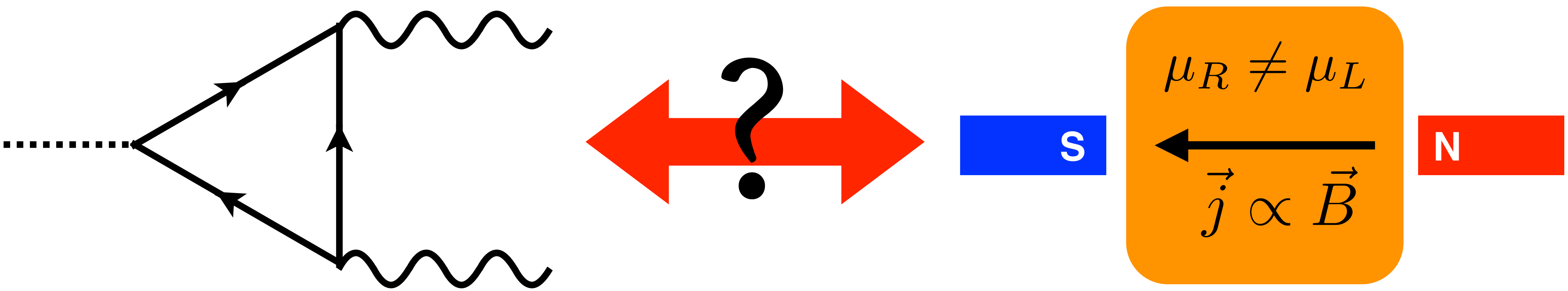
Anomaly matching for chiral transport phenomena



Masaru Hongo (Univ. of Illinois at Chicago)

Osaka University particle physic theory group seminar, 2020/10/20

Based on MH Ann. Phys. (2017), MH-Hidaka arXiv:1902.09166 [hep-th]

Physics in 2020s = Hydro

Major premise:

"He (=Nambu) is always 10 years ahead of us" (Zumino)

Minor premise:

"Recently, hydrodynamics is interesting" (Nambu, 2013)

ノーベル賞:南部さん阪大講演　「寝てても発見探し」

毎日新聞　2013年07月17日　03時25分（最終更新　07月17日　04時21分）

南部さんは長く米国で教壇に立っていたが、数年前から同市で暮らしている。記者会見ではさらに、「家に帰っても、寝ているときも何か新しい発見はないかと考え続けている。最近は流体力学が面白い」と物理学への衰えぬ情熱を語っていた。【斎藤広子】

Conclusion:

2020s must be Renaissance of hydrodynamics!!

Hydro Renaissance in 2010s

Two big developments thanks to hep-th (+α) friends!

Field-theoretically speaking, they are

1. Generating functional (imaginary-time formalism)

"Hydrostatic" generating fcn. → Local eq. averaged current

$$Z[g_{\mu\nu}, A_\mu] \;\Rightarrow\; \langle \hat{T}^{\mu\nu}(x) \rangle = \frac{2}{\sqrt{-g}} \frac{\delta \log Z}{\delta g_{\mu\nu}(x)}, \cdots$$

2. Effective Lagrangian (real-time formalism)

$$Z[g_{\mu\nu}, A_\mu] = \int \mathcal{D}\pi_{\text{hydro}} \exp\left(\mathrm{i}\mathcal{S}_{\text{eff}}[\pi_{\text{hydro}}]\right)$$

(corresponds to a construction of chiral Lagrangian in QCD)

hep-th view of hydrodynamics

2. Effective Lagrangian (real-time formalism)

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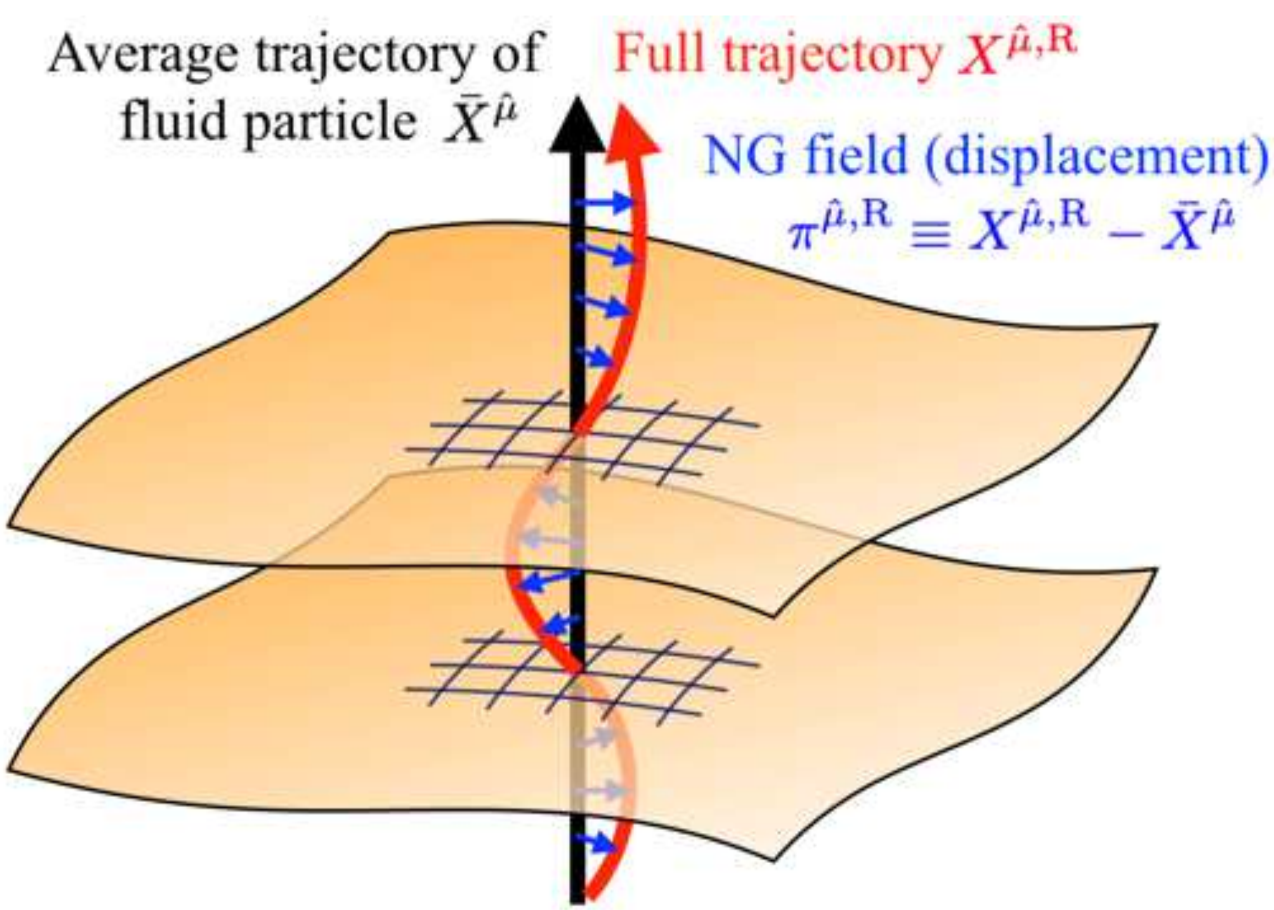
(corresponds to a construction of chiral Lagrangian in QCD)

Hydrodynamics is low-energy EFT of a spacetime filling brane $X^\mu(\sigma^0, \sigma^i)$, enjoying emergent gauge symmetry:

$$\begin{cases} \sigma^0 \to \sigma^0 + f(\sigma^0, \sigma^i) \\ \sigma^i \to \sigma^i + g^i(\sigma^i) \end{cases}$$



[See Crossley et al. arXiv: 1511.03646 [hep-th], MH et al. ongoing work]

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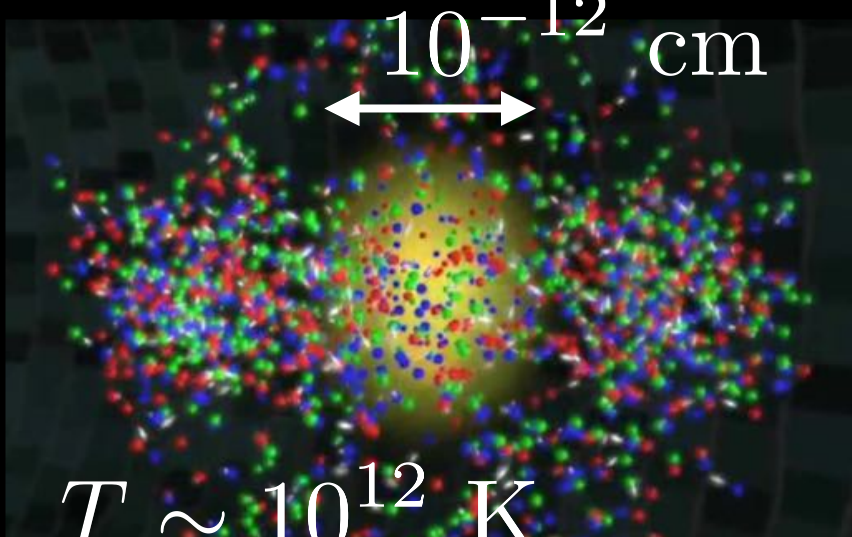
<u>Motivation 1</u>

What is hydrodynamics?

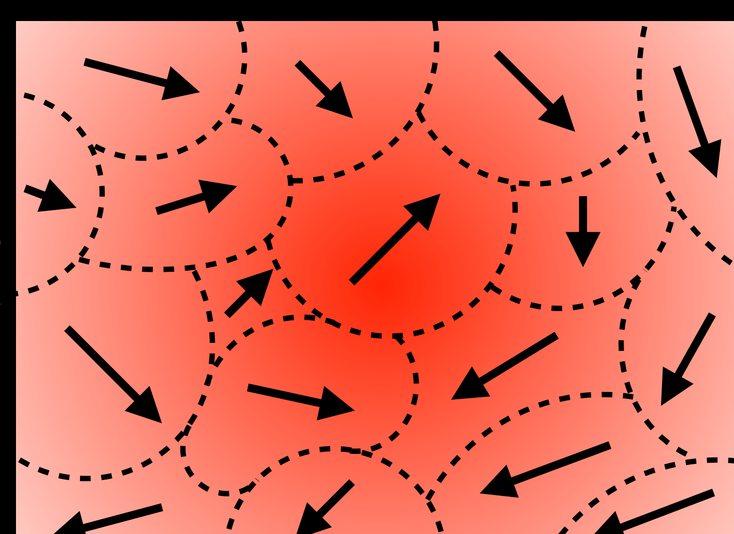
Hydrodynamics is

- **Effective theory** for **macroscopic dynamics**
- **Universal description**, not depending on details
- **Only conserved quantity ~ symmetry of system**

Quark-Gluon Plasma | **Hydro:** $\{\beta(x),\ \vec{v}(x)\}$ | **Neutron Star**



$\simeq$



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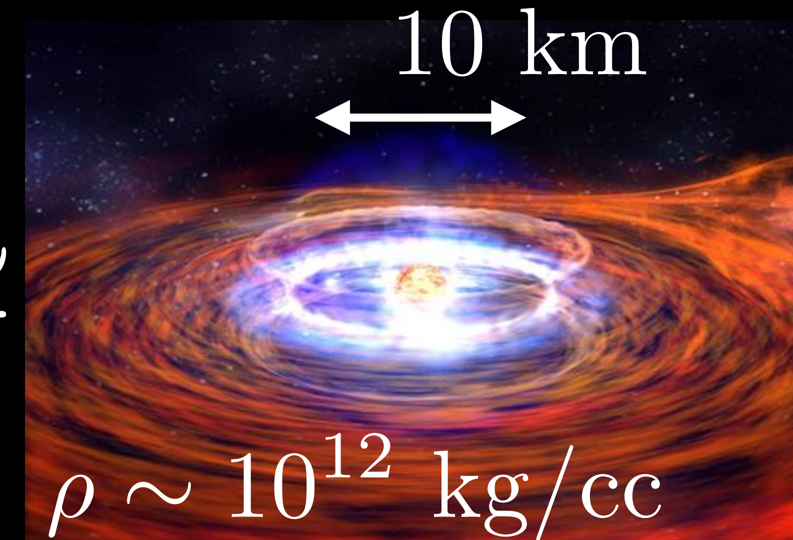
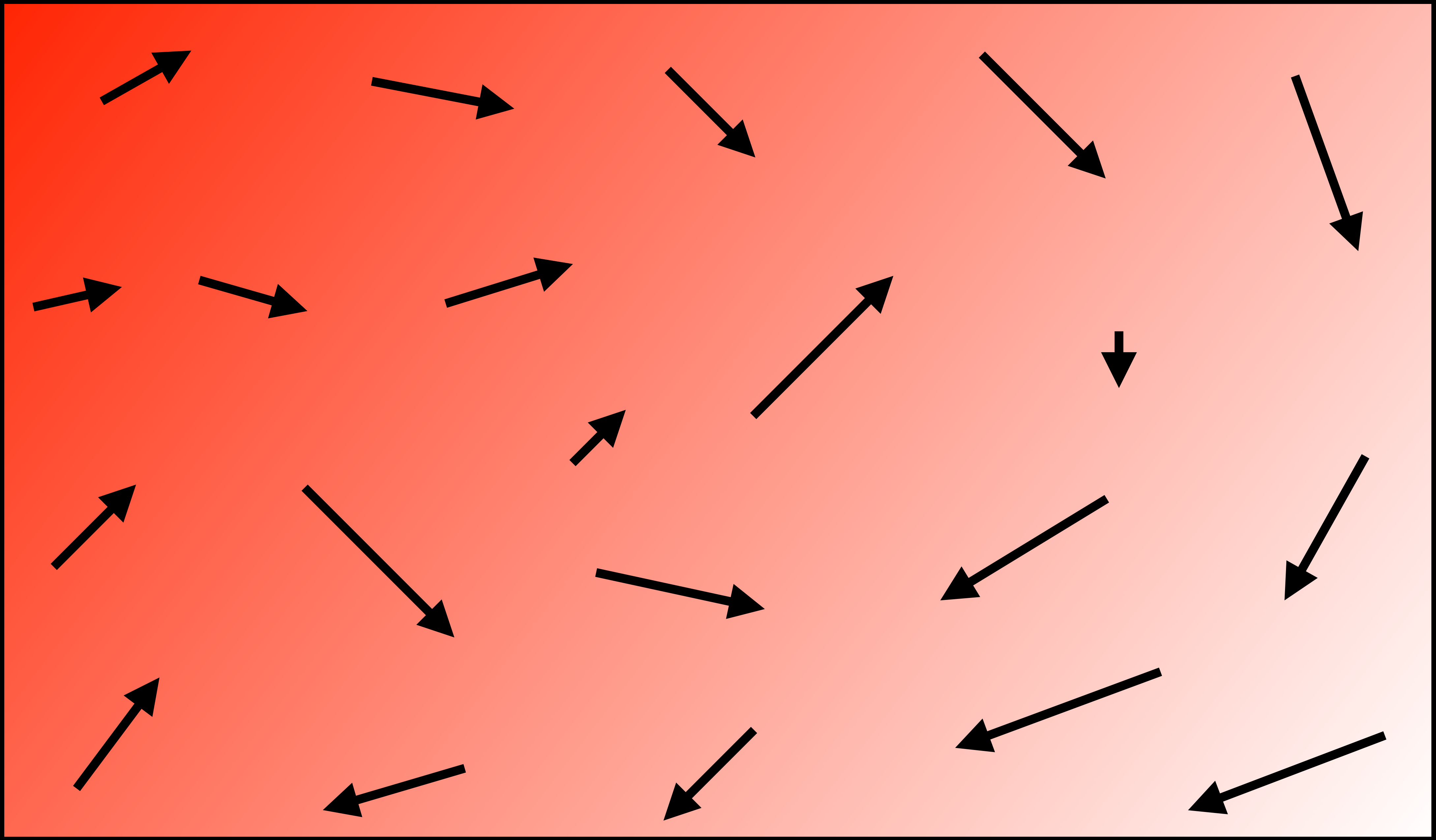


http://www.bnl.gov/rhic/news2/news.asp?a=1403&t=pr

http://newsoffice.mjitugenn.edu/2012/model-bursting-star-0302

Hydrodynamic equation?



