Coleman-Weinberg Abrikosov-Nielsen-Olesen strings

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2205.04394 [hep-ph]

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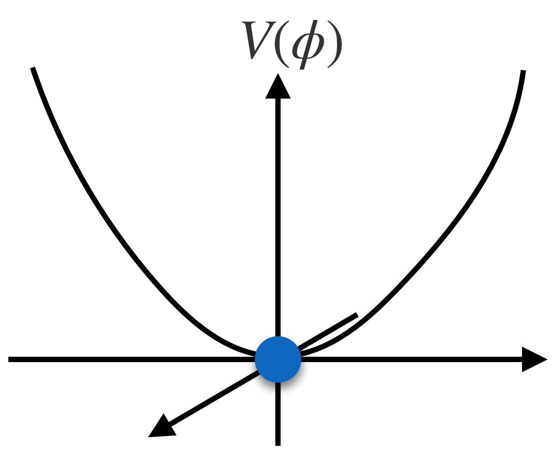
26th July 2022, seminar @ Osaka U.

Introduction

Topological Soliton

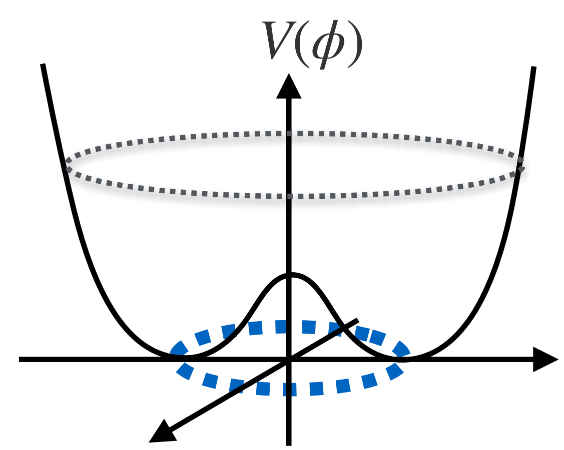
- **Topological soliton is a coherent excitation in field theories.**

- It appears if **vacuum has non-trivial topology.**



trivial

(a point)



non-trivial

(a circle S^1)

- **Why interesting? → Ubiquitous in modern physics!**
  - monopole, vortex string, skyrmion, instanton, etc..

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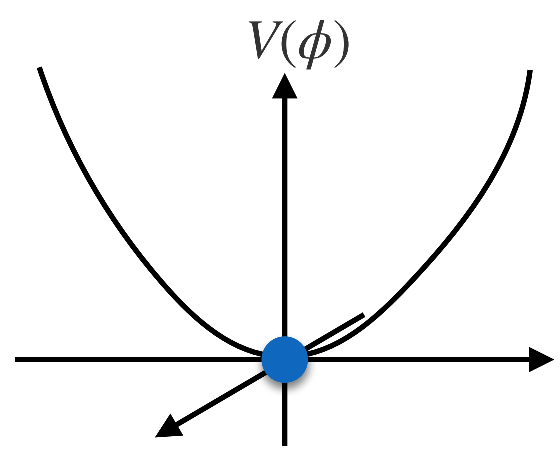

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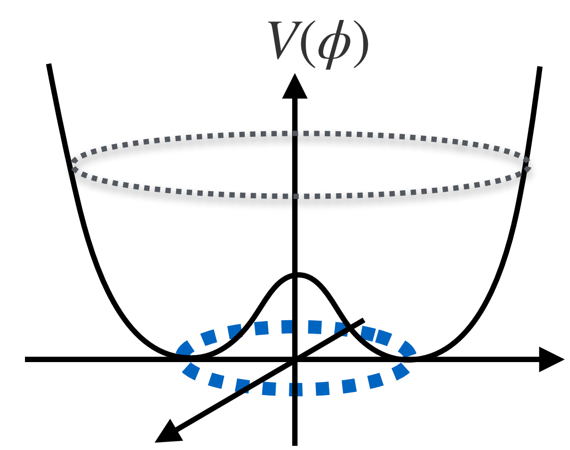

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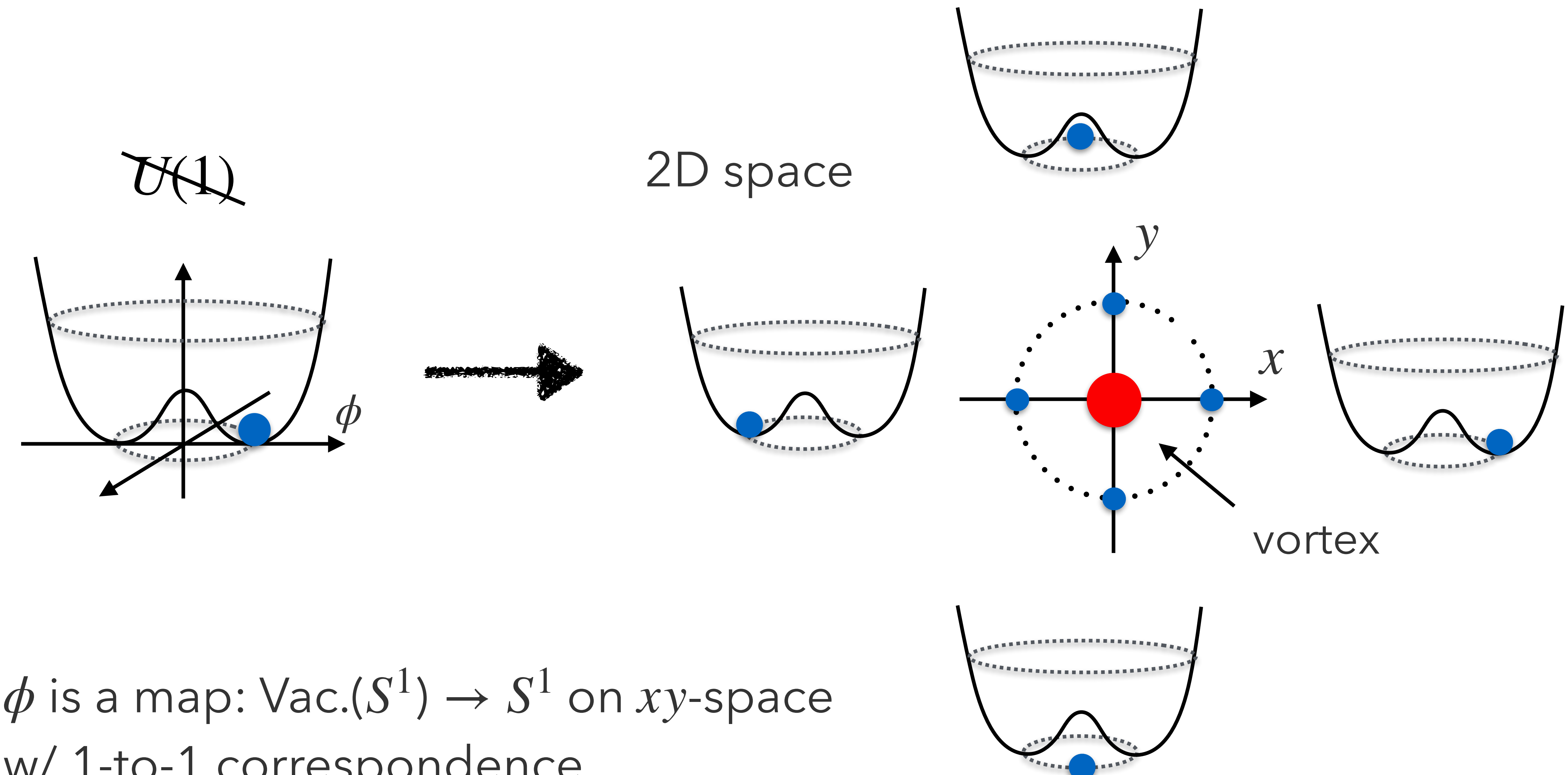
Vortex String

