

# Models of the muonium to antimuonium transition

based on T. Fukuyama, Y. Mimura, & YU,  
PRD**105**, 015026 (2022). [arXiv:2108.10736]

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# Charged Lepton Flavor Violation (CLFV)

- lepton flavor numbers (  $L_e, L_\mu, L_\tau$  ) cf. Lepton number,  $L = L_e + L_\mu + L_\tau$

	$e^-$	$\mu^-$	$\tau^-$	$\nu_e$	$\nu_\mu$	$\nu_\tau$	$e^+$	$\mu^+$	$\tau^+$	$\bar{\nu}_e$	$\bar{\nu}_\mu$	$\bar{\nu}_\tau$	others
$L_e$	+1	0	0	+1	0	0	-1	0	0	-1	0	0	0
$L_\mu$	0	+1	0	0	+1	0	0	-1	0	0	-1	0	0
$L_\tau$	0	0	+1	0	0	+1	0	0	-1	0	0	-1	0

- ◆ lepton flavor violation in charged lepton sector = **CLFV** ( e.g.  $\mu^+ \rightarrow e^+ \gamma$  )

CLFV is a good probe for new physics

because

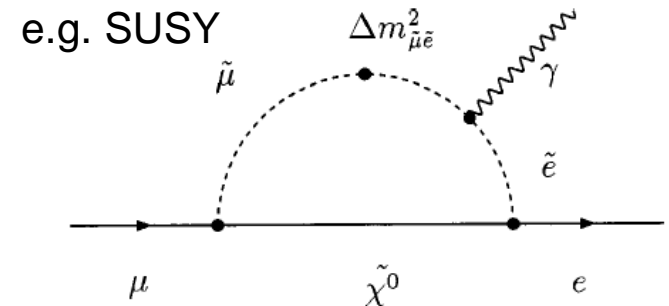
- **forbidden** in SM
- predicted in many theories beyond SM
- tiny contribution of neutrino oscillation:

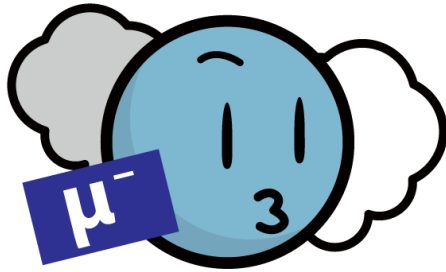
$$\text{Br}(\mu \rightarrow e \gamma) < 10^{-54}$$

- ✓ cannot be observed by current technology  
(does not contaminate the new physics search)



If found, it must be an evidence of new physics !! (not  $\nu$  osci.)





# Muon rare decays

- Long lifetime: easy to treat
- Many muons can be produced ( $\sim 10^9/s$ ).

- LFV processes with  $\Delta L_\mu = -\Delta L_e = \pm 1$

e.g.  $\mu^+ \rightarrow e^+ \gamma$     $\mu^+ \rightarrow e^+ e^+ e^-$     $\mu^- N \rightarrow e^- N$

## ➤ Current upper limits

L. Calibbi & G. Signorelli, Riv. Nuovo Cim. **41**, 1 (2018).

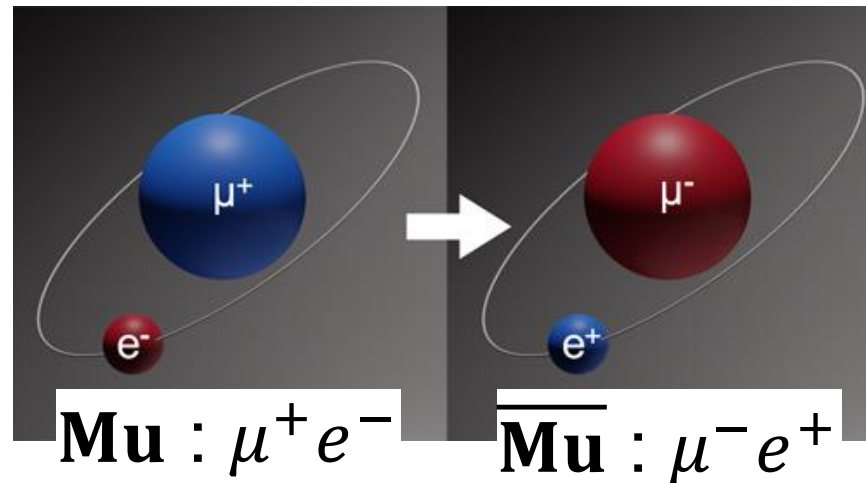
Reaction	Present limit	C.L.	Experiment	Year
$\mu^+ \rightarrow e^+ \gamma$	$< 4.2 \times 10^{-13}$	90%	MEG at PSI	2016
$\mu^+ \rightarrow e^+ e^- e^+$	$< 1.0 \times 10^{-12}$	90%	SINDRUM	1988
$\mu^- \text{Ti} \rightarrow e^- \text{Ti}$	$< 6.1 \times 10^{-13}$	90%	SINDRUM II	1998
$\mu^- \text{Pb} \rightarrow e^- \text{Pb}$	$< 4.6 \times 10^{-11}$	90%	SINDRUM II	1996
$\mu^- \text{Au} \rightarrow e^- \text{Au}$	$< 7.0 \times 10^{-13}$	90%	SINDRUM II	2006



strong experimental constraints

# Muonium( $\text{Mu}$ )-to-Antimuonium( $\overline{\text{Mu}}$ )

Pontecorvo (1957), Weinberg & Feinberg (1961).



- CLFV process with  $\Delta L_\mu = -\Delta L_e = 2$   
 CLFV with  $\Delta L_\mu = -\Delta L_e = \pm 1$  are constrained by  $\mu \rightarrow e\gamma$ ,  $\mu \rightarrow 3e$ , ...  
 Mu-to- $\overline{\text{Mu}}$  can be sizable if a new particle carries **2** units of flavors.
- pure leptonic system ( no hadronic ambiguities )
- J-PARC (Japan, N.Kawamura *et al.*, JPS Conf. Proc. 33, 011120 (2021) )  
 and CSNS (China, MACE collab.) plan future experiments.

$$P < 8.3 \times 10^{-11} \text{ (PSI)} \quad \Rightarrow \quad \mathcal{O}(10^{-14}) \text{ (CSNS)}$$

# Mu-to- $\overline{\text{Mu}}$ transition

cf.  $K-\overline{K}$  mixing

$$|\psi(t)\rangle = \alpha(t)|\text{Mu}\rangle + \beta(t)|\overline{\text{Mu}}\rangle$$

$$i \frac{\partial}{\partial t} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} \\ \mathcal{M}_{21} & \mathcal{M}_{11} \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

New physics !!

