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@Osaka Univ. Seminary

Electroweak Axion String and Superconductivity

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Collaboration with

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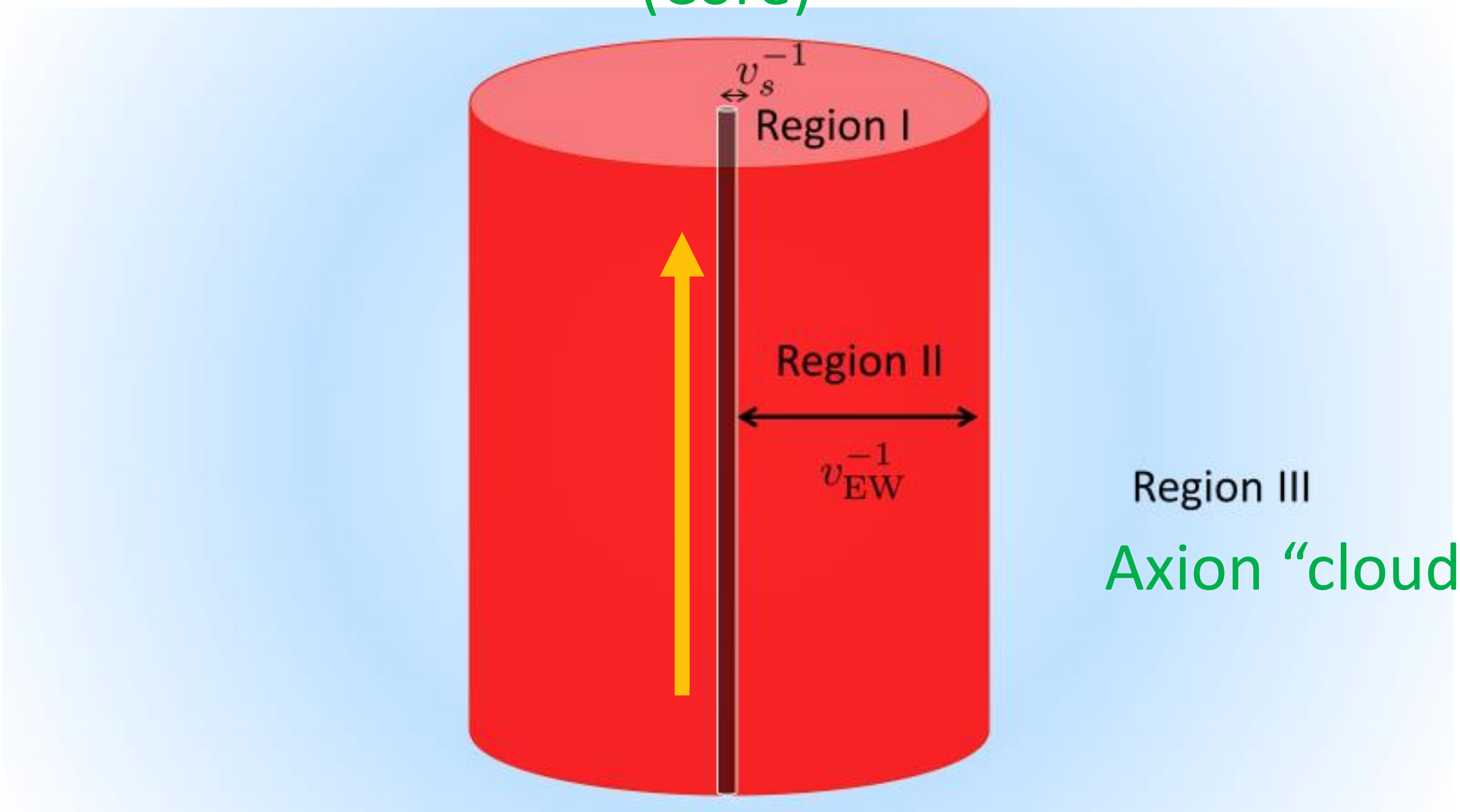
(Kyoto Univ.)

Ref. arXiv:2010.02834[hep-ph]

ELECTROWEAK AXION STRING

Axion string core
(Core)



Axion “cloud”

Electroweak flux tube

ELECTROWEAK AXION STRING

Electroweak Axion String ~ Frankfurt

Electroweak flux tube
(Frankfurt)

Axion “cloud”
(Delicious smell)

Axion string core
(Stick)

TALK STRUCTURE

1. Introduction
2. DFSZ Model
3. Type-A, B, C EW Axion Strings
4. Superconductivity
5. Summary

1. Introduction

STRONG CP-PROBLEM

- QCD Lagrangian

$$\mathcal{L}_{\rm QCD} = -\frac{1}{2g_s^2}{\rm tr}(G_{\mu\nu})^2 + \frac{i\theta}{8\pi^2}{\rm tr}(G_{\mu\nu}\tilde{G}^{\mu\nu}) + \mathcal{L}_{\rm matter}$$

- The θ-term produces the non-zero electric di-pole moment of neutron.
- However, the di-pole moment is highly restricted by the experiments.

[C. A. Baker et al. '06]

$$|d_n| < 2.9 \times 10^{-26}\, e\, {\rm cm}$$

[P. G. Harris et al. '99]

$$\boxed{|\theta| \lesssim 10^{-10}}$$

Why so small ?

PQ MECHANISM

[Peccei-Quinn ’77]

- Introducing anomalous $U(1)$ sym. so called Peccei-Quinn (PQ) sym.
- SSB of $U(1)_{\mathrm{PQ}}$ → NG boson appears (axion a)

$$\theta_{\mathrm{eff}} = \frac{\langle a \rangle}{f_a} + \theta \to 0$$

Promoting parameter θ to a dynamical field

- Realizations
  - KSVZ model [Kim ’79, Shifman-Vainshtein-Zakhrov, ’80]

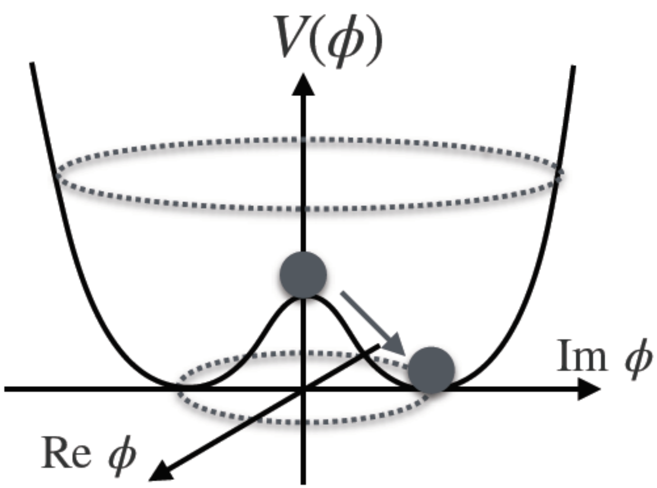
$$\boxed{\text{Extra heavy quark + singlet PQ scalar}}$$

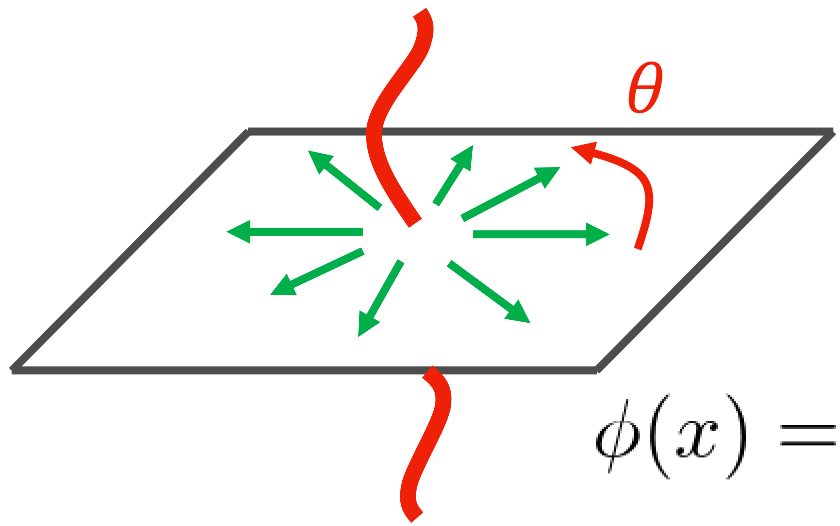
  - DFSZ model [Zhitnitsky, ’80, Dine-Fischler-Srednicki, ’83]

$$\boxed{\text{Two Higgs doublets + singlet PQ scalar}}$$

AXION STRING

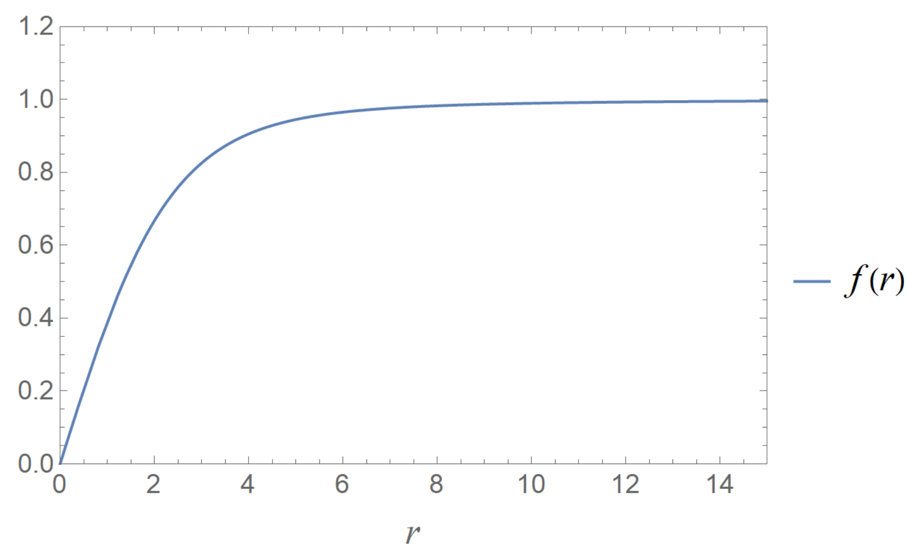
- Global vortex string associated with $U(1)_{\mathrm{PQ}}$ breaking





$$\phi(x) = v_\phi f(r) e^{i\theta}$$

- Field profile of the vortex string



SUPERCONDUCTING STRINGS

- Axion string contains fermionic zero mode inside it.

[Lazarides-Shafi ’85, Lazarides-Panagiotakopoulos-Sfafi ’88, Ganolis-Lazarides ’89, Iwazaki ‘97]

$$\mathcal{L} \supset -y\phi\overline{\Psi_0}\Psi_0, \quad \phi(x) \sim v_\phi e^{i\theta}$$

$$\mathrm{Ind}(i\not{D} + y\phi(x)) = 1$$

[Witten ’80
Jackiw-Rossi ’81,
Callan-Harvey ’85]

- The confined zero mode carries electric current.

