Muon g-2

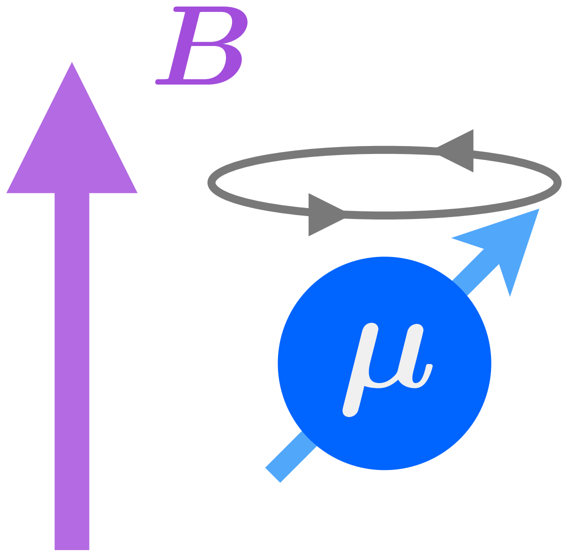
Motoi Endo (KEK)

Osaka seminar, 2022.6.17

What is magnetic moment (g-factor)?

Magnetic interaction with spin

$$\mathcal{H} = -\vec{\mu} \cdot \vec{B}$$



Magnetic moment $\propto$ spin

$$\vec{\mu} = -g \frac{e}{2m} \vec{S}$$

Tree-level prediction

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - m)\psi - \frac{1}{4}(F_{\mu\nu})^2 - e\bar{\psi}\gamma^\mu\psi A_\mu \longrightarrow g = 2$$

Radiative correction — "anomalous" magnetic moment, g-2

$$g \neq 2 \quad \Rightarrow \quad \boxed{a_\ell = \frac{g_\ell - 2}{2}}$$

flavor, CP conserved
super-precise

History of measurement (after 60's)

Authors	Lab	Muon Anomaly	
Garwin et al. '60	CERN	0.001 13(14)	
Charpak et al. '61	CERN	0.001 145(22)	
Charpak et al. '62	CERN	0.001 162(5)	
Farley et al. '66	CERN	0.001 165(3)	
Bailey et al. '68	CERN	0.001 166 16(31)	
Bailey et al. '79	CERN	0.001 165 923 0(84)	
Brown et al. '00	BNL	0.001 165 919 1(59)	(μ^+)
Brown et al. '01	BNL	0.001 165 920 2(14)(6)	(μ^+)
Bennett et al. '02	BNL	0.001 165 920 4(7)(5)	(μ^+)
Bennett et al. '04	BNL	0.001 165 921 4(8)(3)	(μ^-)

Long-standing target for >50 years!

$a_\mu^{\rm BNL} - a_\mu^{\rm SM} = 26.8(7.3) \times 10^{-10}$ — 3.5σ deviation

2021 News

Fermilab (Run-1, ~ BNL E821) [2104.03281]

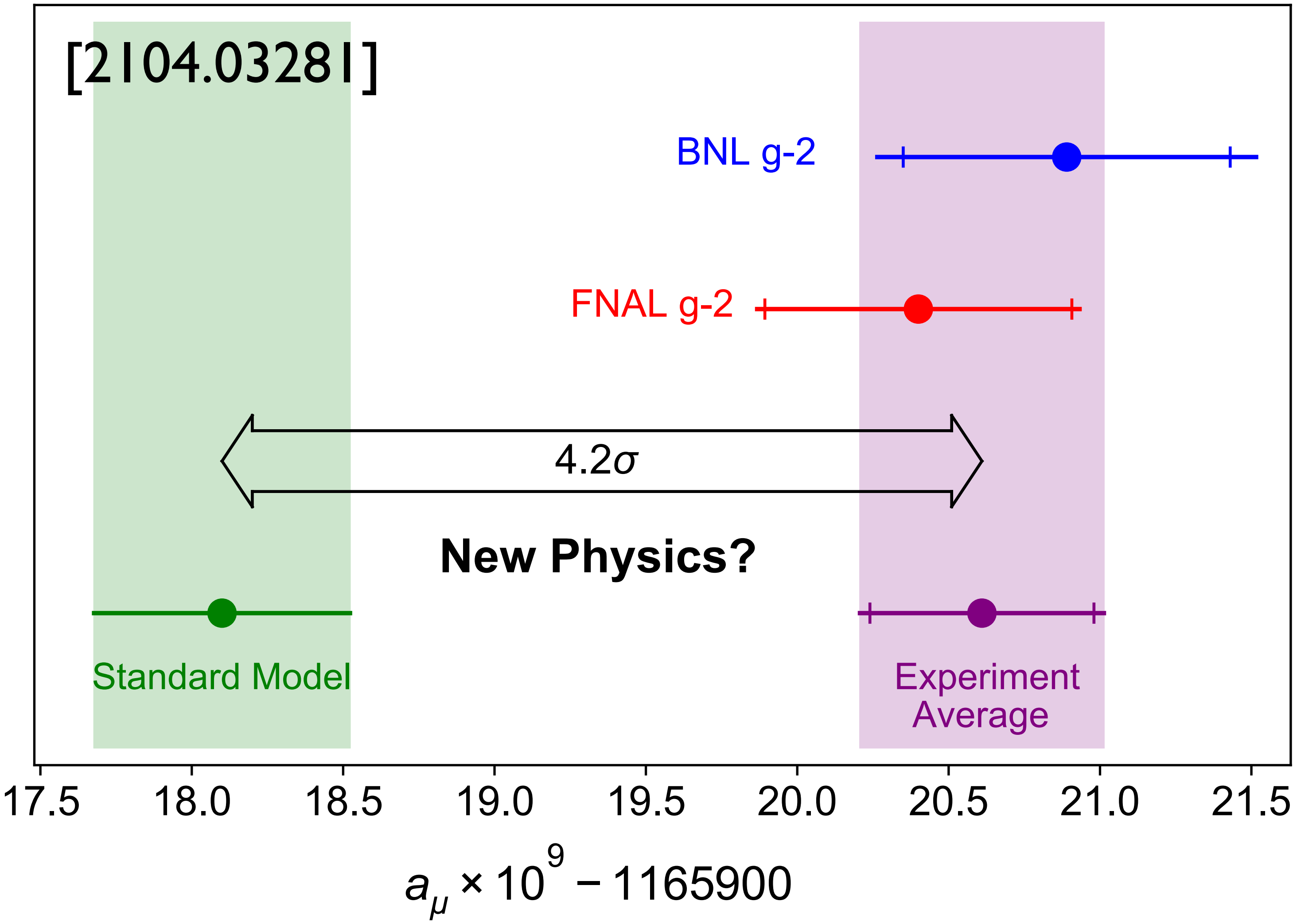
$$a_\mu^{\mathrm{FNAL}} = (11\,659\,204.0 \pm 5.1_{\mathrm{stat}} \pm 1.9_{\mathrm{sys}}) \times 10^{-10}$$

Confirmed BNL result: $a_\mu^{\mathrm{BNL}} = (11\,659\,208.9 \pm 5.4_{\mathrm{stat}} \pm 3.3_{\mathrm{sys}}) \times 10^{-10}$

$$a_\mu^{\mathrm{BNL+FNAL}} = (11\,659\,206.1 \pm 4.1) \times 10^{-10}$$

$$a_\mu^{\mathrm{BNL+FNAL}} - a_\mu^{\mathrm{SM}} = (25.1 \pm 5.9) \times 10^{-10}$$

4.2σ deviation



Today's talk

- Overview of measurement
- Recent progress and issue on SM prediction
- New physics interpretation

Experiment: principle

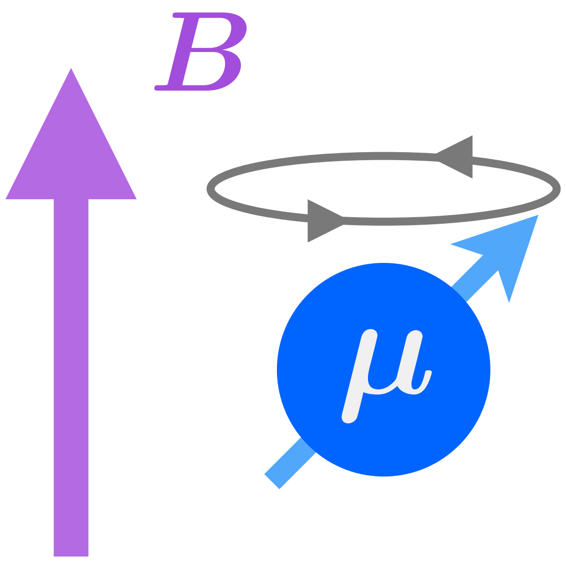
Lamor (spin) precession

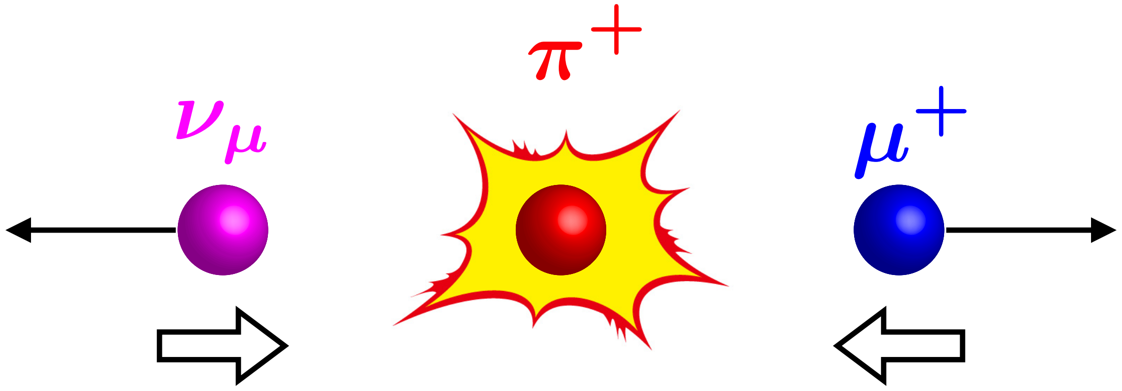
Polarized muon from
pion decay

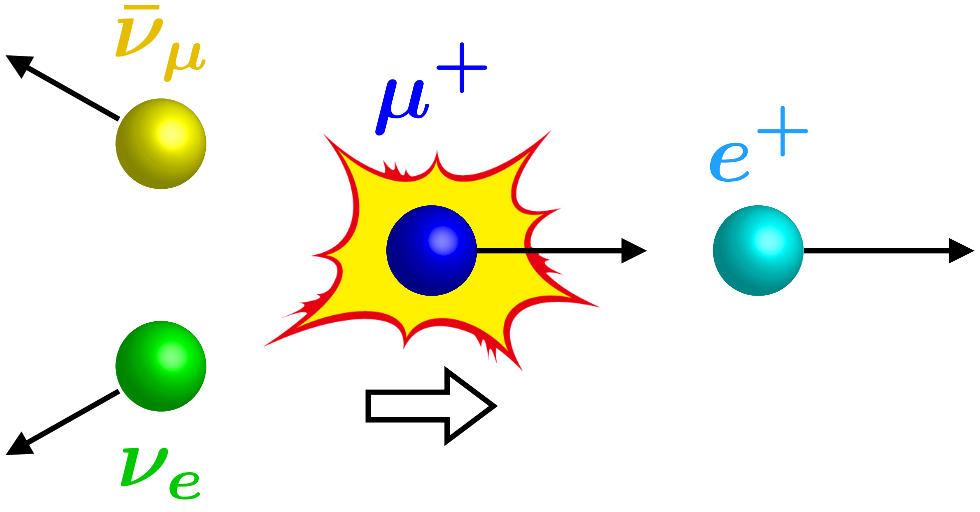
Muons in storage ring with
well-calibrated magnet

Energetic electron/positron
emission parallel to muon spin

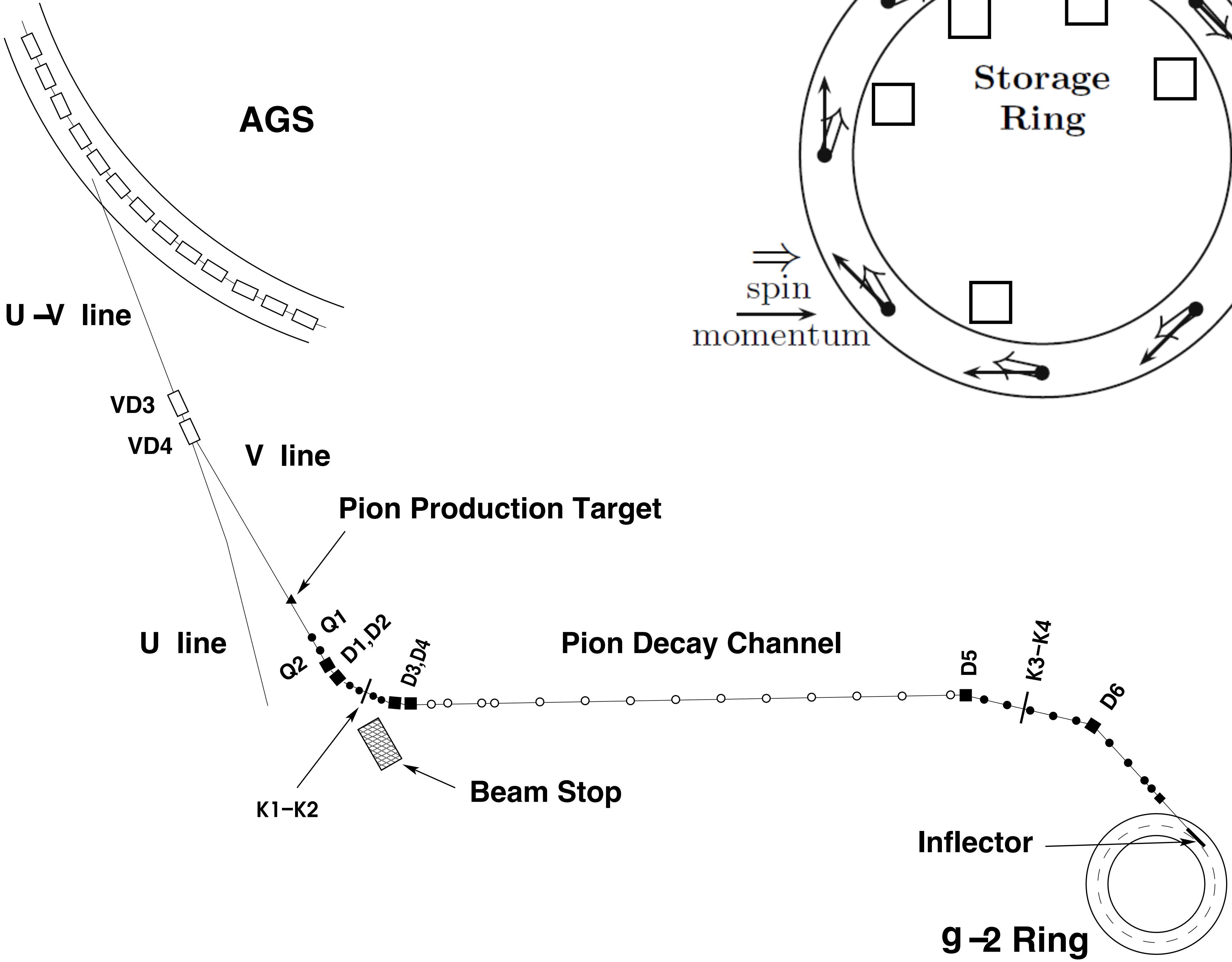
of $e^{+/-}$ events increases when
muon spin points to detector