$$\mathcal{M}_{ij} = M_{ij} - i\Gamma_{ij}/2$$

$$M = M^\dagger, \Gamma = \Gamma^\dagger$$



Transition probability

$$P(\text{Mu} \rightarrow \overline{\text{Mu}}) \simeq 2\tau^2 |\mathcal{M}|^2$$

$\tau$  : Mu lifetime  $\simeq 2.2 \mu\text{s}$

$$\mathcal{M} \equiv \sqrt{\mathcal{M}_{12}\mathcal{M}_{21}}$$

$$|\text{Mu}(0)\rangle = |\text{Mu}\rangle$$

$$\Downarrow i \frac{\partial}{\partial t} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} \\ \mathcal{M}_{21} & \mathcal{M}_{11} \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$|\text{Mu}(t)\rangle = f_+(t) |\text{Mu}\rangle + \sqrt{\frac{\mathcal{M}_{21}}{\mathcal{M}_{12}}} f_-(t) |\overline{\text{Mu}}\rangle$$

$$f_{\pm}(t) = \frac{e^{-i\lambda_+} \pm e^{-i\lambda_-}}{2}$$

$$\lambda_{\pm} = M - i \frac{\Gamma}{2} \pm \frac{1}{2} \left( \Delta M - i \frac{\Delta \Gamma}{2} \right)$$

$$\Delta M - i \frac{\Delta \Gamma}{2} = 2 \sqrt{\mathcal{M}_{12} \mathcal{M}_{21}}$$

Mu-to- $\overline{\text{Mu}}$  transition probability at time  $t$  :

$$P(\text{Mu} \rightarrow \overline{\text{Mu}}; t) \sim |\langle \overline{\text{Mu}} | \text{Mu}(t) \rangle|^2 \simeq e^{-\Gamma t} \sin^2 \frac{\Delta M}{2} t$$

(assuming CP conservation or  $|\Gamma_{12}/M_{12}| \ll 1$ ,  $|\frac{\mathcal{M}_{21}}{\mathcal{M}_{12}}| = 1$ )

Integrated probability of Mu-to- $\overline{\text{Mu}}$  transition :

$$\begin{aligned} P(\text{Mu} \rightarrow \overline{\text{Mu}}) &= \int_0^\infty dt \Gamma P(\text{Mu} \rightarrow \overline{\text{Mu}}; t) \\ &= \int_0^\infty dt \Gamma e^{-\Gamma t} \sin^2 \frac{\Delta M}{2} t = \frac{1}{2} \frac{(\Delta M)^2}{(\Delta M)^2 + \Gamma^2} \end{aligned}$$

$$\Delta M \ll \Gamma \rightarrow \simeq \frac{(\Delta M)^2}{2\Gamma^2} = \boxed{2\tau^2 |\mathcal{M}|^2} \quad \Gamma : \text{Mu decay width}$$

$\tau$  : muonium lifetime  $\simeq 2.2 \mu\text{s}$

# Effective interactions

cf. R. Conlin & A. A. Petrov, PRD102, 095001 (2020).

$$-\mathcal{L}_{\text{Mu}-\overline{\text{Mu}}} = \sum_i \frac{G_i}{\sqrt{2}} Q_i$$

$G_i$  : coupling constants

$$Q_1 = [\bar{\mu}\gamma_\alpha(1 - \gamma_5)e][\bar{\mu}\gamma^\alpha(1 - \gamma_5)e] \quad \text{LL vector}$$

$$Q_2 = [\bar{\mu}\gamma_\alpha(1 + \gamma_5)e][\bar{\mu}\gamma^\alpha(1 + \gamma_5)e] \quad \text{RR vector}$$

$$Q_3 = [\bar{\mu}\gamma_\alpha(1 - \gamma_5)e][\bar{\mu}\gamma^\alpha(1 + \gamma_5)e] \quad \text{LR vector}$$

$$Q_4 = [\bar{\mu}(1 - \gamma_5)e][\bar{\mu}(1 - \gamma_5)e] \quad \text{LL scalar}$$

$$Q_5 = [\bar{\mu}(1 + \gamma_5)e][\bar{\mu}(1 + \gamma_5)e] \quad \text{RR scalar}$$

※ Any 4-Fermi type operators can be written by the linear combination of the five operators ( ∴ Fierz identity )



# Four muonium states

- Muonium has four hyperfine 1S states by combination of spins of the bound muon and electron

$\text{Mu}(F, m)$

magnitude of total angular mom.  $\nearrow$   
 $\nearrow$  z component of total angular mom.

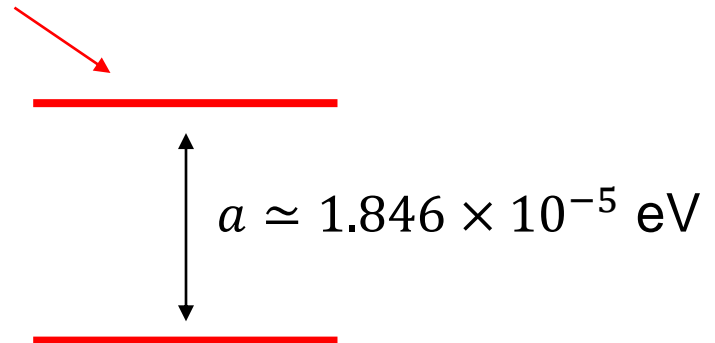
$$(F, m) = \left\{ \begin{array}{l} (1, +1) \\ (1, 0) \\ (1, -1) \\ (0, 0) \end{array} \right\}$$

triplet (ortho)  
 singlet (para)

$$E(\text{Mu}; 1, 0) = E(\text{Mu}; 1, \pm 1) = E_0 + \frac{a}{4}$$

$$E(\text{Mu}; 0, 0) = E_0 - \frac{3}{4}a$$

$E_0$ : averaged 1S energy



- Mu-to- $\overline{\text{Mu}}$  transition rate depends on the initial Mu states.

For the triplet ( $F = 1$ ) states,

$$\mathcal{M}_{1,0} = \mathcal{M}_{1,\pm 1} = \frac{8|\varphi(0)|^2}{\sqrt{2}} \left( G_1 + G_2 + \frac{1}{2} G_3 - \frac{1}{4} G_4 - \frac{1}{4} G_5 \right)$$

For the singlet ( $F = 0$ ) state,

$$\mathcal{M}_{0,0} = \frac{8|\varphi(0)|^2}{\sqrt{2}} \left( G_1 + G_2 - \frac{3}{2} G_3 - \frac{1}{4} G_4 - \frac{1}{4} G_5 \right)$$

$$\varphi(0) = \sqrt{\frac{(m_{\text{red}}\alpha)^3}{\pi}} : \text{overlap of lepton wave function}$$

- Mu-to- $\overline{\text{Mu}}$  transition rate depends on the magnetic field.

The magnetic field makes... 

- ① nonzero energy gap of  $\text{Mu}(1, \pm 1)$  and  $\overline{\text{Mu}}(1, \pm 1)$
- ② mixing of  $\text{Mu}(1,0)$  and  $\text{Mu}(0,0)$

① nonzero energy gap of  $\text{Mu}(1, \pm 1)$  &  $\overline{\text{Mu}}(1, \pm 1)$

Magnetic field  $B\hat{z}$  makes an energy gap  $\Delta E$   
between  $\text{Mu}(1, \pm 1)$  &  $\overline{\text{Mu}}(1, \pm 1)$ .

