On-shell amplitudes in generalized HEFT

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Based on **1904.07618** (PRD)
and **2102.08519**

in collabolation with
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Big questions

- SM cannot explain
  - Why EWSB scale ~ O(100 GeV)
  - What dark matter is

    ⋮

- Physics beyond SM is needed.

Searches for BSM signals

- Direct searches
- Indirect searches

How to organize BSM signals ?

Can we be systematic ?

Talk plan

- Scalar sector 1904.07618 (PRD)
- Fermion sector 2102.08519

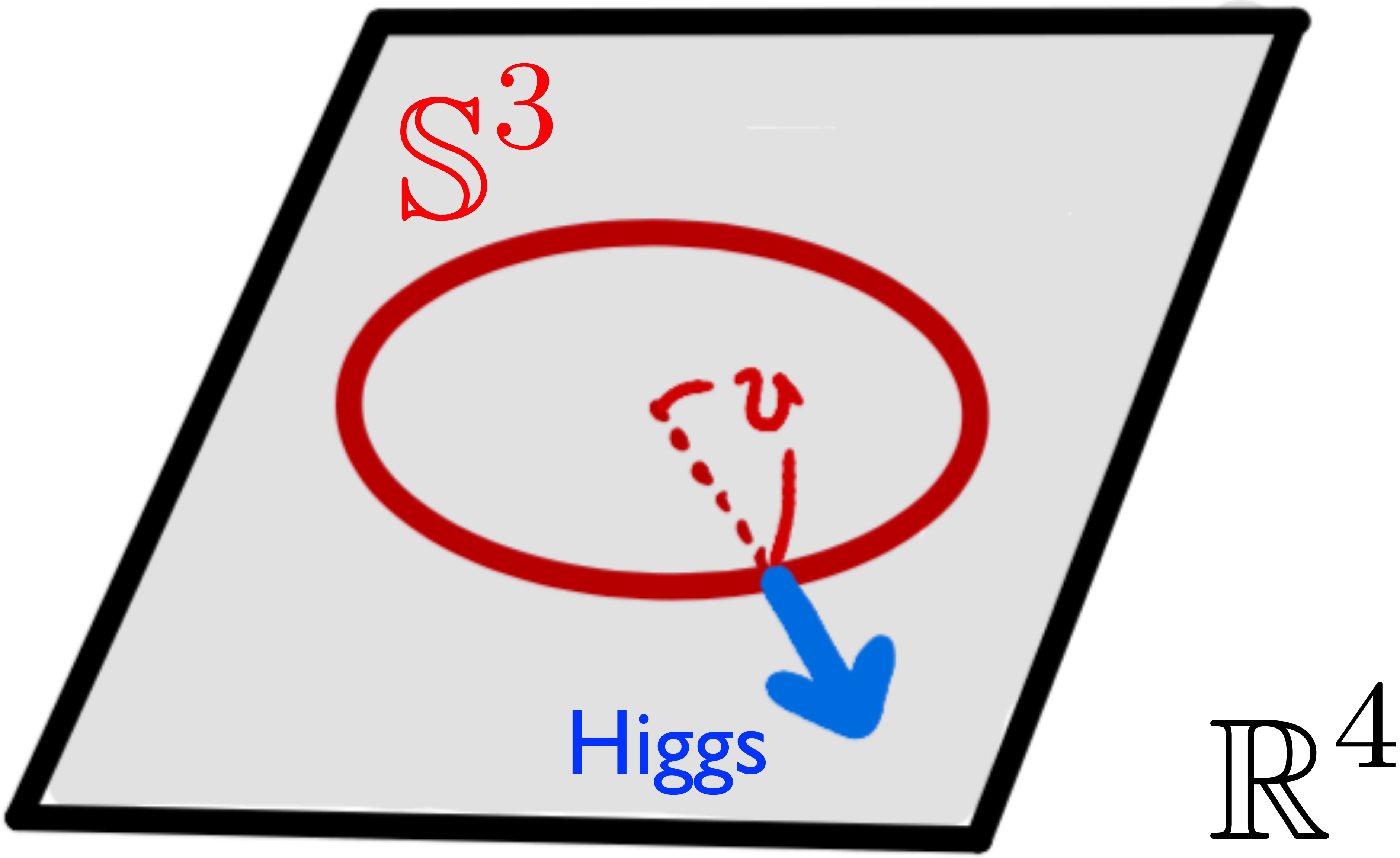
SM Higgs

$$\mathcal{L}_{\rm SM} = \frac{1}{2}(\partial_\mu \vec{\phi}) \cdot (\partial^\mu \vec{\phi}) - \frac{\lambda}{2}(\vec{\phi} \cdot \vec{\phi} - v^2)$$

$$\vec{\phi} = (\phi^1, \phi^2, \phi^3, \phi^4)$$

- Field space : $\mathbb{R}^4$
- Vacuum : $\mathbb{S}^3 \quad (\vec{\phi} \cdot \vec{\phi} = v^2)$