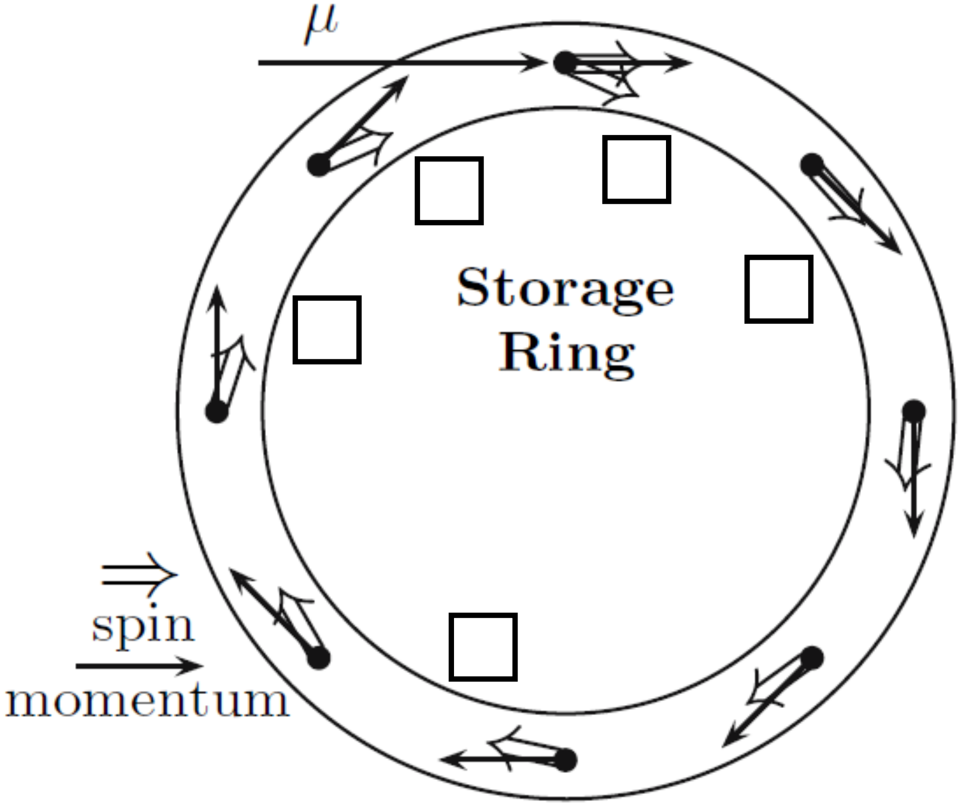






BNL & FNAL experiment





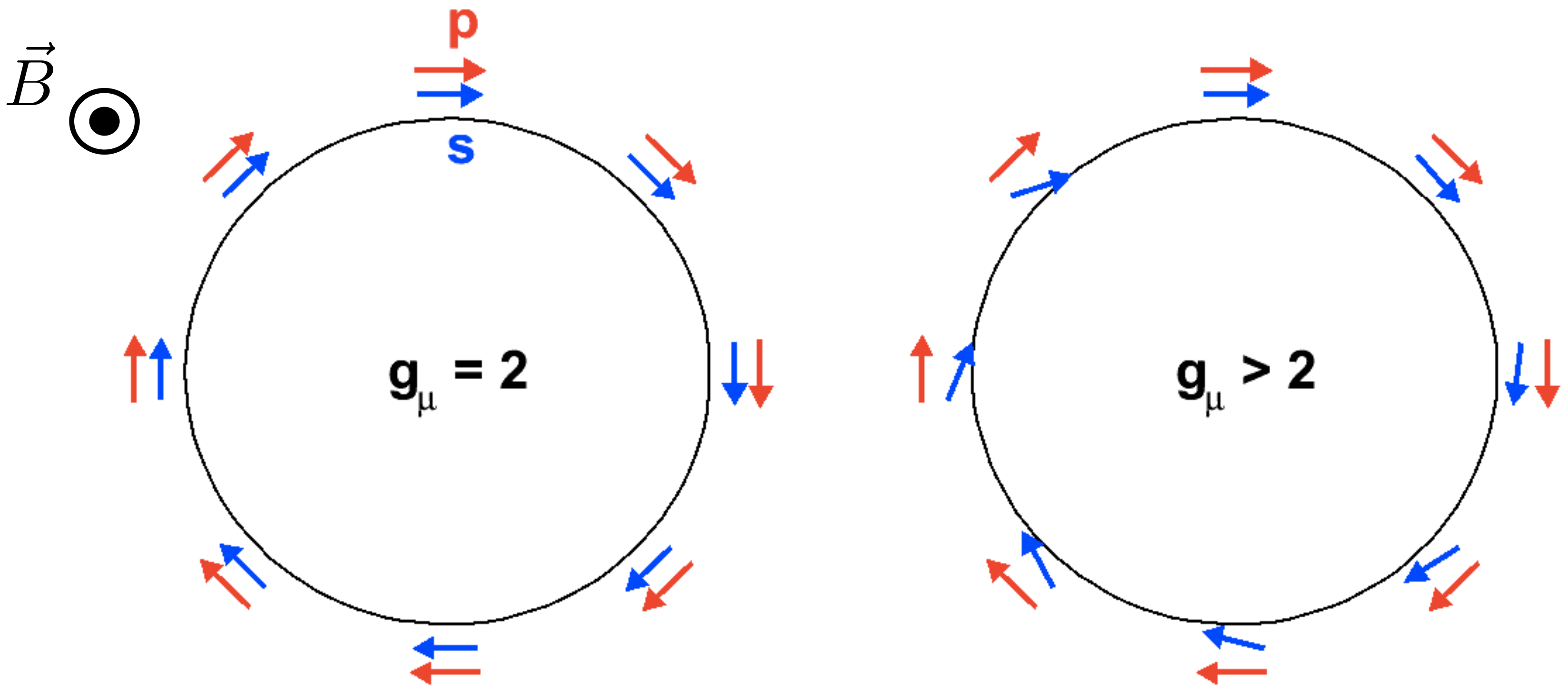
Measurement

Spin precession is faster than cyclotron precession for g>2

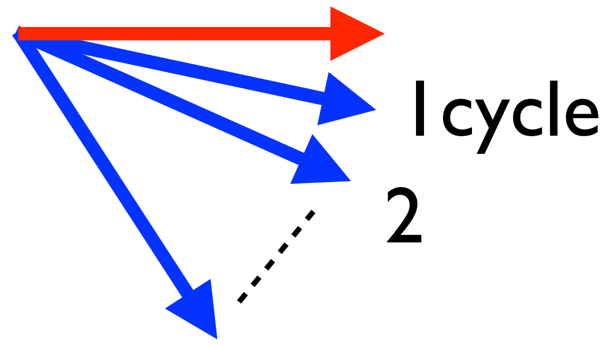
$$\vec{\omega}_a = \vec{\omega}_S - \vec{\omega}_C = -\frac{e}{m}\left[a_\mu \vec{B} - \cancel{\left(a_\mu - \left(\frac{mc}{p}\right)^2\right)} \frac{\vec{\beta}\times\vec{E}}{c}\right]$$

drop at p=3.094GeV

Precise measurement of ω_a and B (NMR)

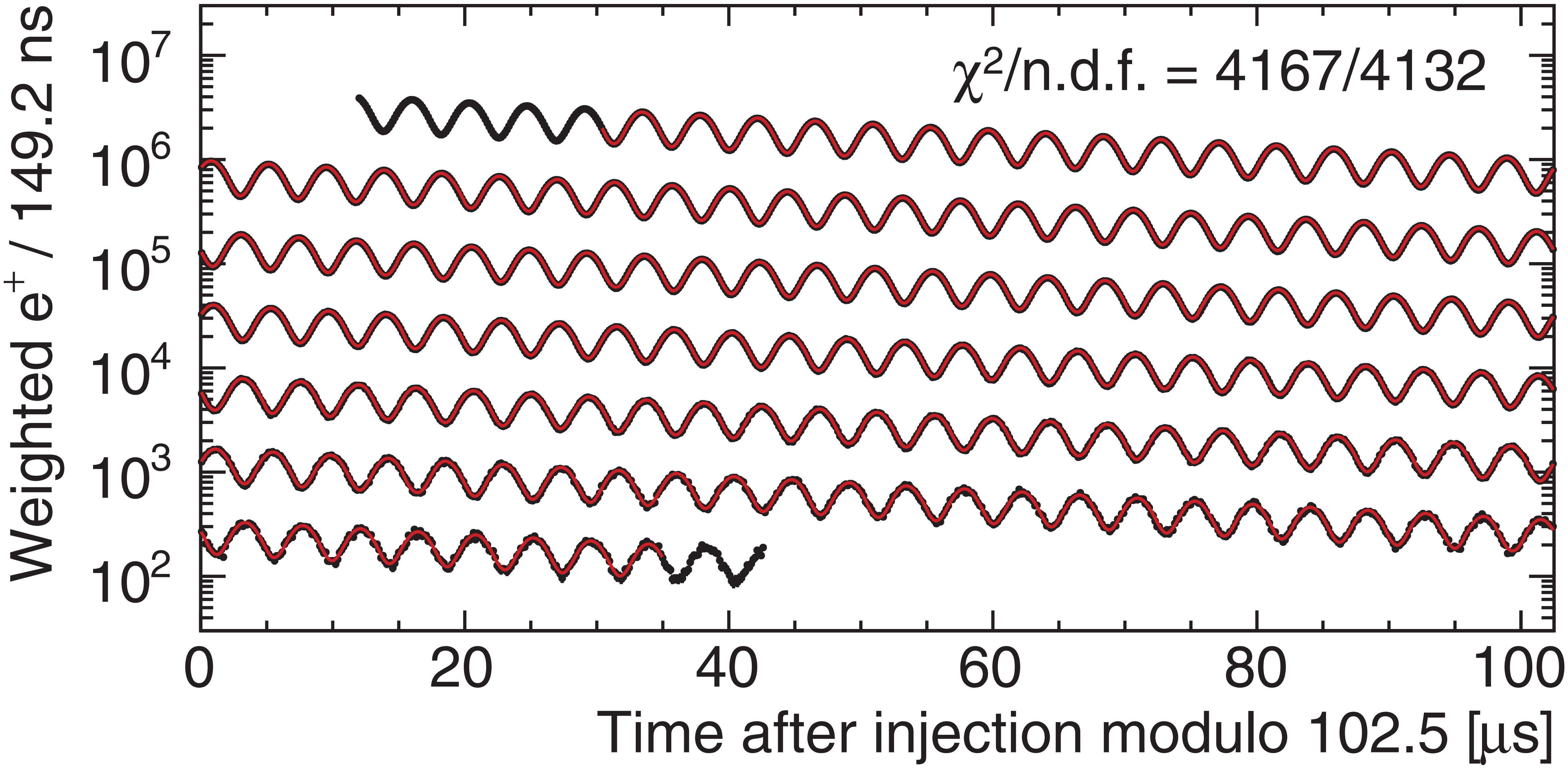


spin rotates 372°
per each cycle



Result

$$a_\mu^{\text{FNAL}} = (11\,659\,204.0 \pm 5.1_{\text{stat}} \pm 1.9_{\text{sys}}) \times 10^{-10}$$



Latest result

Fermilab (Run-1, ~ BNL E821) [2104.03281]

$$a_\mu^{\mathrm{FNAL}} = (11\,659\,204.0 \pm 5.1_{\mathrm{stat}} \pm 1.9_{\mathrm{sys}}) \times 10^{-10}$$

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4.2σ deviation

Experimental prospect

Fermilab E989 on going: same storage ring as BNL E821

Goal: 1/4 x BNL uncertainty

Statistics ~ 21.5 x BNL, Improve systematics

Run-4 finished: ~13 x BNL → Run-5 on-going

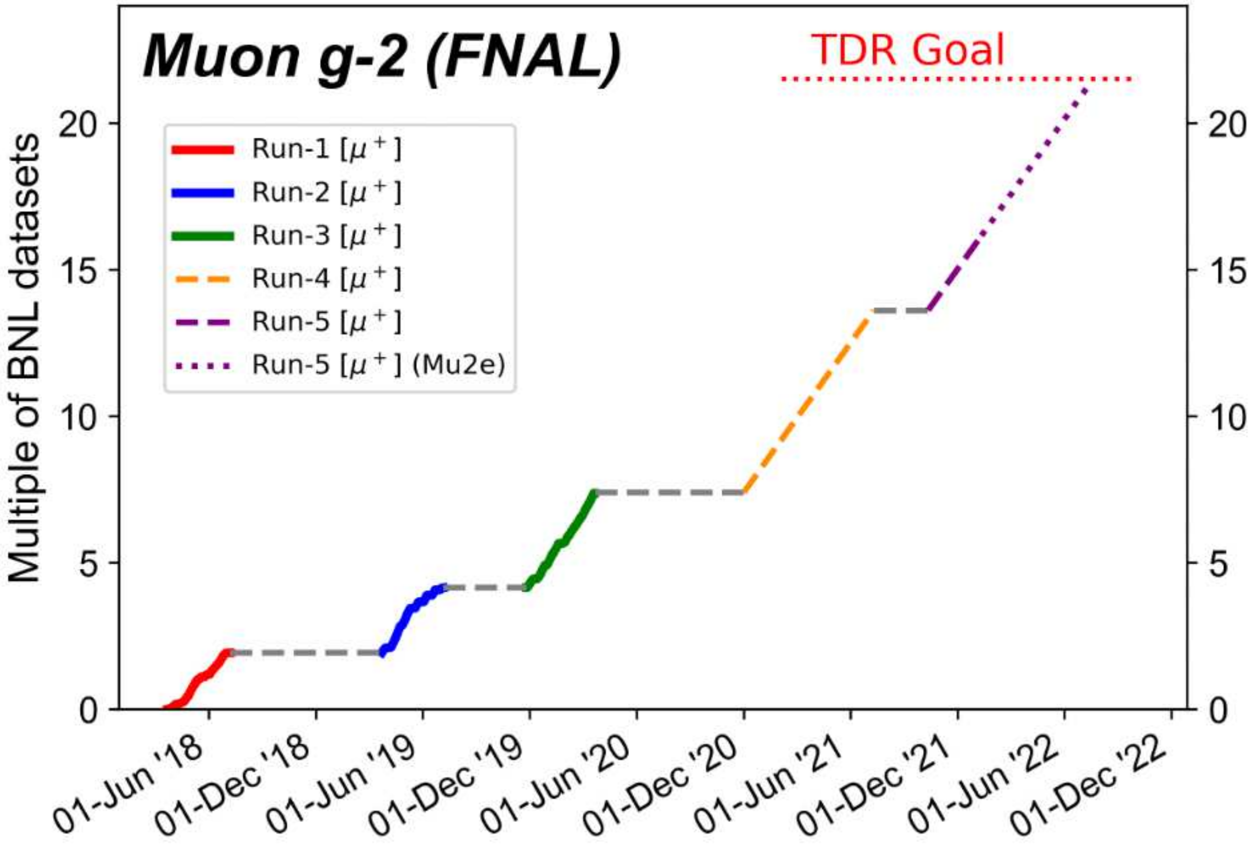
Run-2+3: expected stat. uncertainty ~ 200ppb (still > syst.)

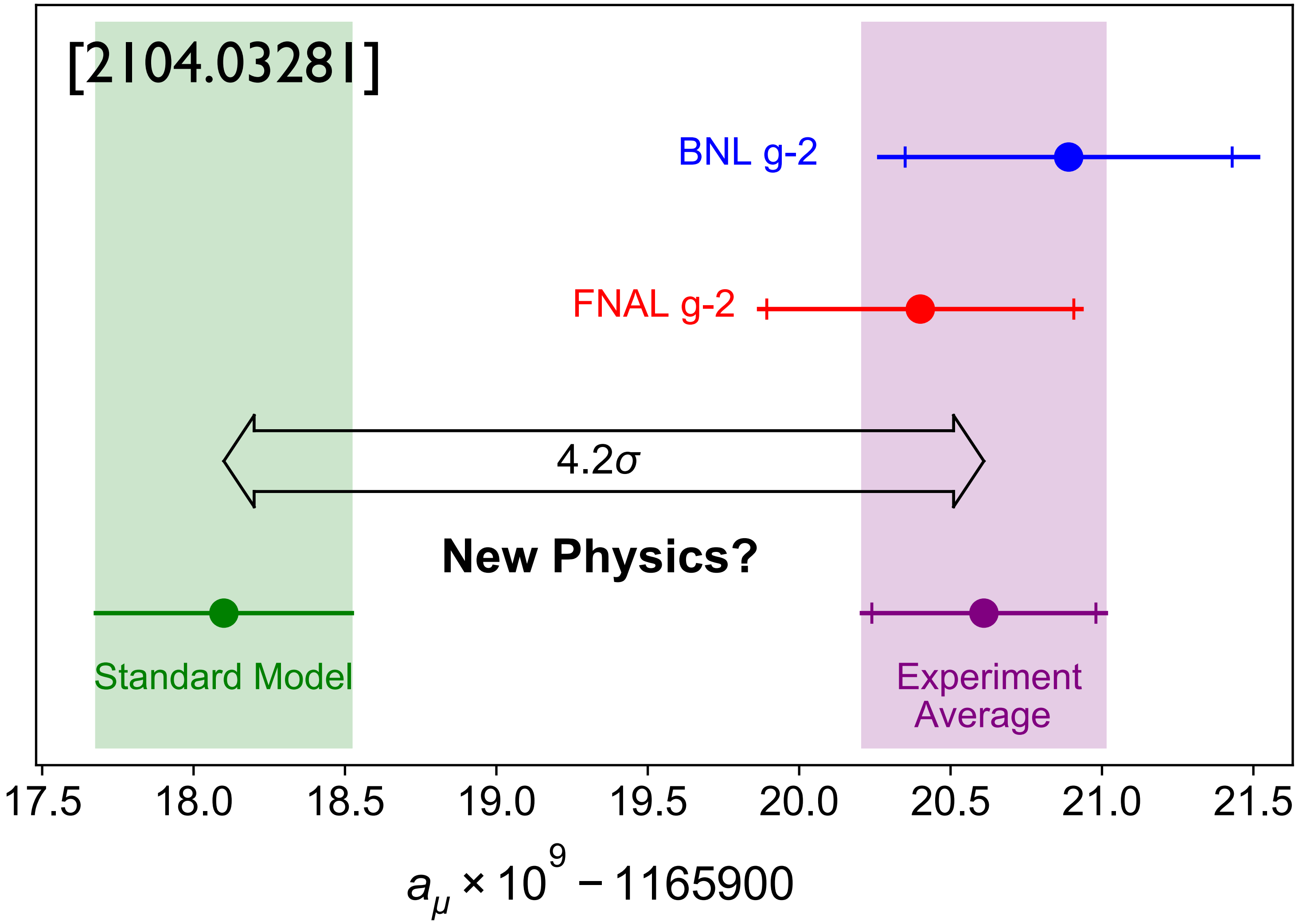
→ this year (?)

J-PARC in progress

Independent method

start in 2027 (?)





Today’s talk

- Overview of measurement
- Recent progress and issue on SM prediction
- New physics interpretation

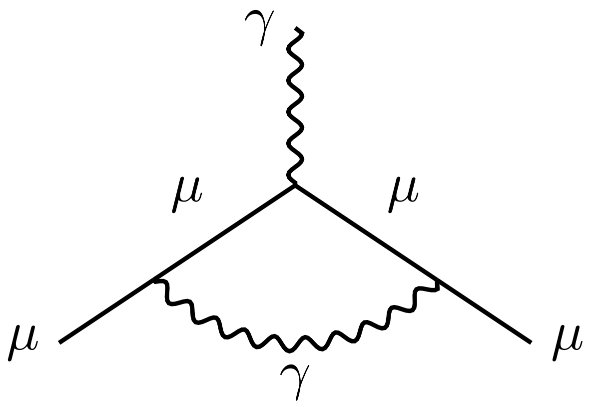
SM prediction

White paper (WP): current consensus of SM value [2006.04822]

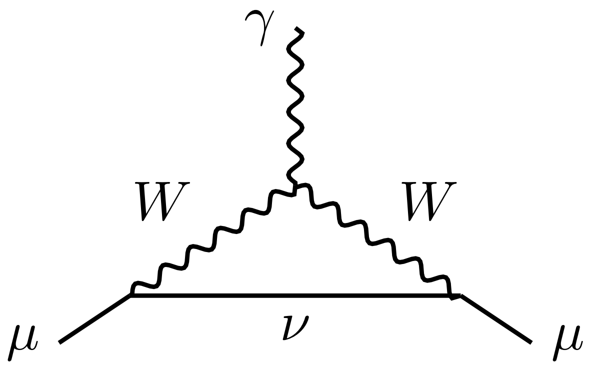
Contribution	Value ×10^{10}	
QED	11 658 471.8931	± 0.0104
EW	15.36	± 0.10
HVP	684.5	± 4.0
HLbL	9.2	± 1.8
Total	11 659 181.0	± 4.3

Recent issue:

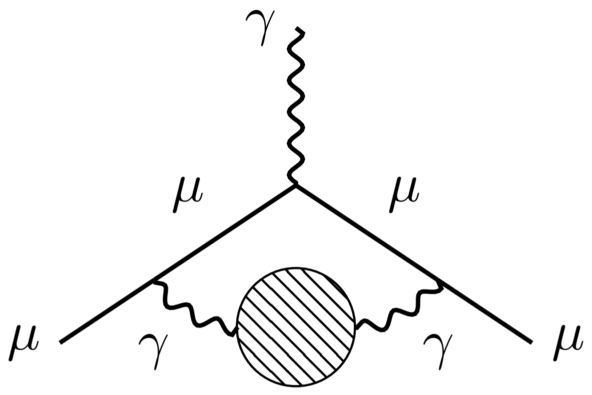
discrepancy between WP and new lattice result of HVP



