Precise Calculation of the Decay Rate of False Vacuum with Multi-Field Bounce

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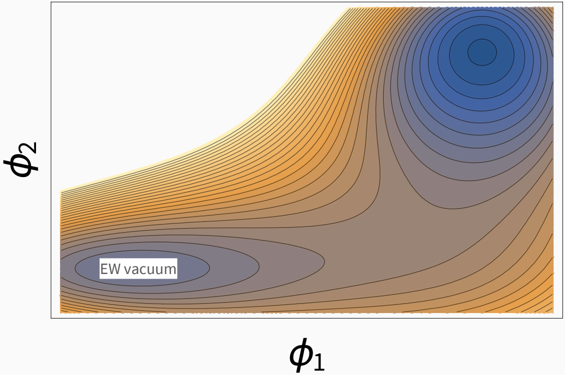
Introduction

Vacuum Stability

What if the electroweak vacuum is not the global minimum?



e.g. color and/or charge breaking minima in supersymmetric models, the standard model, ...

Bubble Nucleation Rate

A meta-stable vacuum finally decays into a lower vacuum.

Source: Nikolodion / iStock / Getty Images Plus

The decay of the vacuum is triggered by the nucleation of a bubble of the lower vacuum through quantum tunneling

$$\text{Lifetime}^{-1} = \underset{\text{inside the Universe}}{\text{Nucleation Rate}}$$

Bubble Nucleation Rate

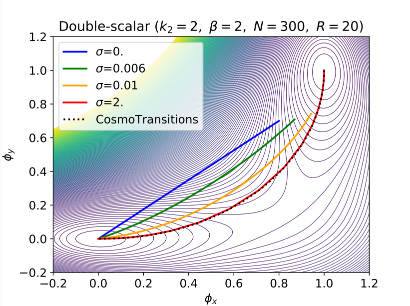
$$\text{Nucleation Rate} : \gamma = \mathcal{A}e^{-\mathcal{B}}$$

per unit volume

$\mathcal{A}$: Quantum corrections to $\mathcal{B}$

$\mathcal{B}$: Bounce action

[*T. Banks, C. M. Bender, T. T. Wu, '73; C. G. Jr. Callan, S. R. Coleman, '77*]



Bounce: $\bar{\phi}$
$O(4)$ symmetric solution to the Euclidean equations of motion connecting the two vacua

Bounce calculated by the gradient flow

[*S. Chigusa, T. Moroi, YS, '20*]

Precise Calculation of Vacuum Decay Rates

To determine the overall factor and to cancel the renormalization scale uncertainty, one needs to calculate $\mathcal{A}$

[*M. Endo, T. Moroi, M. M. Nojiri, YS, '16*]

$$\underbrace{\text{Nucleation Rate}}_{\text{per unit volume}} : \gamma = \mathcal{A} e^{-\mathcal{B}}$$

At the one-loop level,

$$\mathcal{A} \sim \left| \frac{\det' \mathcal{M}}{\det \widehat{\mathcal{M}}} \right|^{-1/2}$$

$$\mathcal{M} = \frac{d^2 S}{d\chi_i d\chi_j}[\bar{\phi}], \ \widehat{\mathcal{M}} = \frac{d^2 S}{d\chi_i d\chi_j}[v]$$

where χ_i's are fluctuations, v is the false vacuum and $\det'$ indicates that zero modes are subtracted appropriately

Current Status

- The standard model (order estimation for gauge zero modes)
  [*G. Isidori, G. Ridolfi, A. Strumia, '01*]
- Stau instability in the MSSM (without gauge fields)
  [*M. Endo, T. Moroi, M. M. Nojiri, YS, '16*]
- Treatment of the gauge zero modes for single-field bounce
  [*M. Endo, T. Moroi, M. M. Nojiri, YS, '17*]
- The standard model (complete one-loop)
  [*A. Andreassen, W. Frost, M. D. Schwartz, '17; S. Chigusa, T. Moroi, YS, '17,'19*]
- The standard model + additional fields (single-field bounce)
  [*S. Chigusa, T. Moroi, YS, '19*]
- The DFSZ axion model (multi-field bounce, but reducible to single-field bounce)
  [*S. Oda, YS, D.S. Takahashi, '19*]

new! Treatment of the gauge zero modes for multi-field bounce
[*S. Chigusa, T. Moroi, YS, '20*]

