

Observable Gravitational Waves in Minimal Scotogenic Model

Kohei Fujikura (TokyoTech)

This work in collaboration with:

Debasish Borah (IIT Guwahati)

Arnab Dasgupta (SeoulTech)

Sin Kyu Kang (SeoulTech)

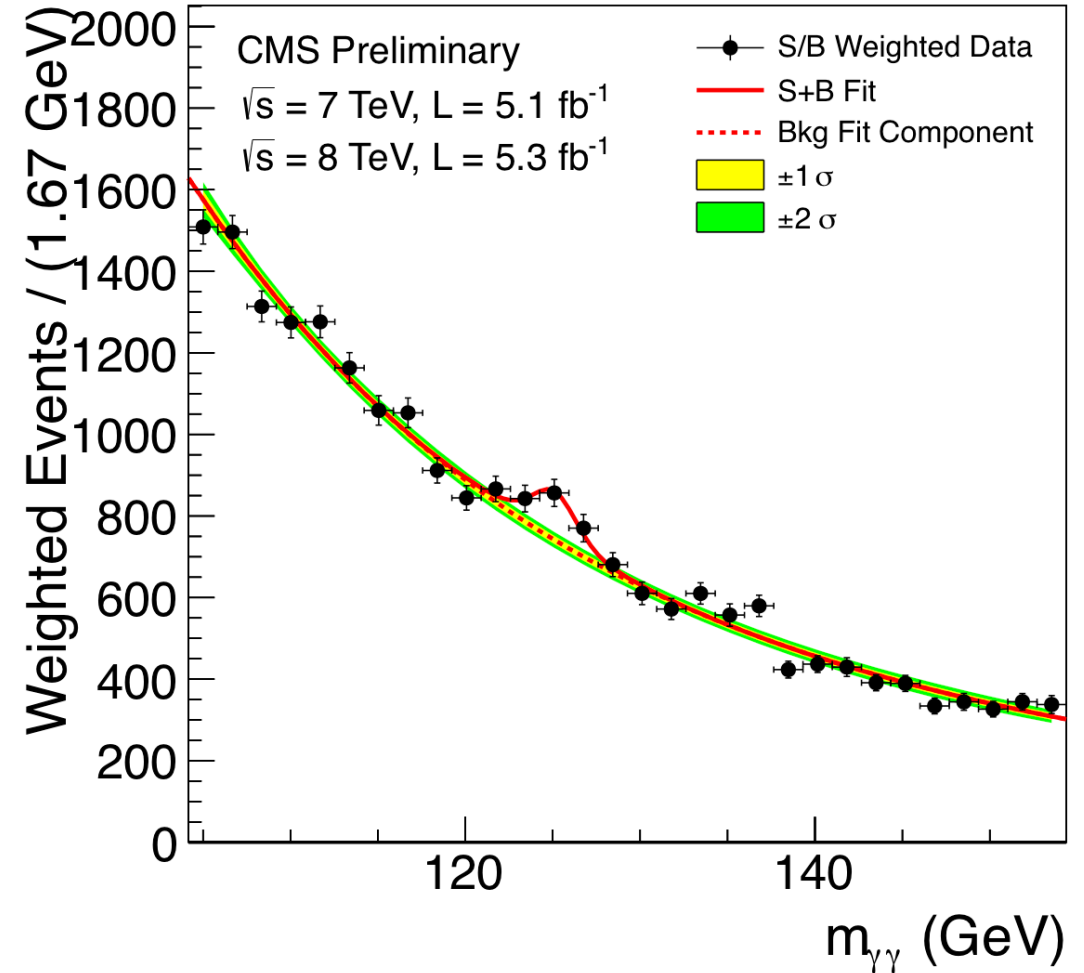
Devabrat Mahanta (IIT Guwahati)

Based on arXiv:2003.02276

The Standard Model

✓ The SM is in agreement with collider experiments.

✓ In 2012, the SM-like Higgs boson was discovered by the LHC.



CMS (2012)

Problems in the SM

The SM cannot explain following phenomena:

- **Dark matter**

SM does not have DM candidate.

- **Neutrino masses**

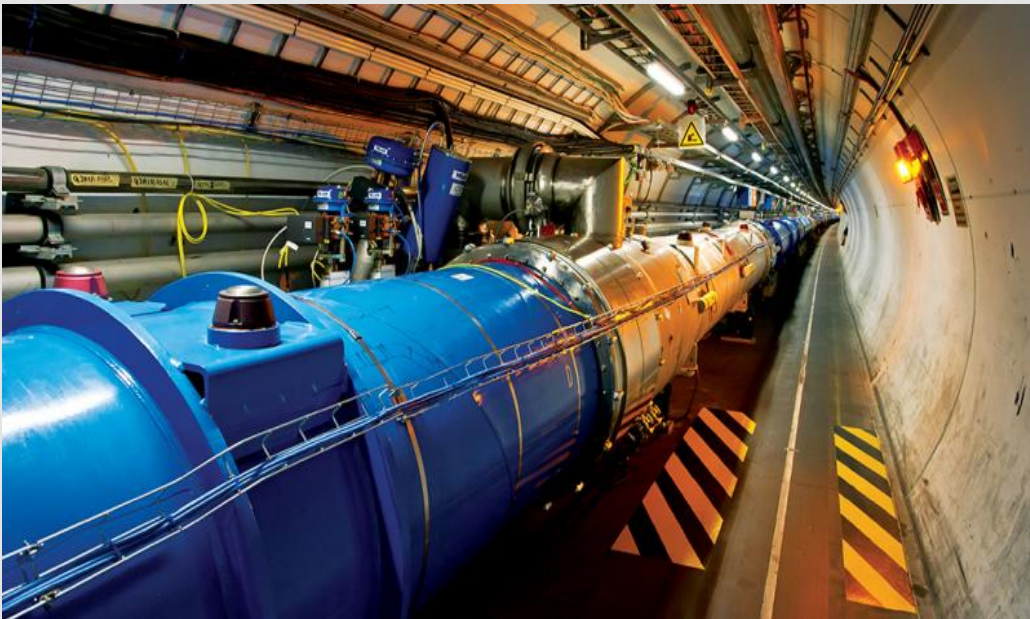
SM cannot explain tiny neutrino masses.

We need a beyond standard model physics!

The subject today

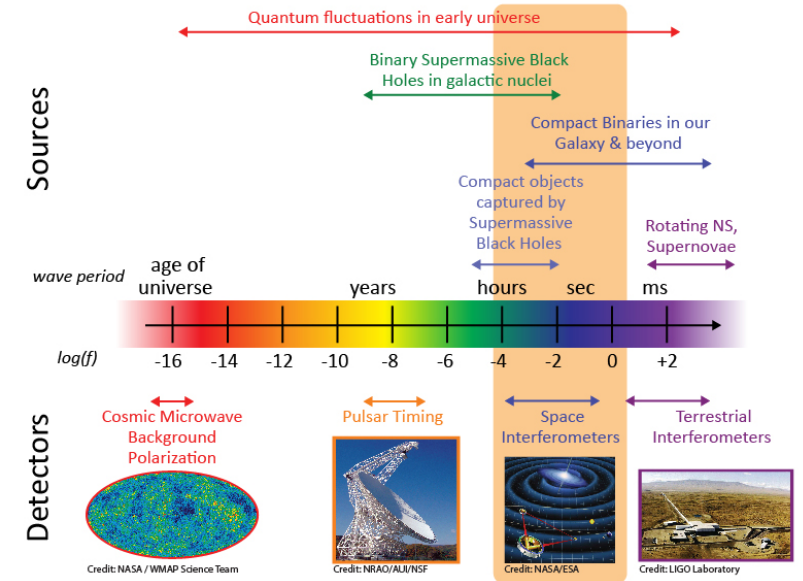
Scotogenic Model

Collider searches



Gravitational wave

The Gravitational Wave Spectrum



Outline

- **Minimal Scotogenic Model**
- **A First-Order Electroweak Phase Transition**
- **Results**

Scotogenic Model

SM + additional $SU(2)_w$ doublet + 3 right-handed neutrinos can explain following phenomena:

[E. Ma (2006)]

- Dark matter
- Neutrino masses

The Scalar Potential

$$V = \lambda_{\text{SM}} \left(|H|^2 - \frac{v_{\text{SM}}^2}{2} \right)^2 + m_1^2 |S|^2 + \lambda_1 |H|^2 |S|^2$$

SM Higgs potential

$$+ \lambda_2 |H^\dagger S|^2 + [\lambda_3 (H^\dagger S)^2 + \text{h.c.}] + \lambda_S |S|^4$$

H : SM Higgs doublet S : $SU(2)_W$ doublet

Suppose that S does not acquire VEV. (Inert scalar doublet)

Invariant under the Z_2 symmetry: $S \rightarrow -S$

S is stable, i.e., cannot decay into the SM particles.

Three Right-handed Neutrinos

$$\mathcal{L}_{\text{int}} \supset \frac{1}{2} (M_N)_{ij} N_i N_j + (Y_{ij} \bar{L}_i \tilde{S} N_j)$$

N_i : right – handed neutrino \bar{L} : SM leptons

Interactions with new particles preserve Z_2 symmetry!

$$N_i \rightarrow -N_i \quad S \rightarrow -S$$

Interactions with SM particles:

- The inert scalar S : $SU(2)_W \times U(1)_Y$ gauge interactions and scalar-Higgs couplings!
- Right-handed neutrinos: Yukawa couplings with SM leptons!

Two DM candidates

- **A neutral component of the inert scalar.**
- **The lightest right-handed neutrino.**

Scalar Masses

$$S = \begin{pmatrix} H^+ \\ H_0 \end{pmatrix}$$

Charged component

Neutral component
(WIMP dark matter)

$$H_0 = \frac{H + iA}{\sqrt{2}}$$

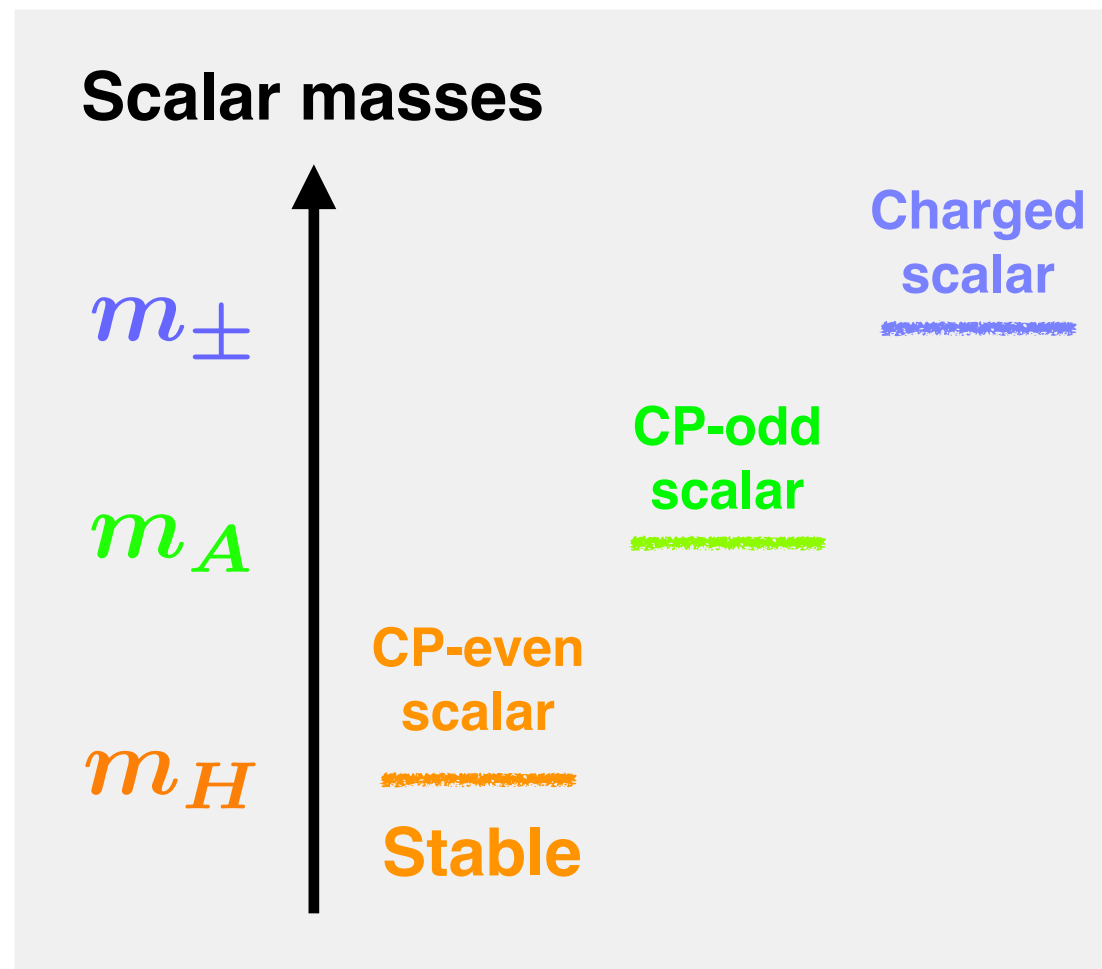
H: CP-even scalar

A: CP-odd scalar

$$m_{\pm}^2 = m_1^2 + \frac{1}{2} \lambda_1 v_{\text{SM}}^2$$

$$m_A^2 = m_1^2 + \frac{1}{2} (\lambda_1 + \lambda_2 - 2\lambda_3) v_{\text{SM}}^2$$

$$m_H^2 = m_1^2 + \frac{1}{2} (\lambda_1 + \lambda_2 + 2\lambda_3) v_{\text{SM}}^2$$



Weakly Interacting Massive Particle (WIMP)

H (or A) scalar can be a DM candidate!

- DM is thermally produced.

- Freeze-out occurs when

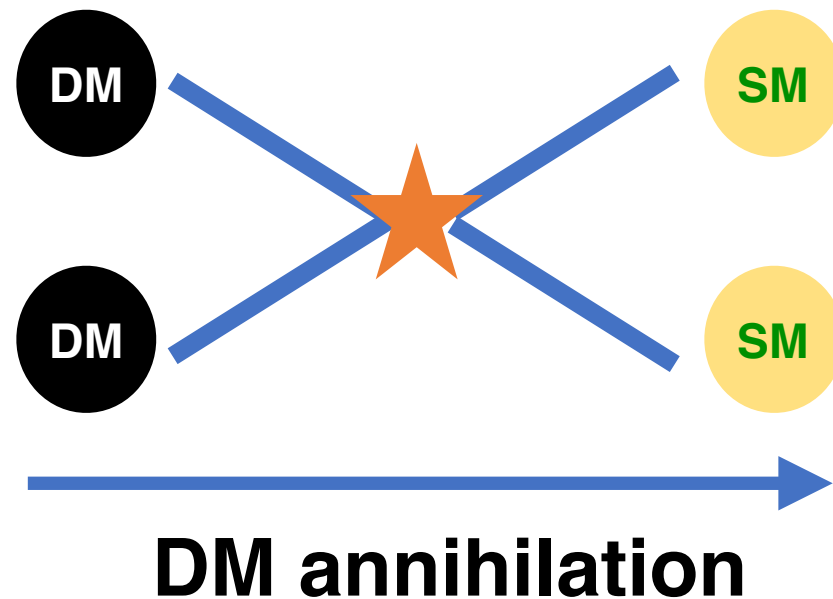
$$\Gamma(T) \simeq H(T)$$

Reaction rate

Hubble parameter

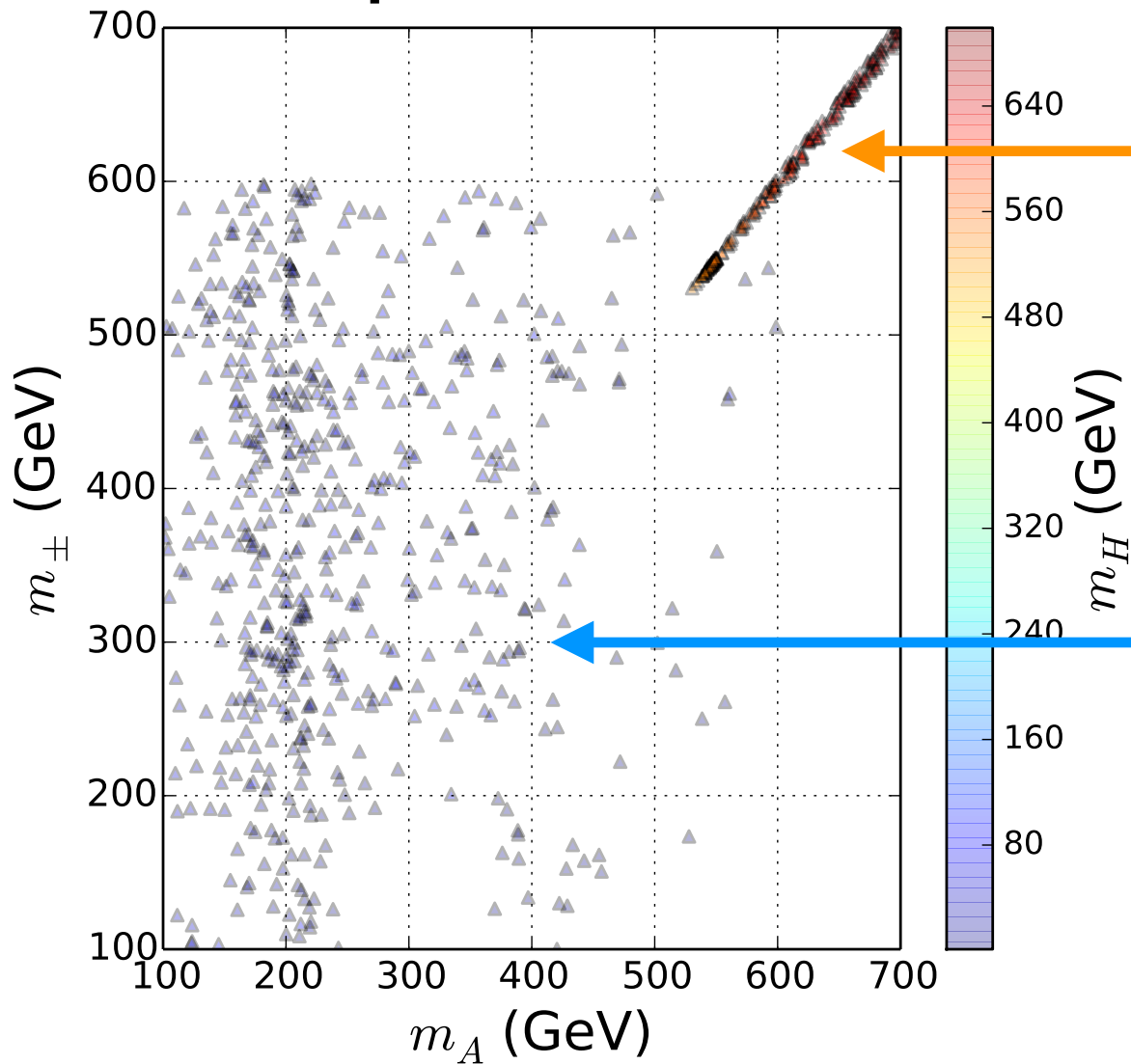
- A correct DM abundance can be obtained with weak scale DM mass.

