SU(N)-natural inflation

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My work

- Axion-gauge dynamics in inflation

Today's talk

"SU(N)-natural inflation"

T. Fujita, H. Nakatsuka, K. Mukaida, KM [arXiv: 2110.03228]

"Statistically-Anisotropic Tensor Bispectrum from Inflation"

"Gravitational wave trispectrum in the axion-SU(2) model"

My work

■ Other papers (Self-introduction)

- Primordial black hole

  "Formation of supermassive primordial black holes by Affleck-Dine mechanism"

  "Strong clustering of primordial black holes from Affleck-Dine mechanism"

- Rotation of the CMB polarization (= CMB Birefringence)

  "Probing axionlike particles via cosmic microwave background polarization"

  "Detection of isotropic cosmic birefringence and its implications for axionlike particles including dark energy"

- Big Bang Nucleosynthesis

  "Big Bang Nucleosynthesis constraints on sterile neutrino and lepton asymmetry of the universe"

  "Big-bang nucleosynthesis with sub-GeV massive decaying particles"

Contents

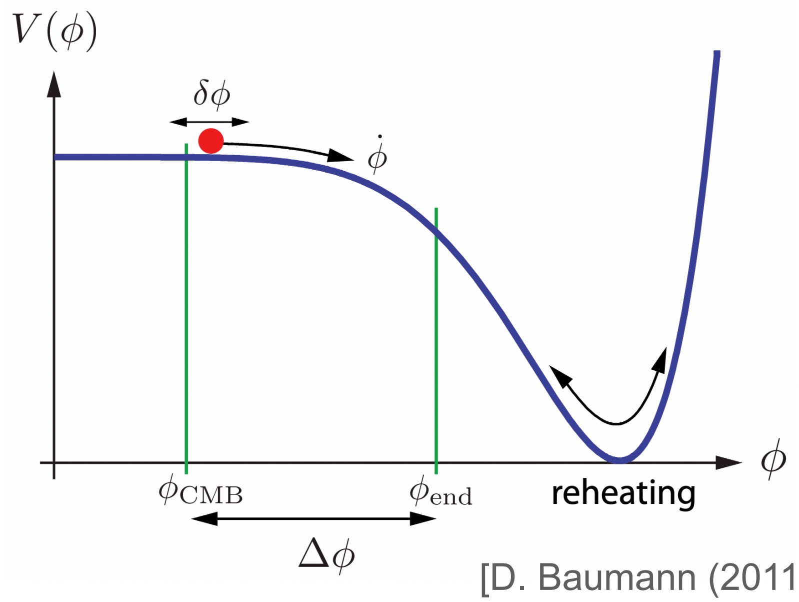
Contents

■ Slow-roll inflation

Inflation explains the origin of the cosmological fluctuations:

$$\mathcal{P}_\zeta \sim \frac{V}{\epsilon M_{\rm Pl}^4} \sim 10^{-9},\ \ n_s \sim 1 - 2\epsilon - \eta \sim 0.96$$

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \sim \frac{M_{\rm Pl}^2}{2}\left(\frac{V'}{V}\right)^2, \eta \equiv \frac{\dot{\epsilon}}{H\epsilon} \sim M_{\rm Pl}^2 \frac{V''}{V}$$



[D. Baumann (2011)]

Slow-roll inflation requires a flat potential.
Considering radiative corrections, a fine-tuning problem arises.

→ "Natural inflation"

[K. Freese, J. A. Frieman, and A. V. Olinto (1990)]

Introduction

- Natural inflation [K. Freese, J. A. Frieman, and A. V. Olinto (1990)]

Pseudo Nambu-Goldstone boson such as an axion
→ The shift symmetry protects the flatness of the potential.

$$V(\phi) = \Lambda^4 \left[1 + \cos\left(\frac{\phi}{f}\right) \right]$$

The success of natural inflation requires

$$f \gtrsim 3M_{\rm Pl}, \ \Lambda \sim m_{\rm GUT} \sim 10^{15}\,{\rm GeV}$$

[K. Freese and W. H. Kinney (2004)]

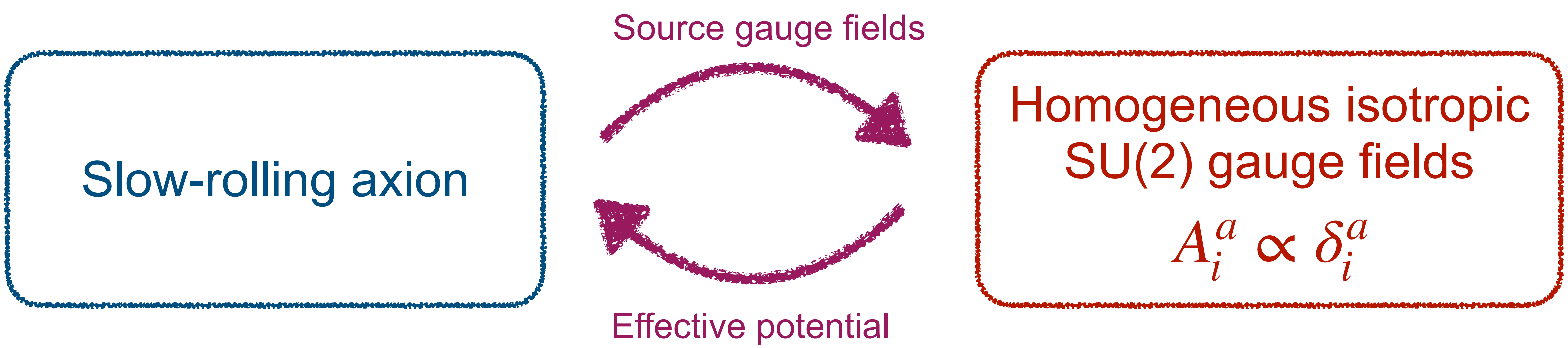
Axion inflation with a sub-Planckian decay constant?
One of the possibilities is

“Chromo-natural inflation”

[P. Adshead and M. Wyman (2012)]

- Chromo-natural inflation (CNI) [P. Adshead and M. Wyman (2012)]

$$\mathscr{L} = \underbrace{\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)}_{\text{axion/inflaton}} \underbrace{- \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}}_{\text{SU(2) gauge fields}} + \underbrace{\frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}}_{\text{Axion-gauge fields coupling}}$$



SU(2) gauge fields induce the effective potential of inflaton.

→ More slow-roll inflation

We can obtain $\mathscr{P}_\zeta$ and n_s compatible with Planck with $f < M_{\rm Pl}$.

Introduction

- Chromo-natural inflation (CNI) [P. Adshead and M. Wyman (2012)]

$$\mathscr{L} = \underbrace{\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)}_{\text{axion/inflaton}} \underbrace{- \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}}_{\text{SU(2) gauge fields}} + \underbrace{\frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}}_{\text{Axion-gauge fields coupling}}$$

Due to SU(2) gauge field background,
"tensor" perturbations of gauge fields are amplified.

→ Gravitational waves are overproduced.
Chromo-natural inflation fails…

If the axion is a spectator, this model can predict observable GWs.

[E. Dimastrogiovanni, M. Fastello, T. Fujita (2016)]

Introduction

- Extend Chromo-natural inflation

SU(2) group is the minimal choice for nonzero gauge field VEV.

→ How about other groups?