→ superconducting string [Witten ’85]

$$\partial_\mu J^\mu = \frac{e^2}{16\pi} E_z$$

- Max. amount of the current:

$$J_{\max} \sim M_{\mathrm{fermion}}$$

Phenomenology, cosmology
see
Fukuda-Manohar-Murayama-Telem
arXiv:2010.02763 [hep-ph]

MESSAGES OF OUR WORK

- Axion strings in DFSZ model should be more complicated after EW symmetry breaking.

  → We call **electroweak(EW) axion strings**

- EW axion string can be superconducting string containing large current without heavy fermions!

  → **DFSZ model is as interesting as KSVZ model!**

2. DFSZ Model

DFSZ MODEL

[Zhitnitsky '80 ,Dine-Fischler-Srednicki '81]

- Particle contents

Two Higgs doublet model + SM singlet scalar

	H_1	H_2	S
$SU(2)_W$	**2**	**2**	**1**
$U(1)_Y$	1	1	0
$U(1)_{\rm PQ}$	X_1	X_2	X_s

- Scalar potential $V(H,S) = V_H + V_S + V_{\rm mix}$

$$\begin{aligned} V_H =& m_{11}^2|H_1|^2 + m_{22}^2|H_2|^2 + \frac{\beta_1}{2}|H_1|^4 + \frac{\beta_2}{2}|H_2|^4 \\ &+ \beta_3|H_1|^2|H_2|^2 + \beta_4(H_1^\dagger H_2)(H_2^\dagger H_1) \end{aligned}$$

$$V_S = -m_S^2|S|^2 + \lambda_S|S|^4$$

$$V_{\rm mix} = \left(\kappa S^2 H_1^\dagger H_2 + \text{h.c.}\right) + \kappa_{1S}|S|^2|H_1|^2 + \kappa_{2S}|S|^2|H_2|^2$$

- Charge relation

$$\boxed{2X_S - X_1 + X_2 = 0}$$

DFSZ MODEL

- Covariant derivative

$$D_\mu H_i = \partial_\mu H_i - \frac{ig}{2} W_\mu^a \sigma_a H_i - \frac{ig'}{2} Y_\mu H_i$$

- Scalar VEVs

$$\langle H_1 \rangle = \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle H_2 \rangle = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \quad \langle S \rangle = v_s$$

- Electroweak scale

$$\boxed{v_{\mathrm{EW}}^2 = 2(v_1^2 + v_2^2)}$$

DFSZ MODEL

- Electroweak gauge sector

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ Y_\mu \end{pmatrix}$$

mixing angle

$$\cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}}, \quad \sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}$$

- Gauge boson mass

$$m_W = \frac{g v_{\mathrm{EW}}}{2}, \quad m_Z = \frac{g v_{\mathrm{EW}}}{2\cos\theta_W}$$

DFSZ MODEL

- Symmetry breaking pattern

$$SU(2)_W \times U(1)_Y \times U(1)_{\mathrm{PQ}} \longrightarrow U(1)_{\mathrm{EM}}$$

- Scalar fields
  - 3 scalars → these are massive (h, h_2, h_3)

  The lightest h is identified to the 125 GeV SM-like Higgs
  - 3 pseudo-scalars

  → massive + axion + NGB eaten by Z-boson
  - 2 complex scalars

  → charged Higgs + NGB eaten by W-boson

PQ CHARGE

- The mass matrix of pseudo-scalars

$$\kappa \begin{pmatrix} -\frac{v_2 v_s^2}{v_1} & v_s^2 & 2v_2 v_s \\ v_s^2 & -\frac{v_1 v_s^2}{v_2} & -2v_1 v_s \\ 2v_2 v_s & -2v_1 v_s & -4v_1 v_2 \end{pmatrix}$$

- PQ transformations are defined so that the $U(1)_{\mathrm{PQ}}$ current does not couple to the Z-boson

$$\boxed{H_1 \mapsto e^{2i\alpha \sin^2\beta} H_1, \quad H_2 \mapsto e^{-2i\alpha \cos^2\beta} H_2, \quad S \mapsto e^{i\alpha} S}$$

- PQ charge

H_1	H_2	S
$2\sin^2\beta$	$-2\cos^2\beta$	1

3. Type-A, B, C EW Axion Strings

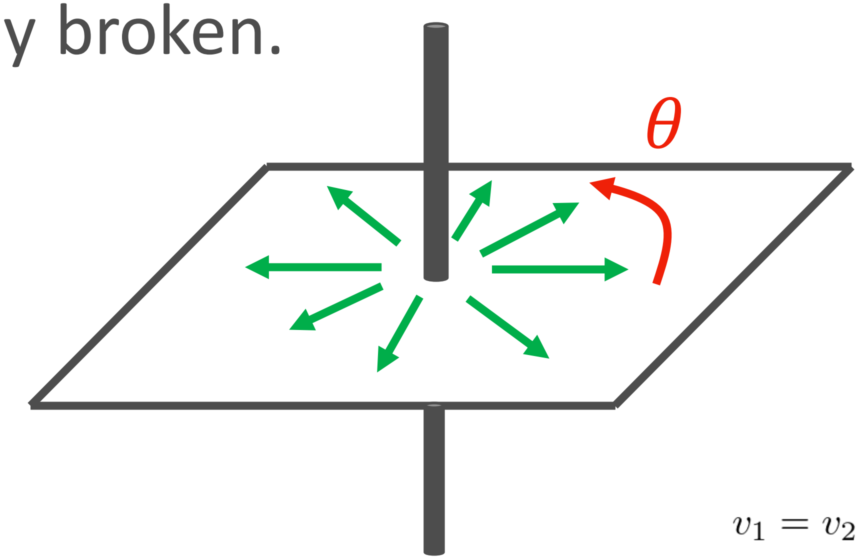
AXION STRING in DFSZ MODEL

- Assume that the electroweak sym. is not broken, but $U(1)_{\mathrm{PQ}}$ is spontaneously broken.

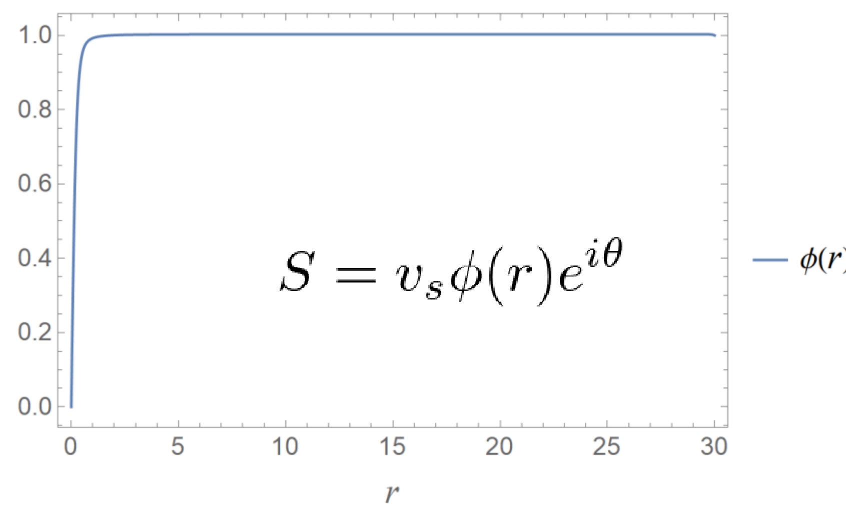
$$S \sim v_s e^{i\theta}$$

$$H_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$H_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

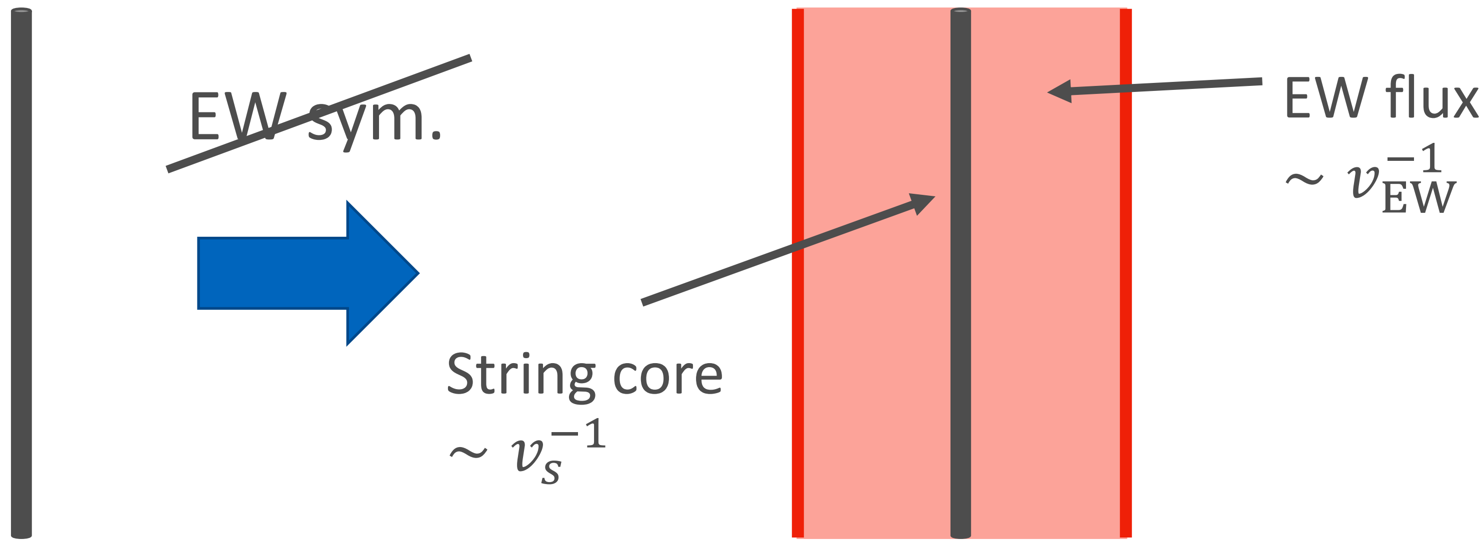


$$v_1 = v_2 = v = \frac{1}{10} v_s$$



ELECTROWEAK AXION STRINGS

- Electroweak sym. is also broken
- The Higgs doublet also develop non-zero VEVs



Axion string

Electroweak axion string

- There are three types of EW axion strings depending on the winding patterns of Higgs → Type-A, B, <u>C</u>

