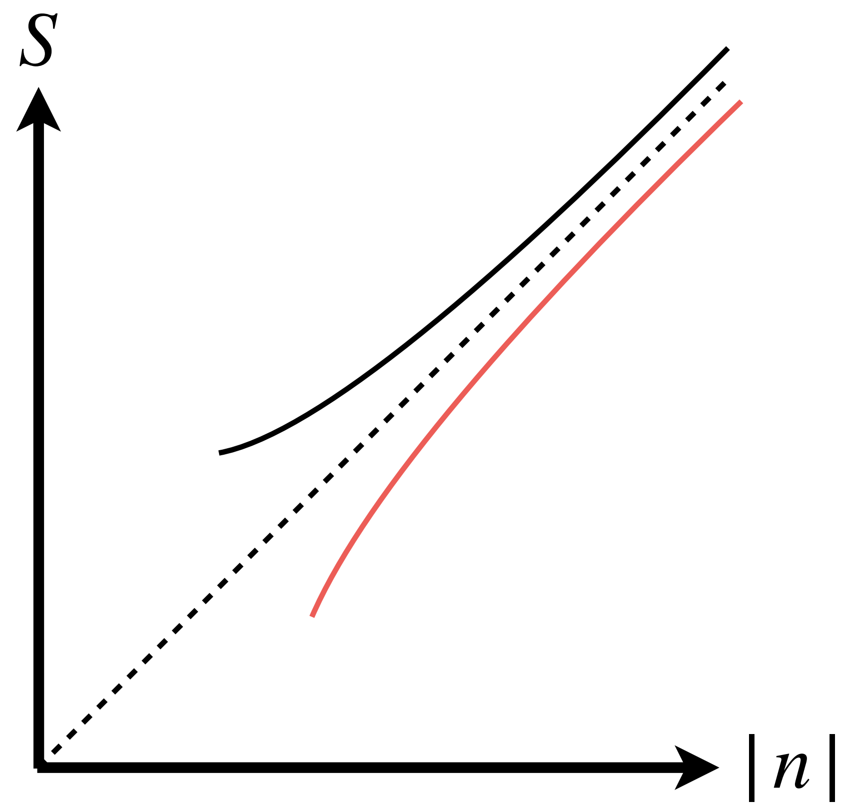
Euclidean (semi)wormhole in Einstein-axion theory:

$$ds^2 = \frac{dr^2}{1-(r_0/r)^4} + r^2 d\Omega_3^2, \quad r_0^4 = \frac{n^2 f^2}{24\pi^4 M_{\rm Pl}^6}$$ (n : axion charge, f : decay const.)

※ each semiwormhole (instanton) has an action $S = |n| \frac{\sqrt{6}\pi}{4} \cdot \frac{M_{\rm Pl}}{f}$

※ this fixes the $\mathcal{O}(1)$ constant in the WGC bound: $S \leq \frac{\sqrt{6}\pi}{4} \cdot \frac{|n| M_{\rm Pl}}{f}$

higher derivative corrections



action-charge relation of semiwormholes

- - - - - - - - - : Einstein-axion $S = |n| \frac{\sqrt{6}\pi}{4} \cdot \frac{M_{\rm Pl}}{f}$

———— : corrected one w/ $\alpha < 0$

———— : corrected one w/ $\alpha > 0$

※ WGC requires an instanton w/ $S \leq |n| \frac{\sqrt{6}\pi}{4} \cdot \frac{M_{\rm Pl}}{f}$

graviton-axion EFT up to four-derivatives

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\rm Pl}^2}{2} R - \frac{1}{2} \partial_\mu a \partial^\mu a + \alpha (\partial_\mu a \partial^\mu a)^2 + \beta_1 W_{\mu\nu\rho\sigma}^2 + \beta_2 \, a \, W_{\mu\nu\rho\sigma} \widetilde{W}^{\mu\nu\rho\sigma} \right]$$

+ appropriate boundary terms

※ modify wormhole solutions and so their action: $\Delta S = -24\pi^2 M_{\rm Pl}^4 \alpha + \mathcal{O}(1/n)$

if the α operator has a positive coefficient $\alpha > 0$,

macroscopic (semi)wormholes satisfy the WGC bound.