Considering the energy gap  $\Delta E$  of  $\text{Mu}$  and  $\overline{\text{Mu}}$ ,  
the transition rate is

$$P(\text{Mu}(1, \pm 1) \rightarrow \overline{\text{Mu}}) = \frac{2\tau^2 |\mathcal{M}_{1, \pm 1}|^2}{1 + (\tau\Delta E)^2}$$

$$\tau\Delta E = 3.8 \times 10^5 \times \frac{B}{\text{Tesla}}$$

∴ The contribution of  $m = \pm 1$  is suppressed  
in a magnetic field stronger than  $\mu\text{T}$ .

cf. terrestrial magnetism 30-60  $\mu\text{T}$

## ② mixing of $Mu(1,0)$ and $Mu(0,0)$

transition amplitude of  $m = 0$  states in a magnetic field  $B$

$$\mathcal{M}_{0,0}^B \simeq \frac{1}{2} \left( \mathcal{M}_{0,0} - \mathcal{M}_{1,0} + \frac{\mathcal{M}_{0,0} + \mathcal{M}_{1,0}}{\sqrt{1 + X^2}} \right)$$

$$\mathcal{M}_{1,0}^B \simeq \frac{1}{2} \left( -\mathcal{M}_{0,0} + \mathcal{M}_{1,0} + \frac{\mathcal{M}_{0,0} + \mathcal{M}_{1,0}}{\sqrt{1 + X^2}} \right)$$

$$X = \frac{\mu_B B}{a} \left( g_e + \frac{m_e}{m_\mu} g_\mu \right) \simeq 6.31 \frac{B}{\text{Tesla}}$$

Thus, the transition probability in a magnetic field is

$$P = 2\tau^2 \left( |c_{0,0}|^2 |\mathcal{M}_{0,0}^B|^2 + |c_{1,0}|^2 |\mathcal{M}_{1,0}^B|^2 + \sum_{m=\pm 1} |c_{1,m}|^2 \frac{|\mathcal{M}_{1,\pm 1}|^2}{1 + (\tau\Delta E)^2} \right)$$

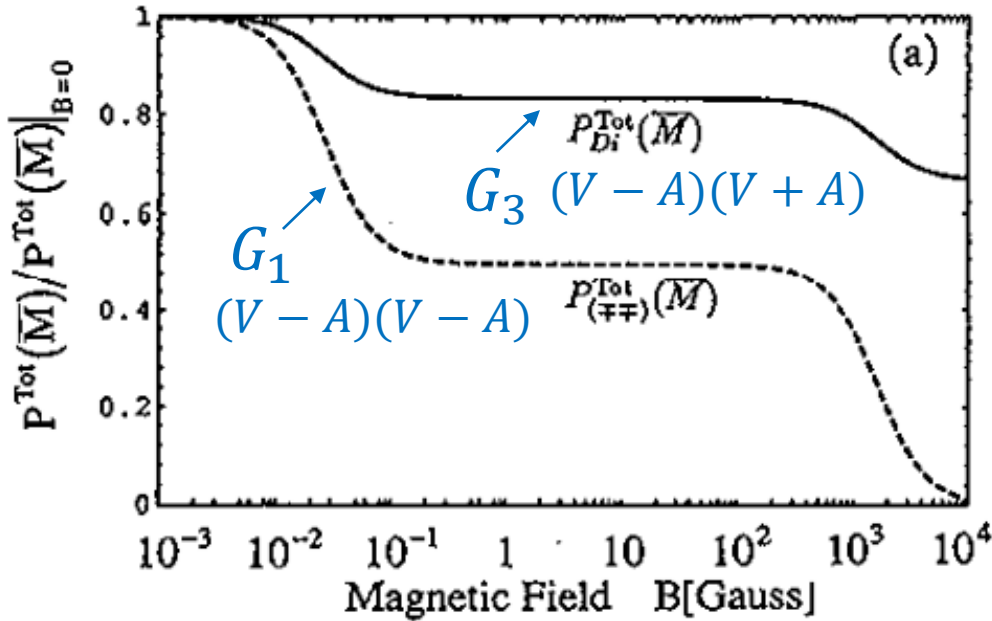
$$|c_{F,m}|^2 : \text{population of the state } (F, m) \quad \sum_{F,m} |c_{F,m}|^2 = 1$$

# Transition rate vs. magnetic field

K.Horikawa & K.Sasaki, PRD53, 560 (1996).

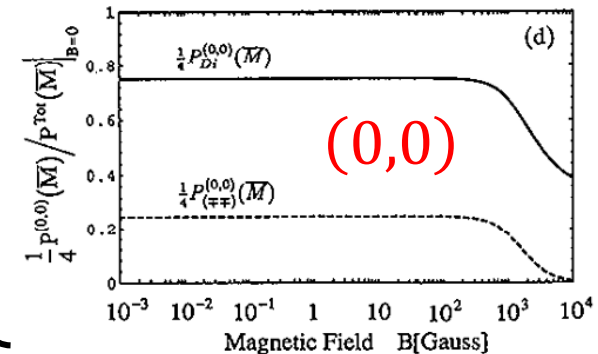
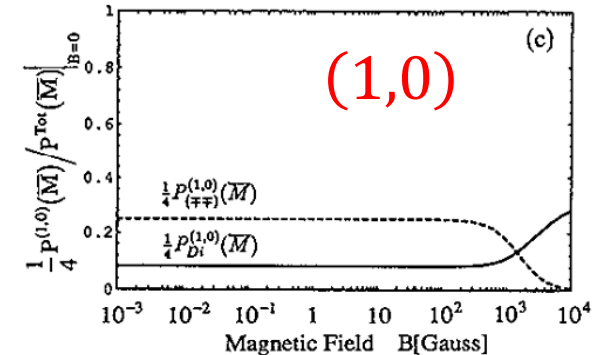
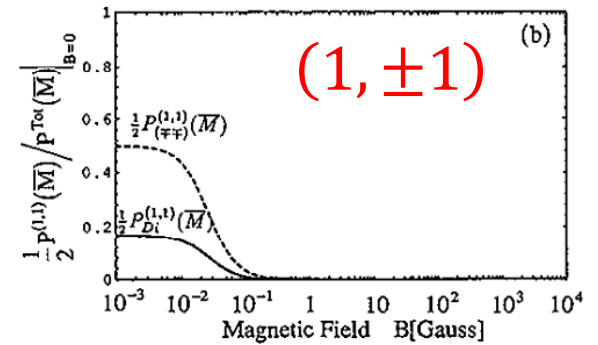
$10^4$  Gauss = 1 Tesla

Total



$(1, \pm 1)$  contribution vanishes.

$(1,0)$  &  $(0,0)$  mixes.