- **Vortex string appears by SSB of $U(1)$**



ϕ is a map: Vac.(S^1) $\to$ S^1 on xy-space
w/ 1-to-1 correspondence

→ winding # = 1, vortex is topologically protected

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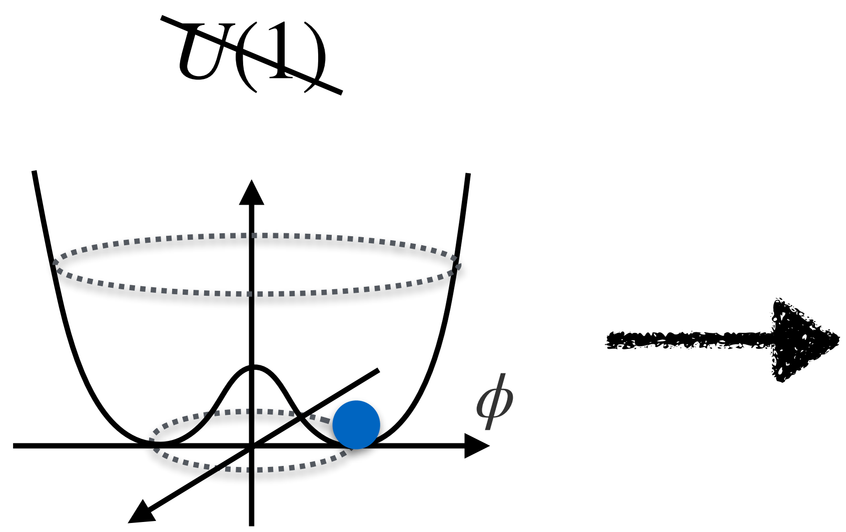

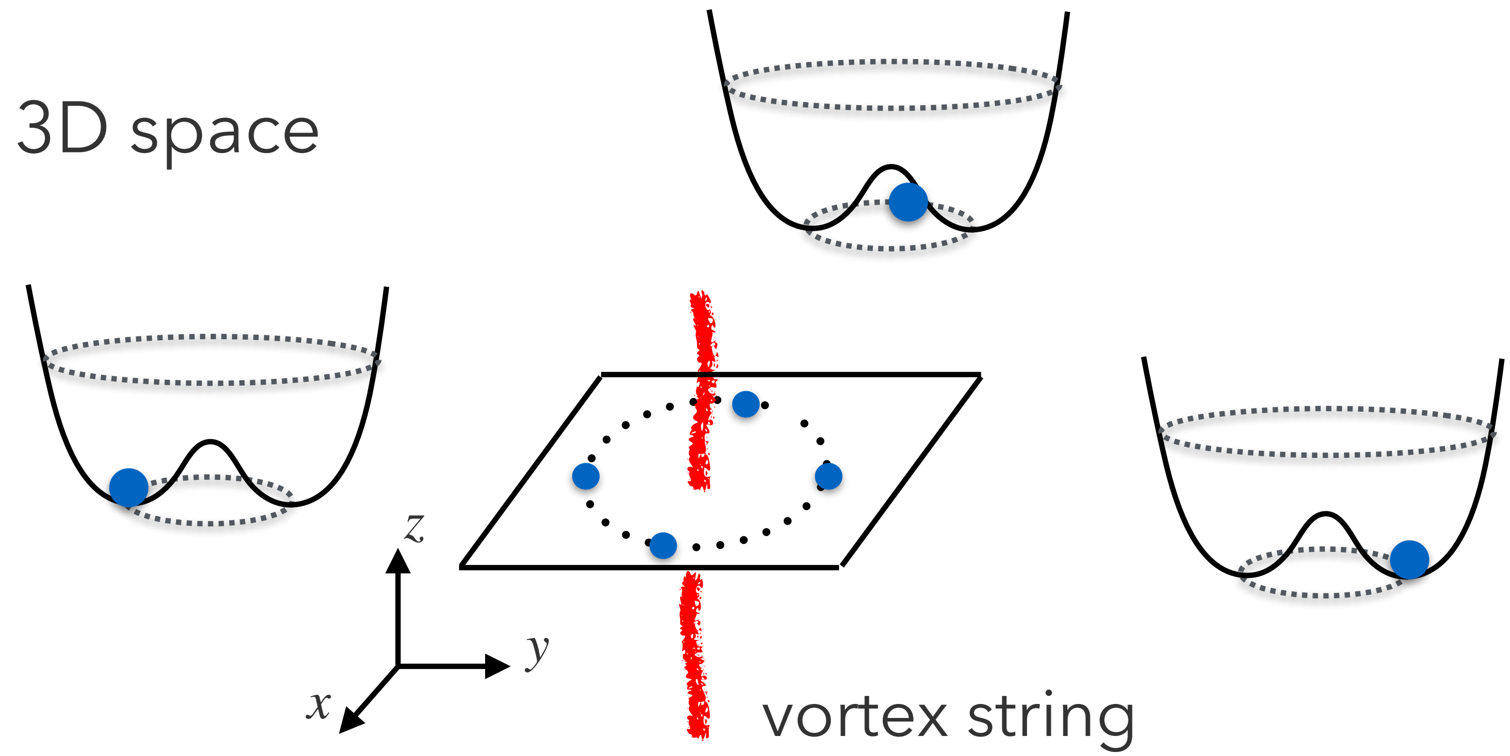

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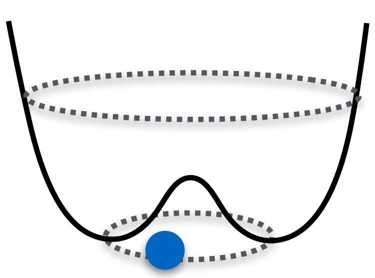

