CFT duals of three-dimensional de Sitter gravity

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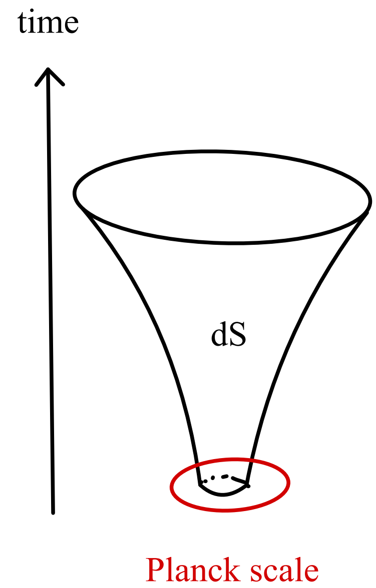
Introduction

De Sitter space

- Spacetime with positive cosmological constant
- Relevant to our Universe
  - Inflationary era
  - Present expanding universe

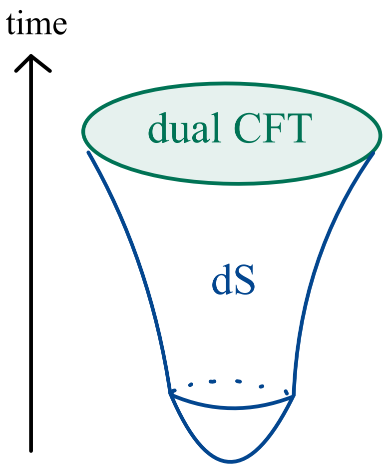


Motivation

Want to understand quantum gravity on dS spacetime!

Approaches to dS quantum gravity

- String theory : dS is hard to construct
- Holography: describe QG on dS by a dual field theory
  - dS / CFT correspondence [Strominger 01, Witten 01, Maldacena 02, …]



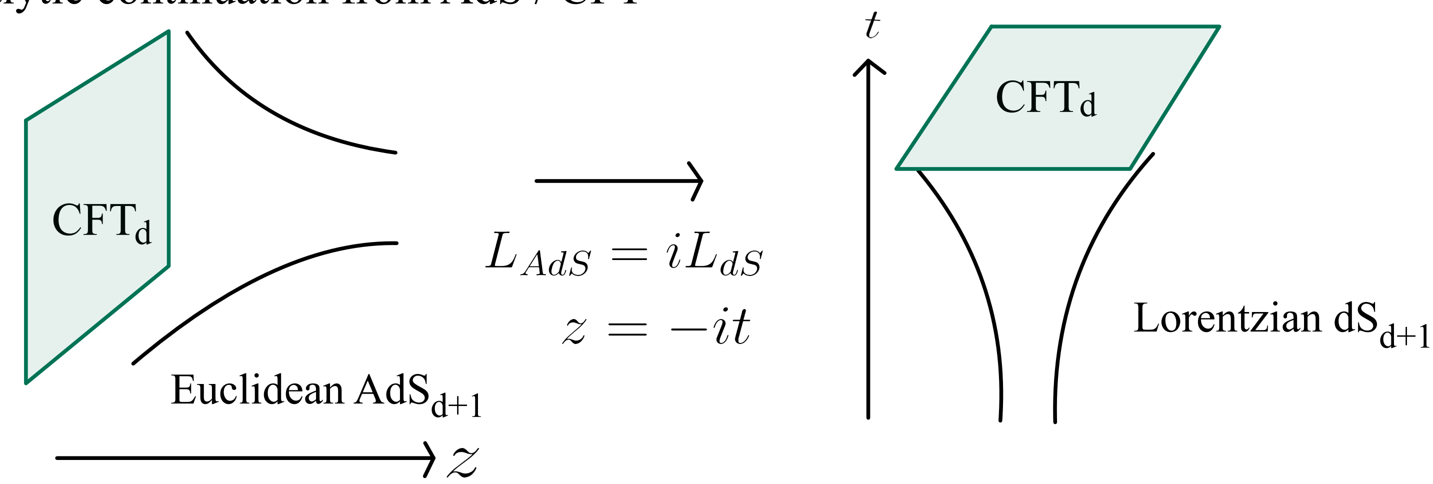
$$\Psi_{\mathrm{dS}}\left[\phi^{(0)}\right] = Z_{\mathrm{CFT}}\left[\phi^{(0)}\right]$$