# Current experimental constraints by PSI

In a magnetic field  $B = 0.1$  Tesla,

$$P < 8.3 \times 10^{-11}$$

L. Willmann *et al.*, PRL82, 49 (1999).



( considering the magnetic effects )

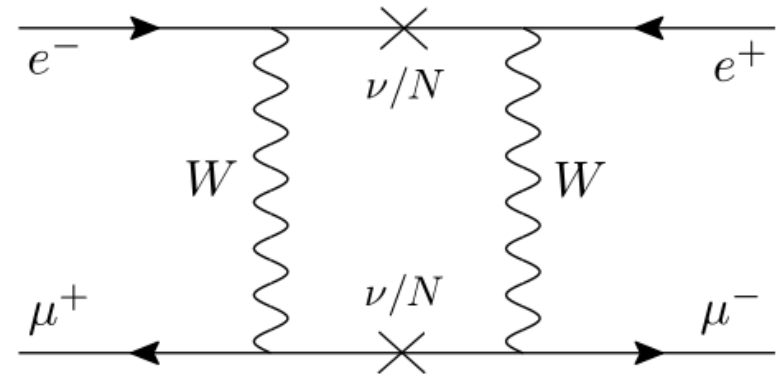
cf. K. Horikawa & K. Sasaki, PRD53, 560 (1996),  
W. S. Hou & G. G. Wong, PLB357, 145 (1995).

$$\underline{|G_i| \lesssim 3.0 \times 10^{-3} G_F}$$

# Classification of new particles & interactions to generate Mu-to- $\overline{\text{Mu}}$ transition

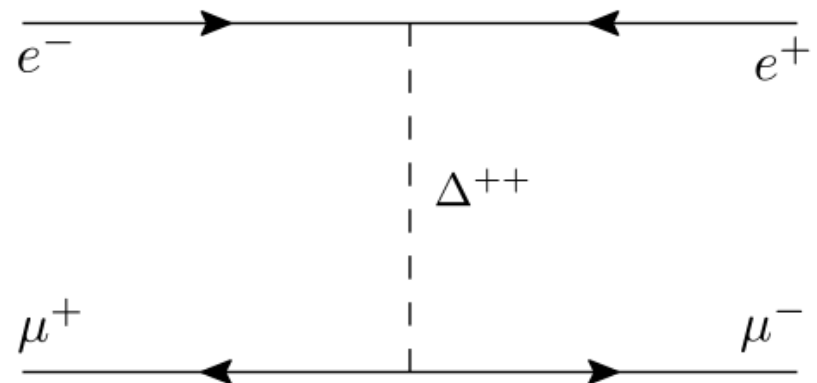
$$1. \Delta L_e = \Delta L_\mu = 0$$

- mass term of SM singlet which violates lepton number
- loop
- e.g. Majorana  $\nu$  mass



$$2. (\Delta L_e, \Delta L_\mu) = (\pm 2, 0), (0, \pm 2)$$

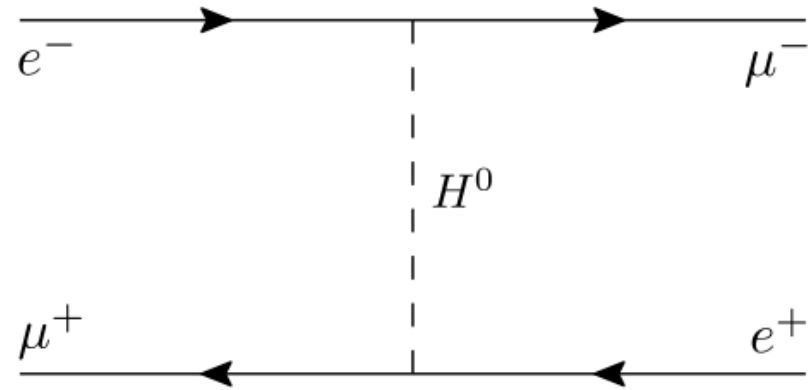
- doubly-charged mediator
- tree
- LNV is not needed.
- If mediator couples to  $W^+W^+$ ,  $0\nu 2\beta$  is induced.



cf. T. Fukuyama, Y. Mimura, & YU, PRD106, 055041 (2022).

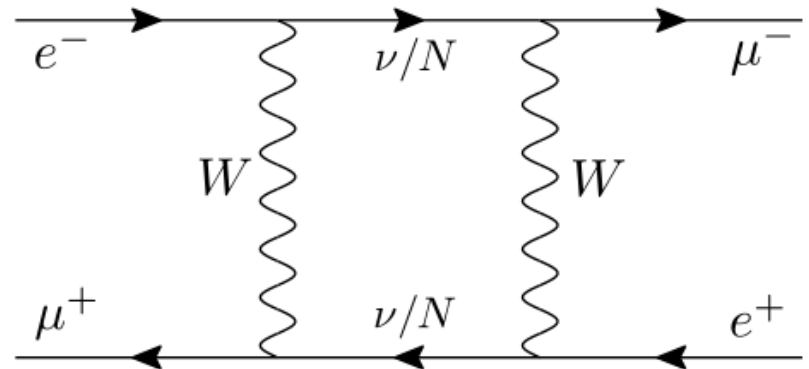
### 3. $\Delta L_e = -\Delta L_\mu = \pm 1$

- neutral mediator
- tree
- LNV is not needed.
- If a diagonal coupling  $\bar{\ell}\ell H^0$  exists,  $\mu \rightarrow e\gamma$  is induced.



### 4. $(\Delta L_e, \Delta L_\mu) = (\pm 1, 0), (0, \pm 1)$

- loop
- model-independently, strongly constrained by  $\mu \rightarrow e\gamma$  or  $\mu \rightarrow 3e$





# Summary of models

- Tree level

Model	$G_1$	$G_2$	$G_3$	$G_4$	$G_5$
Type I + II hybrid seesaw	✓	–	–	–	–
Left-right model with $SU(2)_R$ triplet	–	✓	–	–	–
Inert Higgs doublet	–	–	✓	△	△
$R$ -parity violating SUSY	–	–	✓	–	–
Dilepton gauge boson	–	–	✓	–	–
Neutral flavor gauge boson	✓	✓	✓	–	–

✓ :  $G_i/G_F \sim O(10^{-3})$  is allowed      △ : suppressed by LFV bounds

- $\Delta L_e - \Delta L_\mu = \pm 1$

Model	$ G_1 /G_F$	$ G_2 /G_F$	$ G_3 /G_F$
Heavy singlet neutrino	$\lesssim O(10^{-8})$	–	–
Left-right model without $SU(2)_R$ triplet	$\lesssim O(10^{-8})$	$\lesssim O(10^{-8})$	$\lesssim O(10^{-10})$
SUSY (Gaugino loop)	$\lesssim O(10^{-8})$	–	–
Leptoquark	$\lesssim O(10^{-8})$	$\lesssim O(10^{-8})$	$\lesssim O(10^{-8})$

# Neutrino mass models

- radiative neutrino mass model (loop induced by LNV)

Model	$ G_1 /G_F$	$ G_2 /G_F$
Charged Higgs(ino)	$\lesssim O(10^{-5})$	–
KNT model	–	$\lesssim O(10^{-5})^{(*)}$
AKS model	–	$\lesssim O(10^{-6})$