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Vortex String

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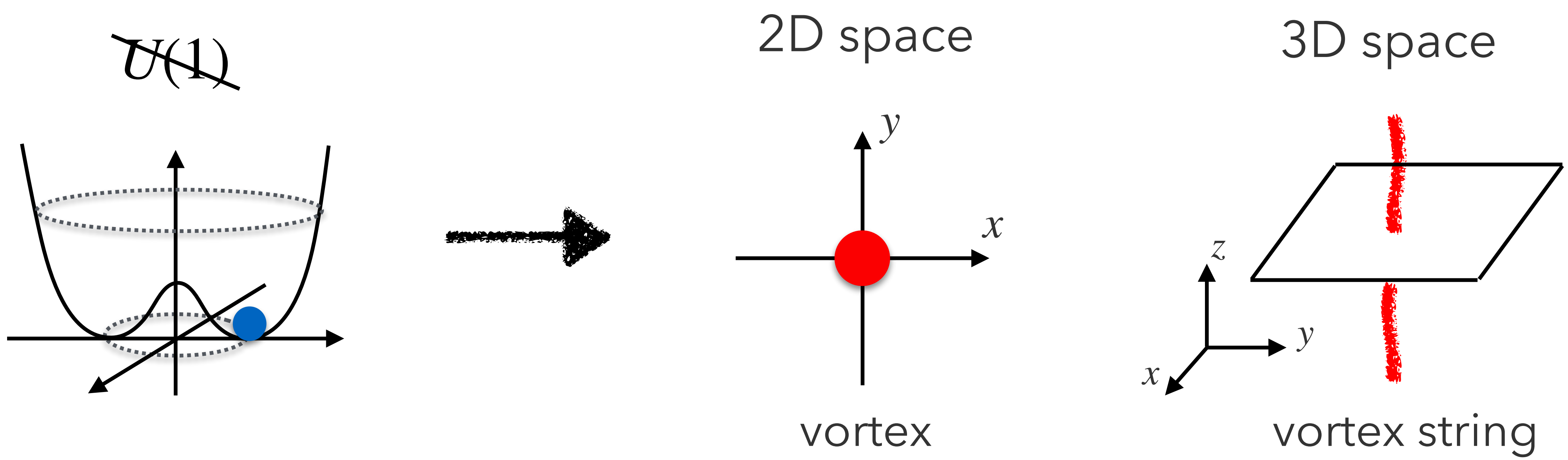

- **Vortex string appears in many systems:**
    - cosmic string in particle physics / cosmology
    - quantum vortex in superfluid
    - magnetic flux tube in superconductor
    - color superconductor vortex in neutron star

The (most) important question:

interaction of vortex strings

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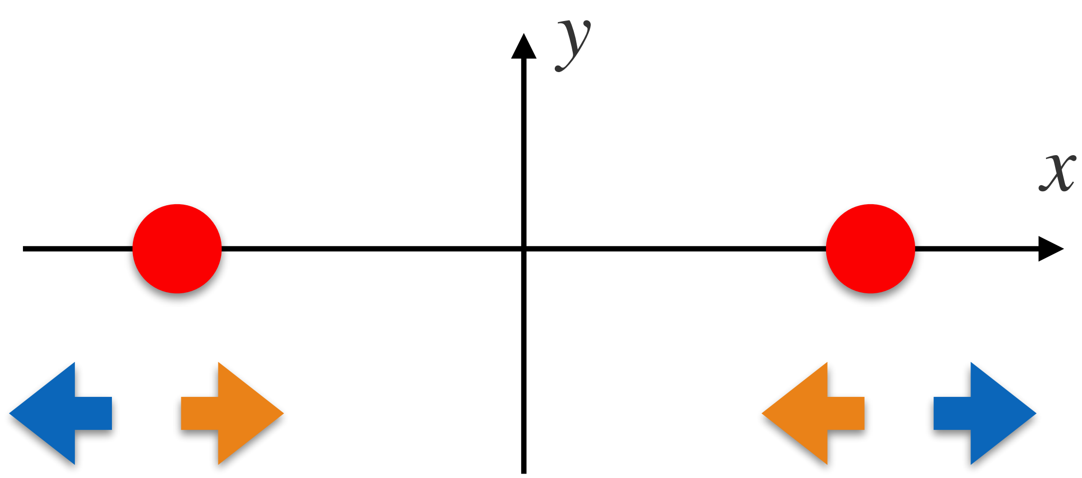
interaction of vortex strings

repulsive / attractive?

long- / short-range?

Interaction of Vortex Strings

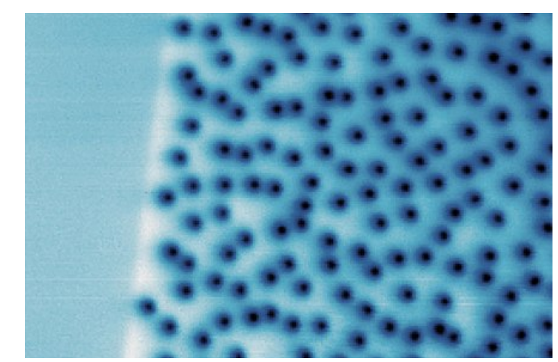
- **Two parallel vortex strings on 2D slice**



repulsive? **attractive?**

cf.) vortex-antivortex is always attractive

(wikipedia)



- In superconductor,

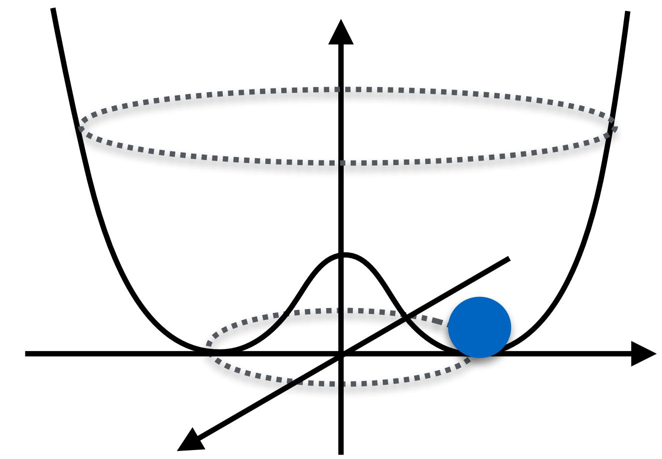
  { repulsive (type II) → formation of Abrikosov lattice
  { attractive (type I) → easy to destroy superconductivity

- For cosmic strings, the interaction determines the dynamics.