Theoretical structure of hydro

Conservation laws

$$\nabla_\mu \langle \hat{T}^{\mu\nu}(x) \rangle = 0, \quad \nabla_\mu \langle \hat{J}^{\mu}(x) \rangle = 0$$

Consider (3+1)d relativistic theory with U(1) symmetry:

# of EoM :	4	+	1	=	5
# of d.o.f. :	10 ($T^{\mu\nu}$)	+	4 (J^μ)	=	14

} does **not** form a closed set of equations!!

➡ To solve conservation laws, constitutive relations is needed;
Spatial components needs to be expressed by temporal ones

$$T^{ij} = T^{ij}[T^{0\mu}, J^0], \; J^i = J^i[T^{0\mu}, J^0]$$

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# of EoM :	4	+	1	=	5
Indep. # of d.o.f. :	10 - 6 ($T^{\mu\nu}$)	+	4 - 3 (J^{μ})	=	14 - 9

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We can solve conservation law + constitutive rel.!!

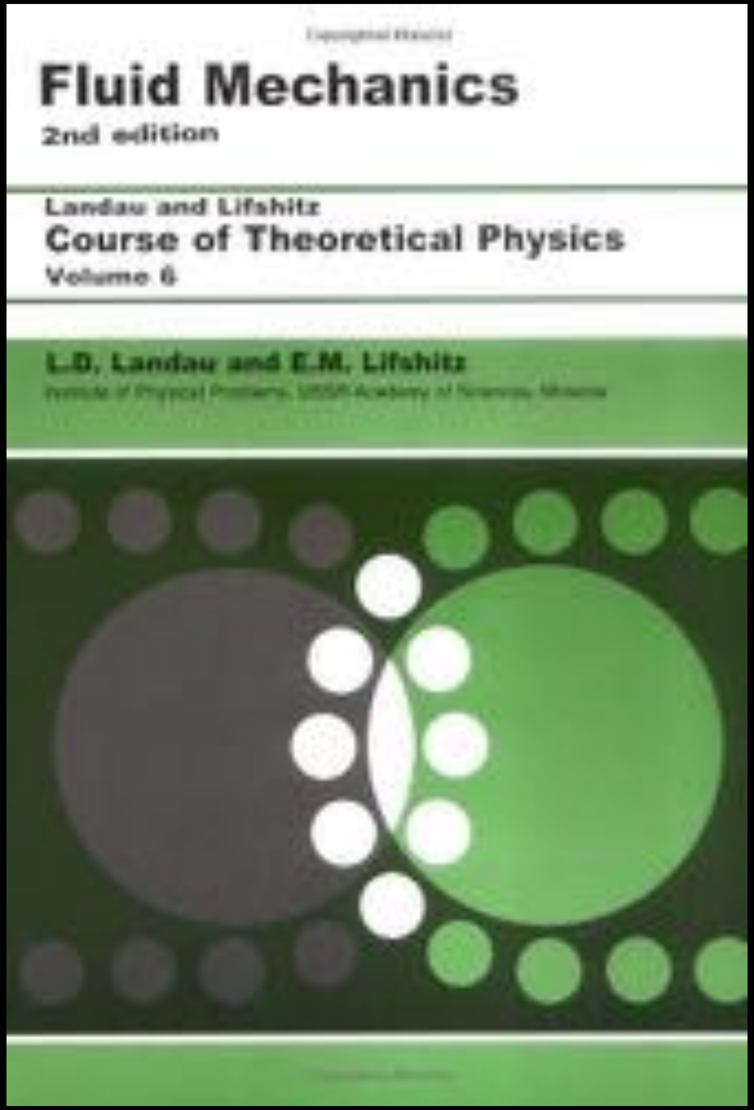
➡ To solve conservation laws, constitutive relations is needed;
Spatial components needs to be expressed by temporal ones

$$T^{ij} = T^{ij}[T^{0\mu}, J^0], \ J^i = J^i[T^{0\mu}, J^0]$$

Today's main Question

Q. Why $T^{\mu\nu} = (e+p)u^\mu u^\nu + pg^{\mu\nu} + \cdots$?

Answer 1.



Answer2. My talk

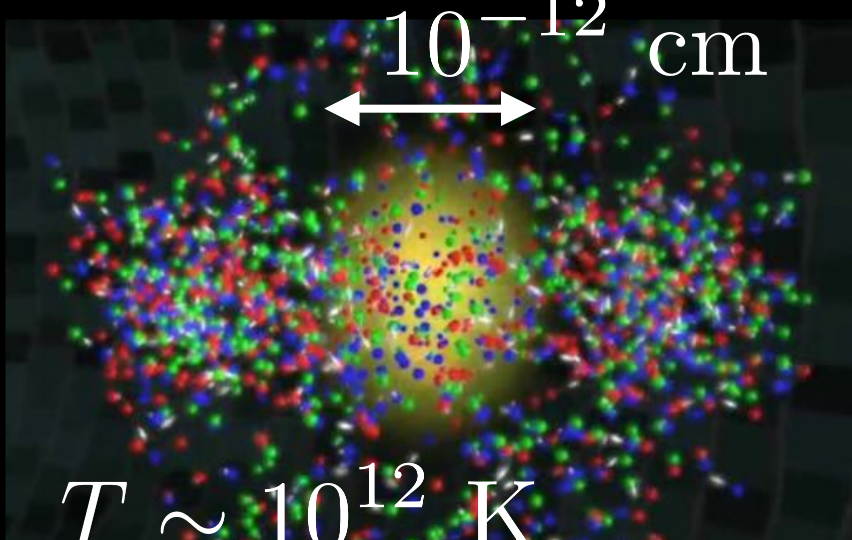
Motivation 2

Hydro and anomaly

Hydrodynamics is

- **Effective theory** for **macroscopic dynamics**
- **Universal description**, not depending on details
- **Only conserved quantity ~ ~~symmetry~~ of system**

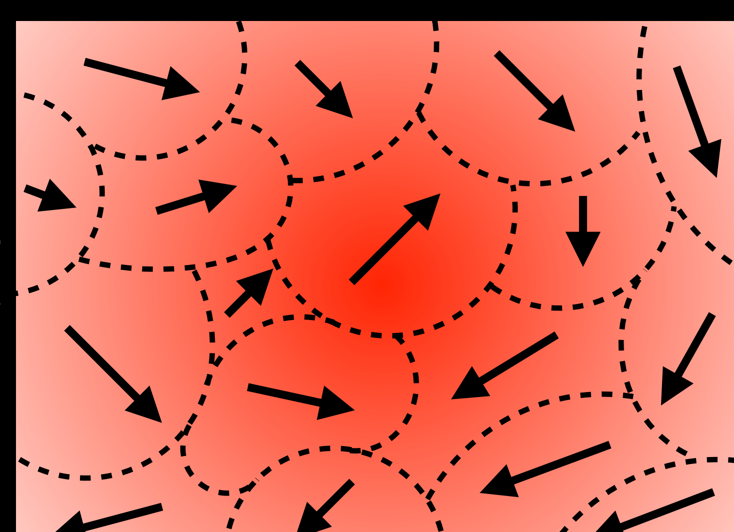
Quark-Gluon Plasma



http://www.bnl.gov/rhic/news2/news.asp?a=1403&t=pr

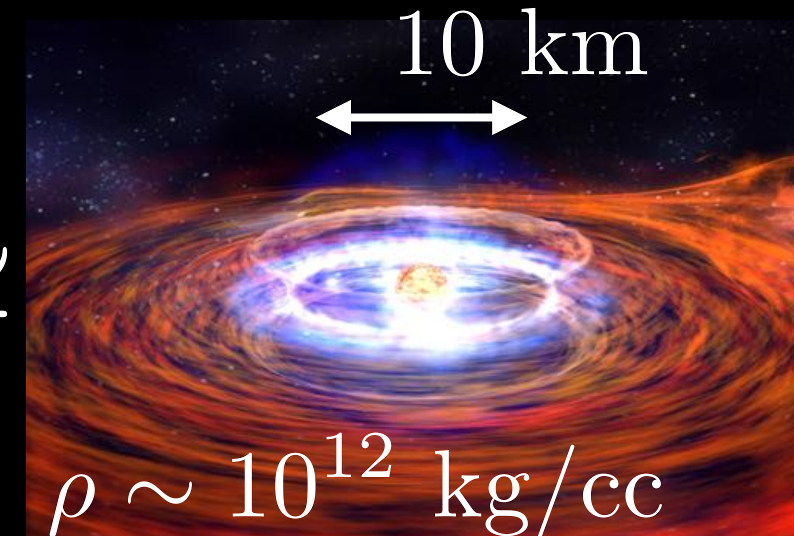
$\simeq$

Hydro: $\{\beta(x),\ \vec{v}(x)\}$



$\simeq$

Neutron Star

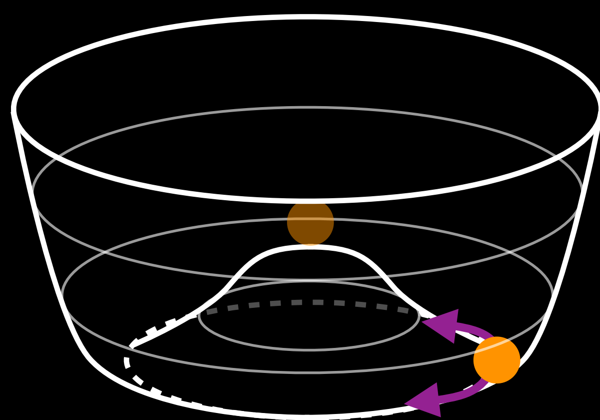


http://newsoffice.mjitugenn.edu/2012/model-bursting-star-0302

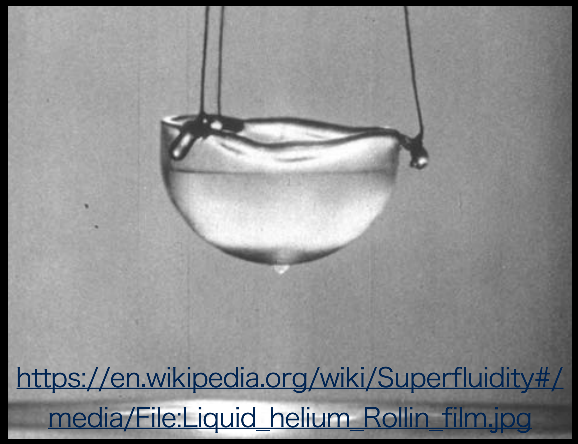
Symmetry breaking & Hydro

- Spontaneous symmetry breaking

Micro：Selecting vacuum

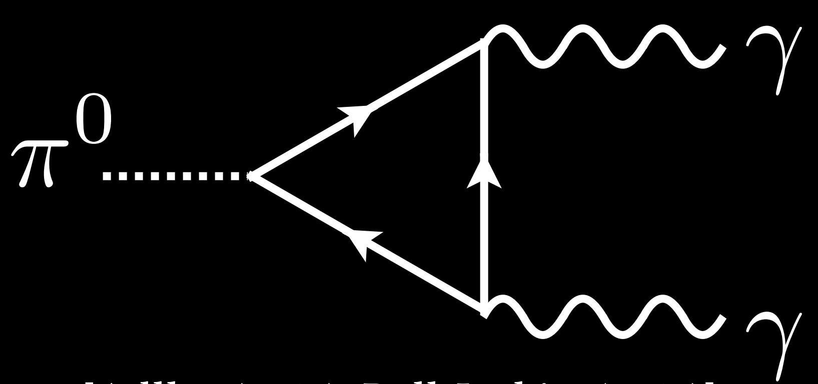


Macro：Superfluid



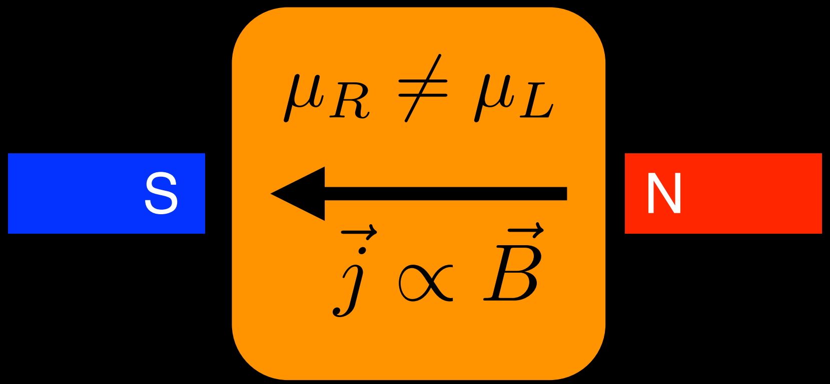
- **Symmetry breaking by quantum anomaly**