Contents

Zero Modes

Zero Modes

When there is a symmetry that is broken only by the bounce, there appears a zero mode

- Translational zero modes
- Dilatational zero mode (scale invariant theory)
- Gauge zero modes (unbroken at the false vacuum)

Since zero modes have zero eigenvalues,

$$\mathcal{A}|_{\mathrm{w/zero}} \sim \left|\frac{\det \mathcal{M}}{\det \widehat{\mathcal{M}}}\right|^{-1/2} = \left|\frac{\omega_1\omega_2\cdots}{\hat{\omega}_1\hat{\omega}_2\cdots}\right|^{-1/2} = \infty$$

$$\omega_a, \hat{\omega}_a : \text{Eigenvalues of } \mathcal{M}, \widehat{\mathcal{M}}, (\exists b, \omega_b = 0)$$

So, it is mandatory to subtract these zero modes

Symmetry

Let us consider a generic symmetry that transforms

$$\begin{pmatrix} A_\mu = 0 \\ \phi = \bar{\phi} \end{pmatrix} \to \begin{pmatrix} 0 \\ \bar{\phi} \end{pmatrix} + \sum_A z_A \mathcal{F}_A + \mathcal{O}(z^2)$$

with

$$\lim_{x_\mu x_\mu \to \infty} \mathcal{F}_A(x) = 0$$

If this symmetry is broken only by $\bar{\phi}$,

$$\mathcal{M}\mathcal{F}_A = 0$$

This is the origin of the zero modes

Saddle-point approximation

We expand the fields around the bounce as

$$\begin{pmatrix} A_\mu \\ \phi \end{pmatrix} = \begin{pmatrix} 0 \\ \bar{\phi} \end{pmatrix} + \sum_a c_a \mathcal{G}_a$$

where

$$\mathcal{M}\mathcal{G}_a = \omega_a \mathcal{G}_a$$
$$\langle \mathcal{G}_a | \mathcal{G}_b \rangle = \delta_{ab}$$

If all ω_a's are non-zero, we can use the saddle-point approximation as

$$\int \mathcal{D}A\mathcal{D}\phi e^{-S[A,\phi]} = \int \left(\prod_a \frac{dc_a}{\sqrt{2\pi}} \right) e^{-S[A,\phi]}$$
$$\simeq e^{-S[0,\bar{\phi}]} \int \left(\prod_a \frac{dc_a}{\sqrt{2\pi}} \right) e^{-\frac{1}{2}\sum_b \omega_b c_b^2}$$

Reinterpretation of Zero Modes

When there are zero modes $\omega_a = 0, a \in \mathcal{I}_0$,

$$\int \mathcal{D}A\mathcal{D}\phi e^{-S[A,\phi]} = \int \left(\prod_a \frac{dc_a}{\sqrt{2\pi}} \right) e^{-S[A,\phi]}$$

$$\simeq e^{-S[0,\bar{\phi}]} \int \left(\prod_{a \in \mathcal{I}_0} \frac{dc_a}{\sqrt{2\pi}} \right) \left(\det{}' \mathcal{M} \right)^{-1/2}$$

The remaining integration variables are related to z_A's as

$$c_a = \sum_A \langle \mathcal{G}_a | \mathcal{F}_A \rangle z_A$$

Then,

$$\int \left(\prod_{a \in \mathcal{I}_0} \frac{dc_a}{\sqrt{2\pi}} \right) [\dots] = \int \left(\prod_A \frac{dz_A}{\sqrt{2\pi}} \right) \sqrt{\det_{AB} \langle \mathcal{F}_A | \mathcal{F}_B \rangle} [\dots]$$

Translational Zero Modes

The translation of the bounce does not change the action

$$\bar{\phi}(x) \to \bar{\phi}(x+y) = \bar{\phi}(x) + y_\mu \partial_\mu \bar{\phi}(x) + \mathcal{O}(y^2)$$