"SU(N) gauge natural inflation"

In this talk, I will mainly discuss the background solution.

→ There are multiple isotropic solutions.

First, let us see the details of CNI.

Contents

Chromo-natural inflation

- Equations of motion

$$\mathscr{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}$$

Using the temporal gauge $A^a_0 = 0$ and an ansatz $A^a_i(t) = \delta^a_i a(t)Q(t)$, we obtain the background EoMs:

$$\ddot{\phi} + 3H\dot{\phi} + \partial_\phi V(\phi) = -\frac{3g}{f}Q^2\left(\dot{Q} + HQ\right),$$

$$\ddot{Q} + 3H\dot{Q} + \left(\dot{H} + 2H^2\right)Q + 2g^2Q^3 = \frac{g}{f}Q^2\dot{\phi}$$

The motion of ϕ sources the gauge field Q.

Chromo-natural inflation

- Equations of motion

$$\ddot{\phi} + 3H\dot{\phi} + \partial_\phi V(\phi) = -\frac{3g}{f}Q^2\left(\dot{Q} + HQ\right),$$

$$\ddot{Q} + 3H\dot{Q} + \left(\dot{H} + 2H^2\right)Q + 2g^2Q^3 = \frac{g}{f}Q^2\dot{\phi}$$

In the slow-roll limit, the background solution is

$$m_Q \equiv \frac{gQ}{H} \simeq \left(\frac{-g^2 f \partial_\phi V}{3H^4}\right)^{1/3},$$

$$\xi \equiv \frac{\dot{\phi}}{2fH} \simeq m_Q + m_Q^{-1}$$

Chromo-natural inflation

Points of the BG solution

$$A_i^a(t) = \delta_i^a a(t) Q(t), \quad m_Q \equiv \frac{gQ}{H} \simeq \left(\frac{-g^2 f \partial_\phi V}{3H^4}\right)^{1/3},$$

$$\xi \equiv \frac{\dot{\phi}}{2fH} \simeq m_Q + m_Q^{-1}.$$

- The gauge fields have a rotationally invariant configuration:

$$T^{ij}_{\rm gauge} \propto \delta^{ij}$$

- The motion of ϕ is balanced with the back reaction from Q.
- This solution is an attractor solution. [A. Maleknejad and E. Erfani (2013)]
- $N \sim \mathcal{O}(60)$ can be achieved with $\Delta\phi < M_{\rm Pl}$.

Chromo-natural inflation

Tensor perturbations

Consider tensor perturbations to see the GW production:

$$g_{ij} = -a^2(\delta_{ij} + \underset{\sim}{h_{ij}}), \quad A_i^a = \bar{A}_i^a + \delta A_i^a = \bar{A}_i^a + \underset{\sim}{t_{ia}} + \cdots,$$

Since $\bar{A}_i^a$ is invariant under the diagonal rotation of i and a:

$$\bar{A}_i^a \to U^{ij} U^{ab} \underbrace{\bar{A}_j^b}_{\propto\, \delta_j^b} = \bar{A}_i^a, \qquad (\ U\text{: rotation matrix}\ )$$

we can consider δA_j^a as a spatial tensor.

Then, δA_j^a linearly couple to the tensor metric perturbation h_{ij}.

Tensor perturbations

The linear EoMs of the "tensor" perturbations are

small

$$\psi_{ij}'' - \partial_k\partial_k\psi_{ij} - \frac{2}{\tau^2}\psi_{ij} = \frac{2\sqrt{\epsilon_B}}{m_Q\tau}t_{ij}' + \frac{2\sqrt{\epsilon_B}}{\tau}\epsilon^{ikl}\partial_l t_{jk} + \frac{2\sqrt{\epsilon_B}m_Q}{\tau^2}t_{ij},$$

$$t_{ij}'' - \partial_k\partial_k t_{ij} + \frac{2(2m_Q + m_Q^{-1})}{\tau}\epsilon^{ikl}\partial_l t_{jk} + \frac{2(m_Q^2+1)}{\tau^2}t_{ij} = \mathcal{O}(\psi_{ij})\,.$$

$$\psi_{ij} \equiv \frac{aM_{\rm Pl}}{2}h_{ij}, \quad \delta A_i^a = t_{ai} + \cdots, \quad a\mathrm{d}\tau = \mathrm{d}t,$$

$$\tau \simeq -\frac{1}{aH}, \quad \epsilon_B \equiv \frac{g^2Q^4}{M_{\rm Pl}^2H^2},$$

Chromo-natural inflation

- Tensor perturbations

$$i\epsilon^{ikl}k^l e_{jk}^{R/L}(\mathbf{k}) = \pm\, k e_{ij}^{R/L}(\mathbf{k})$$

$$\psi_{ij}(\tau,\mathbf{x}) = \int \frac{\mathrm{d}^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} \left[e_{ij}^R(\hat{\mathbf{k}}) \Psi_{\mathbf{k}}^R(\tau) + e_{ij}^L(\hat{\mathbf{k}}) \Psi_{\mathbf{k}}^L(\tau) \right],$$

$$t_{ij}(\tau,\mathbf{x}) = \int \frac{\mathrm{d}^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} \left[e_{ij}^R(\hat{\mathbf{k}}) T_{\mathbf{k}}^R(\tau) + e_{ij}^L(\hat{\mathbf{k}}) T_{\mathbf{k}}^L(\tau) \right],$$

$$\partial_\tau^2 \Psi^{R/L} + \left[k^2 - \frac{2}{\tau^2} \right] \Psi^{R/L} = \frac{2\sqrt{\epsilon_B}}{m_Q \tau} \partial_\tau T^{R/L} \pm \frac{2k\sqrt{\epsilon_B}}{\tau} T^{R/L} + \frac{2\sqrt{\epsilon_B} m_Q}{\tau^2} T^{R/L},$$

$$\partial_\tau^2 T^{R/L} + \left[k^2 \pm \frac{2k(2m_Q + m_Q^{-1})}{\tau} + \frac{2(m_Q^2+1)}{\tau^2} \right] T^{R/L} = \mathcal{O}\left(\Psi^{R/L}\right).$$

T^R experiences a tachyonic instabilitiy.

Ψ^R is sourced by T^R.

Chromo-natural inflation

■ Tensor perturbations

$$\partial_\tau^2 \Psi_1^{R/L} + \left[k^2 - \frac{2}{\tau^2}\right] \Psi_1^{R/L} = \frac{2\sqrt{\epsilon_B}}{m_Q \tau} \partial_\tau T_1^{R/L} \pm \frac{2k\sqrt{\epsilon_B}}{\tau} T_1^{R/L} + \frac{2\sqrt{\epsilon_B} m_Q}{\tau^2} T_1^{R/L},$$

$$\partial_\tau^2 T_1^{R/L} + \left[k^2 \pm \frac{2k(2m_Q + m_Q^{-1})}{\tau} + \frac{2(m_Q^2 + 1)}{\tau^2}\right] T_1^{R/L} = \mathcal{O}\left(\Psi_1^{R/L}\right).$$

$$T_1^R(\tau, k) = \frac{1}{\sqrt{2k}} e^{\pi(2m_Q + m_Q^{-1})/2} W_{\beta,\alpha}(2ik\tau),$$