indeed, $\alpha > 0$ follows from analyticity, unitarity and locality

of UV scattering amplitudes (positivity bounds) [Adams et al '06]

$\rightarrow$ an existence proof of (the mild form of) WGC

caveat: applicable only when gravitational Regge states are negligible

[see Hamada-TN-Shiu '18 for details]

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Generalization to graviton-axion-dilaton system

[Andriolo-Huang-TN-Ooguri-Shiu '20]

graviton-axion-dilaton EFT

Einstein-axion-dilaton action

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\rm Pl}^2}{2} R - \frac{1}{2} e^{\lambda\phi} \partial_\mu a \partial^\mu a - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi \right]$$

- we focus on $|\lambda| < 4/\sqrt{6}$, otherwise no regular wormholes

four-derivative terms relevant to our problem

$$\Delta\mathscr{L} = \alpha_1 e^{2\lambda\phi} (\partial_\mu a \partial^\mu a)^2 + \alpha_2 (\partial_\mu \phi \partial^\mu \phi)$$
$$+ \alpha_3 e^{\lambda\phi} (\partial_\mu a \partial^\mu a)(\partial_\nu \phi \partial^\nu \phi) + \alpha_4 e^{\lambda\phi} (\partial_\mu a \partial^\mu \phi)^2$$