  → affect cosmological history

Eg.) Abrikosov-Nielsen-Olesen string

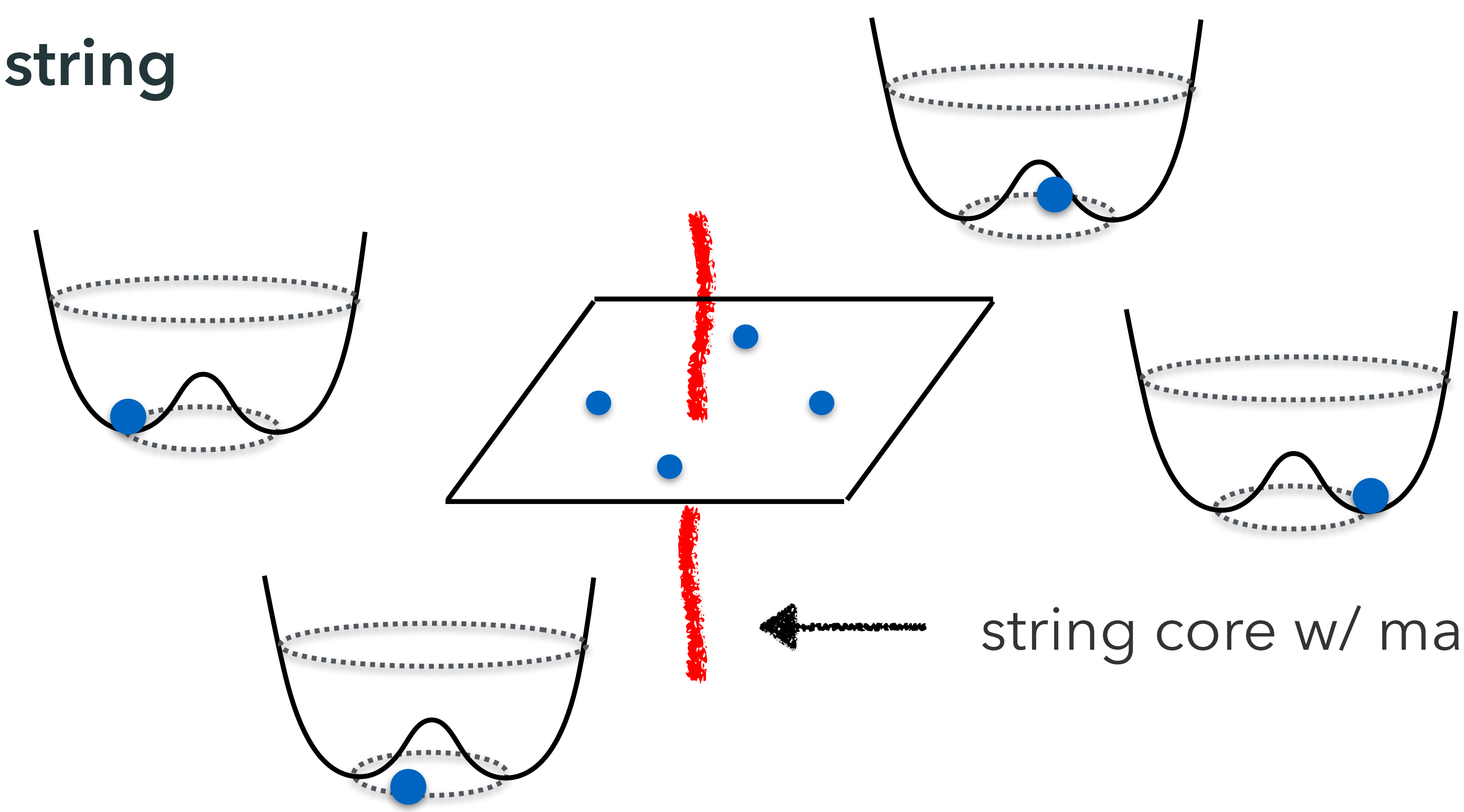
- (3+1)D Abelian-Higgs model w/ U(1) gauge sym



$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu \phi|^2 + m^2|\phi|^2 - \lambda|\phi|^4$$

$$\langle\phi\rangle = v \longrightarrow \cancel{U(1)} \qquad \begin{cases} m_\phi = 2\sqrt{\lambda}v \\ m_A = \sqrt{2}gv \end{cases}$$

- **ANO string**



string core w/ mag. flux $2\pi/g$

Eg.) Abrikosov-Nielsen-Olesen string

- **Interaction force between two ANO strings**

determined by one parameter: $\beta \equiv \frac{m_\phi^2}{m_A^2} = \frac{4\lambda v^2}{2g^2v^2}$

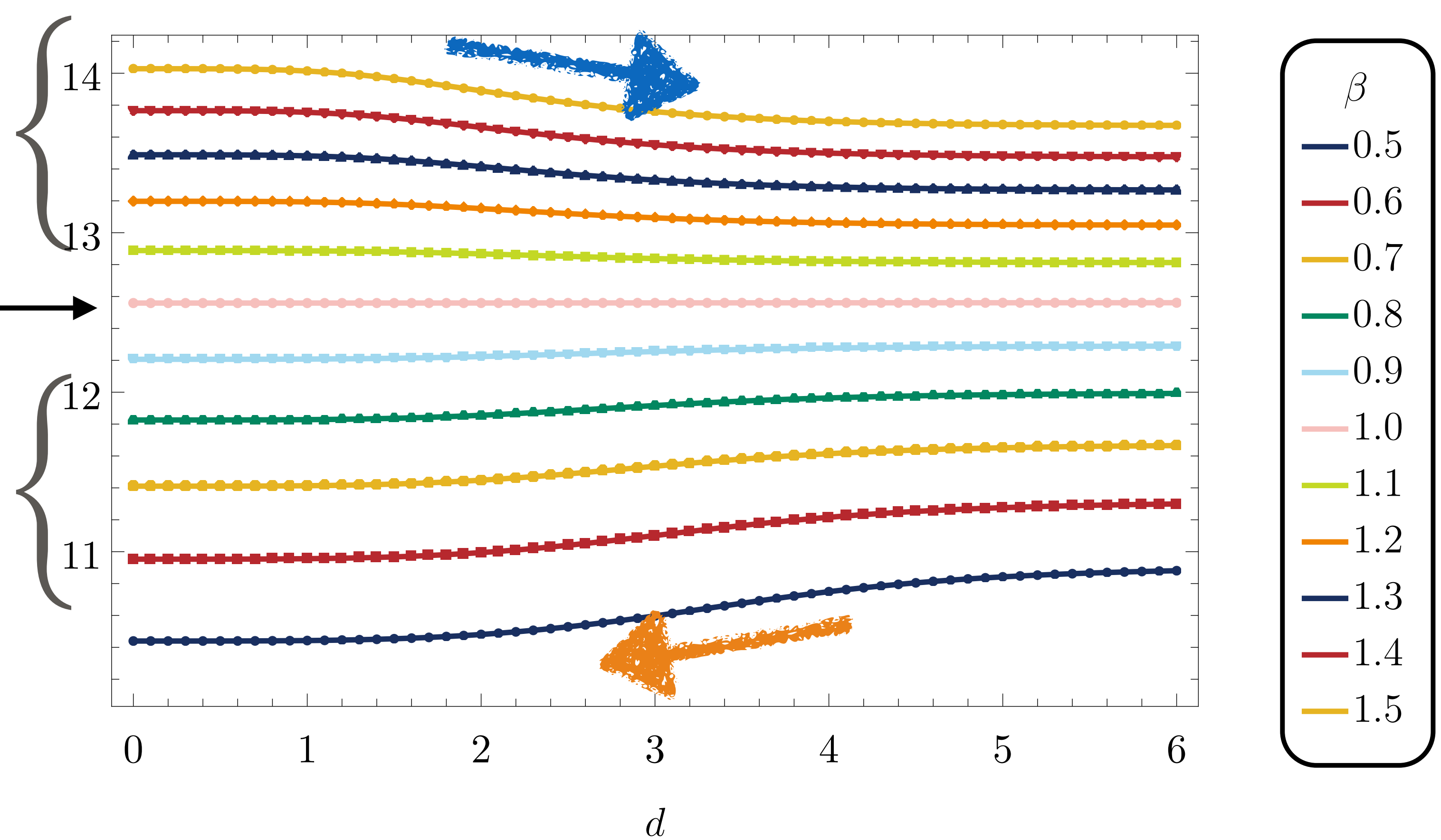
The force can be **repulsive** or **attractive** for $\beta > 1$ and $\beta < 1$.

interaction potential

$\beta > 1$
(type II)

