Some Exact Results in QCD-like and Chiral Gauge Theories

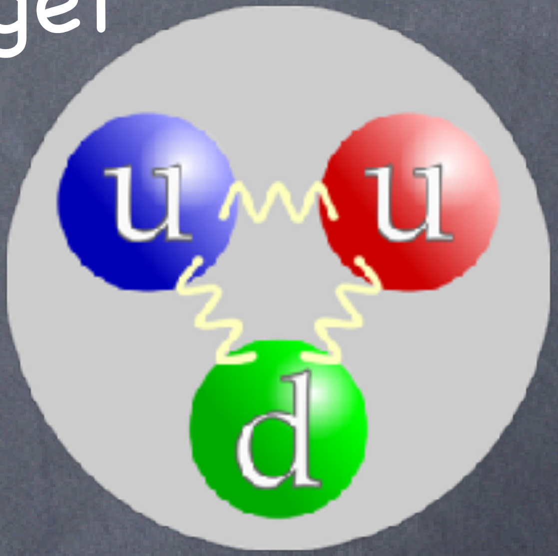
Hitoshi Murayama (Berkeley, Kavli IPMU)

Osaka University Seminar, July 13, 2021

Introduction

Can we solve QCD?

- When we first learn about quarks, we get told we can never see them
  - Internet Scam?



Dear friend,

I am Andre Ouedraogo, a banker by profession from Burkina Faso in West Africa and currently holding the post of Director Auditing and Accounting unit of the bank. It's my urgent need for a foreign partner that made me to contact you for this business. I have the opportunity of transferring the left over funds ($11.5 million) of one of my bank clients who died along with his entire family on 31 July 2000 in a plane crash. You can confirm the genuineness of the deceased death by clicking on this website.
http://news.bbc.co.uk/1/hi/world/europe/859479.stm
I need a foreign partner who will support me because i can not claim this money alone without a foreign partner since the deceased client (the owner of the fund) was a foreigner.

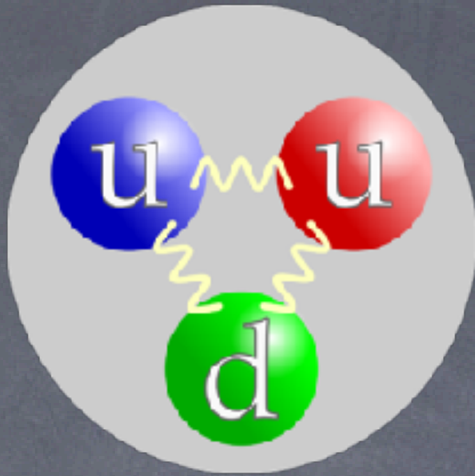
This fund ($11.5 million) will be shared between us in the ratio of 60/40. I agreed that 40% of this money will be for you as a respect to the provision of a foreign account while 60% will be for me and I want to assure you that this transaction is absolutely legal and risk free since i work in this bank and i have all the necessary information that might be needed. Before we proceed, i would like to know your ability to handle this over there in your country.
Please tell me more about the political/economic stability/monetary policy of your country. I need to know all these because i don't want to have problem with the Government of your country.

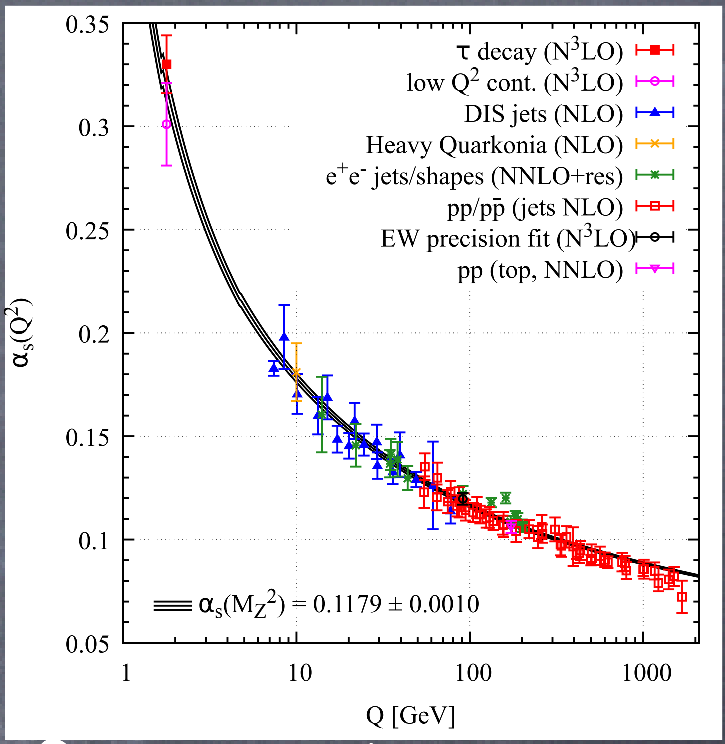
Kindly update me with the
following information because i want to know you more before we proceed on this transaction. Hope you will understand the importance of this request.

1. Your full name........................
2. Your age/sex ....
3. your occupation ....
4. Your residential address ....
5. Your nationality ....
6. Your private phone number ....
7. Your fax number ....

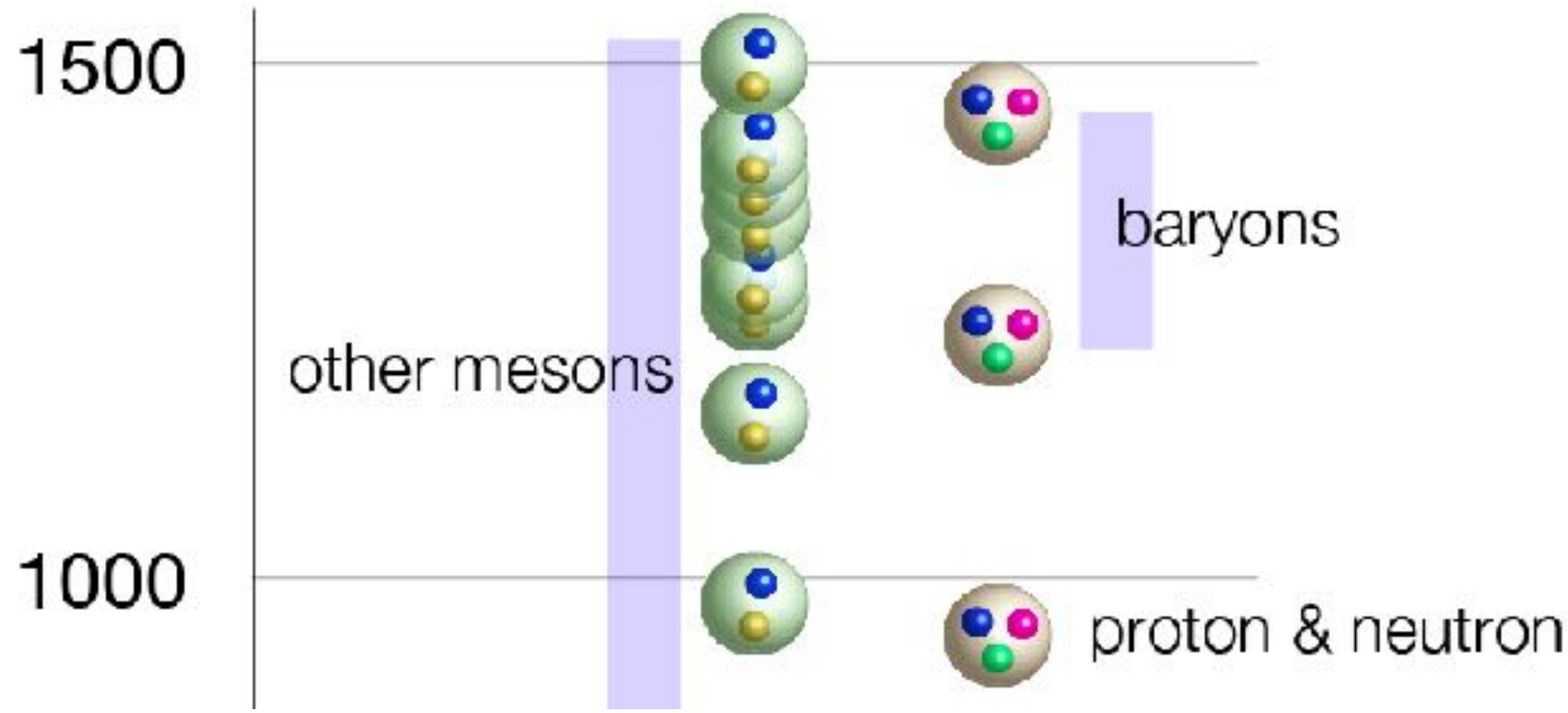
I will be waiting for your response.
Thanks for your understanding.
Have a great day.
Yours.
Andre Ouedraogo

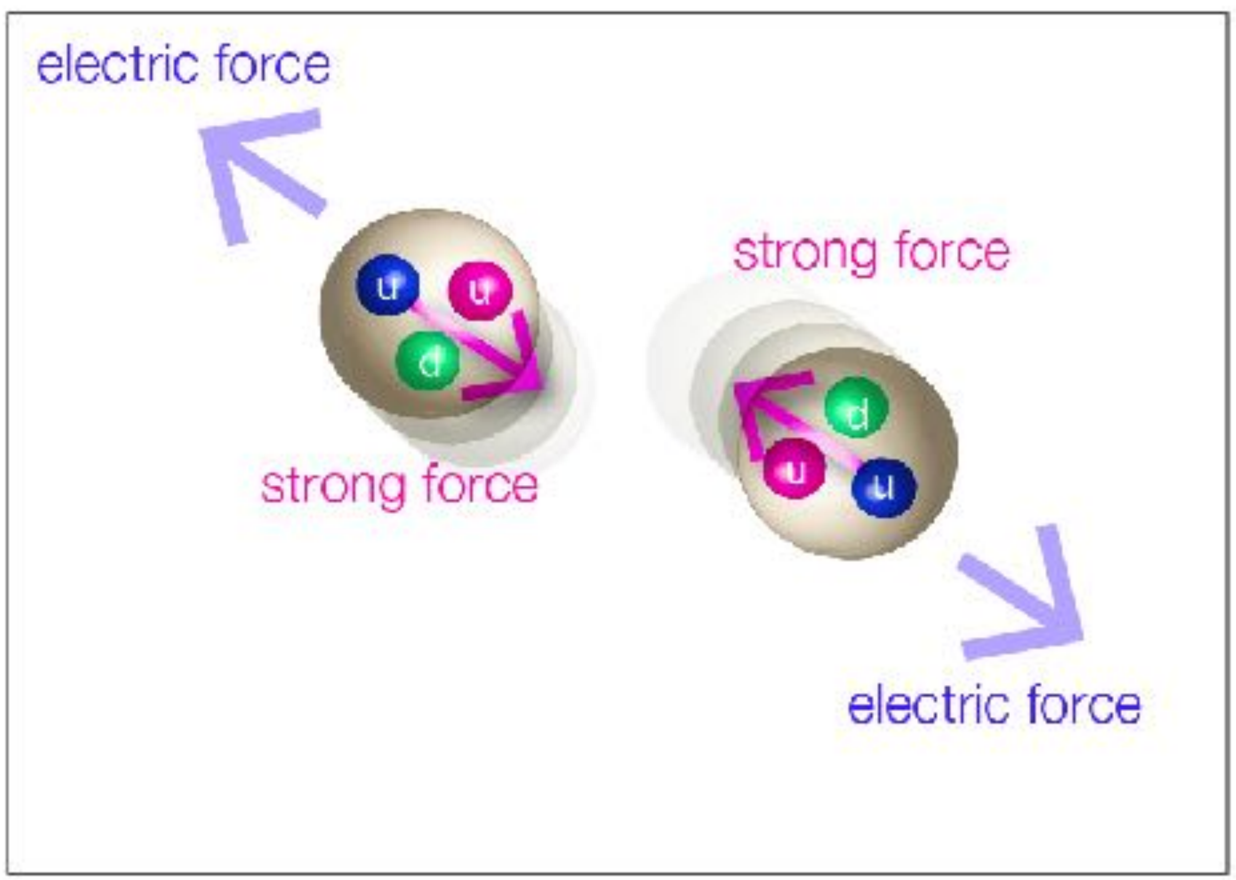
Can we solve QCD?