- suppressed terms w/Weyl tensor, which do not correct the action

- also see our paper for more general dilaton couplings

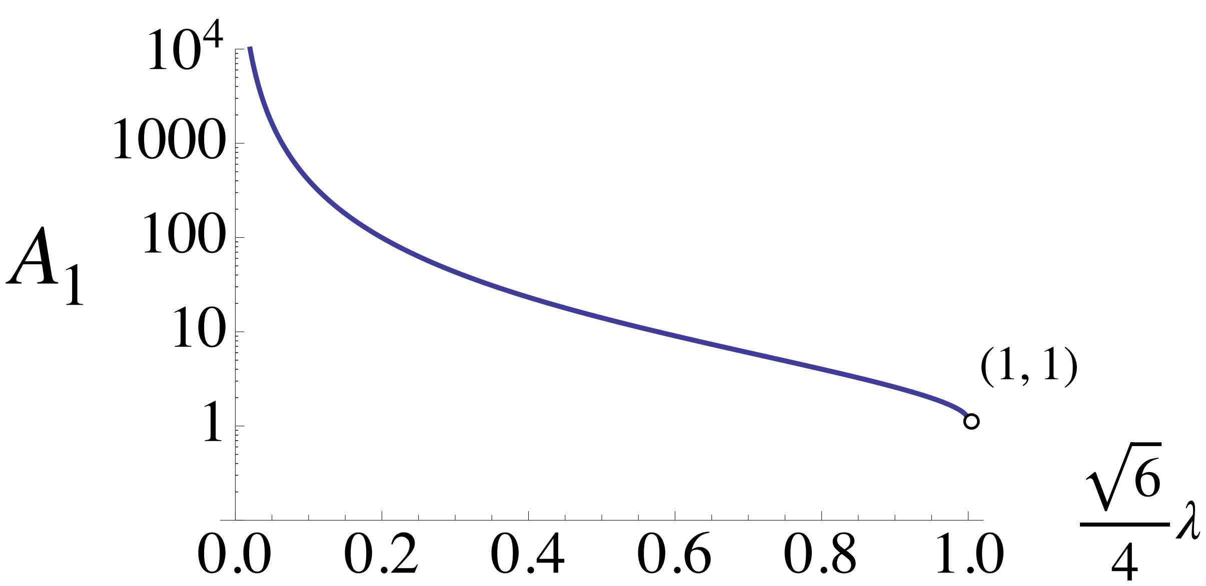
Corrections to (semi)wormhole action

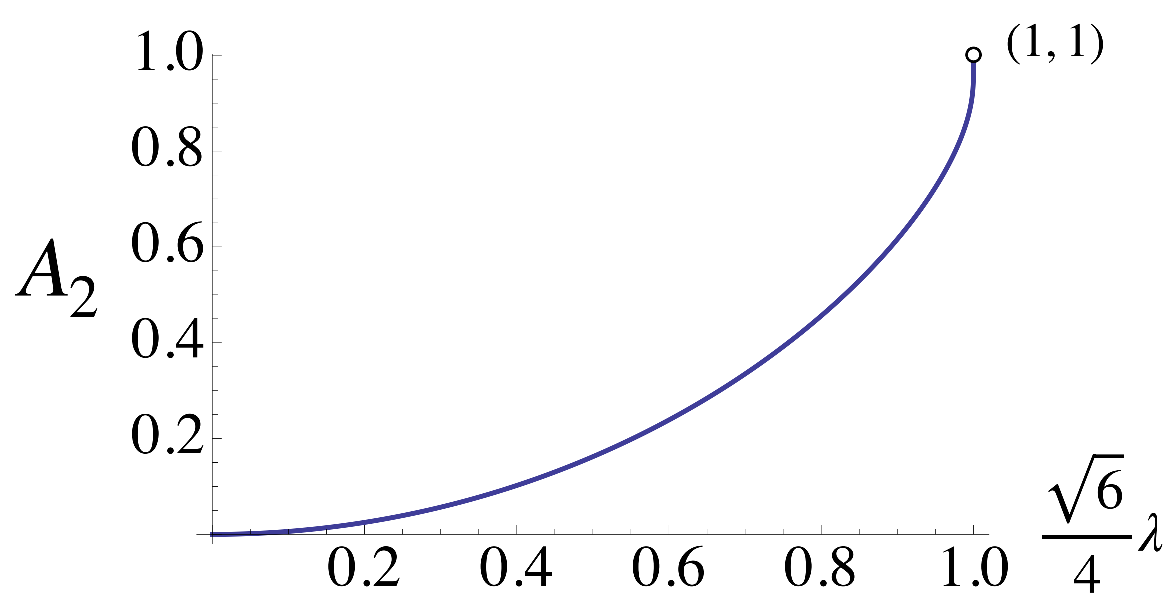
four-derivative corrections to the (semi)wormhole action

$$\Delta S = 36\pi^2 M_{\rm Pl}^4 \int_0^{\pi/2} dt\, \cos^3 t \left[-\alpha_1 \sec^4\!\left[\frac{\sqrt{6}}{4}\lambda\cdot t\right] - \alpha_2 \tan^4\!\left[\frac{\sqrt{6}}{4}\lambda\cdot t\right] + (\alpha_3+\alpha_4)\sec^2\!\left[\frac{\sqrt{6}}{4}\lambda\cdot t\right]\tan^2\!\left[\frac{\sqrt{6}}{4}\lambda\cdot t\right]\right] + \mathcal{O}(1/n)$$

- the condition for $\Delta S < 0$ and so WGC reads

$\alpha_3 + \alpha_4 < A_1(\lambda)\,\alpha_1 + A_2(\lambda)\,\alpha_2$ ($A_{1,2}$: λ-dep. positive coefficients)