Superconducting string

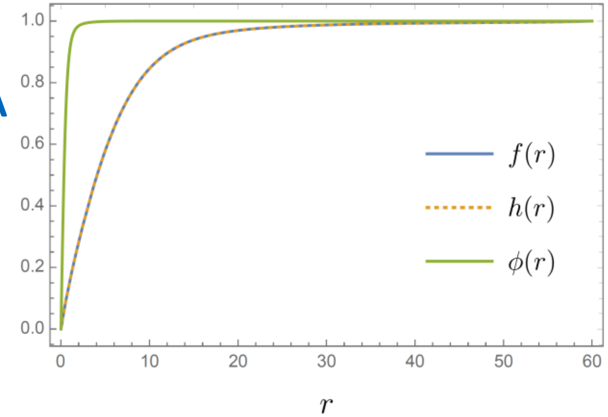
ELECTROWEAK AXION STRINGS

- Assumptions:

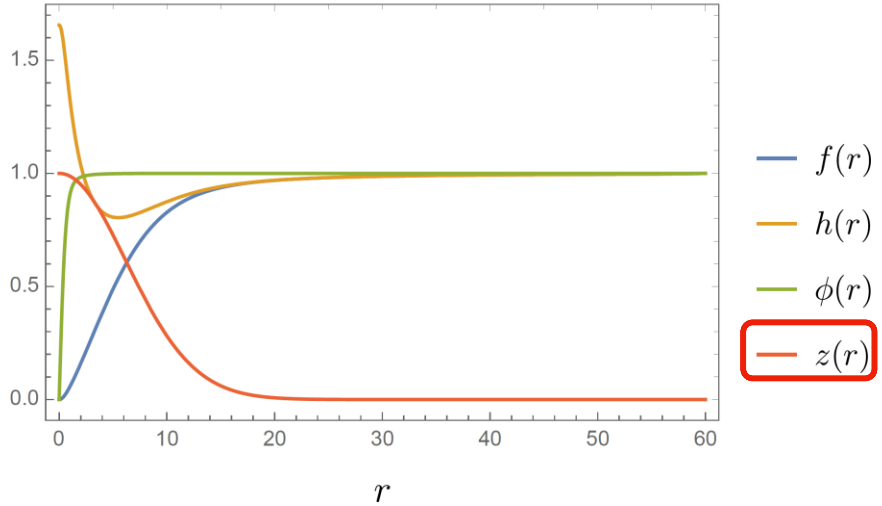
$$v_1 = v_2 = v = \frac{1}{10} v_s$$ but qualitive behavior will not change.

- Field profiles

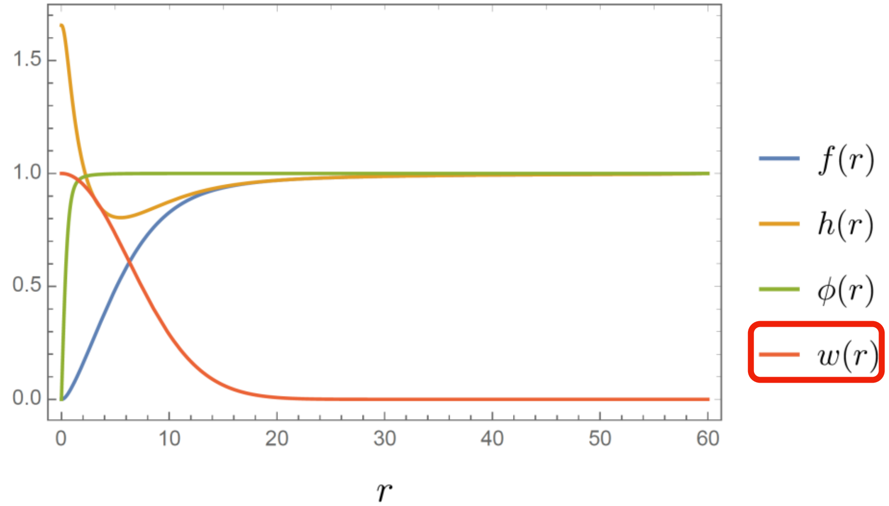
Type-A



Type-B



Type-C



TYPE-A EW AXION STRING

- Type-A EW axion string is characterized by

H_1	H_2	S
$2\sin^2\beta$	$-2\cos^2\beta$	1

$$S \sim v_s e^{i\theta}$$

$$V_{\rm mix} \supset \kappa S^2 H_1^\dagger H_2 + \text{h.c.}$$

$$H_1 \sim e^{i\theta} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}$$

$$H_2 \sim e^{-i\theta} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

TYPE-A EW AXION STRING

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H_1	H_2	S
$2\sin^2\beta$	$-2\cos^2\beta$	1

$$S \sim v_s e^{i\theta}$$

$$H_1 \sim e^{i\theta}\begin{pmatrix} 0 \\ v_1 \end{pmatrix} = \underline{e^{2i\theta\sin^2\beta}}\,\underline{e^{-i\theta\sigma_3\cos 2\beta}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}$$

$$H_2 \sim e^{-i\theta}\begin{pmatrix} 0 \\ v_2 \end{pmatrix} = \underline{e^{-2i\theta\cos^2\beta}}\,\underline{e^{-i\theta\sigma_3\cos 2\beta}}\begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

$U(1)_{\rm PQ}$ winding $\quad$ $U(1)_Z$ winding

NB: $U(1)_Z$ winding is needed to ensure single-valuedness unless $\tan\beta = 1$.

TYPE-A EW AXION STRING

- There is the non-zero Z-boson field.

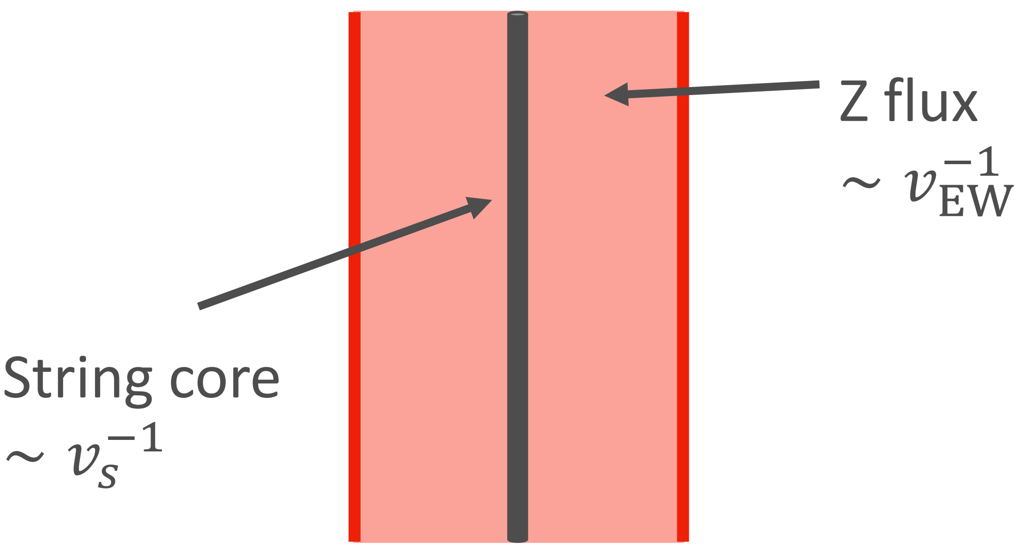
$$g_Z \equiv \sqrt{g^2 + g'^2}$$

$$S \sim v_s e^{i\theta}$$

$$H_1 \sim e^{2i\theta \sin^2 \beta} e^{-i\theta\sigma_3 \cos 2\beta} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}$$

$$H_2 \sim e^{-2i\theta \cos^2 \beta} e^{-i\theta\sigma_3 \cos 2\beta} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

$$\boxed{Z_\theta \sim \frac{-2\cos 2\beta}{g_Z}}$$



Z-flux

$$\boxed{\Phi_Z = \int F^Z = \frac{-4\pi \cos\beta}{g_Z}}$$

NB: $\tan\beta = 1 \Rightarrow \Phi_Z = 0$

TYPE-B EW AXION STRINGS

- Type-B EW axion string is characterized by

H_1	H_2	S
$2\sin^2\beta$	$-2\cos^2\beta$	1

$$S \sim v_s e^{i\theta}$$

$$H_1 \sim e^{2i\theta}\begin{pmatrix} 0 \\ v_1 \end{pmatrix} = \underbrace{e^{2i\theta \sin^2\beta}}_{U(1)_{\rm PQ}\text{ winding}}\underbrace{e^{-2i\theta\sigma_3\cos^2\beta}}_{U(1)_Z\text{ winding}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}$$

$$H_2 \sim \begin{pmatrix} 0 \\ v_2 \end{pmatrix} = \underbrace{e^{-2i\theta\cos^2\beta}}_{U(1)_{\rm PQ}\text{ winding}}\underbrace{e^{-2i\theta\sigma_3\cos^2\beta}}_{U(1)_Z\text{ winding}}\begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

- Z-boson and Z-flux

$$\boxed{Z_\theta \sim \frac{-4\cos^2\beta}{g_Z}} \qquad \boxed{\Phi_Z = \frac{-8\pi\cos^2\beta}{g_Z}}$$