The corresponding zero modes are

$$\mathcal{F}_\mu = \begin{pmatrix} 0 \\ \partial_\mu \bar{\phi} \end{pmatrix}$$

Then,

$$\int \left(\prod_{a \in \mathcal{I}_0} \frac{dc_a}{\sqrt{2\pi}} \right) [\dots] = \int \left(\prod_\mu dy_\mu \right) \frac{\mathcal{B}^2}{4\pi^2} [\dots]$$
$$= VT \frac{\mathcal{B}^2}{4\pi^2} [\dots]$$

(Recall that γ is the probability par unit time and unit volume)

Gauge Zero Modes

Similarly, the (global) symmetry transformation would be

$$\bar{\phi}(x) \to e^{\theta_a T^a}\bar{\phi}(x) = \bar{\phi}(x) + \theta_a T^a \bar{\phi}(x) + \mathcal{O}(\theta^2)$$

where $\bar{\phi}$ is a real vector and

$$T^{aT} = -T^a,\ [T^a, T^b] = -f^{abc}T^c$$

The corresponding zero modes would be

$$\mathcal{F}_a = \begin{pmatrix} 0 \\ T^a\bar{\phi} \end{pmatrix}$$

However, this is NOT the case when one uses the following gauge fixing

$$\mathcal{L}_{\mathrm{BG}}^{(\mathrm{GF})} = \frac{1}{2\xi}\left(\partial_\mu A_\mu^a + \xi g_a \phi^T T^a \bar{\phi}\right)^2$$

(It is called as the background gauge and is often used in the numerical calculation)

Gauge Zero Modes

The correct gauge zero modes in the background gauge are

$$\mathcal{F}_a = \begin{pmatrix} \tilde{A}^a_\mu(x) \\ \sum_b \tilde{G}^{ab}(x) T^b \bar{\phi}(x) \end{pmatrix}$$

where $\tilde{A}^a_\mu$ and $\tilde{G}^{ab}$ are some functions

So, we have problems

- After the rotation, the guage bosons obtain VEVs and the functional determinant looks different for different θ
- The rotation angle is position-dependent
- It is not clear what measure of θ we should use

Fermi Gauge

Such a problem is absent if we use the Fermi gauge,

$$\mathcal{L}^{(\mathrm{GF})} = \frac{1}{2\xi}\left(\partial_\mu A^a_\mu\right)^2$$

Then, the zero mode is simply given by

$$\mathcal{F}_a = \begin{pmatrix} 0 \\ T^a \bar{\phi} \end{pmatrix}$$

So, we can interpret the integral over the zero modes as the group space integral

However, the numerical calculation becomes difficult due to severe cancellations

Proposal

We propose a hybrid method to compute the prefactor, $\mathcal{A}$

1. Calculate the prefactor in the background gauge with a certain way of the gauge zero mode subtraction
2. Convert the obtained prefactor into that in the Fermi gauge with the gauge zero modes being subtracted appropriately

To obtain the conversion relation, we need semi-analytic expressions in both gauges

Functional Determinants

Setup

We consider

$$\mathcal{L}_E = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \frac{1}{2}(D_\mu\phi)_i(D_\mu\phi)_i + V(\phi) + \mathcal{L}^{(\mathrm{GF})} + \mathcal{L}^{(\mathrm{ghost})}$$

where

$$D_\mu\phi = (\partial_\mu + g_a A^a_\mu T^a)\phi$$

We assume

- The rank of the gauge boson mass matrix is unchanged during the bounce (but can be different from that in the false vacuum)
- There are no zero modes except for the gauge zero modes and the translational zero modes
- The bounce approaches to the false vacuum exponentially

Partial Wave Expansion

Since the bounce is $O(4)$ symmetric, we use the hyperspherical functions, $Y_{\ell m_1 m_2}$, as the basis of angular functions

Then, the prefactor is decomposed as

$$\mathcal{A} = \left[\prod_{\ell=0}^{\infty}\left(\frac{\det \mathcal{M}_\ell^{(c\bar{c})}}{\det \widehat{\mathcal{M}}_\ell^{(c\bar{c})}}\right)^{(\ell+1)^2}\right]\left(\frac{\det' \mathcal{M}_0^{(S\varphi)}}{\det \widehat{\mathcal{M}}_0^{(S\varphi)}}\right)^{-1/2}\left(\frac{\det' \mathcal{M}_1^{(SL\varphi)}}{\det \widehat{\mathcal{M}}_1^{(SL\varphi)}}\right)^{-2}$$
$$\times\left[\prod_{\ell=2}^{\infty}\left(\frac{\det \mathcal{M}_\ell^{(SL\varphi)}}{\det \widehat{\mathcal{M}}_\ell^{(SL\varphi)}}\right)^{-(\ell+1)^2/2}\right]\left[\prod_{\ell=1}^{\infty}\left(\frac{\det \mathcal{M}_\ell^{(T)}}{\det \widehat{\mathcal{M}}_\ell^{(T)}}\right)^{-(\ell+1)^2}\right]$$