Whittaker function

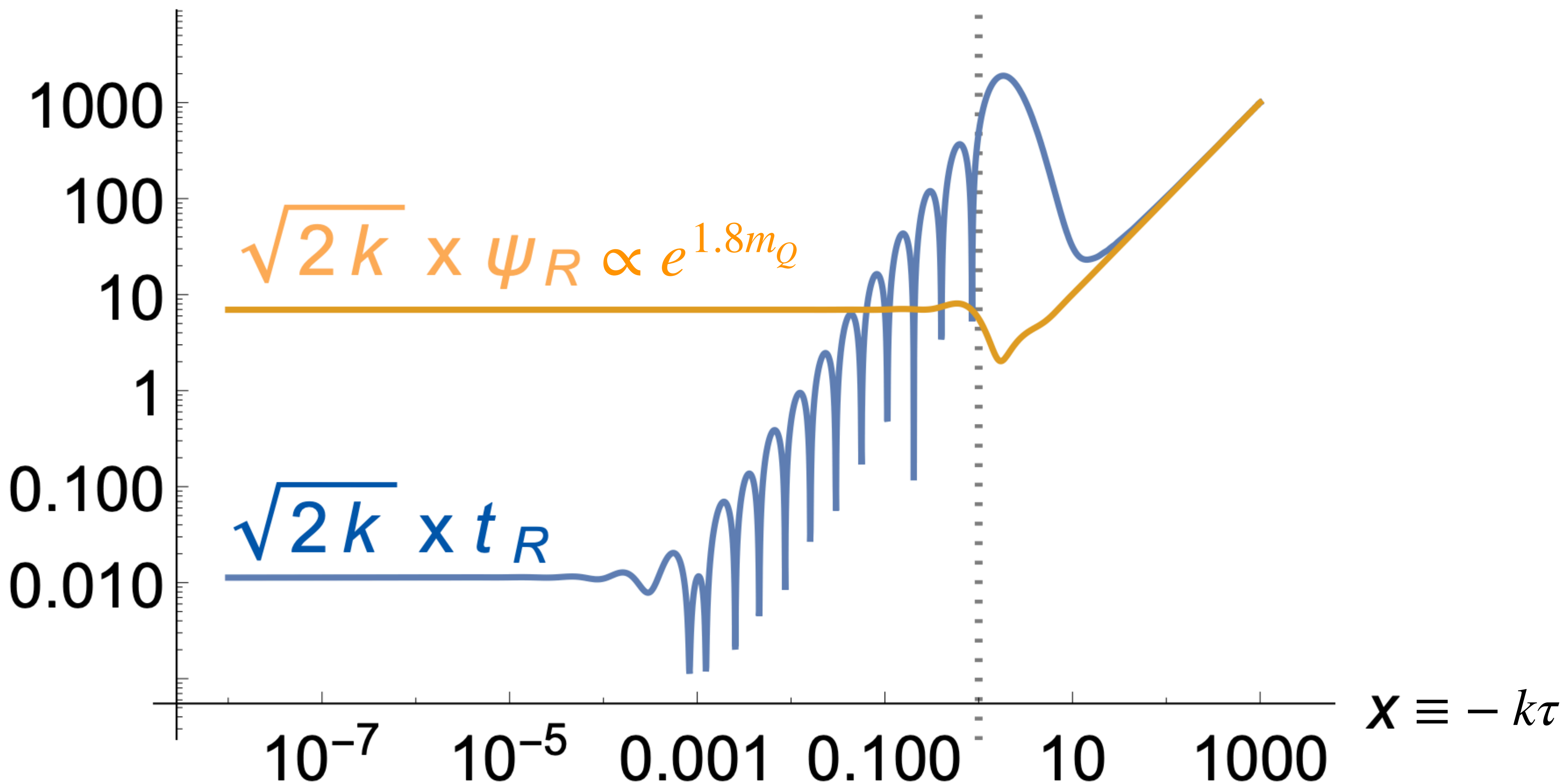
$$\Psi_1^R(\tau, k) = \int_{-\infty}^{\infty} \mathrm{d}\eta \, G_\psi(\tau, \eta, k) \mathscr{D}(\eta, k) T_1^R(\eta, k),$$

$$\alpha \equiv -i\sqrt{2m_Q^2 + 7/4}, \quad \beta \equiv -i(2m_Q + m_Q^{-1}), \quad \mathscr{D}(\eta, k) \equiv \frac{2\sqrt{\epsilon_B}}{m_Q \eta} \partial_\eta + \frac{2\sqrt{\epsilon_B}}{\eta^2}(m_Q + k\eta),$$

$$G_\psi(\tau, \eta, k) \equiv \frac{\Theta(\tau - \eta)}{k^3 \tau \eta} \left[k(\eta - \tau)\cos\left(k(\tau - \eta)\right) + (1 + k^2 \tau \eta)\sin\left(k(\tau - \eta)\right)\right],$$

Chromo-natural inflation

- Tensor perturbations



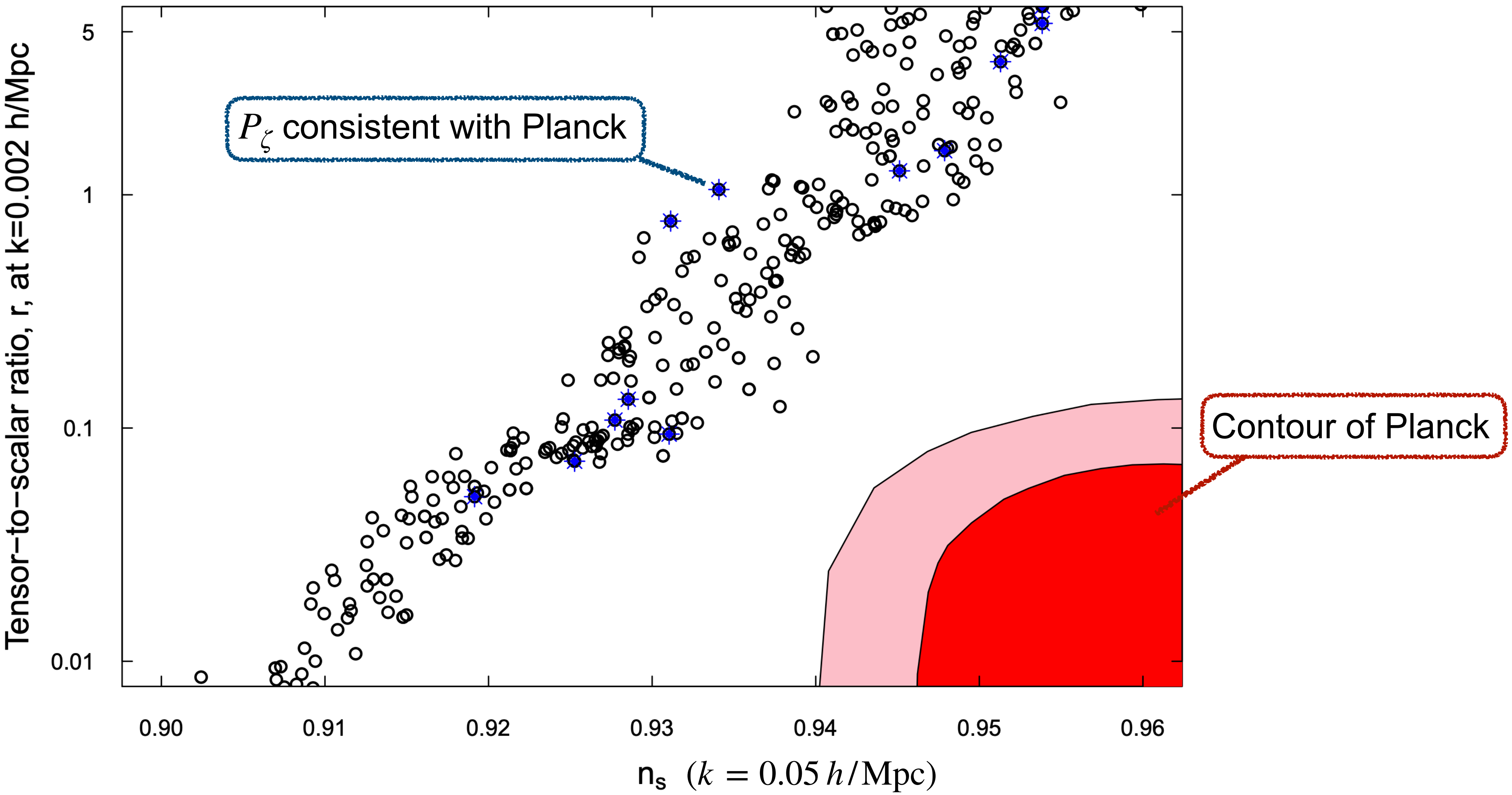
[E. Dimastrogiovanni, M. Fasiello, and T. Fujita (2016)]