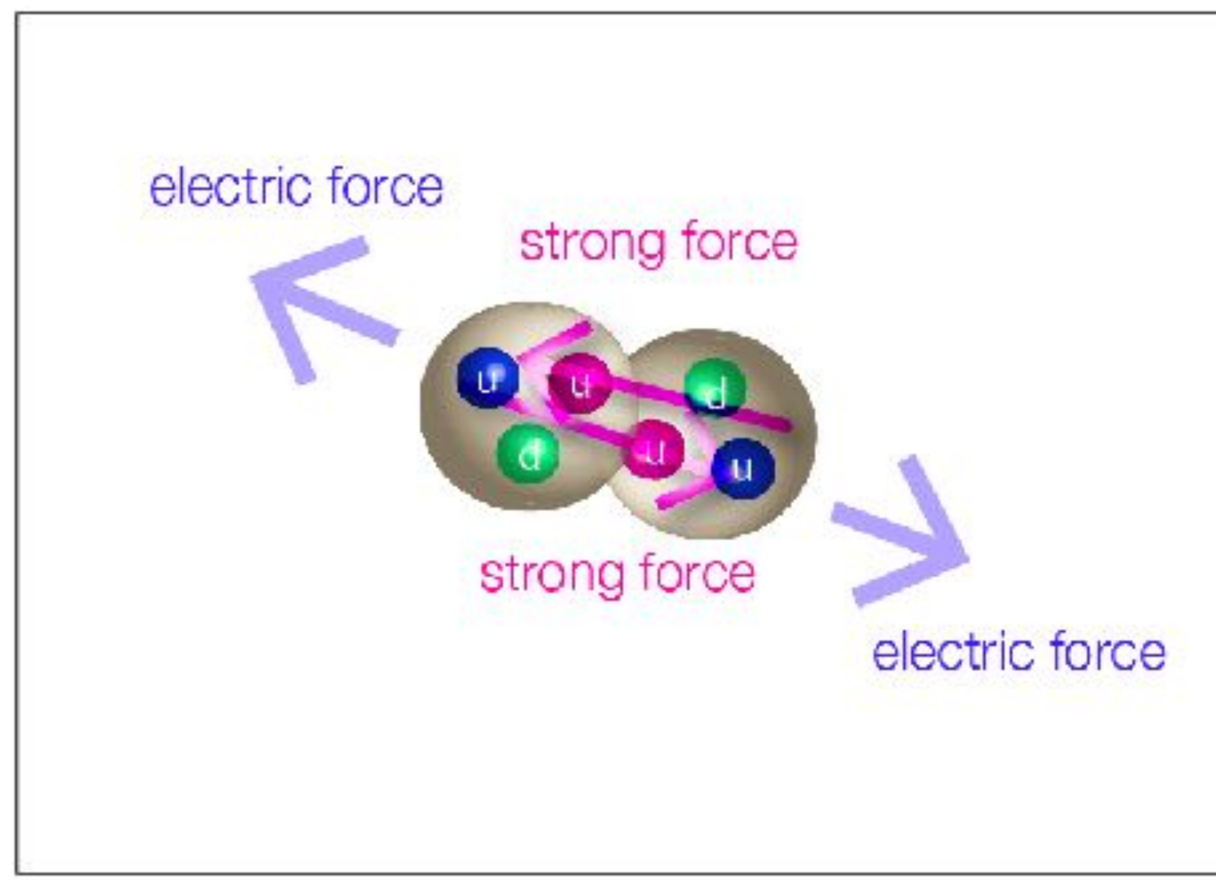


- When we first learn about quarks, we get told we can never see them
  - Internet Scam?
  - Confinement!
  - $\beta<0$ and asymptotic freedom
  - only suggestive, doesn't prove confinement
- Another puzzle: proton and pion are made of same quarks
  - why pion ≈ massless ≪ proton?
- very mysterious!





If pions are heavy



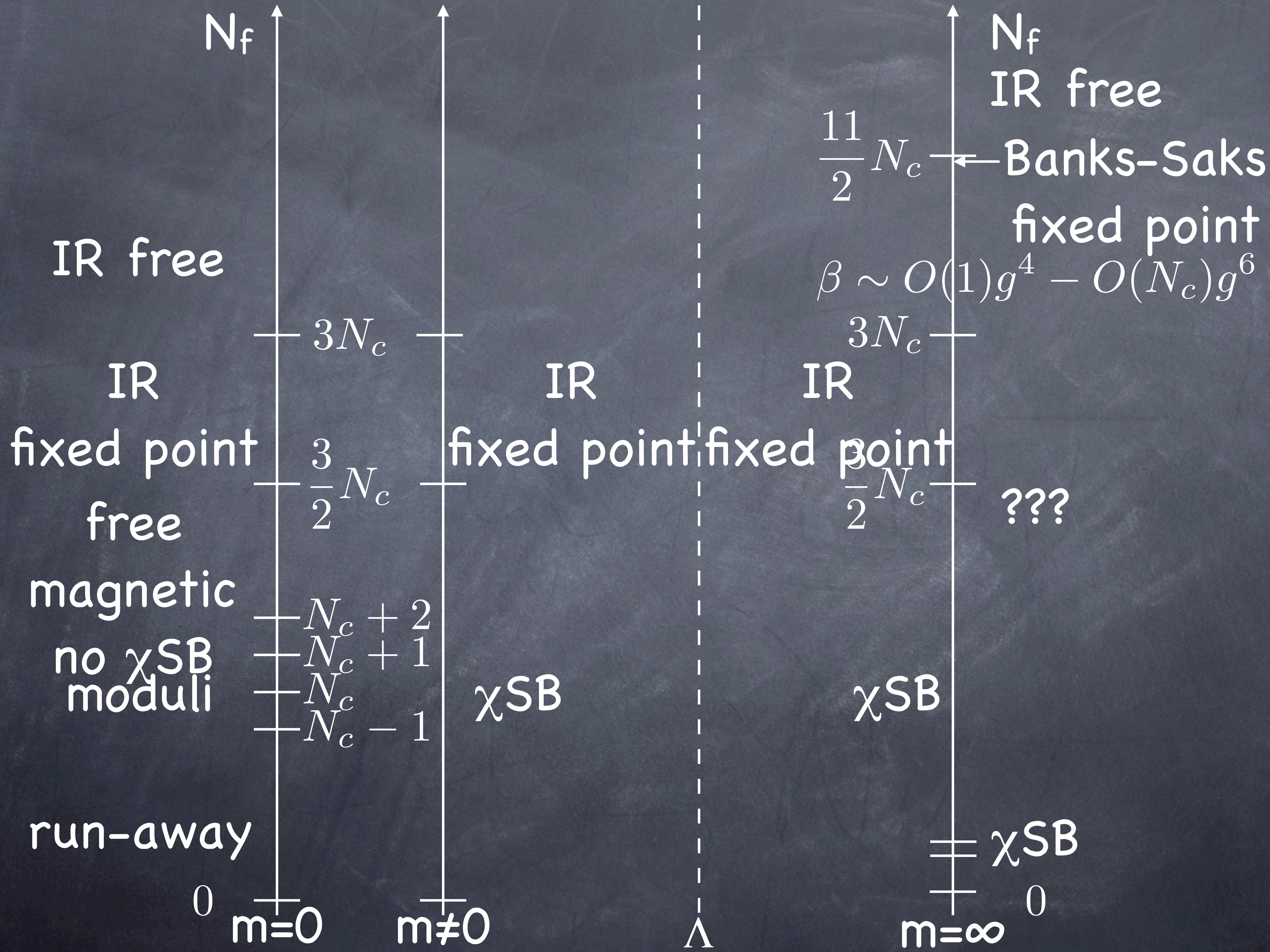
With real light-weight pions

Feeling better

- Qualitative picture makes us feel better
- Confinement
  - dual Meißner effect (Mandelstam)
  - assume monopole condensation
  - quarks confined by electric flux tube
- Chiral symmetry breaking (Nambu)
  - massless QCD invariant under $SU(N_f)_L \times SU(N_f)_R \times U(1)_B$
  - assume broken to $SU(N_f)_V \times U(1)_B$
  - pion = Nambu-Goldstone boson = massless
- but still not derived from QCD!

Feeling even better but not there yet

- Progress in understanding QCD
- Confinement (Seiberg-Witten)
  - N=2 SYM has Coulomb branch $u=\mathrm{Tr}\Phi^2$
  - singularities = massless monopole/dyon
  - N=1 perturbation $W = \mu u - (u-\Lambda^2)M^+M^-$
  - $M^+=M^-=\sqrt{\mu}\neq 0$: monopole condensation!
  - can further perturb to N=0 with $m_\lambda \neq 0$
- Chiral symmetry breaking
  - N=2 doesn't have χS $W = \sqrt{2}\tilde{Q}_i \Phi Q^i$
  - N=1 (Seiberg) has too unusual phases



Nf
IR free
IR fixed point
free magnetic
no χSB
moduli
run-away
$3N_c$
$\frac{3}{2}N_c$
N_c+2
N_c+1
N_c
N_c-1
0
m=0
m≠0
IR fixed point
χSB
Λ
IR fixed point
χSB
Nf
IR free
$\frac{11}{2}N_c$
Banks-Saks fixed point
$\beta \sim O(1)g^4 - O(N_c)g^6$
$3N_c$
$\frac{3}{2}N_c$
???
χSB
0
m=∞

Main result

- We start with N=1 QCD à la Seiberg
- Most schemes to bring N=1 to N=0 do not have sufficient predictive power
- Anomaly mediation (AMSB) is uniquely suited thanks to its UV insensitivity
- Even with infinitesimal AMSB, the theory collapses to expectations in non-SUSY QCD
- N=1 SUSY + AMSB: great tool to study non-SUSY gauge theories in 4D
- Now we can "derive" χSB from QCD!