Each fluctuation operator is shown in the following slides

(the hatted operators are obtained by $\bar{\phi} \to v$)

Definitions

We define

$$\Delta_\ell = \partial_r^2 + \frac{3}{r}\partial_r - \frac{L^2}{r^2}$$
$$L = \sqrt{\ell(\ell+2)}$$
$$M_{ia} = -g_a[T^a\bar{\phi}]_i$$
$$\Omega_{ij} = \frac{d^2V}{d\phi_i d\phi_j}$$

where r is the radius from the center of the bounce

Faddeev-Popov Ghost and (T)-modes

Fermi gauge

$$\mathcal{M}_\ell^{(c\bar{c})} = -\Delta_\ell$$
$$\mathcal{M}_\ell^{(T)} = -\Delta_\ell + M^T M$$

Background gauge

$$\mathcal{M}_{\mathrm{BG},\ell}^{(c\bar{c})} = -\Delta_\ell + \xi M^T M$$
$$\mathcal{M}_{\mathrm{BG},\ell}^{(T)} = -\Delta_\ell + M^T M$$

All of them are $n_G \times n_G$ matrices with n_G being the number of the gauge bosons

$(S\varphi)$-modes ($\ell = 0$)

Fermi gauge

$$\mathcal{M}_0^{(S\varphi)} = \begin{pmatrix} -\frac{1}{\xi}\Delta_1 + M^T M & (M')^T - M^T \partial_r \\ 2M' + M\frac{1}{r^3}\partial_r r^3 & -\Delta_0 + \Omega \end{pmatrix}$$

Background gauge

$$\mathcal{M}_{\mathrm{BG},0}^{(S\varphi)} = \begin{pmatrix} -\frac{1}{\xi}\Delta_1 + M^T M & 2(M')^T \\ 2M' & -\Delta_0 + \Omega + \xi M^T M \end{pmatrix}$$

$(SL\varphi)$-modes ($\ell > 0$)

Fermi gauge

$$\mathcal{M}_\ell^{(SL\varphi)} = \begin{pmatrix} -\Delta_\ell + \frac{3}{r^2} + M^T M & -\frac{2L}{r^2} & (M')^T - M^T \partial_r \\ -\frac{2L}{r^2} & -\Delta_\ell - \frac{1}{r^2} + M^T M & -\frac{L}{r} M^T \\ 2M' + M \frac{1}{r^3} \partial_r r^3 & -\frac{L}{r} M & -\Delta_\ell + \Omega \end{pmatrix}$$
$$+ \left(1 - \frac{1}{\xi}\right) \begin{pmatrix} \partial_r^2 + \frac{3}{r}\partial_r - \frac{3}{r^2} & -L\left(\frac{1}{r}\partial_r - \frac{1}{r^2}\right) & 0 \\ L\left(\frac{1}{r}\partial_r + \frac{3}{r^2}\right) & -\frac{L^2}{r^2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Background gauge

$$\mathcal{M}_{\mathrm{BG},\ell}^{(SL\varphi)} = \begin{pmatrix} -\Delta_\ell + \frac{3}{r^2} + M^T M & -\frac{2L}{r^2} & 2(M')^T \\ -\frac{2L}{r^2} & -\Delta_\ell - \frac{1}{r^2} + M^T M & 0 \\ 2M' & 0 & -\Delta_\ell + \Omega + \xi M^T M \end{pmatrix}$$
$$+ \left(1 - \frac{1}{\xi}\right) \begin{pmatrix} \partial_r^2 + \frac{3}{r}\partial_r - \frac{3}{r^2} & -L\left(\frac{1}{r}\partial_r - \frac{1}{r^2}\right) & 0 \\ L\left(\frac{1}{r}\partial_r + \frac{3}{r^2}\right) & -\frac{L^2}{r^2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Theorem