TYPE-B EW AXION STRING

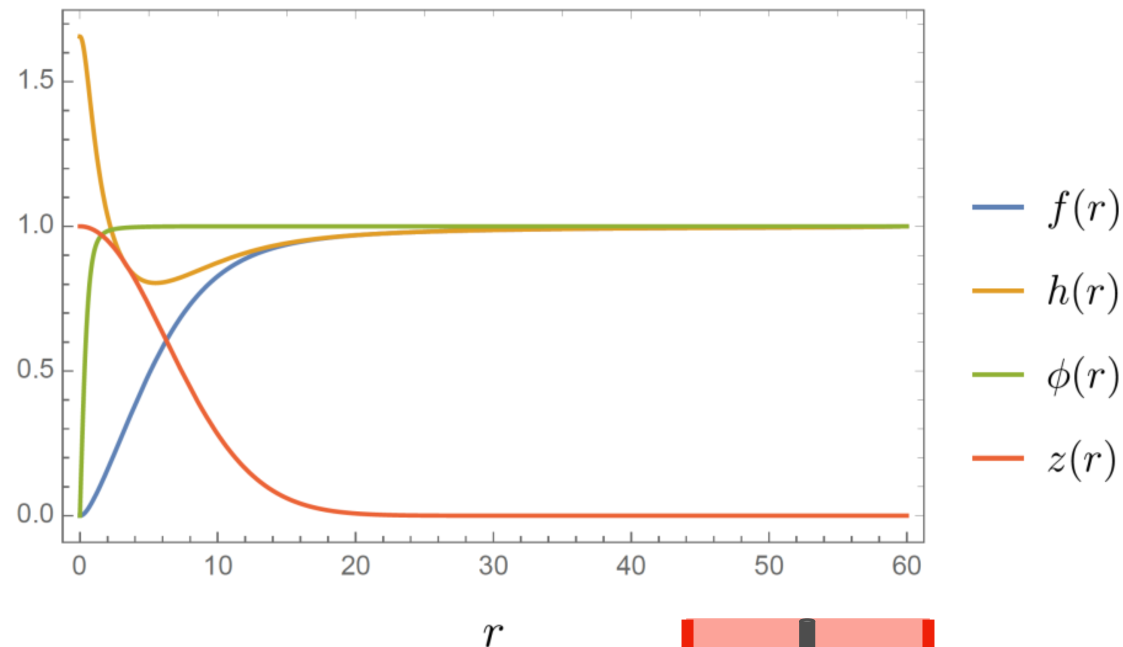
- Profile of type-B EW axion string

$$S = v_s e^{i\theta}\phi(r),$$

$$H_1 = v_1 e^{2i\theta}\begin{pmatrix} 0 \\ f(r) \end{pmatrix},$$

$$H_2 = v_2 \begin{pmatrix} 0 \\ h(r) \end{pmatrix},$$

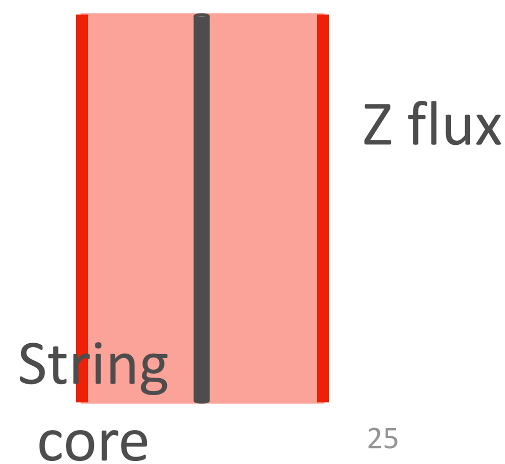
$$Z_\theta = \frac{-4\cos^2\beta}{g_Z}(1 - z(r)).$$



- Boundary condition

$$f(0) = \phi(0) = 0,\ \partial_r h|_{r=0} = 0,\ z(0) = 0$$

$$\phi(\infty) = f(\infty) = h(\infty) = z(\infty) = 1$$



ELECTROWEAK AXION STRING

Can we get “superconducting” strings ?

TYPE-C EW AXION STRING

- The last one, type-C, may seem more exotic:

$$S \sim v_s e^{i\theta}$$

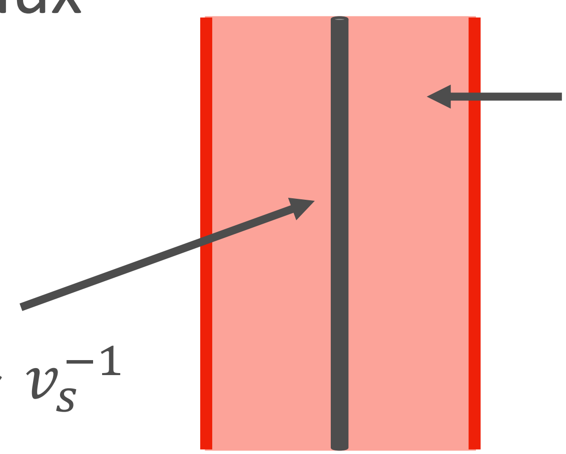
$$H_1 \sim \frac{v_1}{2}\begin{pmatrix} e^{2i\theta} - 1 \\ e^{2i\theta} + 1 \end{pmatrix} = e^{2i\theta \sin^2\beta} e^{-i\theta \cos 2\beta \sigma_Z} e^{i\theta\sigma_1}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}$$

$$H_2 \sim \frac{v_2}{2}\begin{pmatrix} 1 - e^{-2i\theta} \\ 1 + e^{-2i\theta} \end{pmatrix} = \underbrace{e^{-2i\theta \cos^2\beta}}_{U(1)_{\mathrm{PQ}}\ \text{winding}} \underbrace{e^{-i\theta \cos 2\beta \sigma_Z}}_{U(1)_Z\ \text{winding}} \underbrace{e^{i\theta\sigma_1}}_{U(1)_{W^1}\ \text{winding}}\begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

- This contains Z-flux and W-flux

$$\Phi_Z = \frac{-4\pi \cos 2\beta}{g_Z}$$

$$\Phi_{W^1} = \frac{-4\pi}{g}$$



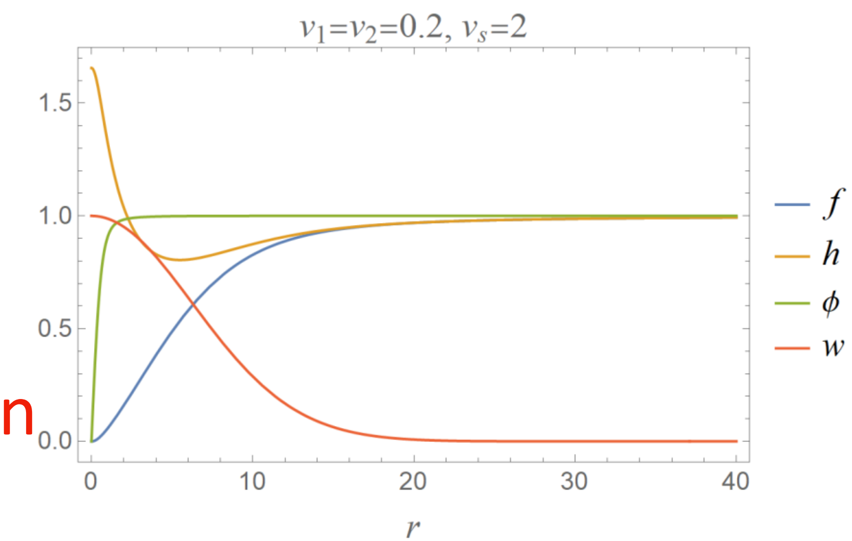
TYPE-C EW AXION STRING

- Inside the string, the field configuration is described by the "smeared ansatz":

$$H_1 = \frac{1}{2} v_1 e^{i\theta} \begin{pmatrix} f(r) e^{2i\theta} - h(r) \\ f(r) e^{2i\theta} + h(r) \end{pmatrix}, \quad H_2 = \frac{1}{2} v_2 \begin{pmatrix} h(r) - f(r) e^{-2i\theta} \\ h(r) + f(r) e^{-2i\theta} \end{pmatrix}$$

- EW sym.

$$\hat{Q}_{\mathrm{EM}} H_i \propto f - h \begin{cases} \to 0 & (r \to \infty) \\ \neq 0 & (r \lesssim v_{\mathrm{EW}}^{-1}) \end{cases}$$



- $U(1)_{EM}$ is spontaneously broken

  → **Superconducting**

- Current carrier: charged components of Higgs and W-boson

EW AXION STRINGS

- We have seen three types of the EW axion strings

  Type-A (w/ Z-flux), type-B (w/ Z-flux), type-C (w/ Z and W-flux)

- The next question…

  Which types of the EW string is realized ?

  Type-C string is exotic… Is this really?

- The tensions of the strings depends on the Higgs couplings.

STRING TENSIONS

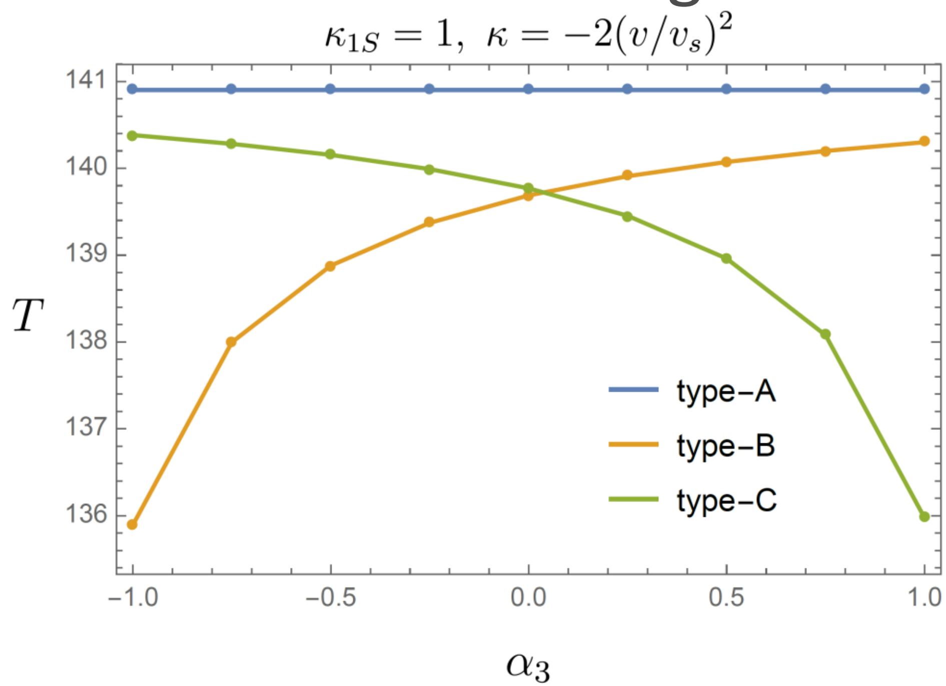
- Compare the tensions of the three EW axion strings

$$\alpha_3 = \frac{1}{8}(\beta_1 + \beta_2 - 2\beta_3)$$

$v_s = 10v_1,\ m_h^2 = (125\ \mathrm{GeV})^2,\ \tan\beta = 1,$

$\kappa_{1S} = \kappa_{2S},\ \kappa = -2(v/v_s)^2,\ \lambda_S = 1$

Length unit: $v_1 = 0.2$



- Type-C string can be the most stable string

  → favored in some parameter space

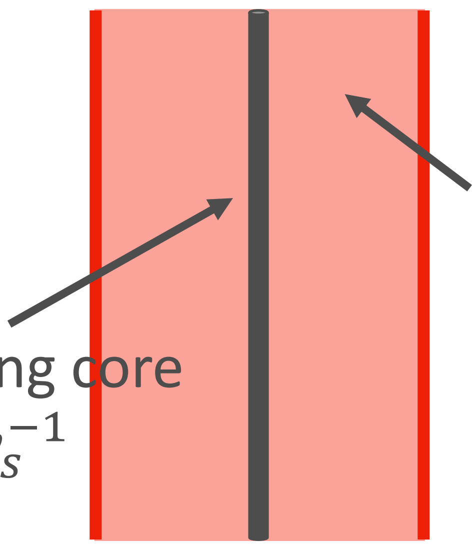
- The axion string necessarily become type-C string after the electroweak sym. breaking.

4. Superconductivity

SUPERCURRENT OF TYPE-C STRING

- How should we evaluate the current traveling along the EW axion string?