Micro：π^0 decay



[Adller (1969), Bell-Jackiw (1969)]

Macro：Anomalous transport



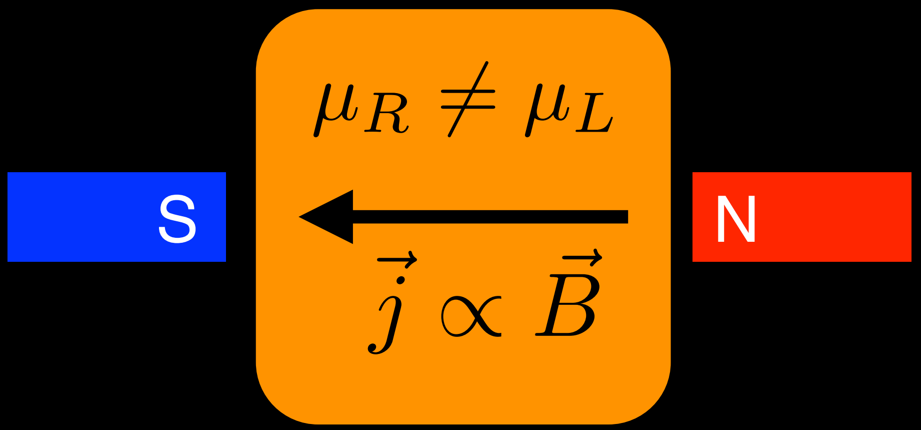
[Erdmenger et al. (2008), Son-Surowka (2009)]

Anomaly-induced chiral transport

[Fukushima et al.(2008), Vilenkin (1980)]

- **Chiral Magnetic Effect (CME)**

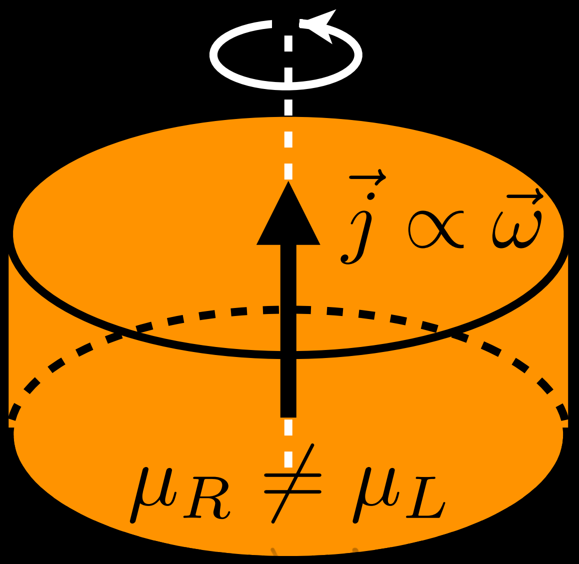
$$\vec{j} = \frac{e\mu_5}{2\pi^2}\vec{B}$$



[Erdmenger et al. (2008), Son-Surowka (2009)]

- **Chiral Vortical Effect (CVE)**

$$\vec{j} = \frac{\mu\mu_5}{2\pi^2}\vec{\omega}$$



Derivation of chiral transport

- Fluid/gravity (AdS/CFT) correspondence [Erdmenger et al. 2008]

- Phenomenological entropy-current analysis [Son-Surowka 2009]

- Linear response theory at one-loop order [Landsteiner et al, 2011]

- Chiral kinetic theory with Berry phase [J-H Gao et al, 2012 Son-Yamamoto, 2012, Stephanov-Yin, 2012, …]

- Anomaly matching for thermodynamic functional [Jensen et al, 2012, Banerjee et al, 2012, (See Hongo-Hidaka, 2019 for a review)]

- Anomalous commutation relation in current algebra [Hongo-Sogabe-Yamamoto, ongoing]

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’t Hooft anomaly matching

◆ Def. of ’t Hooft anomaly

$$Z[A+\mathrm{d}\theta] = \mathrm{e}^{\mathrm{i}\mathcal{A}[A;\theta]} Z[A]$$

A : Bkg. gauge field for global G-symmetry

($\mathrm{i}\mathcal{A}[A;\theta]$ cannot be removed by gauge-inv. local counter term)

◆ ’t Hooft anomaly matching

$\mathrm{i}\mathcal{A}[A;\theta]$ is RG inv. ➡ If present in UV, it restrict IR physics!!

➡ Trivial (non-degenerate) vacuum is excluded!

(- Classical Ex. : vacuum of massless QCD (would) break chiral symmetry
- Modern Ex. : Higher-form/discrete symmetry (topological phases/$\theta=\pi$ QCD))

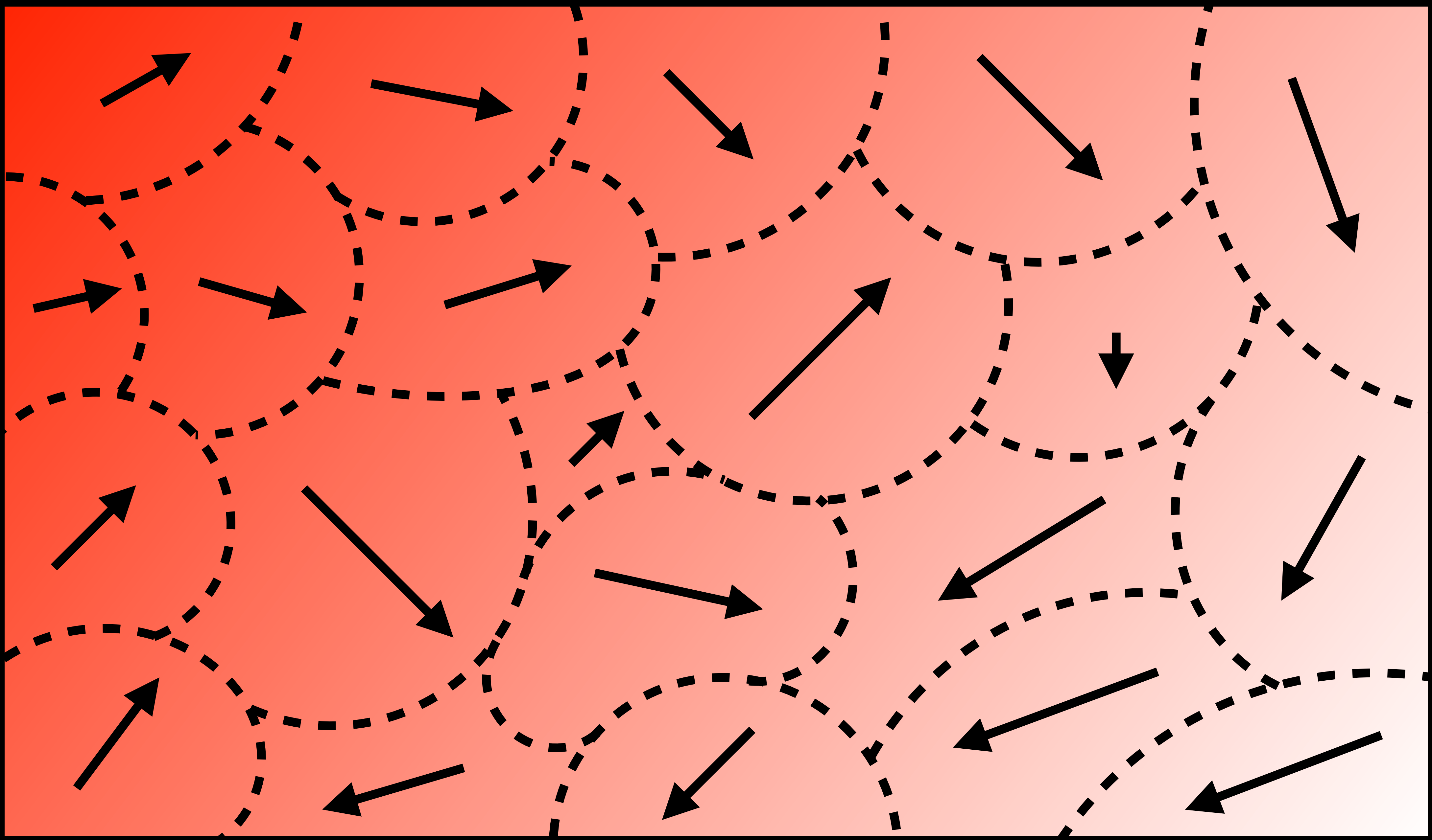
➡ How we can apply this to transport??

Formulation

QFT at local equilibrium

Based on MH Ann. Phys. (2017), MH-Hidaka Particles (2019)

Local thermal equilibrium



Determined only by **local temperature, local velocity...** at that time

($\beta(x)$, $\vec{v}(x)$ **is assumed to be smooth functions w.r.t.** x)

How to describe local thermal equil.

$T = \text{const.}$

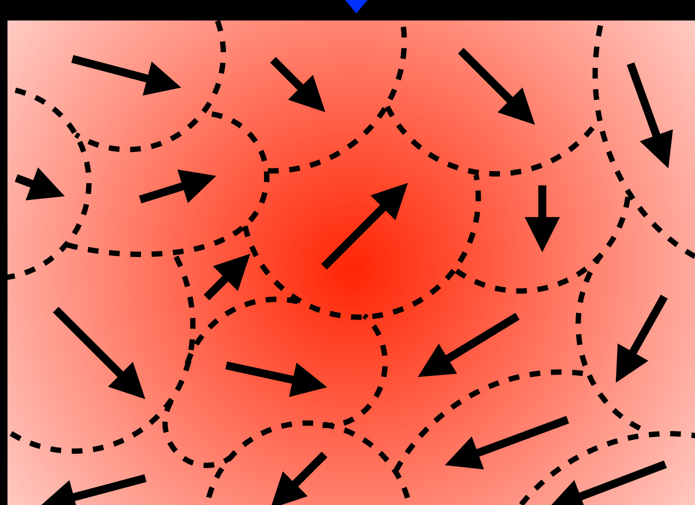
Global thermal equilibrium:

Gibbs distribution:

$$\hat{\rho}_G = e^{-\beta \hat{H} - \Psi[\beta]}, \quad \Psi[\beta] \equiv \log \mathrm{Tr} e^{-\beta \hat{H}}$$

Localize

Local thermal equilibrium:



$\{\beta(x),\ \vec{v}(x)\}$

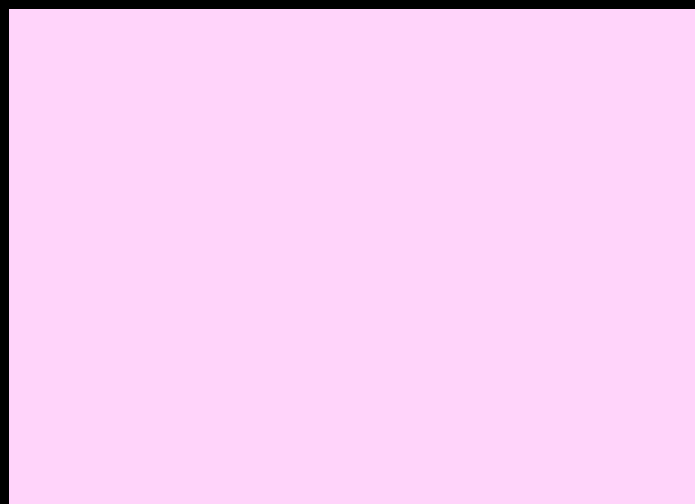
Local Gibbs (LG) distribution:

$$\hat{\rho}_{LG} = e^{-\hat{K} - \Psi[\beta^\mu(x), \nu(x)]}$$

$$\hat{K} = -\int d^3x \left(\beta^\mu(\boldsymbol{x}) \hat{T}^0{}_\mu(\boldsymbol{x}) + \nu(\boldsymbol{x}) \hat{J}^0(\boldsymbol{x}) \right)$$

“Derivation” of LG distribution

Gibbs distribution



What is the state with maximizing information entropy: $S(\hat{\rho}) = -\mathrm{Tr}\hat{\rho}\log\hat{\rho}$

under constraints:

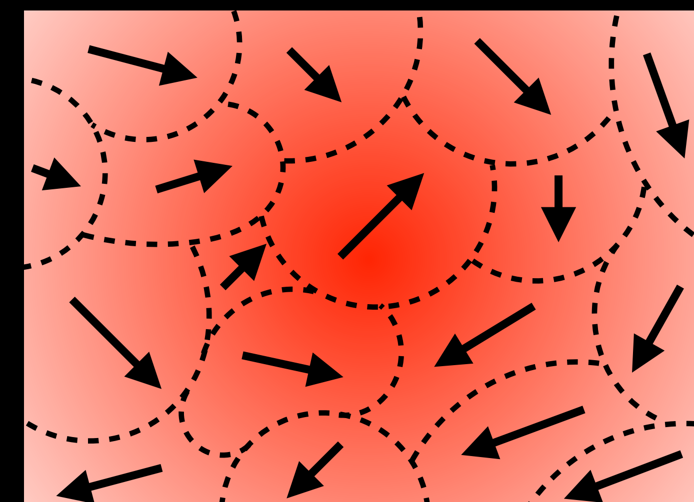
$$\langle \hat{H} \rangle = E = \text{const.}, \ \langle \hat{N} \rangle = N = \text{const.}$$

Answer:

$$\hat{\rho}_{\mathrm{G}} = e^{-\beta\hat{H}-\nu\hat{N}-\Psi[\beta,\nu]}$$

Lagrange multipliers: $\Lambda^a = \{\beta, \nu = \beta\mu\}$

Local Gibbs distribution



What is the state with maximizing information entropy: $S(\hat{\rho}) = -\mathrm{Tr}\hat{\rho}\log\hat{\rho}$

under constraints:

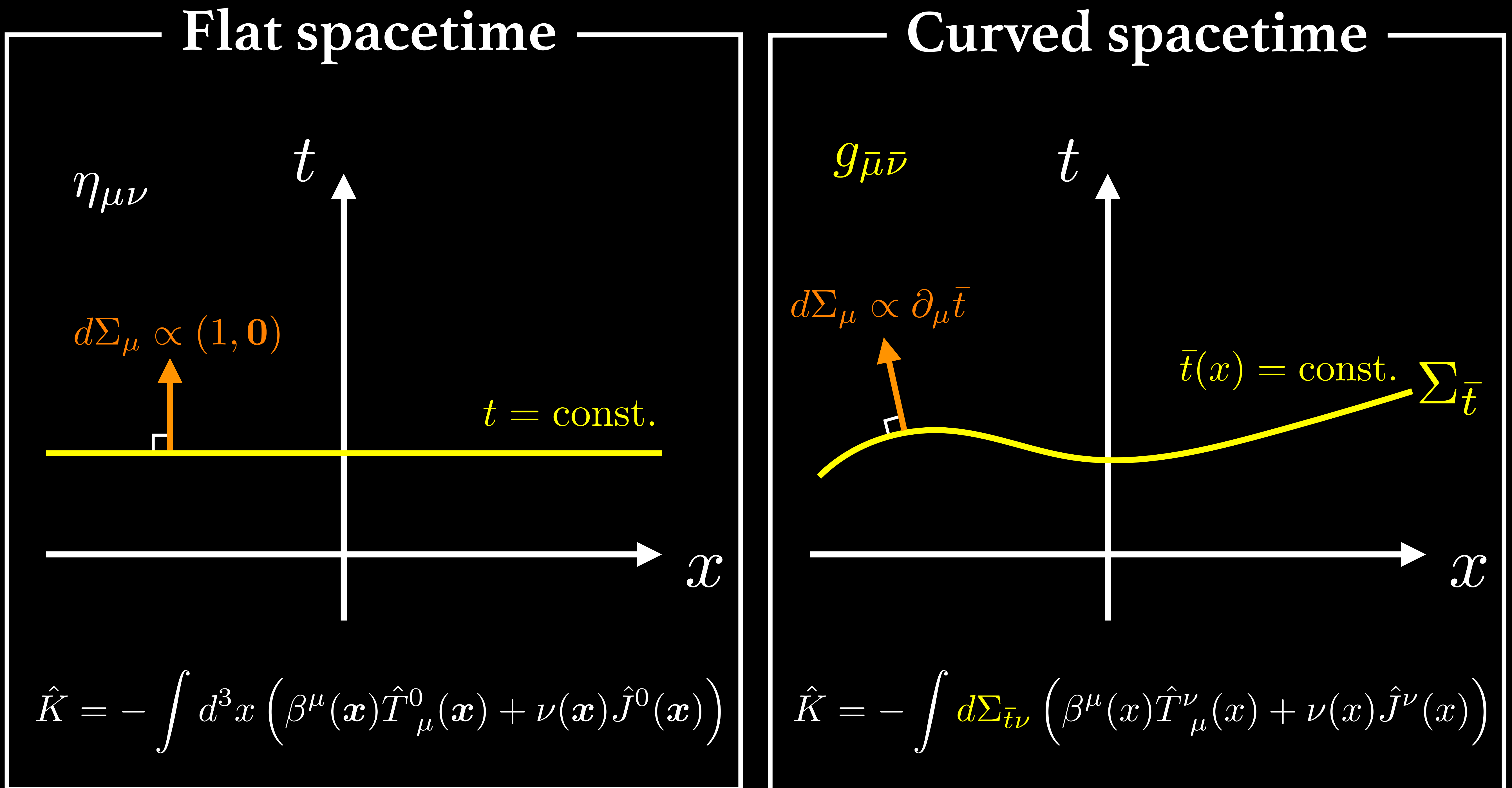
$$\langle \hat{T}^0{}_\mu(x) \rangle = p_\mu(x), \ \langle \hat{J}^0(x) \rangle = n(x)$$

Answer:

$$\hat{\rho}_{\mathrm{LG}} = e^{-\int d^{d-1}x(\beta^\mu\hat{T}^0{}_\mu+\nu\hat{J}^0)-\Psi[\beta^\mu,\nu]}$$

Lagrange multipliers: $\lambda^a(x) = \{\beta^\mu(x), \nu(x)\}$

Introducing background metric

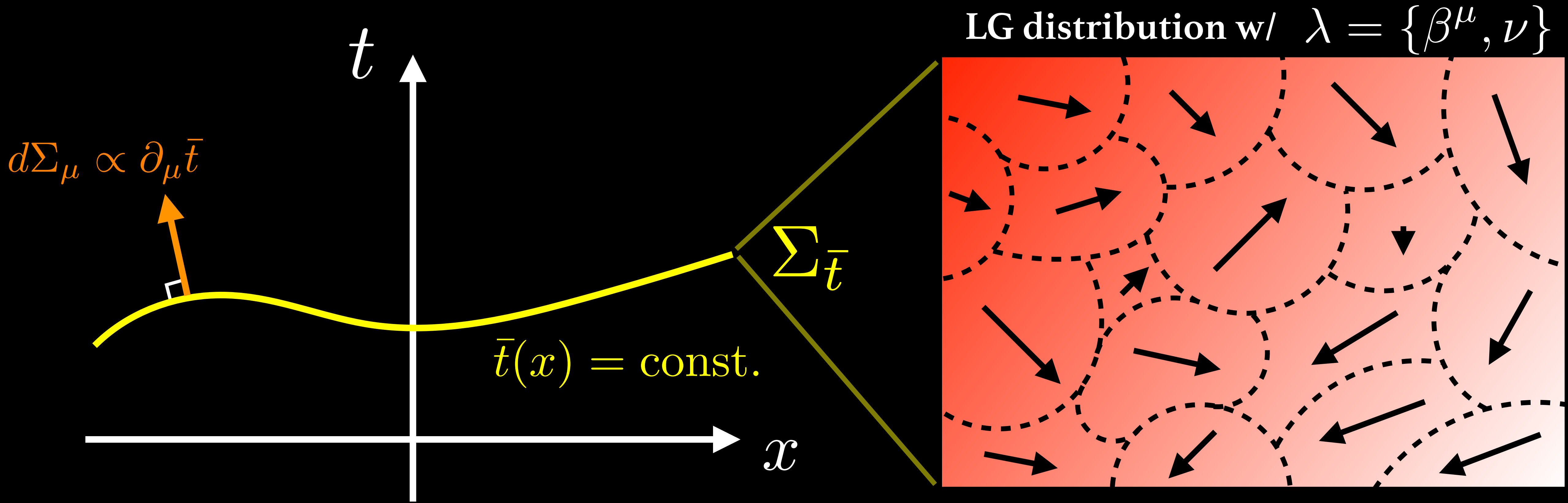


→ ① Formulation becomes manifestly covariant

② Background metric plays a role as external field coupled to $T^{\mu\nu}$

Local thermodynamic potential

[Banerjee et al.(2012), Jensen et al.(2012) , Haehl et al. (2015), MH(2017)]



◆ **Massieu-Planck fcn. (= log Z) as generating functional**

$$\Psi[\bar{t};\lambda] \equiv \log \mathrm{Tr} \exp \left[\int d\Sigma_{\bar{t}\nu} \left(\beta^\mu(x) \hat{T}^\nu{}_\mu(x) + \nu(x) \hat{J}^\nu(x) \right) \right]$$

$$\langle \hat{T}^{\mu\nu}(x) \rangle_{\bar{t}}^{\mathrm{LG}} = \frac{2}{\beta' \sqrt{\gamma}} \frac{\delta \Psi[\bar{t};\lambda]}{\delta g_{\mu\nu}(x)}, \quad \langle \hat{J}^{\mu}(x) \rangle_{\bar{t}}^{\mathrm{LG}} = \frac{1}{\beta' \sqrt{\gamma}} \frac{\delta \Psi[\bar{t};\lambda]}{\delta A_{\mu}(x)}$$

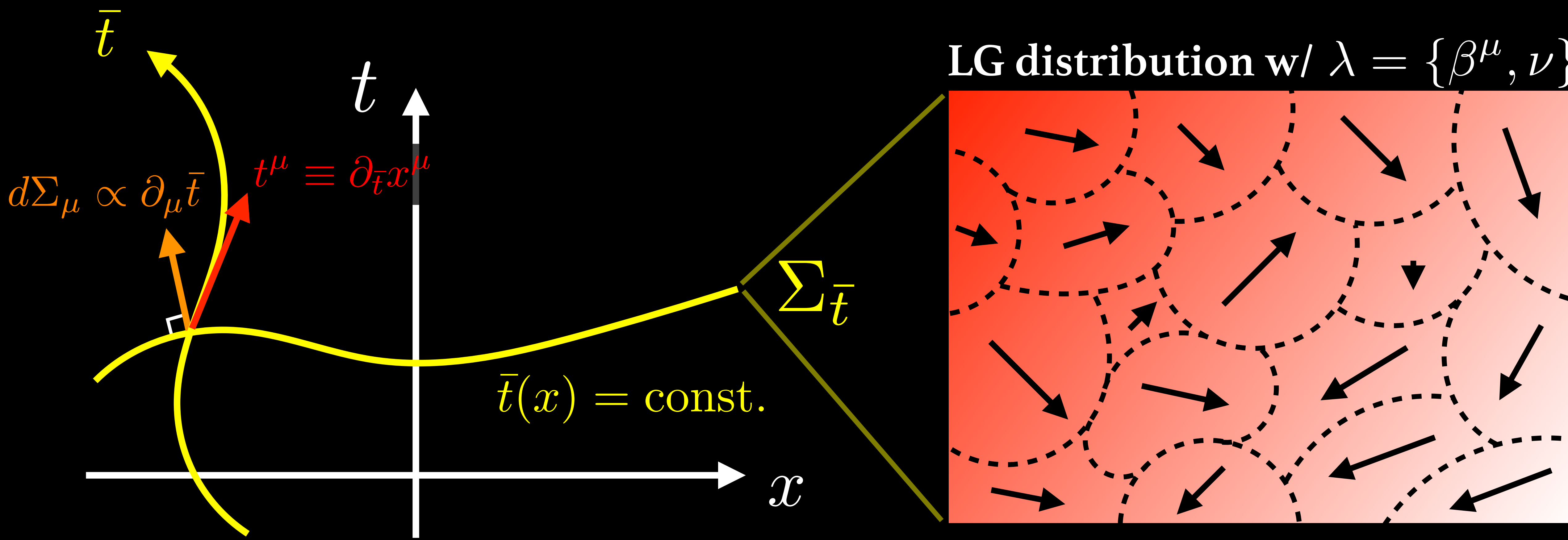
Variation formula for local equil.

[Banerjee et al.(2012), Jensen et al.(2012) , Haehl et al. (2015), MH(2017), ...]

Variational formula in "hydrostatic gauge"

$$\langle \hat{T}^{\mu\nu}(x) \rangle_{\bar{t}}^{\mathrm{LG}} = \frac{2}{\sqrt{-g}} \frac{\delta}{\delta g_{\mu\nu}(x)} \Psi[\bar{t};\lambda], \quad \langle \hat{J}^{\mu}(x) \rangle_{\bar{t}}^{\mathrm{LG}} = \frac{1}{\sqrt{-g}} \frac{\delta}{\delta A_{\mu}(x)} \Psi[\bar{t};\lambda]$$

(Local) Thermodynamic Potential

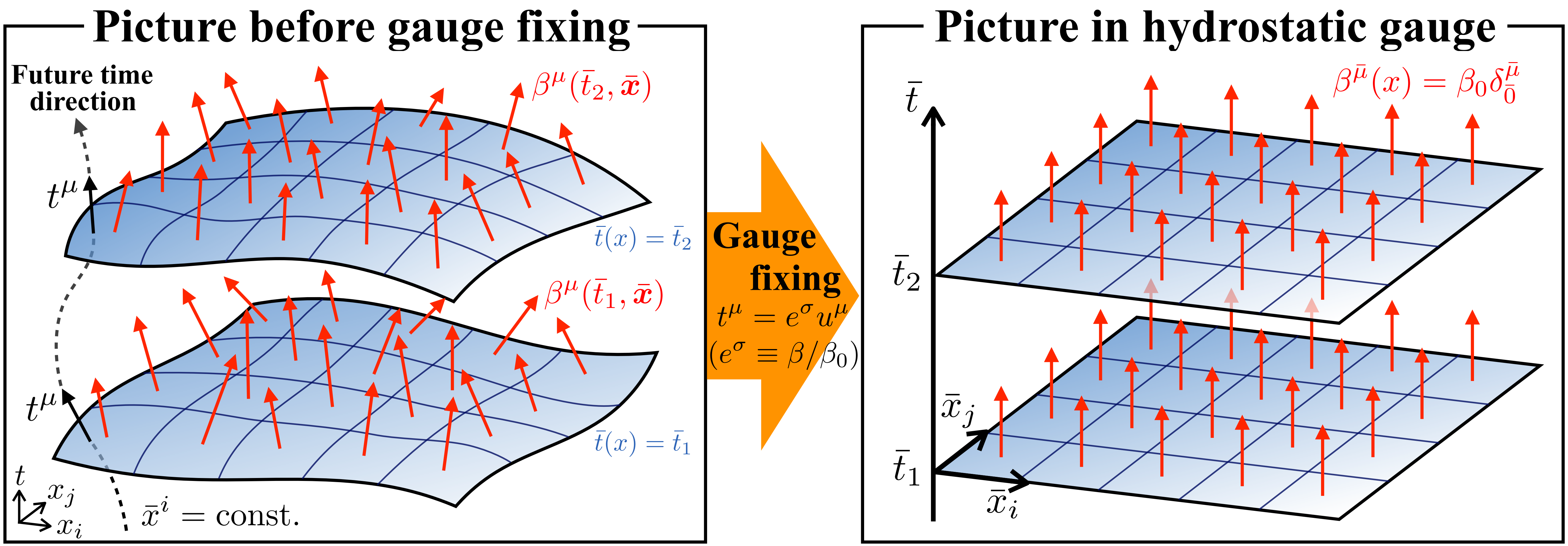


Masseiu-Planck functional

$$\Psi[\bar{t};\lambda] \equiv \log \mathrm{Tr} \exp \left[\int d\Sigma_{\bar{t}\nu} \left(\beta^\mu(x) \hat{T}^\nu{}_\mu(x) + \nu(x) \hat{J}^\nu(x) \right) \right]$$

$$= \log \mathrm{Tr} \exp \left[- \int d^3\bar{x} \sqrt{-g} \left(\beta^{\bar{\mu}}(\bar{x}) \hat{T}^{\bar{0}}{}_{\bar{\mu}}(\bar{x}) + \nu(\bar{x}) \hat{J}^{\bar{0}}(\bar{x}) \right) \right]$$

Hydrostatic gauge fixing



We can choose the time direction vector $t^\mu(x) \equiv \partial_{\bar{t}} x^\mu$

Hydrostatic gauge fixing

Let us choose $t^\mu(x) = \beta^\mu(x)/\beta_0, \ A_{\bar{0}}(x) = \nu(x)$

Variational formula for local equil.