- neutrino mass model (tree by doubly charged scalar)

Model	$ G_1 /G_F$	$ G_2 /G_F$
Type-II seesaw	$\lesssim O(10^{-5})^{(\#)}$	–
Zee-Babu model	–	$\lesssim O(10^{-3})$
Cocktail model	–	$\lesssim O(10^{-5})$

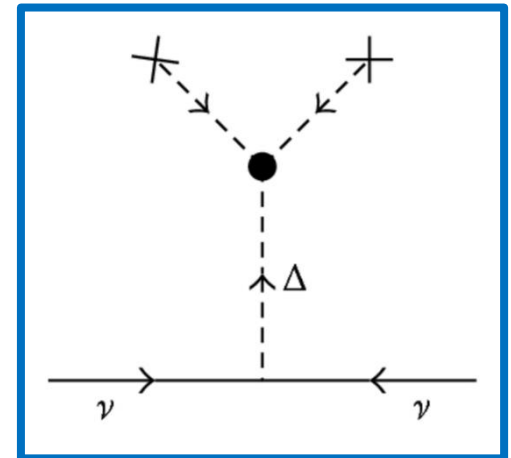
# Example 1: Type-II seesaw

- SU(2) triplet scalar  $\Delta \rightarrow \nu$  mass

$$-\mathcal{L} = \left( \frac{1}{2} \kappa_{ij}^L \overline{(\ell_{iL})^c} \ell_{jL} \Delta_L + \mu_{\Delta} H H \Delta_L^* + h.c. \right) + M_{\Delta}^2 |\Delta_L|^2$$

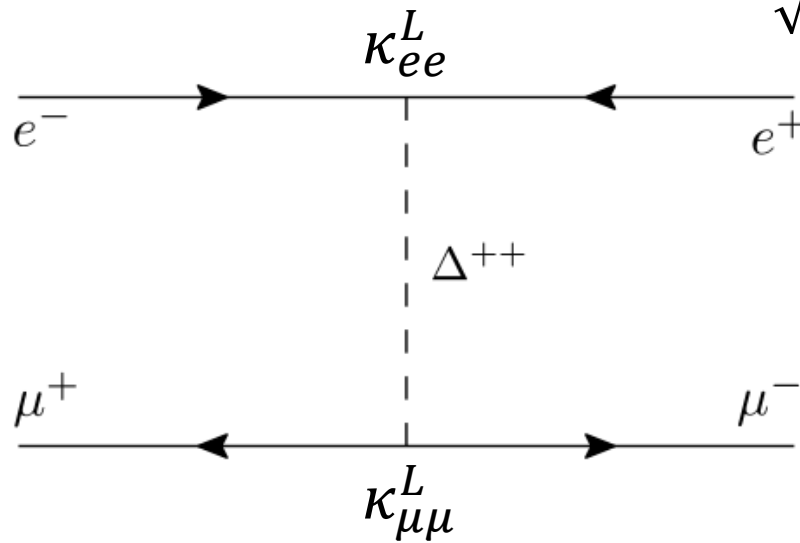
$$(\Delta_L)_{ab} = \begin{pmatrix} \Delta_L^{++} & \Delta_L^+/\sqrt{2} \\ \Delta_L^+/\sqrt{2} & \Delta_L^0 \end{pmatrix}$$

$$\begin{aligned} \overline{(\ell_{iL})^c} \ell_{jL} \Delta_L &= \overline{\nu_{iL}^c} \nu_{jL} \Delta_L^0 - \frac{1}{\sqrt{2}} \overline{\nu_{iL}^c} e_{jL} \Delta_L^+ \\ &\quad - \frac{1}{\sqrt{2}} \overline{e_{iL}^c} \nu_{jL} \Delta_L^+ + \overline{e_{iL}^c} e_{jL} \Delta_L^{++} \end{aligned}$$



$$M_{\nu}^{\text{II}} = \kappa^L \langle \Delta_L^0 \rangle = -\kappa^L \mu_{\Delta} \langle H^0 \rangle^2 / M_{\Delta}^2$$

- Mu- $\overline{\text{Mu}}$  (tree)



$$\frac{G_1}{\sqrt{2}} = -\frac{\kappa_{ee}^L \kappa_{\mu\mu}^{L*}}{32M_\Delta^2} = -\frac{(M_\nu^{\text{II}})_{ee} (M_\nu^{\text{II}})_{\mu\mu}^*}{32v_L^2 M_\Delta^2}$$

- constrained by  $\mu \rightarrow 3e$

$$\text{Br}(\mu \rightarrow 3e) < 1.0 \times 10^{-12}$$

$$\frac{G_1}{G_F} = \frac{\sqrt{\text{Br}(\mu \rightarrow 3e)}}{2\sqrt{2}} \left| \frac{\kappa_{\mu\mu}^L}{\kappa_{e\mu}^L} \right| \lesssim 3.5 \times 10^{-7} \left| \frac{\kappa_{\mu\mu}^L}{\kappa_{e\mu}^L} \right|$$

⇒ The off-diagonal components of  $M_\nu^{\text{II}}$  should be small  
(to obtain the sizable Mu- $\overline{\text{Mu}}$  rate).

- If type-II contribution dominates neutrino masses,

$$M_{\nu}^{\text{II}} = U_{\text{PMNS}}^* \text{diag}(m_1 e^{i\alpha_1}, m_2 e^{i\alpha_2}, m_3) U_{\text{PMNS}}^{\dagger}$$

$$\Rightarrow \frac{\kappa_{e\mu}^L}{\kappa_{\mu\mu}^L} \sim \mathcal{O}\left(\theta_{13}, \sqrt{\Delta m_{\text{sol}}^2 / \Delta m_{\text{atm}}^2}\right)$$

$$\sim 10\text{-}20\%$$

$$\Rightarrow \frac{G_1}{G_F} < \mathcal{O}(10^{-6})$$

$$\text{cf. current constraint: } |G_i|/G_F \lesssim 3.0 \times 10^{-3}$$

However, the neutrino mass can be reproduced even if  $\kappa_{e\mu}^L \rightarrow 0$ .