$$m_{DM} \sim \mathcal{O}(1 \sim 100) \text{ GeV}$$



Scalar DM

Assumption: 100% DM



$m_{DM} > 550\text{GeV}$
High DM mass regime

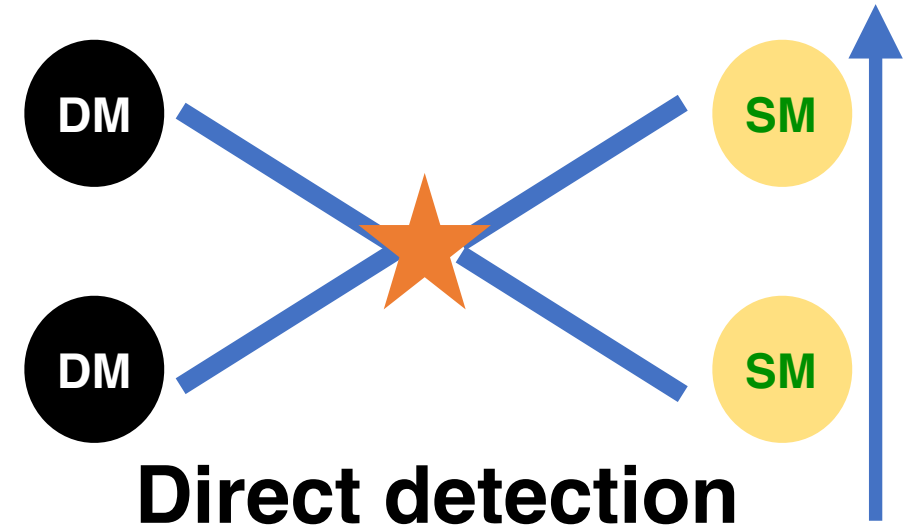
$m_{DM} < 80\text{GeV}$
Low DM mass regime

The Direct Detection Constraint

- DM interacts with SM particles (Nucleon) via the SM Higgs.
- Spin-independent (SI) cross-section with nucleon:

$$\sigma_{SI} \sim \lambda_H^2 \frac{\mu^2 m_n^2}{m_h^4 m_{DM}^2}$$

$$m_{DM}^2 = m_1^2 + \frac{1}{2} \lambda_H v_{SM}^2$$



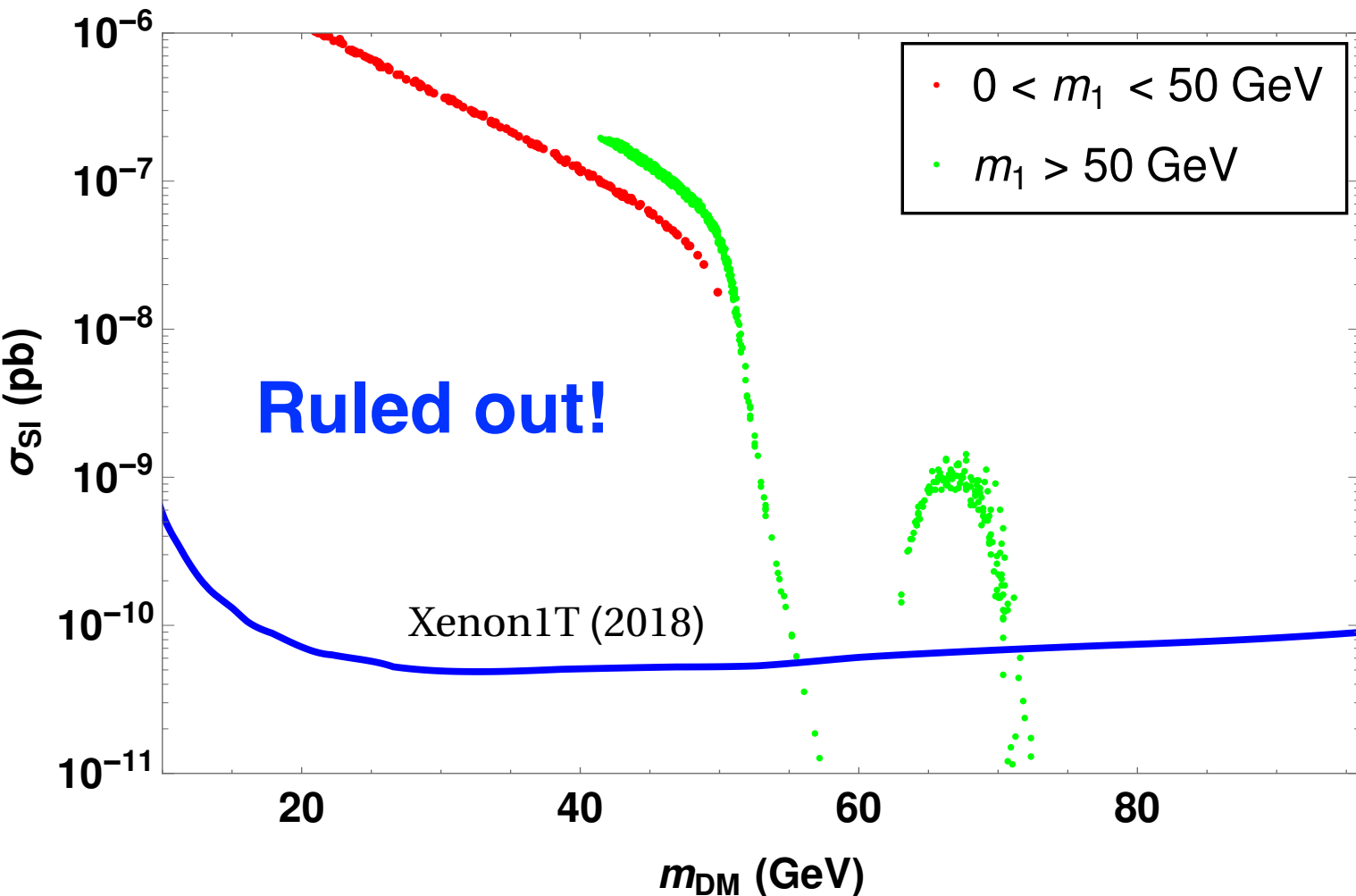
$$\mu = m_n m_{DM} / (m_n + m_{DM})$$

m_h : SM Higgs mass

m_n : Nucleon mass

$$\lambda_H = \lambda_1 + \lambda_2 + 2\lambda_3$$

Direct Detection Constraint (Scalar DM)



$$\lambda_H = \lambda_1 + \lambda_2 + 2\lambda_3$$

$$m_{DM}^2 = m_1^2 + \frac{1}{2}\lambda_H v_{SM}^2$$

**A small dark matter coupling is needed!
(A small m_1 is ruled out!)**

Two DM candidates

- **A neutral component of the inert scalar.**

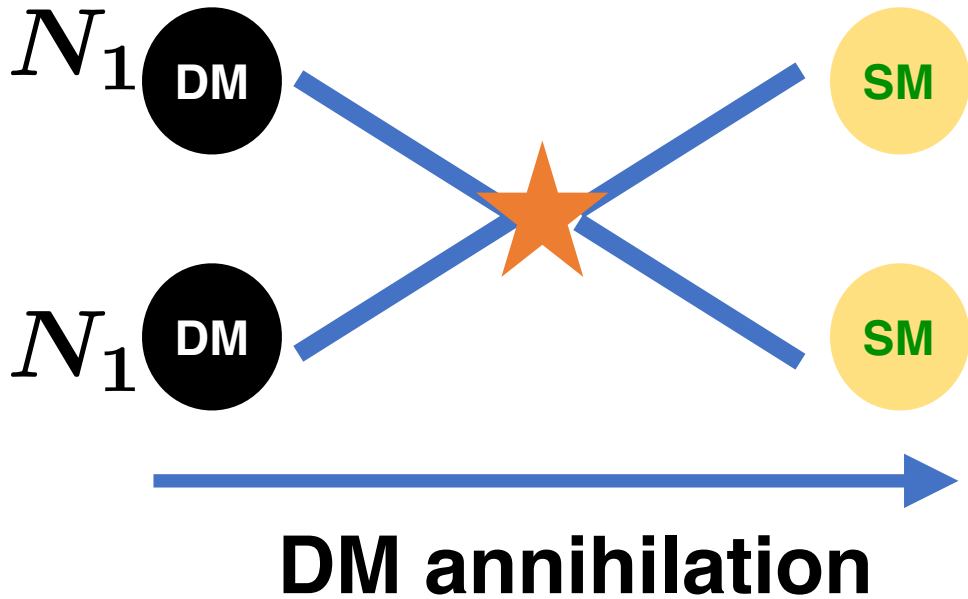
The direct detection constraint is stringent.

H (or A) boson should be the lightest state.

- **The lightest right-handed neutrino.**

Fermion DM (1)

There is no restriction on scalar masses!
(The charged scalar can be light.)



Masses for Z_2 odd particles

m_H

m_A

m_{\pm}

M_N

CP-even
scalar

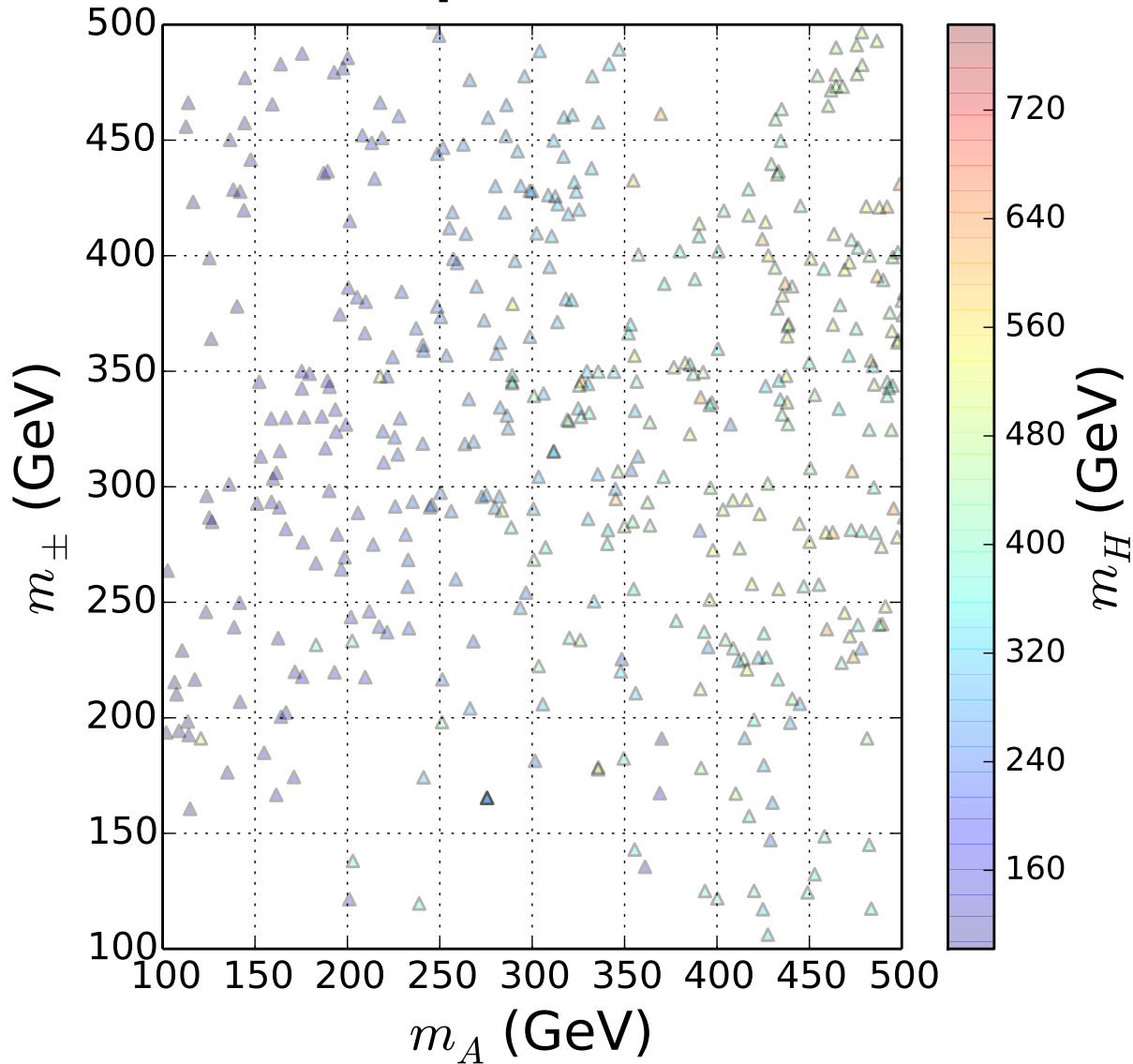
CP-odd
scalar

Charged
scalar

Right-handed
neutrino

Fermion DM (2)

Assumption: 100% DM

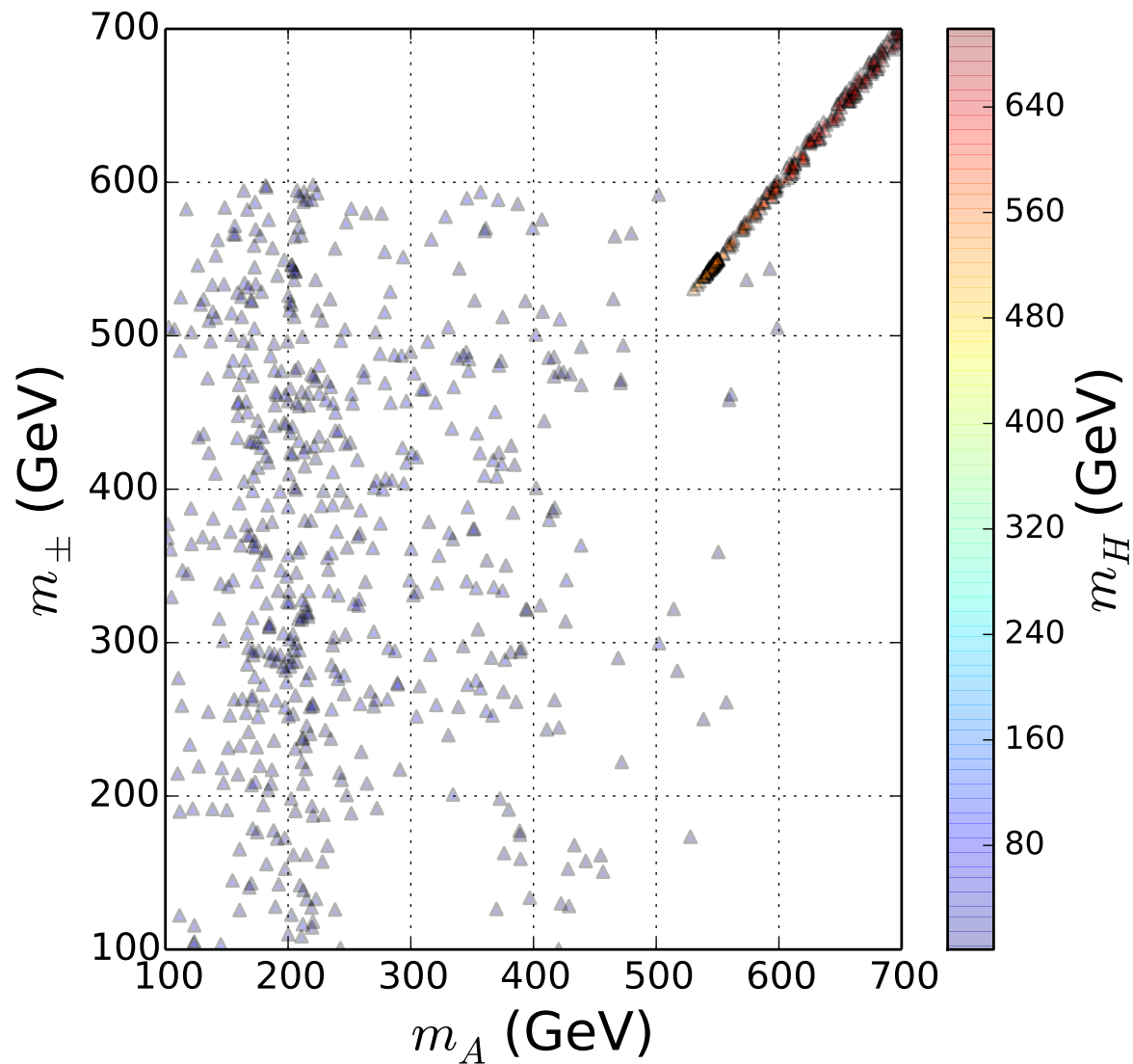


The charged scalar can be lighter than neutral component.