dS wave function

CFT partition function

Expected property of CFT dual to dS

- Analytic continuation from AdS / CFT



For $d = 2$

Central charge

$$c = \frac{3L_{AdS}}{2G_N} = i\frac{3L_{dS}}{2G_N}$$

CFT with
imaginary central charge!

We propose a concrete model of dS/CFT

Outline

- de Sitter basics
- dS / CFT correspondence
- Our proposal

Einstein gravity on dS_3	$\Longleftrightarrow$	$SU(2)_k$ WZW in $k \to -2$ limit

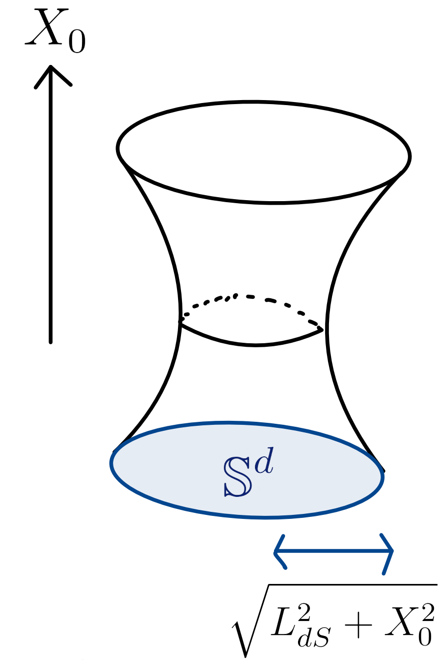
- Tests of the proposal
- Higher - spin generalization
- Summary & future problems

de Sitter basics

$(d+1)$- dimensional de Sitter (dS_{d+1})

$=$ hypersurface in $\mathbb{R}^{1,d+1}$

$$-X_0^2 + X_1^2 + \cdots + X_{d+1}^2 = L_{dS}^2$$



- $SO(1,d+1)$ symmetry
- positive cosmological constant $\Lambda = \dfrac{d(d-1)}{2L_{dS}^2}$

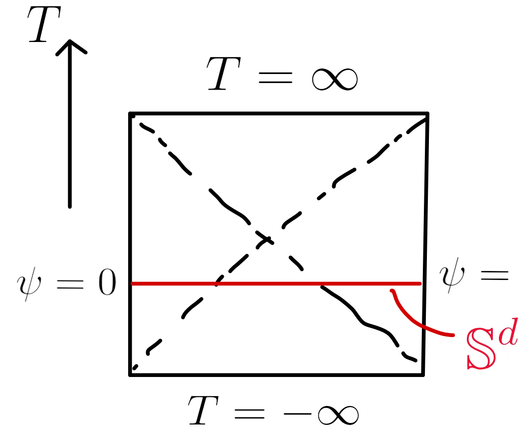
$$I = \frac{1}{16\pi G_N}\int d^{d+1}x\sqrt{-g}(R-2\Lambda)$$

Global coordinates

$$ds^2 = L_{dS}^2\left(-dT^2 + \cosh^2 T d\Omega_d^2\right)$$

$$d\Omega_d^2 = d\psi^2 + \sin^2\psi d\Omega_{d-1}^2$$

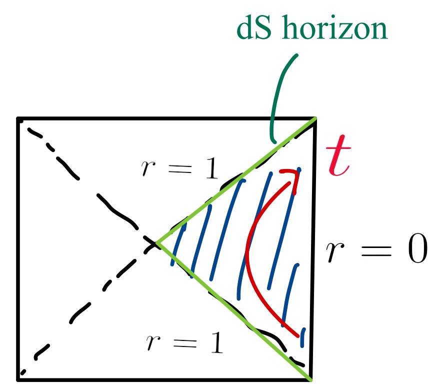
- cover whole region



Static coordinates

$$ds^2 = L_{dS}^2\left[-\left(1-r^2\right)dt^2 + \frac{dr^2}{1-r^2} + r^2 d\Omega_{d-1}^2\right]$$