Outline

- Introduction
- Anomaly Mediated Supersymmetry Breaking
- $SU(N_c)$ QCD
- confinement and χSB in $SO(N_c)$ QCD
- chiral $SU(N_c)$ with anti-symmetric tensor
- chiral $SU(N_c)$ with symmetric tensor
- Conclusion

Anomaly-Mediated Supersymmetry Breaking

Randall, Sundrum (1998)

Giudice, Luty, HM, Rattazzi (1998)

Our Needs

- We'd like to connect N=1 SUSY results by Seiberg to non-SUSY gauge theories
  - decouple gauginos and squarks!
  - SUSY breaking m_λ and $m_{\tilde{Q}}$
- But we need to deal with composites such as mesons and baryons
- In particular, we need to know signs of their mass-squared to understand symmetry breaking patterns and universality classes

Gaugino Mass Perturbation

- Series of papers by N. Evans, S. Hsu, M. Schwetz + S. Selipsky (1995)
- Gaugino mass:

$$\int d^2\theta \mathcal{S} W_\alpha W^\alpha \qquad \mathcal{S} = \frac{8\pi^2}{g^2} + i\vartheta + \theta^2 \frac{16\pi^2}{g^2} m_\lambda$$

- non-perturbative superpotential is a function of $\Lambda^{3N_c - N_f} = \mu e^{-\mathcal{S}} = \mu e^{-8\pi^2/g^2} e^{-i\vartheta} \left(1 - \theta^2 \frac{16\pi^2}{g^2} m_\lambda\right)$
- but Kähler potential arbitrary function of S

$$M^{ij} = \tilde{Q}^i Q^j \qquad \int d^4\theta \mathcal{S}^* \mathcal{S} M^*_{ij} M^{ij}$$

- we don't even know the sign of meson mass^2
- avoid problem with large enough quark mass
- can't discuss chiral symmetry!

$$W = m_Q \tilde{Q} Q$$
$$m_Q^2 > m_\lambda^2$$

external symmetry

- external symmetries constrain how symmetry breaking parameters enter Lagrangian
- e.g. quark mass in chiral Lagrangian

$$\mathcal{L}_{\rm QCD} = -\frac{1}{2}{\rm Tr}G_{\mu\nu}G^{\mu\nu} + \overline{Q_L}i\not{D}Q_L + \overline{Q_R}i\not{D}Q_R - \overline{Q_L}MQ_R - \overline{Q_R}M^\dagger Q_L$$

- quark mass M "transforms" as (N_f,N_f^*) under $SU(N_f)_L \times SU(N_f)_R$
- Chiral Lagrangian a function of $U = \frac{1}{f_\pi \mu^2} Q_L \overline{Q_R}$

$$\mathcal{L}_\chi = \frac{f_\pi^2}{4}{\rm Tr}\partial_\mu U^\dagger \partial^\mu U + f_\pi \mu^2 {\rm Tr} M^\dagger U + c.c.$$

- switch from UV to IR description, but we know how symmetry breaking works

background spacetime

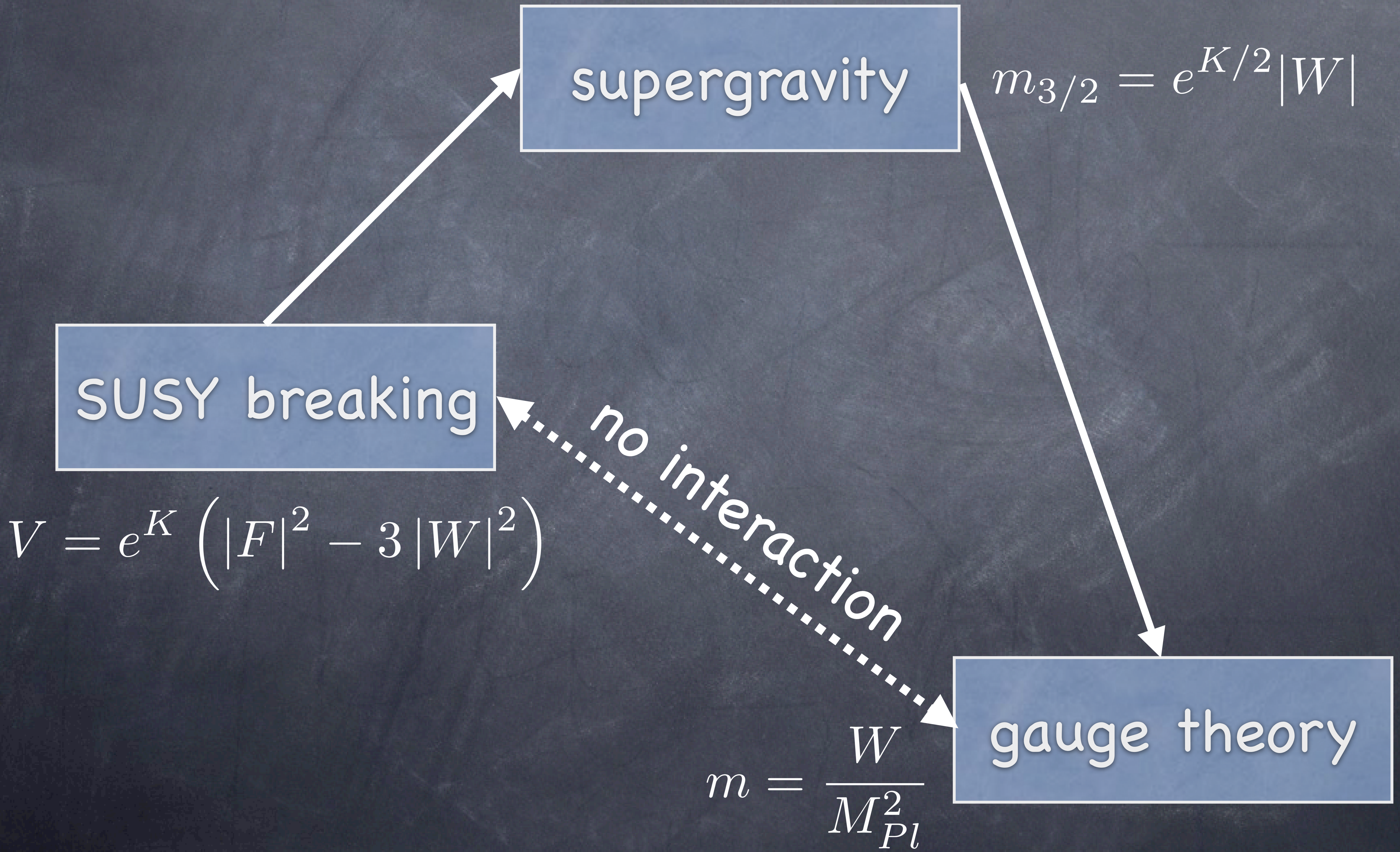
- Good example of external symmetry is general coordinate invariance
- consider QCD in a background metric

$$\mathcal{L}_{\rm QCD} = \sqrt{-g}\left(-\frac{1}{2}g^{\mu\rho}g^{\nu\sigma}{\rm Tr}G_{\mu\nu}G_{\rho\sigma} + \bar{Q}e^{\mu}_{a}\gamma^{a}iD_{\mu}Q\right)$$

- switch from UV to IR description
- we know how chiral Lagrangian couples to the background spacetime!

$$\mathcal{L}_{\chi} = \sqrt{-g}\left(\frac{f_\pi^2}{4}g^{\mu\nu}{\rm Tr}\partial_\mu U^\dagger\partial_\nu U + f_\pi\mu^2{\rm Tr}M^\dagger U + c.c.\right)$$

Sequestering



Weyl compensator

- "superspacetime background"

$$\int d^4\theta \mathcal{E}^* \mathcal{E} K - \int d^2\theta \mathcal{E}^3 W$$

$$\mathcal{E} = 1 + \theta^2 m$$

- all effects of SUSY breaking encoded in $\mathcal{E}$
- it can be removed by conformal transformation $\phi \to \phi/\mathcal{E}$ if no mass params

$$\mathcal{L} = \int d^4\theta \mathcal{E}^* \mathcal{E} \phi^* \phi - \int d^2\theta \mathcal{E}^3 \lambda \phi^3$$

$$\to \mathcal{L} = \int d^4\theta \phi^* \phi - \int d^2\theta \lambda \phi^3$$

Weyl compensator

- "superspacetime background"

$$\int d^4\theta \mathcal{E}^* \mathcal{E} K - \int d^2\theta \mathcal{E}^3 W$$

$$\mathcal{E} = 1 + \theta^2 m \qquad \phi \to \phi/\mathcal{E}$$

- dimensionful parameters receive SUSY breaking

$$\mathcal{L} = \int d^4\theta \mathcal{E}^* \mathcal{E} \phi^* \phi - \int d^2\theta \mathcal{E}^3 \left(\frac{1}{2} M \phi^2 + \lambda \phi^3 \right)$$

$$\to \mathcal{L} = \int d^4\theta \phi^* \phi - \int d^2\theta \left(\frac{1}{2} M \mathcal{E} \phi^2 + \lambda \phi^3 \right)$$

$$V_{\mathrm{AMSB}} = -m \left(\phi \frac{\partial W}{\partial \phi} - 3W \right)$$

Superconformal Anomaly

- Cutoff scale also acquires SUSY breaking

$$Z\left(\frac{\mu}{M}\right) \to \mathcal{Z}\left(\frac{\mu}{M\mathcal{E}}\right) = Z\left(1 + \gamma\frac{1}{2}\ln\frac{\mu^2}{M^*\mathcal{E}^*M\mathcal{E}} + \frac{1}{2}\dot{\gamma}\frac{1}{4}\ln^2\frac{\mu^2}{M^*\mathcal{E}^*M\mathcal{E}} + \cdots\right)$$

$$\int d^4\theta \mathcal{Z}\phi^*\phi = Z\left(F^*F + \gamma\frac{1}{2}(m^*\phi^*F + m\phi F^*) + \frac{1}{2}\dot{\gamma}m^*m\phi^*\phi\right)$$

$$V = -\frac{1}{4}\dot{\gamma}_i m^* m\phi_i^*\phi_i - \frac{1}{2}(\gamma_i + \gamma_j + \gamma_k)m\lambda_{ijk}\phi_i\phi_j\phi_k + c.c.$$

- determined only by physics at the energy scale of interest
- UV insensitivity!

Superconformal Anomaly

- Cutoff scale also acquires SUSY breaking

$$\frac{1}{g^2}\left(\frac{\mu}{M}\right) \to \frac{1}{g^2}\left(\frac{\mu}{M\mathcal{E}}\right) = \frac{1}{g^2} - \theta^2 \frac{\beta(g^2)}{g^4} m$$

$$\int d^2\theta \frac{1}{g^2}\left(\frac{\mu}{M\mathcal{E}}\right) W_\alpha W^\alpha \supset -\frac{\beta(g^2)}{4g^4} m\lambda\lambda$$

$$m_\lambda = -\frac{\beta(g^2)}{2g^2} m$$

- determined only by physics at the energy scale of interest
- UV insensitivity!

AMSB Summary

- Tree-level piece on dimensionful parameters

$$V_{\rm AMSB} = -m\left(\phi\frac{\partial W}{\partial \phi} - 3W\right)$$

- loop-level piece from running

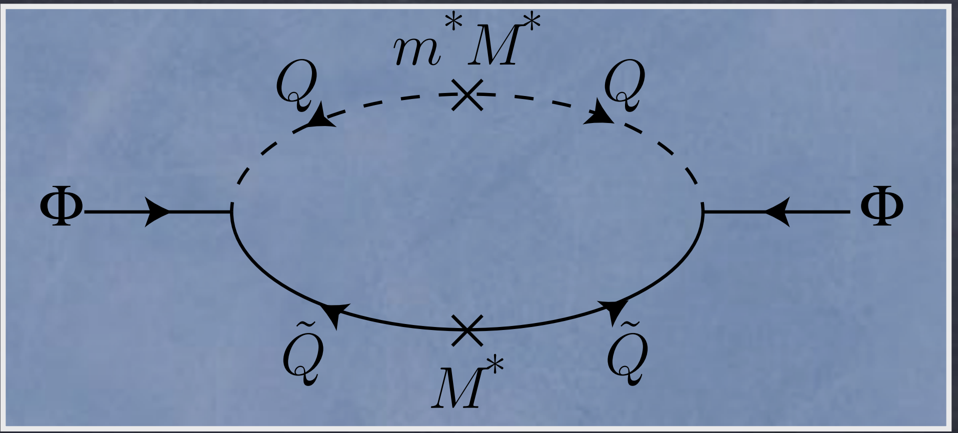
$$M_i = -\frac{\beta_i(g^2)}{2g_i^2}m_{3/2}, \quad m_i^2 = -\frac{\dot{\gamma}_i}{4}m_{3/2}^2, \quad A_{ijk} = -\frac{1}{2}(\gamma_i + \gamma_j + \gamma_k)m_{3/2}$$

- determined only by physics at the energy scale of interest
- UV insensitivity!