SM Higgs



(Minimal) Composite Higgs

Agashe-Contino-Pomarol (2005)

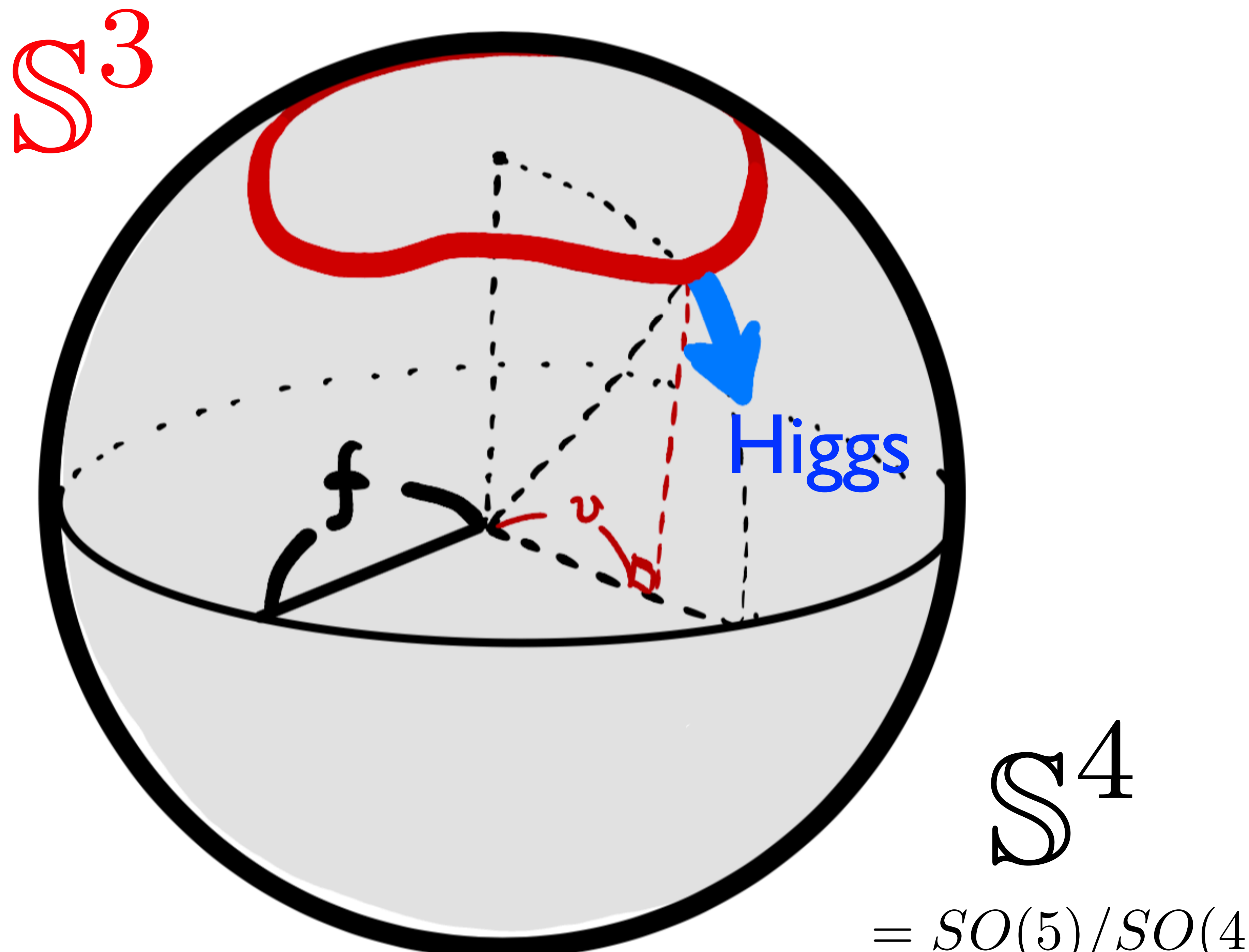
Energy

f $\quad SO(5) \to SO(4)$

4 NGBs = (h, π^1, π^2, π^3)

(Minimal) Composite Higgs

Agashe-Contino-Pomarol (2005)