- cover the shaded region

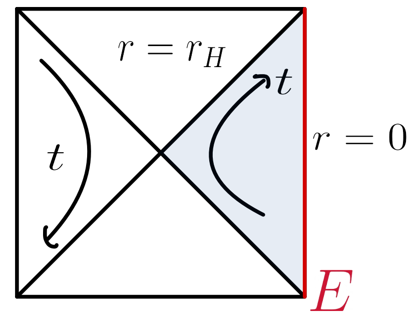


dS horizon : $r = 1$

dS_3 BH $(d=2)$

$$ds^2 = L_{\rm dS}^2 \left[-(1-8G_N E - r^2)\, dt^2 + \frac{dr^2}{1-8G_N E - r^2} + r^2 d\psi^2 \right]$$

$$(0 \leq r \leq r_H, \psi \sim \psi + 2\pi)$$

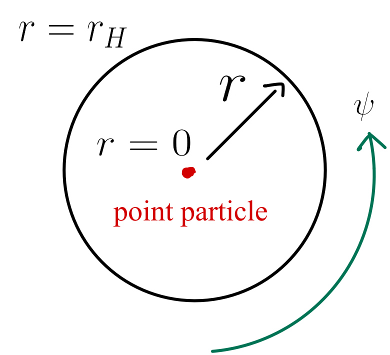


- horizon : $r_H = \sqrt{1 - 8G_N E}$

- conical deficit at $r = 0$

$$\psi \sim \psi + 2\pi - \delta$$

$$\delta = 2\pi \left(1 - \sqrt{1 - 8G_N E} \right)$$



→ point particle of energy E

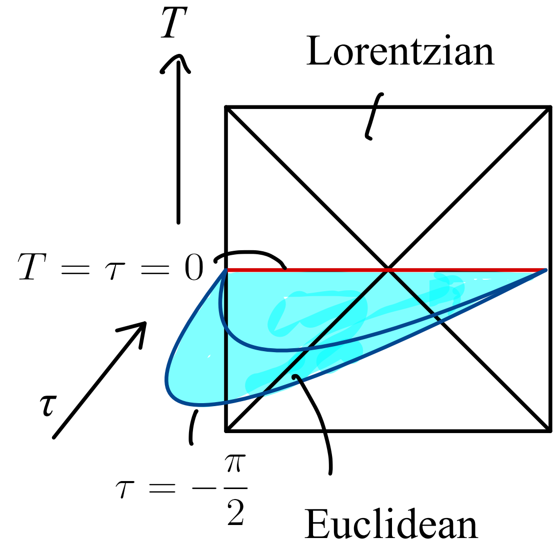
dS thermodynamics

- Global dS

$$ds^2 = L_{dS}^2 \left(-dT^2 + \cosh^2 T d\Omega_d^2\right)$$

Analytic continuation to Euclidean time

$$T \to -i\tau \quad \text{at} \quad T = \tau = 0$$



- Euclidean dS = sphere

$$ds_E^2 = L_{dS}^2 \left[d\tau^2 + \cos^2 \tau d\Omega_d^2\right]$$

$$-\frac{\pi}{2} \leq \tau \leq \frac{\pi}{2}$$

dS BH temperature & entropy

- Near $r = r_H$

$$ds^2 \approx L_{dS}^2 \left[\rho^2 d\left(r_H t_E\right)^2 + d\rho^2 + r_H^2 d\psi^2\right]$$

$$\rho \equiv \sqrt{\frac{2}{r_H}\left(r - r_H\right)}$$

- No conical singularity at $\rho = 0 \Leftrightarrow t_E \sim t_E + \frac{2\pi}{r_H}$