UV insensitivity

$$M_i = -\frac{\beta_i(g^2)}{2g_i^2} m_{3/2}, \quad m_i^2 = -\frac{\dot{\gamma}_i}{4} m_{3/2}^2, \quad A_{ijk} = -\frac{1}{2}(\gamma_i + \gamma_j + \gamma_k) m_{3/2}$$

- Surprising result: AMSB depends only on physics at the energy scale of interest
- No matter how complicated the UV physics is, they all disappear from low-energy soft SUSY breaking
- e.g., decouple a massive matter field:
  - Changes the beta function
  - one-loop threshold correction precisely account for the change in gaugino mass



UV insensitivity cont.

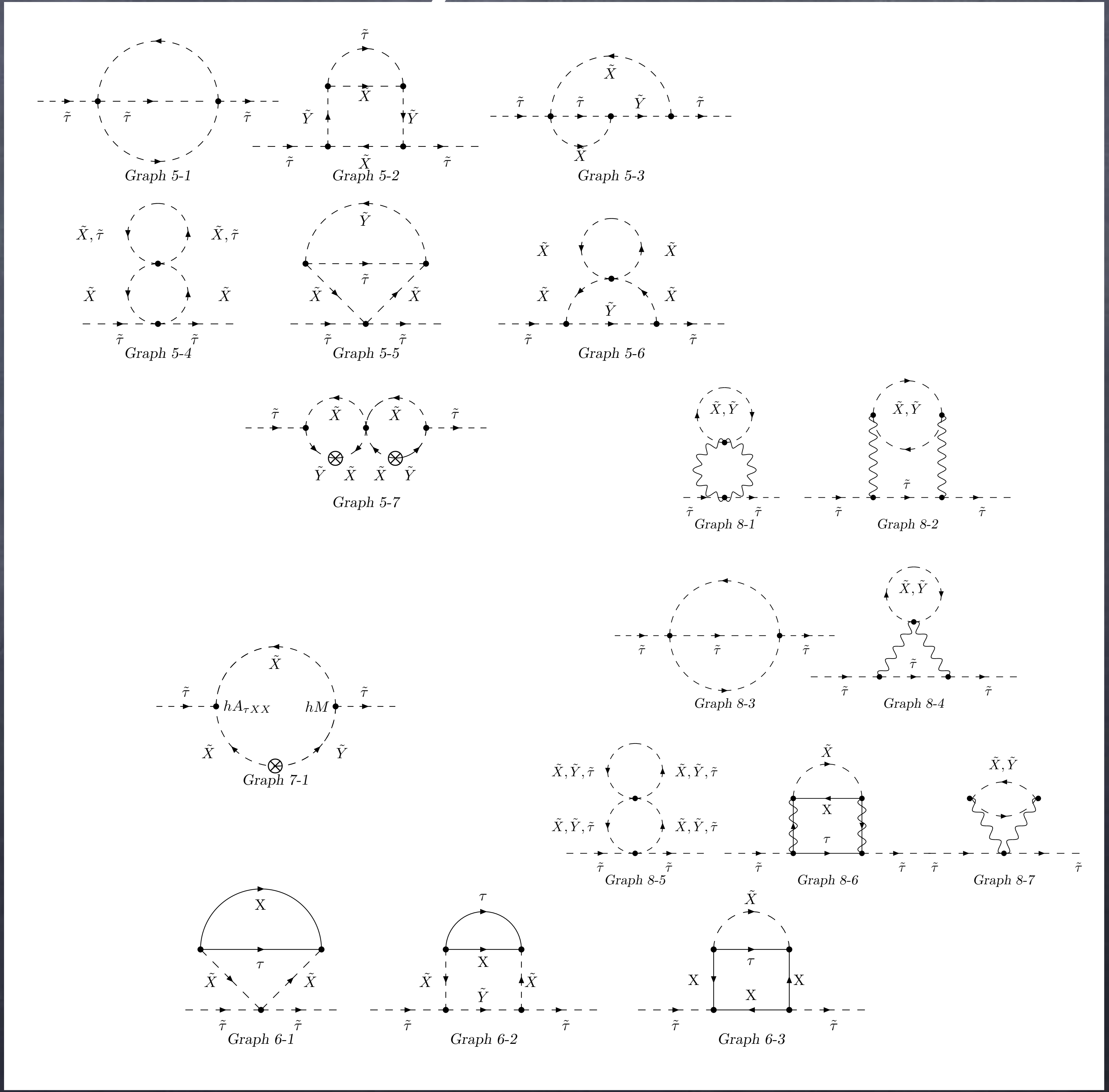
- decouple a massive matter field
- two-loop threshold correction precisely account for the change in the anomalous dimension and hence the scalar mass

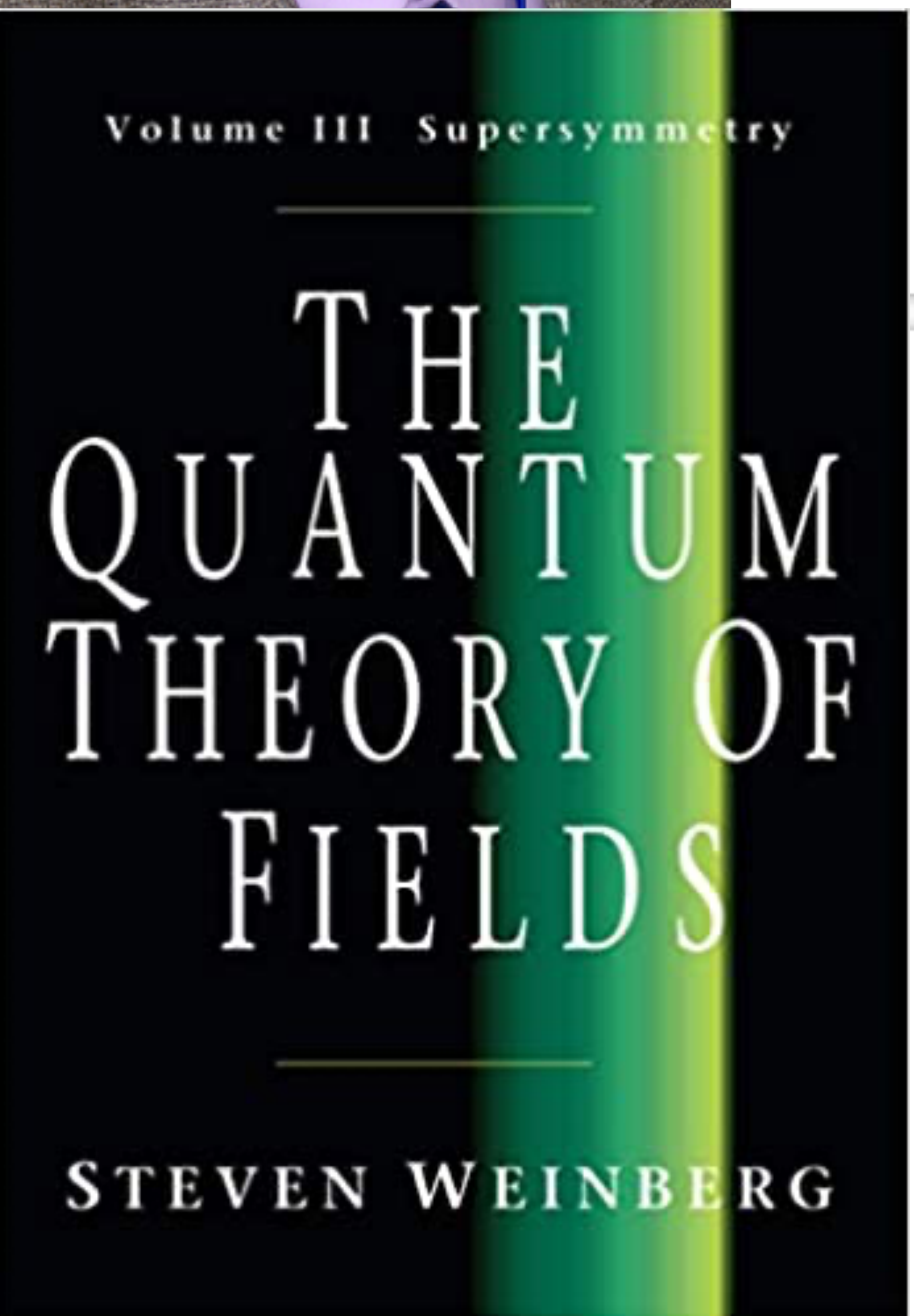
$$M_i = -\frac{\beta_i(g^2)}{2g_i^2} m_{3/2}$$

$$m_i^2 = -\frac{\dot{\gamma}_i}{4} m_{3/2}^2,$$

$$A_{ijk} = -\frac{1}{2}(\gamma_i + \gamma_j + \gamma_k) m_{3/2}$$

Boyda, HM, Pierce 2001





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Riley for finding countless books and articles, and to Jan Duffy for many helps. I am grateful to Maureen Storey of Cambridge University Press for working to ready this book for publication, and especially to my editor, Rufus Neal, for his continued friendly good advice.

STEVEN WEINBERG

Austin, Texas
May, 1999

It is sometimes convenient to introduce a complex gravitino mass, defined as

$$\tilde{m}_g \equiv \frac{2\kappa}{3}\Big(\langle s\rangle + i\langle p\rangle\Big)\,, \tag{31.3.20}$$

whose absolute magnitude is the physical gravitino mass (31.3.19).

31.4 Anomaly-Mediated Supersymmetry Breaking

In Section 28.3 the possibility was raised that supersymmetry may be broken in some sort of hidden sector of superfields that do not carry the $SU(3) \times SU(2) \times U(1)$ quantum numbers of the standard model, and communicated to observable particles gravitationally. In this section we will deal with one class of supersymmetry-breaking effects in the minimum supersymmetric standard model, those of first order in $\kappa \equiv \sqrt{8\pi G}$. This includes the gaugino masses and the parameters A_{ij} and B in the Lagrangian density (28.4.1). Other supersymmetry-breaking effects such as squark and slepton squared masses are of second order in κ, and will be taken up in Section 31.7, when we consider gravity-mediated supersymmetry breaking using the general supergravity formalism described in Section 31.6.