[Banerjee et al.(2012), Jensen et al.(2012) , Haehl et al. (2015), MH(2017), …]

Variational formula in "hydrostatic gauge"

$$\langle \hat{T}^{\mu\nu}(x)\rangle_{\bar{t}}^{\mathrm{LG}} = \frac{2}{\sqrt{-g}}\frac{\delta}{\delta g_{\mu\nu}(x)}\Psi[\bar{t};\lambda], \quad \langle \hat{J}^{\mu}(x)\rangle_{\bar{t}}^{\mathrm{LG}} = \frac{1}{\sqrt{-g}}\frac{\delta}{\delta A_{\mu}(x)}\Psi[\bar{t};\lambda]$$

Proof. Consider time derivative of $\Psi[\lambda]$

$$\partial_{\bar{t}}\Psi[\bar{t};\lambda] = \int d^{d-1}\bar{x}\sqrt{-g}\left(\nabla_\mu\beta_\nu\langle\hat{T}^{\mu\nu}\rangle_{\bar{t}}^{\mathrm{LG}} + (\nabla_\mu\nu + F_{\nu\mu}\beta^\nu)\langle\hat{J}^\mu\rangle_{\bar{t}}\right)$$

$$= \int d^{d-1}\bar{x}\sqrt{-g}\left(\frac{1}{2}(\nabla_\mu\beta_\nu + \nabla_\nu\beta_\mu)\langle\hat{T}^{\mu\nu}\rangle_{\bar{t}}^{\mathrm{LG}} + (\beta^\nu\nabla_\nu A_\mu + A_\nu\nabla_\mu\beta^\nu)\langle\hat{J}^\mu\rangle_{\bar{t}}\right)$$

$$= \int d^{d-1}\bar{x}\sqrt{-g}\left(\frac{1}{2}\pounds_\beta g_{\mu\nu}\langle\hat{T}^{\mu\nu}\rangle_{\bar{t}}^{\mathrm{LG}} + \pounds_\beta A_\mu\langle\hat{J}^\mu\rangle_{\bar{t}}\right)$$

On the other hand, since $t^\mu = \beta^\mu$, we can express the LHS as

$$\partial_{\bar{t}}\Psi[\bar{t};\lambda] = \int d^{d-1}\bar{x}\left(\pounds_\beta g_{\mu\nu}\frac{\delta\Psi}{\delta g_{\mu\nu}} + \pounds_\beta A_\mu\frac{\delta\Psi}{\delta A_\mu}\right)$$

Matching them gives the above variation formula! $\square$

Q. How can we calculate $\Psi \equiv \log Z$?

Thermal QFT in a Nutshell

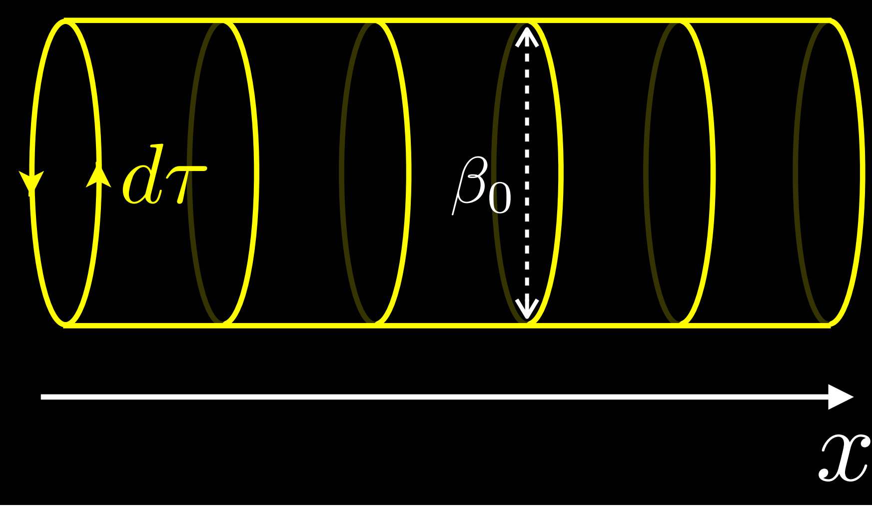
Global equil. β_0

$$T = \text{const.}$$

Path int.

Thermal QFT (Matsubara formalism)

[Matsubara, 1955]



QFT in the flat spacetime with size β_0

Gibbs dist.: $$\hat{\rho}_G = \frac{e^{-\beta(\hat{H}-\mu\hat{N})}}{Z} = e^{-\beta(\hat{H}-\mu\hat{N})-\Psi[\beta,\nu]}$$

Thermodynamic potential with Euclidean action

$$\Psi[\beta,\nu] = \log \mathrm{Tr}\, e^{-\beta(\hat{H}-\mu\hat{N})} = \log \int d\varphi \langle \pm\varphi | e^{-\beta(\hat{H}-\mu\hat{N})} | \varphi \rangle$$

$$= \log \int_{\varphi(\beta)=\pm\varphi(0)} \mathcal{D}\varphi\, e^{+S_E[\varphi]}, \quad S_E[\varphi] = \int_0^\beta d\tau \int d^3x\, \mathcal{L}_E(\varphi, \partial_\mu\varphi)$$

Local Thermal QFT

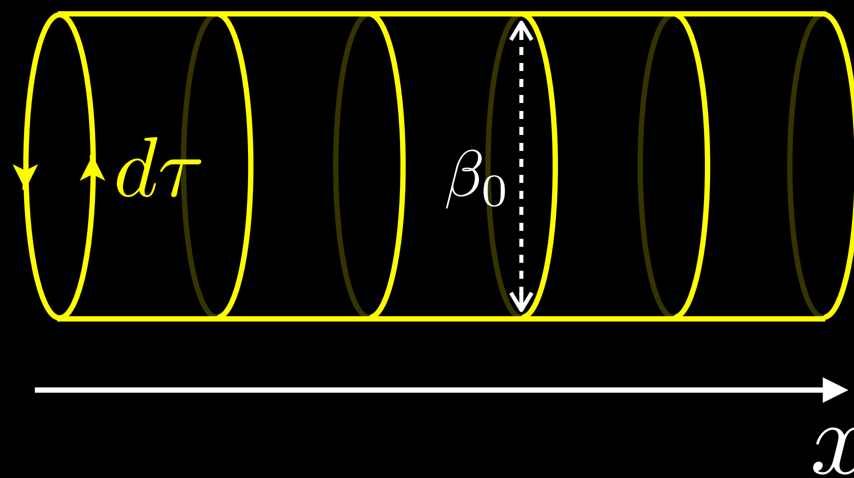
Global equil. β_0

$$T = \text{const.}$$

Path int.

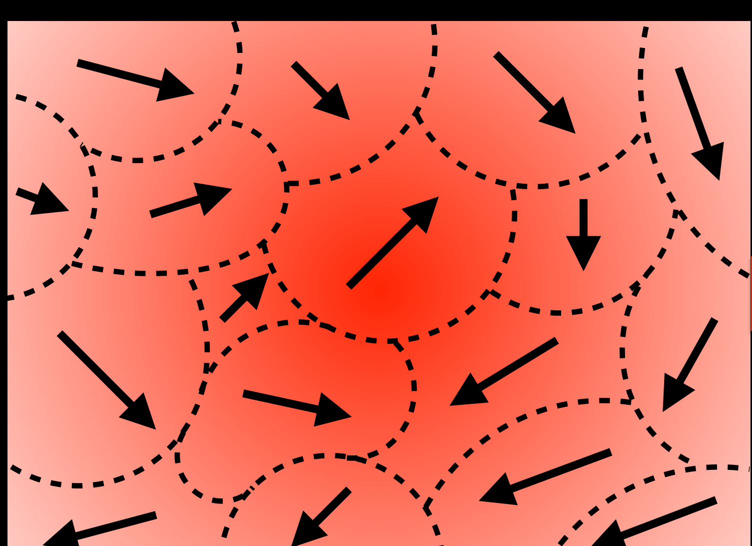
Thermal QFT (Matsubara formalism)

[Matsubara, 1955]



QFT in the flat spacetime with size β_0

Local equil. $\{\beta(x),\ \vec{v}(x)\}$



Path int.

Local Thermal QFT

Case study I: Scalar field

$$\mathcal{L} = -\frac{g^{\bar\mu\bar\nu}}{2}\partial_{\bar\mu}\phi\partial_{\bar\nu}\phi - V(\phi)$$

$$\Rightarrow \quad \hat{T}^{\mu\nu} \equiv \frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g_{\mu\nu}} = \partial^\mu\hat\phi\partial^\nu\hat\phi + g^{\mu\nu}\mathcal{L}(\hat\phi, \partial_\rho\hat\phi)$$

$$\Psi[\bar t;\lambda] = \log \mathrm{Tr}\exp\left[-\int d^{d-1}\bar x\sqrt{-g}\beta^\mu(x)\hat{T}^{\bar 0}{}_\mu(x)\right]$$
$$= \log\int \mathcal{D}\phi \exp\left(S_E[\phi,\beta^\mu]\right) = \log\int \mathcal{D}\phi \exp\left(S_E[\phi,\tilde g]\right)$$

$$S[\phi,\beta^\mu] = \int_0^{\beta_0} d\tau \int d^3\bar x\sqrt{-g}e^\sigma u^{\bar 0}\left[-\frac{e^{-2\sigma}}{2u^{\bar 0}u_{\bar 0}}(i\dot\phi)^2 - \frac{-e^{-\sigma}u^{\bar i}}{u^{\bar 0}u_{\bar 0}}(i\dot\phi)\partial_{\bar i}\phi - \frac{1}{2}\left(\gamma^{\bar i\bar j} + \frac{u^{\bar i}u^{\bar j}}{u^{\bar 0}u_{\bar 0}}\right)\partial_{\bar i}\phi\partial_{\bar j}\phi - V(\phi)\right]$$
$$= \int_0^{\beta_0} d\tau \int d^3\bar x\sqrt{-\tilde g}\left[-\frac{\tilde g^{\bar\mu\bar\nu}}{2}\partial_{\bar\mu}\phi\partial_{\bar\nu}\phi - V(\phi)\right] \qquad \left(e^{\sigma(\bar x)} \equiv \beta(\bar x)/\beta_0\right)$$

Ψ in terms of thermal metric

$$\Psi[\bar{t};\lambda] = \log \int \mathcal{D}\phi \exp\left(S_E[\phi,;\tilde{g}]\right)$$

Thermal metric

$$\tilde{g}_{\bar{\mu}\bar{\nu}} = \begin{pmatrix} -e^{2\sigma} & e^{\sigma} u_{\bar{j}} \\ e^{\sigma} u_{\bar{i}} & \gamma_{\bar{i}\bar{j}} \end{pmatrix}$$

$$(e^{\sigma(\bar{x})} \equiv \beta(\bar{x})/\beta_0)$$

Inverse thermal metric

$$\tilde{g}^{\bar{\mu}\bar{\nu}} = \begin{pmatrix} \dfrac{e^{-2\sigma}}{u^{\bar{0}}u_{\bar{0}}} & -\dfrac{e^{-\sigma}u^{\bar{j}}}{u^{\bar{0}}u_{\bar{0}}} \\ -\dfrac{e^{-\sigma}u^{\bar{i}}}{u^{\bar{0}}u_{\bar{0}}} & \gamma^{\bar{i}\bar{j}} + \dfrac{u^{\bar{i}}u^{\bar{j}}}{u^{\bar{0}}u_{\bar{0}}} \end{pmatrix}$$

◆ Interpretation of above result

$\Psi[\bar{t};\lambda]$ **is described by QFT in "curved spacetime" s. t.**

$$d\tilde{s}^2 = -e^{2\sigma}(d\tilde{t} + a_{\bar{i}}dx^{\bar{i}})^2 + \gamma'_{\bar{i}\bar{j}}dx^{\bar{i}}dx^{\bar{j}}$$

$$(a_{\bar{i}} \equiv e^{-\sigma}u_{\bar{i}}, \quad \gamma'_{\bar{i}\bar{j}} \equiv \gamma_{\bar{i}\bar{j}} + u_{\bar{i}}u_{\bar{j}}, \quad d\tilde{t} = -id\tau)$$

Case study 2: Dirac field

$$\mathcal{L} = -\frac{1}{2}\bar{\psi}\left(\gamma^a e_a{}^{\bar{\mu}}\overrightarrow{D}_{\bar{\mu}} - \overleftarrow{D}_{\bar{\mu}}\gamma^a e_a{}^{\bar{\mu}}\right)\psi - m\bar{\psi}\psi$$

Symmetric energy-momentum tensor

$$T^{\bar{\mu}}{}_{\bar{\nu}} = -\delta^{\bar{\mu}}_{\bar{\nu}}\mathcal{L} - \frac{1}{4}\bar{\psi}(\gamma^{\bar{\mu}}\overrightarrow{D}_{\bar{\nu}} + \gamma_{\bar{\nu}}\overrightarrow{D}^{\bar{\mu}} - \overleftarrow{D}_{\bar{\nu}}\gamma^{\bar{\mu}} - \overleftarrow{D}^{\bar{\mu}}\gamma_{\bar{\nu}})\psi$$

◆ Result of path integral

$$\Psi[\bar{t};\lambda] \equiv \log \operatorname{Tr}\exp\left[\int d\Sigma_{\bar{t}\nu}\left(\beta^\mu(x)\hat{T}^\nu{}_\mu(x) + \nu(x)\hat{J}^\nu(x)\right)\right]$$

$$= \log\int \mathcal{D}\bar{\psi}\mathcal{D}\psi\exp\left(S_E[\psi,\bar{\psi};\tilde{e}]\right)$$