· dS BH temperature :

$$T_{BH} = \frac{r_H}{2\pi L_{dS}} = \frac{\sqrt{1 - 8G_N E}}{2\pi}$$

· dS BH entropy:

$$S_{BH} = \frac{A}{4G_N} = \frac{\pi L_{dS}}{2G_N}\sqrt{1 - 8G_N E}$$

Partition function

- Euclidean partition function of gravity

$$\log Z_{\mathrm{grav}} = -I_E = \frac{1}{16\pi G_N}\int_{M_3} d^3x\sqrt{g}(R-2\Lambda) \xrightarrow[\text{on-shell}]{} \frac{\mathrm{Vol}\,(M_3)}{4\pi G_N L_{dS}^2}$$

• dS BH :

$$\log Z_{\mathrm{grav}} = \frac{\pi L_{dS}\sqrt{1-8G_N E}}{2G_N}$$

cf) This is consistent with thermodynamic identity

$$\log Z_{\mathrm{grav}} = -\frac{1}{T_{BH}}E_{BH} + S_{BH} = S_{BH}$$

($E = 0$ for a compact space in GR)

dS / CFT correspondence

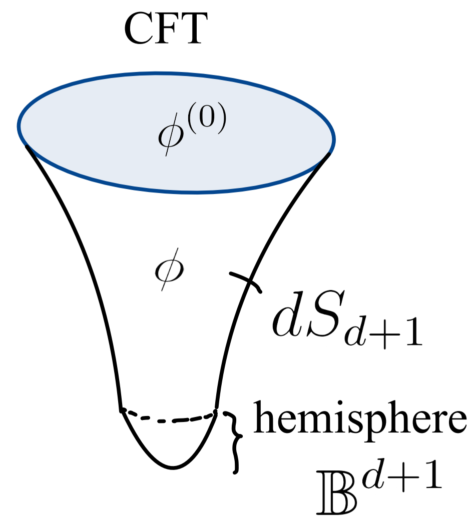
[Strominger 01, Maldacena 02]

<u>"Dictionary"</u>

$$\Psi_{\rm dS}\left[\phi^{(0)}\right] \quad = \quad Z_{\rm CFT}\left[\phi^{(0)}\right]$$

wave function
w/ future b.c.

CFT partition function
w/ source $\phi^{(0)}$
coupled to CFT op.



$\Rightarrow$

$$\langle O(x)O(y)\rangle_{CFT} = \left.\frac{\delta^2\Psi_{dS}}{\delta\phi^{(0)}(x)\delta\phi^{(0)}(y)}\right|_{\phi^{(0)}=0}$$

Example

dS_4/CFT_3 [Anninos-Hartman-Strominger 11]

Vasiliev higher-spin gravity on dS_4

$\Updownarrow$

Euclidean $Sp(N)$ model in 3d

CFT is NOT unitary!

Obtained by analytic continuation $N \to -N$

of Vasiliev HS on AdS_4 /$O(N)$ model [Klebanov-Polyakov 04]

Analytic continuation from Euclidean AdS to dS

- Euclidean Poincare AdS :

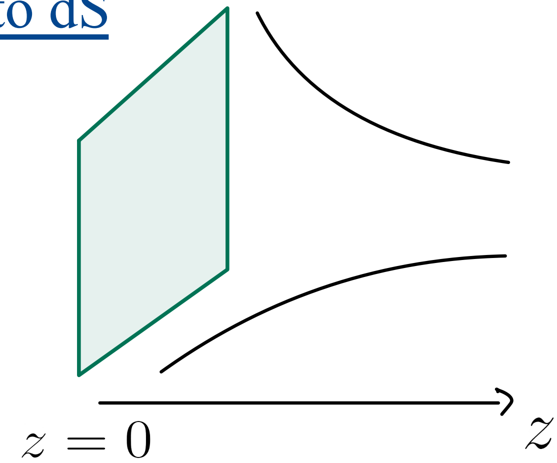
$$ds^2_{AdS} = L^2_{AdS} \frac{dz^2 + d\vec{x}^2}{z^2}$$