$\Rightarrow$  Next, let's consider such a special case that  $\kappa_{e\mu}^L$  is tiny.

$$\kappa^L \langle \Delta_L^0 \rangle = U_{\text{PMNS}}^* \text{diag}(m_1 e^{i\alpha_1}, m_2 e^{i\alpha_2}, m_3) U_{\text{PMNS}}^\dagger$$

In case  $\kappa_{e\mu}^L \rightarrow 0$ ,

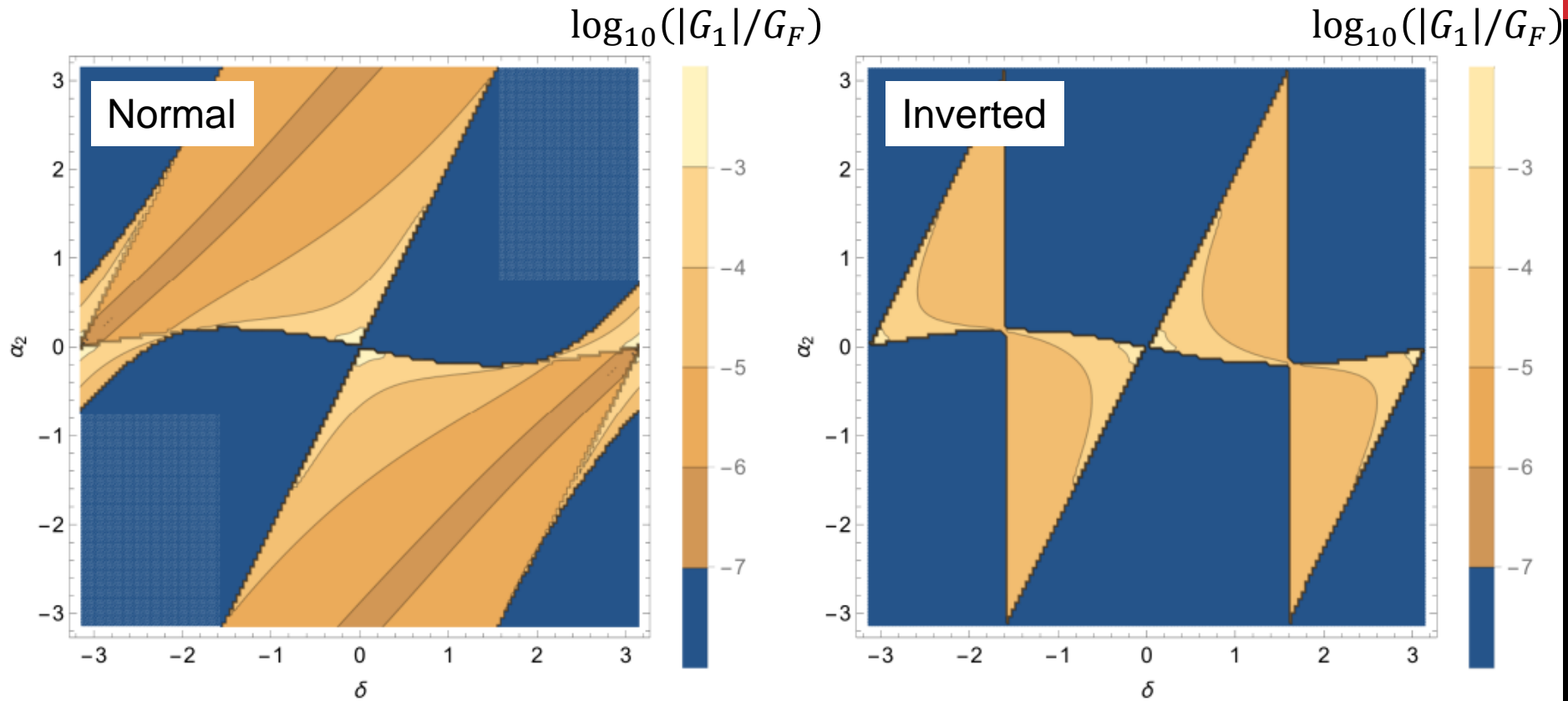
1.  $m_1 e^{i\alpha_1} \simeq m_2 e^{i\alpha_2} (\simeq m_3)$

2.  $\sum U_{ei}^* U_{\mu i}^* m_i e^{i\alpha_i}$  is accidentally canceled

Choosing  $m_1 e^{i\alpha_1}$  to realize  $\kappa_{e\mu}^L = 0$ ,

$M_\nu$  can be represented by a function of Dirac phase  $\delta$  & Majorana phase  $\alpha_2$ .

Considering LFV constraints from  $\mu$  decay &  $\tau$  decay,  
we calculate the upper limit of  $G_1$ .

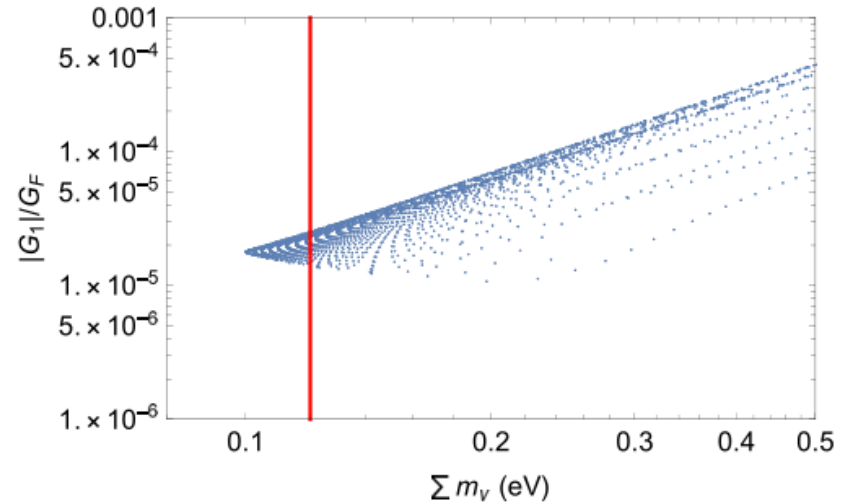
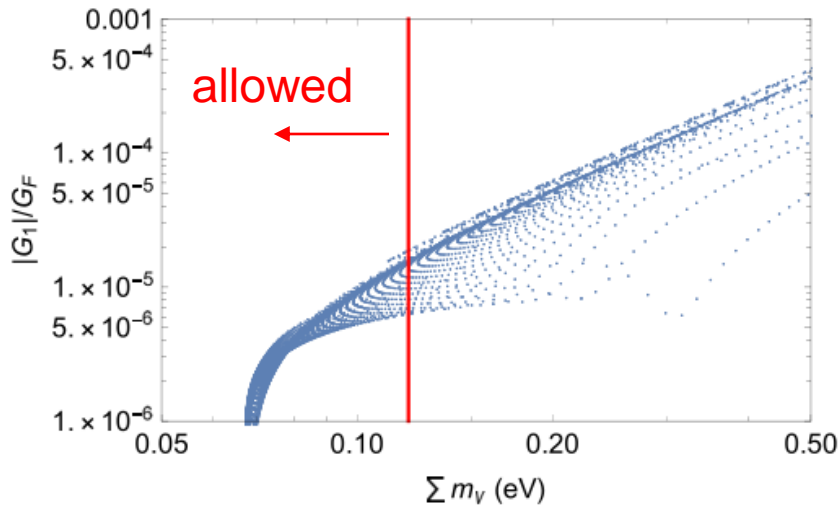


$\alpha_2 \sim 0, \delta \sim 0$  or  $\pi \Rightarrow G_1 \rightarrow \text{large}$  (mass degenerate):  $G_1/G_F \sim 10^{-3}$  is allowed.

Normal :  $\kappa_{e\mu}^L = 0$  can be realized in a wide region without mass degenerate.

Inverted : When  $m_1 \sim m_2$ ,  $G_1$  can be large if the solution of  $\kappa_{e\mu}^L = 0$  exists.