Chromo-natural inflation

Constraints on CNI [P. Adshead, E. Martinec, and M. Wyman (2013)]

CNI is excluded by (n_s, r).

For acceptable scalar spectra, the gravitational waves are overproduced.



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SU(N)-natural inflation

- Background gauge fields

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu,$$

$$\tilde{F}^{a\mu\nu} = \frac{1}{2\sqrt{-\tilde{g}}} \epsilon^{\mu\nu\rho\sigma} F^a_{\rho\sigma},$$

Consider SU(N) gauge group:

$$\mathscr{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) + \frac{\phi}{4f} F^a_{\mu\nu} \tilde{F}^{a\mu\nu}$$

Assume $a \propto e^{Ht}$ and $\xi \equiv \dfrac{\dot{\phi}}{2fH} = \text{const}\,.$

Parametrize A^a_i by $M^a_i(t) \equiv \dfrac{g}{a(t)H} A^a_i(t)$.

EoM of background gauge fields

$$\frac{\ddot{M}^a_i}{H^2} + \frac{3}{H} \dot{M}^a_i + 2M^a_i + f^{bac} f^{bde} M^c_j M^d_i M^e_j - \xi \epsilon_{ijk} f^{abc} M^b_j M^c_k = 0,$$

SU(N)-natural inflation

- Background gauge fields

$$\rho_A = \frac{H^4}{4g^2}\left[f^{abc}f^{ade}M_i^bM_j^cM_i^dM_j^e + 2\left(\frac{\dot{M}_i^a}{H} + M_i^a\right)^2\right]$$

EoM of static background gauge fields

$$\frac{\ddot{M}_i^a}{H^2} + \frac{3}{H}\dot{M}_i^a + 2M_i^a + f^{bac}f^{bde}M_j^cM_i^dM_j^e - \xi\epsilon_{ijk}f^{abc}M_j^bM_k^c = 0,$$

We have considered homogeneous and static solutions.

But this is still difficult to solve…

→ We assume that “electromagnetic” fields are “parallel”.

SU(N)-natural inflation

- Parallelism of electromagnetic fields

E_i and B_i are coupled through the Chern-Simons term:

$$\phi F^a_{\mu\nu}\tilde{F}^{a\mu\nu} \rightarrow \propto \xi E^a_i B^a_i$$

→ E_i and B_i source each other.

→ We assume $E^a_i \propto B^a_i \quad (i = x, y, z)$

On the other hand, for constant M^a_i,

$$E^a_i = -\frac{aH^2}{g}M^a_i, \quad B^a_i = -\frac{a^2H^2}{2g}\epsilon_{ijk}f^{abc}M^b_j M^c_k$$

Then,

$$B_i = \frac{ig\epsilon^{ijk}}{H^2}\left[E_j, E_k\right]$$

$E_i = E^a_i T^a, B_i = B^a_i T^a$

T^a : SU(N) generators

$$\mathrm{Tr}\left[T^a T^b\right] = \frac{\delta^{ab}}{2}$$

SU(N)-natural inflation

- Parallelism of electromagnetic fields

E_i and B_i are coupled through the Chern-Simons term:

$$\phi F^a_{\mu\nu}\tilde{F}^{a\mu\nu} \to \propto \xi E^a_i B^a_i$$

→ E_i and B_i source each other.

→ We assume $E^a_i \propto B^a_i \quad (i = x, y, z)$

Using $E_i \propto B_i \propto M_i \propto \mathcal{T}_i$,

$$B_i = \frac{ig\epsilon^{ijk}}{H^2}\left[E_j, E_k\right] \to [\mathcal{T}_i, \mathcal{T}_j] = i\lambda\epsilon_{ijk}\mathcal{T}_k$$

$\{\mathcal{T}_i\}$ generates an SU(2) subalgebra.

$\mathcal{T}_i$: SU(N) generators

$$\mathrm{Tr}\left[\mathcal{T}_i\mathcal{T}_j\right] = \frac{\delta_{ij}}{2}$$

SU(N)-natural inflation

- Consequence of parallelism

EoM

$$2M_i^a + f^{bac} f^{bde} M_j^c M_i^d M_j^e - \xi \epsilon_{ijk} f^{abc} M_j^b M_k^c = 0$$

$$M_i^a = \sigma_i n_i^a, \quad n_i^a n_j^a = \delta_{ij}, \quad [n_i, n_j] = i\lambda \epsilon_{ijk} n_k$$

$$2\sigma_i + \lambda^2 \sigma_i \sum_{l \neq i} \sigma_l^2 - \xi \lambda \sum_{j,k} |\epsilon_{ijk}| \sigma_j \sigma_k = 0 \quad (i = x, y, z)$$

$$\sigma = \sigma_x = \sigma_y = \sigma_z = 0, \frac{\xi \pm \sqrt{\xi^2 - 4}}{2\lambda}$$

cf.) $\sigma = \left(\xi + \sqrt{\xi^2 - 4}\right)/2$ for CNI

SU(N)-natural inflation

- Isotropy

For an arbitrary spatial rotation R,
there is a corresponding gauge transformation G :

$$^{\forall}R, ^{\exists}G : R_{ij}M_j^a = G^{ab}M_i^b$$

The solutions we found:

$$M_i^a = \sigma\, n_i^a, \quad \sum_a n_i^a n_j^a = \delta_{ij}, \quad [n_i, n_j] = i\lambda \sum_k \epsilon_{ijk} n_k$$

satisfies the isotropy condition.