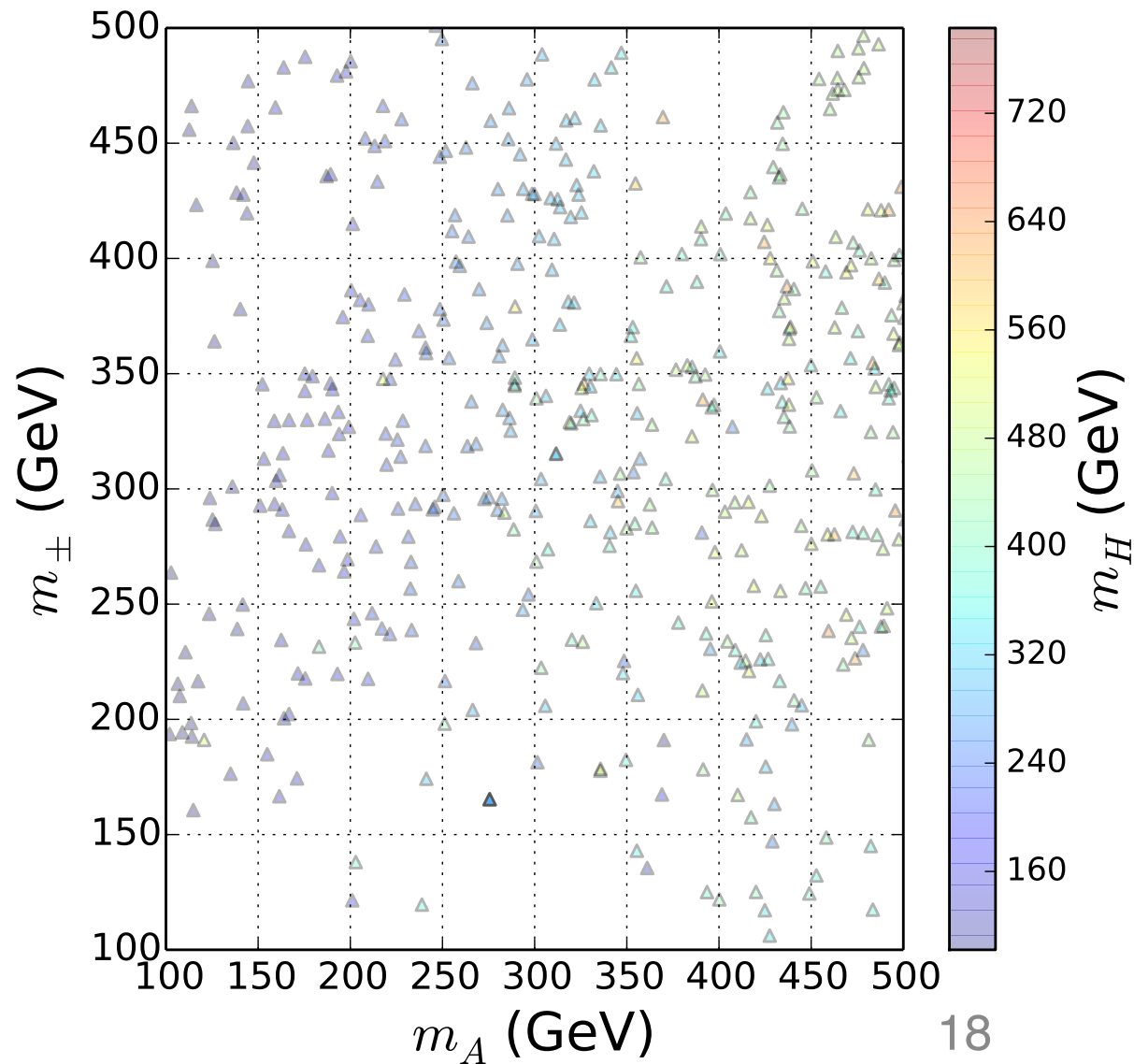
The direct detection constraint is not relevant!
(The fermion DM does not couple to the SM Higgs at tree level!)

Scalar VS Fermion DM

Scalar DM



Fermion DM



Two DM candidates

- **A neutral component of the inert scalar**

The direct detection constraint is stringent.

H (or A) boson should be the lightest state.

- **The lightest right-handed neutrino**

The direct detection constraint is not relevant.

The charged scalar can be lighter than neutral component.

Two DM candidates

- **A neutral component of the inert scalar**

The direct detection constraint is stringent.

H (or A) boson should be the lightest state.

- **The lightest right-handed neutrino**

The direct detection constraint is not relevant.

The charged scalar can be lighter than neutral component.

However, Yukawa interactions with SM leptons are sources of
lepton flavor violation processes.

(Discuss later)

Scotogenic Model

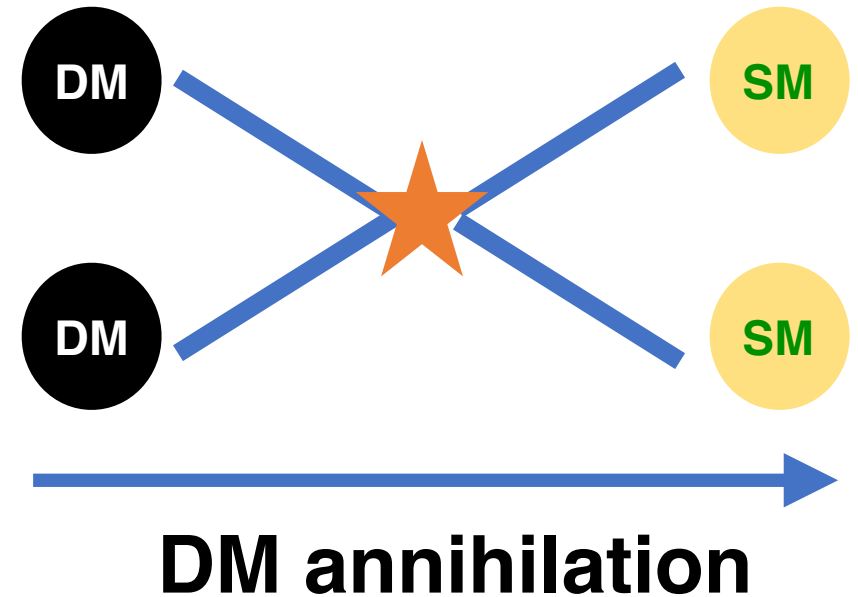
SM + additional $SU(2)_W$ doublet + 3 right-handed neutrinos can explain following phenomena:

- **Dark matter**

The inert (scalar) dark matter
The lightest right-handed neutrino

- **Neutrino masses**

[E. Ma (2006)]



Neutrino masses

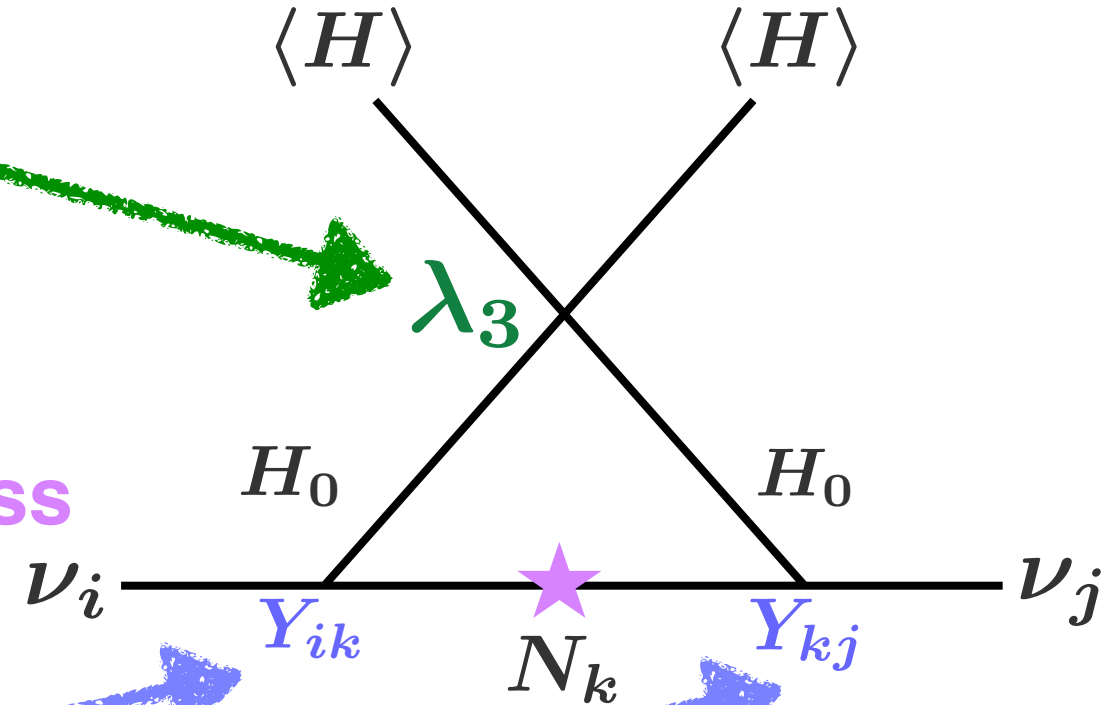
Neutrino masses are generated radiatively!

Higgs-scalar doublet interaction

$$\mathcal{L}_{\text{int}} \supset \lambda_3 (H^\dagger S)^2 + \text{h.c.}$$

Yukawa interaction and Majorana mass

$$\mathcal{L}_{\text{int}} \supset \frac{1}{2} (M_N)_{ij} N_i N_j + (Y_{ij} \bar{L} \tilde{S} N_j + \text{h.c.})$$



Neutrino masses

$$(m_\nu)_{ij} = \text{Diagram} \simeq \frac{\lambda_3 v_{\text{SM}}^2}{16\pi^2} \frac{Y_{ik} Y_{kj}}{M_k}$$

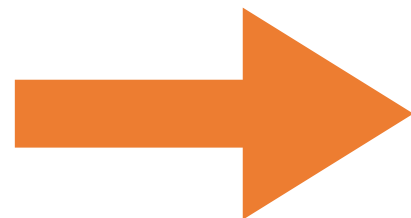
The diagram shows a one-loop Feynman diagram for neutrino mass generation. It features two external neutrino lines, ν_i and ν_j , connected by a horizontal line. Two internal Majorana neutrino lines, N_k , form a loop. The vertices are labeled with Y_{ik} and Y_{kj} in blue. Two Higgs boson lines, H_0 , connect the Majorana neutrino lines to two Higgs vacuum expectation value lines, $\langle H \rangle$, at the top. A green λ_3 is placed in the center of the loop.

One-loop suppression!

[E. Ma 2006]

Thanks to the one-loop suppression, tiny neutrino masses are generated with **TeV scale Majorana masses!**

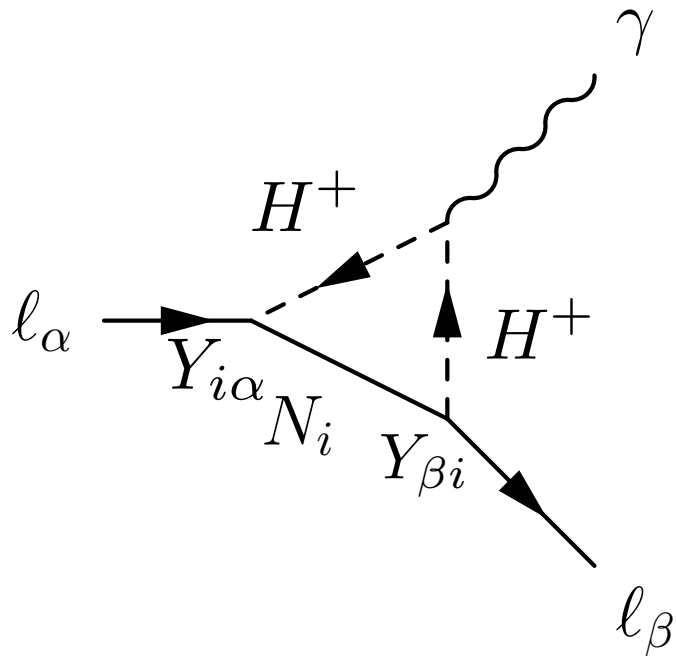
e.g.) $m_\nu \sim 1\text{eV}$
 $\lambda_3 = 0.01$
 $Y \sim \mathcal{O}(0.1)$



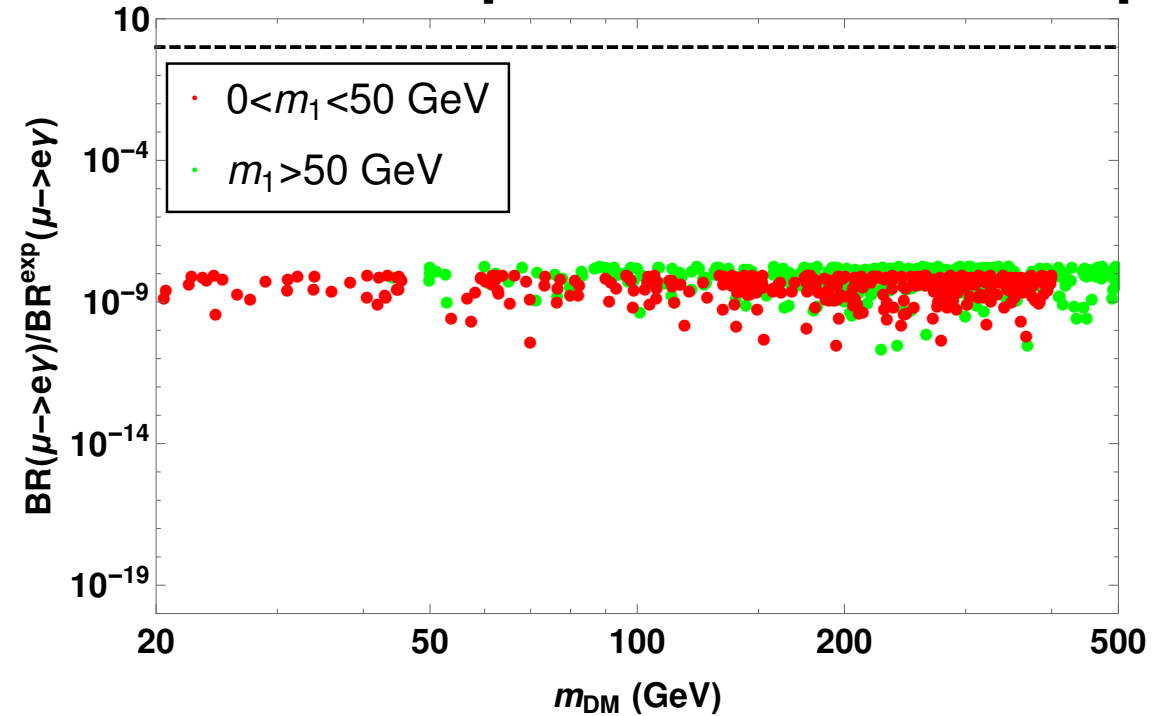
$M \sim 1\text{TeV}$

Lepton Flavor Violation (LFV)

In the SM, LFV is generated at one-loop which is suppressed by tiny neutrino masses.