- cosmological bound  $\sum m_\nu < 0.12 \text{ eV}$



$$\Rightarrow |G_1|/G_F \lesssim \mathcal{O}(10^{-5})$$

- If the contribution from type-I cancels  $\kappa_{e\mu}^L$ ,  
an arbitrary size of  $\text{Mu}-\overline{\text{Mu}}$  can be generated.

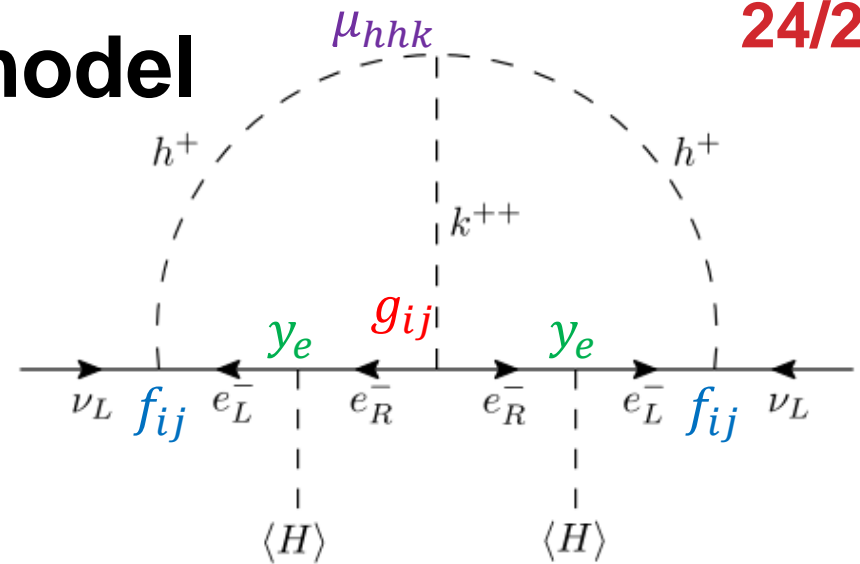
$$\text{e.g. } \kappa_{ee}^L \sim \kappa_{\mu\mu}^L \sim 0.3, M_\Delta = 600 \text{ GeV} \Rightarrow \frac{|G_1|}{G_F} \sim 1 \times 10^{-3}$$



# Example2: Zee-Babu model

one of radiative neutrino models

neutrino mass is generated by two-loop



$$-\mathcal{L} \supset \left( f_{ij} \bar{\ell}_i^c \cdot \ell_j h^+ + g_{ij} \bar{e}_i e_j^c k^{--} + \mu_{hhk} h^+ h^+ k^{--} + h.c. \right) \\ + m_h^2 h^- h^+ + m_k^2 k^{--} k^{++}$$

$f$  : anti-symmetric for  $i, j$        $g$  : symmetric for  $i, j$

$$M_\nu = \frac{1}{M_0} f M_e g M_e f^T$$

$$\frac{1}{M_0} = \frac{\mu_{hhk}}{48\pi^2 \max(m_h^2, m_k^2)} \tilde{I}$$

$$M_e = \text{diag}(m_e, m_\mu, m_\tau)$$

rank-2 ( $m_{\text{lightest}} = 0$ )     $\because f$  is anti-symmetric

## Normal ordering case

$$U = (u_1, u_2, u_3) : \text{PMNS matrix}$$

$$M_\nu = U^* \text{diag}(0, m_2, m_3) U^\dagger = m_2 u_2^* u_2^\dagger + m_3 u_3^* u_3^\dagger$$

where

$$u_1 = \begin{pmatrix} c_{12}c_{13} \\ -s_{12}c_{23} - e^{i\delta}c_{12}s_{13}s_{23} \\ s_{12}s_{23} - e^{i\delta}c_{12}s_{13}c_{23} \end{pmatrix}, \quad u_2 = \begin{pmatrix} s_{12}c_{13} \\ c_{12}c_{23} - e^{i\delta}s_{12}s_{13}s_{23} \\ -c_{12}s_{23} - e^{i\delta}s_{12}s_{13}c_{23} \end{pmatrix}, \quad u_3 = \begin{pmatrix} e^{-i\delta}s_{13} \\ c_{13}s_{23} \\ c_{13}c_{23} \end{pmatrix}$$



The structure of  $g$  is partly determined to reproduce the PMNS matrix.

$$M_\nu = \frac{1}{M_0} f M_e g M_e f^T \quad f = f_0 \begin{pmatrix} 0 & U_{\tau 1} & -U_{\mu 1} \\ -U_{\tau 1} & 0 & U_{e 1} \\ U_{\mu 1} & -U_{e 1} & 0 \end{pmatrix}$$

$f_0$  : proportionality coefficient

$$\frac{f_0^2}{M_0} M_e g M_e = m_2 u_3 u_3^T + m_3 u_2 u_2^T + a_1 u_1 u_1^T + a_2 (u_1 u_2^T + u_2 u_1^T) + a_3 (u_1 u_3^T + u_3 u_1^T)$$

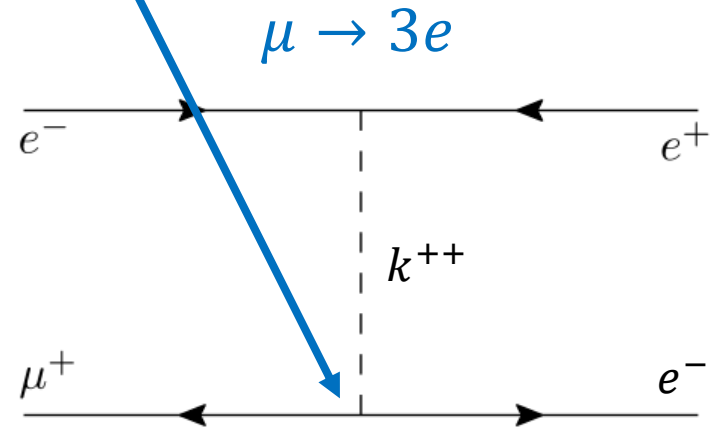
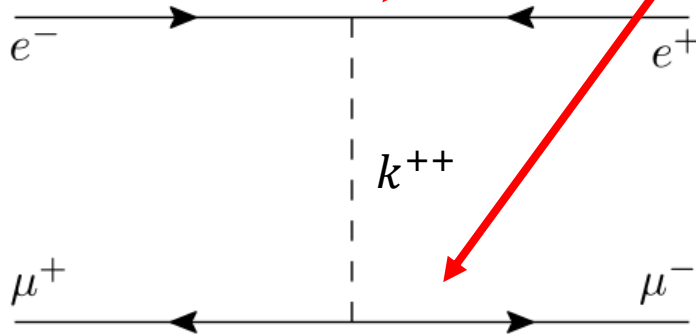
$a_1, a_2, a_3$ : free parameters

$$-\mathcal{L} \supset g_{ij} \bar{e}_i e_j^c k^{--}$$

$$g = \begin{pmatrix} g_{ee} & g_{e\mu} & g_{e\tau} \\ g_{e\mu} & g_{\mu\mu} & g_{\mu\tau} \\ g_{e\tau} & g_{\mu\tau} & g_{\tau\tau} \end{pmatrix}$$

$\tau^- \rightarrow e^+ \ell \ell'$   
 $\tau^- \rightarrow \mu^+ \ell \ell'$

Mu-to-Mu



In analysis, we set  $a_1, a_2, a_3$  as follows:

- adjust  $g_{ee}$  using one degrees of freedom of  $a_1, a_2, a_3$   
( $g_{ee}$  tends to be larger than 1 due to  $g_{ee} m_e^2 \sim g_{\mu\mu} m_\mu^2$ .)
- eliminate  $g_{e\mu}$  &  $g_{e\tau}$  using the rest two degrees

⇒  $g_{\mu\tau}$  cannot be eliminated:  $\tau^- \rightarrow \mu^+ e^- e^-$  &  $\tau \rightarrow 3\mu$  are predicted.