ψ in terms of thermal vielbein

$$\Psi[\bar{t};\lambda] = \log \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left(S_E[\psi,\bar{\psi};\tilde{e}]\right)$$

◆ **Euclidean action with thermal vielbein**

$$S_E[\psi,\bar{\psi};\tilde{e}] = \int_0^{\beta_0} d\tau \int d^3\bar{x}\tilde{e}\left[-\frac{1}{2}\bar{\psi}\left(\gamma^a \tilde{e}_a{}^{\bar{\mu}}\overrightarrow{D}_{\bar{\mu}} - \overleftarrow{D}_{\bar{\mu}}\gamma^a\tilde{e}_a{}^{\bar{\mu}}\right)\psi - m\bar{\psi}\psi\right]$$

Thermal vielbein : $\tilde{e}_{\bar{0}}{}^a = e^{\sigma}u^a,\ \ \tilde{e}_{\bar{i}}{}^a = e_{\bar{i}}{}^a \quad (e^{\sigma} \equiv \beta(x)/\beta_0)$

◆ **Interpretation of above result**

$\Psi[\bar{t};\lambda]$ **is described by QFT in "curved spacetime" s. t.**

$$d\tilde{s}^2 = \tilde{e}_{\bar{\mu}}{}^a\tilde{e}_{\bar{\nu}}{}^b\eta_{ab}dx^{\bar{\mu}}dx^{\bar{\nu}} = -e^{2\sigma}(d\tilde{t} + a_{\bar{i}}dx^{\bar{i}})^2 + \gamma'_{\bar{i}\bar{j}}dx^{\bar{i}}dx^{\bar{j}}$$

$$(a_{\bar{i}} \equiv e^{-\sigma}u_{\bar{i}}, \quad \gamma'_{\bar{i}\bar{j}} \equiv \gamma_{\bar{i}\bar{j}} + u_{\bar{i}}u_{\bar{j}}, \quad d\tilde{t} = -id\tau)$$

Local Thermal QFT

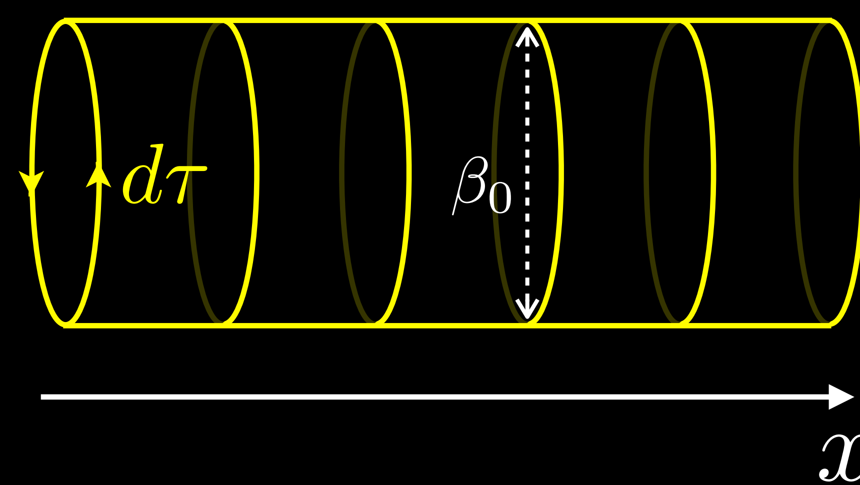
Global equil. β_0

$$T = \text{const.}$$

Path int.

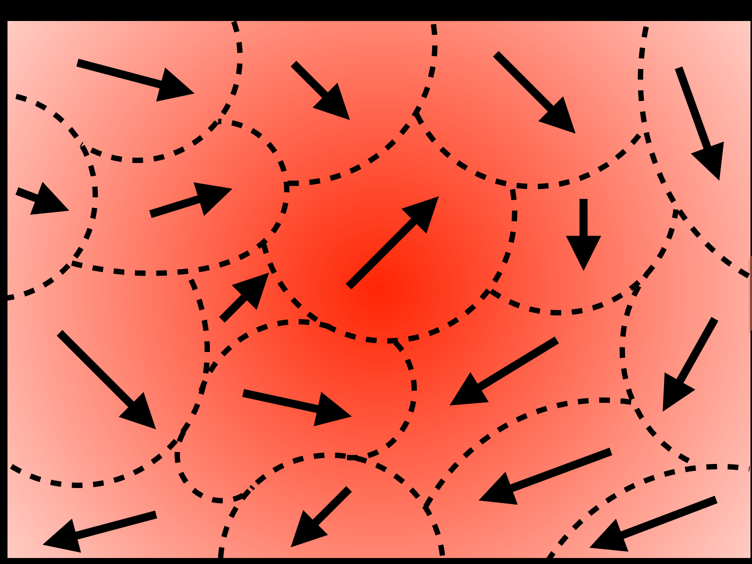
Thermal QFT (Matsubara formalism)

[Matsubara, 1955]



QFT in the flat spacetime with size β_0

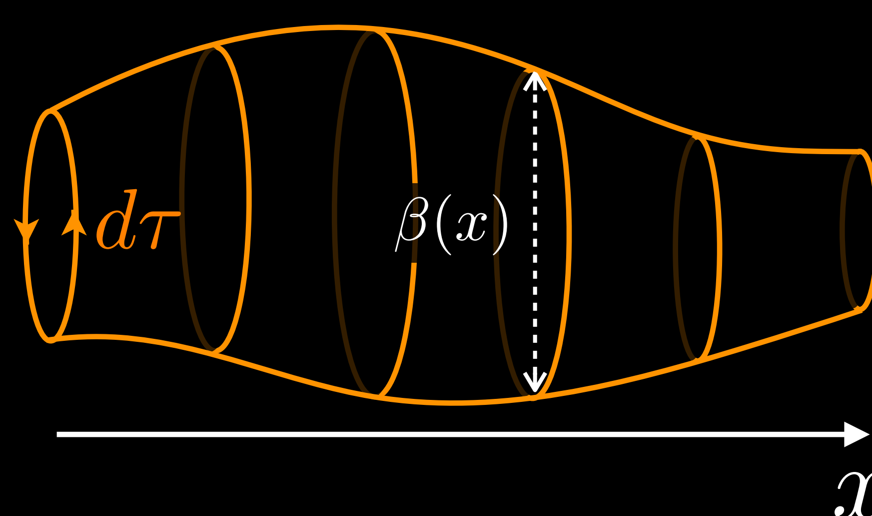
Local equil. $\{\beta(x),\ \vec{v}(x)\}$



Path int.

Local Thermal QFT

[Hayata-Hidaka-MH-Noumi PRD(2015)]
[MH (2017)]



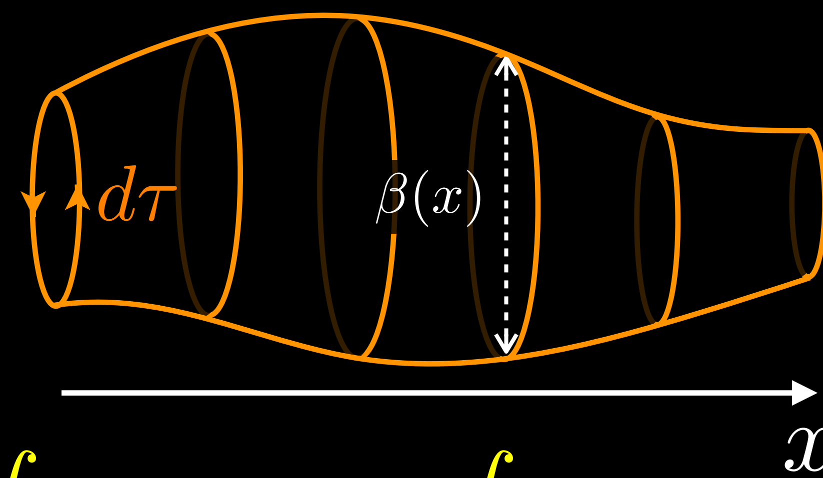
QFT in the "curved spacetime" with "line element"

$$d\tilde{s}^2 = d\tilde{s}^2(\beta, \vec{v})$$

Two ways to construct $\Psi \equiv \log Z$

1. Use diffeo & gauge invariance!

- Ψ is expressed by $\{\tilde{g}_{\mu\nu}, \tilde{A}_\mu\}$
- Ψ is diffeo & gauge invariant!



➡ Ψ is expressed in terms of $\beta = \oint d\tilde{s},\ \beta\mu = \oint \tilde{A},\ \tilde{R},\ \tilde{F}_{\mu\nu}$

2. Use symmetry from imaginary-time nature!

- Ψ is spatial diffeomorphism invariant
- Ψ is Kaluza-Klein gauge invariant!

➡ $\Psi \equiv \log Z$ should respect these two symmetries!!

[cf. Hydrostatic partition function method Banerjee et al.(2012), Jensen et al.(2012)]

Two ways to construct $\Psi \equiv \log Z$

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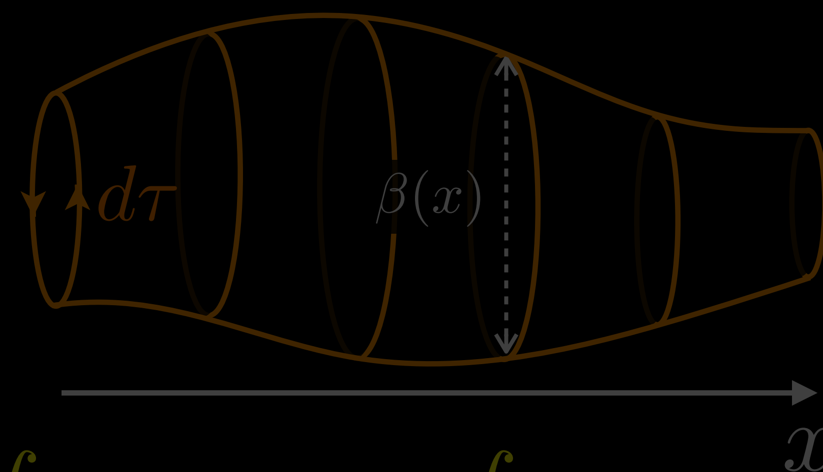

➡ Ψ is expressed in terms of $\beta = \oint d\tilde{s},\ \beta\mu = \oint \tilde{A},\ \tilde{R},\ \tilde{F}_{\mu\nu}$

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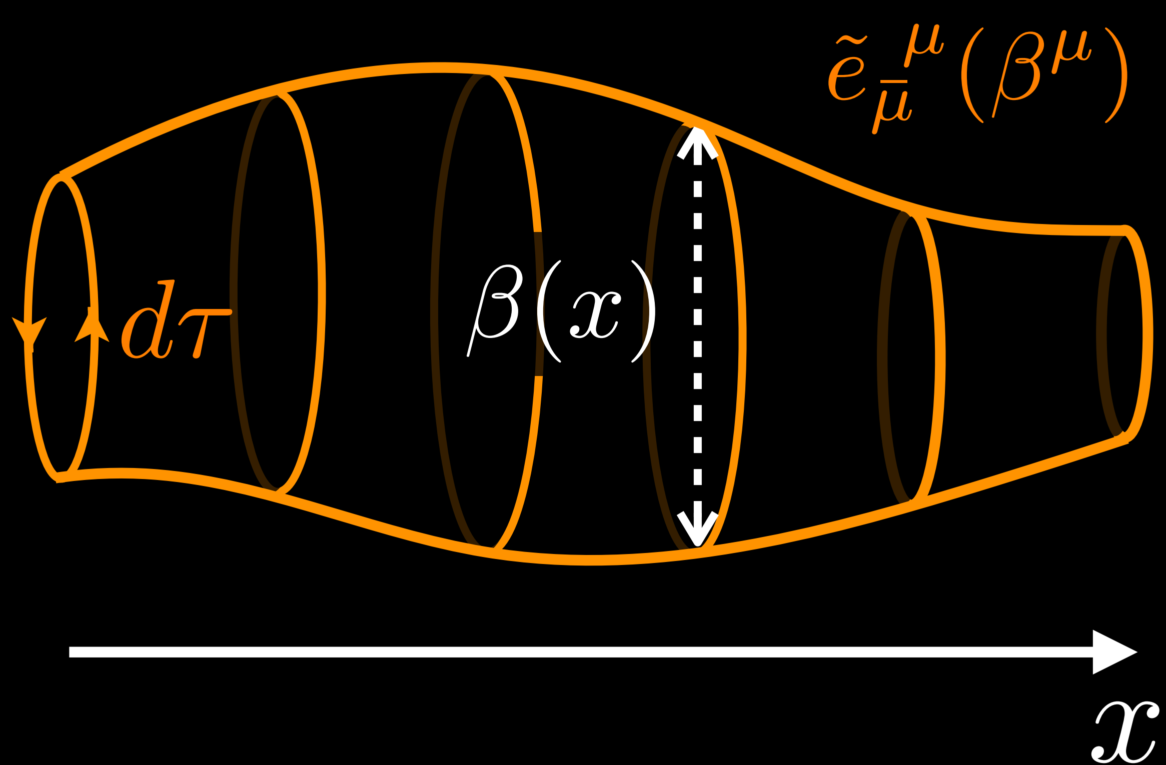
➡ $\Psi \equiv \log Z$ should respect these two symmetries!!

[cf. Hydrostatic partition function method Banerjee et al.(2012), Jensen et al.(2012)]

Kaluza-Klein gauge symmetry

$$d\tilde{s}^2 = -e^{2\sigma}(d\tilde{t} + a_{\bar{i}}dx^{\bar{i}})^2 + \gamma'_{\bar{i}\bar{j}}dx^{\bar{i}}dx^{\bar{j}} \ \ (d\tilde{t} = -id\tau)$$



Parameters λ don't depend on imaginary time τ.