We can find the effects of gravity-mediated supersymmetry breaking to first order in κ by simply replacing the component fields of the gravitational supermultiplet in the interaction (31.1.34) with their expectation values. The only ones of these component fields that can acquire non-vanishing vacuum expectation values from the spontaneous breakdown

SU(N_c) QCD ($N_c \geq 3$)

HM, 2104.01179

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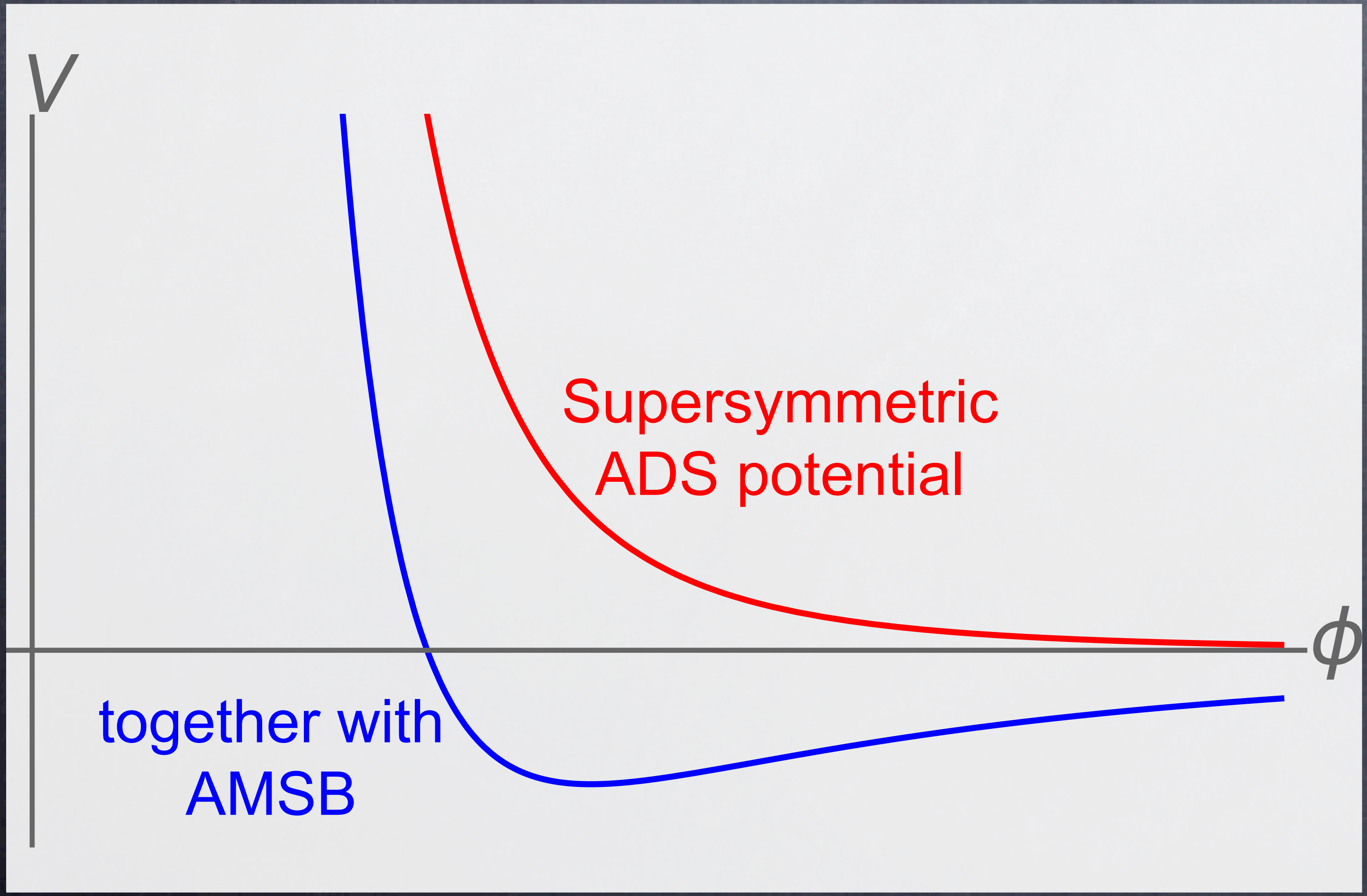
$N_f < N_c$

- run-away superpotential for $M^{ij} = \tilde{Q}^i Q^j$

$$W = (N_c - N_f)\left(\frac{\Lambda^{3N_c-N_f}}{\det M}\right)^{1/(N_c-N_f)} \qquad M^{ij} = \delta^{ij}\phi^2$$

$$V = \left|2N_f\frac{1}{\phi}\left(\frac{\Lambda^{3N_c-N_f}}{\phi^{2N_f}}\right)^{1/(N_c-N_f)}\right|^2 - (3N_c - N_f)m\left(\frac{\Lambda^{3N_c-N_f}}{\phi^{2N_f}}\right)^{1/(N_c-N_f)} + c.c.$$



$$M_{ij} = \Lambda^2\left(\frac{4N_f(N_c+N_f)}{3N_c-N_f}\frac{\Lambda}{m}\right)^{(N_c-N_f)/N_c}\delta_{ij}$$

$SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$

χSB! Proving Nambu

mesino loop→WZW term

$N_f=1$ special

no NGB, gapped

$N_c+2 \leq N_f \leq 3N_c/2$

- "free magnetic phase"
- $SU(N_c)$ becomes strong, binds baryons, which break up into dual quarks

$$B^{i_1,\cdots i_{N_c}} = Q^{i_1}_{\alpha_1}\cdots Q^{i_{N_c}}_{\alpha_{N_c}}\epsilon^{\alpha_1\cdots\alpha_{N_c}} = \epsilon^{i_1\cdots i_{N_c} i_{N_c+1}\cdots i_{N_f}} b_{i_{N_c+1}\cdots i_{N_f}}$$

$$b_{i_{N_c+1}\cdots i_{N_f}} = q^{\beta_1}_{i_{N_c+1}}\cdots q^{\beta_{N_f-N_c}}_{i_{N_f}}\epsilon_{\beta_1\cdots\beta_{N_f-N_c}}$$

- "magnetic" IR-free $SU(N_f-N_c)$ gauge theory

$$W = \frac{1}{\mu}M^{ij}q_i\tilde{q}_j \to \lambda\tilde{M}^{ij}q_i\tilde{q}_j$$

- λ dimensionless, only loop-level AMSB

$N_c+2 \leq N_f \leq 3N_c/2$

- "magnetic" IR-free $SU(N_f-N_c)$ gauge theory

$$W = \frac{1}{\mu} M^{ij} q_i \tilde{q}_j \to \lambda \tilde{M}^{ij} q_i \tilde{q}_j$$

- λ dimensionless, only loop-level AMSB

$$V = \lambda^2 \left|q_i \tilde{q}_j\right|^2 + \lambda^2 \left|M^{ij} q_i\right|^2 + \lambda^2 \left|M^{ij} \tilde{q}_j\right|^2$$

$$+ m_M^2 \left|M^{ij}\right|^2 + m_q^2 \left(\left|q_i\right|^2 + \left|\tilde{q}_j\right|^2\right) + A\lambda M^{ij} q_i \tilde{q}_j + c.c.$$

$$M = V \left(\begin{array}{c|c} 1_{(N_f-N_c)\times(N_f-N_c)} & 0_{(N_f-N_c)\times N_c} \\ \hline 0_{N_c\times(N_f-N_c)} & 0_{N_c\times N_c} \end{array} \right)$$

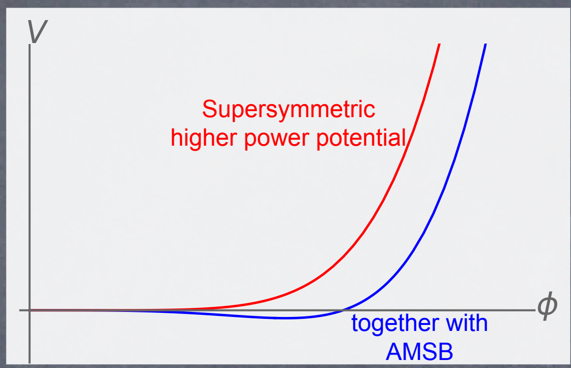
$$q = \tilde{q} = v \left(\begin{array}{c} 1_{(N_f-N_c)\times(N_f-N_c)} \\ \hline 0_{N_c\times(N_f-N_c)} \end{array} \right)$$

- breaks $SU(N_f)_L \times SU(N_f)_R \times U(1)_B$ to $SU(N_f-N_c) \times SU(N_c)_L \times SU(N_c)_R$?
- vacuum energy loop-suppressed: $V \approx -\left(\frac{\lambda^2}{16\pi^2}\right)^4 m^4$

$N_c+2 \le N_f \le 3N_c/2$

- "magnetic" IR-free $SU(N_f-N_c)$ gauge theory

$$W = \frac{1}{\mu} M^{ij} q_i \tilde{q}_j \to \lambda \tilde{M}^{ij} q_i \tilde{q}_j$$



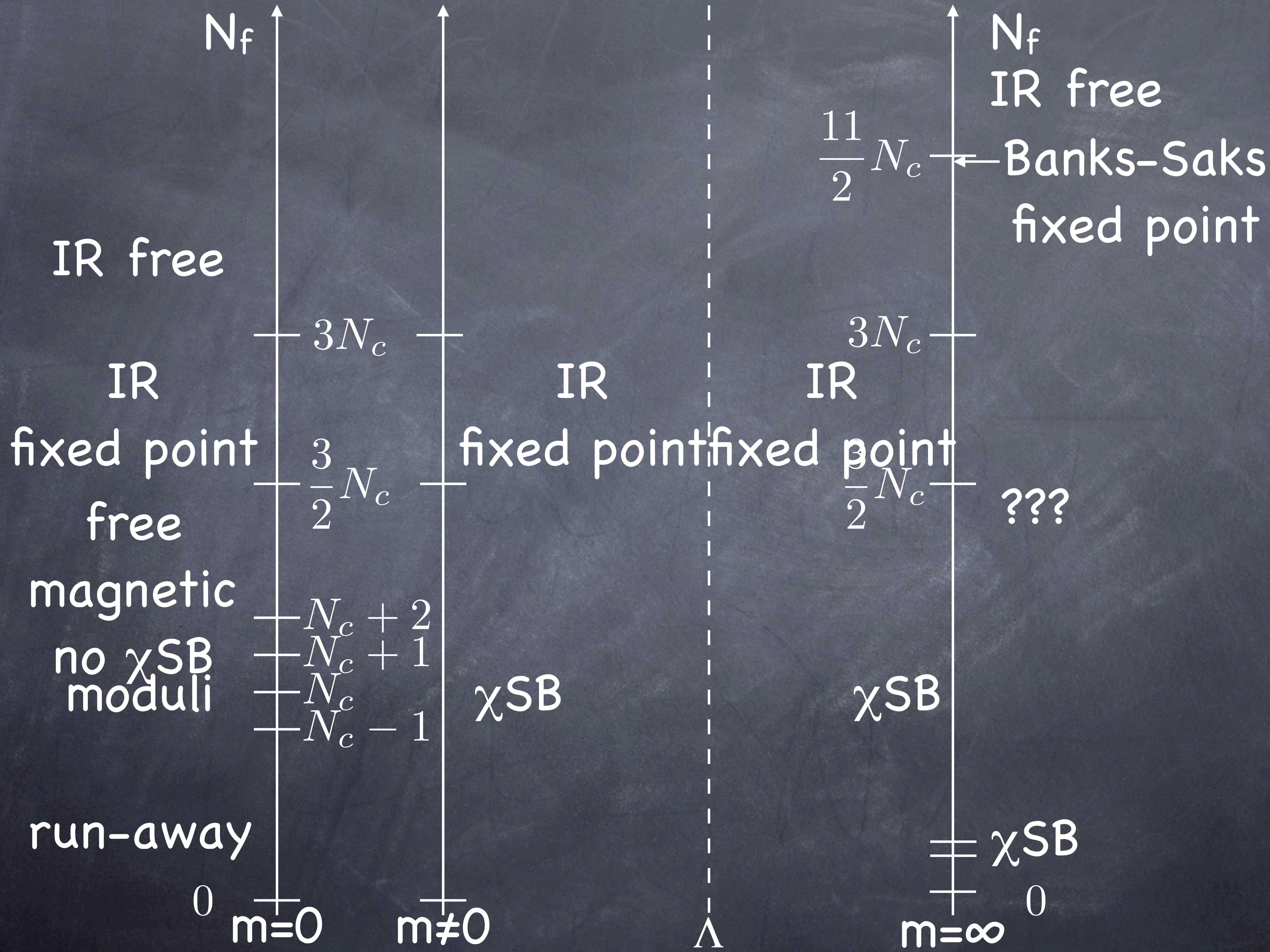
- much lower global minimum
- go along the meson direction with rank $M=N_f$
- integrate out dual quarks with $M^{ij} = \phi \delta^{ij}$
- pure $SU(N_f-N_c)$ YM forms gaugino condensate

$$W = (N_f - N_c) \left(\frac{\kappa^{N_f} \det M}{\Lambda^{3N_c - 2N_f}} \right)^{1/(N_f - N_c)}$$

$$V = N_f \Lambda^4 \left| \frac{\kappa \phi}{\Lambda} \right|^{2N_c/(N_f - N_c)} - (2N_f - 3N_c) m \Lambda^3 \left(\frac{\kappa \phi}{\Lambda} \right)^{N_f/(N_f - N_c)} + c.c.$$

$$\phi = \kappa^{-1} \Lambda \left(\frac{2N_f - 3N_c}{N_c} \frac{m}{\Lambda} \right)^{(N_f - N_c)/(2N_c - N_f)} \ll \Lambda$$