[Alford-Benson-Coleman-March-Russell-Wilczek '91]



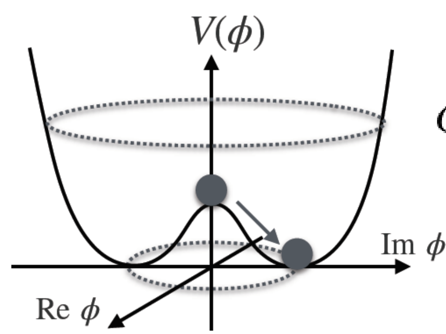
Type-C
electroweak axion string

Background

$$\tilde{S}(r,\theta), \widetilde{H}_{1,2}, (r,\theta), \widetilde{W}_\mu(r,\theta)$$

$U(1)_{\mathrm{EM}}$ is broken in the axion string
→ there exists the zero mode

Cf.



$$\phi = (v_\phi + \rho)\boxed{\exp(i\pi/v_\phi)}$$

SUPERCURRENT OF TYPE-C STRING

[YA-Hamada-Yoshioka '20]

- We consider the (z,t) dependent zero mode $\eta(z,t)$. $\tilde{S}, \widetilde{H}_i, \widetilde{W}_\mu, \tilde{Y}_\mu$ denote the type-C backgrounds.

$$S = \tilde{S}$$

$$H_i = \exp\left[i\hat{Q}_{\mathrm{EM}}\eta(z,t)\xi(r)\right]\tilde{H}_i$$

$$W_\mu = \tilde{W}_\mu - \frac{\eta(z,t)}{g} D_\mu\left(\xi(r)n\right)$$

$$Y_\mu = \tilde{Y}_\mu + \frac{\eta(z,t)}{g'}\partial_\mu \xi(r)$$

$$n^a \equiv \frac{H^\dagger \sigma_a H_1}{H_1^\dagger H_1} = \frac{H_2^\dagger \sigma_a H_2}{H_2^\dagger H_2}$$

- EOMs

$$\left(D_\nu W^{\nu\mu}\right)^a = -j_W^{\mu,a}, \quad \partial_\nu Y^{\nu\mu} = -j_Y^\mu, \quad D_\mu D^\mu H_i = -\frac{\delta V}{\delta H_i^\dagger}$$

SUPERCURRENT OF TYPE-C STRING

- Linearized EOM at large r

$$(\partial_z^2 - \partial_t^2)\eta(z,t) = 0, \quad \xi(r) \sim \log r$$

$\eta(z,t)$ is massless excitation along the string

- Static solution → $\eta(z,t) = \omega z$ with a constant ω

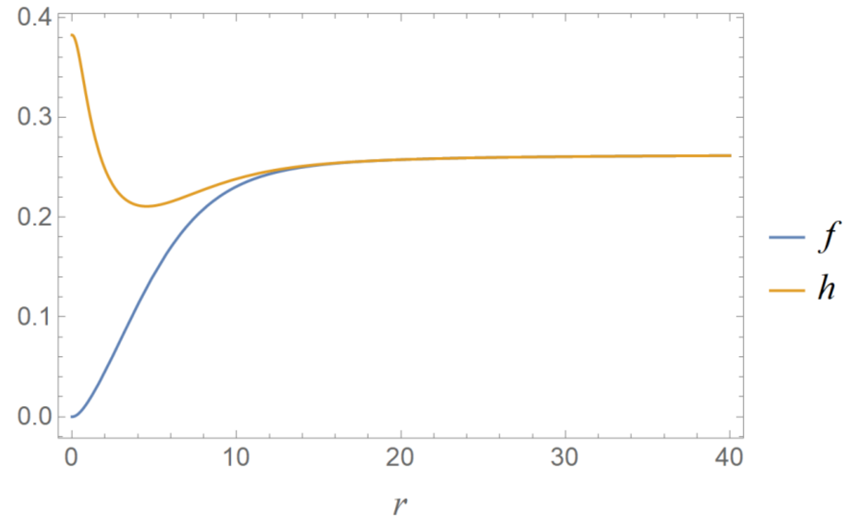
- EM field strength and electric current

$$F_{rz}^{\mathrm{EM}} \sim -\frac{\omega}{er}, \quad J_{\mathrm{EM}} \equiv -2\pi r F_{rz}^{\mathrm{EM}} \sim \frac{2\pi\omega}{e}$$

MAXIMUM CURRENT IN TYPE-C

- Zero mode η gives "a mass" term to the Higgs fields:

$$\mathcal{L} \supset -(\omega\xi)^2 \frac{v^2}{2}(f-h)^2$$



- $U(1)_{\mathrm{EM}}$ is restored and superconducting state is destroyed. (current quenching)
- Current becomes maximum when this effect is balanced with the negative mass terms in the Higgs potential.

$$m_1^2 \sim C_1\, v^2 + C_2\, v_s^2$$

- Large current can travel along the string

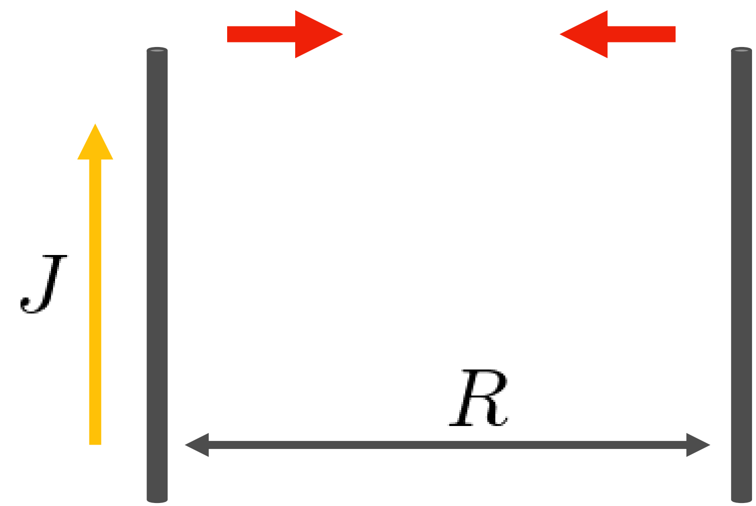
$$\boxed{J_{\max} \sim v_s}$$

INTERACTIONS between STRINGS

- Current-induced interaction between the strings

Attractive force

$$F_{\rm mag} \sim \frac{v_s^2}{e^2 R^2}$$



- The axion-induced interaction is repulsive force

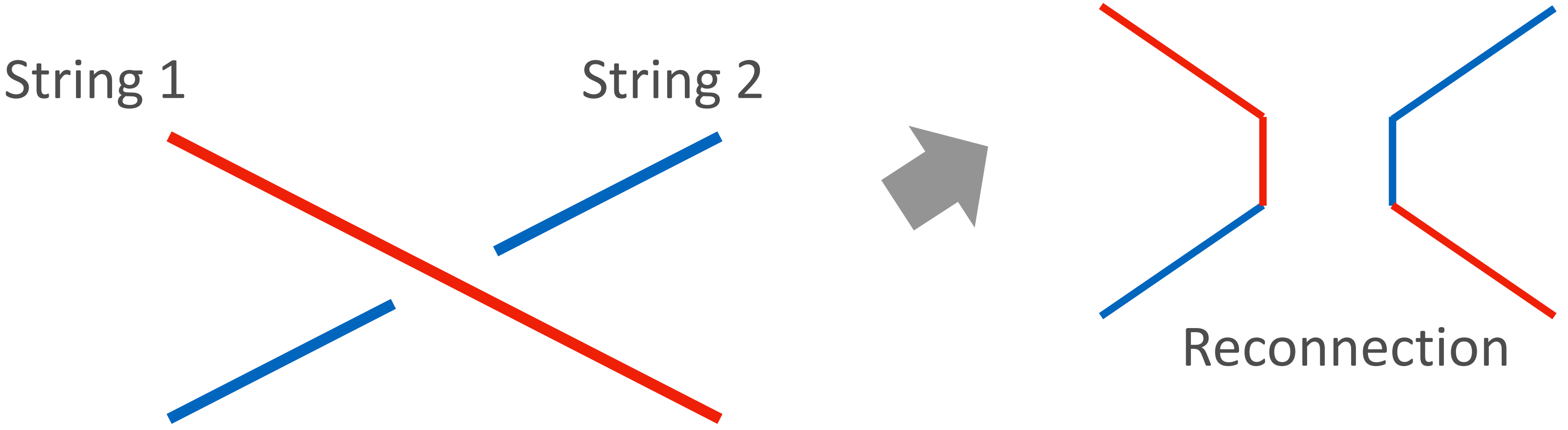
$$F_{\rm axion} \sim \frac{v_s^2}{R^2}$$

- Magnetic interaction could affect the time evolution of the string network. → **Y-junction ?**

Y-(SHAPED) JUNCTION

[Betterncourt-Kibble ’94, Betterncourt-Laguna-Matzner ’96]

- It has been believed that axion strings always reconnect.



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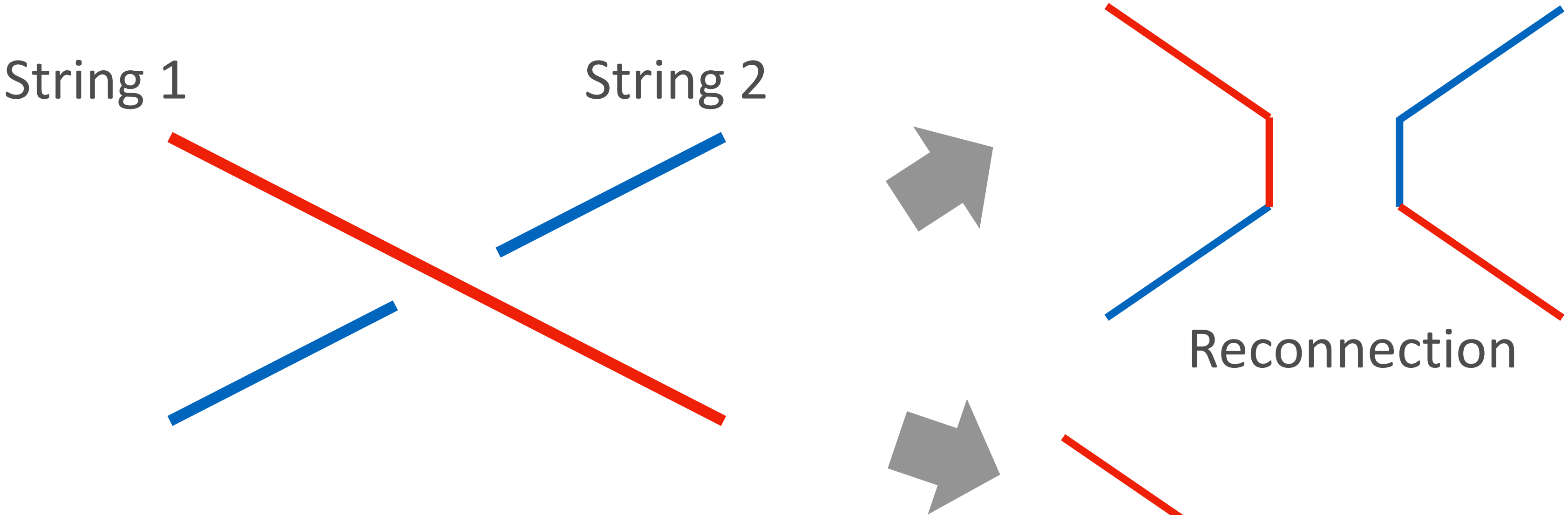
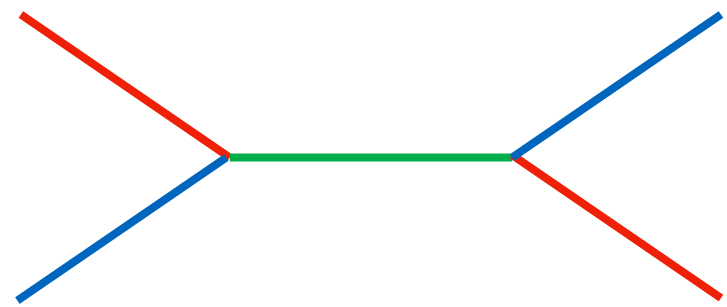




- However, the superconducting axion strings can form Y-junction.