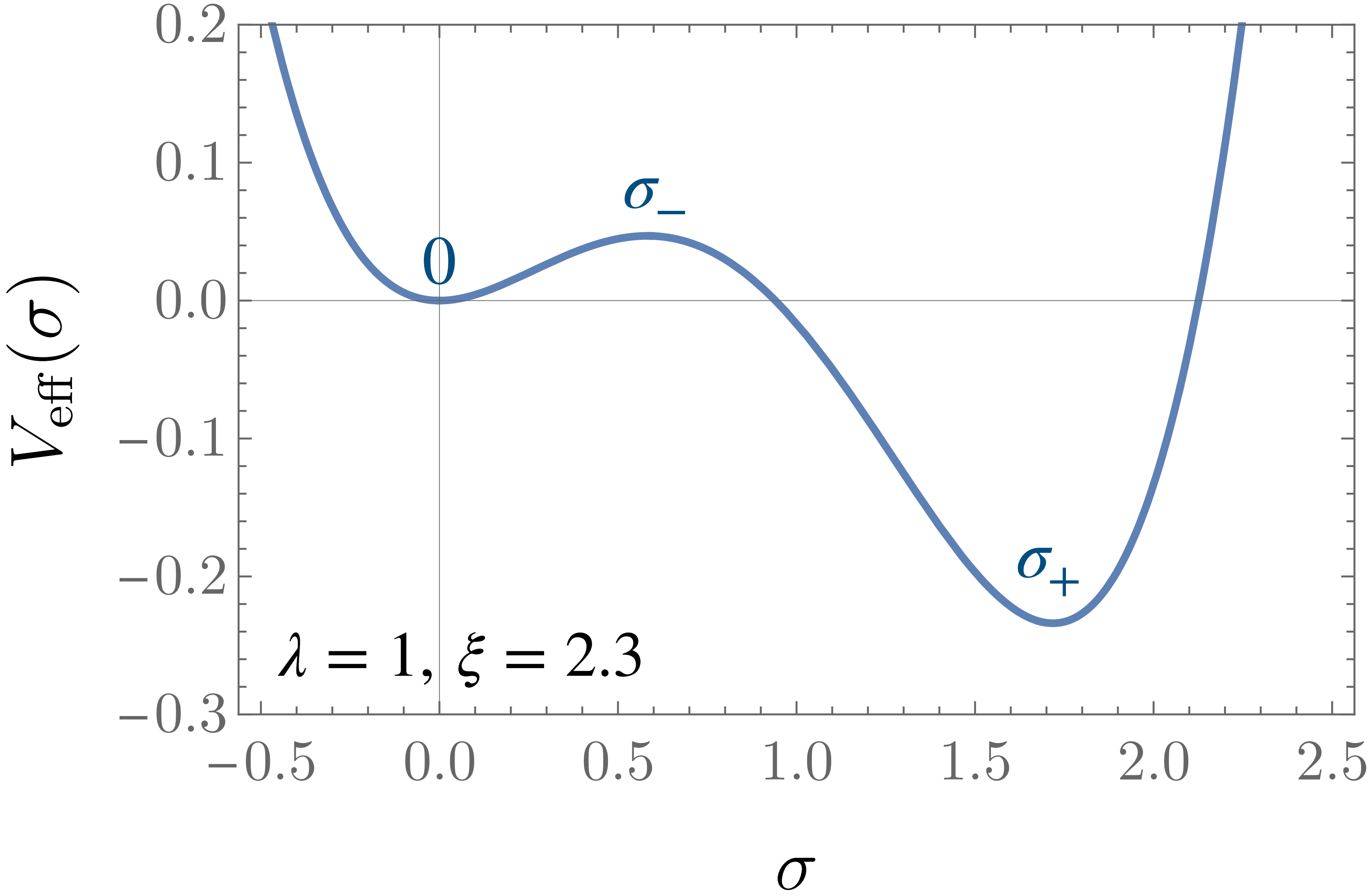
$$\because \quad \underbrace{\delta M^a_{i,\mathrm{rot}} T^a}_{\text{Spatial rotation}} = \theta \epsilon^{ijz} \sigma n_j^a T^a = \frac{i\theta}{\lambda}\left[n_z, \sigma n_j\right] = \underbrace{\delta M^a_{i,\mathrm{gauge}} T^a}_{\text{Gauge transformation}}$$

$$\text{SSB: } SO(3) \times \underbrace{SU(2)}_{\subset SU(N)} \rightarrow SO(3)$$

- Stability

Consider the effective potential for σ.

→ $\sigma = \sigma_+$ is stable.



$$V_{\rm eff}[\sigma] = \frac{1}{2}\sigma^2 - \frac{1}{3}\xi\lambda\sigma^3 + \frac{1}{4}\lambda^2\sigma^4$$

$$\sigma = 0, \frac{\sigma_\pm}{\lambda}$$

$$\sigma_\pm = \frac{\xi \pm \sqrt{\xi^2 - 4}}{2}$$

SU(N)-natural inflation

- Construction of the solutions : $N = 3$

First, determine the SU(2) subgroup in SU(N):

$$SU(3) \supset SU(2), \qquad \mathbf{3} = \mathbf{3}$$

$$n_i = \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Using the generators of the SU(2) subalgebra $\{n_i\}$,

$$M_i^a = \sigma\, n_i$$

Amplitude σ is determined by the commutation relation of $\{n_i\}$:

$$[n_x, n_y] = \frac{i}{2} n_z \rightarrow \lambda = \frac{1}{2} \rightarrow \sigma = 2\sigma_+$$

$$\sigma_\pm = \frac{\xi \pm \sqrt{\xi^2 - 4}}{2}$$

SU(N)-natural inflation

- Examples: $N = 3$

$$SU(3) \supset SU(2) \times U(1) \quad \mathbf{3} = \mathbf{2} + \mathbf{1}$$

$$n_i = \frac{1}{2}\begin{pmatrix} \sigma_i & 0 \\ 0 & 0 \end{pmatrix} \qquad \lambda = 1 \quad \sigma = \sigma_+$$

$$SU(3) \supset SU(2) \quad \mathbf{3} = \mathbf{3}$$

$$n_i = \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\lambda = \frac{1}{2} \qquad \sigma = 2\sigma_+$$

SU(N)-natural inflation

- Examples: $N = 4$

$$SU(4) \supset SU(2) \times U(1) \quad \mathbf{4} = \mathbf{3} + \mathbf{1}$$

$$n_z = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad \lambda = \frac{1}{2} \qquad \sigma = 2\sigma_+$$

$$SU(4) \supset SU(2) \quad \mathbf{4} = \mathbf{4}$$

$$n_z = \frac{1}{\sqrt{10}}\begin{pmatrix} \frac{3}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{3}{2} \end{pmatrix} \qquad \lambda = \frac{1}{\sqrt{10}} \qquad \sigma = \sqrt{10}\sigma_+$$

SU(N)-natural inflation

- Examples: $N = 4$

$$SU(4) \supset SU(2) \times SU(2) \quad \mathbf{4} = (\mathbf{2}, \mathbf{1}) + (\mathbf{1}, \mathbf{2})$$

$$n_z = \frac{1}{2}\begin{pmatrix} \sigma_i & 0 \\ 0 & 0 \end{pmatrix}, \quad \frac{1}{2}\begin{pmatrix} 0 & 0 \\ 0 & \sigma_i \end{pmatrix}$$

$$\lambda = 1$$

$$\sigma = \sigma_+, \sqrt{2}\sigma_+$$

$$SU(4) \supset SU(2) \times SU(2) \quad \mathbf{4} = (\mathbf{2}, \mathbf{2})$$

$$n_z = \frac{1}{2\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad \frac{1}{2\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\lambda = \frac{1}{\sqrt{2}}$$

$$\sigma = \sqrt{2}\sigma_+, 2\sigma_+$$

SU(N)-natural inflation

- General N

$$SU(N) \supset SU(2) \times \cdots \qquad \mathbf{N} = \mathbf{m} + \cdots \qquad (m = 2, \cdots, N)$$

$$n_z = \Lambda(m)\,\mathrm{diag}\left(\frac{m-1}{2}, \frac{m-3}{2}, \cdots, -\frac{m-1}{2}, 0, \cdots, 0\right)$$

$$\lambda = \Lambda(m) = \left(\frac{m(m^2-1)}{6}\right)^{-1/2} \qquad \sigma = \frac{\sigma_+}{\Lambda(m)} \sim \sqrt{\frac{m(m^2-1)}{6}}\sigma_+$$