- $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$ $V \approx -\Lambda^4 \left(\frac{m}{\Lambda} \right)^{2N_c/(2N_c - N_f)}$



N_f
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IR free
$\frac{11}{2}N_c$
Banks-Saks fixed point
$3N_c$
$\frac{3}{2}N_c$
???
χSB
0
m=∞

$3N_c/2 < N_f < 3N_c$

- theories flow to IR fixed point
- d=3R/2

$$Z_Q(\mu) = Z_{\tilde{Q}}(\mu) = \left(\frac{\mu}{\mu'}\right)^{(3N_c - N_f)/N_f} Z_{Q,\tilde{Q}}(\mu'),$$

$$Z_M(\mu) = \left(\frac{\mu}{\mu'}\right)^{(6N_c - 4N_f)/N_f} Z_M(\mu'),$$

$$Z_q(\mu) = Z_{\tilde{q}}(\mu) = \left(\frac{\mu}{\mu'}\right)^{(2N_f - 3N_c)/N_f} Z_{q,\tilde{q}}(\mu')$$

- NSVZ exact beta function vanishes $\beta = \beta'_*(g^2 - g_*^2)$
- loop-level AMSB power suppressed

$$m_\lambda(\mu) = -m\beta'_* \frac{g^2(\mu') - g_*^2}{2g^2(\mu)} \left(\frac{\mu}{\mu'}\right)^{\beta'_*}$$

- IR fixed point with "emergent SUSY"=SCFT

$3N_c/2 < N_f < 3N_c$ (revised)

- loop-level AMSB power suppressed $\beta = \beta'_*(g^2 - g_*^2)$

$$m_\lambda(\mu) = -m\beta'_* \frac{g^2(\mu') - g_*^2}{2g^2(\mu)} \left(\frac{\mu}{\mu'}\right)^{\beta'_*}$$

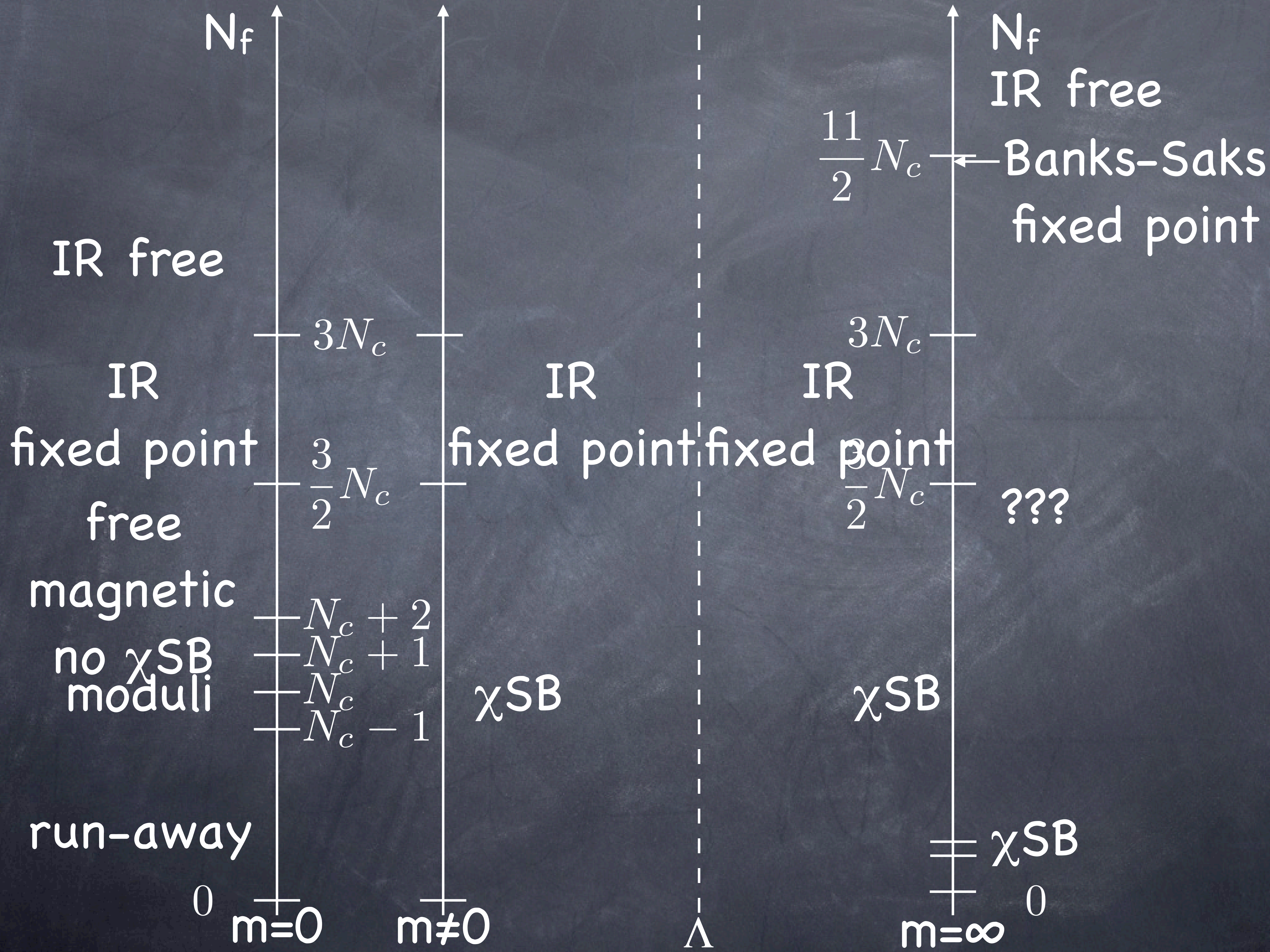
- IR fixed point with "emergent SUSY"=SCFT
  - if SUSY breaking operators are irrelevant

$$\frac{m_\lambda(\mu)}{\mu} \to 0 \qquad \frac{m_Q^2(\mu)}{\mu^2} \to 0$$

- I don't know how to compute $\beta'(g_*)$ non-perturbatively
- based on Banks-Zaks fixed point analysis in the large N_c limit, it appears
  - $1.5N_c<N_f<1.62N_c$: chiral symmetry breaking
  - $1.62N_c<N_f<1.65N_c$: SCFT
  - $1.65N_c<N_f<3N_c$: CFT

Friedland, de Gouvêa, HM
hep-th/9810020

HM + Bea Noether
Digvijay Roy Varier
in preparation

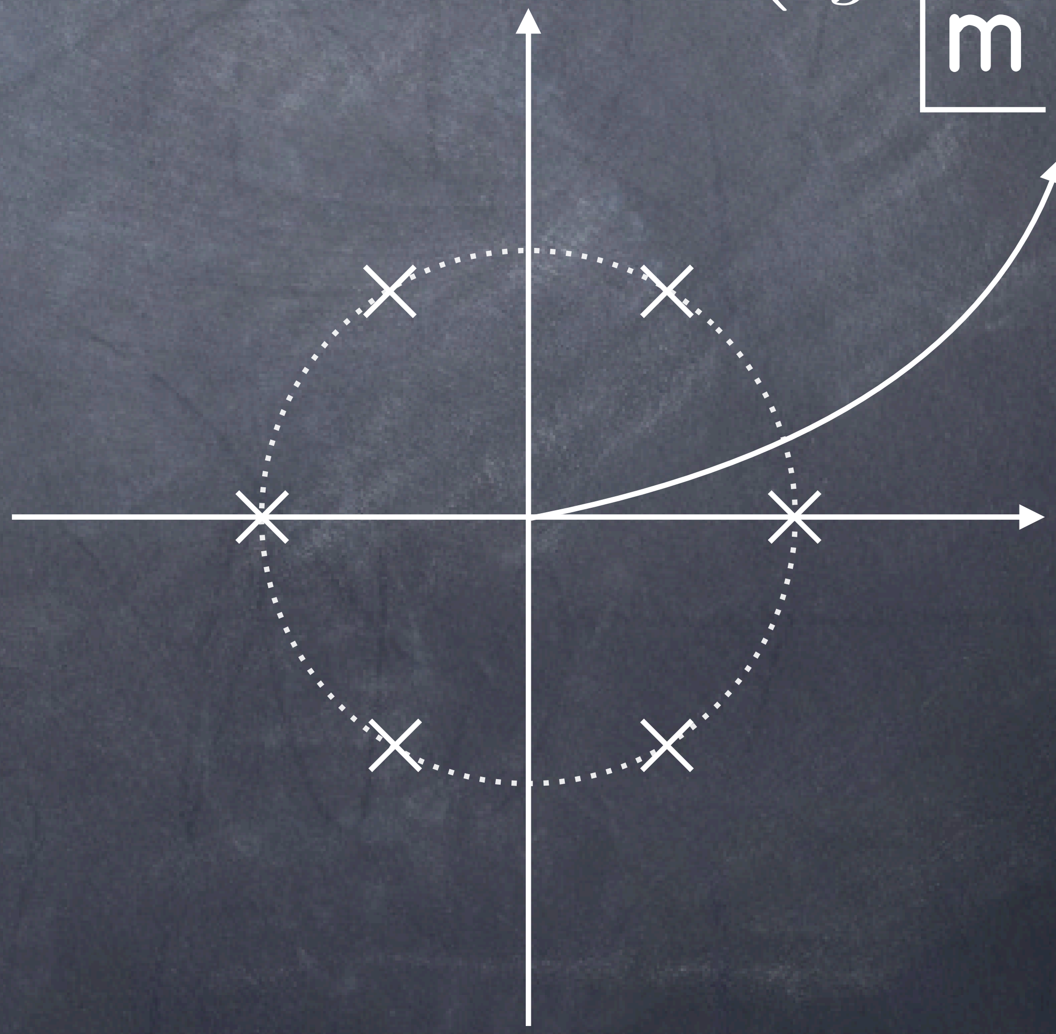


N_f
IR free
IR fixed point
free magnetic
no χSB
moduli
run-away
$3N_c$
$\frac{3}{2}N_c$
N_c+2
N_c+1
N_c
N_c-1
0
m=0
m≠0
IR fixed point
χSB
Λ
IR fixed point
χSB
N_f
IR free
$\frac{11}{2}N_c$
Banks-Saks fixed point
$3N_c$
$\frac{3}{2}N_c$
???
χSB
0
m=∞

phase transition?