"Kaluza-Klein" gauge tr.

$$\begin{cases} \tilde{t} \to \tilde{t} + \chi(\bar{\boldsymbol{x}}) \\ \bar{\boldsymbol{x}} \to \bar{\boldsymbol{x}} \\ a_{\bar{i}}(\bar{\boldsymbol{x}}) \to a_{\bar{i}}(\bar{\boldsymbol{x}}) - \partial_{\bar{i}}\chi(\bar{\boldsymbol{x}}) \end{cases}$$

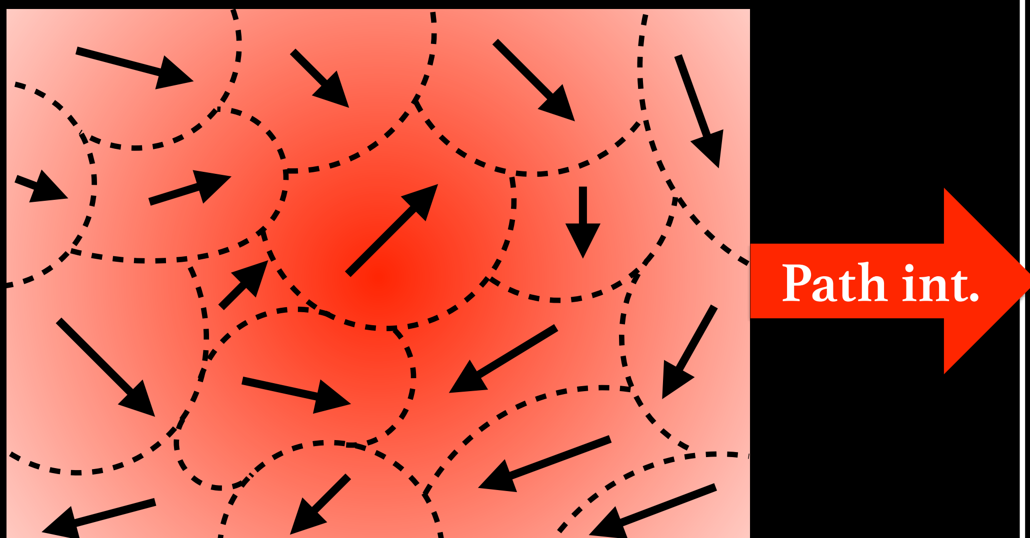
$$\Psi[\lambda] = \log \int \mathcal{D}\bar{\psi}\mathcal{D}\psi e^{S[\psi,\bar{\psi},\tilde{e}]} \ni$$

$$(f_{\bar{i}\bar{j}} \equiv \partial_{\bar{i}}a_{\bar{j}} - \partial_{\bar{j}}a_{\bar{i}})$$

- (good) $f^{\bar{i}\bar{j}}f_{\bar{i}\bar{j}}, \cdots$
- (bad) $a_{\bar{i}}, \ a_{\bar{i}}a^{\bar{i}}, \cdots$

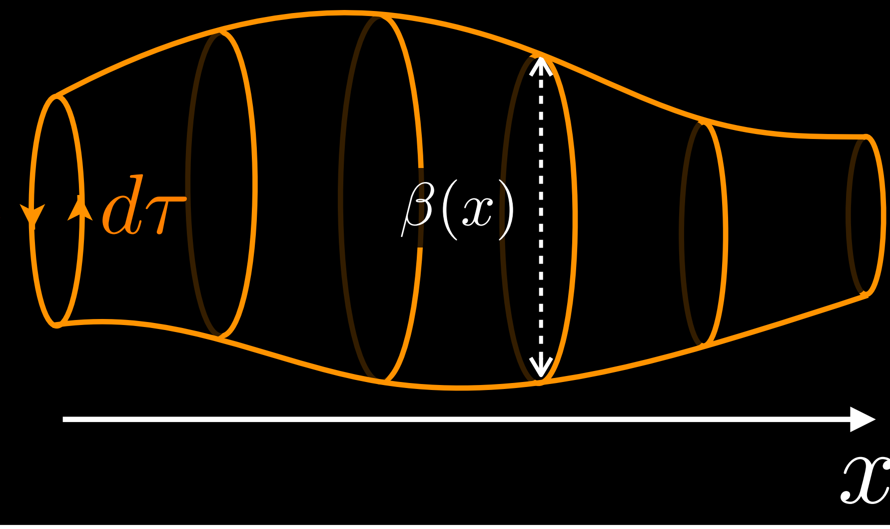
Short Summary: Local Thermal QFT

Local equil. $\{\beta(x),\ \vec{v}(x)\}$



Local Thermal QFT

[Hayata-Hidaka-MH-Noumi PRD(2015)]
[MH (2017)]



QFT in the "curved spacetime" with "line element"

$$d\tilde{s}^2 = d\tilde{s}^2(\beta, \vec{v})$$

$$\Psi[\bar{t};\lambda] \equiv \log \mathrm{Tr} \exp \left[\int d\Sigma_{\bar{t}\nu} \Big(\beta^\mu(x) \hat{T}^\nu{}_\mu(x) + \nu(x) \hat{J}^\nu(x) \Big) \right]$$

① $\Psi[\lambda]$ plays a role as the generating functional: $\langle \hat{T}^{\mu\nu}(x) \rangle^{\mathrm{LG}} = \dfrac{2}{\sqrt{-g}} \dfrac{\delta}{\delta g_{\mu\nu}(x)} \Psi[\lambda]$

② $\Psi[\lambda]$ is written in terms of QFT in curved spacetime

$$d\tilde{s}^2 = -e^{2\sigma}(d\tilde{t} + a_{\bar{i}} dx^{\bar{i}})^2 + \gamma'_{\bar{i}\bar{j}} dx^{\bar{i}} dx^{\bar{j}}$$

Symmetry = Spatial diffeomorphism + Kaluza-Klein gauge

Q. How can we calculate $\Psi \equiv \log Z$?

A. Symmetry-based derivative exp.!

Case 1

Hydro without anomaly

Derivative expansion of ψ

Derivative expansion of ψ

$$\Psi[\beta^\mu, \nu] = \boxed{\Psi^{(0)}[\beta^\mu, \nu]} + \boxed{\Psi^{(1)}[\beta^\mu, \nu, \partial]} + \mathcal{O}(\partial^2) + \cdots$$

$\simeq \beta p$ — Symmetry property

$= 0$ — **Parity-even system**

Non-dissipative constitutive relation

$$\langle \hat{T}^{\mu\nu}(x) \rangle_{\bar{t}}^{\mathrm{LG}} = \frac{2}{\sqrt{-g}} \frac{\delta}{\delta g_{\mu\nu}(x)} \Psi[\bar{t}; \lambda] = T^{\mu\nu}_{(0)}[\lambda(x)] + \boxed{T^{\mu\nu}_{(1)}[\lambda(x), \nabla\lambda(x)]} + \cdots$$

$$\langle \hat{J}^{\mu}(x) \rangle_{\bar{t}}^{\mathrm{LG}} = \frac{2}{\sqrt{-g}} \frac{\delta}{\delta A_{\mu}(x)} \Psi[\bar{t}; \lambda] = J^{\mu}_{(0)}[\lambda(x)] + \boxed{J^{\mu}_{(1)}[\lambda(x), \nabla\lambda(x)]} + \cdots$$

$= 0$

Recipe for Masseiu-Planck fcn.

[Banerjee et al.(2012), Jensen et al.(2012)]

Masseiu-Planck functional

$$\Psi[\lambda] = \log \int \mathcal{D}\phi e^{S[\phi,\tilde{g}]} = \underset{\mathcal{O}(p^0)}{\Psi^{(0)}[\lambda]} + \underset{\mathcal{O}(p^1)}{\Psi^{(1)}[\lambda,\partial]} + \mathcal{O}(\partial^2)$$

- **Building blocks**：$\lambda = \{e^{\sigma},\ a_{\bar{i}},\ \mu,\ A_{\bar{i}}\}$

- **Symmetry**：Spatial diffeo, Kaluza-Klein, Gauge

  $A_{\bar{i}}$ ：not Kaluza-Klein inv. ➡ $A'_{\bar{i}} \equiv A_{\bar{i}} - \mu a_{\bar{i}}$

- **Power counting scheme**： $\lambda = \mathcal{O}(p^0)$

  $f_{\bar{i}\bar{j}} \equiv \partial_{\bar{i}} a_{\bar{j}} - \partial_{\bar{j}} a_{\bar{i}} = \mathcal{O}(p^1)$ ➡ $ff = \mathcal{O}(p^2)$

$\Psi^{(0)}$: Order $\mathcal{O}(p^0)$

[Banerjee et al.(2012), Jensen et al.(2012)]

Masseiu-Planck functional

$$\Psi[\lambda] = \log \int \mathcal{D}\phi e^{S[\phi,\tilde{g}]} = \underbrace{\Psi^{(0)}[\lambda]}_{\mathcal{O}(p^0)} + \underbrace{\Psi^{(1)}[\lambda, \partial]}_{\mathcal{O}(p^1)} + \mathcal{O}(\partial^2)$$

- Building blocks : $\lambda = \{e^\sigma, \cancel{a_{\bar{i}}}, \mu, \cancel{A'_{\bar{i}}}\}$

$$\Psi^{(0)}[\lambda] = \int_0^{\beta_0} d\tau \int d^3\bar{x}\sqrt{\gamma'} e^\sigma p(\beta, \mu)$$

→ **Perfect fluid**

$$\langle \hat{T}^{\mu\nu}(x)\rangle_{\bar{t}}^{\mathrm{LG}} = (e+p)u^\mu u^\nu + p\eta^{\mu\nu}$$

$$\langle \hat{J}^{\mu}(x)\rangle_{\bar{t}}^{\mathrm{LG}} = n u^\mu$$

<u>Case 2</u>

Hydro **with** anomaly

Recipe for Masseiu-Planck fcn.

Weyl fermion : $\mathcal{L} = \frac{i}{2}\xi^\dagger \left(e_m^{\ \mu} \sigma^m \overrightarrow{D}_\mu - \overleftarrow{D}_\mu \sigma^m e_m^{\ \mu} \right) \xi$

$$\Psi[\lambda] = \log \int \mathcal{D}\xi^\dagger \mathcal{D}\xi e^{S[\xi,\xi^\dagger,A,\tilde{e}]} = \underbrace{\Psi^{(0)}[\lambda]}_{\mathcal{O}(p^0)} + \underbrace{\Psi^{(1)}[\lambda,\partial]}_{\mathcal{O}(p^1)} + \mathcal{O}(\partial^2)$$

- **Building blocks** : $\lambda = \{e^\sigma,\ a_{\bar{i}},\ \mu_R,\ A'_{\bar{i}}\}$

- **Symmetry** : Spatial diffeo, Kaluza-Klein, U(1)$_R$-gauge

  $A_{\bar{i}}$: not Kaluza-Klein inv. ➡ $A'_{\bar{i}} \equiv A_{\bar{i}} - \mu_R a_{\bar{i}}$

- **Power counting scheme** : $\lambda = \mathcal{O}(p^0)$

  $f_{\bar{i}\bar{j}} \equiv \partial_{\bar{i}} a_{\bar{j}} - \partial_{\bar{j}} a_{\bar{i}} = \mathcal{O}(p^1)$ ➡ $ff = \mathcal{O}(p^2)$

Derivative expansion of Ψ

(2) Derivative expansion of ψ

$$\Psi[\beta^\mu,\nu] = \Psi^{(0)}[\beta^\mu,\nu] + \Psi^{(1)}[\beta^\mu,\nu,\partial] + \mathcal{O}(\partial^2) + \cdots$$

$\simeq \beta p$ — Symmetry property

$= 0$ **Parity-even system**

$\neq 0$ **Parity-odd system**

Non-dissipative constitutive relation

$$\langle \hat{T}^{\mu\nu}(x)\rangle_{\bar{t}}^{\mathrm{LG}} = \frac{2}{\sqrt{-g}}\frac{\delta}{\delta g_{\mu\nu}(x)}\Psi[\bar{t};\lambda] = T^{\mu\nu}_{(0)}[\lambda(x)] + T^{\mu\nu}_{(1)}[\lambda(x),\nabla\lambda(x)] + \cdots$$

$$\langle \hat{J}^{\mu}(x)\rangle_{\bar{t}}^{\mathrm{LG}} = \frac{2}{\sqrt{-g}}\frac{\delta}{\delta A_{\mu}(x)}\Psi[\bar{t};\lambda] = J^{\mu}_{(0)}[\lambda(x)] + J^{\mu}_{(1)}[\lambda(x),\nabla\lambda(x)] + \cdots$$

$= 0$ $\neq 0$

$$-\frac{1}{24\pi^2}\varepsilon^{\mu\nu\rho\sigma}A_\nu F_{\rho\sigma}$$ Bardeen-Zumino current

$\psi^{(0)}$: Order $\mathcal{O}(p^0)$

Weyl fermion : $\mathcal{L} = \frac{i}{2}\xi^\dagger \left(e_m^{\ \mu} \sigma^m \overrightarrow{D}_\mu - \overleftarrow{D}_\mu \sigma^m e_m^{\ \mu} \right) \xi$

$$\Psi[\lambda] = \log \int \mathcal{D}\xi^\dagger \mathcal{D}\xi e^{S[\xi,\xi^\dagger,A,\tilde{e}]} = \underbrace{\Psi^{(0)}[\lambda]}_{\mathcal{O}(p^0)} + \underbrace{\Psi^{(1)}[\lambda,\partial]}_{\mathcal{O}(p^1)} + \mathcal{O}(\partial^2)$$

- Building blocks : $\lambda = \{e^\sigma, ~~a_{\bar{i}}~~, \mu_R, ~~A'_{\bar{i}}~~\}$

$$\Psi^{(0)}[\lambda] = \int_0^{\beta_0} d\tau \int d^3\bar{x}\sqrt{\gamma'} e^\sigma p(\beta, \mu_R)$$

Perfect fluid

$$\langle \hat{T}^{\mu\nu}(x) \rangle_{\bar{t}}^{\mathrm{LG}} = (e+p)u^\mu u^\nu + p\eta^{\mu\nu}$$

$$\langle \hat{J}_R^{\mu}(x) \rangle_{\bar{t}}^{\mathrm{LG}} = n_R u^\mu$$

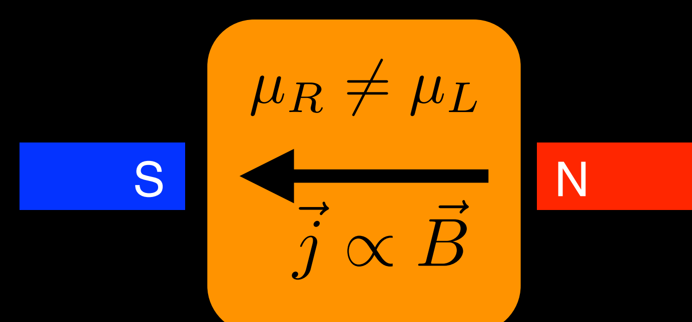
$\psi^{(\mathrm{I})}$: Order $\mathcal{O}(p)$

Weyl fermion : $\mathcal{L} = \frac{i}{2}\xi^\dagger \left(e_m^{\ \mu} \sigma^m \overrightarrow{D}_\mu - \overleftarrow{D}_\mu \sigma^m e_m^{\ \mu} \right) \xi$

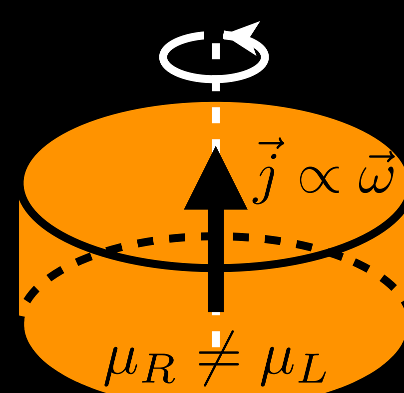
$$\Psi[\lambda] = \log \int \mathcal{D}\xi^\dagger \mathcal{D}\xi e^{S[\xi,\xi^\dagger,A,\tilde{e}]} = \underbrace{\Psi^{(0)}[\lambda]}_{\mathcal{O}(p^0)} + \underbrace{\Psi^{(1)}[\lambda,\partial]}_{\mathcal{O}(p^1)} + \mathcal{O}(\partial^2)$$

- Building blocks : $\lambda = \{e^\sigma,\ a_{\bar{i}},\ \mu_R,\ A'_{\bar{i}}\}$

$\int c_1 A' \mathrm{d} A'$ →



$\int c_2 A' \mathrm{d} a$ →



Question

How can we determine c_I and c_2?