Contents

Numerical simulations

Setting

- Solve the EoM with time derivatives : $\left(a \propto e^{Ht}, \quad \xi \equiv \frac{\dot{\phi}}{2fH} = \text{const}\,.\right)$

$$\frac{\ddot{M}_i^a}{H^2} + \frac{3}{H}\dot{M}_i^a + 2M_i^a + f^{bac}f^{bde}M_j^c M_i^d M_j^e - \xi \epsilon_{ijk} f^{abc} M_j^b M_k^c = 0,$$

- Initial condition is random variables :

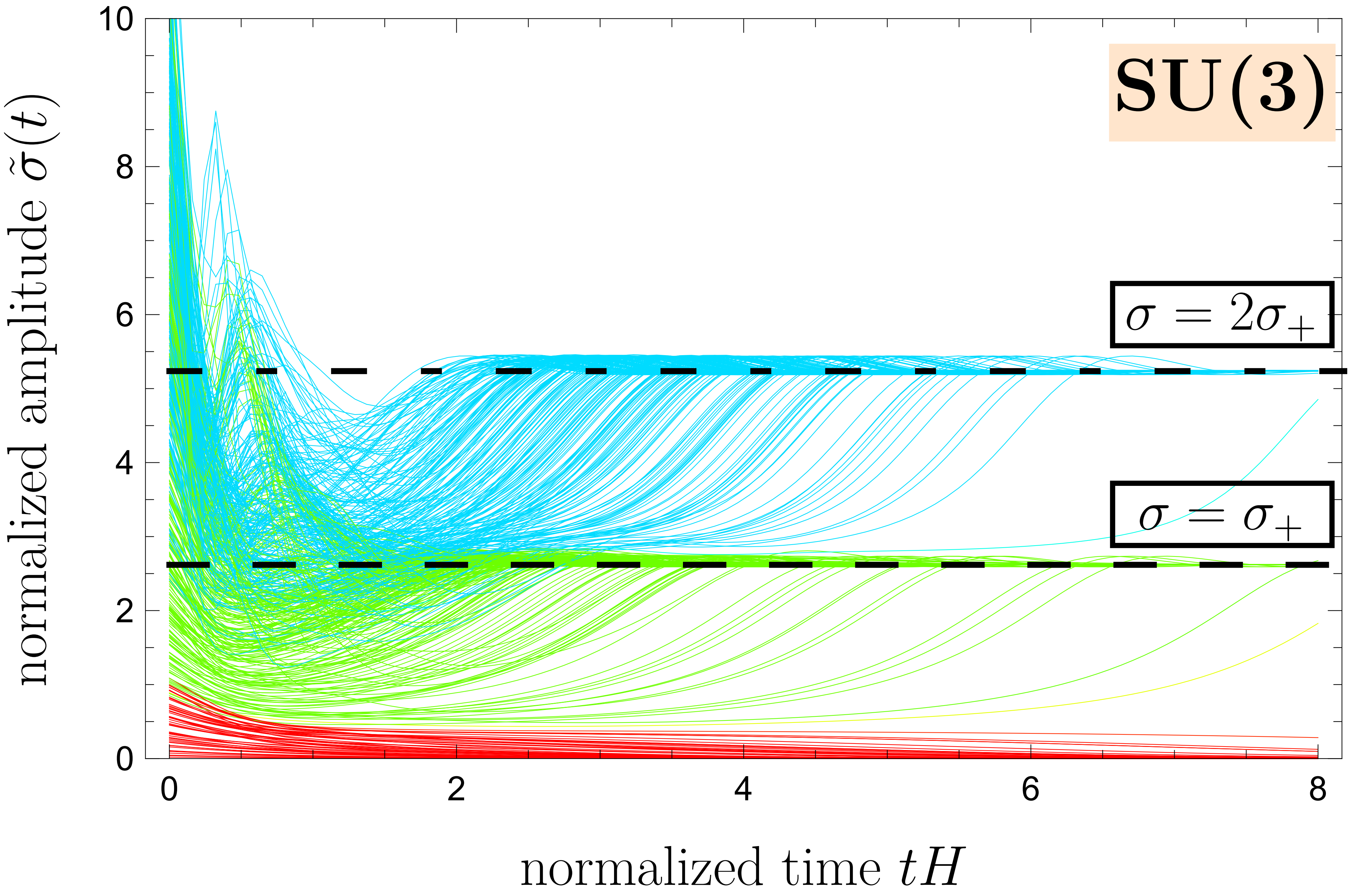
$$\dot{A}_i^a = 0, \quad M_i^a(t_0) = \text{(Gaussian distribution)}$$

- We characterize the results by the averaged amplitude:

$$\tilde{\sigma}^2 = \frac{1}{3}\mathrm{Tr}[M_i M_i]$$

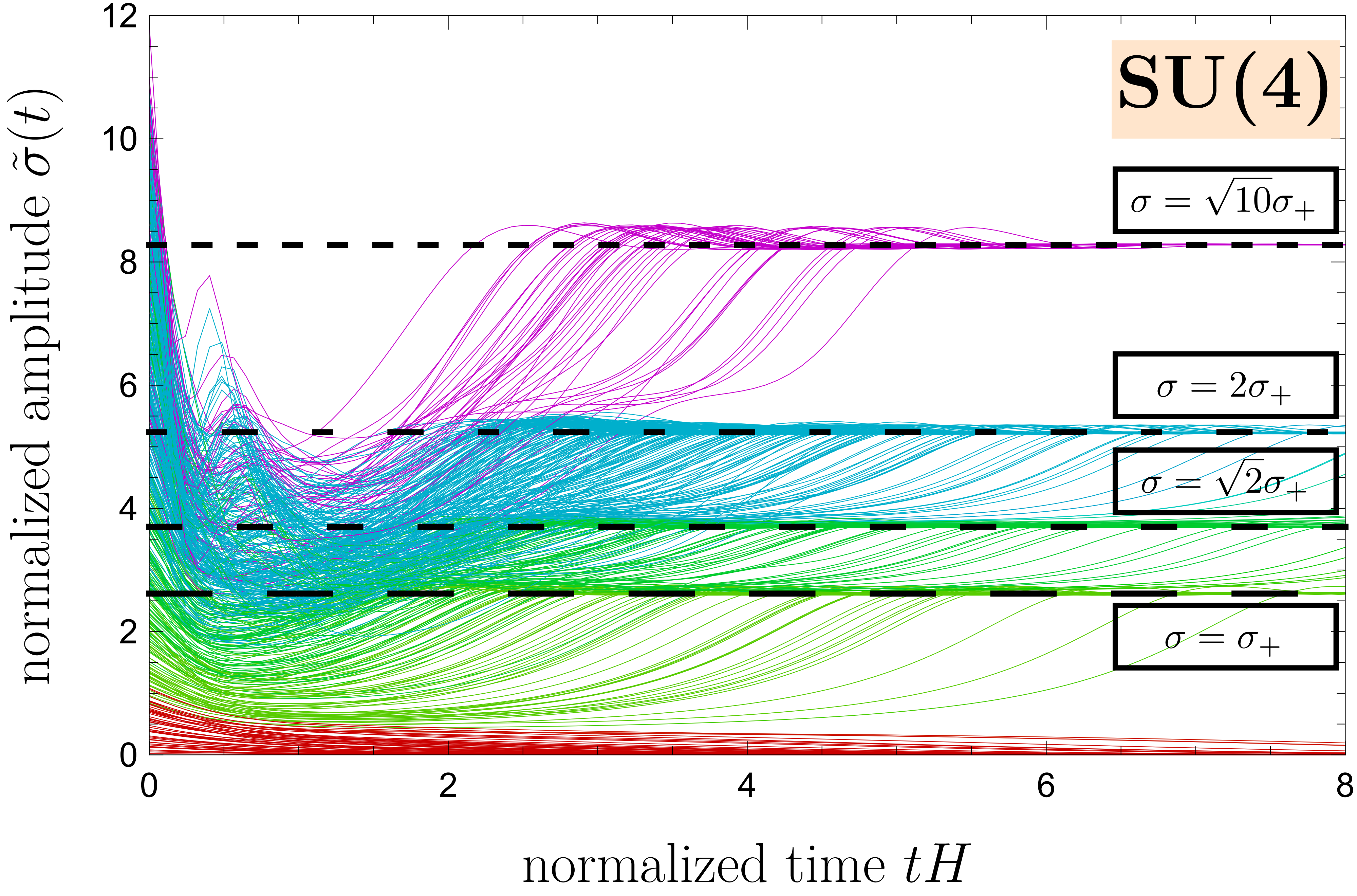
Numerical simulations

- Background dynamics

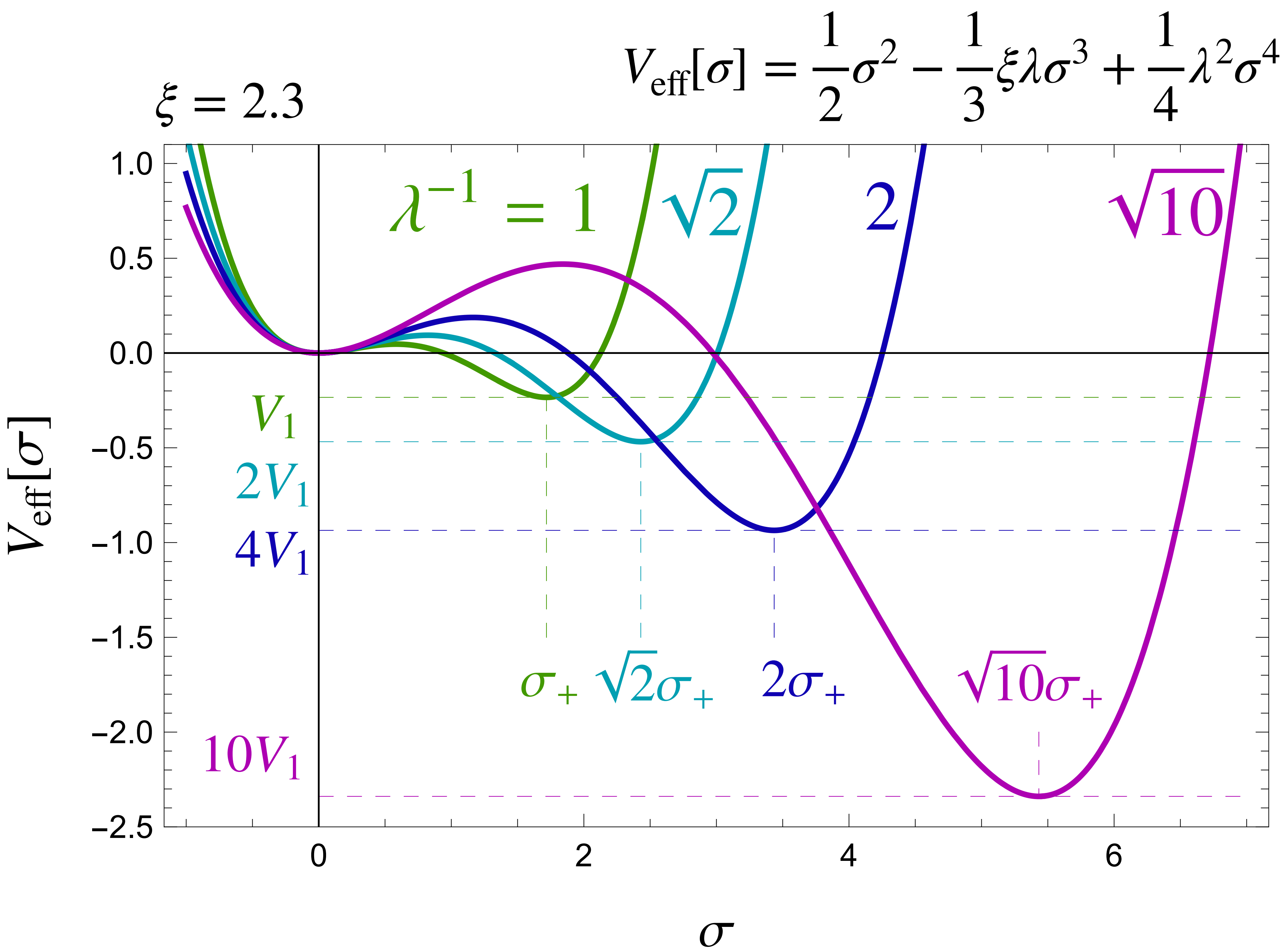


Numerical simulations

- Background dynamics



- Stability



$V_{\rm eff}[\sigma] = \frac{1}{2}\sigma^2 - \frac{1}{3}\xi\lambda\sigma^3 + \frac{1}{4}\lambda^2\sigma^4$
$\xi = 2.3$
$\lambda^{-1} = 1$
$\sqrt{2}$
2
$\sqrt{10}$
V_1
$2V_1$
$4V_1$
$10V_1$
σ_+
$\sqrt{2}\sigma_+$
$2\sigma_+$
$\sqrt{10}\sigma_+$
$V_{\rm eff}[\sigma]$
σ
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
0
2
4
6

Contents

Perturbations in SU(N)

- GW production

Decompose the gauge fields as

$$A_i = \bar{A}_i + \underbrace{\delta A_i^{\bar{j}} \mathcal{T}_{\bar{j}}}_{\textbf{3}\text{ rep.}} + \underbrace{\delta A_i^A \mathcal{T}^A}_{\text{the other reps.}}$$