The ratio of the functional determinants can be calculated in the following way

Let $\mathcal{M}$ and $\widehat{\mathcal{M}}$ be $n \times n$ fluctuation operators. Then,

$$\frac{\det \mathcal{M}}{\det \widehat{\mathcal{M}}} = \left(\lim_{r \to 0} \frac{\det \Psi(r)}{\det \widehat{\Psi}(r)} \right)^{-1} \left(\lim_{r \to \infty} \frac{\det \Psi(r)}{\det \widehat{\Psi}(r)} \right)$$

where

$$\Psi(r) = \begin{pmatrix} \psi^{(1)}(r) & \cdots & \psi^{(n)}(r) \end{pmatrix}$$

$$\widehat{\Psi}(r) = \begin{pmatrix} \widehat{\psi}^{(1)}(r) & \cdots & \widehat{\psi}^{(n)}(r) \end{pmatrix}$$

with independent regular solutions,

$$\mathcal{M}\psi^{(I)}(r) = 0$$

$$\widehat{\mathcal{M}}\widehat{\psi}^{(I)}(r) = 0$$

See [*M. Endo, T. Moroi, M. M. Nojiri, YS, '17*]

Semi-analytic Results

Decomposition

Solutions of $\mathcal{M}_\ell^{(SL\varphi)}\Psi_\ell^{(SL\varphi)} = 0$ are decomposed as

$$\Psi_\ell^{(SL\varphi)} = \begin{pmatrix} \partial_r\chi \\ \frac{L}{r}\chi \\ M\chi \end{pmatrix} + \begin{pmatrix} (M^TM)^{-1}\left[\frac{L}{r}\eta - 2(M')^T\lambda\right] \\ (M^TM)^{-1}\frac{1}{r^2}\partial_r r^2\eta \\ \lambda \end{pmatrix} + \begin{pmatrix} [\partial_r(M^TM)^{-1}]\zeta \\ 0 \\ M(M^TM)^{-1}\zeta \end{pmatrix}$$

$$\begin{aligned}
\mathcal{M}_\ell^{(c\bar{c})}\chi =&[\partial_r(M^TM)^{-1}]\frac{L}{r}\eta - \frac{2}{r^3}\partial_r r^3(M^TM)^{-1}(M')^T\lambda \\
&- \mathcal{M}_0^{(c\bar{c})}(M^TM)^{-1}\zeta - \frac{1}{r^3}\partial_r r^3(M^TM)^{-1}\partial_r\zeta + \xi\zeta \\
\mathcal{M}_\ell^{(c\bar{c})}\zeta =&0
\end{aligned}$$

$$\begin{aligned}
\Delta_\ell\eta =& M^TM\left[\eta - \{\partial_r(M^TM)^{-1}\}\frac{1}{r^2}\partial_r r^2\eta\right] - \frac{2L}{r}M'^T\lambda + \frac{L}{r}M^TM[\partial_r(M^TM)^{-1}]\zeta \\
\Delta_\ell\lambda =& \Omega\lambda - 4M'(M^TM)^{-1}M'^T\lambda + \frac{2L}{r}M'(M^TM)^{-1}\eta - 2M'(M^TM)^{-1}\zeta' \\
&+ M\left[-\frac{2}{r^3}\partial_r r^3(M^TM)^{-1}M'^T\lambda + \frac{L}{r}\{\partial_r(M^TM)^{-1}\}\eta - \{\partial_r(M^TM)^{-1}\}\zeta'\right]
\end{aligned}$$

where $M\lambda = 0$

Relations ($\ell > 0$)

We have evaluated the $r \to 0$ behavior and the $r \to \infty$ behavior carefully and obtain a gauge-invariant semi-analytic expression