[SPheno 3.1: arXiv:1104.1573]

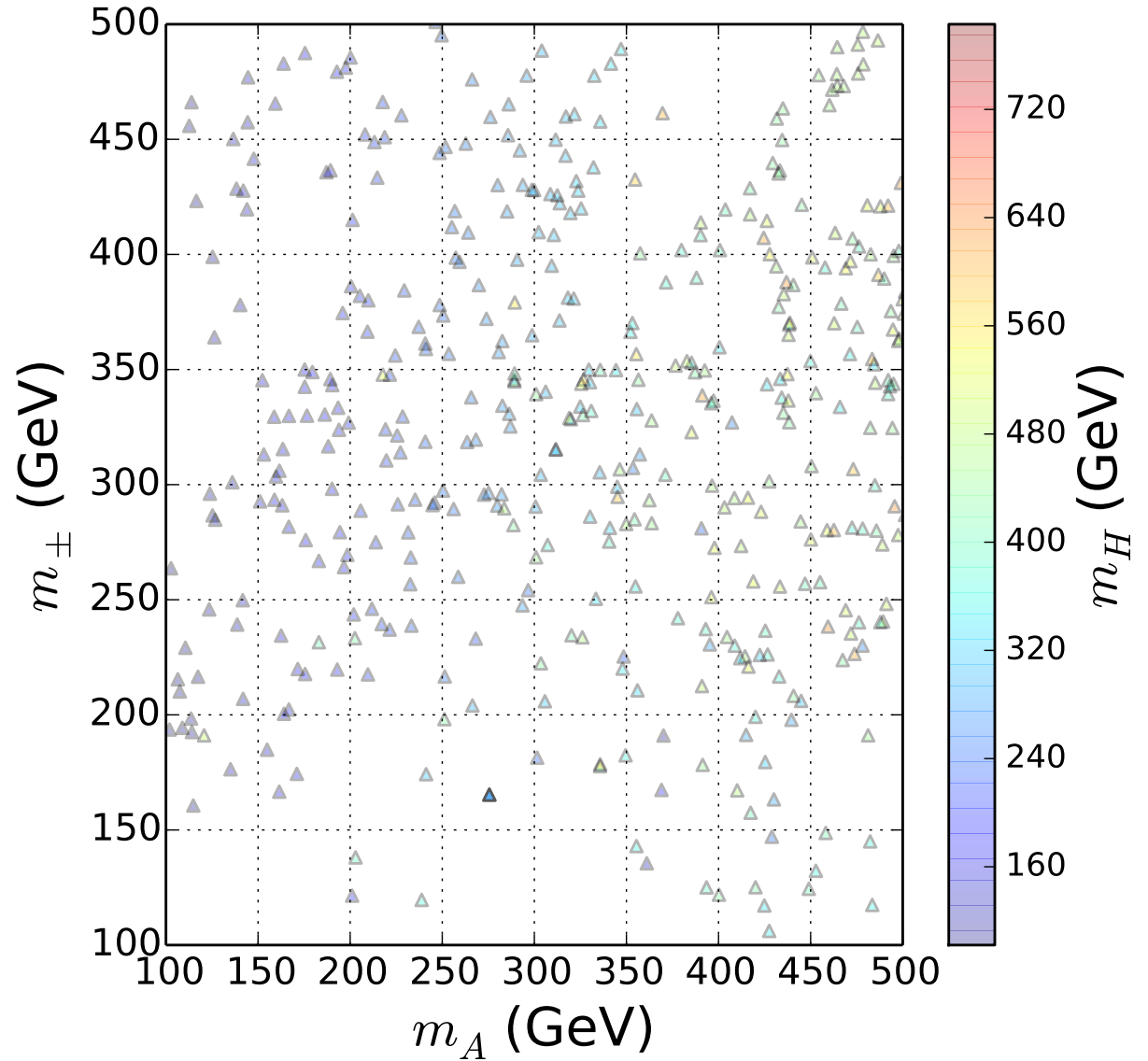
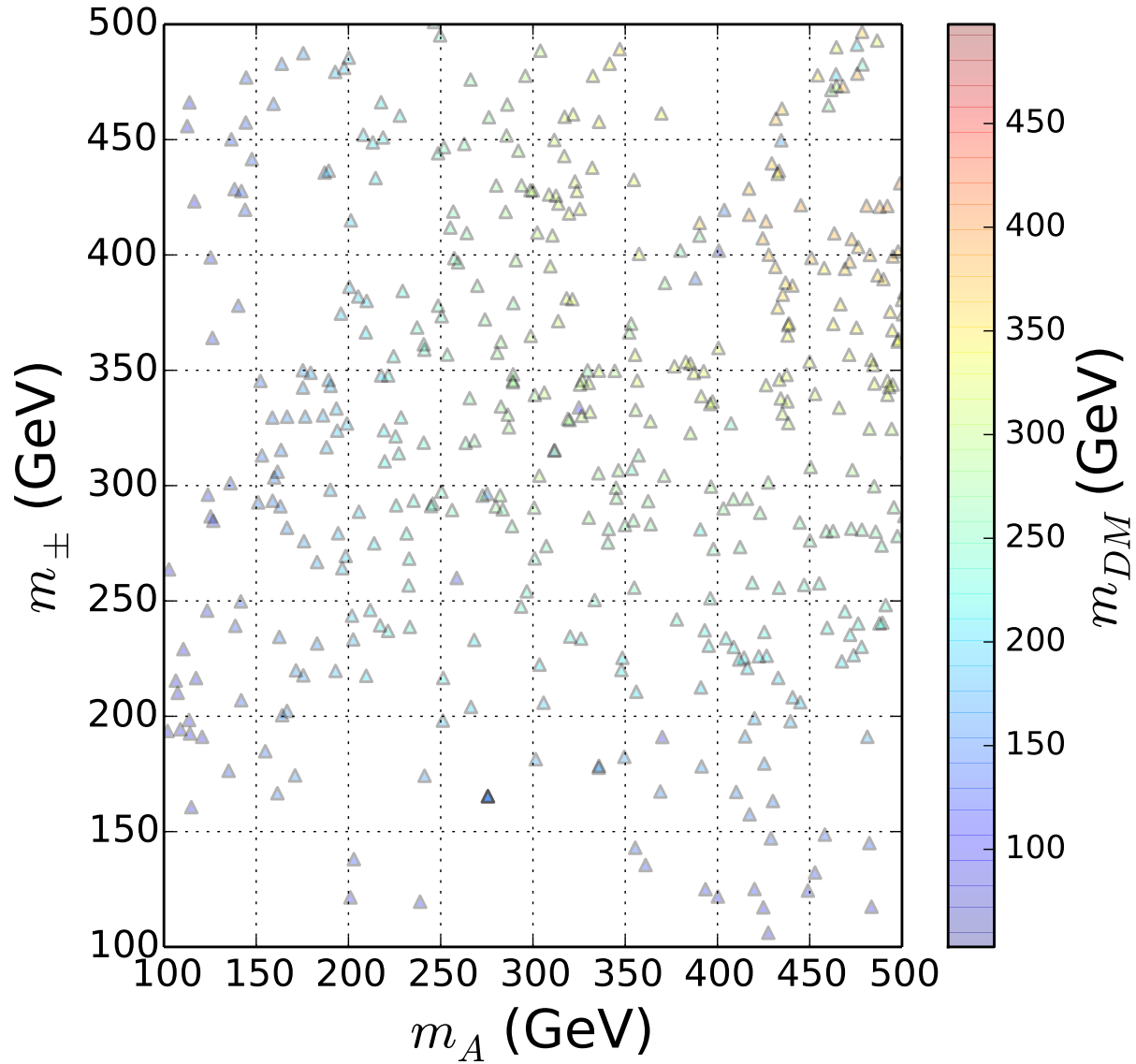


LFV constraint can be evaded with small Yukawa couplings!

A correct DM abundance is obtained by coannihilation with the charged scalar.

[arXiv:1312.2840]

Fermion DM (3)



Scotogenic Model

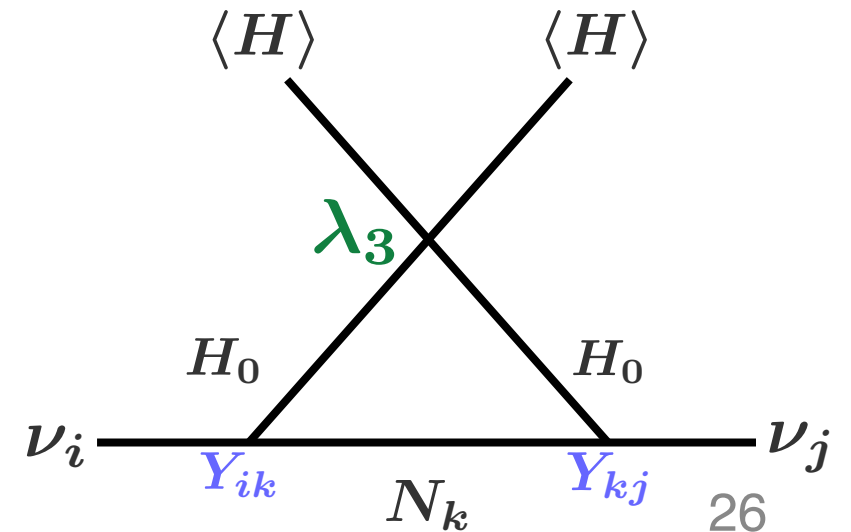
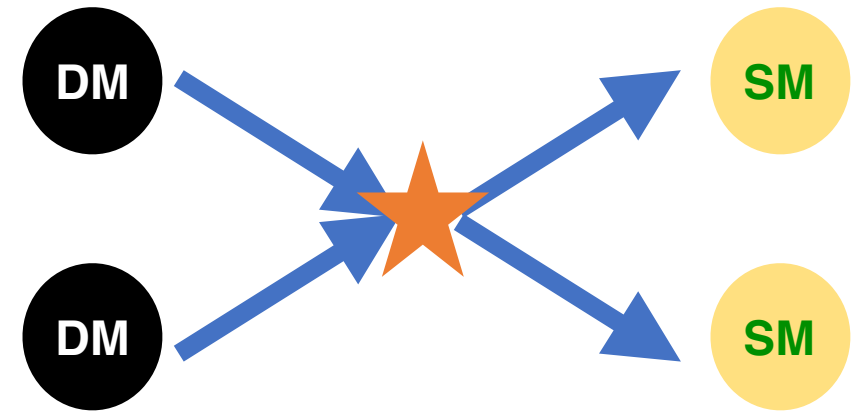
SM + additional $SU(2)_W$ doublet + 3 right-handed neutrinos can explain following phenomena:

- **Dark matter**

The inert (scalar) dark matter
The lightest right-handed neutrino

- **Neutrino masses**

Right-handed neutrino



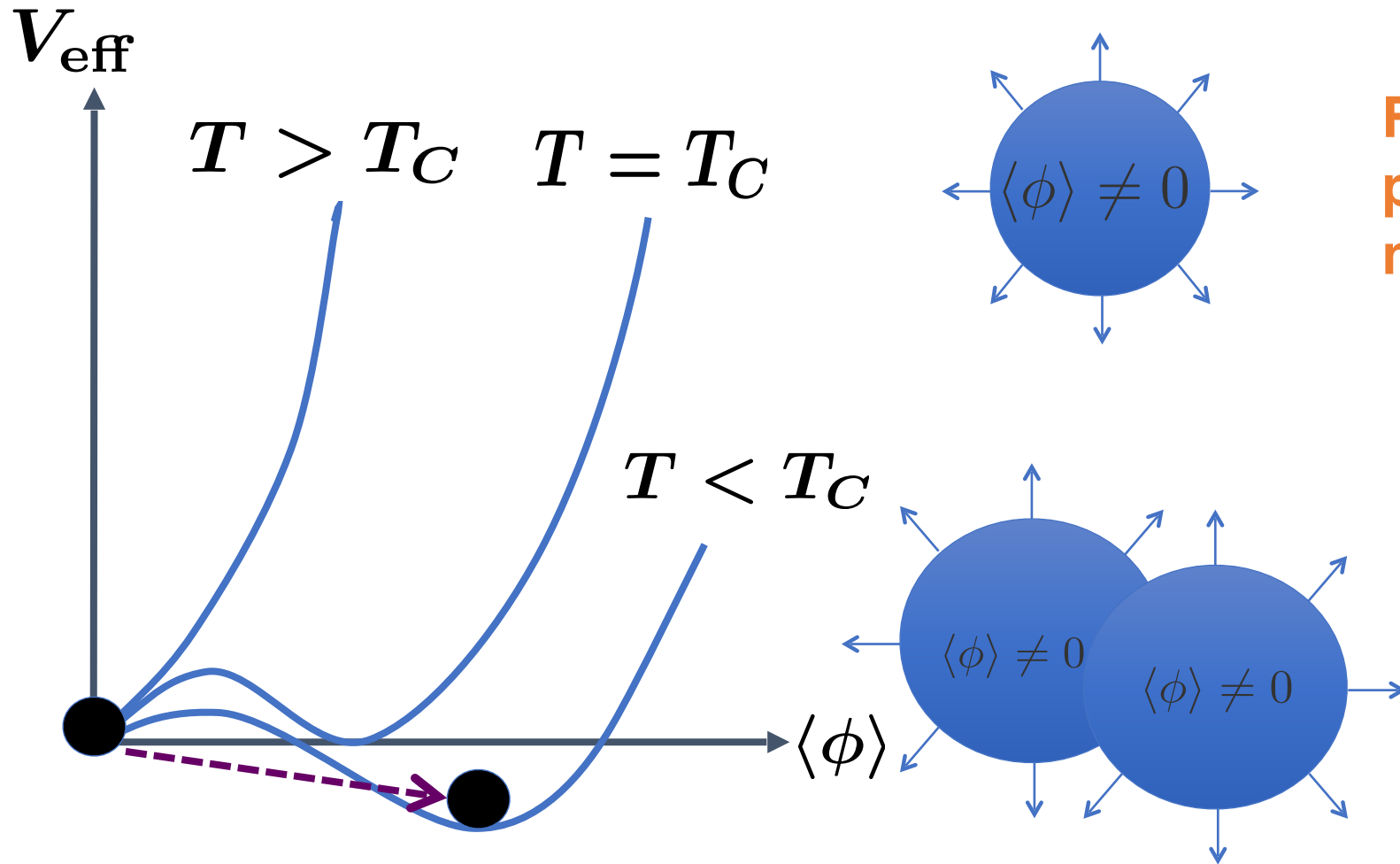
Summary of Scotogenic Model

- **The minimal scotogenic model can explain the DM and tiny neutrino masses.**
- **The lightest neutral inert scalar or the right-handed neutrino can be a DM candidate.**
- **In the scalar DM scenario, the direct detection constraint is stringent.**
- **In the fermion DM scenario, LFV processes give a stringent constraint (depending the size of Yukawa coupling).**

Outline

- **Minimal Scotogenic Model**
- **A First-Order Electroweak Phase Transition**
- **Results**

First-order Phase Transitions



Tunneling!

First-order phase transition proceeds through bubble nucleation.

Three sources of the Gravitational Wave:

Bubble collisions

[Kosowski et al. 1992]

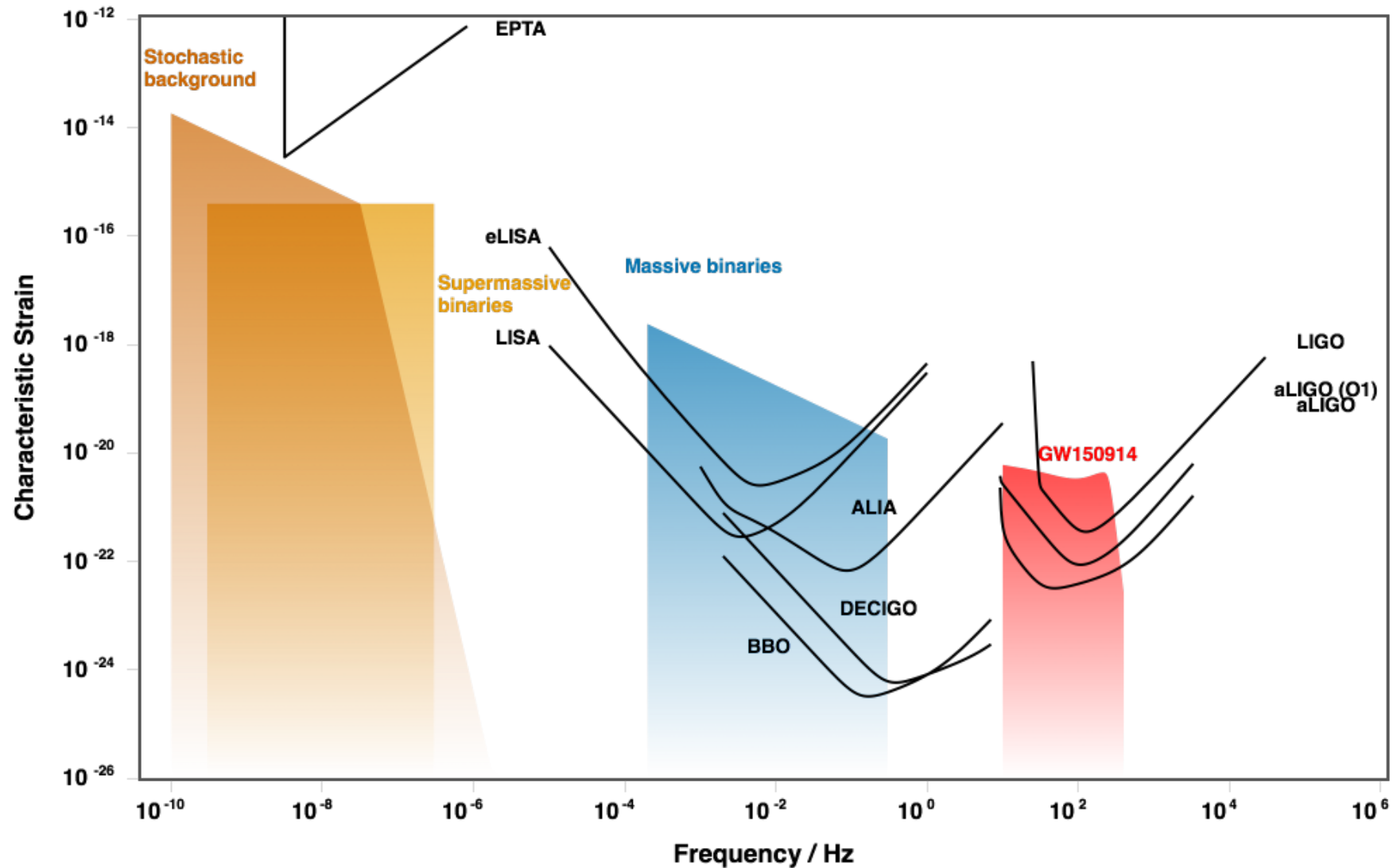
Sound Waves of the plasma

[Hindmarsh et al. 2014]

Turbulence of the plasma

[Kamionkowski et al. 1993]

Detections of Gravitational Wave (GW)



Electroweak Phase Transition (EWPT)

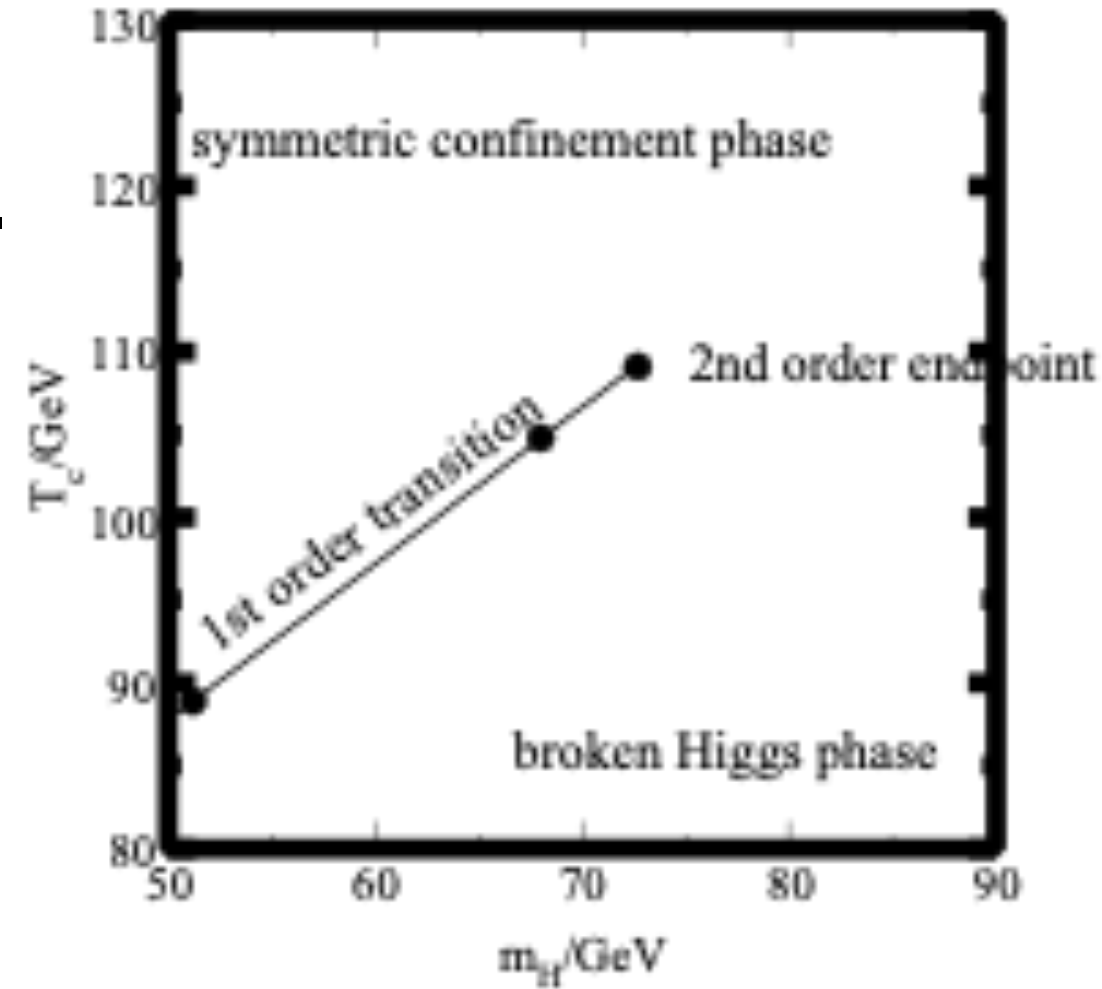
[M. LAINE (2000)]

- EWPT in the SM is not of first order.