$\beta = 1$ (BPS limit)

$\beta < 1$
(type I)



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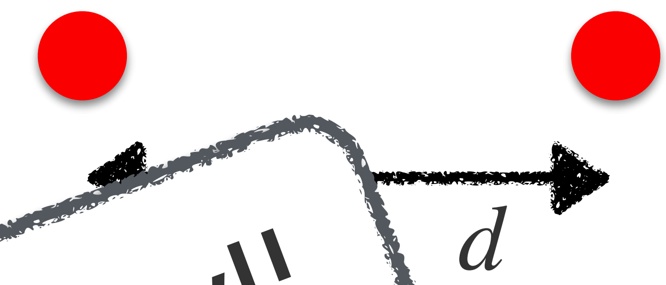


determined by one parameter: $\beta \equiv \frac{m_\phi^2}{m^2} = \frac{4\lambda v^2}{\;\;}$

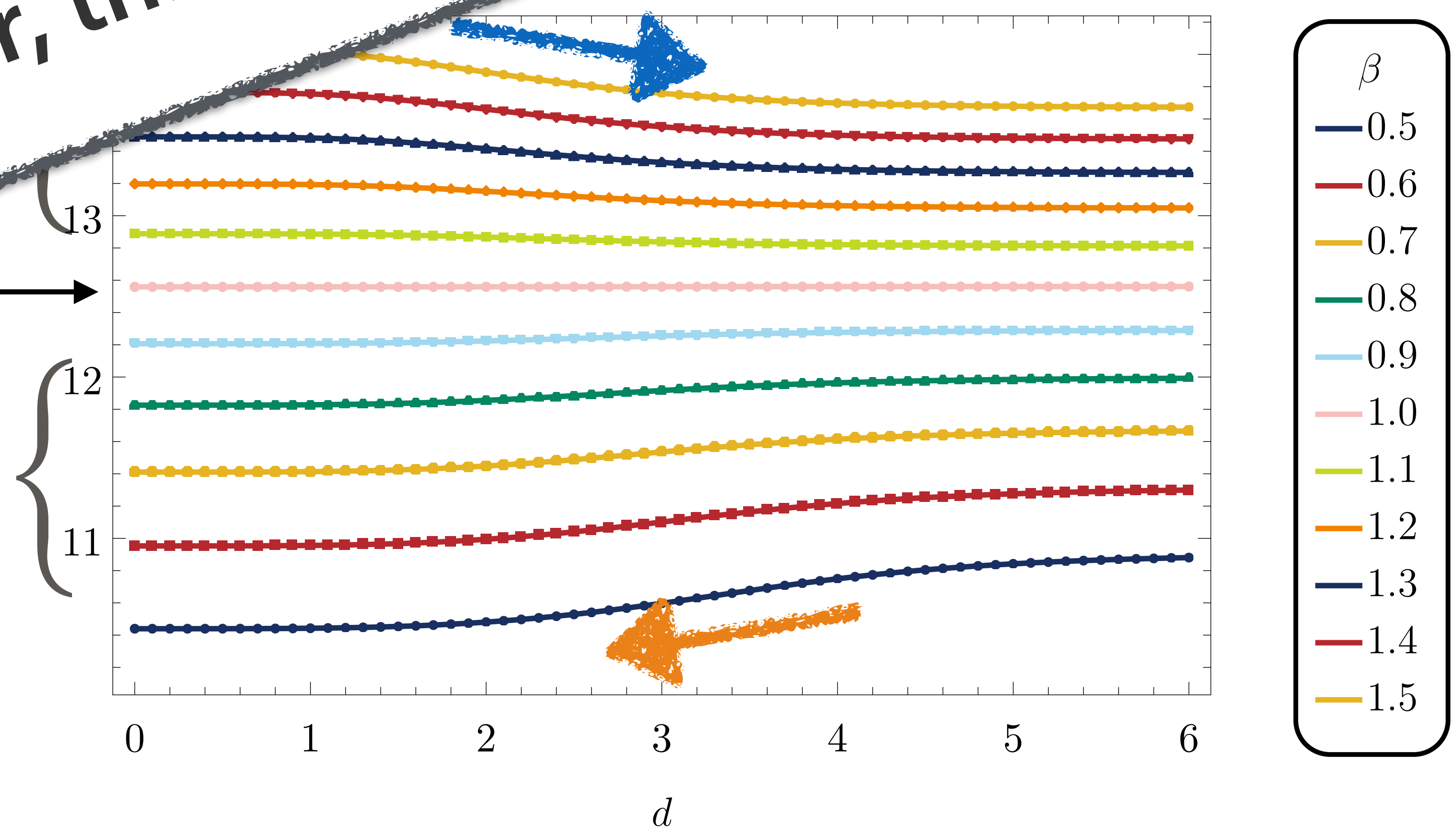
The force can be **repulsive** … nd $\beta < 1$.

…tial

However, this is not the end of story!!

$\beta = 1$ (… limit)

$\beta < 1$
(type I)



Coleman-Weinberg potential

- In particle physics, there are various potentials realizing SSB.

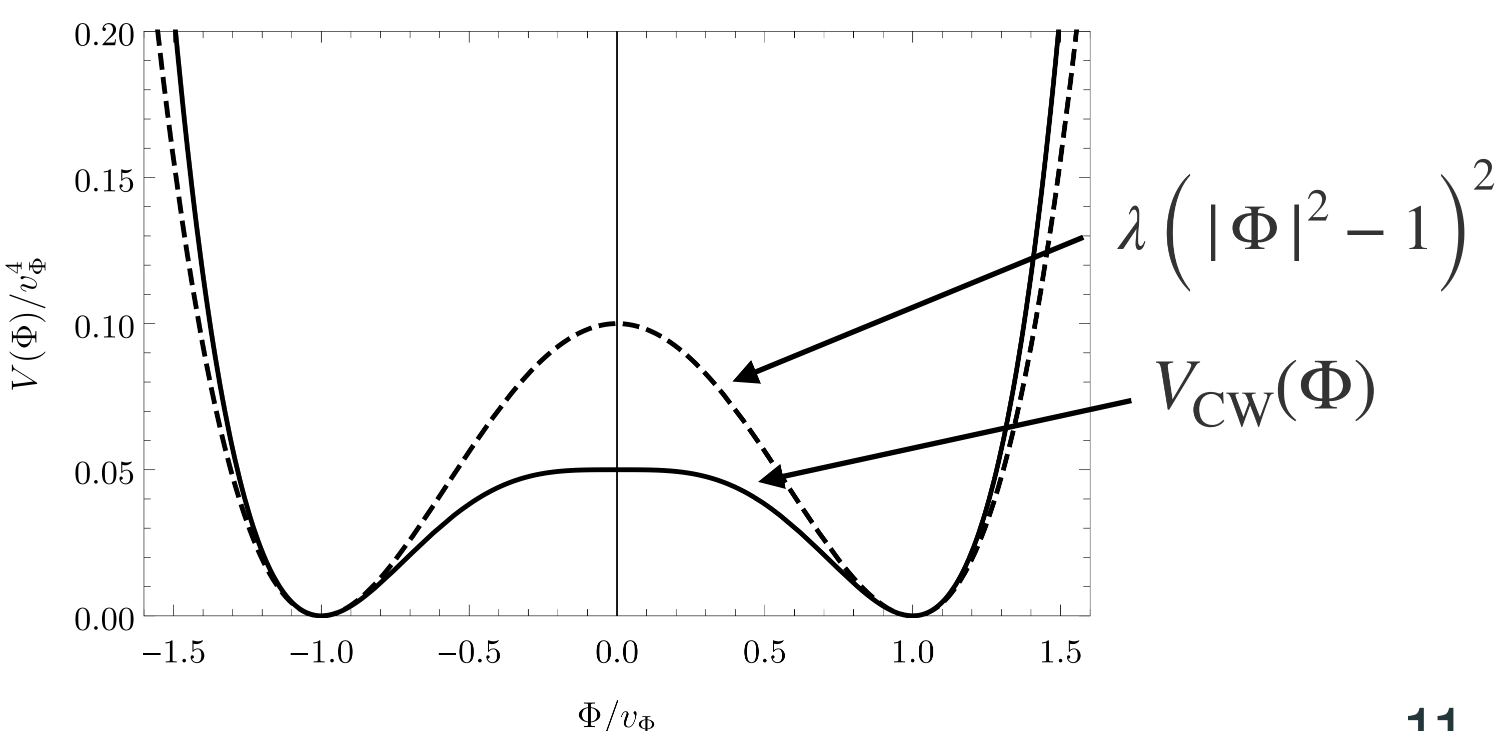
- **Coleman-Weinberg potential w/o quadratic term:**