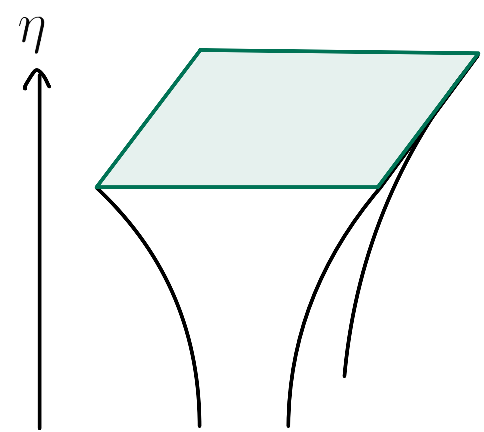
- Lorentzian flat slicing of dS :

$$ds^2_{dS} = L^2_{dS} \frac{-d\eta^2 + d\vec{x}^2}{\eta^2}$$

Analytic continuation :

$$z = -i\eta, \quad L_{AdS} = iL_{dS}$$





dS_3/CFT_2

- AdS_3/CFT_2

central charge $c = \frac{3L_{\rm AdS}}{2G_N}$

$\Downarrow$

- dS_3/CFT_2

central charge $c = i\frac{L_{dS}}{2G_N}$

Non-unitary!

What is the microscopic theory of the CFT dual of dS_3 ?

Our proposal

Einstein gravity on dS_3 with $\frac{L_{dS}}{G_N} \gg 1$

$\Updownarrow$

$SU(2)_k$ WZW model (+ some CFT)