[arXiv:9510020]

- EWPT in the inert doublet model can be of strong first order.

[arXiv:1204.4722]



The scalar potential

$$V = \lambda_{\text{SM}} \left(|H|^2 - \frac{v_{\text{SM}}^2}{2} \right)^2 + m_1^2 |S|^2 + \lambda_1 |H|^2 |S|^2$$

SM Higgs potential

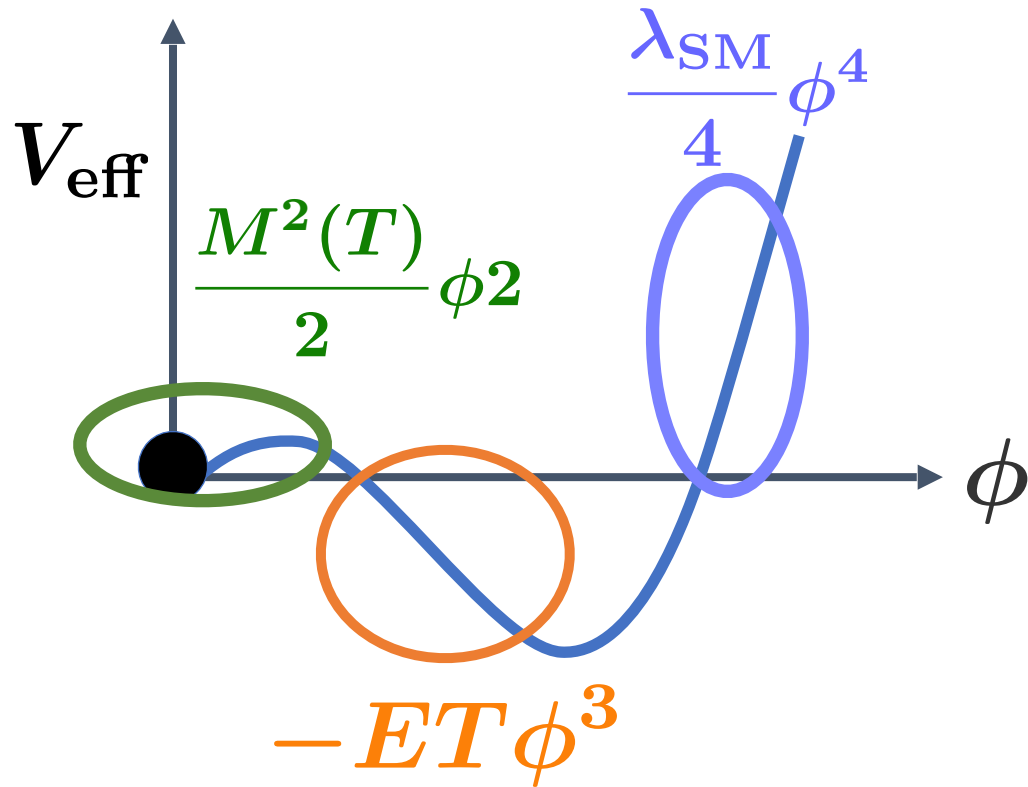
$$+ \lambda_2 |H^\dagger S|^2 + [\lambda_3 (H^\dagger S)^2 + \text{h.c.}] + \lambda_S |S|^4$$

H : SM Higgs doublet S : $SU(2)_W$ doublet

The tree level potential is same as SM.

How is the strength of the phase transition changed?

A Potential Barrier



Thermal effective potential at one-loop order

$$V_{\text{eff}} = \frac{M^2(T)}{2} \phi^2 - ET \phi^3 + \frac{\lambda_{\text{SM}}}{4} \phi^4$$

Thermal fluctuation

**Potential barrier comes from bosons!
(Matsubara zero-modes)**

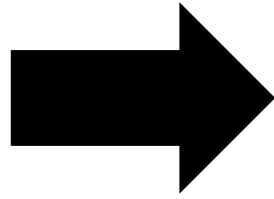
Standard Model: $SU(2)_W \times U(1)_Y$

This model: $SU(2)_W \times U(1)_Y$ + **one scalar doublet**

Overview of Calculation Method

Fix a model

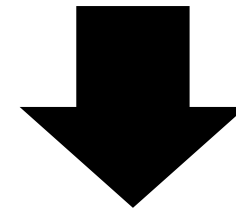
$$\mathcal{L}(\phi, \psi, A_{\mu}^a, \dots)$$



Quantum and Thermal effects

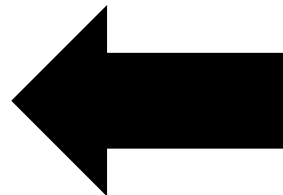
$$V_{\text{CW}}(\phi) \text{ and } V_{\text{Thermal}}(\phi)$$

Solve bounce eq.



GW spectrum

$$\square h_{\mu\nu} = 16\pi G T_{\mu\nu}$$



Bubble dynamics

Bubble wall velocity: \mathcal{V}

Efficiency factor: κ

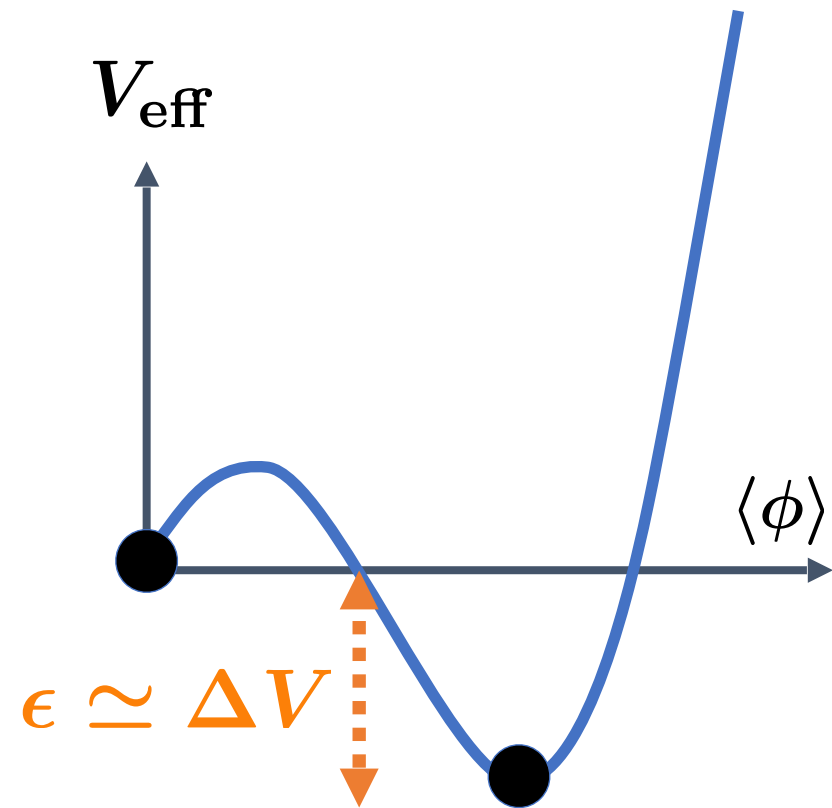
etc..

Important parameters for GW

GW amplitude is related to two important parameters.

Vacuum energy difference: α

Duration of phase transition: β



$$\alpha \sim \frac{\epsilon}{\rho_{\text{rad}}} \quad \Gamma \simeq \Gamma_0 e^{\beta t}$$

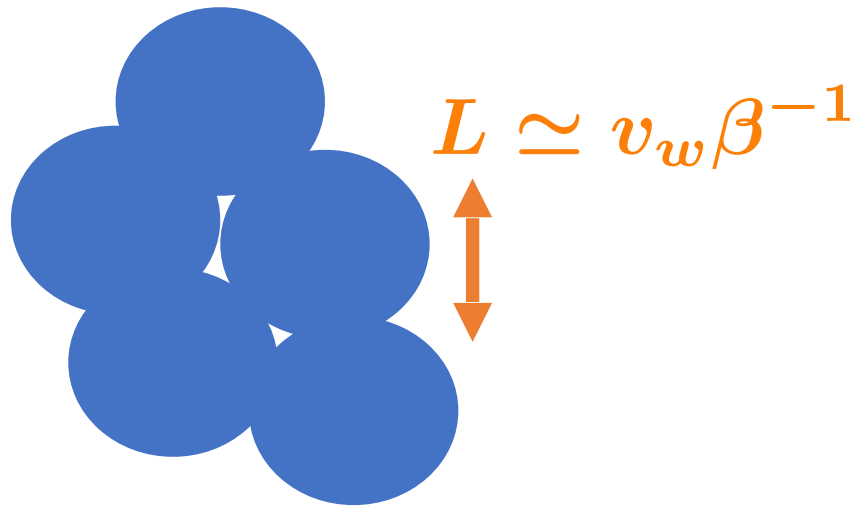
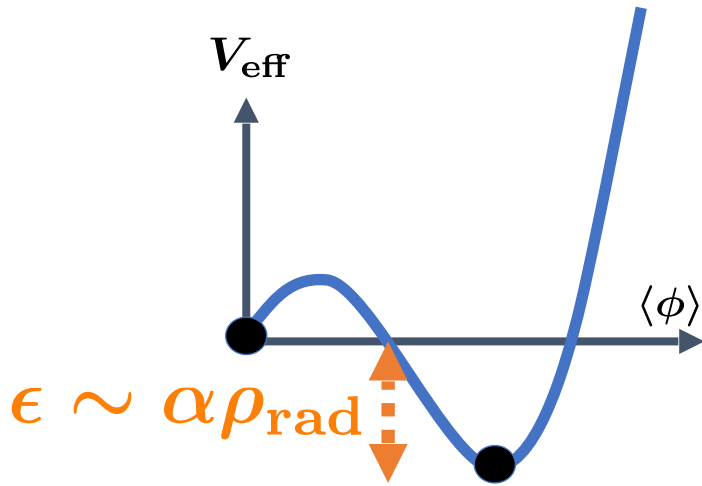
Bubble nucleation rate per unit time per unit volume

Vacuum decay rate

$$\Gamma \sim T^4 e^{-\frac{S_3}{T}}$$

Tunneling occurs at $\Gamma(T_*)/H^4(T_*) \sim 1$

GW amplitude



Bubble size at collision: $L \simeq v_w \beta^{-1}$

$$\Omega_{\text{GW}} \sim \frac{\rho_{\text{GW}}}{\rho_{\text{crit}}} \quad \rho_{\text{crit}} \simeq (1 + \alpha) \rho_{\text{rad}}$$

$$\rho_{\text{GW}} \sim E_{\text{GW}} / L^3 \quad \text{Bubble volume} \sim L^3$$

$$E_{\text{GW}} \sim \int dt P_{\text{GW}} \sim \beta^{-1} P_{\text{GW}}$$

Quadra-pole formula: $P_{\text{GW}} \sim G \dot{E}_{\text{kinetic}}^2$

$$\dot{E}_{\text{kinetic}} \sim \beta E_{\text{kinetic}} \quad E_{\text{kinetic}} \sim \kappa \epsilon L^3$$

efficiency factor: κ $G \sim \frac{H^2}{(1 + \alpha) \rho_{\text{rad}}}$ (Newton const.)

$$\Omega_{\text{GW}} \sim \left(\frac{H}{\beta} \right)^2 \left(\frac{\kappa \alpha}{1 + \alpha} \right)^2 v_w^3$$

Large vacuum energy and long-duration enhance the GW amplitude.

The Scalar Potential

$$V = \lambda_{\text{SM}} \left(|H|^2 - \frac{v_{\text{SM}}^2}{2} \right)^2 + m_1^2 |S|^2 + \lambda_1 |H|^2 |S|^2$$

SM Higgs potential

$$+ \lambda_2 |H^\dagger S|^2 + [\lambda_3 (H^\dagger S)^2 + \text{h.c.}] + \lambda_S |S|^4$$

H : SM Higgs doublet S : $SU(2)_W$ doublet

$$\lambda_{H(A)} \equiv \lambda_1 + \lambda_2 \pm 2\lambda_3$$

$$\lambda_1, \lambda_H, \lambda_A \rightarrow 0$$

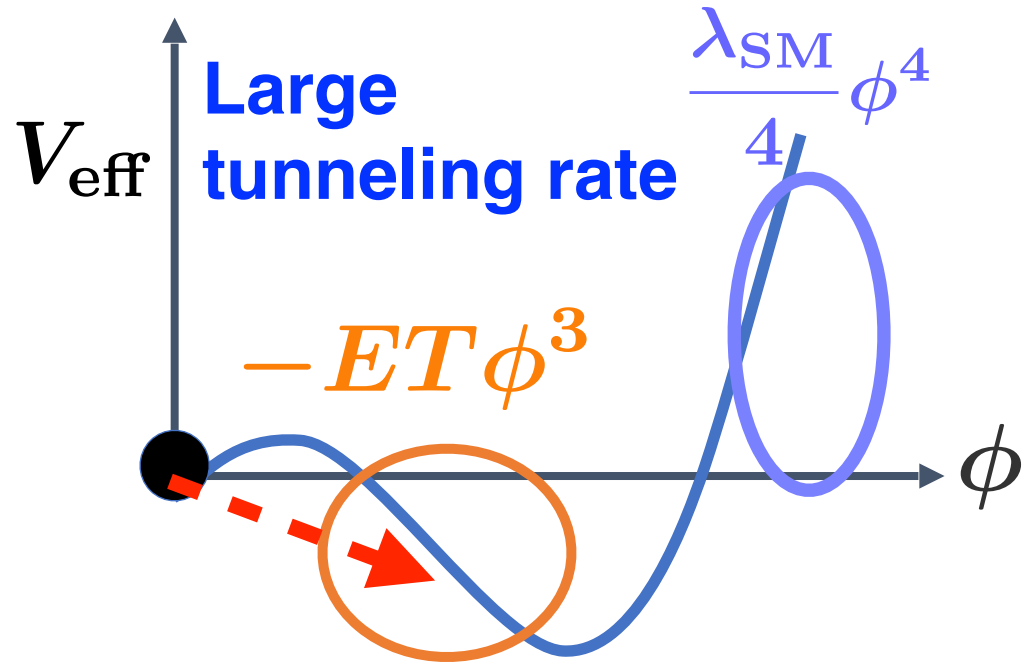
$$m_1 \rightarrow \infty$$

No couplings between SM Higgs and the additional scalar!