$$V_{\rm CW}(\Phi) = \lambda(\Phi)\,|\,\Phi\,|^4 \qquad \lambda(\Phi) = \lambda_{\rm CW}\left(\log\frac{|\,\Phi\,|^2}{v^2} - \frac{1}{2}\right)$$

running quartic coupling

- flatter structure around origin

- well motivated by naturalness (explained later)



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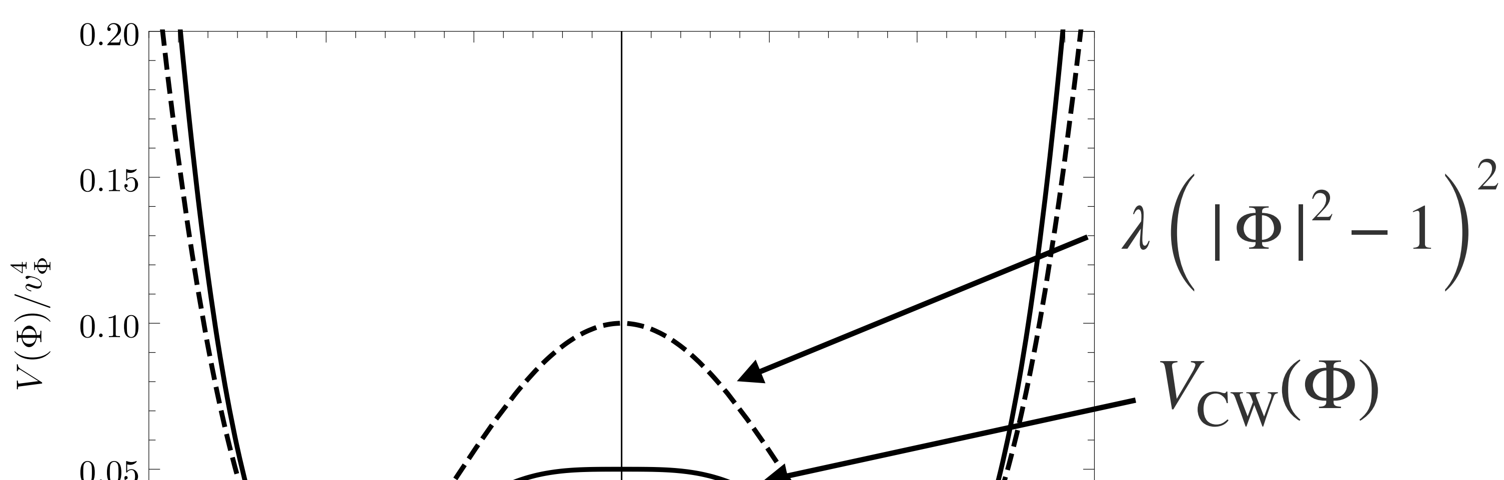


Does this potential affect properties of vortices? → Yes!!

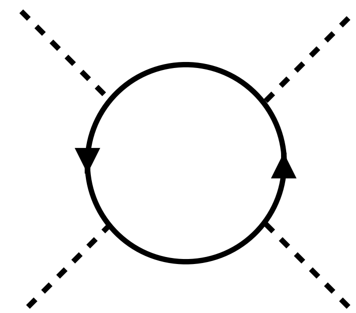
Plan of talk

- Introduction ←Done
- CW-ANO string
- Interaction of CW-ANO string
- Summary

CW-ANO string

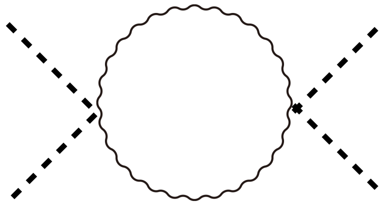
Coleman-Weinberg mechanism [Coleman-Weinberg '73]

- **Coleman-Weinberg potential w/o quadratic term:**



$$V_{\rm CW}(\Phi) = \lambda(\Phi)\,|\,\Phi\,|^4$$

  - tree level → λ=const., **scale invariant, no SSB**



  - quantum effects → $\lambda(\Phi) = \lambda_{\rm CW}\left(\log\frac{|\,\Phi\,|^2}{v^2} - \frac{1}{2}\right)$

    triggers SSB

$\lambda_{\rm CW}, v$ depend on underlying d.o.f.

Scale is generated by quantum effect! (dim. transmutation)

- Many attempts to explain Electroweak scale dynamically in terms of this mechanism (naturalness). [Iso-Okada-Orikasa '09] [Iso-Orikasa '12] [Chun-Jung-Lee '13] [Haruna-Kawai '19] [YH-Tsumura-Yamada '20]
- 1st order p.t. in early universe → gravitational wave

[Jinno-Takimoto '16] [Kubo-Yamada '16] [Iso-Serpico-Shimada '17]

Model

- **3+1 D Abelian-Higgs model w/ two types of potential**

$$S = \int d^4x \left[-\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + |D_\mu \Phi|^2 - V(\Phi) \right] \qquad D_\mu = \partial_\mu + igA_\mu$$

- **usual Quadratic-Quartic**

$$V(\Phi) = \lambda \left(|\Phi|^2 - v^2 \right)^2$$

- **Coleman-Weinberg**

$$V(\Phi) = \lambda \left(\log \frac{|\Phi|^2}{v^2} - \frac{1}{2} \right) |\Phi|^4$$

- Both models spontaneously break $U(1)$ sym and have vortex strings.
  - Quadratic-Quartic → conventional ANO string
  - Coleman-Weinberg → **CW-ANO string!** (main interest)