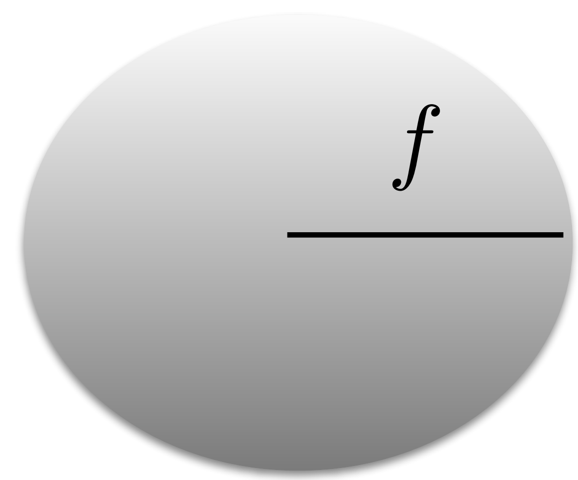


Scalar manifold

SM

$\mathbb{R}^4$

MCHM



$\mathbb{S}^4$

Higgs EFT

- Geometrical form $\phi = (h, \pi^1, \pi^2, \pi^3)$

$$\mathcal{L}_{\rm HEFT} = \frac{1}{2} g_{ij}(\phi)\, \partial_\mu \phi^i \partial^\mu \phi^j - V(\phi)$$

Alonso, Jenkins, Manohar (2015)

- Familiar forms

Cartesian (SMEFT) $\mathcal{L} = \frac{1}{2}(\partial_\mu \vec{\Phi})(\partial^\mu \vec{\Phi}) + C(\Phi)(\vec{\Phi} \cdot \partial_\mu \vec{\Phi})^2 - V(\Phi)$

Polar (HEFT) $\mathcal{L} = \frac{1}{2}(\partial_\mu h)(\partial^\mu h) + \frac{v^2}{4} F(h)\, {\rm Tr}[\partial_\mu U^\dagger \partial^\mu U] - V(h)$

Higgs EFT

- Geometrical form $\phi = (h, \pi^1, \pi^2, \pi^3)$

$$\mathcal{L}_{\rm HEFT} = \frac{1}{2} g_{ij}(\phi)\, \partial_\mu \phi^i \partial^\mu \phi^j - V(\phi)$$

Alonso, Jenkins, Manohar (2015)

Assumption : Extra scalars are decoupled.

We cannot compute on-shell amplitudes of BSM scalars

Adding spin-0

- Generalization with spin-0 particles:

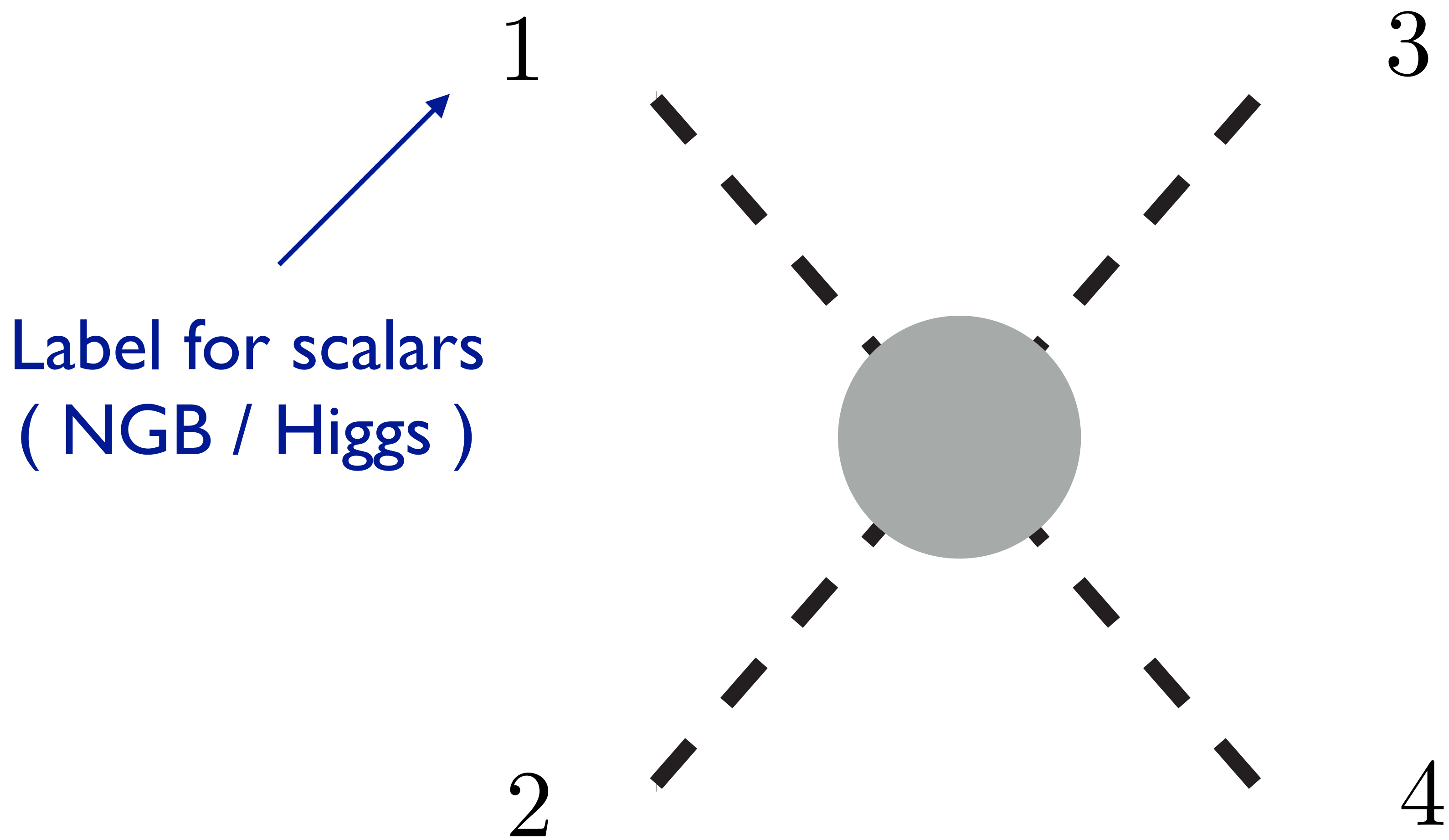
$$\mathcal{L}_{\mathrm{HEFT}} = \frac{1}{2} g_{ij}(\phi)\, \partial_\mu \phi^i \partial^\mu \phi^j - V(\phi)$$

$$\phi = (h, \pi^1, \pi^2, \pi^3, S, H^+, H^-, \cdots)$$

- The "charged" scalar manifold can be constructed by Callen-Coleman-Wess-Zumino (CCWZ) method.

Nagai, Tanabashi, Tsumura. Uchida,
1904.07618 (PRD)

On-shell scalar amplitudes



On-shell scalar amplitudes

$$\mathcal{L} = \frac{1}{2} g_{ij}(\phi)\, \partial_\mu \phi^i \partial^\mu \phi^j - V(\phi)$$

- Particle interaction

$$g_{ij}(\phi) = g_{ij}(\bar{\phi}) + g_{ij,k}(\bar{\phi})\, \varphi^k + \frac{1}{2!} g_{ij,kl}(\bar{\phi})\, \varphi^k \varphi^l + \cdots$$

$$(\phi^i = \bar{\phi}^i + \varphi^i)$$

On-shell scalar amplitudes

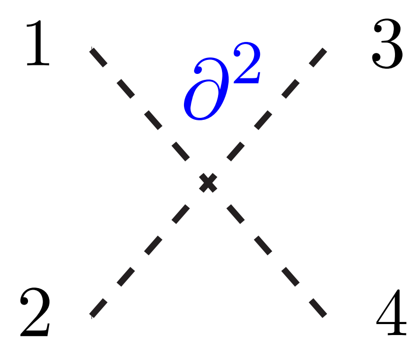
$$\mathcal{L} = \frac{1}{2} g_{ij}(\phi)\, \partial_\mu \phi^i \partial^\mu \phi^j - V(\phi)$$

- "Normal coordinate"