For $\ell > 0$, we have shown

$$\begin{aligned}
&\left(\frac{\det \mathcal{M}_\ell^{(c\bar{c})}}{\det \widehat{\mathcal{M}}_\ell^{(c\bar{c})}}\right)^{(\ell+1)^2} \left(\frac{\det \mathcal{M}_\ell^{(SL\varphi)}}{\det \widehat{\mathcal{M}}_\ell^{(SL\varphi)}}\right)^{-(\ell+1)^2/2} \\
&= \left(\frac{\det \mathcal{M}_{\mathrm{BG},\ell}^{(c\bar{c})}}{\det \widehat{\mathcal{M}}_{\mathrm{BG},\ell}^{(c\bar{c})}}\right)^{(\ell+1)^2} \left(\frac{\det \mathcal{M}_{\mathrm{BG},\ell}^{(SL\varphi)}}{\det \widehat{\mathcal{M}}_{\mathrm{BG},\ell}^{(SL\varphi)}}\right)^{-(\ell+1)^2/2} \\
&= (\text{Gauge-invariant expression})
\end{aligned}$$

(Explicit expression is available in the paper)

Relations ($\ell = 0$)

When there are gauge zero modes, $\det \mathcal{M}_0^{(S\varphi)} = \det \mathcal{M}_{\mathrm{BG},0}^{(S\varphi)} = 0$

In the Fermi gauge, they are subtracted as

$$\det{}' \mathcal{M}_0^{(S\varphi)} = \lim_{\nu \to 0} \frac{1}{\nu^{n_{\mathrm{zero}}}} \det[\mathcal{M}_0^{(S\varphi)} + \nu]$$

We have shown

$$\left(\frac{\det \mathcal{M}_0^{(c\bar{c})}}{\det \widehat{\mathcal{M}}_0^{(c\bar{c})}} \right) \left(\frac{\det' \mathcal{M}_0^{(S\varphi)}}{\det \widehat{\mathcal{M}}_0^{(S\varphi)}} \right)^{-1/2}$$

$$= \sqrt{\frac{\det \mathcal{K}}{\det \mathcal{X}_U}} \left(\frac{\det \mathcal{M}_{\mathrm{BG},0}^{(c\bar{c})}}{\det \widehat{\mathcal{M}}_{\mathrm{BG},0}^{(c\bar{c})}} \right) \left(\lim_{\nu \to 0} \frac{1}{\nu^{n_{\mathrm{zero}}}} \frac{\det \left[\mathcal{M}_{\mathrm{BG},0}^{(S\varphi)} + \nu \begin{pmatrix} \frac{1}{\xi} & 0 \\ 0 & 1 \end{pmatrix} \right]}{\det \widehat{\mathcal{M}}_{\mathrm{BG},0}^{(S\varphi)}} \right)^{-1/2}$$

$$= (\text{Gauge-invariant expression})$$

where $\det \mathcal{K}$ and $\det \mathcal{X}_U$ are calculable objects

Numerical Varification

Numerical Varification

We can explicitly calculate the both sides of

$$\frac{\det \mathcal{M}^{(SL\varphi)}_{\mathrm{BG},\ell}}{\det \widehat{\mathcal{M}}^{(SL\varphi)}_{\mathrm{BG},\ell}} = \text{(semi-analytic expression)}$$

$$\lim_{\nu \to 0} \frac{1}{\nu^{n_{\mathrm{zero}}}} \frac{\det \left[\mathcal{M}^{(S\varphi)}_{\mathrm{BG},0} + \nu \begin{pmatrix} \frac{1}{\xi} & 0 \\ 0 & 1 \end{pmatrix} \right]}{\det \widehat{\mathcal{M}}^{(S\varphi)}_{\mathrm{BG},0}} = \text{(semi-analytic expression)}$$

for $\xi = 1$

$SO(3)$ **Triplets**

Let us consider

$$\phi_1, \phi_2 : SO(3) \text{ triplets}$$
$$S : \text{singlet}$$

The potential is assumed to be

$$V = \frac{\lambda_1}{4}\left(|\phi_1|^2 + \frac{m_1^2}{\lambda_1} - \kappa_1 S^2\right)^2 + \frac{\lambda_2}{4}\left(|\phi_2|^2 + \frac{m_2^2}{\lambda_2} - \kappa_2 S^2\right)^2 + \frac{\lambda_3}{2}(\phi_1 \cdot \phi_2)^2 + V_S$$

where

$$V_S = (1 - \kappa_1^2\lambda_1 - \kappa_2^2\lambda_2)\frac{S^4}{4} - \frac{v_T}{6}(3-\epsilon)\, S^3 + \frac{v_T^2}{4}(1-\epsilon)S^2$$