’t Hooft anomaly matching

◆ Def. of ’t Hooft anomaly

$$Z[A + \mathrm{d}\theta] = \mathrm{e}^{\mathrm{i}\mathcal{A}[A;\theta]} Z[A]$$

A : Bkg. gauge field for global G-symmetry

($\mathrm{i}\mathcal{A}[A;\theta]$ cannot be removed by gauge-inv. local counter term)

◆ ’t Hooft anomaly matching

$\mathrm{i}\mathcal{A}[A;\theta]$ is RG inv. ➡ If present in UV, it restrict IR physics!!

➡ Trivial (non-degenerate) vacuum is excluded!

- Classical Ex. : vacuum of massless QCD (would) break chiral symmetry
- Modern Ex. : Higher-form/discrete symmetry (topological phases/θ=π QCD)

➡ How we can apply this to transport??

Recipe for Masseiu-Planck fcn.

Weyl fermion : $\mathcal{L} = \frac{i}{2}\xi^\dagger \left(e_m^{\ \mu}\sigma^m \overrightarrow{D}_\mu - \overleftarrow{D}_\mu \sigma^m e_m^{\ \mu} \right) \xi$

$$\Psi[\lambda] = \log \int \mathcal{D}\xi^\dagger \mathcal{D}\xi e^{S[\xi,\xi^\dagger,A,\tilde{e}]} = \underbrace{\Psi^{(0)}[\lambda]}_{\mathcal{O}(p^0)} + \underbrace{\Psi^{(1)}[\lambda,\partial]}_{\mathcal{O}(p^1)} + \mathcal{O}(\partial^2)$$

- **Building blocks** : $\lambda = \{e^\sigma,\ a_{\bar{i}},\ \mu_R,\ A'_{\bar{i}}\}$

- **Symmetry** : Spatial diffeo, Kaluza-Klein, U(1)R-gauge (Anomalous!!)

  $A_{\bar{i}}$: not Kaluza-Klein inv. → $A'_{\bar{i}} \equiv A_{\bar{i}} - \mu_R a_{\bar{i}}$

- **Power counting scheme** : $\lambda = \mathcal{O}(p^0)$

  $f_{\bar{i}\bar{j}} \equiv \partial_{\bar{i}} a_{\bar{j}} - \partial_{\bar{j}} a_{\bar{i}} = \mathcal{O}(p^1)$ → $ff = \mathcal{O}(p^2)$

Anomaly and anomaly matching

◆ Definition of ('t Hooft) anomaly

$$Z[A + \mathrm{d}\theta] = \mathrm{e}^{\mathrm{i}\mathcal{A}[A;\theta]} Z[A]$$

A : Background gauge field

System with the single right-handed Weyl fermion

- $U(1)_R$ symmetry: Perturbative chiral anomaly

- $U(1)_R \times U(1)_{KK}$ symmetry: Mixed global anomaly

[See Golkar-Sethi arXiv:1512.02607, Chowdhury-David, arXiv:1604.05003]

➡ $Z[A,a]$ needs to reproduce these anomaly!

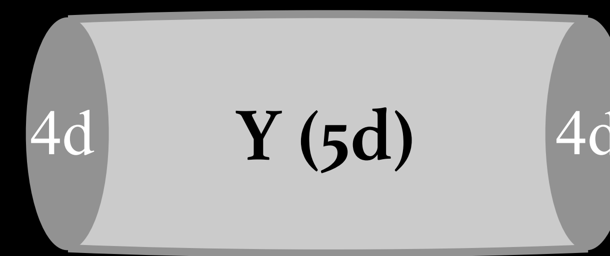
A way to compute anomaly

[See Golkar-Sethi arXiv:1512.02607, Chowdhury-David, arXiv:1604.05003]

Step1. Mapping torus

Large KK gauge trans. : $\tilde{g}_{\mu\nu} \to \tilde{g}'_{\mu\nu}$ interpolates 1d-higher space Y

$$\tilde{g}^{5d}_{\mu\nu}(x^\mu, y) = (1-y)\tilde{g}_{\mu\nu}(x^\mu) + y\tilde{g}'_{\mu\nu}(x^\mu), \quad 0 \le y \le 1$$



Step2. Anomaly = η invariant

$$Z[\tilde{g}'_{\mu\nu}, \tilde{A}'_\mu] = \mathrm{e}^{-\mathrm{i}\pi\eta} Z[\tilde{g}_{\mu\nu}, \tilde{A}_\mu] \text{ with } \not{D}_Y \psi = \lambda_k \psi, \quad \eta \equiv \sum_k \mathrm{sign}(\lambda_k)$$

Step3. Atiyah-Patodi-Singer index theorem

For X(6d) with ∂X(6d) = Y(5d), we can compute eta inv. from

$$\mathrm{ind}\, \not{D}_X = \int_X \hat{A}(R)\mathrm{ch}(F) - \frac{\eta}{2}$$

Anomaly for single Weyl femion

◆ U(1)$_R$ symmetry: Perturbative chiral anomaly

Under U(1)$_R$ gauge trans. $A_0 \to A_0,\ A_i \to A_i + \partial_i \theta(\boldsymbol{x})$

$$\delta_\theta \Psi[\lambda, j; t] = -\frac{C}{3}\int d^3x \theta \varepsilon^{0ijk}\partial_i \mu_R \partial_j A_k \text{ with } C = \frac{1}{4\pi^2}$$

◆ U(1)$_R$×U(1)$_{KK}$ symmetry: Mixed global anomaly

Under large KK gauge trans. $a_i \to a_i + 2\mathrm{i}\beta_0/L$

$$\delta_{\mathrm{lKK}} \Psi[\lambda, j; t] = -\frac{\mathrm{i}\eta}{4}\int_{S^2} \mathrm{d}A' \quad \text{with} \quad \eta = \frac{1}{6}$$ **: eta invariant**

[See Golkar-Sethi arXiv:1512.02607, Chowdhury-David, arXiv:1604.05003]

Anomaly and anomaly matching

System with the single right-handed Well fermion

- $U(1)_R$ symmetry: Perturbative chiral anomaly
- $U(1)_R \times U(1)_{KK}$ symmetry: Mixed global anomaly!

Consistency: $C = \dfrac{1}{4\pi^2}$ $\quad C_1 = \dfrac{\eta}{2} = \dfrac{1}{12}$

◆ Anomalous part of log Z fro Weyl fermion

$$\log Z_{\rm ano} = \frac{C\beta_0}{6}\int \tilde{A}_0\left(\tilde{A}'{\rm d}\tilde{A}' + \frac{1}{2}\tilde{A}_0\tilde{A}'{\rm d}a\right) - \frac{C_1}{\beta_0}\int \tilde{A}'{\rm d}a$$

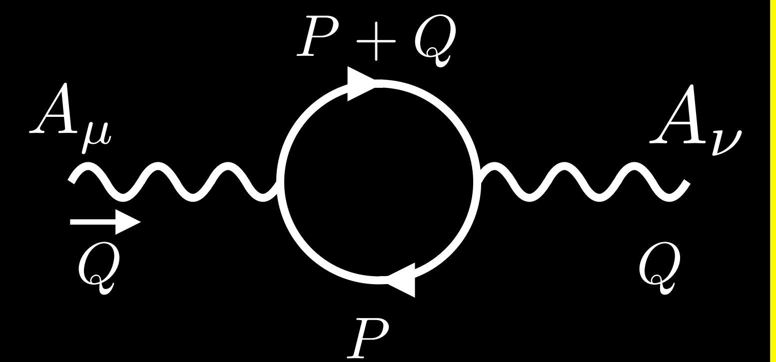
Chiral anomaly — Global anomaly

Derivation of CME/CVE

$$\langle \hat{J}^i_R(x)\rangle^{\rm LG}_{(0,1)} = \frac{1}{\sqrt{-g}}\frac{\delta\Psi^{(1)}}{\delta A_i(x)} - \frac{C}{6}\epsilon^{i\nu\rho\sigma}A_\nu F_{\rho\sigma}$$

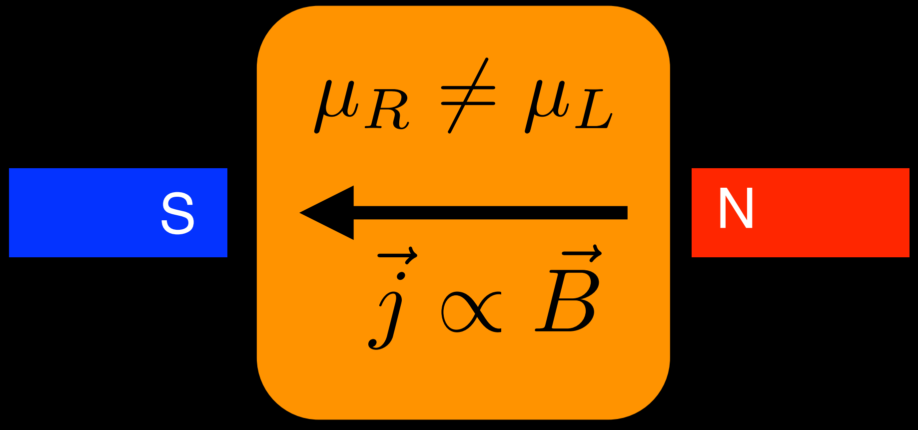
$$= \frac{\mu_R}{4\pi^2}B^i + \left(\frac{\mu_R^2}{4\pi^2} + \frac{T^2}{12}\right)\omega^i$$

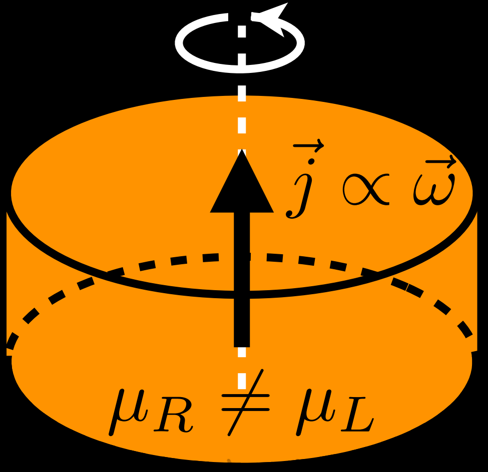
Consistent with e.g.



$$\langle \hat{J}^i_V(x)\rangle^{\rm LG}_{(0,1)} = \frac{\mu_5}{2\pi^2}B^i + \frac{\mu\mu_5}{2\pi^2}\omega^i$$

$$\langle \hat{J}^i_A(x)\rangle^{\rm LG}_{(0,1)} = \frac{\mu}{2\pi^2}B^i + \left(\frac{\mu^2+\mu_5^2}{4\pi^2} + \frac{T^2}{12}\right)\omega^i$$





Anomaly matching in hydro

◆ 't Hooft anomaly matching

$i\mathcal{A}[A;\theta]$ is RG inv. ➡ If present in UV, it restrict IR physics!!

➡ Trivial (non-degenerate) vacuum is excluded!

E ↑

	Ex. massless QCD (T=0)	Ex. Weyl fermion (local eq.)
UV	QCD (quark, gluon)	Weyl fermion (+interaction) with anti-periodic boundary cond.
IR	[Chiral symmetry breaking] Chiral Lagrangian for pion with Wess-Zumino term	Hydrodynamics with chiral transport phenomena

$\log Z|_{T=0}$ is non-local

$\log Z|_{T\neq 0}$ is local!!

(Fermion has KK mass gap!!)

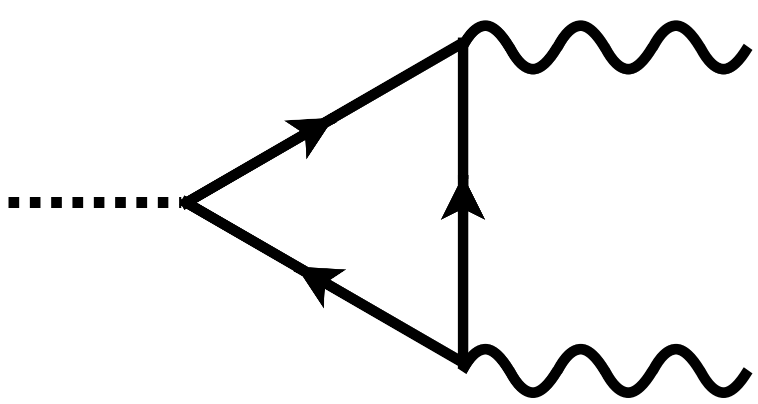
Summary

Slogan

Local equilibrium quantum system ⇔ Theory on a S¹ compactified curved background

If 't Hooft anomaly is present in your model, apply anomaly matching for thermodynamic potential!

Quantum anomaly



Anomaly matching

Chiral transport