Y-junction = bound state

- How frequently? Scaling behavior of the network is spoiled?

5. Summary

SUMMARY

- DFSZ axion string becomes dressed w/ electroweak gauge fluxes after the electroweak symmetry breaking.

Electroweak axion string ~ Frankfurt

- Type-C electroweak axion string breaks $U(1)_{\mathrm{EM}}$ and can be superconducting string using bosonic carriers.
- The maximum current is $J_{\mathrm{max}} \sim v_s$, which is comparable to be scale of the string tension.
- The Y-junctions can be formed.

OUTLOOK

- Study cosmological consequences (vorton, etc.) of superconductivity in the DFSZ model in the same way as [Fukuda et al. arXiv:2010.02763 [hep-ph]]

- Study on Y-junctions may need hard numerical simulations.

  → 2D effective theory ?

- Effect of large Higgs field value around the core

  → BBN ?

Thank you !!

Backup

HIGGS BI-LINEAR FORMALISM

[Grzadkowki-Maniastis-Wudka, '11]

- Introduce the following Higgs matrix:

$$\boxed{H = (i\sigma_2 H_1^*, H_2)}$$

- Covariant derivative

$$D_\mu H = \partial_\mu H - i\frac{g}{2}\sigma_a W_\mu^a H + i\frac{g'}{2} H\sigma_3 Y_\mu$$

- Higgs potential

$$V_H = -m_1^2 \mathrm{tr}|H|^2 - m_2^2 \mathrm{tr}(|H|^2\sigma_3) + \alpha_1 \mathrm{tr}|H|^4 + \alpha_2(\mathrm{tr}|H|^2)^2$$
$$+\alpha_3 \mathrm{tr}(|H|^2\sigma_3|H|^2\sigma_3) + \alpha_4 \mathrm{tr}(|H|^2\sigma_3|H|^2)$$

HIGSS BI-LINEAR FORMALISM

[Grzadkowki-Maniastis-Wudka, ’11]

- Scalar potential

$$V_H = - m_1^2 \mathrm{tr}|H|^2 - m_2^2 \mathrm{tr}(|H|^2\sigma_3) + \alpha_1 \mathrm{tr}|H|^4 + \alpha_2 (\mathrm{tr}|H|^2)^2 \\ + \alpha_3 \mathrm{tr}(|H|^2\sigma_3|H|^2\sigma_3) + \alpha_4 \mathrm{tr}(|H|^2\sigma_3|H|^2),$$

$$V_{\mathrm{mix}} = \left(\kappa S^2 \det H + \mathrm{h.c.}\right) + \frac{1}{2}(\kappa_{1S} + \kappa_{2S})|S|^2 \mathrm{tr}|H|^2 \\ + \frac{1}{2}(\kappa_{1S} - \kappa_{2S})|S|^2 \mathrm{tr}(|H|^2\sigma_3)$$

- Parameter relations

$$m_{11}^2 = m_1^2 + m_2^2, \quad m_{22}^2 = m_1^2 - m_2^2,$$

$$\beta_1 = 2(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4), \quad \beta_2 = 2(\alpha_1 + \alpha_2 + \alpha_3 - \alpha_4),$$

$$\beta_3 = 2(\alpha_1 + \alpha_2 - \alpha_3), \quad \beta_3 = 2(\alpha_3 - \alpha_1)$$

YUKAWA INTERACTIONS

- The Yukawa interactions are also controlled by the PQ charge assignment.

- E.g.

$$\mathcal{L}_{\text{Yukawa}} = -y_U \overline{Q}\left(i\sigma_2 H_1^*\right) u_R - y_D \overline{Q} H_2 d_R - y_e \overline{L} H_2 e_R + \text{h.c.}$$

- In this work, we leave aside the Yukawa terms

$U(1)_{\mathrm{EM}}$ GROUP

[Eto-Hamada-Nitta, '20]

- The unbroken $U(1)_{\mathrm{EM}}$ is defined as

$$\hat{Q}H \equiv -n^a \frac{\sigma_a}{2} H - H \frac{\sigma_3}{2}$$

where

$$n^a \equiv \frac{\sum_{i=1,2} |H_i|^2 n_i^a}{C} \qquad n_1^a \equiv \frac{H_1^\dagger{}_a H_1}{|H_1|^2},\; n_2^a \equiv \frac{H_2^\dagger \sigma_a H_2}{|H_2|^2}$$

Positive normalization factor C defined to satisfy $n^a n^a = 1$

- $U(1)_Z$ subgroup in $SU(2)_W \times U(1)_Y$ is defined as

$$\hat{T}_Z H \equiv -{}^a \frac{\sigma_a}{2} H - \sin^2 \theta_W \hat{Q} H$$

- NB: H denotes the Higgs matrix.

$U(1)_{\mathrm{EM}}$ GROUP

[Eto-Hamada-Nitta, '20]

- Z-boson and photon

$$\begin{cases} Z_\mu \equiv -n^a W_\mu^a \cos\theta_W - Y_\mu \sin\theta_W \\ A_\mu \equiv -n^a W_\mu^a \sin\theta_W + Y_\mu \cos\theta_W \end{cases}$$

- The action of $\hat{Q}$ on Higgs doublets are given by

$$\hat{Q} H_i = \left(-n^a \frac{\sigma_a}{2} + \frac{1}{2}\mathbf{1}\right) H_i,$$

$$\hat{T}_Z H_i = \left(-n^a \frac{\sigma_a}{2} - \sin^2\theta_W \hat{Q}\right) H_i$$

ANALYSIS

- We analyze the EW axion strings in $\tan\beta = 1$ limit.

- The winding number of the strings are assumed to be unity.

EOMs of TYPE-A STRING

$$f''(r) + \frac{f'(r)}{r} - \frac{f(r)}{r^2}$$
$$- \left(2\alpha_{123}\, v^2 f(r)^2 + 2\alpha_2 v^2 h(r)^2 + \kappa_{1S} v_s^2 \phi(r)^2 - m_1^2\right) f(r) - \kappa v_s^2 h(r)\phi(r)^2 = 0,$$
$$h''(r) + \frac{h'(r)}{r} - \frac{h(r)}{r^2}$$
$$- \left(2\alpha_{123}\, v^2 h(r)^2 + 2\alpha_2 v^2 f(r)^2 + \kappa_{1S} v_s^2 \phi(r)^2 - m_1^2\right) h(r) - \kappa v_s^2 f(r)\phi(r)^2 = 0,$$
$$\phi''(r) + \frac{\phi'(r)}{r} - \frac{\phi(r)}{r^2}$$
$$- \left(2\lambda_S v_s^2 \phi(r)^2 + 2\kappa_{1S} v^2 (f(r)^2 + h(r)^2) + 2\kappa v^2 f(r) h(r) - m_S^2\right)\phi(r) = 0.$$

- Boundary conditions

$$f(0) = h(0) = \phi(0) = 0 \qquad f(\infty) = h(\infty) = \phi(\infty) = 1$$

EOMs of TYPE-B STRING

$$f''(r) + \frac{f'(r)}{r} - \frac{(1+z(r))^2}{r^2} f(r)$$
$$- \left(2\alpha_{123}\, v^2 f(r)^2 + 2\alpha_2 v^2 h(r)^2 + \kappa_{1S} v_s^2 \phi(r)^2 - m_1^2\right) f(r) - \kappa v_s^2 h(r) \phi(r)^2 = 0,$$

$$h''(r) + \frac{h'(r)}{r} - \frac{(-1+z(r))^2}{r^2} h(r)$$
$$- \left(2\alpha_{123}\, v^2 h(r)^2 + 2\alpha_2 v^2 f(r)^2 + \kappa_{1S} v_s^2 \phi(r)^2 - m_1^2\right) h(r) - \kappa v_s^2 f(r) \phi(r)^2 = 0,$$

$$\phi''(r) + \frac{\phi'(r)}{r} - \frac{\phi(r)}{r^2}$$
$$- \left(2\lambda_S v_s^2 \phi(r)^2 + \kappa_{1S} v^2 (f(r)^2 + h(r)^2) + 2\kappa v^2 f(r) h(r) - m_S^2\right) \phi(r) = 0,$$

$$z''(r) - \frac{z'(r)}{r} - \frac{g_Z^2 v^2}{2} f(r)^2 (1 + z(r)) - \frac{g_Z^2 v^2}{2} h(r)^2 (-1 + z(r)) = 0.$$

- Boundary conditions

$$f(0) = \phi(0) = 0, \;\; \partial_r h|_{r=0} = 0, \;\; z(0) = 0$$

$$f(\infty) = h(\infty) = \phi(\infty) = z(\infty) = 1$$

EOMs of TYPE-C STRING

$$f''(r) + \frac{f'(r)}{r} - \frac{(1+w(r))^2}{r^2} f(r)$$
$$- \Big(2(\alpha_1+\alpha_2)v^2 f(r)^2 + 2(\alpha_2+\alpha_3)v^2 h(r)^2 + \kappa_{1S} v_s^2 \phi(r)^2 - m_1^2\Big) f(r) - \kappa v_s^2 h(r)\phi(r)^2 = 0,$$

$$h''(r) + \frac{h'(r)}{r} - \frac{(-1+w(r))^2}{r^2} h(r)$$
$$- \Big(2(\alpha_1+\alpha_2)v^2 h(r)^2 + 2(\alpha_2+\alpha_3)v^2 f(r)^2 + \kappa_{1S} v_s^2 \phi(r)^2 - m_1^2\Big) h(r) - \kappa v_s^2 f(r)\phi(r)^2 = 0,$$

$$\phi''(r) + \frac{\phi'(r)}{r} - \frac{\phi(r)}{r^2}$$
$$- \Big(2\lambda_S v_s^2 \phi(r)^2 + \kappa_{1S} v^2 (f(r)^2 + h(r)^2) + 2\kappa v^2 f(r) h(r)\Big)\phi(r) = 0,$$

$$w''(r) - \frac{w'(r)}{r} - \frac{g^2 v^2}{2} f(r)^2 (1+w(r)) - \frac{g^2 v^2}{2} h(r)^2 (-1+w(r)) = 0.$$