Model

- It is convenient to introduce rescaling: $A_\mu \to A_\mu / g \qquad \Phi \to \Phi / g$

$$S = \frac{1}{g^2} \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + |D_\mu \Phi|^2 - V_\beta(\Phi) \right] \qquad D_\mu = \partial_\mu + i A_\mu$$

$$V_\beta(\Phi) = \frac{\beta}{2} \left(|\Phi|^2 - 1 \right)^2 \quad \textbf{(QQ)}$$

$$V_\beta(\Phi) = \frac{\beta}{2} \left(\log |\Phi|^2 - \frac{1}{2} \right) |\Phi|^4 \quad \textbf{(CW)}$$

$$\beta \equiv \frac{m_\phi^2}{m_A^2} = \frac{2\lambda}{g^2}$$

- Tension (=energy per unit length of string):

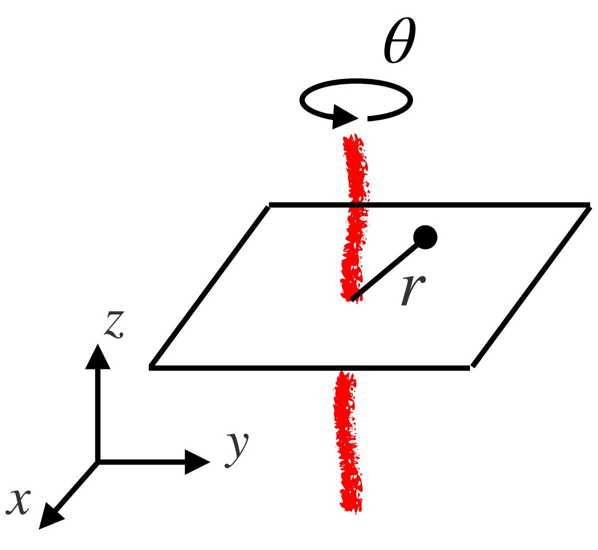
$$T = \frac{dE}{dz} = \int d^2x \left[\frac{1}{2} (\partial_i A_j)^2 + |D_i \Phi|^2 + V_\beta(\Phi) \right]$$

(assuming static and Coulomb gauge)

Axisymmetric string

- **Field configuration:**

$$\Phi(x) = f(r)e^{i\theta} \qquad A_\theta(x) = a(r)$$



$\longrightarrow$ winding # = 1 & magnetic flux $\int d^2x\, B = 2\pi$

- **EOMs for $f(r)$ and $a(r)$:**

$$f'' + \frac{1}{r}f' - \frac{n^2(1-a)^2}{r^2}f - \frac{1}{2}\frac{\partial V}{\partial f} = 0$$

$$a'' - \frac{1}{r}a' + 2(1-a)f^2 = 0$$

- **boundary conditions:**

$$f(0) = a(0) = 0 \qquad f(\infty) = a(\infty) = 1$$

Axisymmetric string

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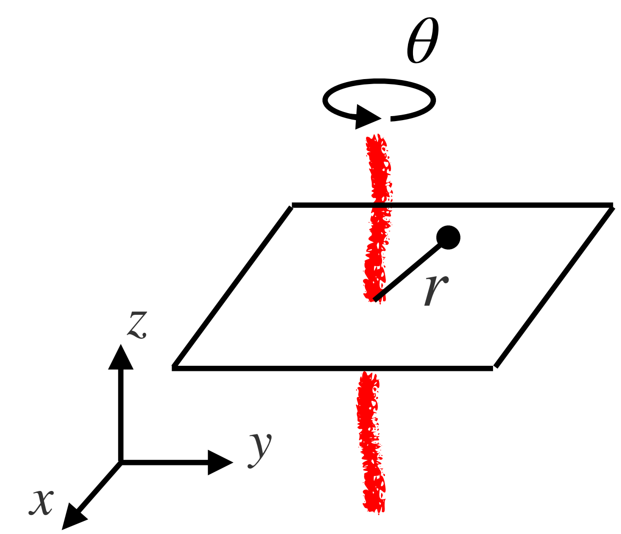
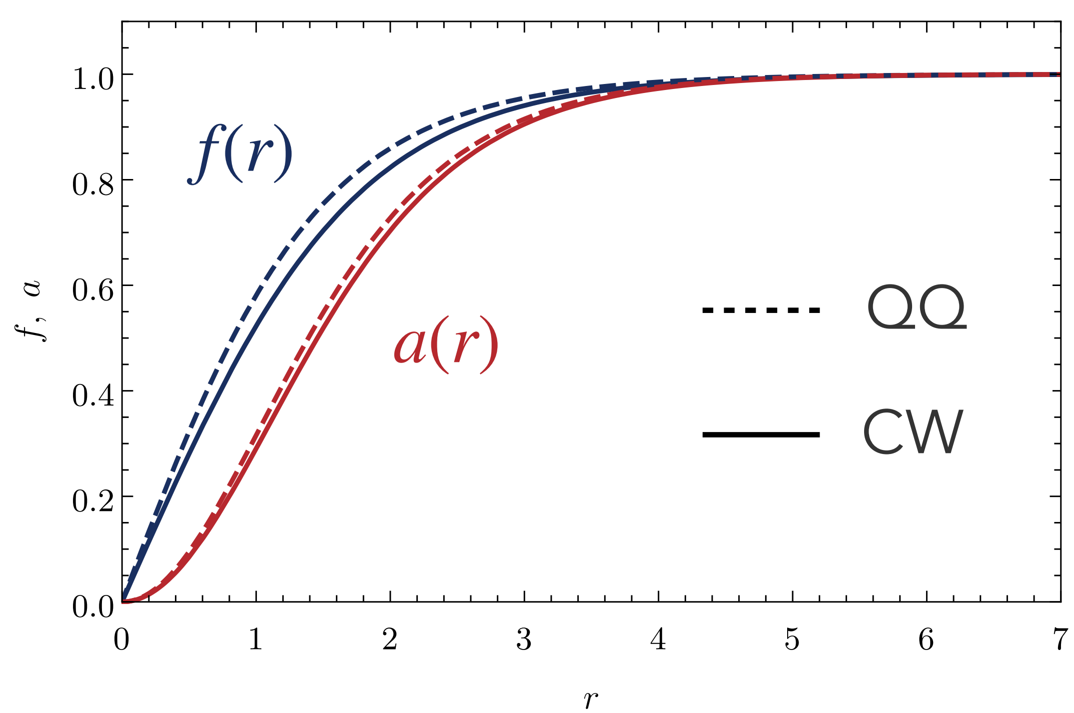