- one may worry about phase transition when m≈Λ
- but they are both holomorphic quantities
- can't compare |m| and |Λ|
- On the complex plane of m, Λ are isolated points
- no phase transition, continuously connected
- "holomorphy argument"

$$m = \frac{W}{M_{Pl}^2}, \quad \Lambda^{b_0} = \mu \exp - \left(\frac{8\pi^2}{g^2} + i\vartheta \right)$$

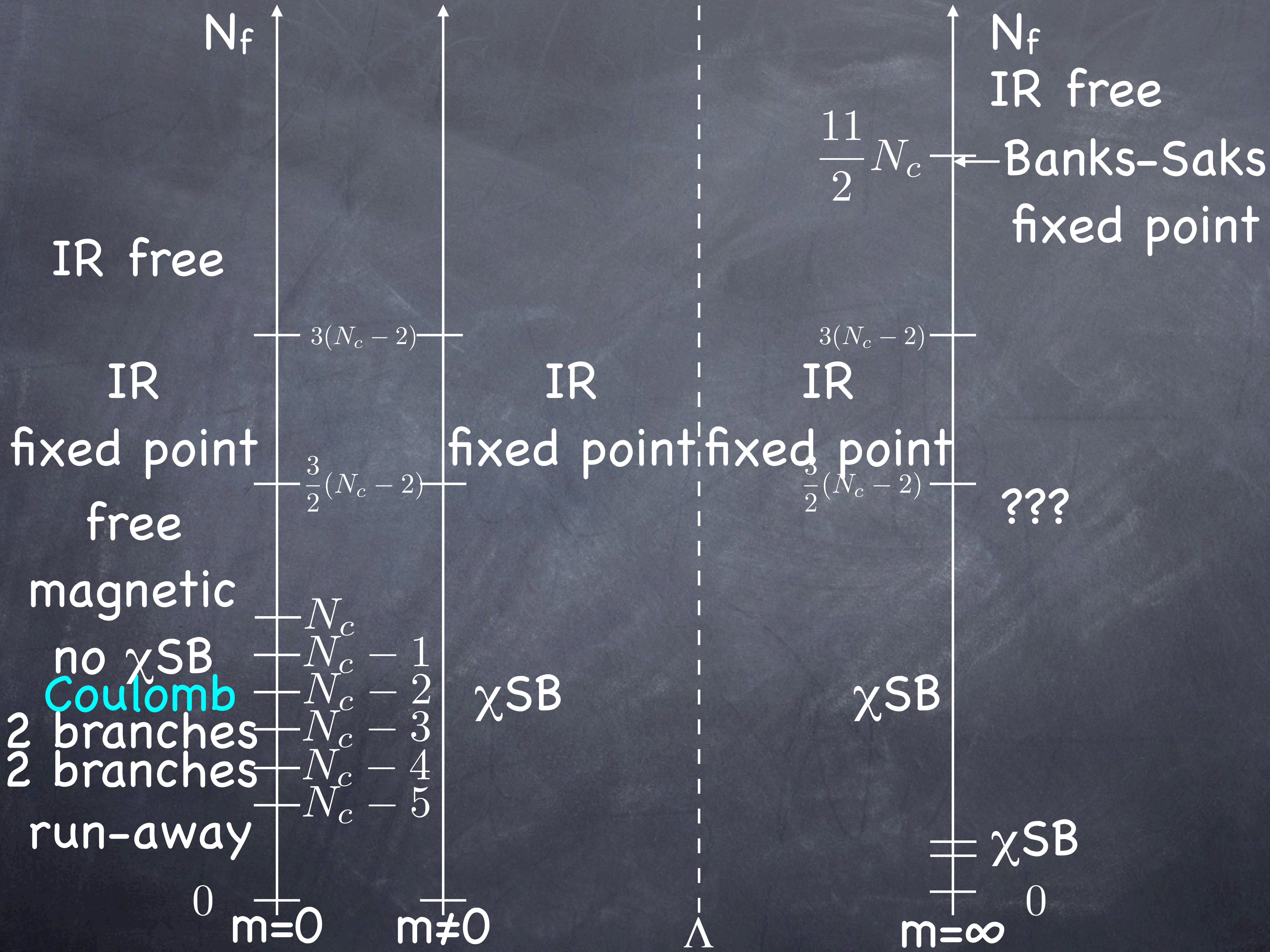


confinement and χSB

Csáki, Gomes, HM, Telem, 2106.10288 + more

confinement vs screening

- We've derived χSB in SU(N_c) QCD
  - it has no confinement
  - massless quarks in the fundamental rep can screen any color charges
  - Wilson loop is perimeter law
- SO(N_c) QCD with quarks in vector rep
  - cannot screen Z_2 center (e.g. spinor rep)
  - rigorous definition of confinement
  - can we see an interplay with χSB?



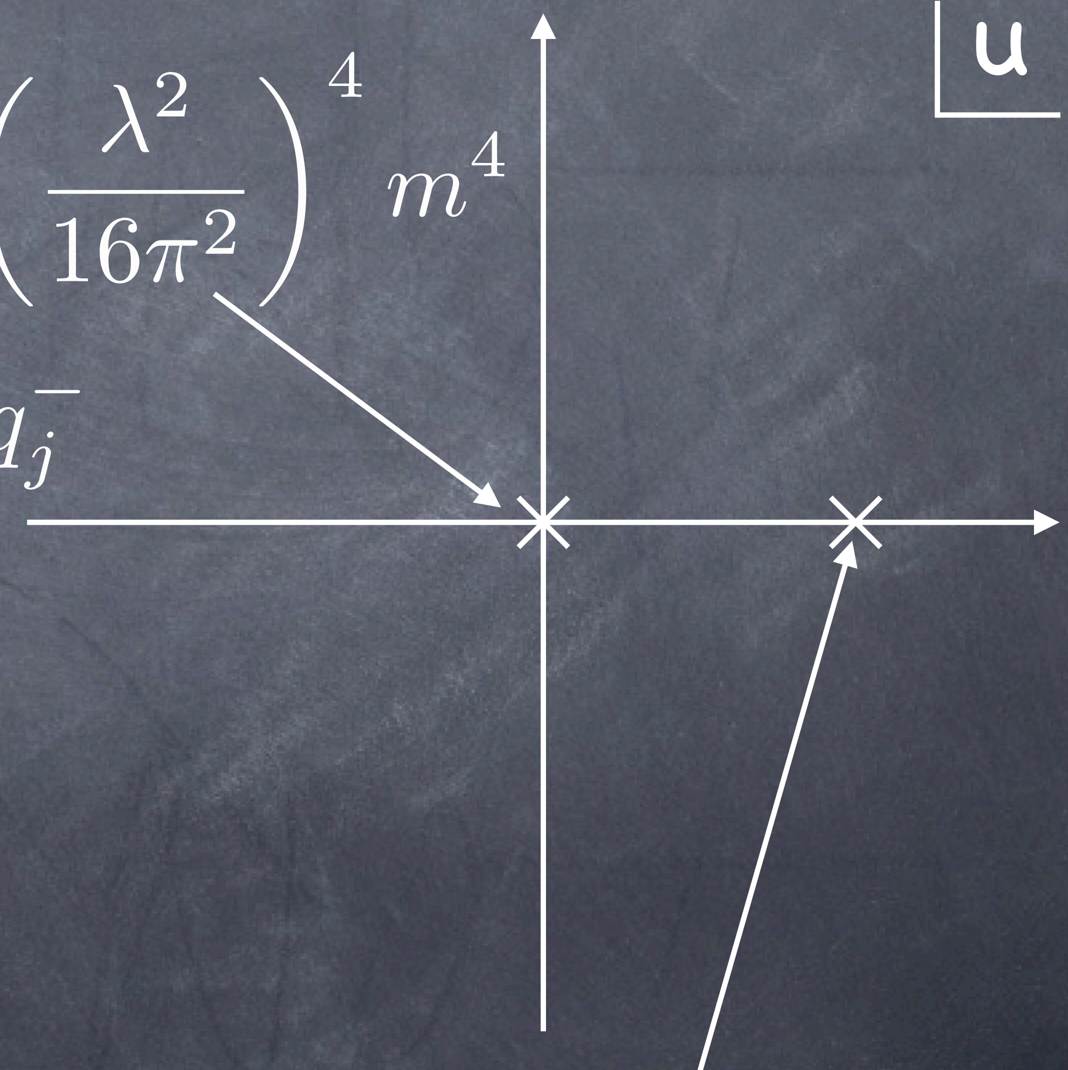
N_f
IR free
IR fixed point
free magnetic
no χSB
Coulomb
2 branches
2 branches
run-away
$3(N_c-2)$
$\frac{3}{2}(N_c-2)$
N_c
N_c-1
N_c-2
N_c-3
N_c-4
N_c-5
0
m=0
m≠0
IR fixed point
χSB
Λ
IR fixed point
χSB
N_f
IR free
$\frac{11}{2}N_c$
Banks-Saks fixed point
$3(N_c-2)$
$\frac{3}{2}(N_c-2)$
???
χSB
0
m=∞

$N_f = N_c - 2$

- for $M^{ij} = Q^i Q^j \neq 0$ with rank $M = N_f$, $SO(N_c)$ is broken to $SO(2)$
- Coulomb branch $u = \det M$
- two singularities
- $u = \det M = 0$
  - dyons: $q_i^{\pm}$ $W = \frac{1}{\mu} M^{ij} q_i^+ q_j^-$
- $u = \det M = \Lambda^{2N_f}$
  - monopoles: $W = (u - \Lambda^{2N_f}) E^+ E^-$

    $$|E^{\pm}| = (m\Lambda)^{1/2}$$
  - both monopoles and meson condense!

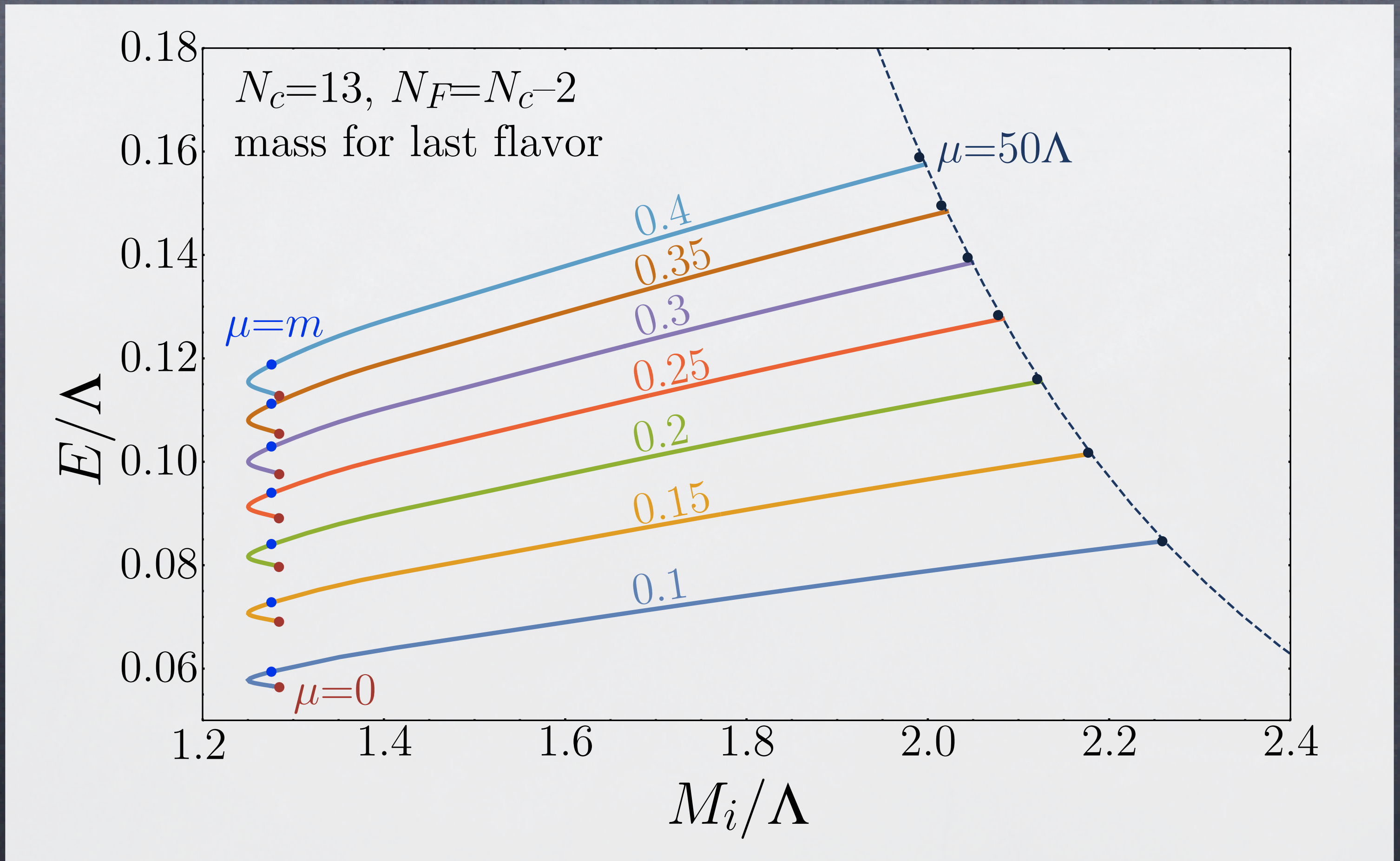
$$V \approx -\left(\frac{\lambda^2}{16\pi^2}\right)^4 m^4$$



$$V = -N_f m^2 \Lambda^2$$

$N_f < N_c - 2$

- add mass m_q to some of the quarks
- can show monopole VEVs persist $m_q \to \infty$
- demonstration of confinement and chiral symmetry breaking for all $N_f \leq N_c - 2$