- Boundary conditions

$$f(0) = \phi(0) = 0,\ \ \partial_r h|_{r=0} = 0,\ \ w(0) = z(0) = 0$$

$$f(\infty) = h(\infty) = \phi(\infty) = w(\infty) = z(\infty) = 1$$

NUMERICAL ANALYSIS

- Parameter sets

$$\alpha_1 = 1, \quad \alpha_2 = -0.3348, \quad \alpha_3 = 0,$$

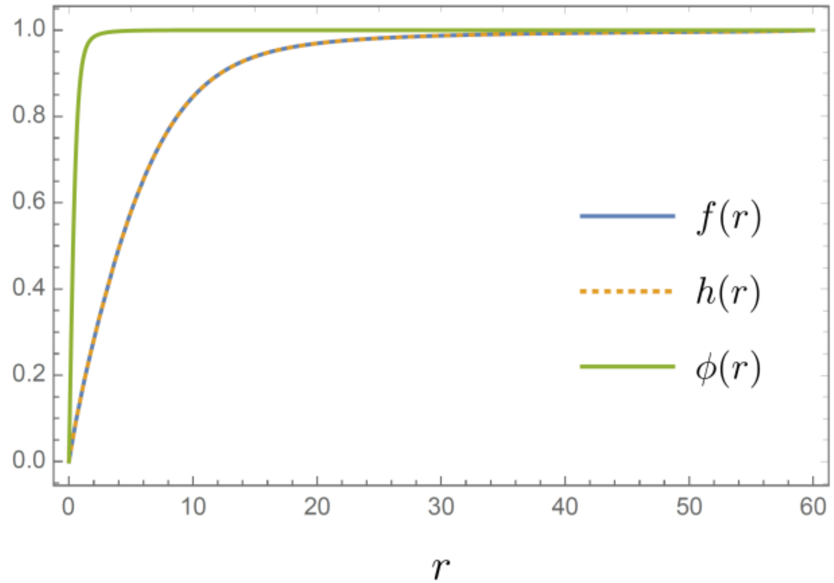
$$\lambda_S = 1, \quad \kappa = -2\left(\frac{v}{v_s}\right)^2, \quad \kappa_{1S} = 0.4$$