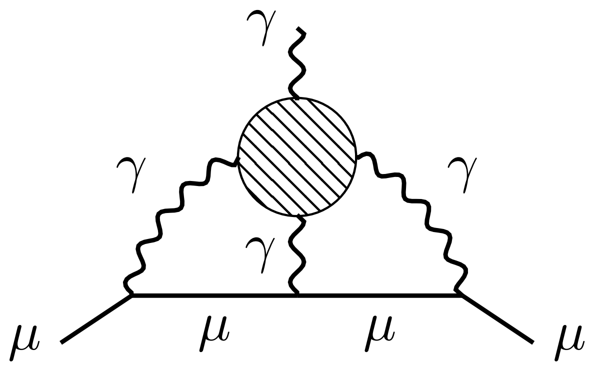
QED



EW



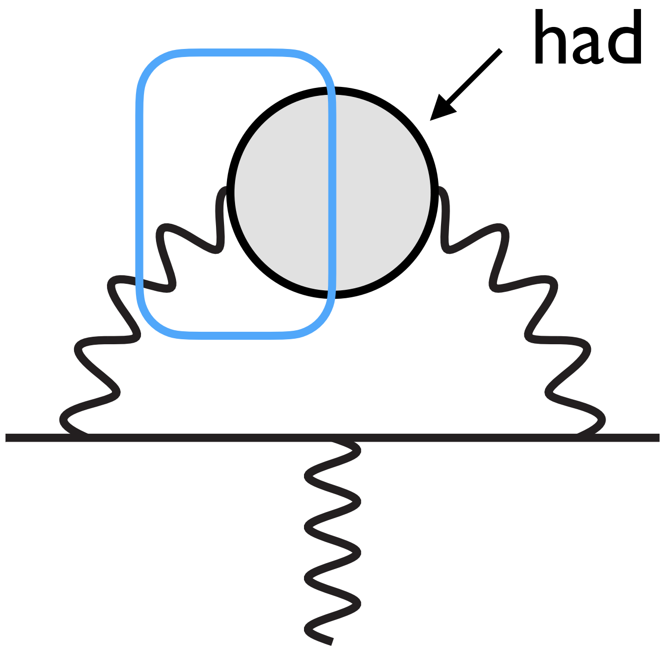
Hadronic VP



Hadronic LbL

Hadronic vacuum polarization

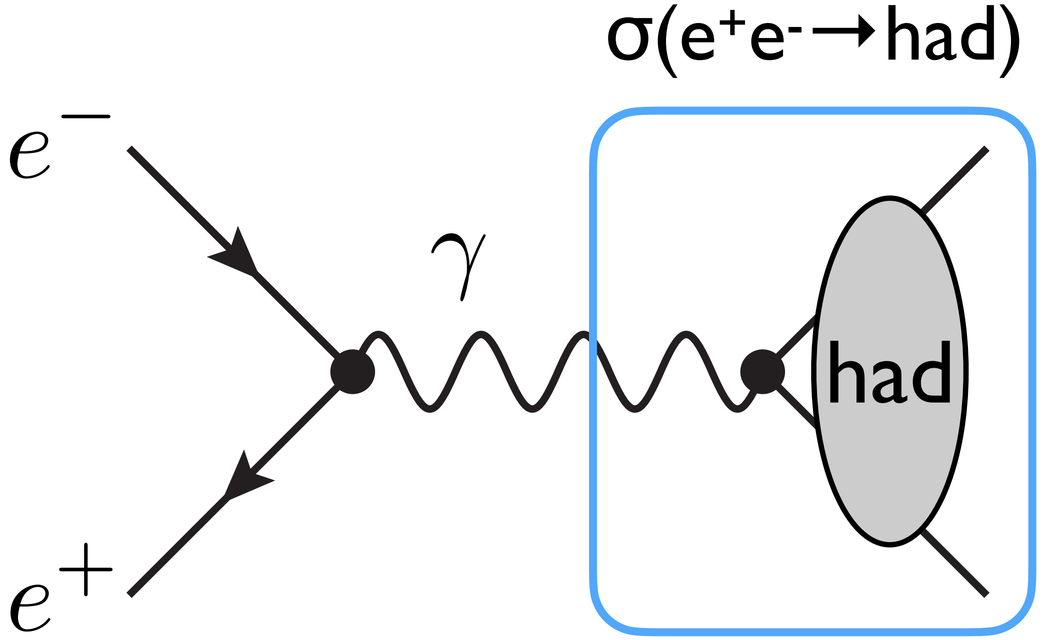
White paper: traditionally determined by hadronic cross section



Gauge sym.

Analyticity

Unitarity



"R-ratio" approach:

$$a_\mu^{\text{HVP,LO}} = \frac{\alpha^2}{3\pi^2}\int_{m_\pi^2}^{\infty}\frac{ds}{s}K(s)R(s)$$

known

experimental data

Status

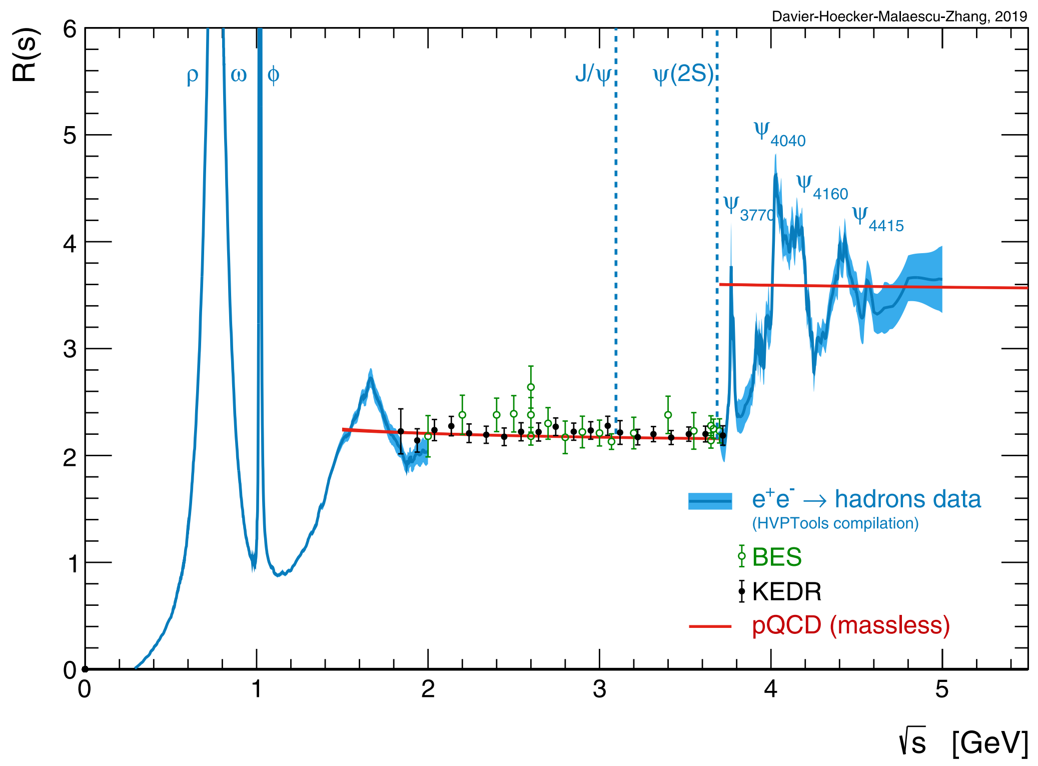
$\pi^+\pi^-$ channel (around ρ/ω region)

~70% of total HVP

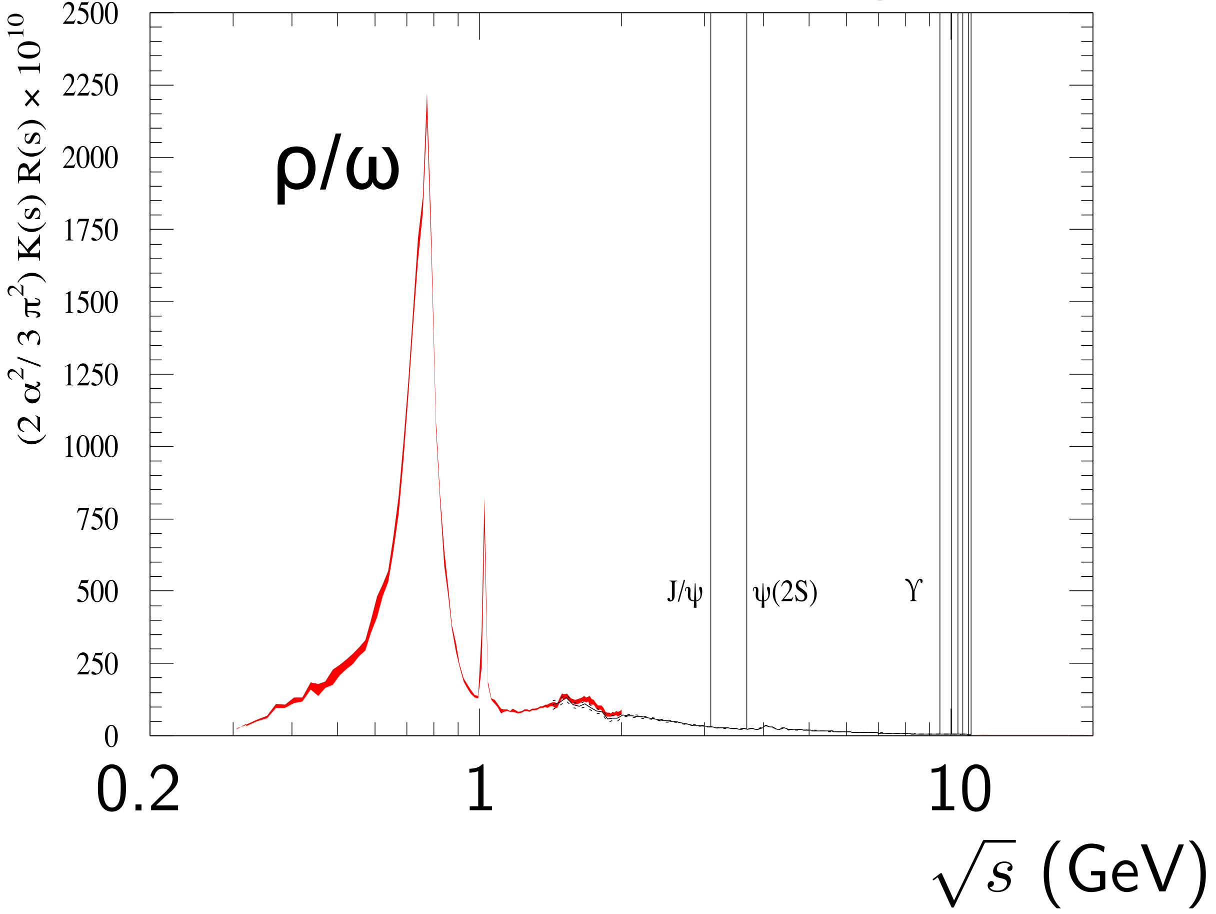
Dominant source of uncertainty

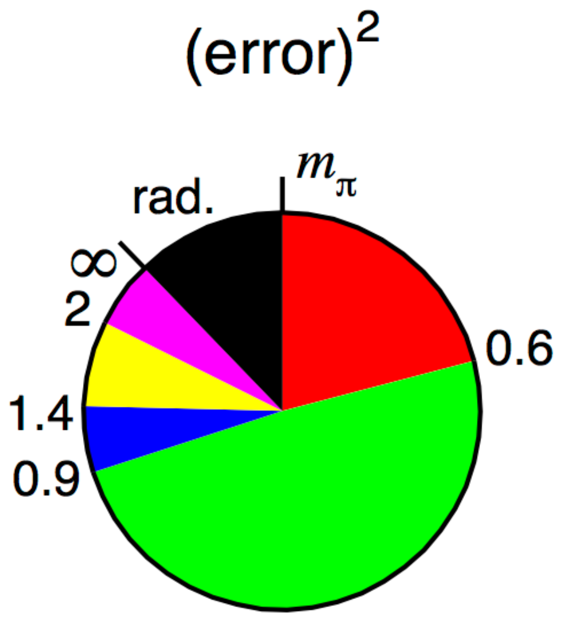
→ long-standing issue

R-ratio: hadronic cross section



contribution to a_μ





Old issue: KLOE/BaBar discrepancy

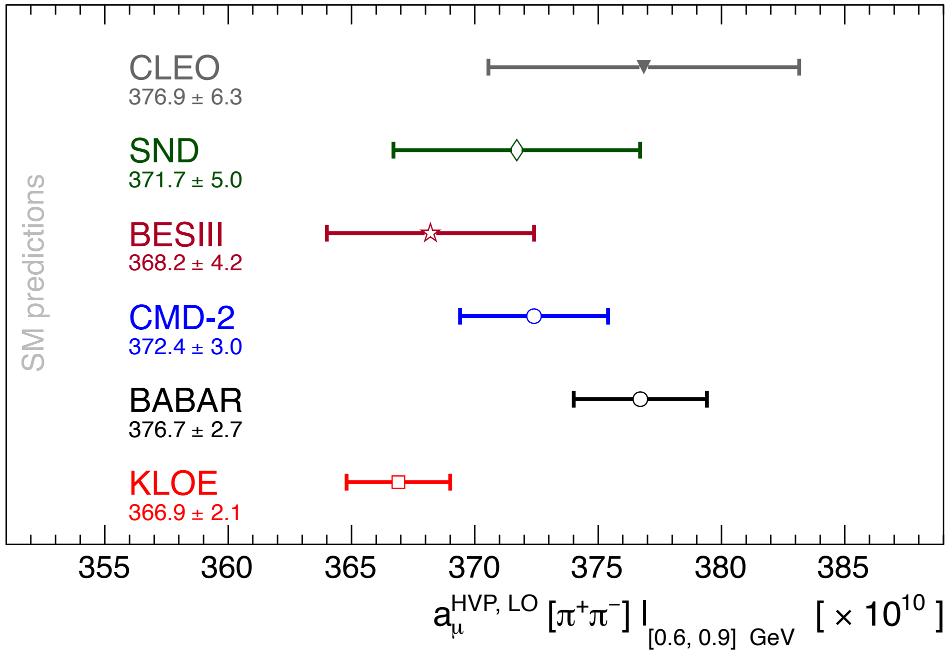
DHMZ19 $694.0 \pm 1.0 \pm 2.5 \pm 2.8$

exp QCD KLOE/BaBar discrepancy

w/o KLOE (→ BaBar) 696.8 ± 3.1

w/o BaBar (→ KLOE) 691.2 ± 3.1

Precision is degraded by ~30% [→ WP]

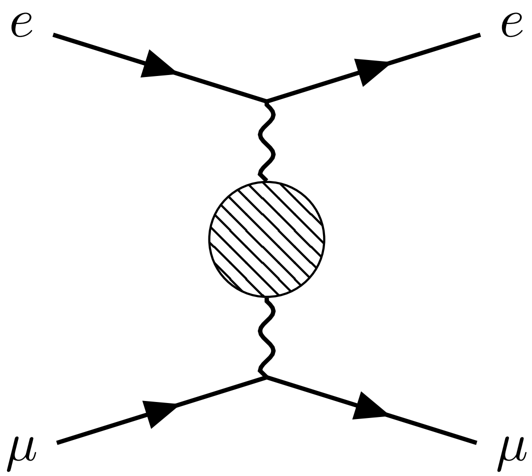


Check by Belle-II, BES-III, CMD-3

Independent test: MUonE

differential cross section of $\mu^+e^- \rightarrow \mu^+e^-$

space-like approach



New issue: BMW lattice result

Lattice evaluation of HVP ($J_\mu = i\sum_f Q_f \bar{\psi}_f \gamma_\mu \psi_f$: quark current)

$$a_\mu^{\text{HVP LO}} = \sum_{t=0}^{\infty} w_t C(t) \quad \text{w/.} \quad C(t) = \frac{1}{3}\sum_{\vec{x}} \sum_{j=0,1,2} \langle J_j(\vec{x},t) J_j(0) \rangle \leftarrow \text{lattice}$$

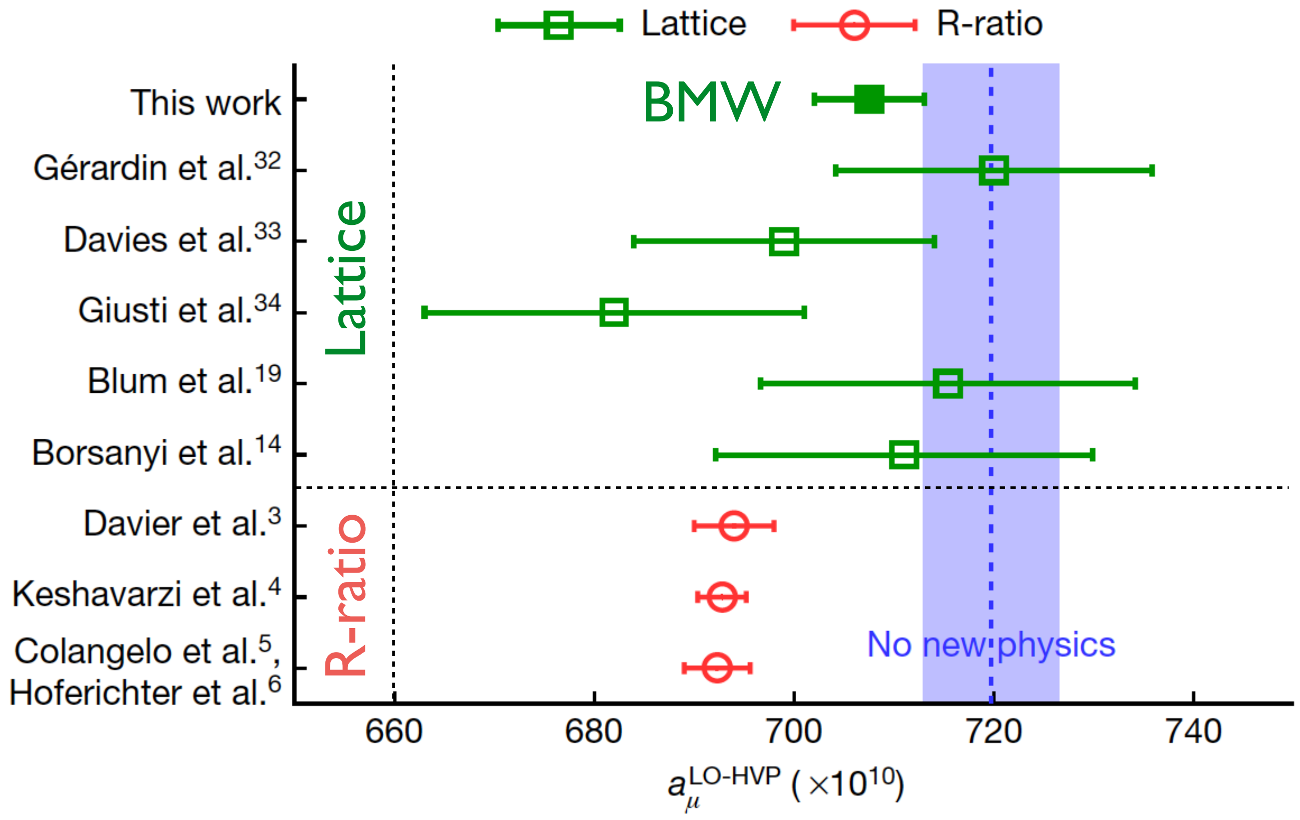
BMW collaboration [2002.12347]