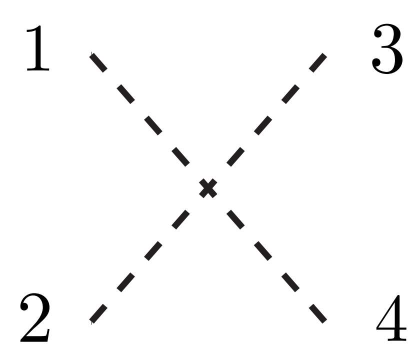
$$g_{ij}(\phi) = \delta_{ij} + \frac{2}{3} R_{iklj}(\bar{\phi})\, \varphi^k \varphi^l + R_{iklj;m}\, \varphi^k \varphi^l \varphi^m + \cdots$$

$$(\phi^i = \bar{\phi}^i + \varphi^i)$$

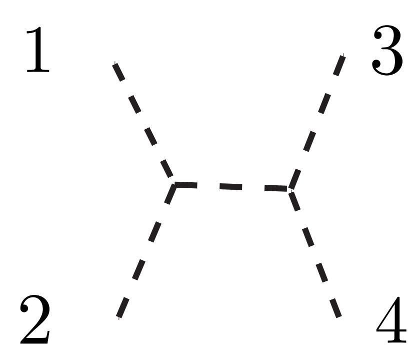
On-shell scalar amplitudes



$$\sim \; R_{1234}\, s$$

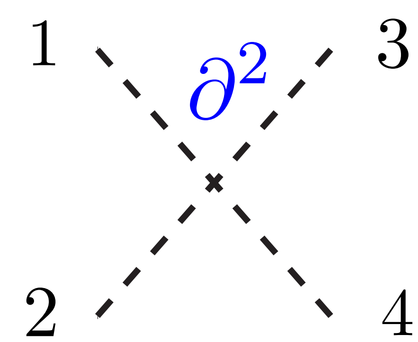


$$\sim \; V_{;(1234)}$$



$$\sim \; \sum_i V_{;(12i)} V_{;(i34)} \frac{1}{s}$$

Perturbative Unitarity Violation



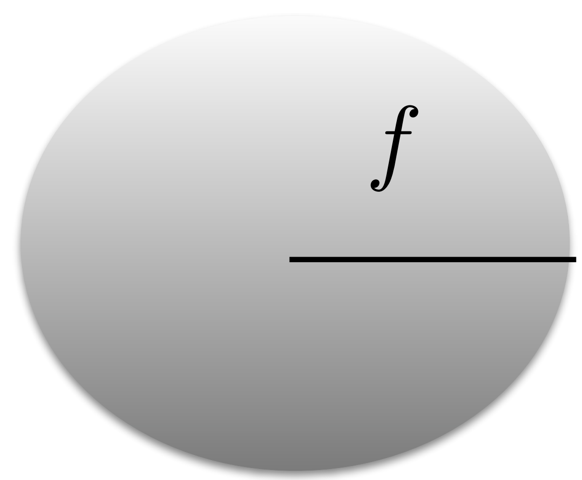
$$\sim \; R_{1234}\, s$$

- Perturvative unitarity is violated if the scalar manifold is *curved*.

Typical PUV scale $\propto$ (Curvature)$^{-1/2}$

Perturbative Unitarity Violation

e.g. Minimal Composite Higgs Model



"Resonances" appear at E ~ 4πf

Typical PUV scale $\propto$ (Curvature)$^{-1/2}$

Summary so far

- Scalar sector: Φ = (NGB, Higgs)

$$\mathcal{L} = \frac{1}{2} g_{ij}(\phi)\, \partial_\mu \phi^i \partial^\mu \phi^j - V(\phi)$$

- On-shell amplitudes are expressed by covariant quantities on the scalar manifold.

- Perturbative unitarity is violated at $\Lambda \sim 1/\sqrt{R}$

Talk plan

- Scalar sector 1904.07618 (PRD)
- Fermion sector 2102.08519

Fermionic extension

$$\mathcal{L} = \frac{1}{2} g_{ij}(\phi)\, \partial_\mu \phi^i \partial^\mu \phi^j - V(\phi)$$

- SUSY !