The couplings of the other possible terms are set to zero

Parameters

First example

$v_1 = v_2 = v_S = 0$

$$\lambda_1 = 0.2,\ \lambda_2 = 0.5,\ \kappa_1 = \kappa_2 = 0.8,$$
$$m_1^2 = 1,\ m_2^2 = 3,\ v_T = 8,\ \epsilon = 0.01,$$
$$g = 1, \lambda_3 = 0.5$$

Second example

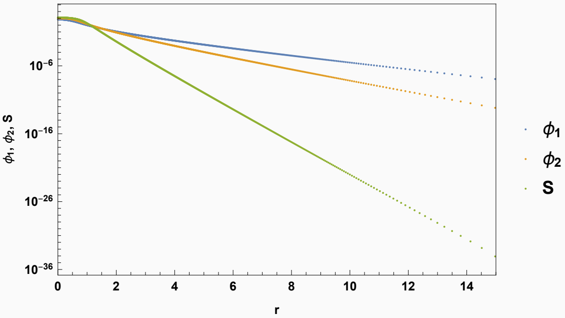
$v_1 = \sqrt{5},\ v_2 = v_S = 0$

$$\lambda_1 = 0.2,\ \lambda_2 = 0.5,\ \kappa_1 = \kappa_2 = 0.8,$$
$$m_1^2 = -1,\ m_2^2 = 3,\ v_T = 8,\ \epsilon = 0.01,$$
$$g = 1, \lambda_3 = 0.5$$

Bounce

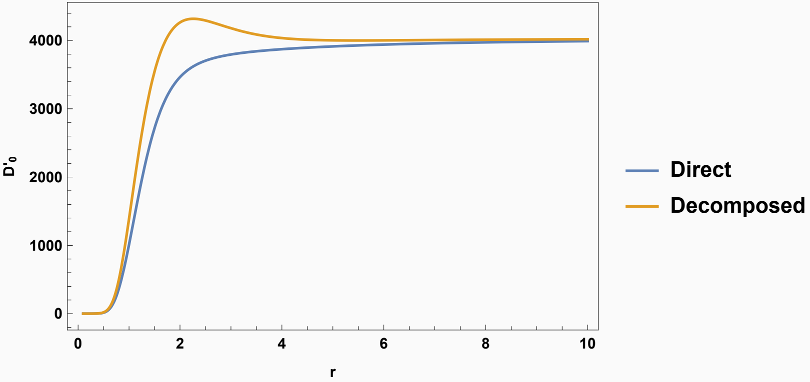
Using the gradient flow method, a very precise bounce is obtained

First example ($\mathcal{B} = 576.808625(16)$)

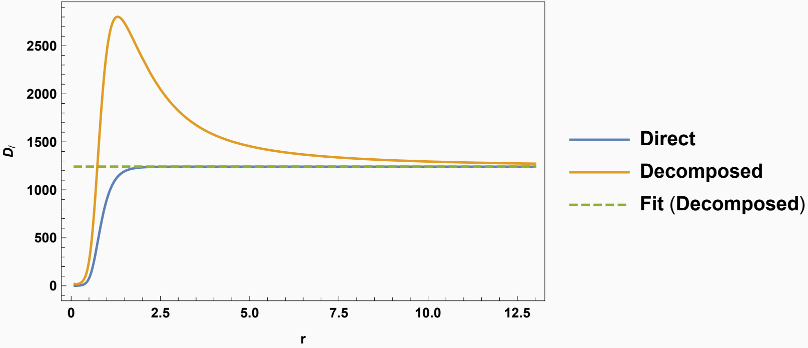


Solved 14 times farther than the typical size of the bounce and 10^{-36} times smaller than the typical field value