Implications of positivity bounds

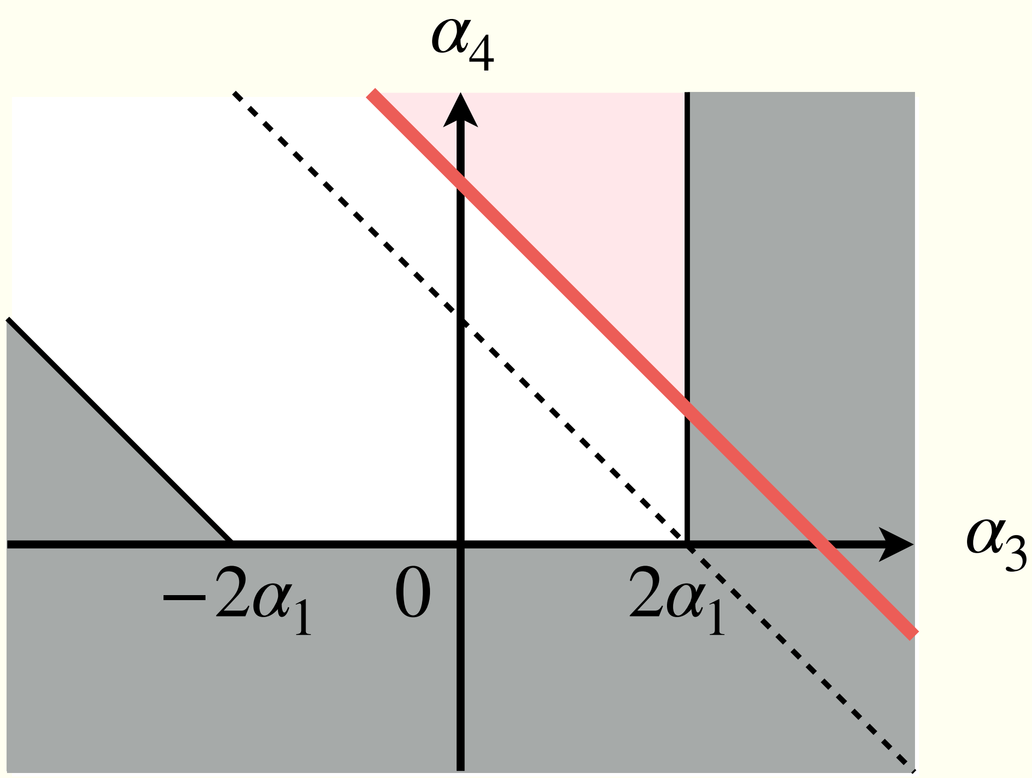
positivity of $aa \to aa,\ \phi\phi \to \phi\phi,\ a\phi \to a\phi$: $\alpha_1, \alpha_2, \alpha_4 > 0$

scattering of superpositions of a, ϕ :

2-para. family of bounds → its envelop gives $-\alpha_4 - 2\sqrt{\alpha_1\alpha_2} < \alpha_3 < 2\sqrt{\alpha_1\alpha_2}$

→ large positive α_4 violates WGC bound $\alpha_3 + \alpha_4 < A_1\alpha_1 + A_2\alpha_2$

projection onto $\alpha_2 = \alpha_1$ plane for illustration



prohibited by positivity

allowed by positivity, but WGC is not satisfied

satisfy positivity & WGC

$\alpha_3 + \alpha_4 = \left(A_1 + A_2\right)\alpha_1 > 2\alpha_1$

positivity is not enough to demonstrate WGC

Q. any additional UV input which implies WGC?

$SL(2,R)$ duality symmetry of axion & dilaton

is an example for such UV information!

Implications of duality constraints

\# $SL(2,R)$ transformation in our convention:

$$\tau \to \frac{a\tau + b}{c\tau + d} \quad (a, b, c, d \in R,\ ad - bc = 1) \quad \text{w/ } \tau = \frac{\lambda}{2}a + ie^{-\frac{\lambda}{2}\phi}$$

only two $SL(2,R)$ invariant operators: $\dfrac{(\partial_\mu \tau \partial^\mu \bar{\tau})^2}{(\mathrm{Im}\,\tau)^4}$, $\dfrac{|\partial_\mu \tau \partial^\mu \tau|^2}{(\mathrm{Im}\,\tau)^4}$

in our language it means $\alpha_2 = \alpha_1$, $\alpha_3 + \alpha_4 = 2\alpha_1$

\# under these conditions, we have $\Delta S = -24\pi^2 M_{\mathrm{Pl}}^4 \alpha_1$

→ positivity $\alpha_1 > 0$ implies $\Delta S < 0$ and so WGC!

we have found that positivity alone is not enough,

but positivity + SL(2,R) duality invariance does imply WGC

Implications of duality constraints

\# $SL(2,R)$ transformation in our convention:

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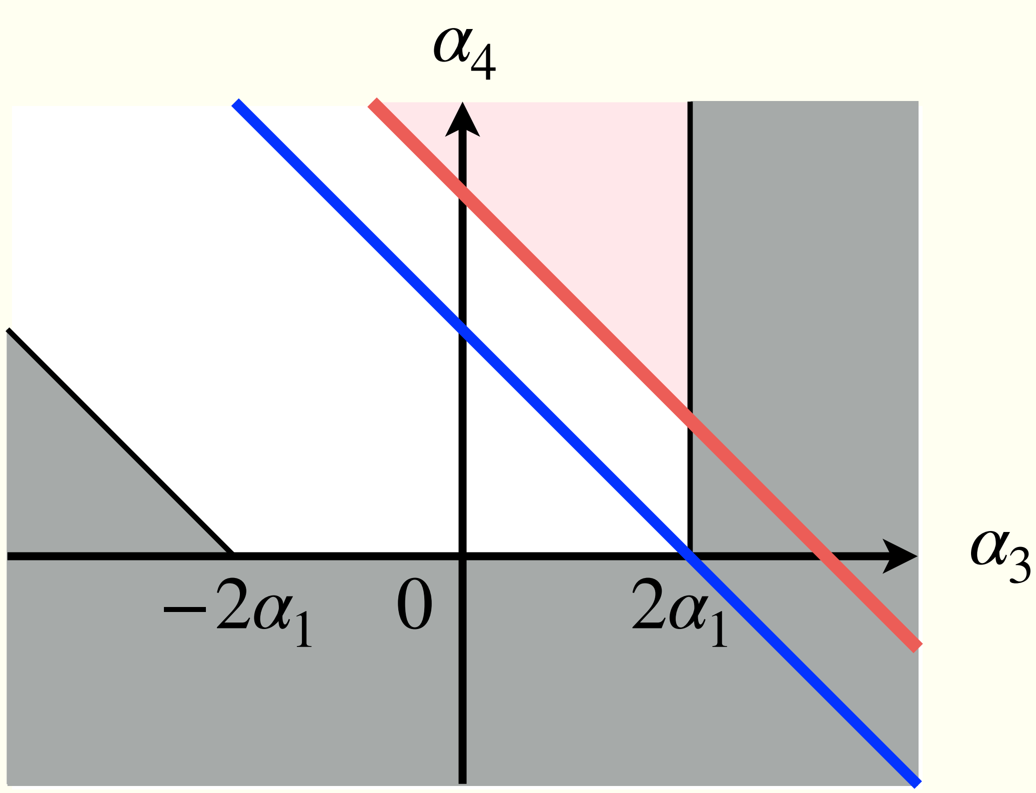
only two $SL(2,R)$ invariant operators: $\dfrac{(\partial_\mu \tau \partial^\mu \bar{\tau})^2}{(\mathrm{Im}\,\tau)^4}, \quad \dfrac{|\partial_\mu \tau \partial^\mu \tau|^2}{(\mathrm{Im}\,\tau)^4}$

in our language it means $\alpha_2 = \alpha_1$, $\alpha_3 + \alpha_4 = 2\alpha_1$

$\boxed{\alpha_2 = \alpha_1}$



allowed by positivity, but WGC is not satisfied

satisfy positivity & WGC

$\alpha_3 + \alpha_4 = (A_1 + A_2)\,\alpha_1 > 2\alpha_1$

SL(2,R) invariant: $\alpha_3 + \alpha_4 = 2\alpha_1$

Related results for BHs

WGC in Einstein-Maxwell-dilaton-axion:

[Loges-TN-Shiu '19]

there exists a parameter space

which is allowed by positivity, but does not satisfy WGC

[Loges-TN-Shiu '20]

duality symmetries are again useful to demonstrate WGC

1. positivity + SL(2,R) → WGC
2. null energy condition + O(d,d;R) → WGC

※ positivity bounds are not applicable for O(d,d;R) case

because gravitational Regge states are not negligible

4. summary and prospects

summary and prospects

positivity implies WGC in graviton-photon and graviton-axion systems

under the assumption that gravitational Regge states are negligible

※ can we incorporate gravitational Regge states in positivity?

but it is not the case once dilaton is turned on:

duality symmetries such as SL(2,R) and O(d,d;R) are useful for WGC

※ are there other UV inputs useful for demonstrating WGC?

we provided evidences for axionic WGC, which constraints axion potential

※ can we generalize our results to potential of other moduli fields?

(cf. scalar WGC, non-SUSY AdS, dS, ⋯)

Thank you!