(Weyl) fermion ↙ ↙

ϕ^i ⇒ $(\phi^i, \phi^{\dagger i^*}, \psi^i, \psi^{\dagger i^*})$

$g_{ij}(\phi)$ ⇒ $g_{ij^*}(\phi, \phi^\dagger)$ Kähler metric

$V(\phi)$ ⇒ $P(\phi)\ \ P^\dagger(\phi^\dagger)$ Superpotential

SUSY HEFT

$$\mathcal{L}_{\text{scalar}} = g_{ij^*}(\partial_\mu \phi^i)(\partial^\mu \phi^{\dagger j^*}) - g^{ij^*} P_{,i} P^\dagger_{,j^*}$$

$$\begin{aligned}
\mathcal{L}_{\text{fermion}} = & \frac{i}{2} g_{ij^*} \psi^{\dagger j^*} \bar{\sigma}^\mu \overleftrightarrow{\partial}_\mu \psi^i \\
& + \frac{i}{2} (g_{ij^*,k} \partial_\mu \phi^k - g_{ij^*,k^*} \partial_\mu \phi^{\dagger k^*})(\psi^{\dagger j^*} \bar{\sigma}^\mu \psi^i) \\
& - P_{;ij} (\psi^i \psi^j) + h.c. \\
& - \frac{1}{8} R_{ik^* jl^*} (\psi^i \psi^j)(\psi^{\dagger i^*} \psi^{\dagger j^*})
\end{aligned}$$

Generalized HEFT

NEW!

$$\mathcal{L}_{\text{scalar}} = \frac{1}{2} g_{ij} (\partial_\mu \phi^i)(\partial^\mu \phi^j) - V(\phi)$$

$$\begin{aligned}
\mathcal{L}_{\text{fermion}} = & \frac{i}{2} g_{\hat{i}\hat{j}^*} \psi^{\dagger \hat{j}^*} \bar{\sigma}^\mu \overleftrightarrow{\partial}_\mu \psi^{\hat{i}} \\
& + \frac{i}{2} (g_{k\hat{j}^*,\hat{i}} - g_{k\hat{i},\hat{j}^*})(\psi^{\dagger j^*} \bar{\sigma}^\mu \psi^i)(\partial_\mu \phi^k) \\
& - \frac{1}{2} M_{\hat{i}\hat{j}} (\psi^i \psi^j) + h.c. \\
& - \frac{1}{4} R_{\hat{i}\hat{k}^*\hat{j}\hat{l}^*} (\psi^{\hat{i}} \psi^{\hat{j}})(\psi^{\dagger \hat{k}^*} \psi^{\dagger \hat{l}^*}) \\
& - \frac{1}{12} R_{\hat{i}\hat{l}\hat{j}\hat{k}} (\psi^{\hat{i}} \psi^{\hat{j}})(\psi^{\hat{k}} \psi^{\hat{l}}) + h.c.
\end{aligned}$$

- can be constructed by CCWZ method. **Nagai,** Tanabashi, Tsumura. Uchida, 2102.08519

Covariance

- Geometric interpretation of "fermionic metric" is not clear in non-SUSY case.

- Anyway, S-matrix should be invariant under

$$\phi^i \to f(\phi)^i{}_j \phi^j \qquad \psi^{\hat{i}} \to \hat{f}(\phi)^{\hat{i}}{}_{\hat{j}} \phi^{\hat{j}}$$

- Let us define covariant quantities under above trans.