The action is also decomposed at the 2nd order according to the representations:

$$L_{2,h^2} = \frac{a^2 M_{\rm Pl}^2}{8} \left(h'_{ij} h'_{ij} - \partial_i h_{jk} \partial_i h_{jk} \right)$$

$$L_{2,h\delta A} = -\frac{\sigma_+}{g\lambda\tau^2} \left[h_{ij} \delta A_j^{\bar{i}'} - \sigma_+ \epsilon^{ijk} h_{il} \partial_k \delta A_l^{\bar{j}} - \frac{\sigma_+}{\tau} h_{ij} \delta A_j^{\bar{i}} \right]$$

$$L_{2,\left(\delta A_i^{\bar{j}}\right)^2} = \frac{1}{2} \left[\left(\delta A_i^{\bar{j}'} \right)^2 - \left(\partial_i \delta A_j^{\bar{k}} \right)^2 \right] - \frac{\sigma_+}{\tau} \epsilon^{i\bar{l}\bar{k}} \partial_i \delta A_j^{\bar{k}} \delta A_j^{\bar{l}} + \frac{\xi}{\tau} \epsilon^{ijk} \left(\delta A_i^{\bar{l}} \partial_j \delta A_k^{\bar{l}} + \frac{\sigma_+}{\tau} \epsilon^{i\bar{l}\bar{m}} \delta A_j^{\bar{l}} \delta A_k^{\bar{m}} \right)$$

$$L_{2,\left(\delta A_i^A\right)^2} = \cdots$$

Perturbations in SU(N)

- GW production

On the GW production, the only difference is λ^{-1} in $L_{2,h\delta A}$.
This difference can be canceled by g.

"Universality" of SU(2) dynamics!

$$L_{2,h^2} = \frac{a^2 M_{\rm Pl}^2}{8}\left(h'_{ij}h'_{ij} - \partial_i h_{jk}\partial_i h_{jk}\right)$$

$$L_{2,h\delta A} = -\frac{\sigma_+}{\underline{g\lambda}\tau^2}\left[h_{ij}\delta A_j^{\bar{i}\prime} - \sigma_+\epsilon^{ijk}h_{il}\partial_k\delta A_l^{\bar{j}} - \frac{\sigma_+}{\tau}h_{ij}\delta A_j^{\bar{i}}\right]$$

$$L_{2,\left(\delta A_i^{\bar{j}}\right)^2} = \frac{1}{2}\left[\left(\delta A_i^{\bar{j}\prime}\right)^2 - \left(\partial_i\delta A_j^{\bar{k}}\right)^2\right] - \frac{\sigma_+}{\tau}\epsilon^{i\bar{l}\bar{k}}\partial_i\delta A_j^{\bar{k}}\delta A_j^{\bar{l}} + \frac{\xi}{\tau}\epsilon^{ijk}\left(\delta A_i^{\bar{l}}\partial_j\delta A_k^{\bar{l}} + \frac{\sigma_+}{\tau}\epsilon^{i\bar{l}\bar{m}}\delta A_j^{\bar{l}}\delta A_k^{\bar{m}}\right)$$

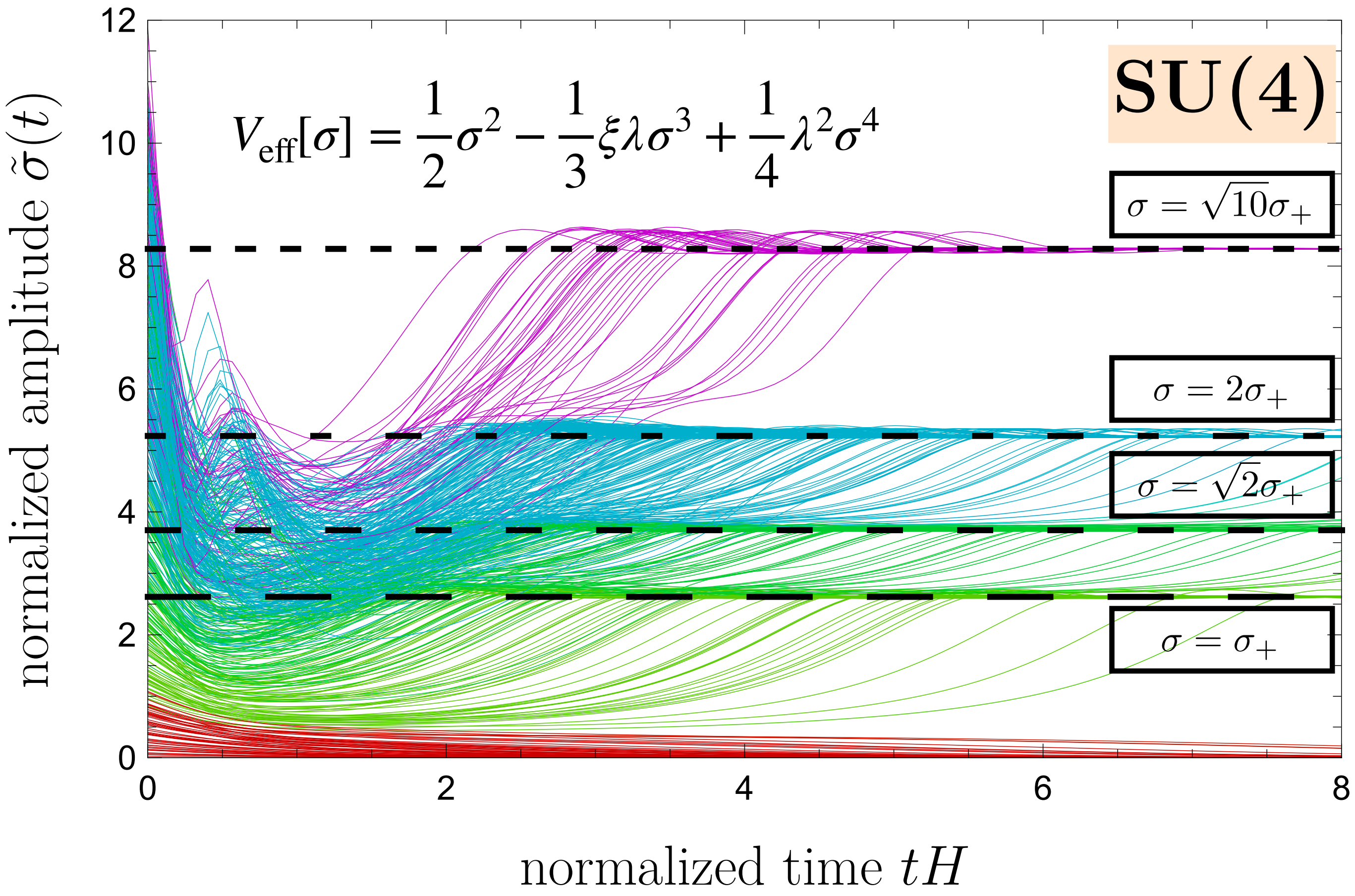
$$L_{2,\left(\delta A_i^{A}\right)^2} = \cdots$$

Perturbations in SU(N)

- Transition of the multiple solutions

  As we have seen, the solutions in SU(N) are the same up to the change of g.

  However, if the transition of the solutions occurs, the SU(N) model can be distinguished from SU(2).



Contents

Summary

- Chromo-natural inflation (CNI) predicts homogeneous and isotropic gauge fields.
- But CNI can produce desirable scalar perturbations with $f < M_{\rm Pl}$ but suffers from the GW overproduction.

- We study the SU(N) version of CNI.
- We can construct homogeneous and isotropic solutions by determining SU(2) subgroups.
- The solutions have different amplitudes corresponding to the choice of SU(2) subgroups.

- We numerically check that the constructed solutions are attractor solutions and explain all the solutions.

- The difference of the solutions can be canceled by g.
- The gauge group can be distinguished by the transition?