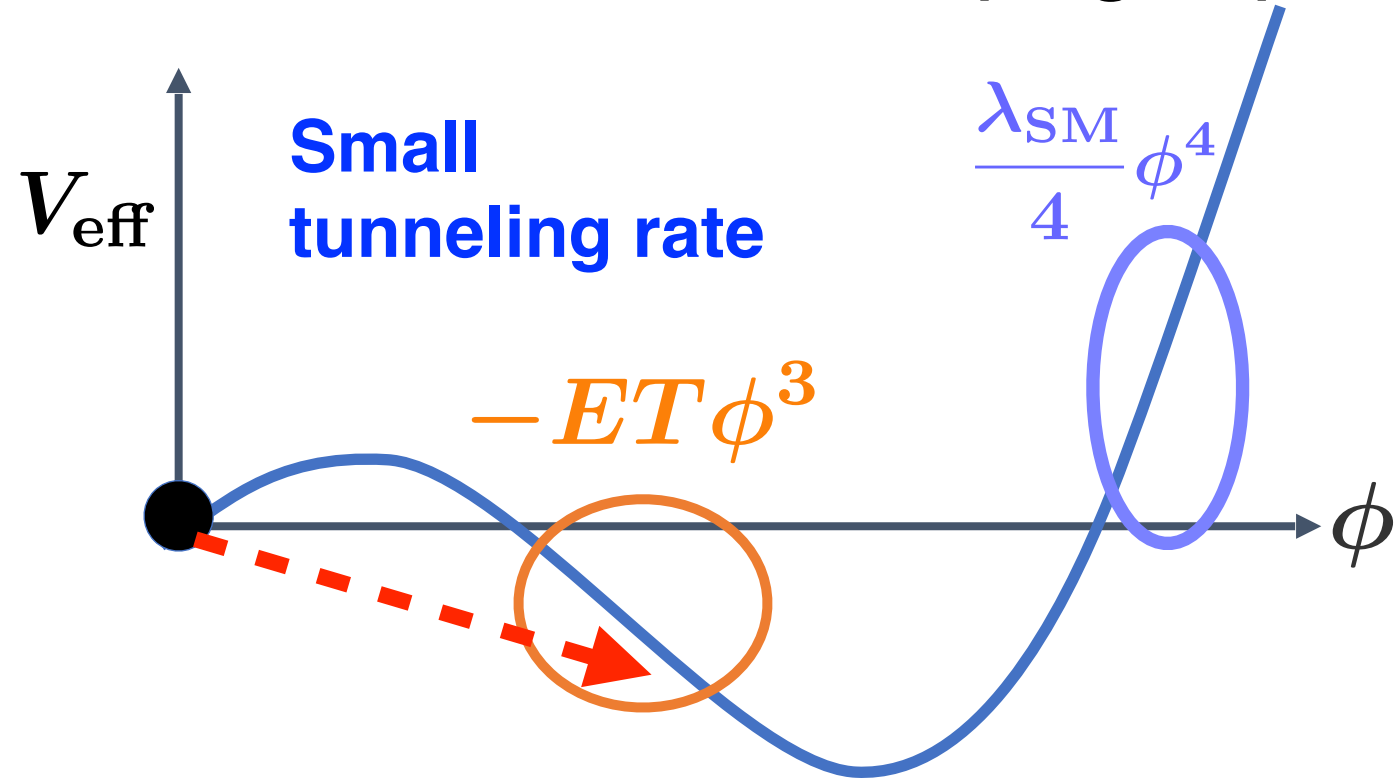
S is heavy so that it does not contribute to the thermal effective potential.

Effective Potential and Tunneling Rate

Standard Model (small E)

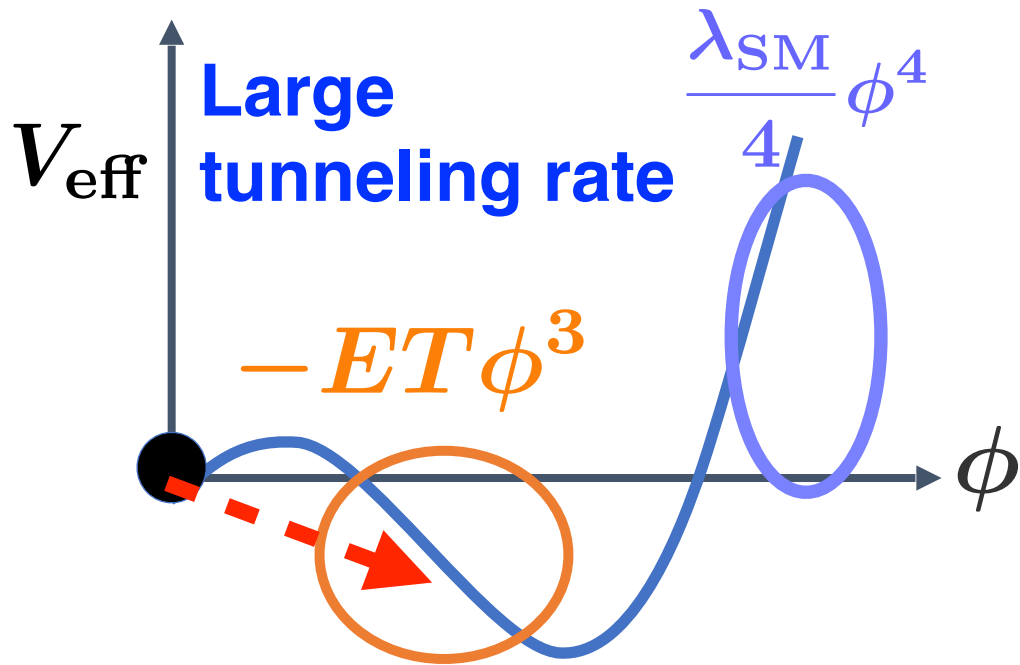


SM + one scalar doublet (large E)



Effective Potential and Tunneling Rate

Standard Model (small E)

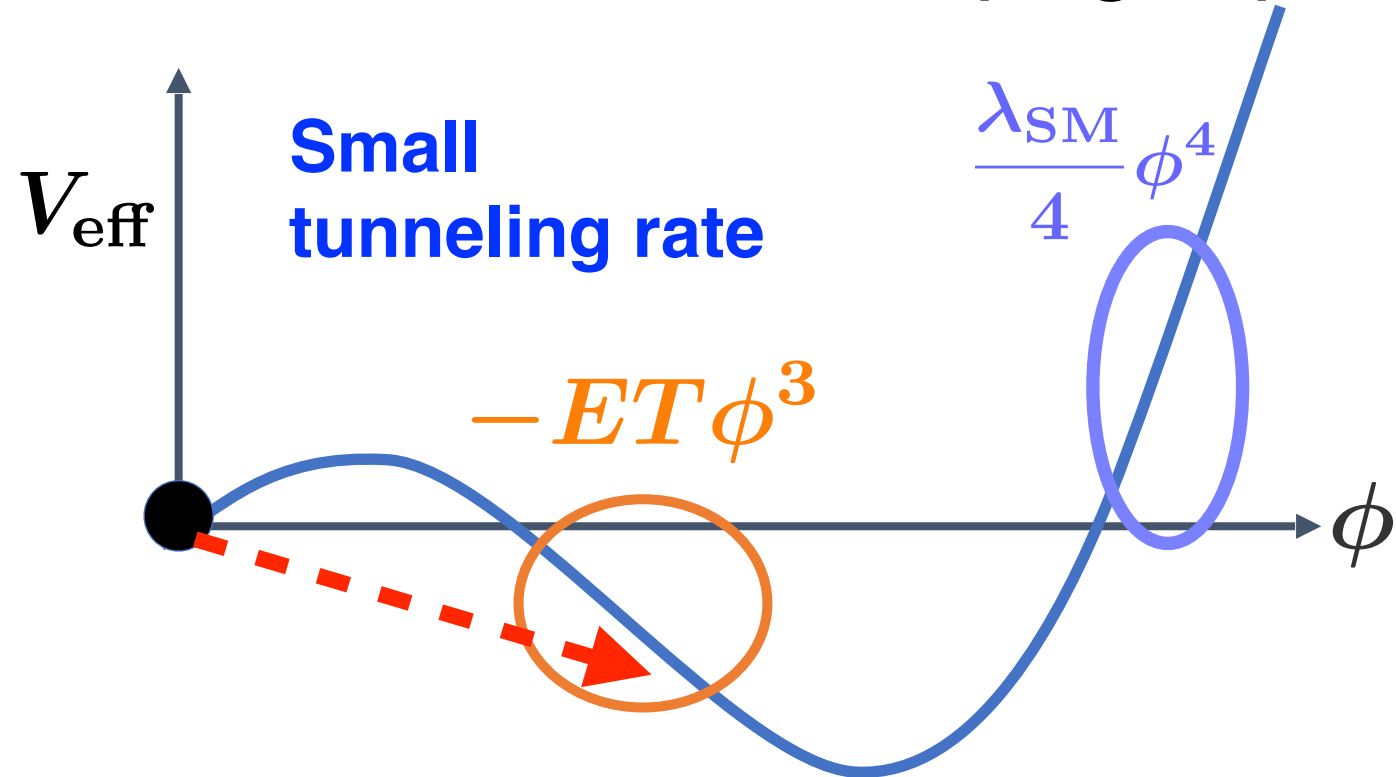


$$\phi(T_*)/T_* \ll 1$$

α :small β :large

Ω_{GW} :small

SM + one scalar doublet (large E)



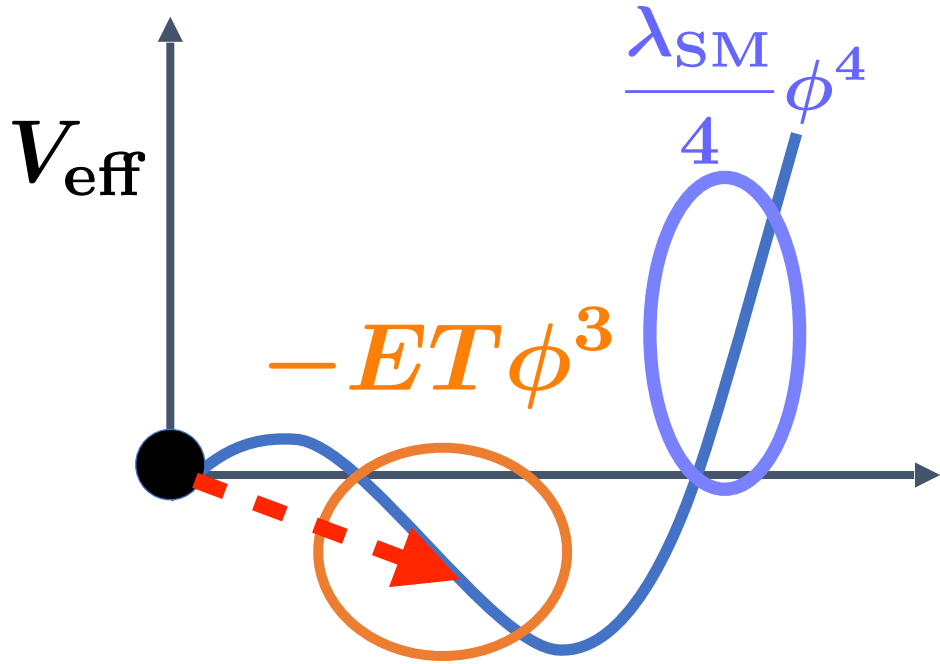
$$\phi(T_*)/T_* \gtrsim 1$$

α :large! β :small!

Ω_{GW} :large!

Effective Potential and Tunneling Rate

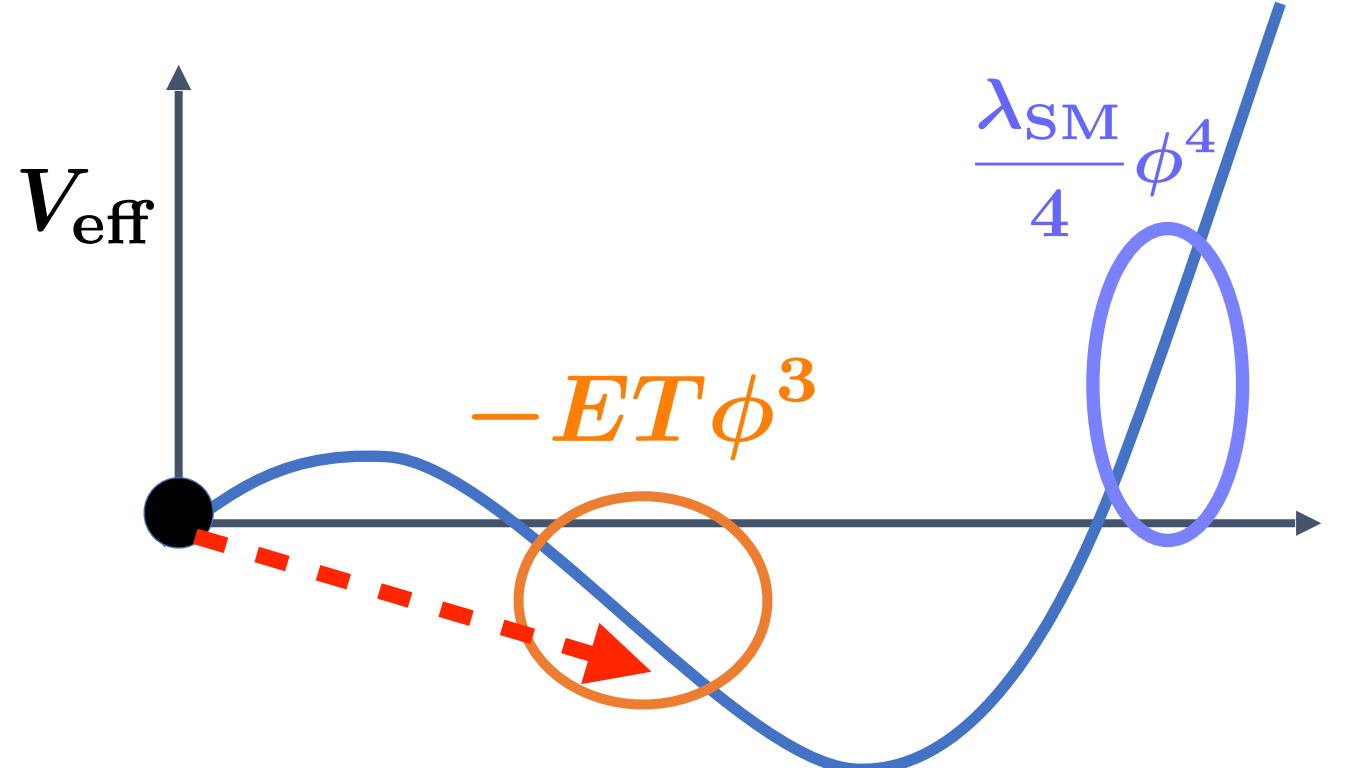
Standard Model (small E)



small: $\lambda_1, \lambda_H, \lambda_A$
 large: m_1

$\Omega_{\text{GW}} : \text{small}$

SM + one scalar doublet (large E)



Large: $\lambda_1, \lambda_H, \lambda_A$
 small: m_1

$\Omega_{\text{GW}} : \text{large!}$

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Parameter Scan

Investigate the parameter regime which satisfies: $\frac{\phi(T_*)}{T_*} > 1$

[cosmoTransitions: arXiv:1109.4189]

Imposing following conditions:

(1) Perturbative conditions

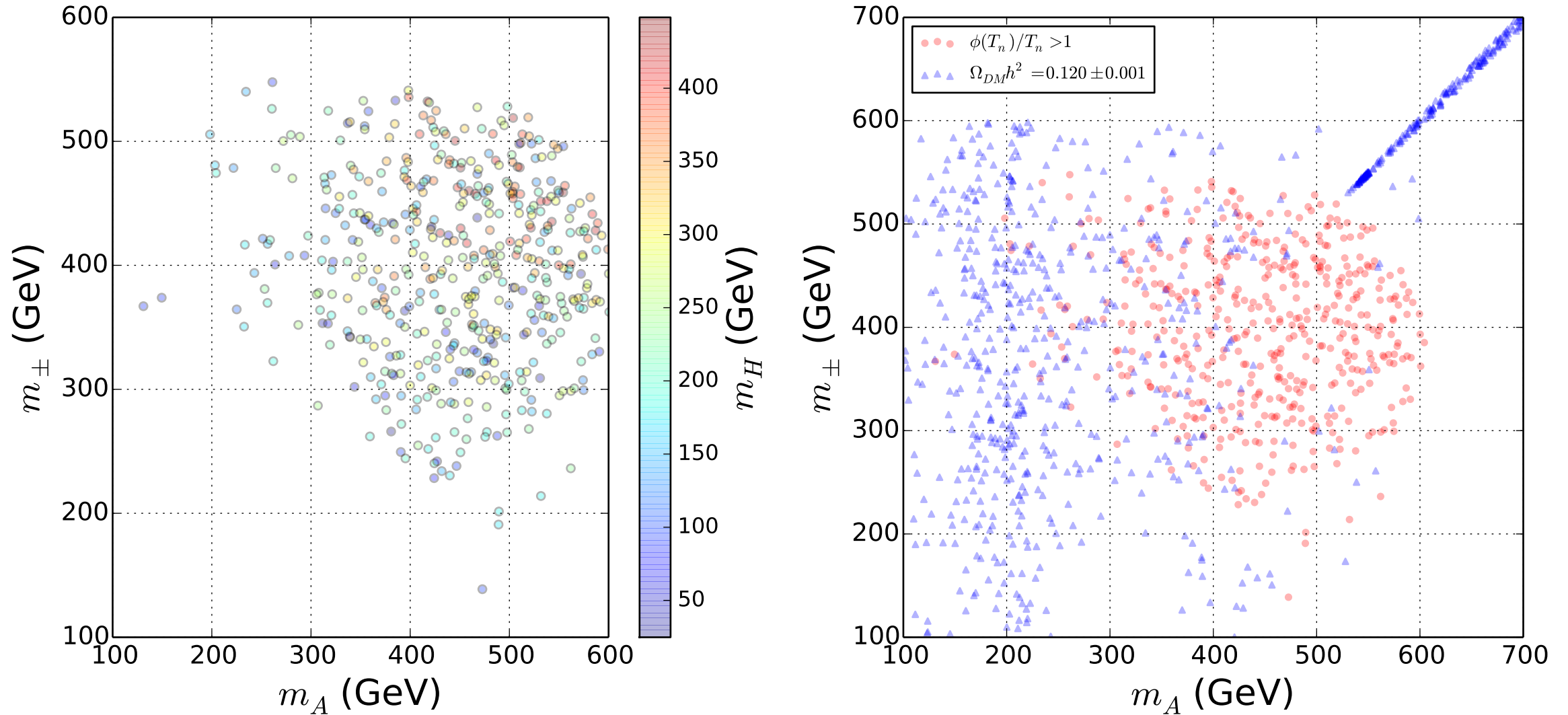
$$|\lambda_i| < 4\pi, |Y_{ij}| < \sqrt{4\pi}, g_i < \sqrt{4\pi}$$