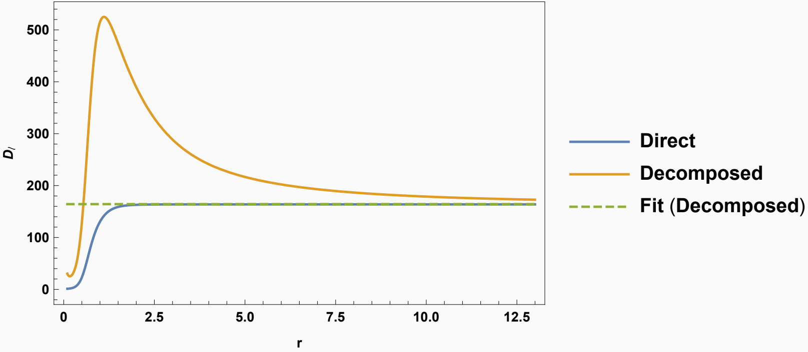
First Example $\ell = 0$



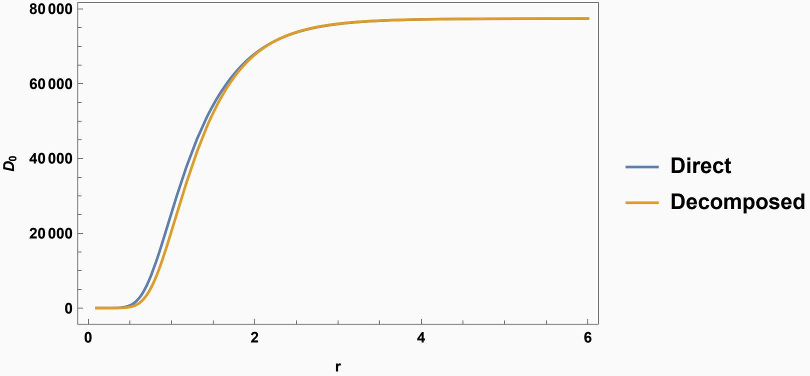
First Example $\ell = 3$



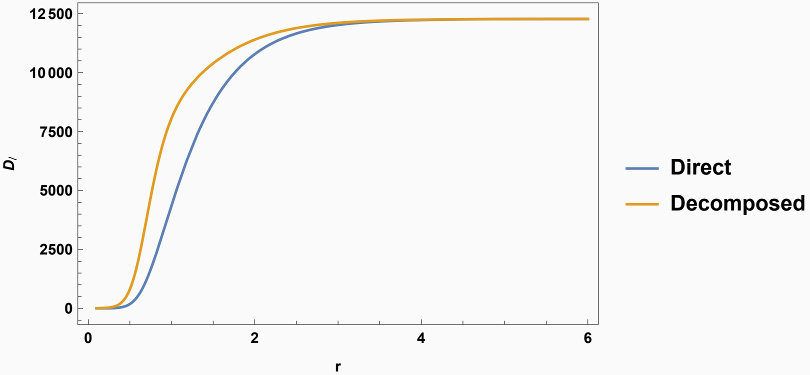
First Example $\ell = 5$



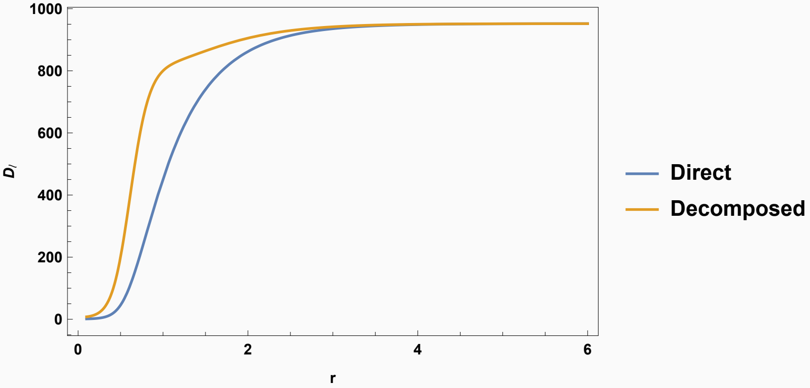
Second Example $\ell = 0$



Second Example $\ell = 3$



Second Example $\ell = 5$



Summary

- The treatment of gauge zero modes has been a long standing problem in the calculation of the prefactor
- The correct treatment has been discovered recently for a single-field bounce and we extended it to a multi-field bounce
- We obtained the semi-analytic expression of the prefactor, which is manifestly gauge invariant
- We calculated in two gauges, the Fermi guage and the background gauge, and find their relations
- They can be used to convert the results in the background gauge, where the numerical calculation becomes stable, to those in the Fermi gauge, where the treatment of the gauge zero modes is feasible
- We have numerially checked our results and found they show very good agreement