No tension w/. experimental result

However, ~2.1σ away from WP (e+e-)

HVP tension has not been resolved

Independent lattice computations
w/. comparable precisions are crucial



cf. A calculation of "window" observable is consistent w/ BMW, and deviated by 3.8σ
from data-driven value [2206.06582]

Consistency with EW precision test

EW observables are also sensitive to R-ratio data (via $\Delta\alpha_{\mathrm{had}}^{(5)}$)

Hypothetical cross section:

R-ratio data

$$\sigma_{\mathrm{had}}(s) = (1+\varepsilon)\sigma_0$$

$\varepsilon > 0$ in $\sqrt{s} \in \sqrt{s_0} \pm \delta/2$

chosen to account for Δa_μ

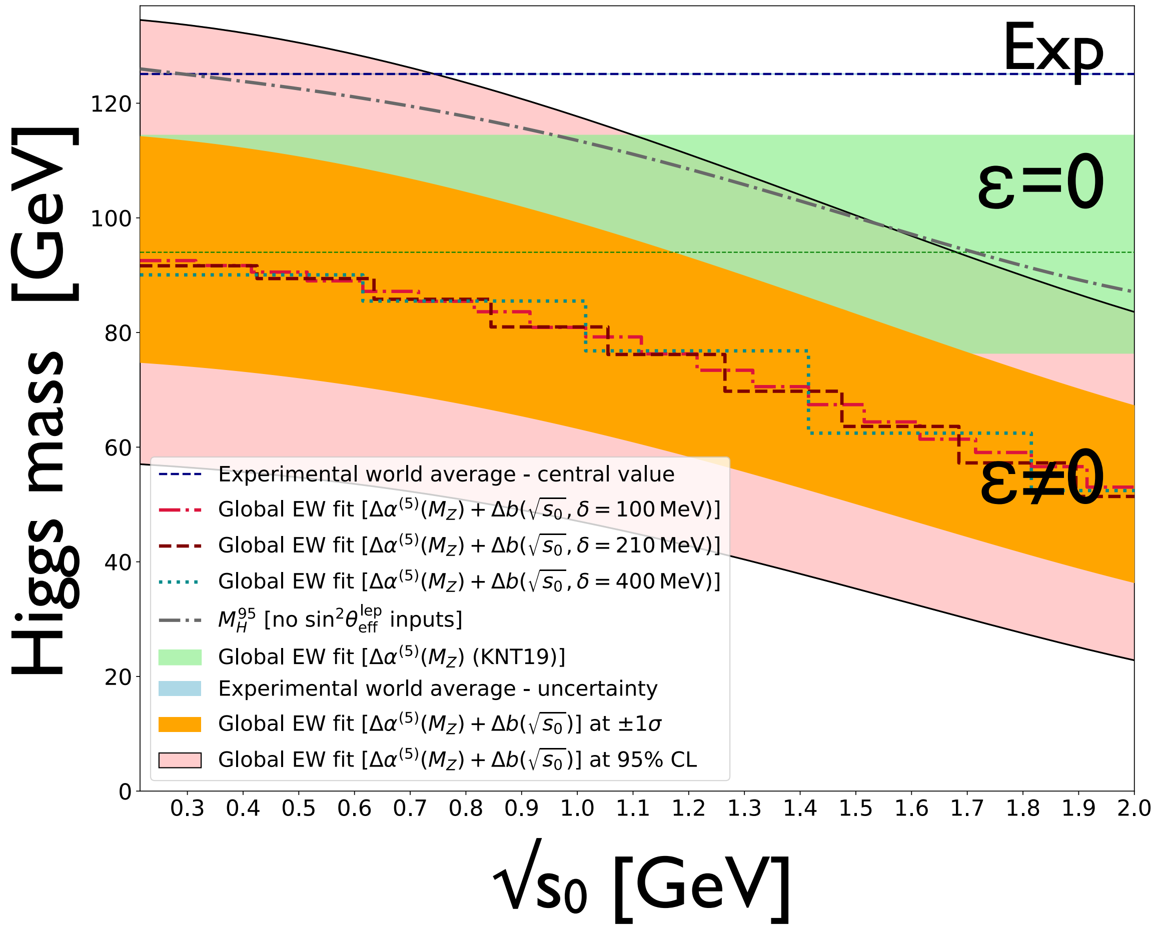
Shift above ~1GeV is excluded.

Shift below ~1GeV is allowed,

but larger than $\sigma(e+e- \to \mathrm{had})$ accuracy

→ Tested in near future, e.g., Belle-II, etc

[2006.12666]



Hadronic light-by-light

Main issue in ~10 years ago

Based on hadronic model before 2014

Dispersive (data-driven) approach

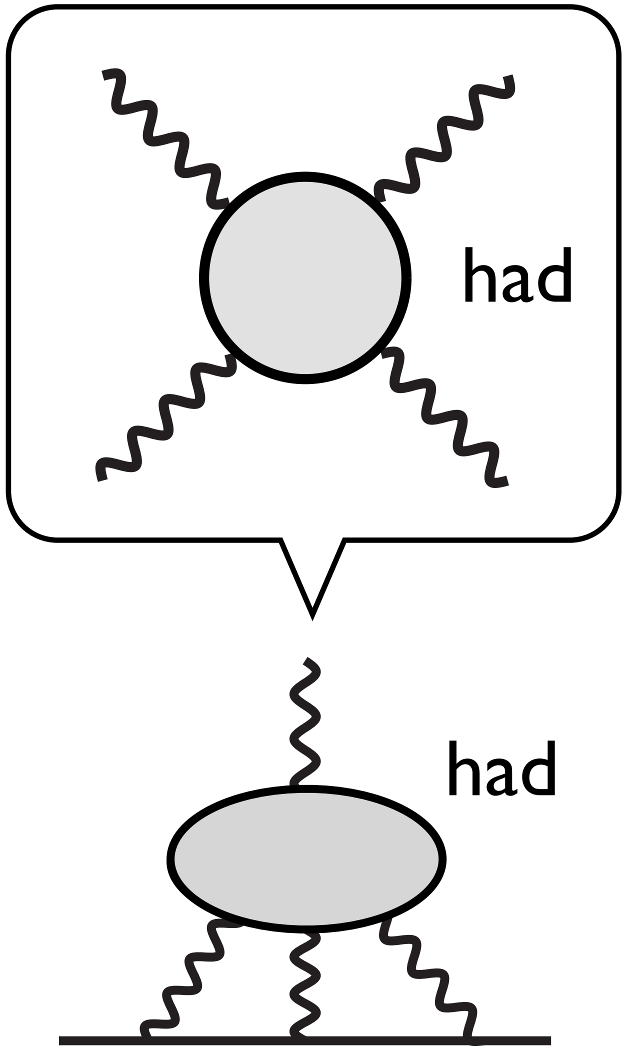
relation with exp. data (cf. e+e- for HVP)

$$a_\mu^{\mathrm{HLbL}} = 9.2(1.8) \times 10^{-10}$$

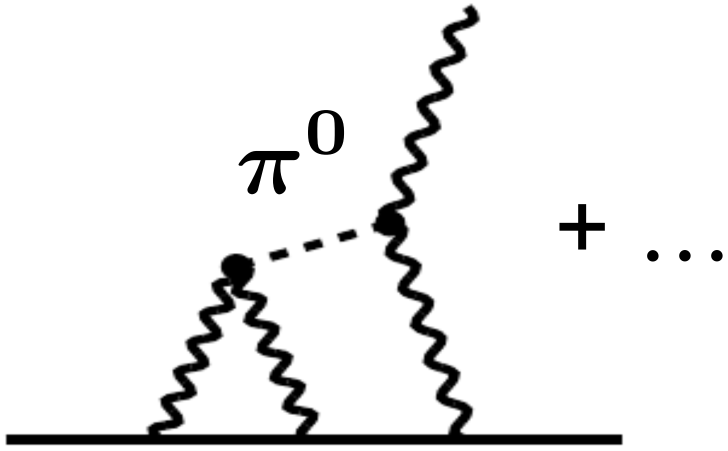
Lattice calculation [Mainz'21]

$$a_\mu^{\mathrm{HLbL}} = 10.48(1.47) \times 10^{-10}$$

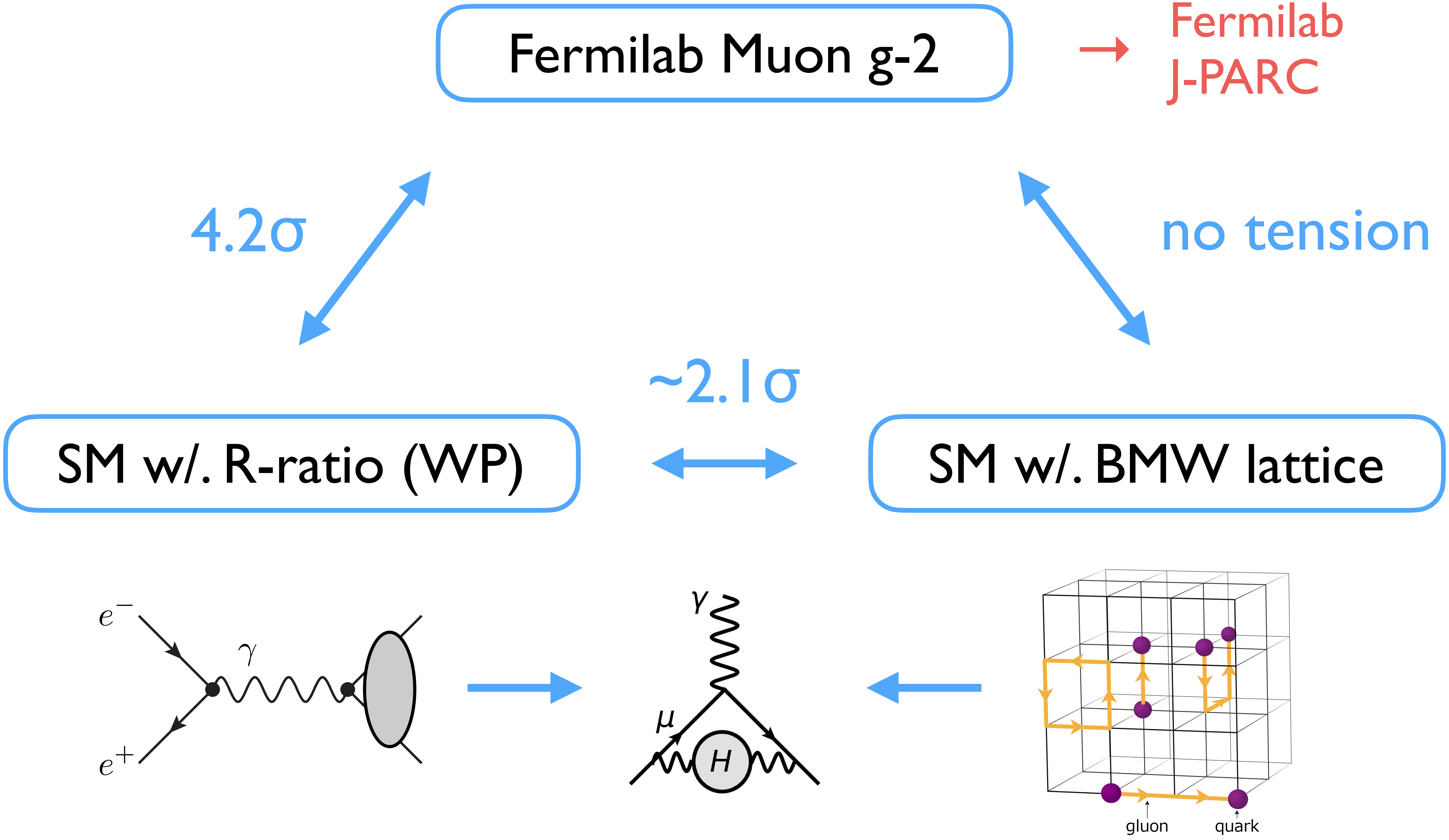
… good agreement with above



hadron model



Status of tensions



Prospect

→ Fermilab
J-PARC

Fermilab Muon g-2

4.2σ

no tension

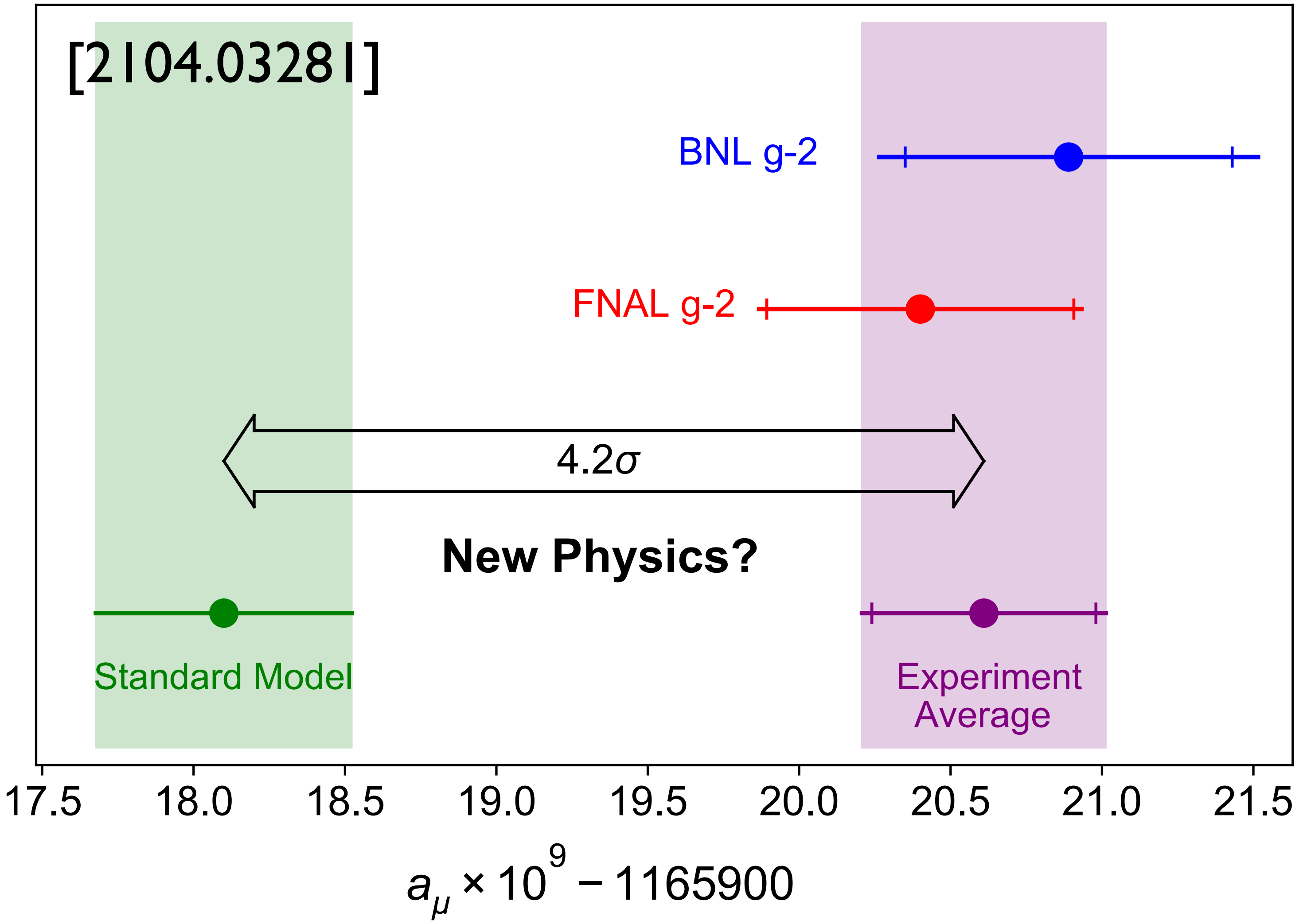
~2.1σ

SM w/. R-ratio (WP)

SM w/. BMW lattice

→ Belle II, BES-III, CMD-3, MUonE, …

→ Independent analysis



Today's talk

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- Recent progress and issue on SM prediction
- New physics interpretation

New physics interpretation

SM based on white paper

$$a_\mu^{\mathrm{BNL+FNAL}} - a_\mu^{\mathrm{SM}} = (25.1 \pm 5.9) \times 10^{-10}$$

4.2σ deviation

Deviation is larger than electroweak contribution

$$a_\mu^{\mathrm{EW}} \simeq 15.4 \times 10^{-10}$$

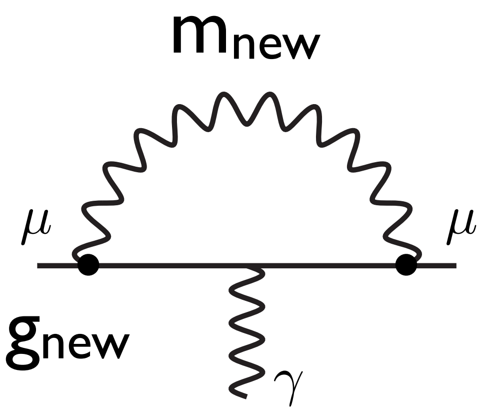
No new particles have been discovered in EW scale

Need mechanism to enhance contribution to muon g-2

Light particle or enhancement

Many models have been proposed

$$a_\mu^{\rm new} \sim \frac{g_{\rm new}^2}{16\pi^2}\frac{m_\mu^2}{m_{\rm new}^2}\,(\times X_{\rm enhance})$$