in $k \to -2 + i\frac{4G_N}{L_{dS}}$ limit

$SU(2)_k$ WZW model

- Wess-Zumino-Witten model

  = sigma model valued on SU(2) with topological term

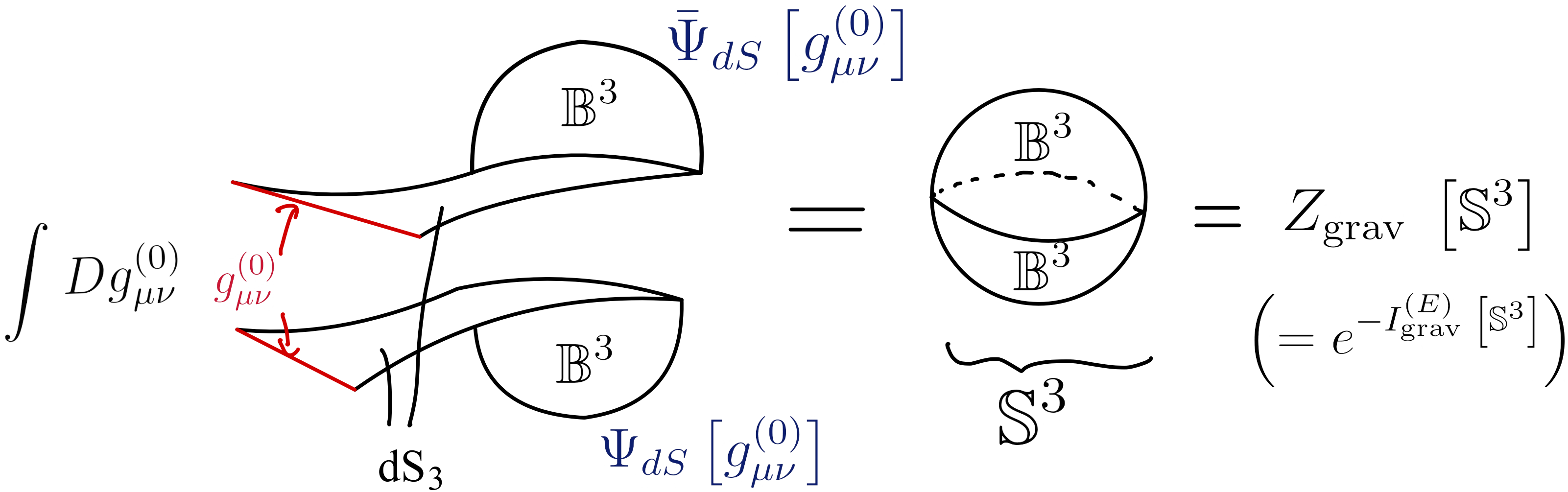
- central charge

$$c = \frac{3k}{k+2} \xrightarrow[k \to -2 + i\frac{4G_N}{L_{dS}}]{} ic^{(g)} \equiv i\frac{3L_{dS}}{2G_N} + O(1)$$

Reproduces the central charge of the dual CFT to dS_3 !

Partition function of dS_3

We will consider pure Einstein gravity



- Lorentzian parts cancel out :

$$\Psi_{dS} \sim \underbrace{e^{iI^{(L)}_{\rm grav}\left[dS_3\right]}}_{\text{phase}}{}^{-I^{(\bar{E})}_{\rm grav}\left[\mathbb{B}^3\right]}$$

In this setup

$$Z_{\text{grav}}\left[\mathbb{S}^3\right] \sim \int Dg^{(0)}_{\mu\nu}\left|\Psi_{dS}\left[g^{(0)}_{\mu\nu}\right]\right|^2$$

$$\sim \left|\Psi_{dS}\left[\mathbb{S}^2\right]\right|^2 \qquad \text{(saddle point approximation)}$$

$$\underset{\text{dS/CFT}}{=} \left|Z_{CFT}\left[\mathbb{S}^2\right]\right|^2$$

Our proposal suggests

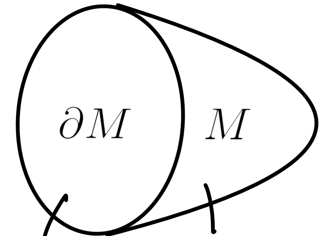
$$\boxed{Z_{\text{grav}}\left[\mathbb{S}^3\right] = \left|Z_{WZW}\left[\mathbb{S}^2\right]\right|^2} \qquad (G_N \to 0)$$

CS / WZW correspondence

[Witten 89, Elitzur-Moore-Schwimmer-Seiberg 89]

G Chern-Simons (CS) theory on 3d manifold M

= chiral G WZW on 2d boundary ∂M



⇒

$SU(2)_k \times SU(2)_{-k}$ CS = non-chiral $SU(2)_k$ WZW

$$I_{SU(2)^2 CS}[M] = I_{\mathrm{SU}(2) WZW}[\partial M]$$

For $M = \mathbb{B}^3, \partial M = \mathbb{S}^2$

$$Z_{SU(2)^2 CS}\left[\mathbb{B}^3\right] = Z_{SU(2) WZW}\left[\mathbb{S}^2\right]$$

Our proposal + CS / WZW correspondence

$$Z_{\text{grav}}\left[\mathbb{S}^3\right] = \left|Z_{SU(2)^2CS}\left[\mathbb{B}^3\right]\right|^2 = \left|Z_{SU(2)CS}\left[\mathbb{S}^3\right]\right|^2$$

- This is the same form as CS formulation of Einstein gravity in 3d!

$$I_{grav} = I_{SU(2)_k \times SU(2)_{-k} CS} \qquad \text{with} \qquad k = \frac{L_{dS}}{4G_N} \gg 1$$

- However, our limit is different from the classical limit

$$k \to -2 + i\frac{4L_{dS}}{G_N}$$

CS partition function

[Witten 88]

Determined by the modular S-matrix of the $SU(2)_k$ WZW

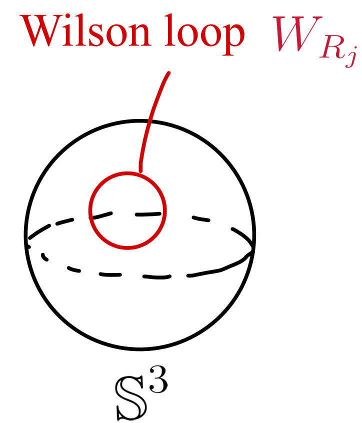
$$S_j^l = \sqrt{\frac{2}{k+2}} \sin\left[\frac{\pi}{k+2}(2j+1)(2l+1)\right]$$