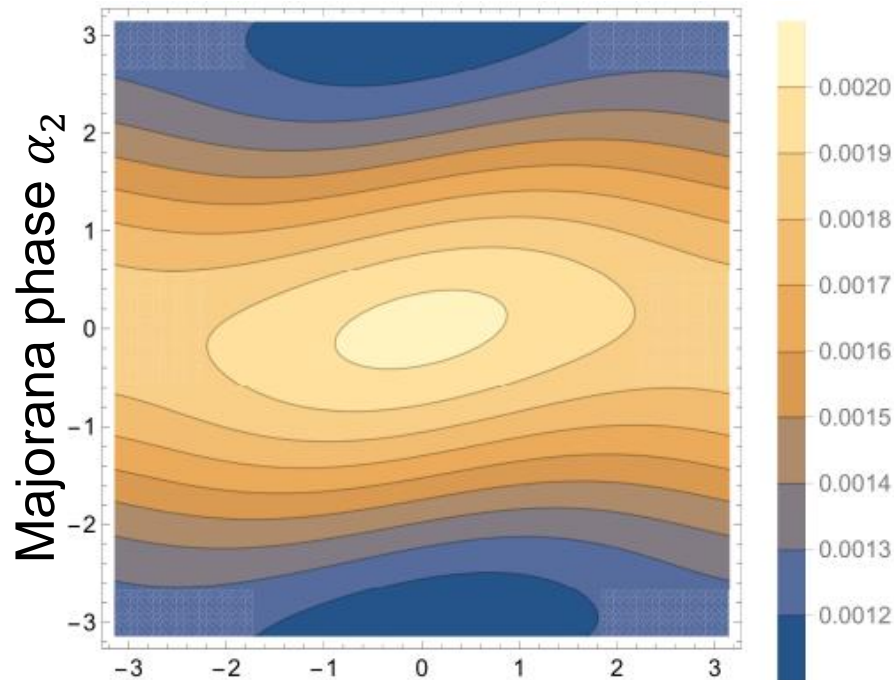
$$g_{ee} = g_{\mu\mu}, g_{e\mu} = g_{e\tau} = 0,$$

$$f_0^2 = 0.002, m_k = 1.2 \text{ TeV}, M_0/(48\pi^2) = 500 \text{ GeV}$$

$$-\mathcal{L} \supset \frac{G_2}{\sqrt{2}} [\bar{\mu}\gamma_\alpha(1 + \gamma_5)e][\bar{\mu}\gamma^\alpha(1 + \gamma_5)e]$$

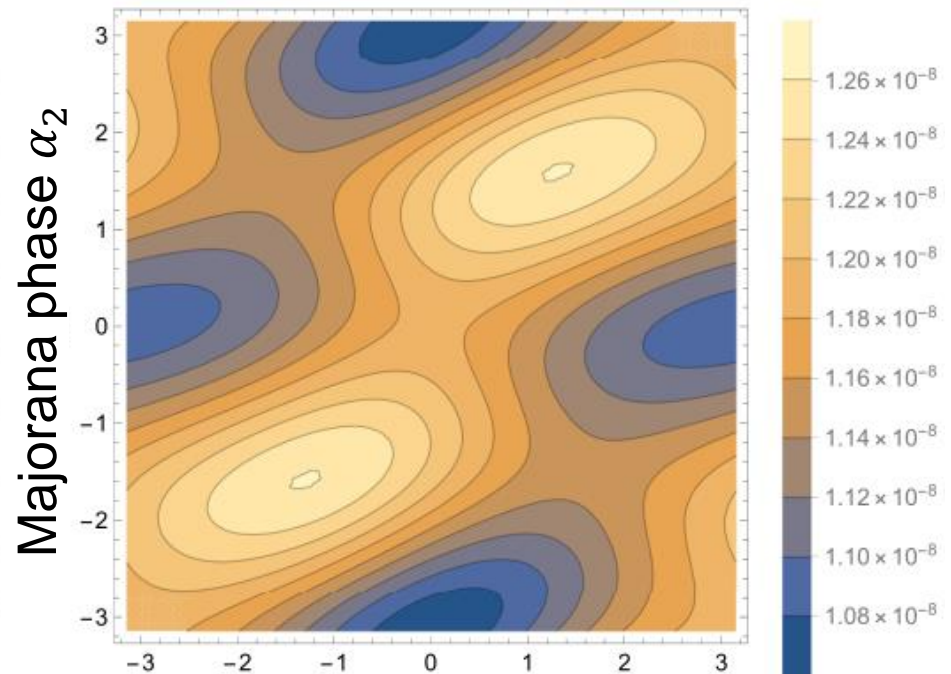
$$|G_2|/G_F$$

$$\text{Br}(\tau \rightarrow 3\mu) = \text{Br}(\tau^- \rightarrow \mu^+ e^- e^-)$$



Dirac phase  $\delta$

$$\Rightarrow \frac{G_2}{G_F} \sim \mathcal{O}(10^{-3})$$



Dirac phase  $\delta$

$$\Rightarrow \text{Br}(\tau \rightarrow 3\mu) \sim \mathcal{O}(10^{-8})$$

(just below the current experimental upper limit)

# Summary

- Mu-to- $\overline{\text{Mu}}$  transition

- ✓ rare process with  $\Delta L_\mu = -\Delta L_e = 2$
- ✓ good probe for the leptonic structure of the new physics model
- ✓ future experiments are planned in Japan & China
- ✓ We investigate how large impacts Mu-to- $\overline{\text{Mu}}$  gives for many models.

T. Fukuyama, Y. Mimura, & Y. Uesaka, PRD**105**, 015026 (2022).

- e.g. Zee-Babu model

- ✓ one of radiative neutrino models ( two loop )
- ✓ Mu-to- $\overline{\text{Mu}}$  rate can be the same as the current limit with reproducing neutrino masses & satisfying other LFV constraints.
- ✓ It is interesting to cross-check  $\tau$  rare decay with Mu-to- $\overline{\text{Mu}}$ .