Light particle = tiny interactions

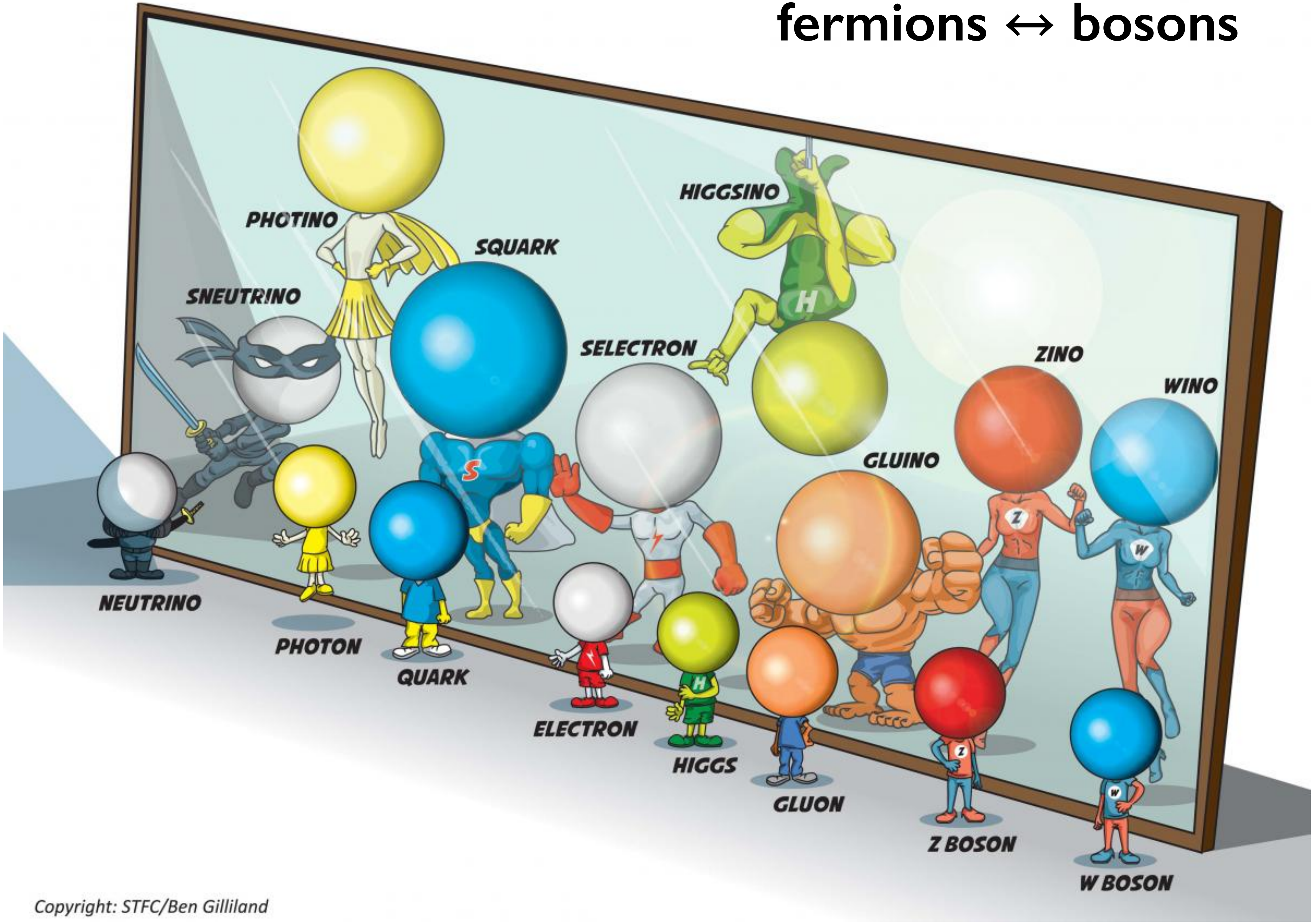
scalar extension, vector extension, axion-like particle, …

Heavy particle = enhancement mechanism

SUSY, Leptoquark, vectorlike fermion, …

Relation with B-anomalies, dark matter, M_W, etc, …

Supersymmetry



fermions ↔ bosons

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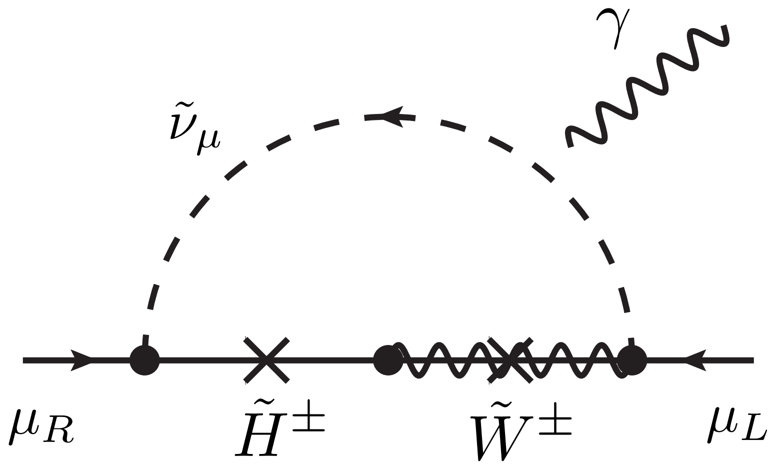
Essence

Smuon (SUSY partner of muon)

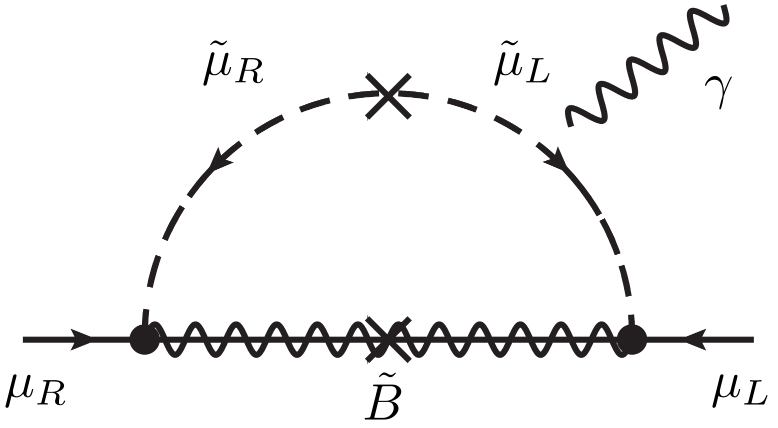
Neutralino, Chargino

(SUSY partner of B, W, H bosons)

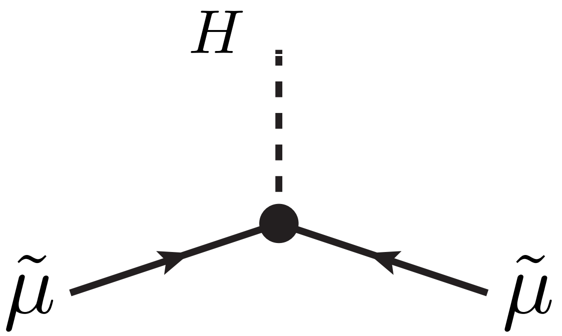


$$a_\mu^{(\mathrm{SUSY})} \sim \frac{g_{\mathrm{EW}}^2}{16\pi^2}\frac{m_\mu^2}{m_{\mathrm{SUSY}}^2}\tan\beta$$

large



$$\text{cf. } \tan\beta \equiv \langle H_u \rangle / \langle H_d \rangle \sim 10$$



$$Y_\mu^{(\mathrm{SUSY})} \simeq Y_\mu^{(\mathrm{SM})}\tan\beta$$

enhancement of chirality flip

Three SUSY scenarios

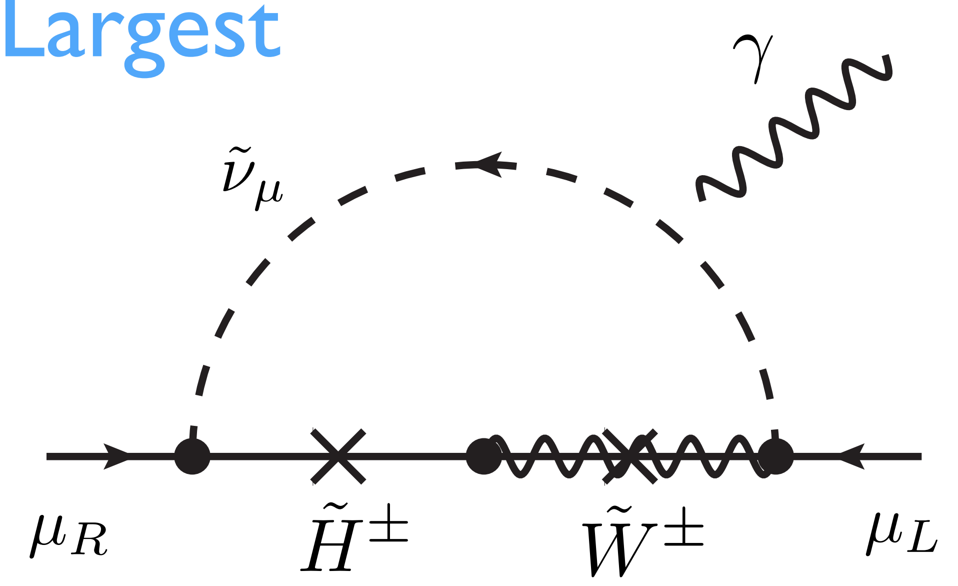
At least 3 light particles

- smuon ("L" and "R")
- neutralino, chargino (B, W, H)

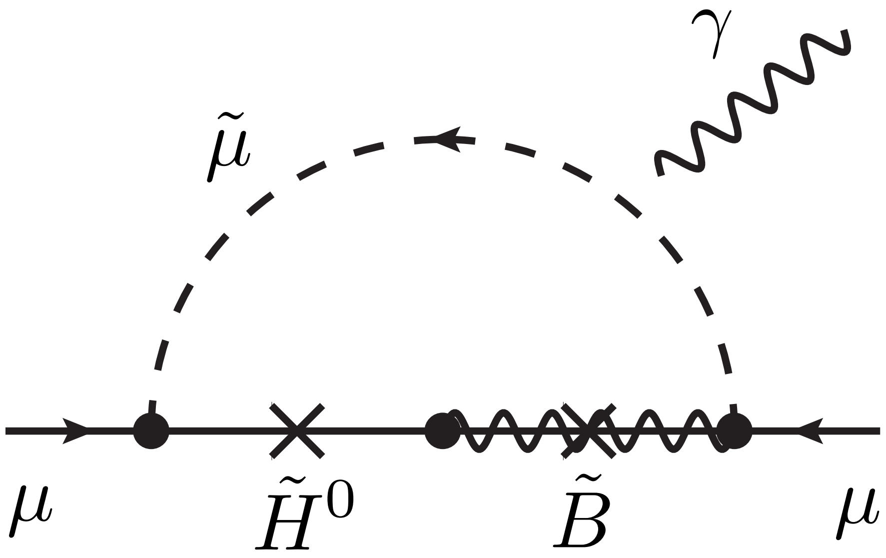
→ 3 scenarios

1. Light Wino/Higgsino

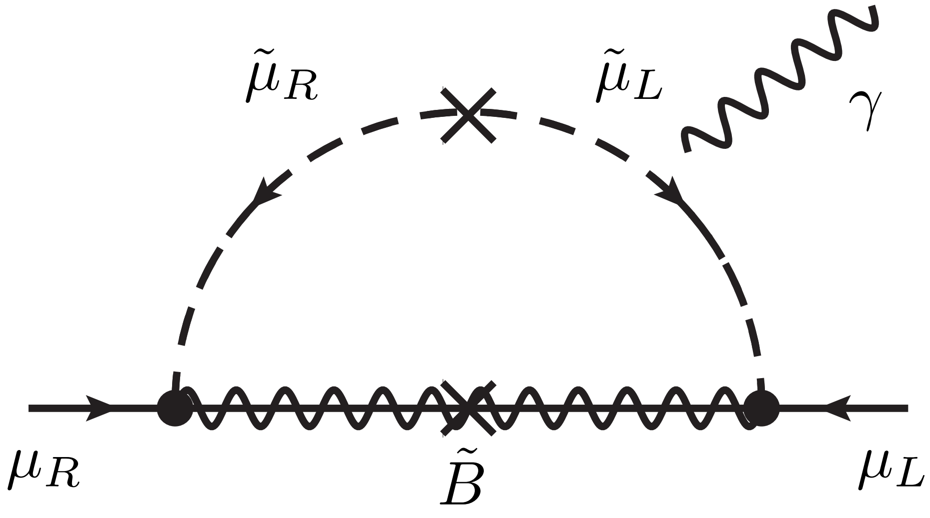
Largest



2. Light Bino/Higgsino



3. Pure Bino



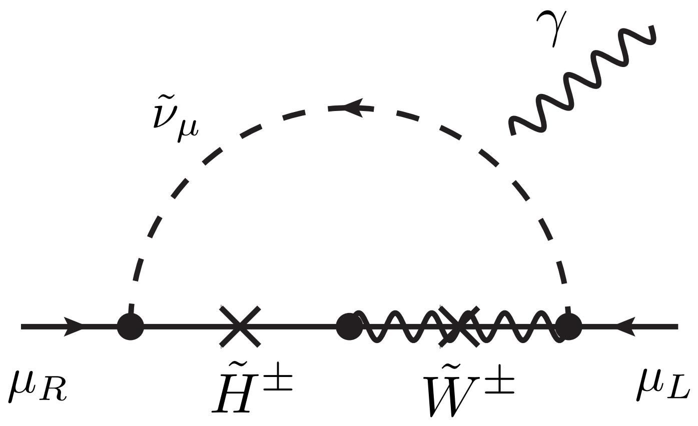
Light Wino/Higgsino scenario

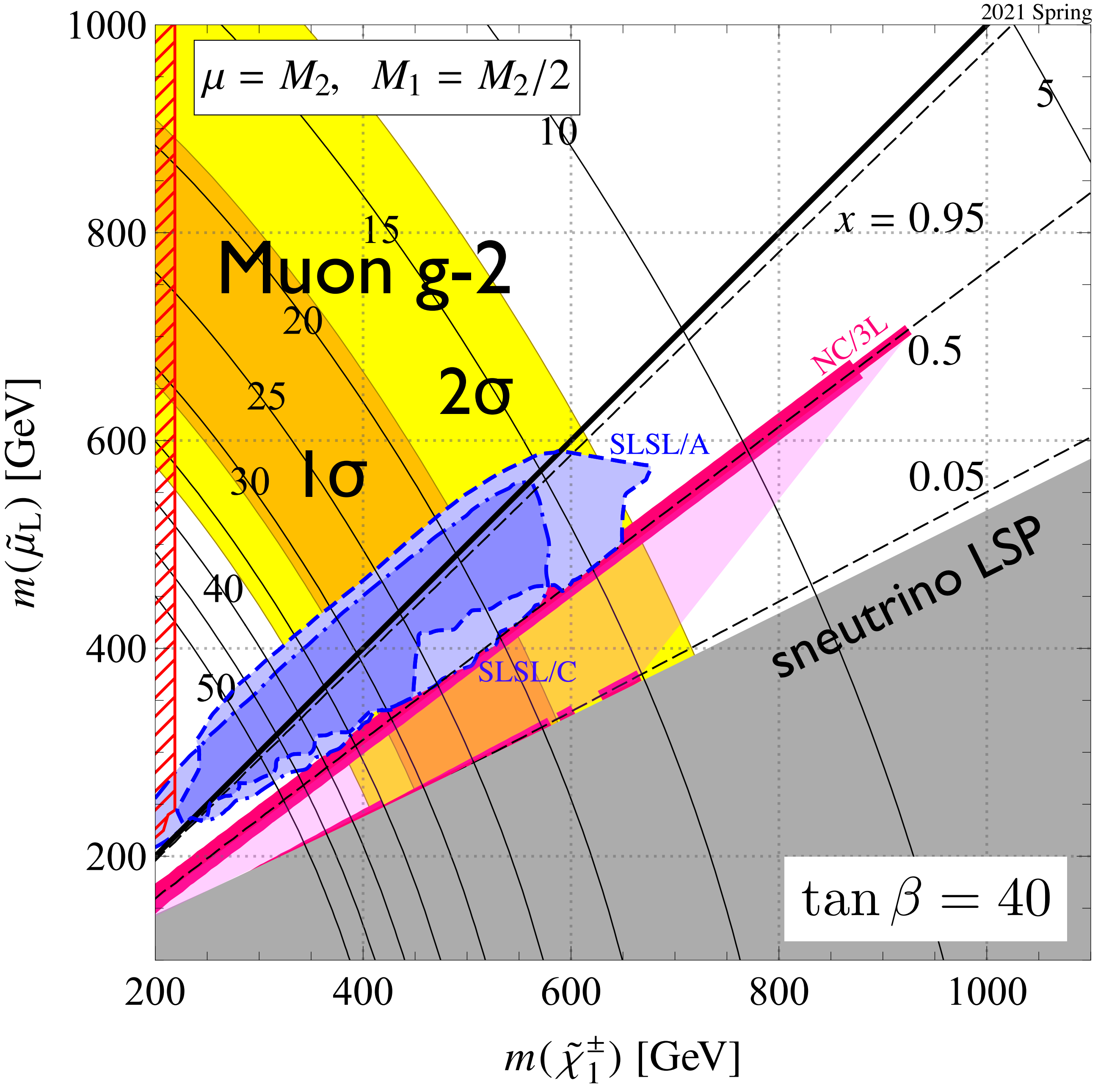
Light left-handed smuon, Wino, Higgsino + Bino (LSP)

Large tanβ

g-2 favors ~100-1000GeV

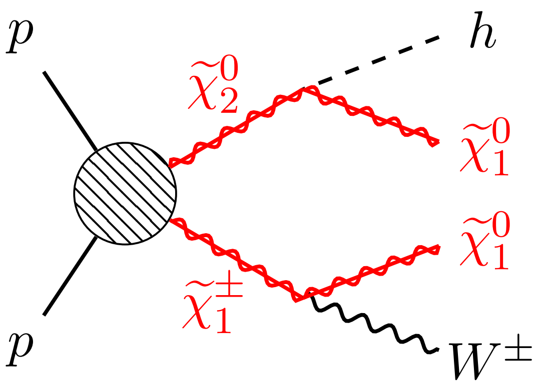
→ **LHC experiment**

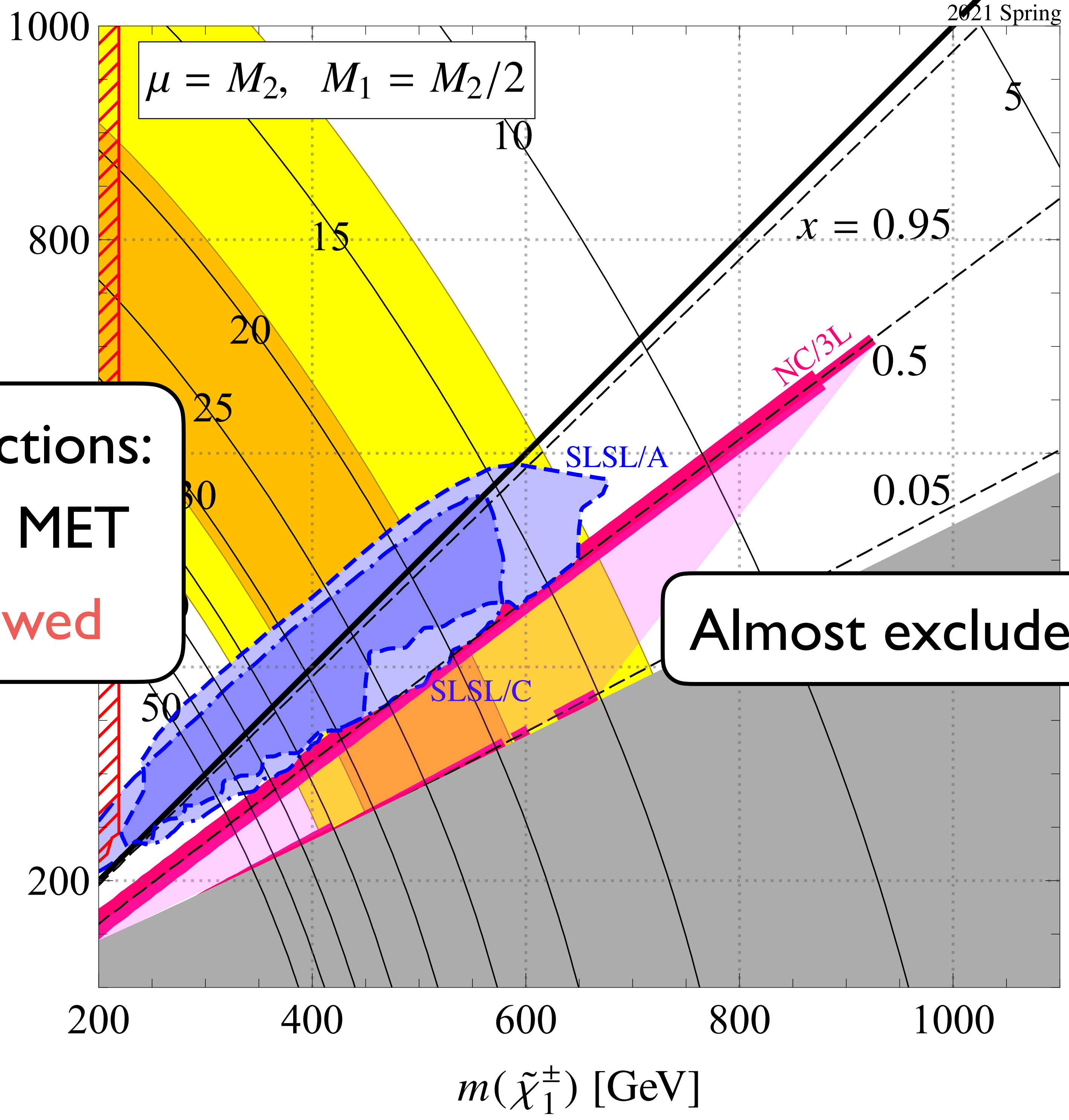




LHC search

Light EWKino ← → **Light slepton**





SM boson productions:

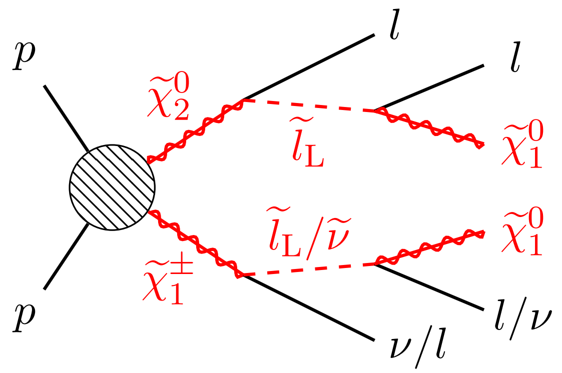
ZW, HW, WW + MET

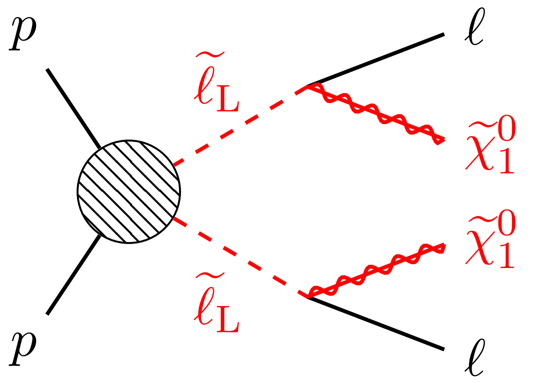
Still widely allowed

Almost excluded

future target

multi-leptpon





$$x = \frac{m_{\tilde{\mu}_L} - m_{\tilde{\chi}_1^0}}{m_{\tilde{\chi}_1^\pm} - m_{\tilde{\chi}_1^0}}$$

ME,Hamaguchi,Iwamoto,Kitahara

Three SUSY scenarios

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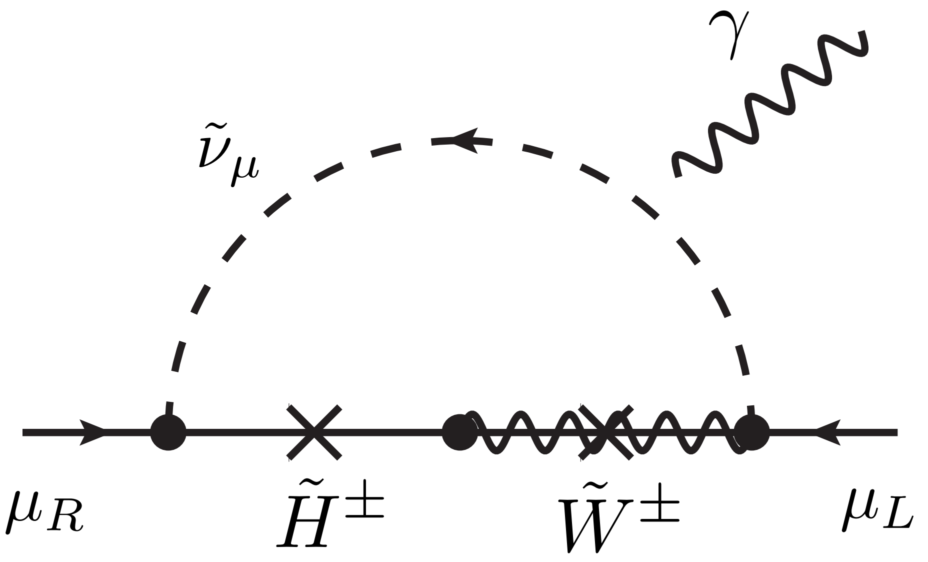
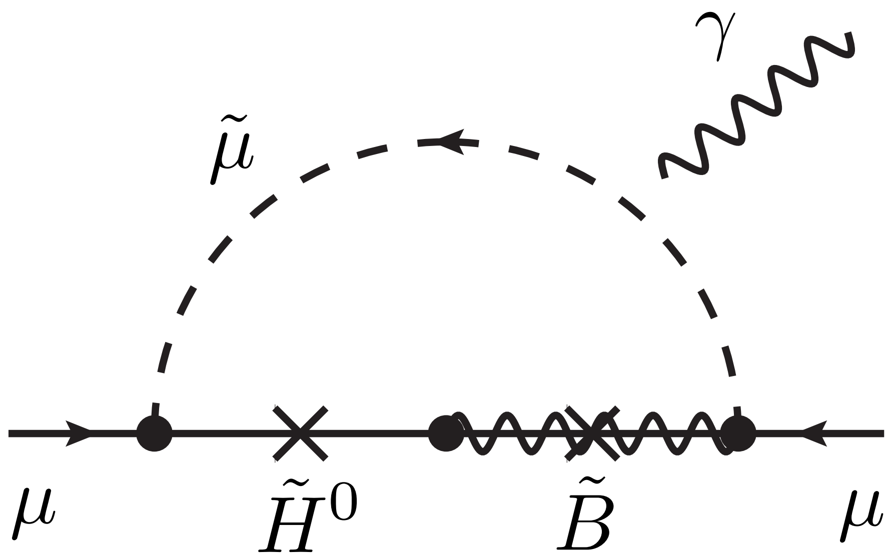
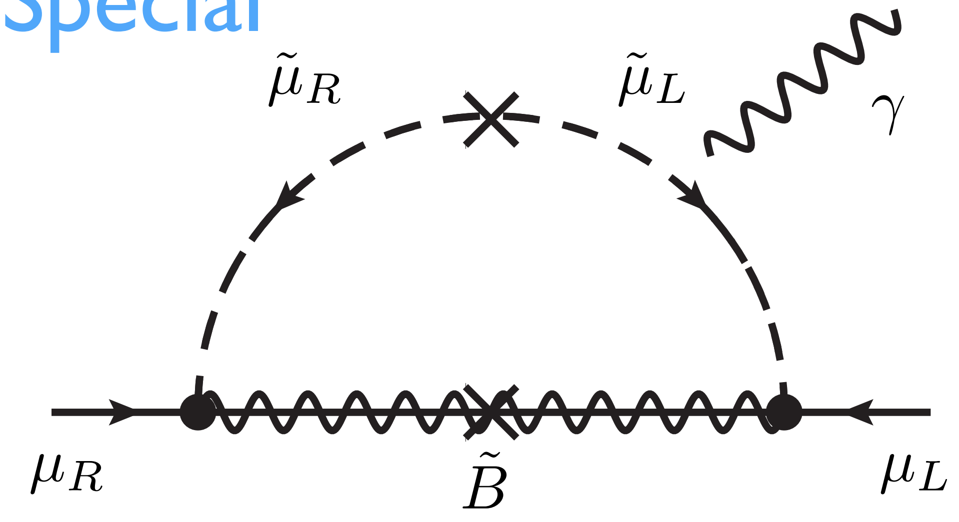


2. Light Bino/Higgsino



3. Pure Bino

Special

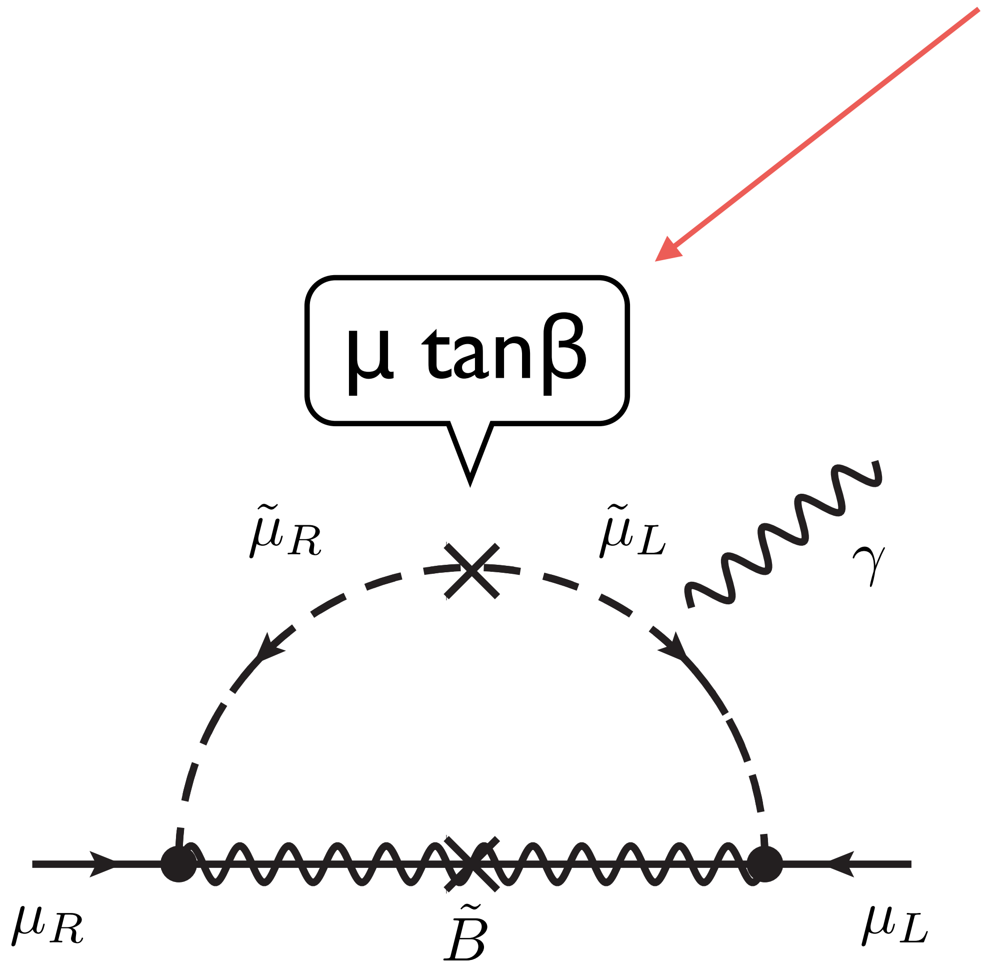


Pure Bino scenario

Light Bino (neutralino), light left- and right-handed smuons

Enhanced by smuon “chirality” flip — large μ tanβ

μ: Higgsino mass, smuon LR mixing



No Higgsinos in loop

Status

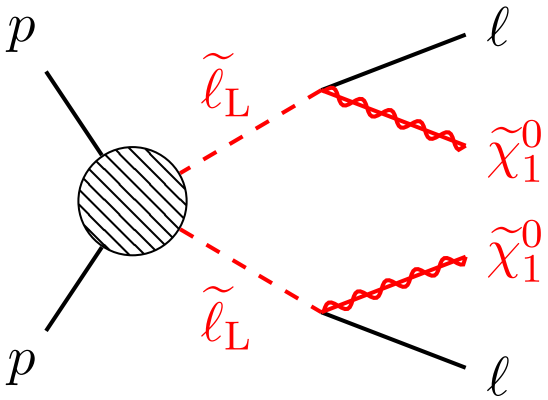
μ is maximized under constraints

Smaller tanβ (~5-10) is favored

* larger tanβ predicts smaller μ, enhancing "light Bino/Higgsino" (destructive) contribution to muon g-2

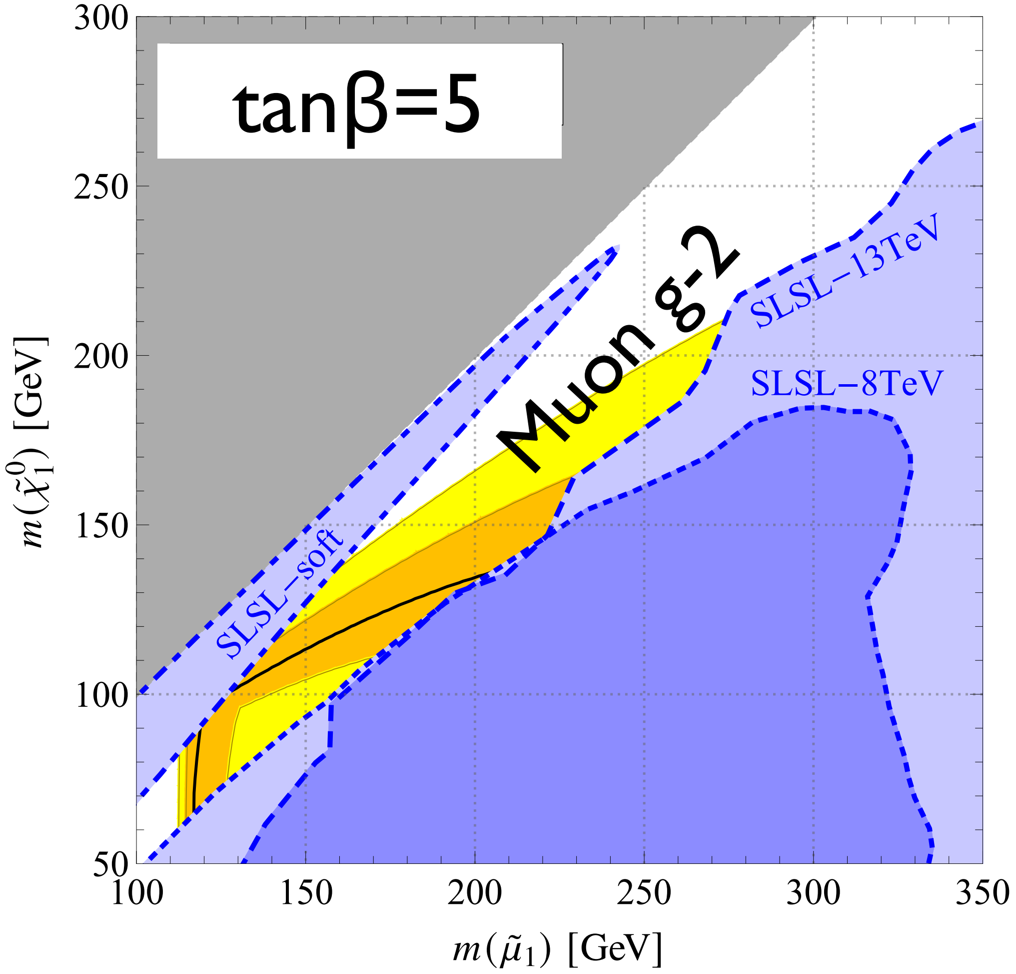
Still allowed by LHC constraints:

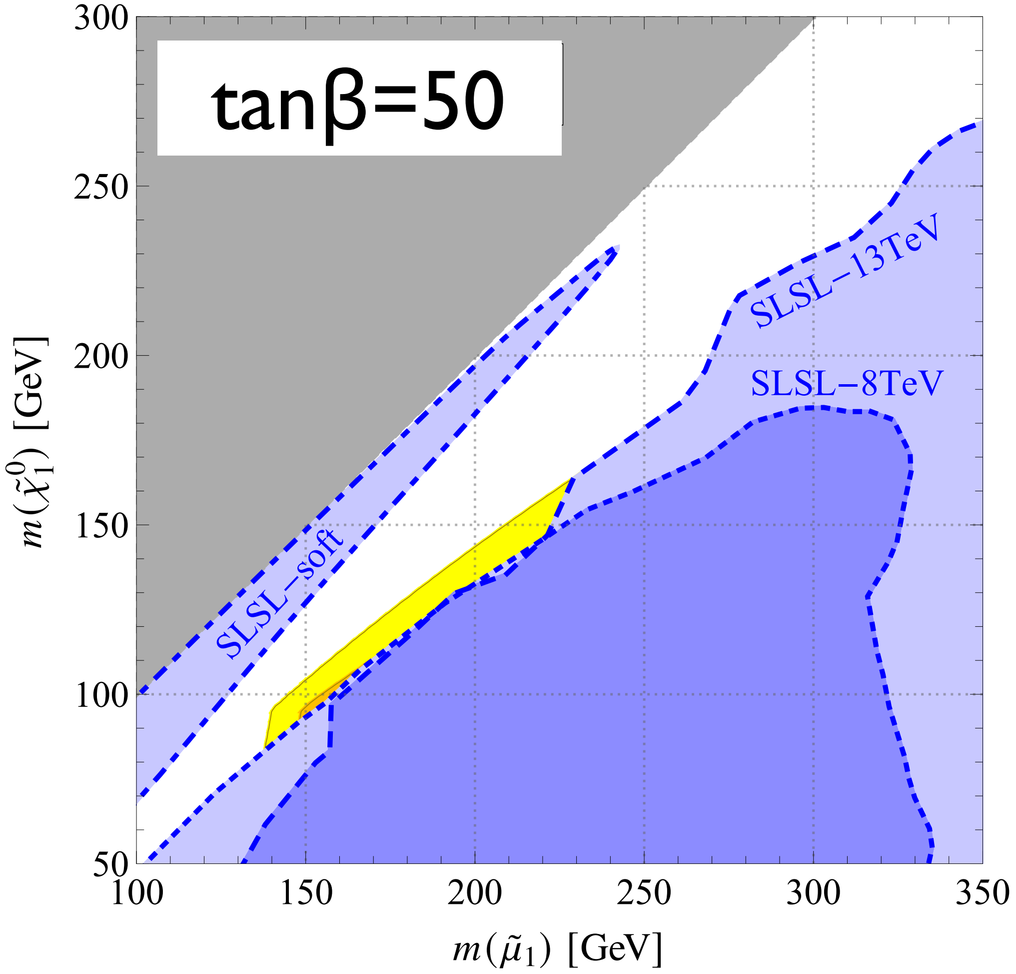
SLSL (-soft) : lepton pair



(+jet)