- Vacuum partition function :

$$Z_{SU(2)CS}\left[\mathbb{S}^3\right] = S_0^0$$

- With Wilson loop :

$$Z_{SU(2)CS}\left[\mathbb{S}^3, R_j\right] = S_0^j$$



Check 1 : Vacuum partition function

- CFT side

$$\left|Z_{\mathrm{SU(2)CS}}\left[\mathbb{S}^3\right]\right|^2 = \left|S_0^0\right|^2 \xrightarrow[k \to -2 + i\frac{6}{c^{(g)}}]{} e^{\frac{\pi}{3}c^{(g)} + O\left(\log c^{(g)}\right)}$$

Agree in $c^{(g)} \to \infty$ limit!

- Gravity side

$$Z_{\mathrm{grav}}\left[\mathbb{S}^3\right] = \exp\left[-I_{\mathrm{grav}}\left[\mathbb{S}^3\right]\right] = e^{\frac{\pi}{3}c^{(g)}}$$

Check 2 : partition function with excitation

- CFT side : Wilson loop

$$\left|Z_{SU(2)CS}\left[\mathbb{S}^3, R_j\right]\right|^2 = \left|S_0^j\right|^2 \xrightarrow[k \to -2 + i\frac{6}{c^{(g)}}]{} e^{\frac{\pi}{3}c^{(g)}(2j+1) + O\left(\log c^{(g)}\right)}$$

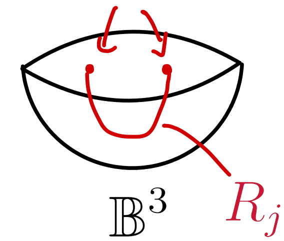
- Gravity side : Euclidean dS BH of energy E_j

$$Z_{\text{grav}}\left[dS_3 BH\right] = e^{\frac{\pi}{3}c^{(g)}\sqrt{1-8G_N E_j}}$$

They agree if $2j+1 = \sqrt{1-8G_N E_j}$

- We use the correspondence

primaries (excitations)



Wilson loop in CS = primary operator in WZW

Primary op. in R_j rep in SU(2) WZW

$$\Delta_j = \frac{2j(j+1)}{k+2} \xrightarrow[k \to -2 + i\frac{6}{c^{(g)}}]{} i\Delta_j^{(g)} = -i\frac{c^{(g)} j(j+1)}{3}$$

↑ conformal dimension

In dS/CFT, we identify $\Delta_j^{(g)} = L_{dS} E_j$

$$\Longrightarrow \quad (2j+1)^2 = 1 - 8G_N E_j$$ as expected!

Higher-spin generalization

Higher-spin gravity on dS_3 in $\frac{L_{dS}}{G_N} \gg 1$

with spin 2, …, N fields

$$\Updownarrow$$

$SU(N)_k$ WZW model (+ some CFT)

in $k \to -N + i\frac{16\epsilon_N^2 G_N}{L_{dS}}$

$$\left(\epsilon_N \equiv \frac{N\left(N^2-1\right)}{12}\right)$$

Evidences

- Perfect match between CFT and gravity partition functions

- Realization as an analytic continuation of
  Gaberdiel-Gopakumar duality

Higher-spin gravity on AdS_3 = W_N-minimal model

$$\frac{SU(N)_k \times SU(N)_1}{SU(N)_{k+1}}$$

dominant part in our limit

Summary & future problems

- We proposed a CFT dual of dS_3 gravity

HS gravity on dS_3 $\simeq$ $SU(N)_k$ WZW in $k \to -N$

- We checked this statement in the classical limit $G_N \to 0$

Future problems

- Quantum corrections in the G_N expansion
- Determine the spectrum of the dual CFT