Covariance

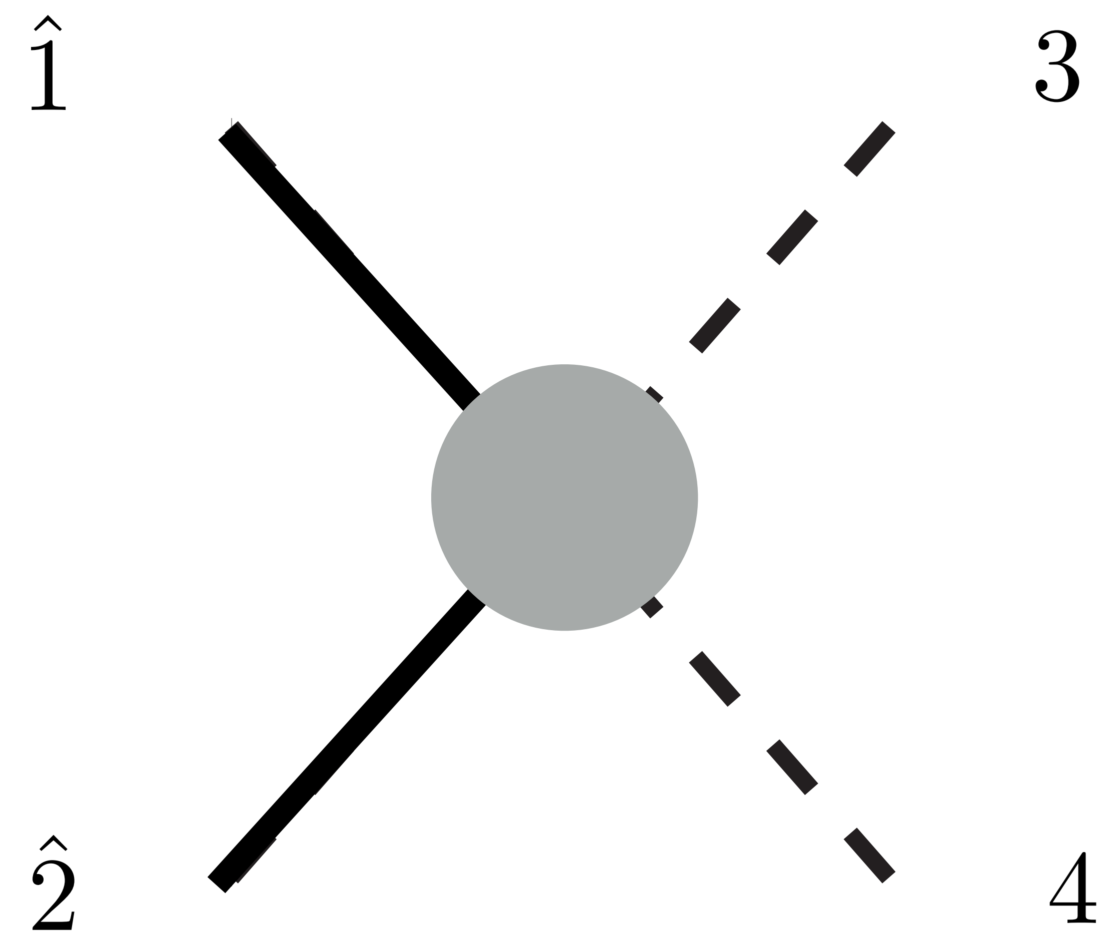
"Affine connection" $\Gamma^{\hat{i}}_{j\hat{k}} := \frac{1}{2} g^{\hat{i}\hat{l}^*} (g_{\hat{k}\hat{l}^*,j} + g_{j\hat{l}^*,\hat{k}} - g_{j\hat{k},\hat{l}^*})$

where $g_{\hat{i}\hat{j}^*;i} := g_{\hat{i}\hat{j}^*,i} - g_{\hat{k}\hat{j}^*}\Gamma^{\hat{k}}_{i\hat{i}} - g_{\hat{i}\hat{k}^*}\Gamma^{\hat{k}^*}_{i\hat{j}^*} = 0$

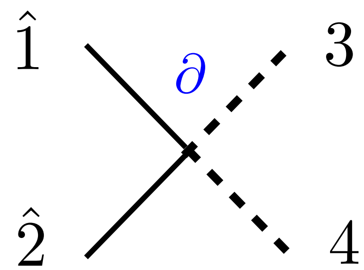
"Curvature tensor" $R^{\hat{i}}_{\hat{j}kl} := \Gamma^{\hat{i}}_{l\hat{j},k} - \Gamma^{\hat{i}}_{k\hat{j},l} + \Gamma^{\hat{i}}_{k\hat{l}}\Gamma^{\hat{l}}_{l\hat{j}} - \Gamma^{\hat{i}}_{l\hat{l}}\Gamma^{\hat{k}}_{l\hat{j}}$

For any covariant vector $a^{\hat{i}}(\phi)$, $a^{\hat{i}}_{;ij} - a^{\hat{i}}_{;ji} = -a^{\hat{k}} R^{\hat{i}}_{\hat{k}ij}$