ME,Hamaguchi,Iwamoto,Kitahara

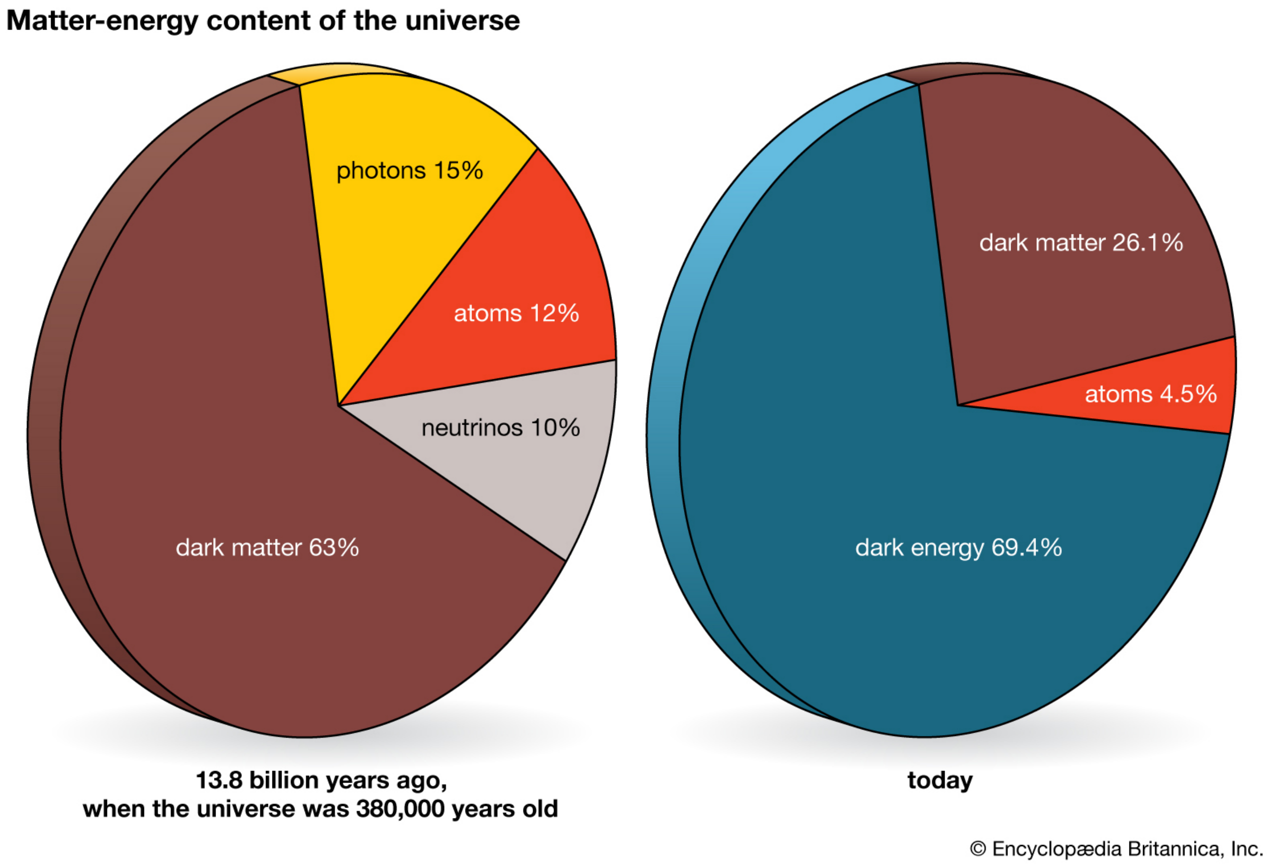




Is mass degeneracy accidental?

SUSY motivations

- naturalness
- dark matter candidate
- grand unified theory
- muon g-2, …

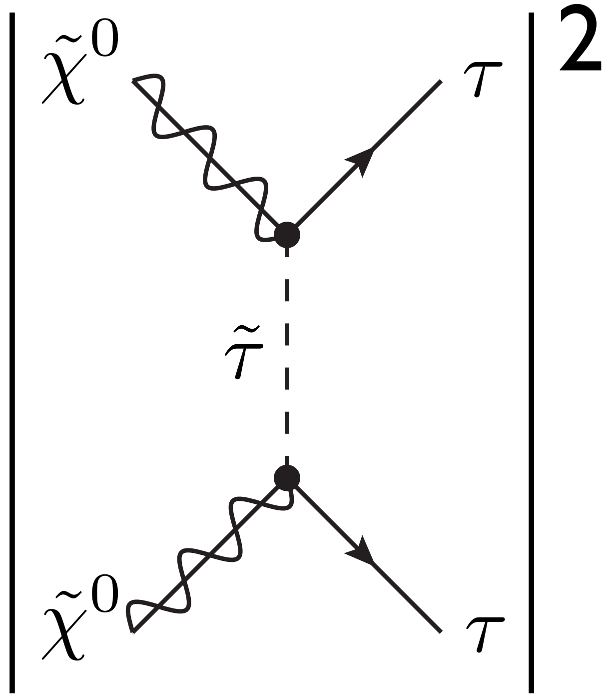


(snapshot observed by CMB)

DM thermal relic abundance

Bino is a well-known WIMP DM candidate

Thermal relic abundance : $\Omega_{\rm DM} \propto \langle \sigma_{\rm ann} v \rangle^{-1}$

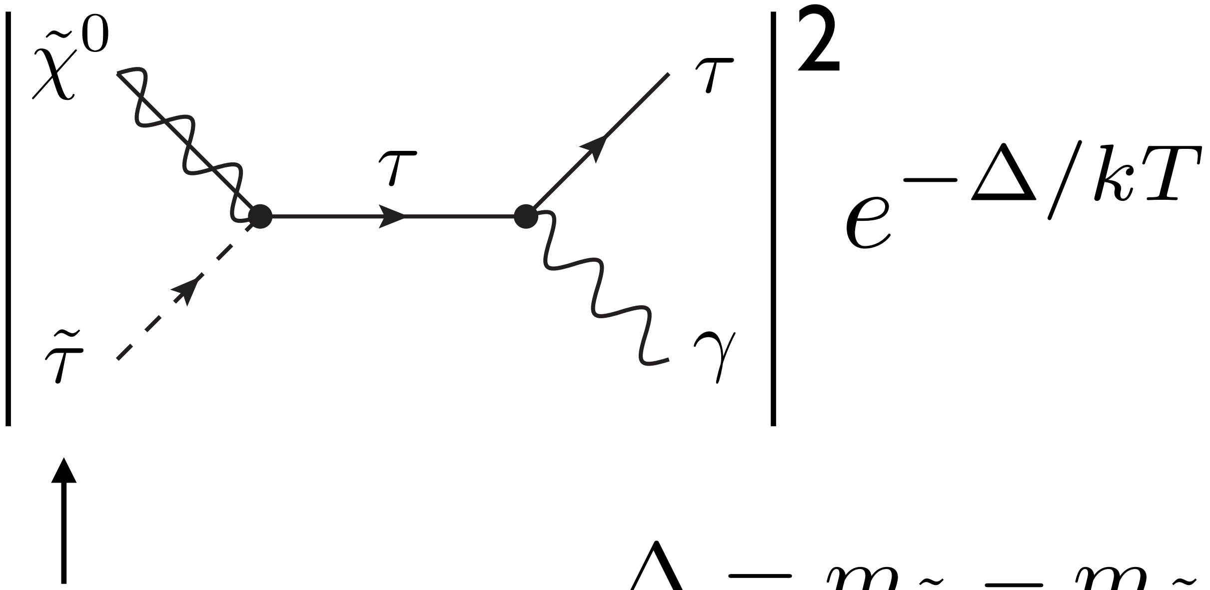


pure Bino

$\Omega_\chi > \Omega_{\rm DM}$

+

"stau coannihilation"



thermal

$\Delta = m_{\tilde{\tau}} - m_{\tilde{\chi}}$

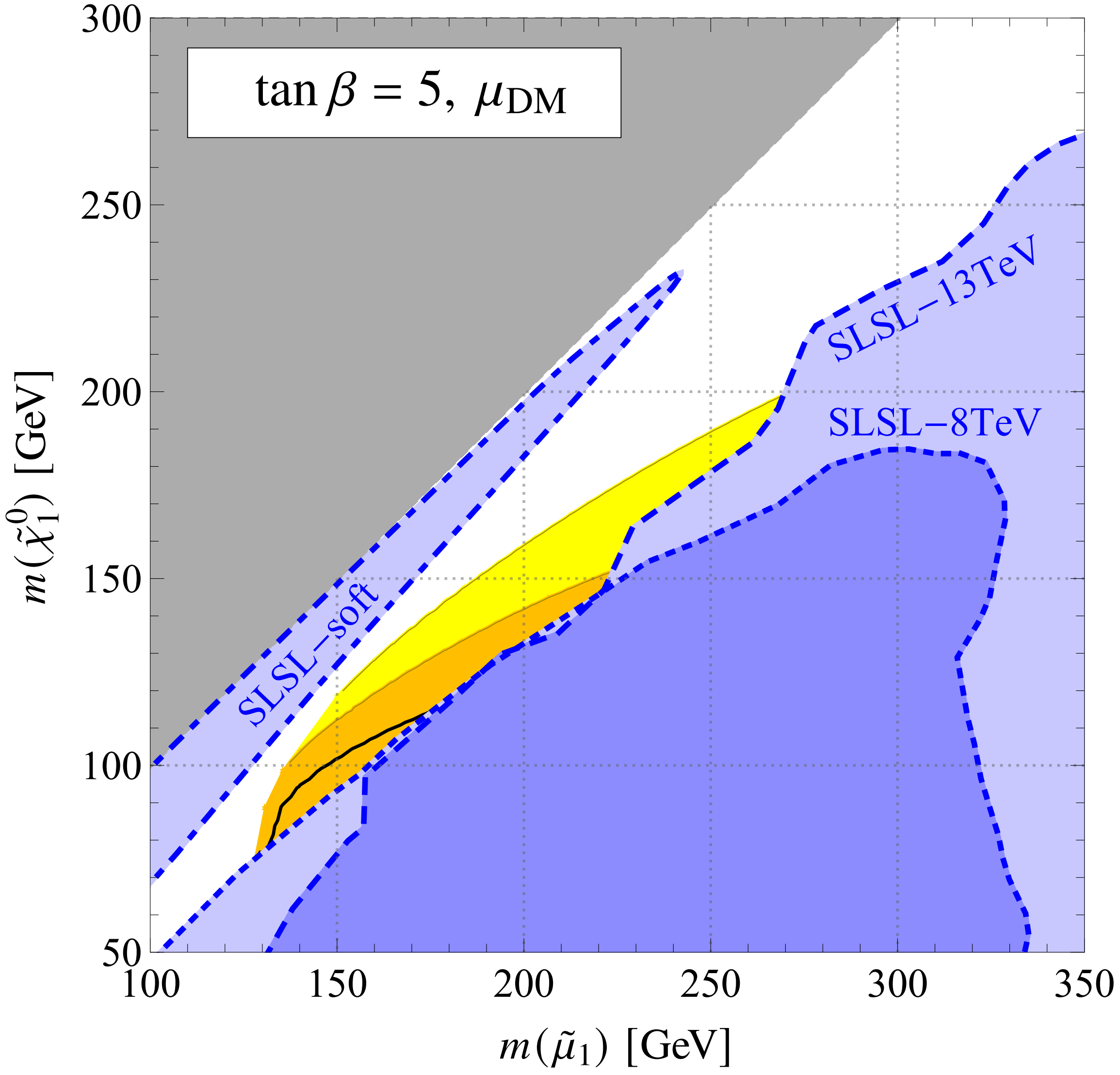
Degeneracy is favored

Muon g-2 and dark matter

μ is determined by DM relic abundance

Wide g-2 regions can be covered by DM direct detection exp.

e.g., XENONnT, PandaX-4T, …



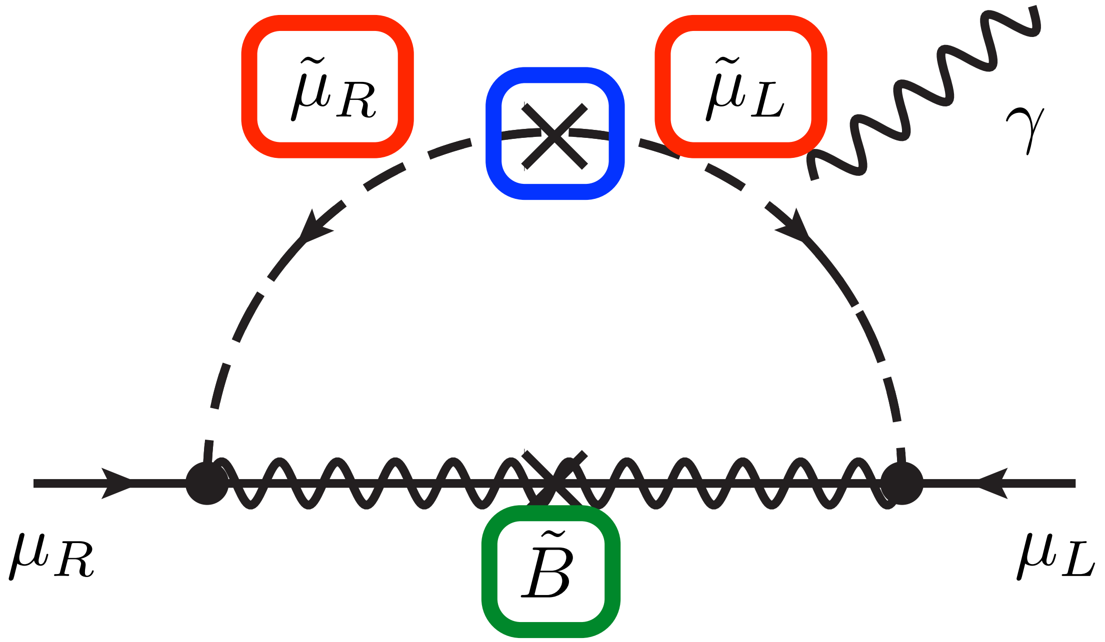
ME, Hamaguchi, Iwamoto, Kitahara

ILC test

Direct test of NP contribution to muon g-2 by *reconstructing* it

Precise measurement of mass and determination of LR mixing

$$\underline{m_{\tilde{\mu}_1},\ m_{\tilde{\mu}_2}},\ \underline{m_{\tilde{\chi}_1^0}},\ \underline{m_{\tilde{\mu}LR}^2}$$



cf. Bino coupling can also be determined by measuring cross section

ME,Hamaguchi,Iwamoto,Kawada,Kitahara,Moroi,Suehara

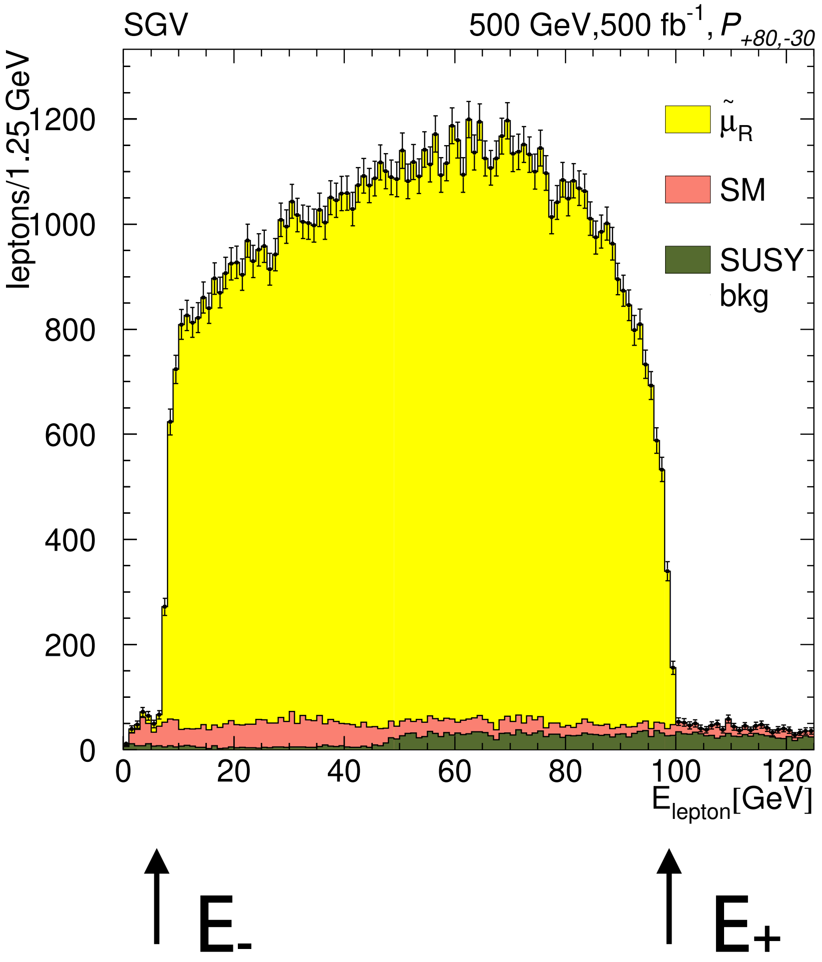
Smuon, neutralino mass determination

Endpoint method

Lepton energy distribution for $\tilde{\ell}^{\pm} \rightarrow \ell^{\pm} \tilde{\chi}_1^0$

$$\begin{aligned} E_{+/-} &= \frac{\sqrt{s}}{4}\left(\frac{m_{\tilde{\ell}}^2 - m_{\tilde{\chi}}^2}{m_{\tilde{\ell}}^2}\right)\left(1 \pm \sqrt{1 - 4\, m_{\tilde{\ell}}^2/s}\,\right) \\ m_{\tilde{\ell}} &= \sqrt{s}\,\frac{\sqrt{E_- E_+}}{E_- + E_+}\,, \\ m_{\tilde{\chi}} &= m_{\tilde{\ell}}\sqrt{1 - \frac{E_- + E_+}{\sqrt{s}/2}}\,. \end{aligned}$$

→ δm ~ 0.1GeV



Smuon LR mixing determination

Smuon LR mixing is proportional to muon mass and thus small

$$m^2_{\tilde{\mu}LR} \simeq -m_\mu \mu \tan\beta$$

(Much) easier to determine <u>stau</u> LR mixing because $m_\tau > m_\mu$

$$m^2_{\tilde{\mu}LR} = \frac{m_\mu}{m_\tau} m^2_{\tilde{\tau}LR}$$

Stau LR mixing

$$m^2_{\tilde{\tau}LR} = \frac{1}{2}\left(m^2_{\tilde{\tau}_1} - m^2_{\tilde{\tau}_2}\right)\sin 2\theta_{\tilde{\tau}}$$

$$\mathcal{M}^2_{\tilde{\tau}} = \begin{pmatrix} m^2_{\tilde{\tau}LL} & m^2_{\tilde{\tau}LR} \\ m^2_{\tilde{\tau}LR} & m^2_{\tilde{\tau}RR} \end{pmatrix}$$

$$\begin{pmatrix} \tilde{\tau}_1 \\ \tilde{\tau}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_{\tilde{\tau}} & \sin\theta_{\tilde{\tau}} \\ -\sin\theta_{\tilde{\tau}} & \cos\theta_{\tilde{\tau}} \end{pmatrix} \begin{pmatrix} \tilde{\tau}_L \\ \tilde{\tau}_R \end{pmatrix}$$