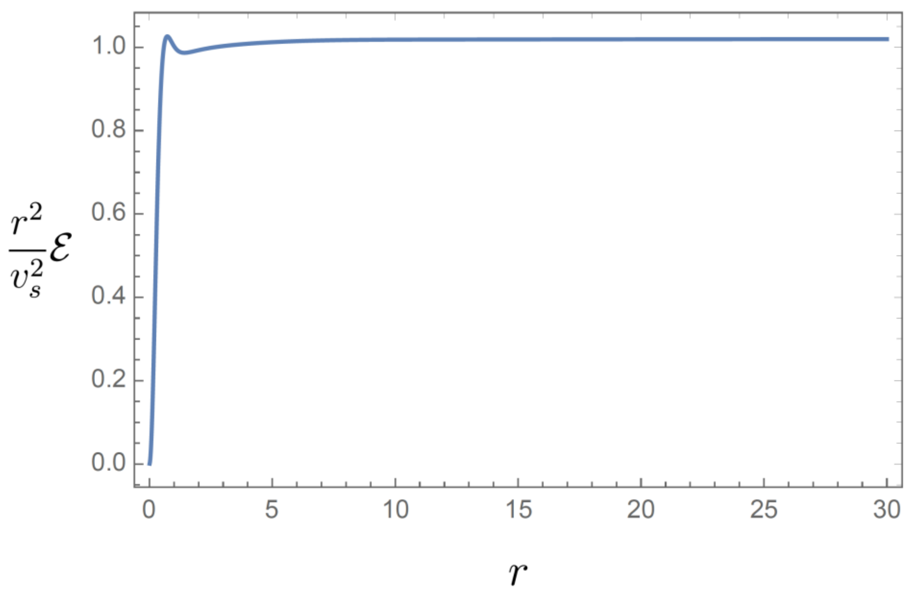
- VEVs of scalar fields

$$v_1 = v_2 = v = \frac{1}{10} v_s$$

PROFILE of STRINGS

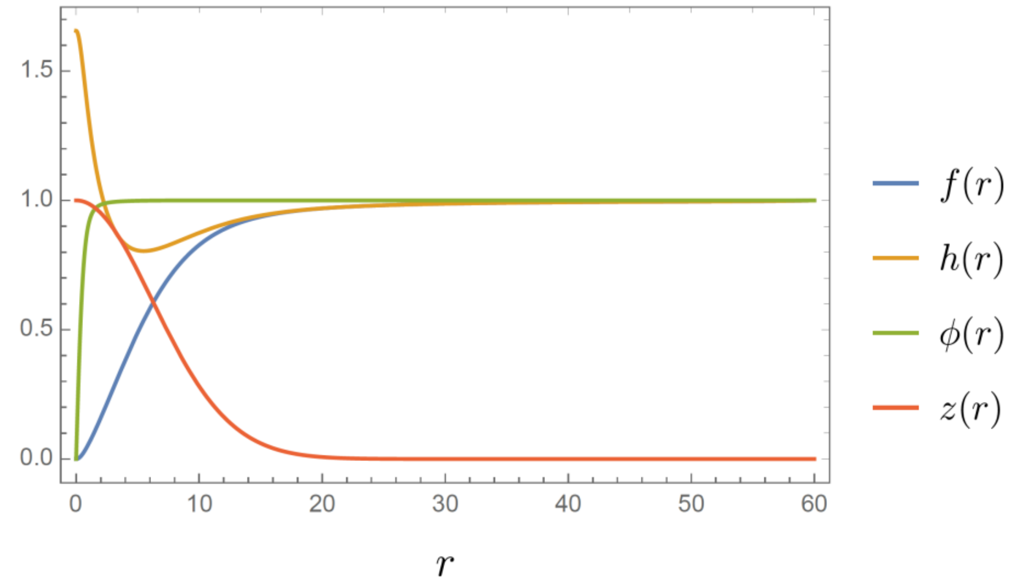
- Type-A

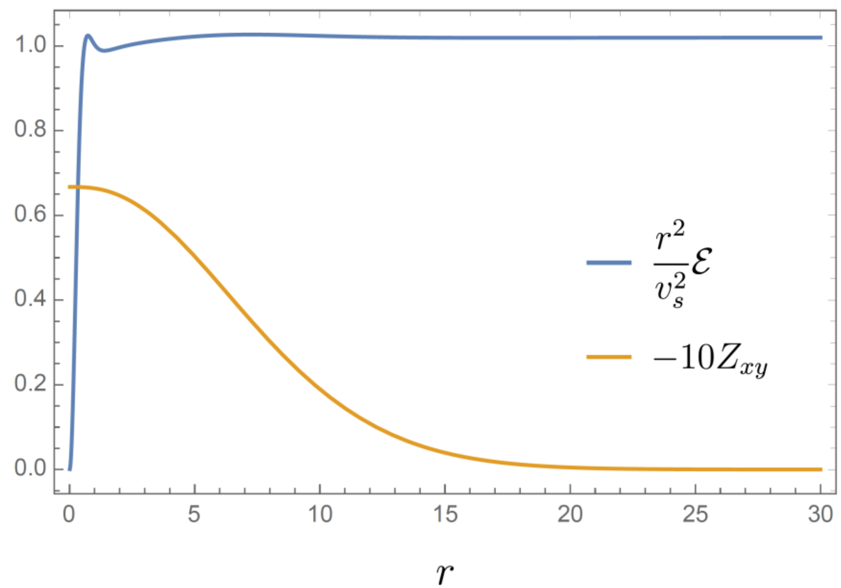




PROFILE of STRINGS

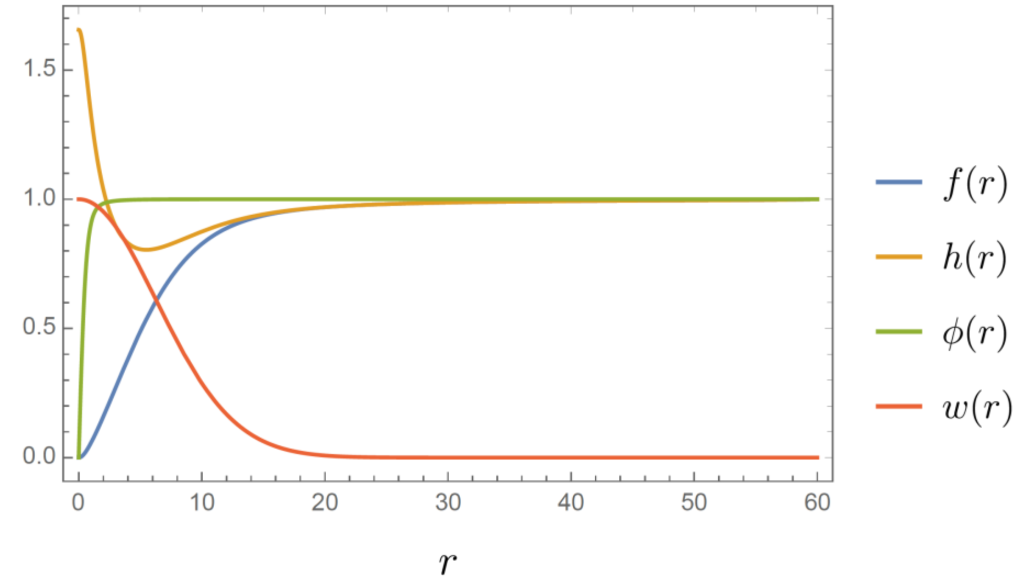
- Type-B

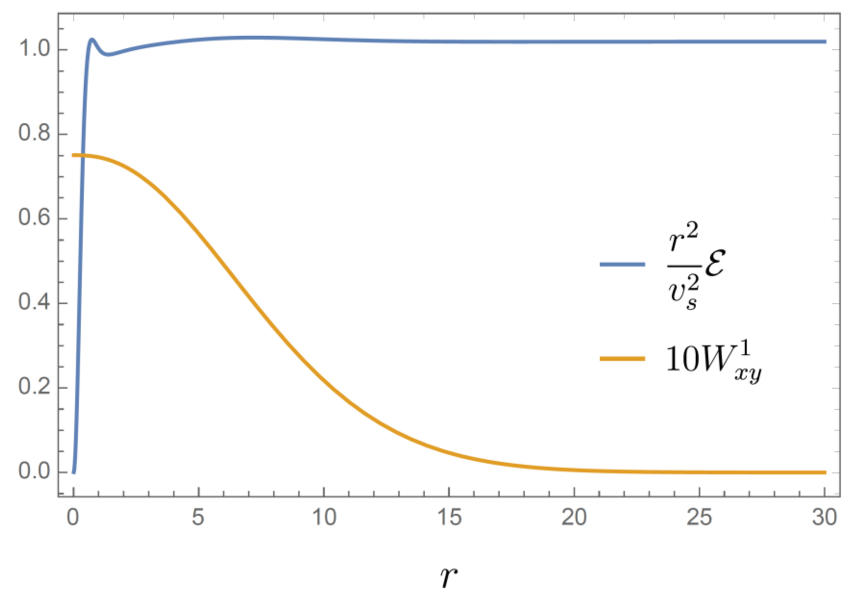




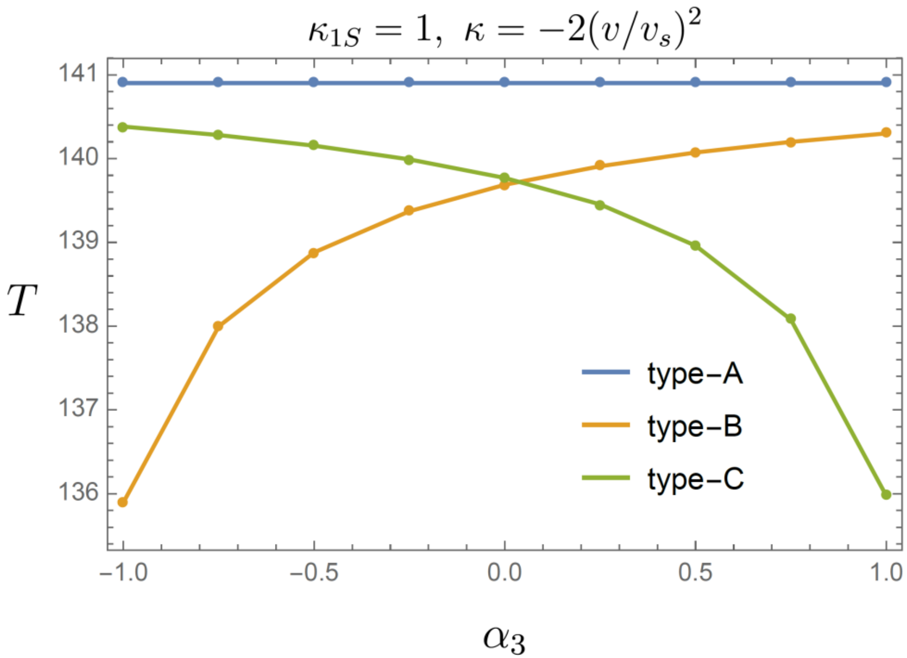
PROFILE of STRINGS

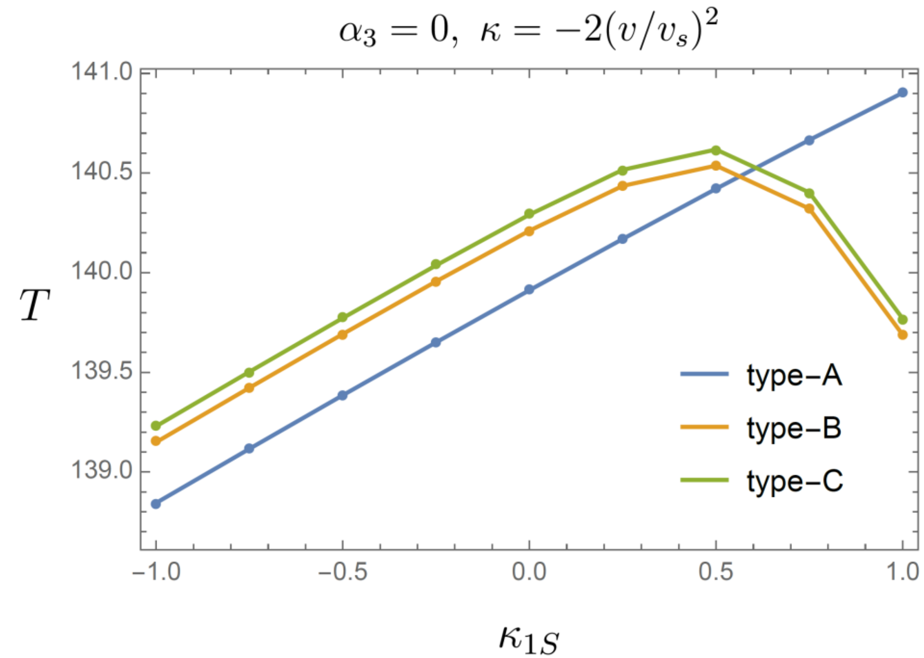
- Type-C





STRING TENSION





Parameters

$$v_s = 10 v_1,\ m_h^2 = (125\ \mathrm{GeV})^2,\ \tan\beta = 1, \qquad \alpha_3 = \frac{1}{8}(\beta_1 + \beta_2 - 2\beta_3)$$

$$\kappa_{1S} = \kappa_{2S},\ \kappa = -2(v/v_s)^2,\ \lambda_S = 1$$

Length unit: $v_1 = 0.2$

SUPERCURRENT OF TYPE-C STRING

- EOMs

$$\partial^\alpha\eta\left(\tilde{D}_j\tilde{D}^j\chi\right)^a = \frac{-g^2}{2}\partial^\alpha\eta\left(\chi^a\mathrm{Tr}|\tilde{H}|^2 + \xi\mathrm{Tr}\left[\tilde{H}^\dagger\sigma_a\tilde{H}\sigma_3\right]\right),$$

$$\partial^\alpha\eta\partial_j\partial^j\xi = \frac{-g'^2}{2}\partial^\alpha\eta\left(\xi\mathrm{Tr}|\tilde{H}|^2 + 2\mathrm{Tr}\left[\tilde{H}^\dagger\chi\tilde{H}\sigma_3\right]\right),$$

$$\tilde{D}^j\chi\ \partial^\alpha\partial_\alpha\eta = 0,$$

$$\partial^j\xi\ \partial^\alpha\partial_\alpha\eta = 0,$$

$$\partial^\alpha\partial_\alpha\eta\left(2\chi\tilde{H} + \xi\tilde{H}\sigma_3\right) = 0.$$

- Linearized EOM for η

$$\partial^\alpha\partial_\alpha\eta = (\partial_t^2 - \partial_z^2)\eta = 0$$

- Zero mode solutions:

$$\eta(z,t) = \eta^\pm(z \pm t)$$

SUPERCURRENT OF TYPE-C STRING

- Static zero mode solution

$$\eta(z) = \omega z$$

- $r \to \infty$, $U(1)_{\mathrm{EM}}$ is restored and the backgrounds satisfy

$$\tilde{H}\sigma_3 + n^a \sigma_a \tilde{H} = 0, \quad n^a = -\frac{\mathrm{tr}(\sigma_3 \tilde{H}^\dagger \sigma_a \tilde{H})}{\mathrm{tr}|\tilde{H}|^2}, \quad (\tilde{D}_\mu n)^a = 0, \quad \mathrm{tr}|\tilde{H}|^2 = 2v^2$$

- EOMs lead to the following equations

$$(\tilde{D}_j \tilde{D}^j \chi)^a = -g^2 v^2 (\chi^a - n^a \xi), \quad \partial_j \partial^j \xi = -g'^2 v^2 (\xi - \chi^a n^a)$$

- Long-range force → $\chi^a - n^a \xi = 0$

$$\boxed{\frac{1}{r}\partial_r(r \partial_r \xi) = 0}$$

CURRENT QUENCHING

- Inside the string $r \to 0$, f and h feel the following mass terms in the Lagrangia:

$$
\begin{aligned}
-\mathcal{L} \supset & \frac{4v^2}{r^2} f^2 + 2m_{11}^2 v^2 (f^2 + h^2) + \omega^2 \frac{v^2}{2} (f - h)^2 \\
= & v^2 \begin{pmatrix} f & h \end{pmatrix} \begin{pmatrix} \frac{4}{r^2} + 2m_{11}^2 + \frac{\omega^2}{2} & -\frac{\omega^2}{2} \\ -\frac{\omega^2}{2} & 2m_{11}^2 + \frac{\omega^2}{2} \end{pmatrix} \begin{pmatrix} f \\ h \end{pmatrix}
\end{aligned}
$$

- Determinant of the mass matrix:

$$
\det M^2 = (2m_{11}^2)^2 + \frac{4v^2}{r^2} \left(2m_{11}^2 + \frac{\omega^2}{2} \right) + \omega^2 m_{11}^2
$$

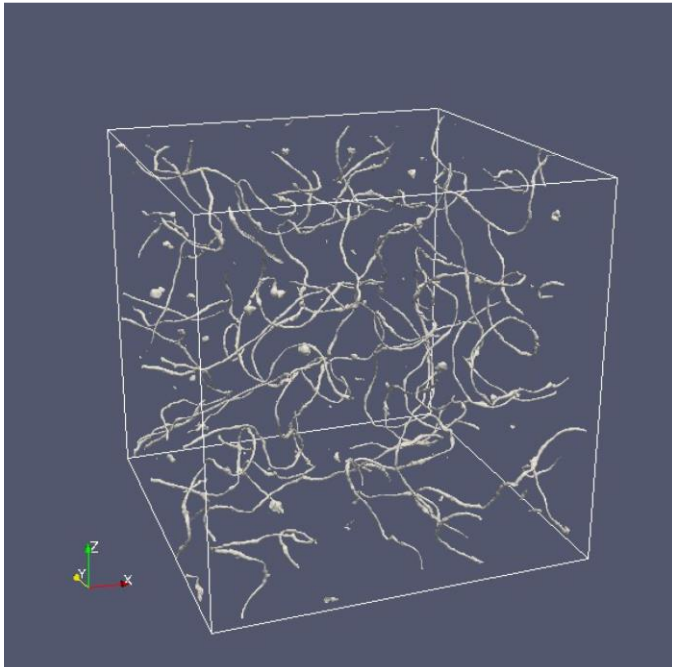
- Avoiding quenching $\Rightarrow \exists$ region where $\det M^2 < 0$

$$
\boxed{|\omega| \lesssim |m_{11}| \sim v_s}
$$

AXION STRING NETWORK

- The axion strings form a network in the universe.

- Studying the time evolution of the network is very important.



[Hiramatsu-san's talk at COSMO-17]

- Y-shaped junctions can affect the time evolution of the string network.

CONSTRAINTS

[Espriu-Mescia-Renau, '15]

- Constraints from the electroweak precision test