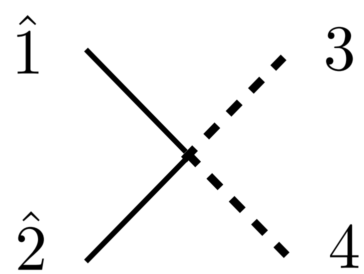
On-shell scalar/fermion amplitudes



On-shell scalar/fermion amplitudes

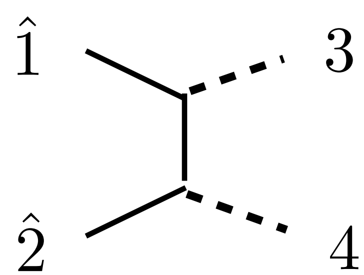


$$\sim R_{\hat{1}\hat{2}34}\, s$$

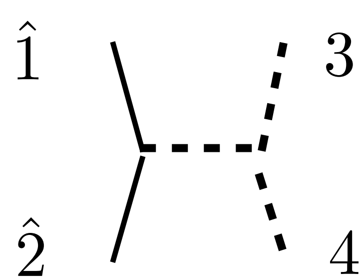


$$\sim M_{\hat{1}\hat{2};(34)}\sqrt{s}$$

Energy growing !



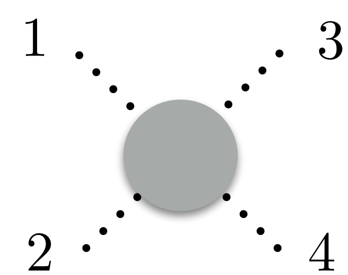
$$\sim \sum_{\hat{i}} M_{\hat{1}\hat{i};3} M_{\hat{i}\hat{2};4}$$



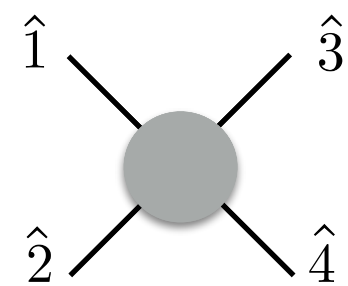
$$\sim \sum_{i} M_{\hat{1}\hat{2};i} V_{;(;34)} \frac{1}{\sqrt{s}}$$

On-shell amplitudes

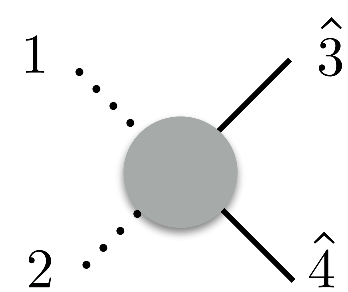
- High-energy on-shell amplitudes probe the "curvature".

SSSS 

$\sim R_{1234}\, s$

FFFF 

$\sim R_{\hat{1}\hat{2}\hat{3}\hat{4}}\, s$

SSFF 

$\sim R_{12\hat{3}\hat{4}}\, s$

Summary and Outlook

- EFT for spin-0 and spin-1/2:

$$
\begin{aligned}
\mathcal{L}_{\text{scalar}} &= \frac{1}{2} g_{ij} (\partial_\mu \phi^i)(\partial^\mu \phi^j) - V(\phi) \\
\mathcal{L}_{\text{fermion}} &= \frac{i}{2}\, g_{\hat{i}\hat{j}^*} \psi^{\dagger \hat{j}^*} \bar{\sigma}^\mu \overleftrightarrow{\partial}_\mu \psi^{\hat{i}} \\
&\quad + \frac{i}{2} (g_{k\hat{j}^*,\hat{i}} - g_{k\hat{i},\hat{j}^*}) (\psi^{\dagger j^*} \bar{\sigma}^\mu \psi^i)(\partial_\mu \phi^k) \\
&\quad - \frac{1}{2} M_{\hat{i}\hat{j}}\, (\psi^i \psi^j) + h.c. \\
&\quad - \frac{1}{4} R_{\hat{i}\hat{k}^*\hat{j}\hat{l}^*} (\psi^{\hat{i}} \psi^{\hat{j}})(\psi^{\dagger \hat{k}^*} \psi^{\dagger \hat{l}^*}) \\
&\quad - \frac{1}{12} R_{\hat{i}\hat{l}\hat{j}\hat{k}} (\psi^{\hat{i}} \psi^{\hat{j}})(\psi^{\hat{k}} \psi^{\hat{l}}) + h.c.
\end{aligned}
$$

- On-shell amplitudes are expressed by covariant quantities on the field space.

Summary and Outlook

- In our papers, we have also studied
  - Couplings to EW gauge bosons
  - CCWZ construction
  - Power counting
  - Normal coordinate on fermionic part