(2) The vacuum stability and unitarity conditions

$$\lambda_S > 0, \lambda_1 + 2\sqrt{\lambda_{\text{SM}}\lambda_S} > 0, \lambda_1 + \lambda_2 - \frac{|\lambda_3|}{2} + 2\sqrt{\lambda_{\text{SM}}\lambda_S} > 0$$

**(3) The lightest state is the CP-even scalar.
(scalar DM case)**

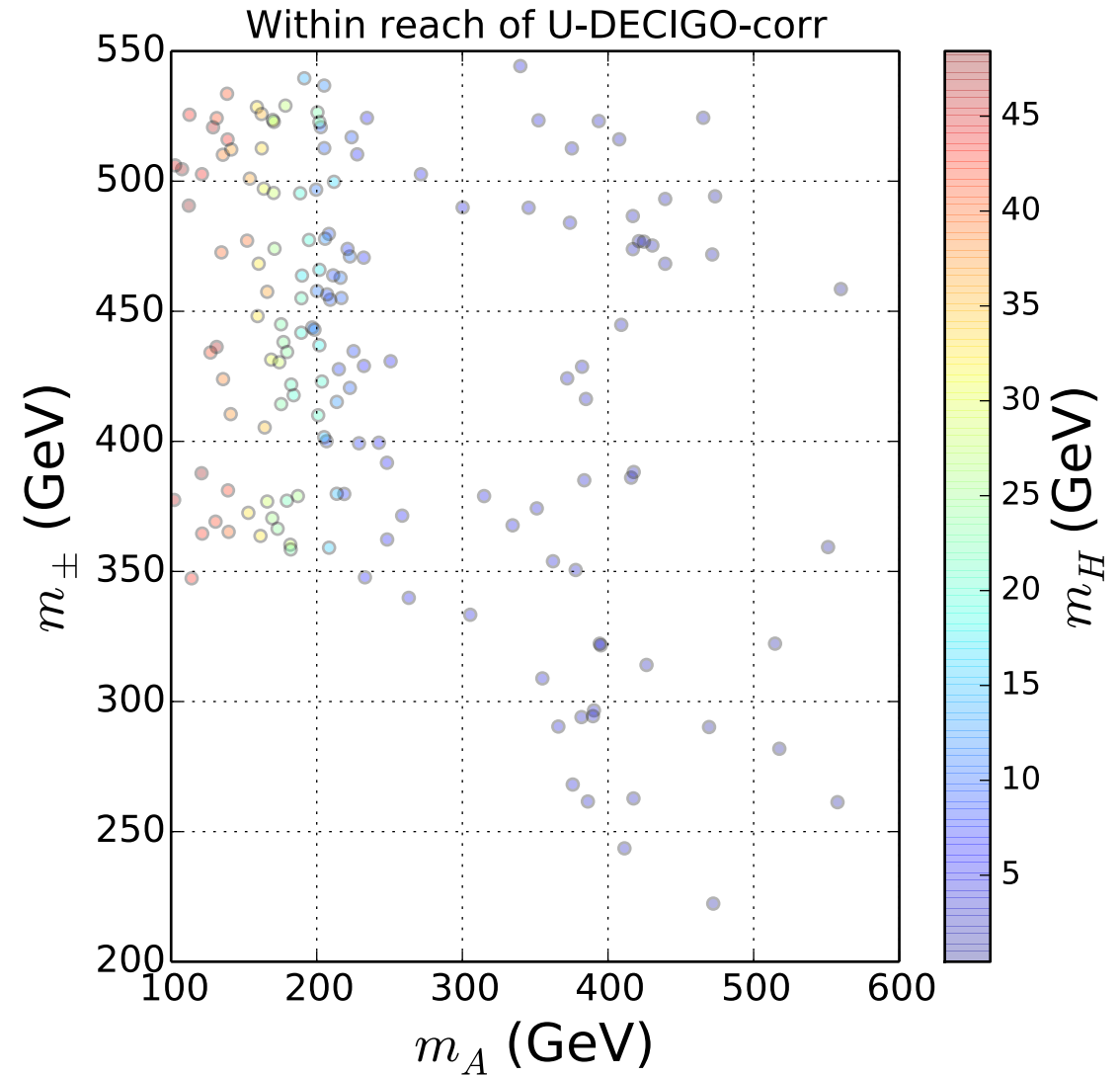
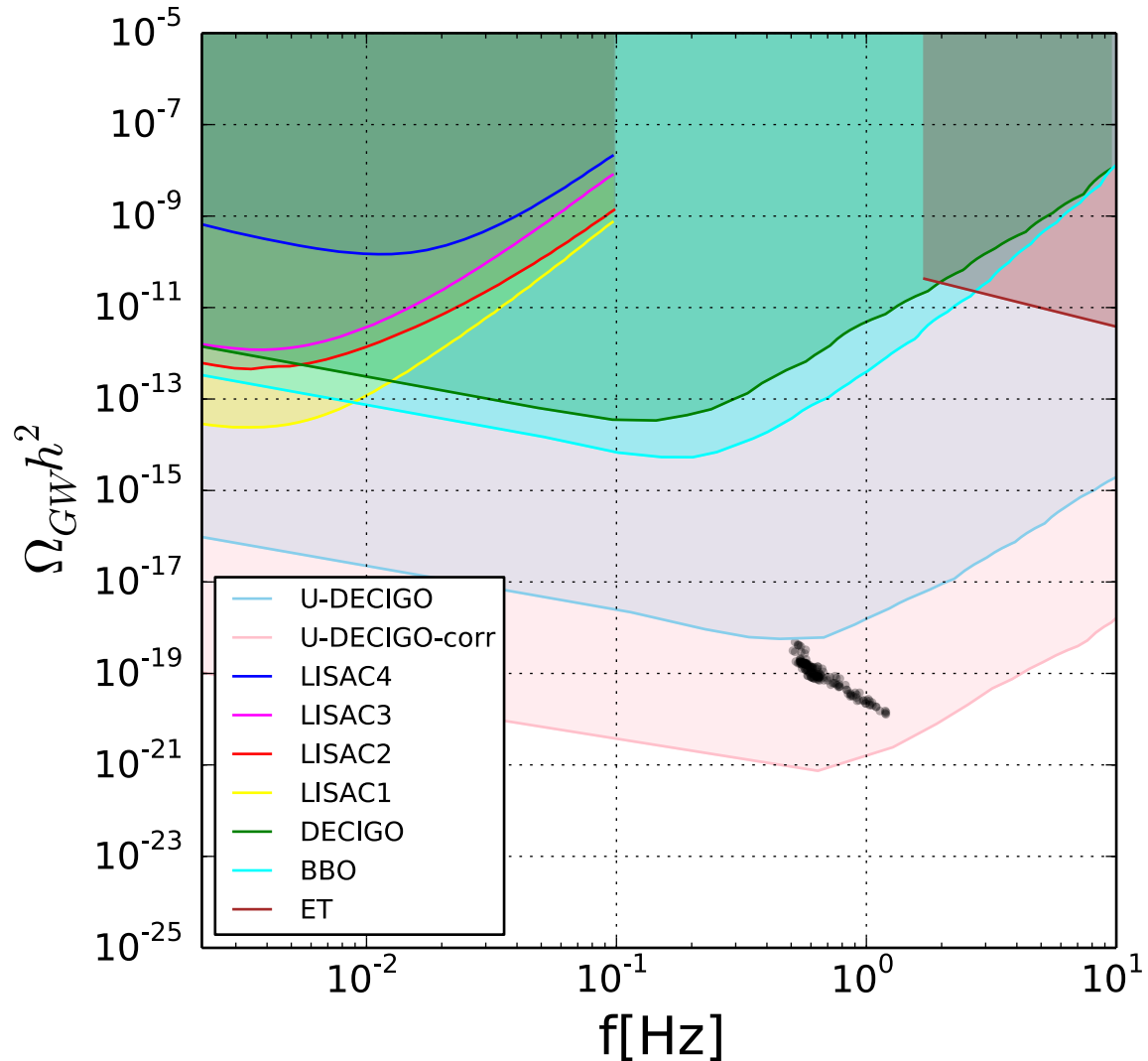
Strong First-Order Phase Transition (Scalar DM)



$\lambda_H < \lambda_{1,A} \simeq \mathcal{O}(1)$ $m_1 \ll 50\text{GeV}$ **A small DM mass is preferred!**

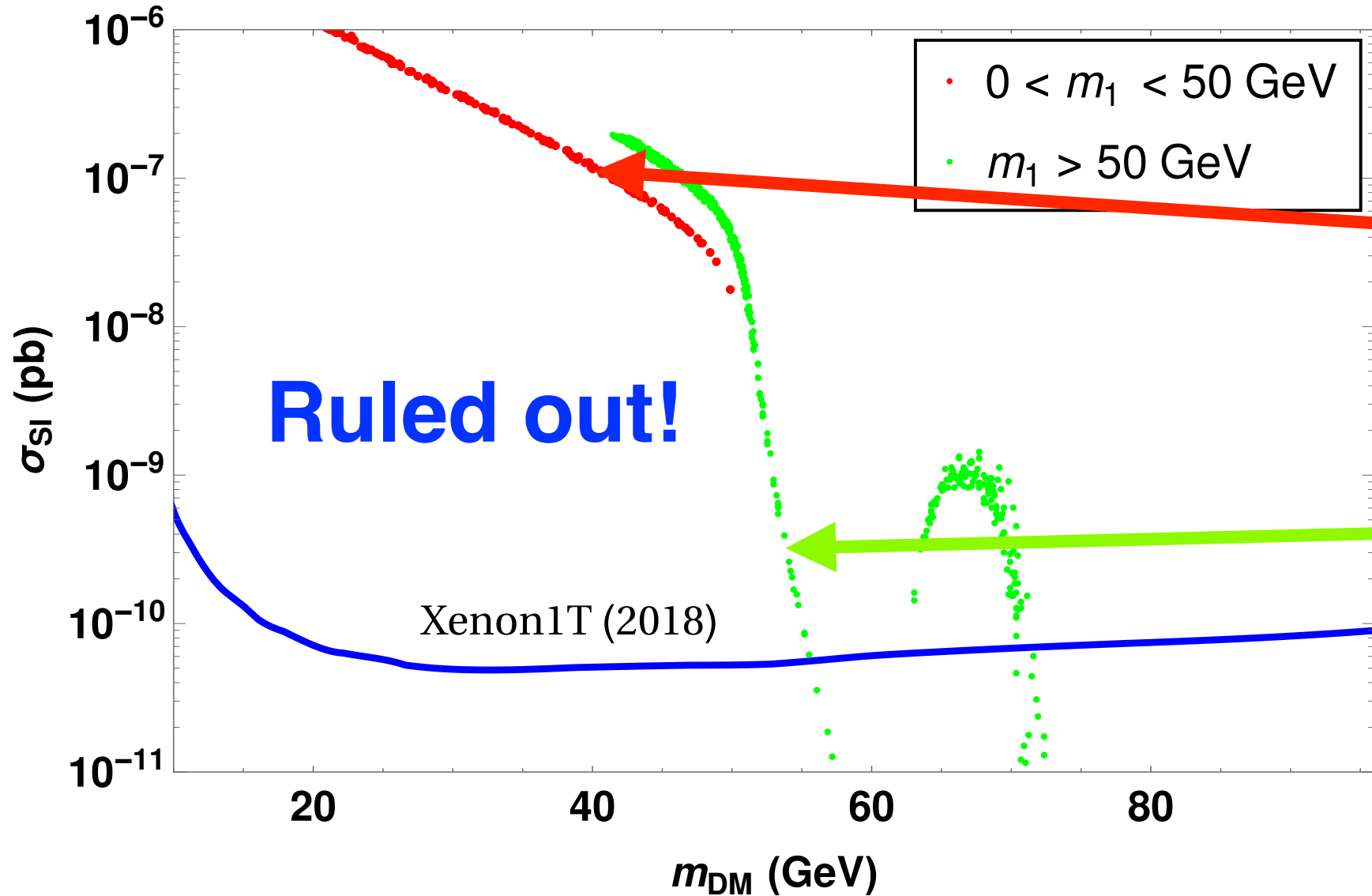
Gravitational Wave Signals (Scalar DM)

Assumption: 100% DM!



$$\lambda_H < \lambda_{1,A} \simeq \mathcal{O}(1) \quad m_1 \ll 50 \text{ GeV}$$

Direct Detection Constraint Revisited



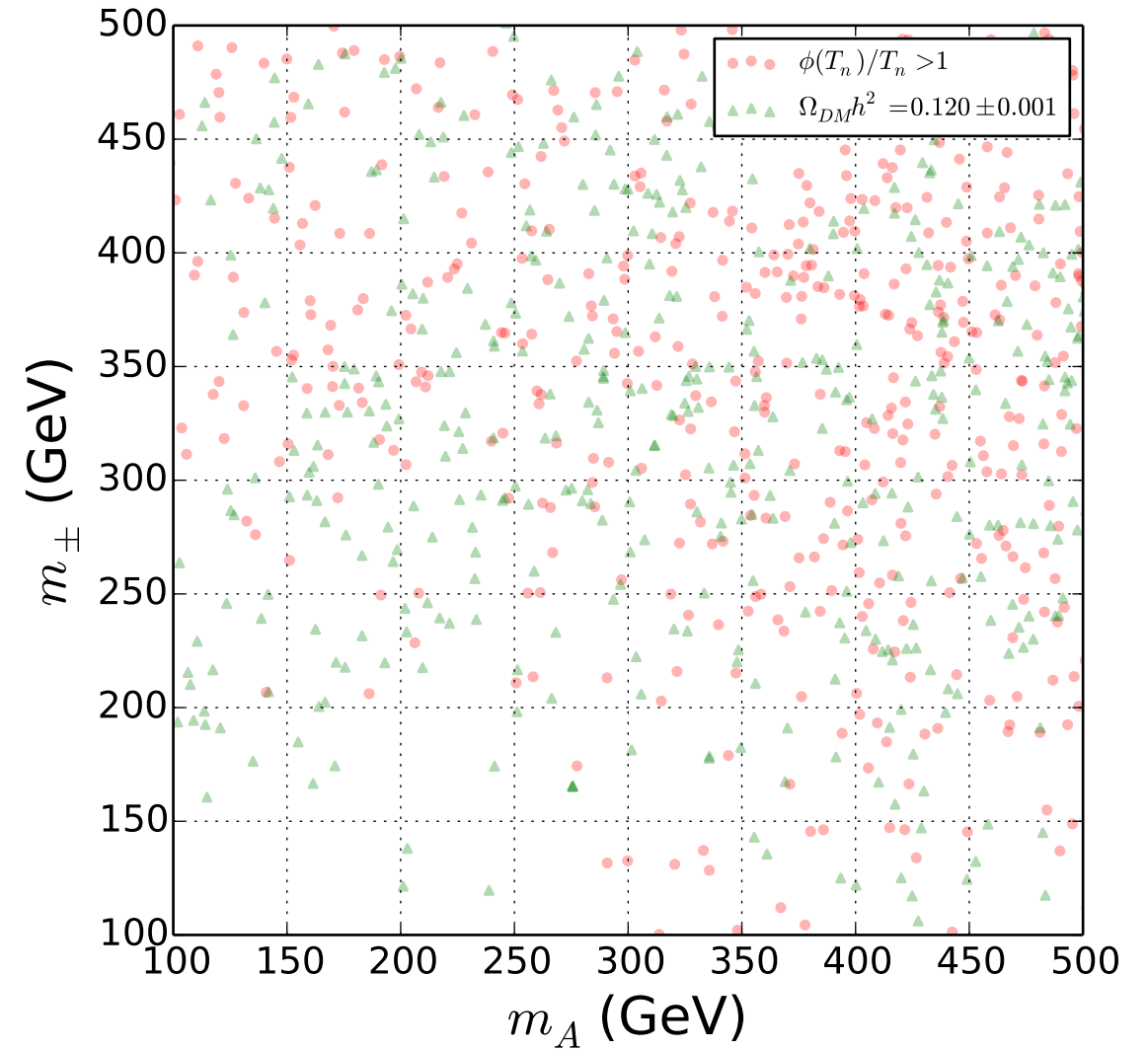
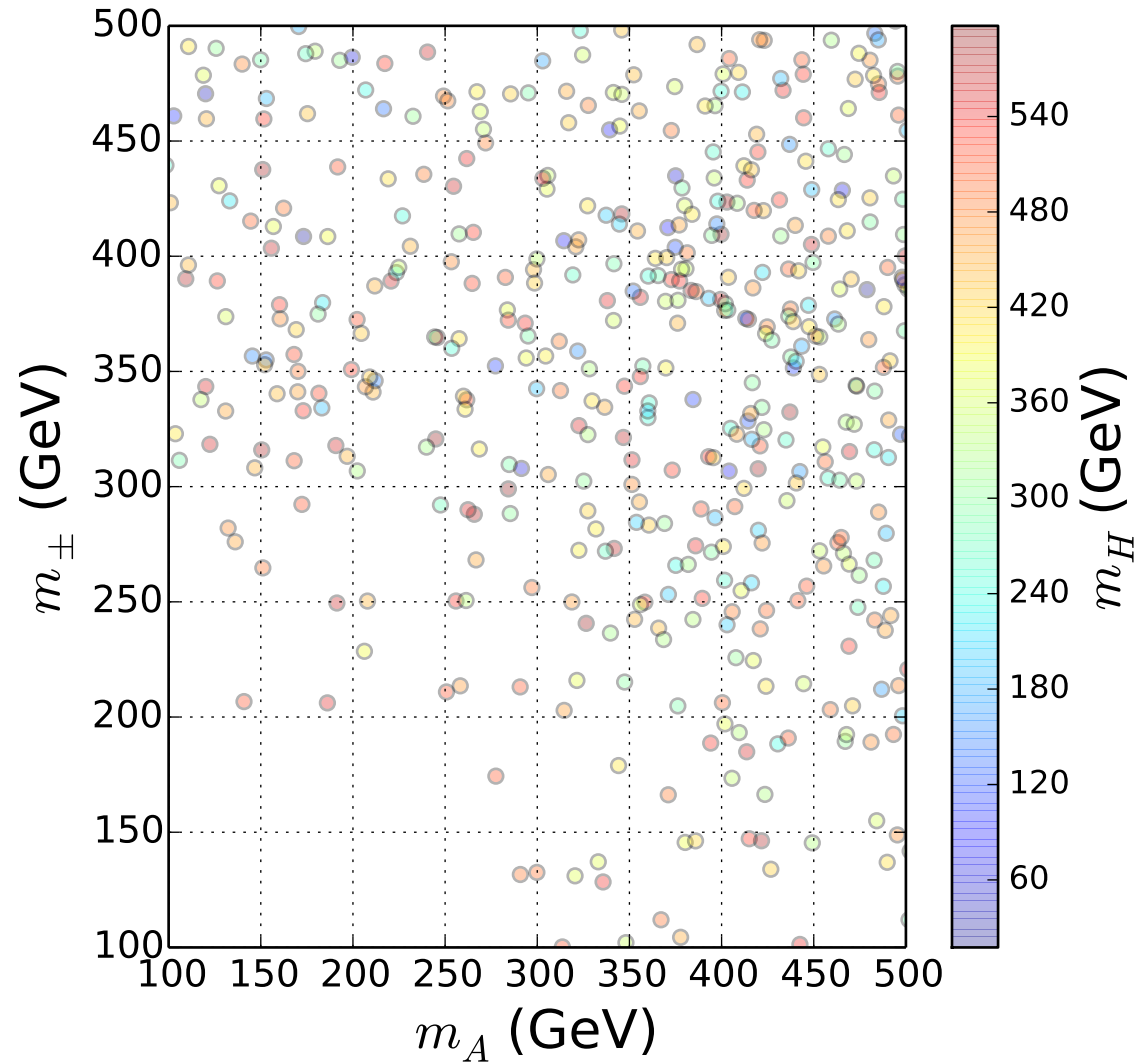
EWPT is of strong first order!