→ Measure <u>stau mass</u> and <u>mixing angle</u>

Stau endpoint and cross section

$m_{\tilde{e}_1}$	$m_{\tilde{e}_2}$	$m_{\tilde{\mu}_1}$	$m_{\tilde{\mu}_2}$	$m_{\tilde{\tau}_1}$	$m_{\tilde{\tau}_2}$	$m_{\tilde{\chi}_1^0}$	$\cos\theta_{\tilde{\mu}}$	$\cos\theta_{\tilde{\tau}}$
155.8	156.7	154.0	158.5	113.2	189.8	99.3	0.631	0.703
$m_{\rm L}$	$m_{\rm R}$	M_1	μ	$\tan\beta$	$\Omega_{\rm DM}h^2$	$a_\mu^{\rm SUSY}$	$a_\mu^{(\tilde{B})}$	
150.0	150.0	100.0	1323	4.94	0.120	27.1×10^{-10}	27.5×10^{-10}	

ILC at √s = 500GeV, ∫L = 1.6ab^{-1} for each polarization

$$(P_{e^-}, P_{e^+}) = (-80\%, +30\%)\;[\text{eLpR}]$$
$$(+80\%, -30\%)\;[\text{eRpL}]$$

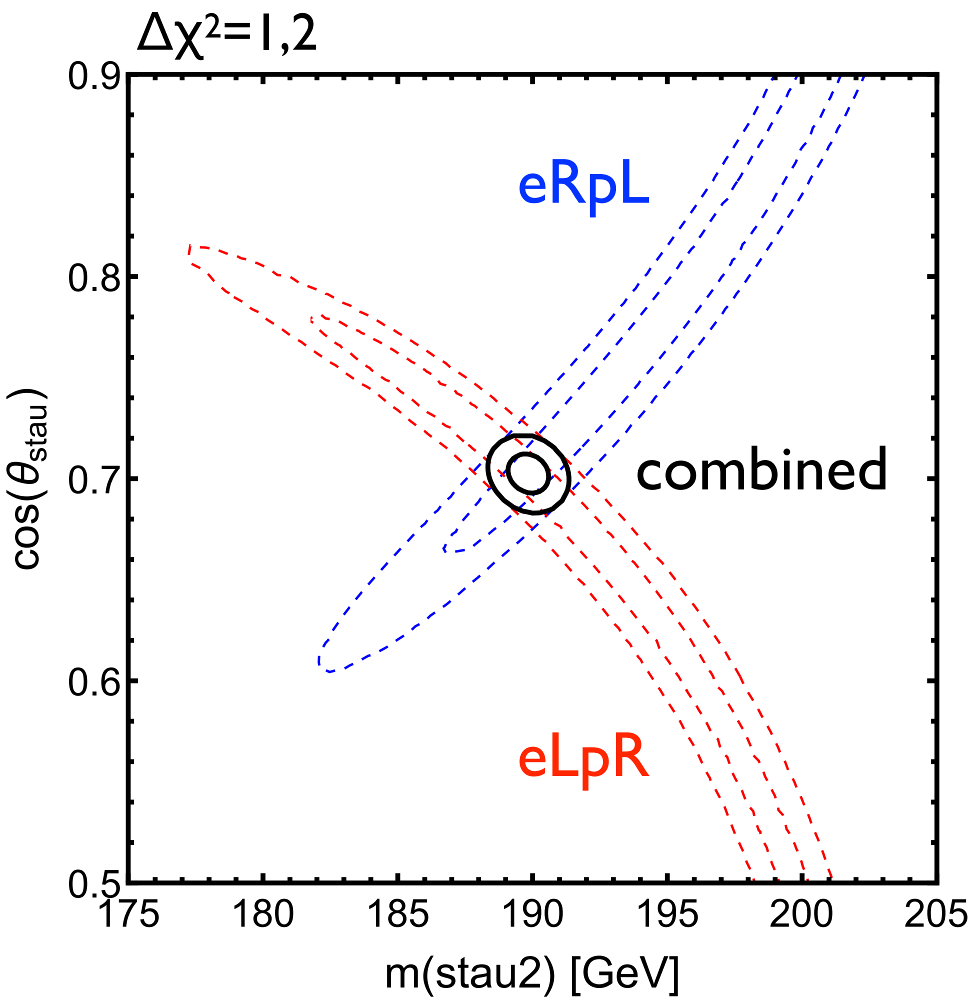
Global fit of endpoints and xsecs:

true

$$m_{\tilde{\tau}_1} = 112.8 \pm 0.2\,\text{GeV} \quad [113.2]$$
$$m_{\tilde{\tau}_2} = 189.9^{+0.8}_{-0.7}\,\text{GeV} \quad [189.8]$$
$$\cos\theta_{\tilde{\tau}} = 0.703 \pm 0.010 \quad [0.703]$$

Stau LR mixing

$$-\,m^2_{\tilde{\tau}LR} = (1.17 \pm 0.01) \times 10^4\,\text{GeV}^2$$



Muon g-2 reconstruction

Smuon mixing angle

$$\sin 2\theta_{\tilde{\mu}} = \frac{2m^2_{\tilde{\mu}LR}}{m^2_{\tilde{\mu}_1} - m^2_{\tilde{\mu}_2}} = \frac{m_\mu}{m_\tau} \frac{2m^2_{\tilde{\tau}LR}}{m^2_{\tilde{\mu}_2} - m^2_{\tilde{\mu}_1}}$$

Muon g-2

$$a_\mu^{(\tilde{B})} = \frac{1}{16\pi^2} \sum_{A=1,2} \frac{m_\mu^2}{m^2_{\tilde{\mu}A}} \left\{ -\frac{1}{12} \left[(\hat{N}_A^{\mu_L})^2 + (\hat{N}_A^{\mu_R})^2 \right] F_1^N \left(\frac{m^2_{\tilde{\chi}^0_1}}{m^2_{\tilde{\mu}A}} \right) - \frac{m_{\tilde{\chi}^0_1}}{3m_\mu} \hat{N}_A^{\mu_L} \hat{N}_A^{\mu_R} F_2^N \left(\frac{m^2_{\tilde{\chi}^0_1}}{m^2_{\tilde{\mu}A}} \right) \right\}$$

LR mixing

(+ leading 2-loop)

Result: reconstruction at 1% accuracy

$$a_\mu^{(\tilde{B})}\Big|_{m^2_{\tilde{\mu}LR}} = (27.6 \pm 0.3) \times 10^{-10}$$ [Error: LR mixing]

$$a_\mu^{(\tilde{B})} = (27.5 \pm 0.4) \times 10^{-10}$$ [Error: All]

Summary

Fermilab confirmed muon g-2 result.

Current discrepancy is 4.2σ, which will be checked soon.

White paper has been published, but there are issues on HVP.

Many new physics models have been proposed.

SUSY is a major target and expected to be checked in near future.

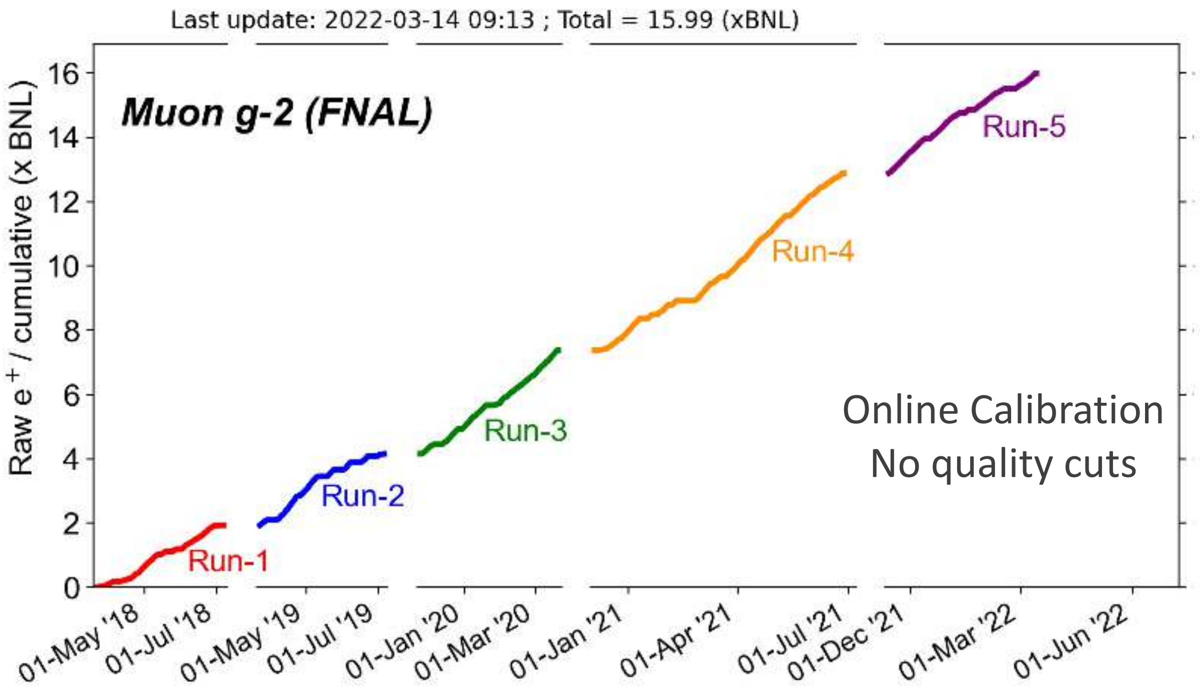
In particular, ILC will be able to reconstruct a_μ at 1% accuracy.

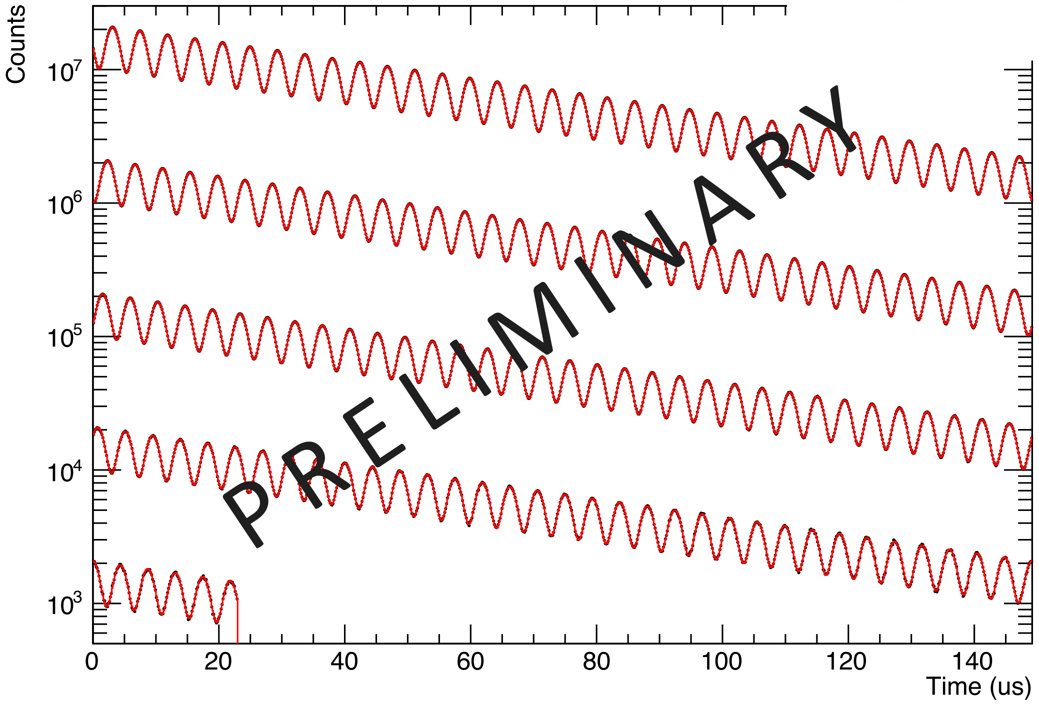
Backup slide

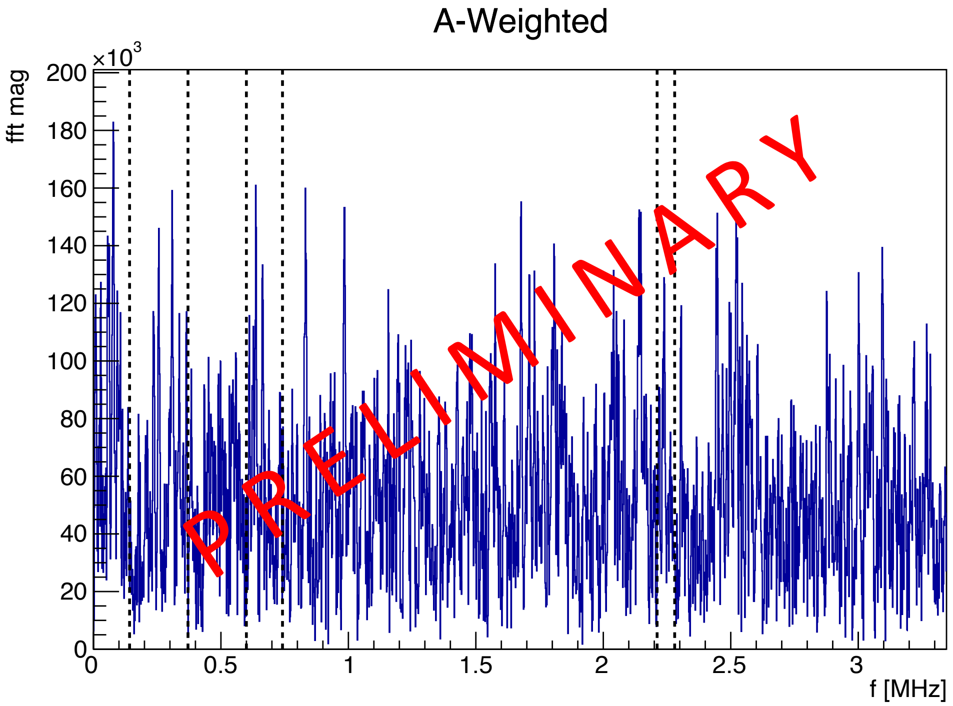
More Statistics

Run-2 and Run-3 analysis proceeding

Expected statistical uncertainty ~200 ppb







Slide by K. Labe, Moriond EW 2022

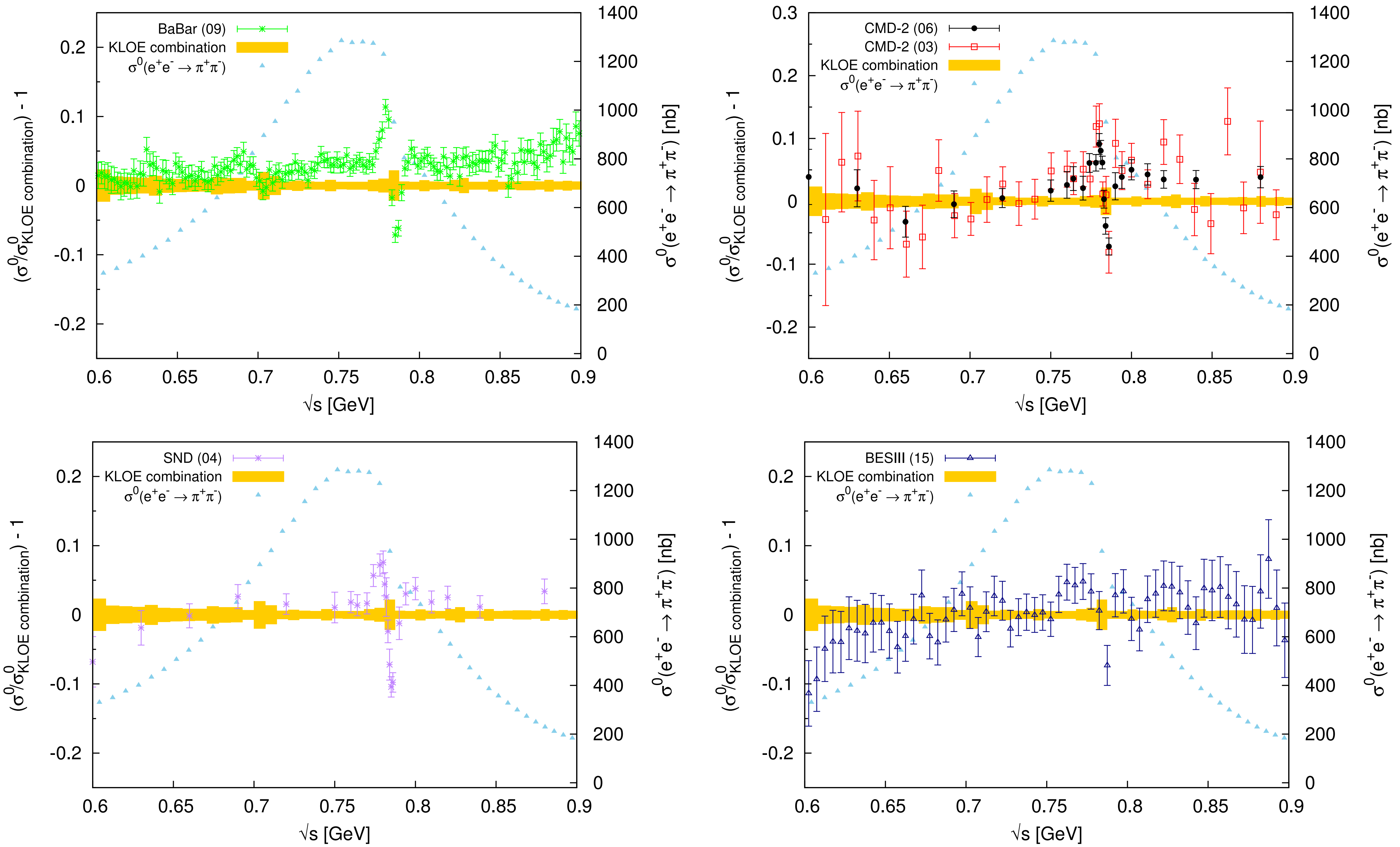


Fig. 13. The $\pi^+\pi^-$ cross section from the KLOE combination compared to the BABAR, CMD-2, SND, and BESIII data points in the 0.6–0.9 GeV range [82]. The KLOE combination is represented by the yellow band. The uncertainties shown are the diagonal statistical and systematic uncertainties summed in quadrature.