Chiral SU(N_c) with A + (N_c−4) $\bar{F}$

Csáki, HM, Telem, 2104.10171

conjectures

- A + (N_c-4)F*: anomaly free chiral gauge th
- SU(N_c-4) x U(1) global symmetry

1. massless composite fermions: $A\bar{F}_{\{i,}\bar{F}_{j\}}$ satisfy 't Hooft anomaly matching conditions without breaking any global symmetries (E. Eichten, R. D. Peccei, J. Preskill, and D. Zeppenfeld (1986))
2. condensate $A\bar{F}_i$ breaks SU(N_c)$_g$xSU(N_c-4)xU(1) to SU(N_c-4)xU(1)xSU(4)$_g$ with massless (½N_c(N_c+1),N_c) fermion $A\bar{F}_{\{i,}\bar{F}_{j\}}$ (S. Dimopoulos, S. Raby, L. Susskind (1980)) "tumbling"

SUSY + AMSB, N_c odd

- non-perturbative run-away superpotential

$$A = \frac{\varphi}{\sqrt{2}} \left(\begin{array}{c|c} J_{(N-5)} & 0 \\ \hline 0 & 0_{5\times 5} \end{array} \right), \qquad \bar{F} = \varphi \left(\begin{array}{c|c} I_{(N-5)} & 0 \\ \hline 0 & 0_{5\times 1} \end{array} \right)$$

$$W = \left(\frac{\Lambda_N^{2N+3}}{(\mathrm{Pf}' A \bar{F} \bar{F})(\mathrm{Pf}' A)} \right)^{3/13}$$

Pouliot (1995)

$$\varphi \approx \Lambda \left(\frac{\Lambda}{m} \right)^{13/(4N-7)} \gg \Lambda$$

- SU(N_c–4)xU(1) broken to Sp(N_c–5)xU(1)
- massless fermions (N_c–4,N_c)

revised tumbling

- $A\bar{F}_i$ breaks $SU(N_c)_g \times SU(N_c-4) \times U(1)$ to $SU(4)_g \times SU(N_c-4) \times U(1)$
- $\bar{F}_{[i,}\bar{F}_{j]}$ breaks it further to $SU(4)_g \times Sp(N_c-5) \times U(1)$
- $SU(4)_g$ becomes strong and A(4) and F*(4*) condense

SUSY + AMSB, N_c even

- non-perturbative run-away superpotential

$$A = \frac{\varphi}{\sqrt{2}} \left(\begin{array}{c|c} J_{(N_c-4)} & 0 \\ \hline 0 & 0_{4\times 4} \end{array} \right), \qquad \bar{F} = \varphi \left(\begin{array}{c} I_{(N_c-4)} \\ \hline 0 \end{array} \right)$$

$$W = \left(\frac{\Lambda^{2N+3}}{(\mathrm{Pf} A \bar{F} \bar{F})(\mathrm{Pf} A)} \right)^{1/3}$$

$$\varphi \approx \Lambda \left(\frac{\Lambda}{m} \right)^{3/2N_c}$$

Pouliot (1995)

- $SU(N_c-4) \times U(1)$ broken to $Sp(N_c-4)$
- no massless fermions

revised tumbling

- $A\bar{F}_i$ breaks $SU(N_c)_g \times SU(N_c-4) \times U(1)$ to $SU(4)_g \times SU(N_c-4) \times U(1)$
- $\bar{F}_{[i,}\bar{F}_{j]}$ breaks it further to $SU(4)_g \times Sp(N_c-4)$
- $SU(4)_g$ becomes strong and A(4) and F*(4*) condense

Chiral SU(N_c) with S + (N_c+4) F*

Csáki, HM, Telem, 2105.03444

conjectures

- S + (N_c+4)F*: anomaly free chiral gauge th
- SU(N_c+4) x U(1) global symmetry

1. massless composite fermions: $S\bar{F}_{[i,}\bar{F}_{j]}$ satisfy 't Hooft anomaly matching conditions without breaking any global symmetries (E. Eichten, R. D. Peccei, J. Preskill, and D. Zeppenfeld (1986))
2. condensate $S\bar{F}_i$ breaks SU(N_c)$_g$xSU(N_c+4)xU(1) to SU(N_c)xSU(4) with massless (N_c*,4) fermion (S. Dimopoulos, S. Raby, L. Susskind (1980)) "tumbling"

SUSY + AMSB

- dual Spin(8) with N_c+4 q^i and one spinor p

$$M_{ij} = S\bar{F}_{\{i,}\bar{F}_{j\}}, \quad U = \det S$$

$$W = \frac{1}{\mu_1^2} M_{ij} q^i q^j + \frac{1}{\mu_2^{N_c-1}} U p p$$

Pouliot
Strassler
(1995)

- rank M=Nc+4, U≠0, integrate out q^i and p

$$W_{\rm dyn} = (\tilde{\Lambda}_L^{18})^{1/6} = \left(\frac{\det\tilde{M}\,\tilde{U}}{\tilde{\Lambda}^{N-17}} \right)^{1/6}$$

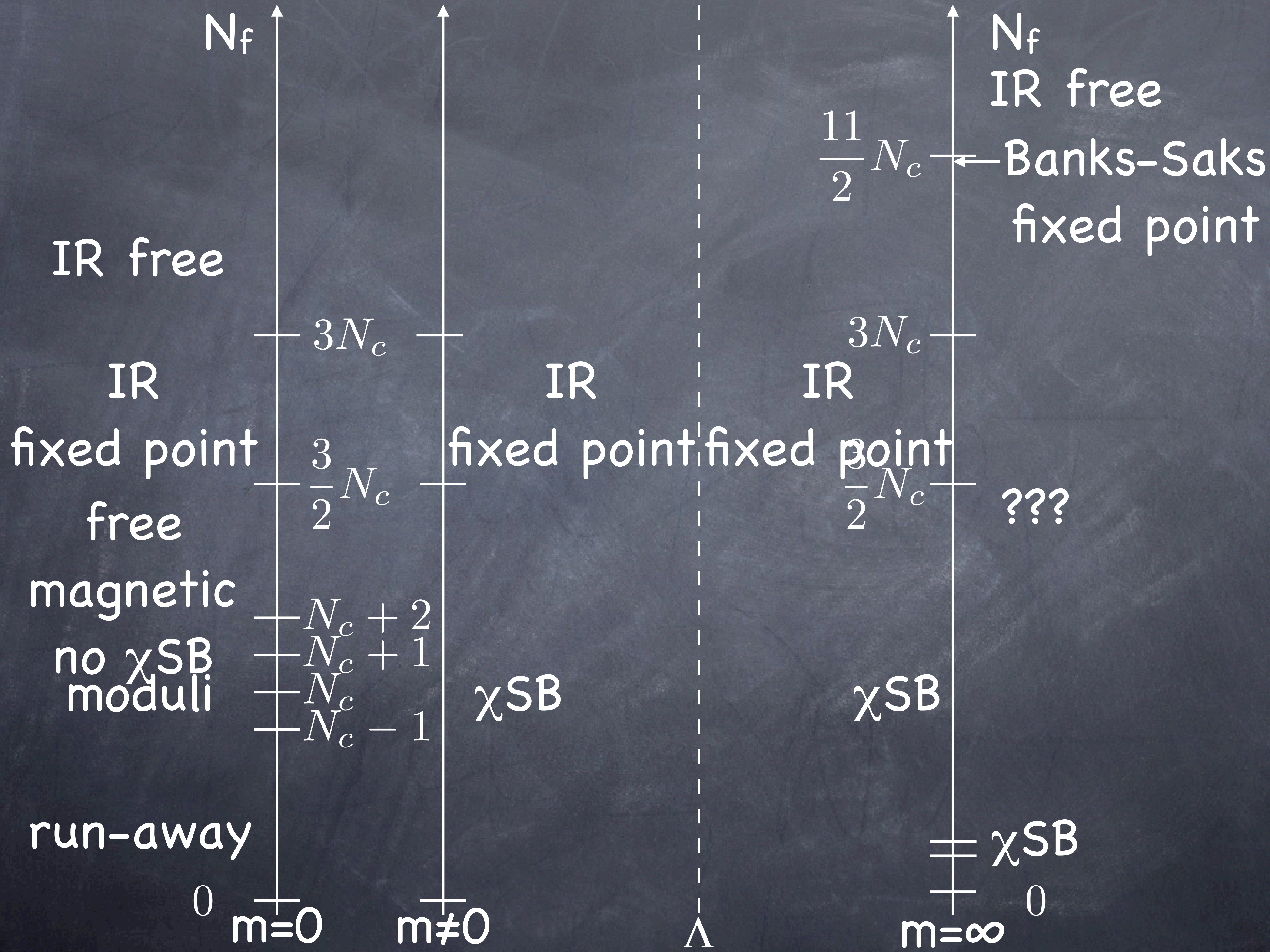
$$\mathcal{L}_{\rm tree} = m \frac{N-17}{6} W_{\rm dyn} + c.c.$$

$$\tilde{M}_{ij} \approx \delta_{ij} m \left(\frac{\tilde{\Lambda}}{m} \right)^{\frac{N-17}{N-11}}, \quad \tilde{U} \approx m \left(\frac{\tilde{\Lambda}}{m} \right)^{\frac{N-17}{N-11}}, \quad V \approx -m^4 \left(\frac{\tilde{\Lambda}}{m} \right)^{\frac{2(N-17)}{N-11}}$$

- SU(N_c+4)xU(1) broken to SO(N_c)
- no massless fermions
- SCFT if N≤16 + R-charge by a-minimization

revised tumbling

- SS in the $\boxplus$ channel breaks $SU(N_c)_g \times SU(N_c+4) \times U(1)$ to $SO(N_c)_g \times SU(N_c+4) \times U(1)$
- $\bar{F}_{\{i,}\bar{F}_{j\}}$ is singlet under $SO(N_c)_g$ and condenses, breaks it further to $SO(N_c+4)$



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$\frac{11}{2}N_c$
Banks-Saks
fixed point
$3N_c$
$\frac{3}{2}N_c$
???
χSB
0
m=∞

Conclusion

- N=1 SUSY + AMSB: a great tool to study non-SUSY gauge theories
- SU(N_c) QCD:
  - χSB ($N_f \leq 3N_c/2$), CFT ($3N_c/2 < N_f < 3N_c$)
  - derived χSB from QCD for the first time!
- SO(N_c) QCD:
  - χSB and confinement by monopole condensation
- chiral gauge theories:
  - different from past conjectures
  - seems to minimize massless fermions