EWP T is “not” of strong first order!

Why?

- **A neutral component of the inert scalar must be lightest!**
- **A large λ_H and small m_1 is needed to enhance the GW signals.**
- **A large λ_H and small m_1 is constrained by the direct detection constraint.**

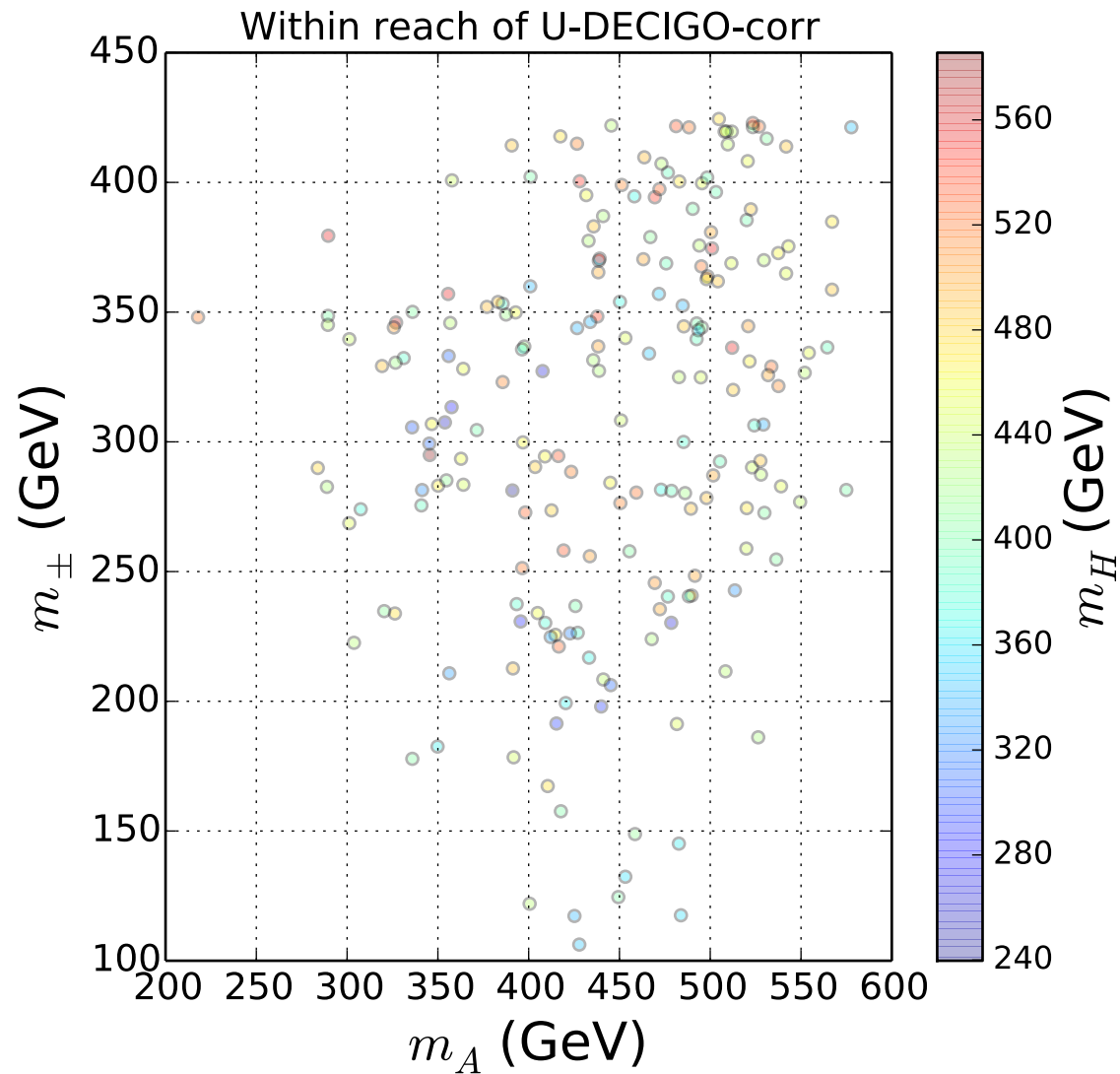
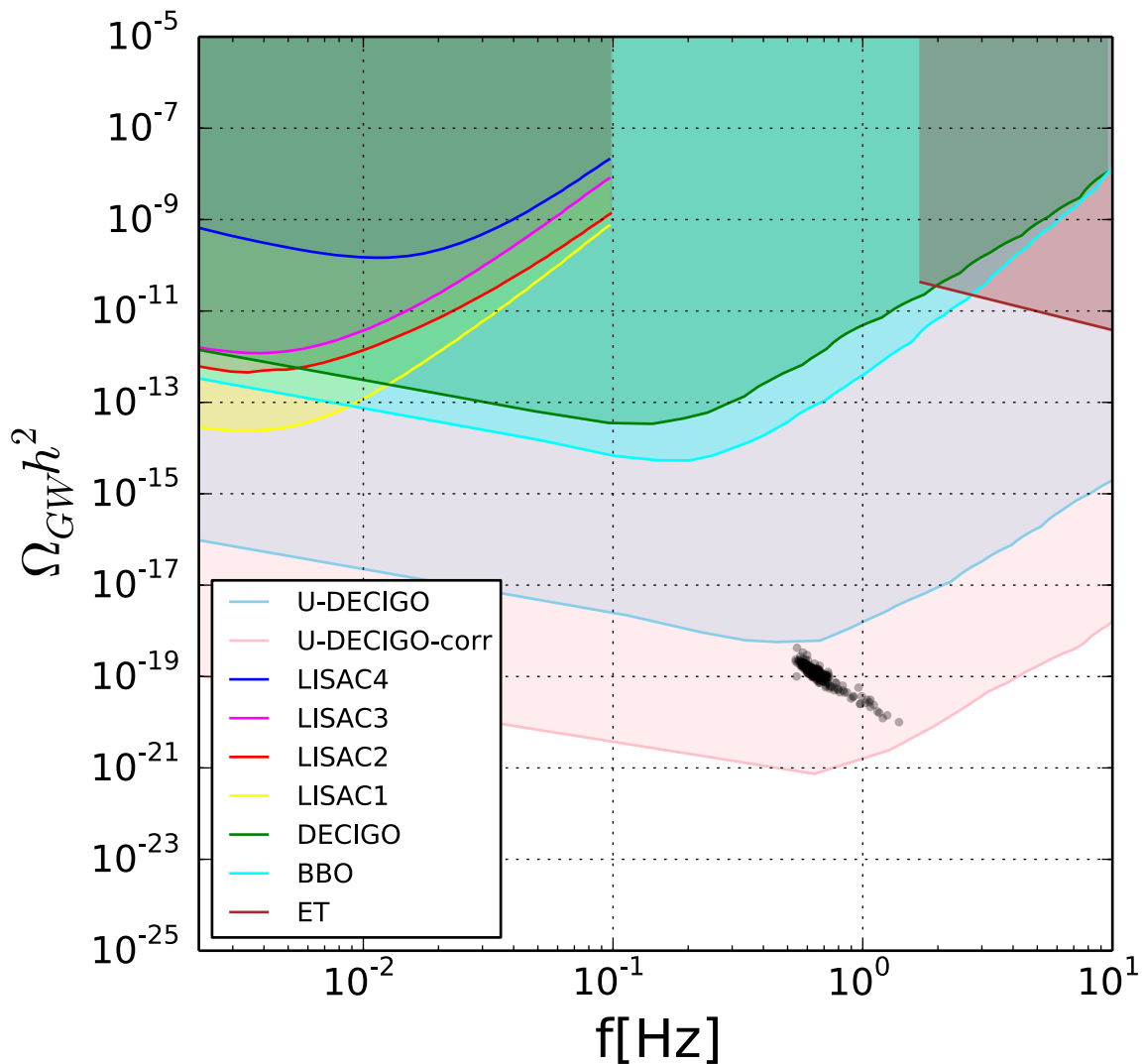
Strong First-Order Phase Transition (Fermion DM)



$$\lambda_{1,H,A} \sim \mathcal{O}(1) \quad m_1 \ll 50 \text{ GeV}$$

Gravitational Wave Signals (Fermion DM)

Assumption: 100% DM!



$$\lambda_{1,H,A} \sim \mathcal{O}(1) \quad m_1 \ll 50 \text{ GeV}$$

Why?

- ~~A neutral component of the inert scalar must be lightest!~~

As long as the right-handed neutrino is lightest, the charged scalar can be light!

- A large λ_H and small m_1 is needed to enhance the GW signals.

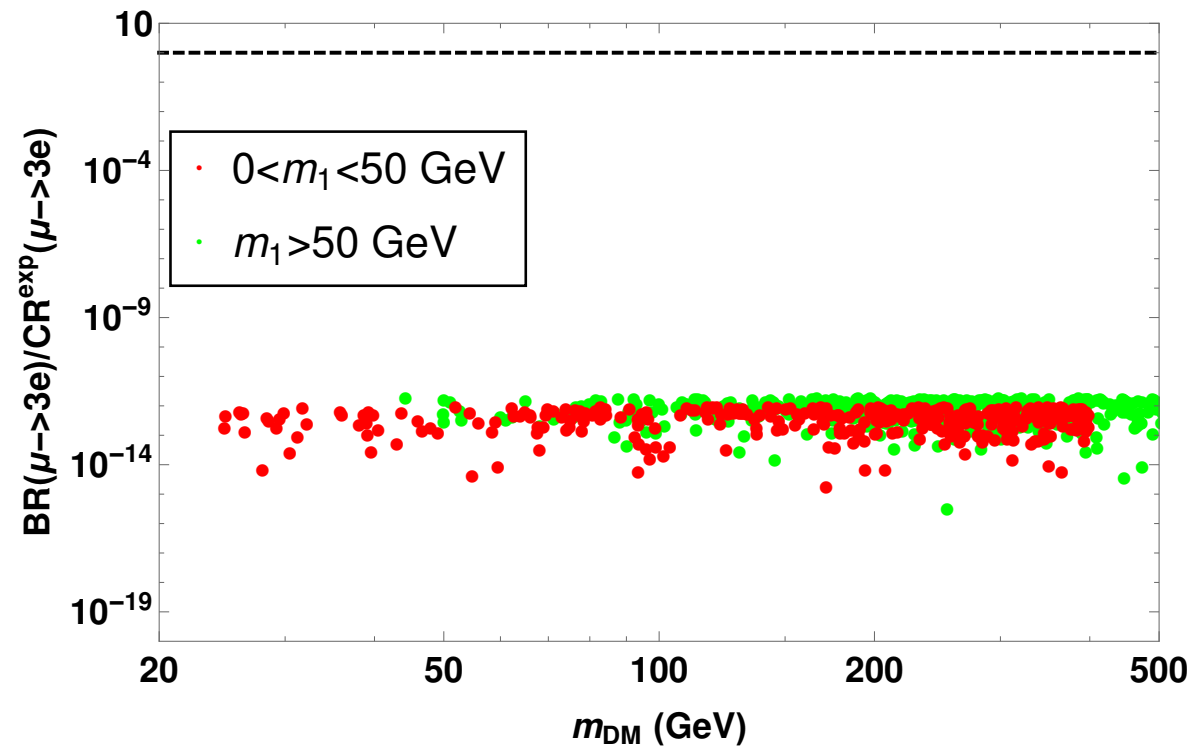
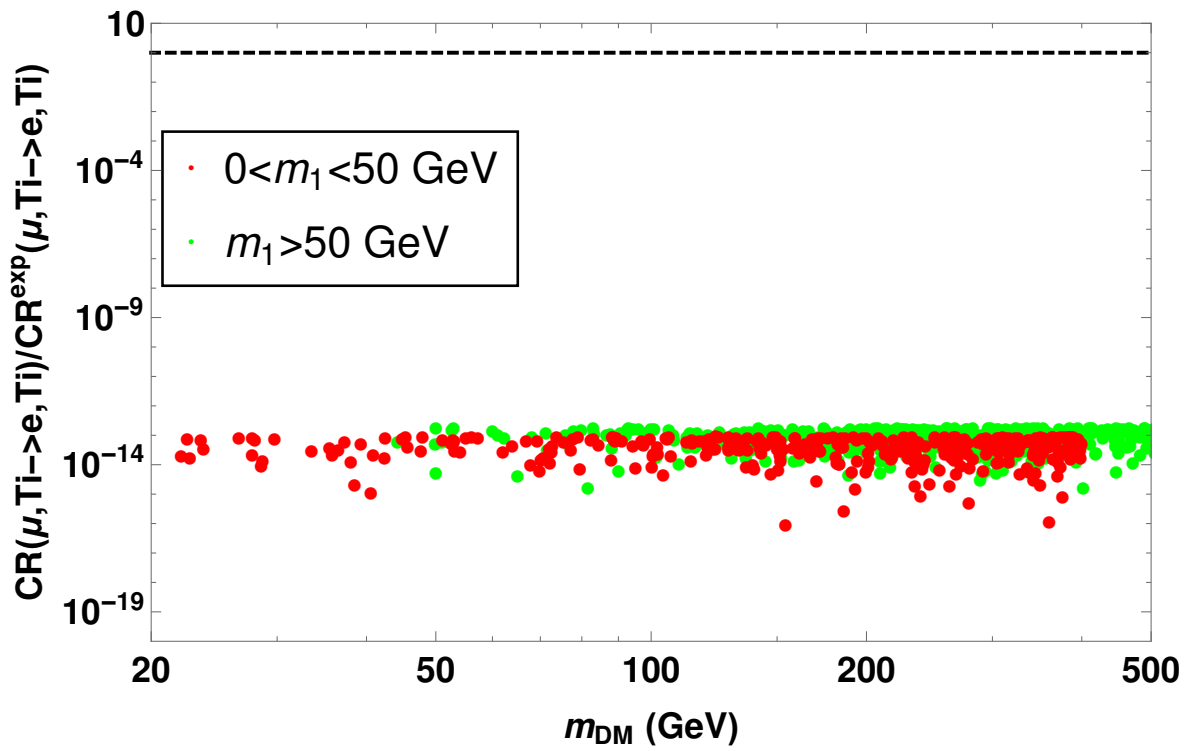
- ~~A large λ_H and small m_1 is constrained by the direct detection constraint.~~

In the fermion DM case, the direct detection constraint is not important!

Summary

- **The inert scalar and RH neutrinos are introduced in minimal scotogenic model.**
- **The scalar DM mass and coupling are stringently constrained by the direct detection experiment.**
- **GW signals within reach of U-DECIGO-corr with 100% DM are almost ruled out by the DM direct detection experiment in the scalar DM scenario.**
- **GW signals within reach of U-DECIGO-corr with 100% DM are **not** ruled out in the fermion DM scenario.**

back up



Negative cubic terms?

$$V_{\text{eff}} \supset -\frac{T}{6\pi}(m_{\pm}^2(\phi) + \Pi_S)^{\frac{3}{2}} - \frac{T}{12\pi}(m_A^2(\phi) + \Pi_S)^{\frac{3}{2}} - \frac{T}{12\pi}(m_H^2(\phi) + \Pi_S)^{\frac{3}{2}}$$

$$\Pi_S = \left(\frac{1}{8}g_2^2 + \frac{1}{16}(g_1^2 + g_2^2) + \frac{1}{2}\lambda_S + \frac{1}{12}\lambda_1 + \frac{1}{24}\lambda_A + \frac{1}{24}\lambda_H \right) T^2.$$

$$m_{\pm}^2(\phi) = m_1^2 + \frac{1}{2}\lambda_1\phi^2 \quad m_A^2(\phi) = m_1^2 + \frac{1}{2}\lambda_A\phi^2 \quad m_H^2(\phi) = m_1^2 + \frac{1}{2}\lambda_H\phi^2$$

$$\Pi_S, \quad m_1 \ll \lambda_{1,H,A}\phi^2$$

Negative cubic term!

$$\Pi_S, \quad m_1 \gg \lambda_{1,H,A}\phi^2$$

Constant term...