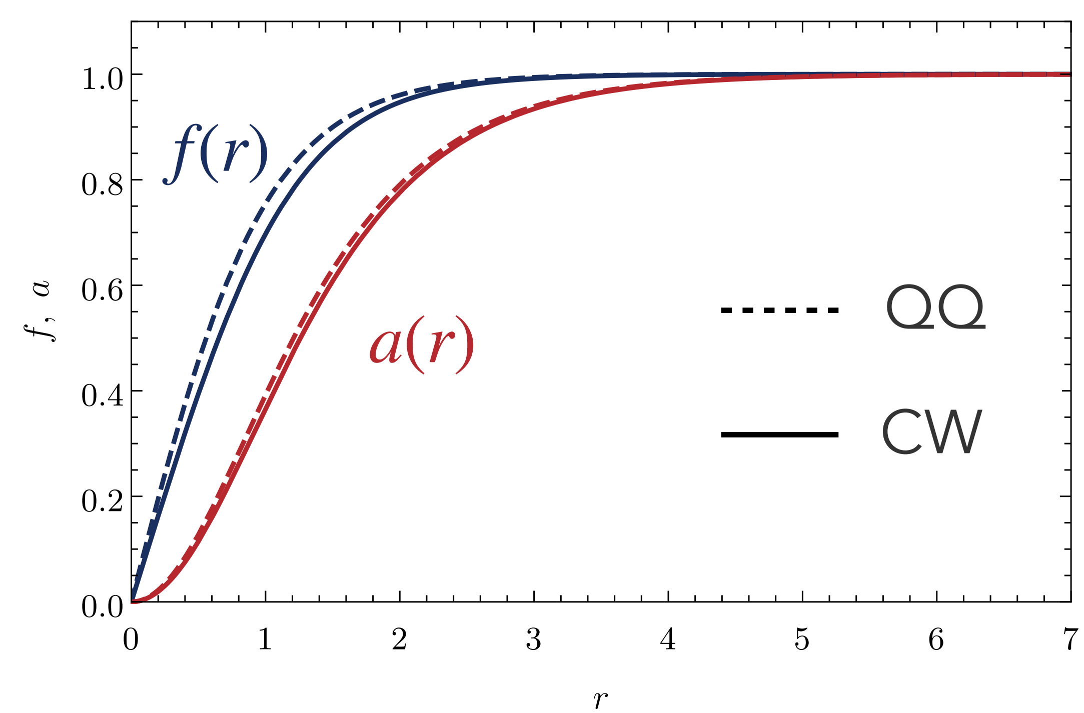
$\longrightarrow$ winding # = 1 & magnetic flux $\int d^2x\, B = 2\pi$

- **no significant difference for the string solutions**

$\beta = 0.5$



$\beta = 1.5$



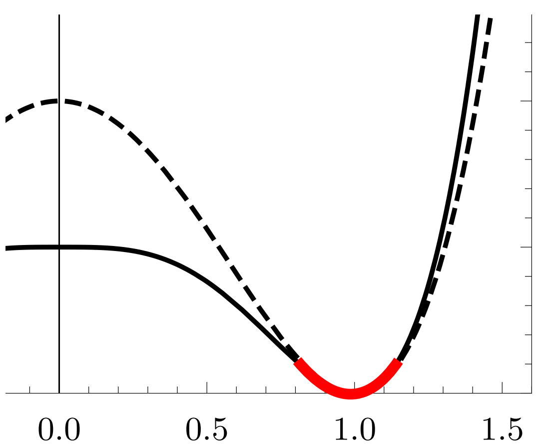
Asymptotics of CW-ANO string

- Asymptotic behavior at $r \to \infty$ can be derived analytically.

- Since $f(r) \simeq 1$ and $a(r) \simeq 1$ at $r \to \infty$, it is useful to write down linearized EOM w.r.t. $\delta f \equiv 1 - f$ and $\delta a \equiv 1 - a$

$$\delta f'' + \frac{1}{r}\delta f' - 2\beta\delta f = \mathcal{O}((\delta f)^2, (\delta a)^2)$$

$$\delta a'' - \frac{1}{r}\delta a' - 2\delta a = \mathcal{O}((\delta f)^2, (\delta a)^2)$$



Only curvature around vac is relevant.

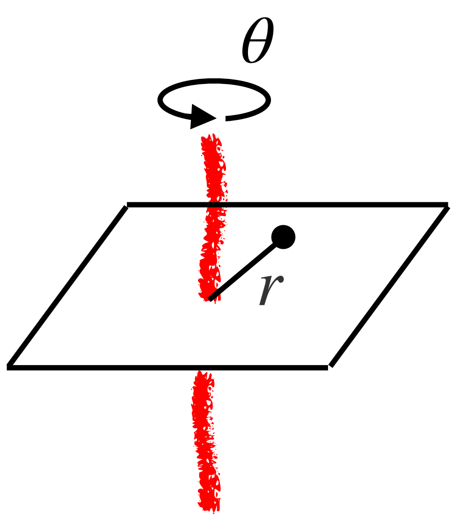
- Asymptotic behavior:

$$\delta f \simeq r^{-1/2} \exp\left[-\sqrt{2\beta} r\right] \qquad \delta a \simeq r^{1/2} \exp\left[-\sqrt{2} r\right]$$

Higher winding

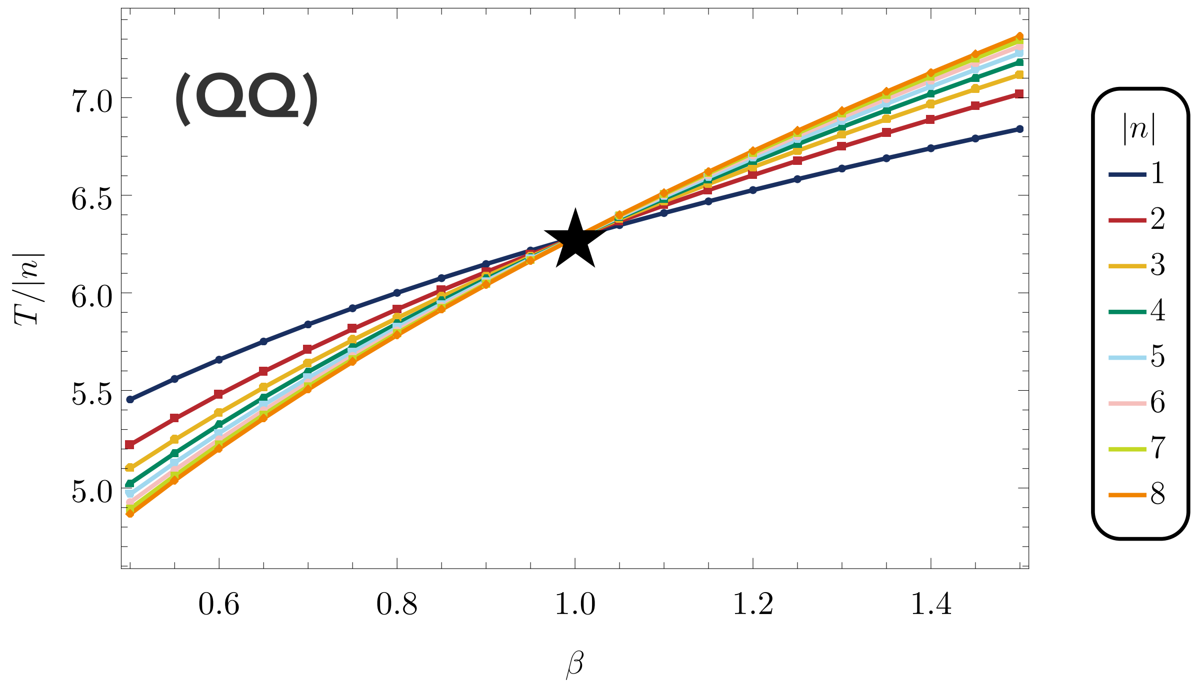
- **Field configuration:**

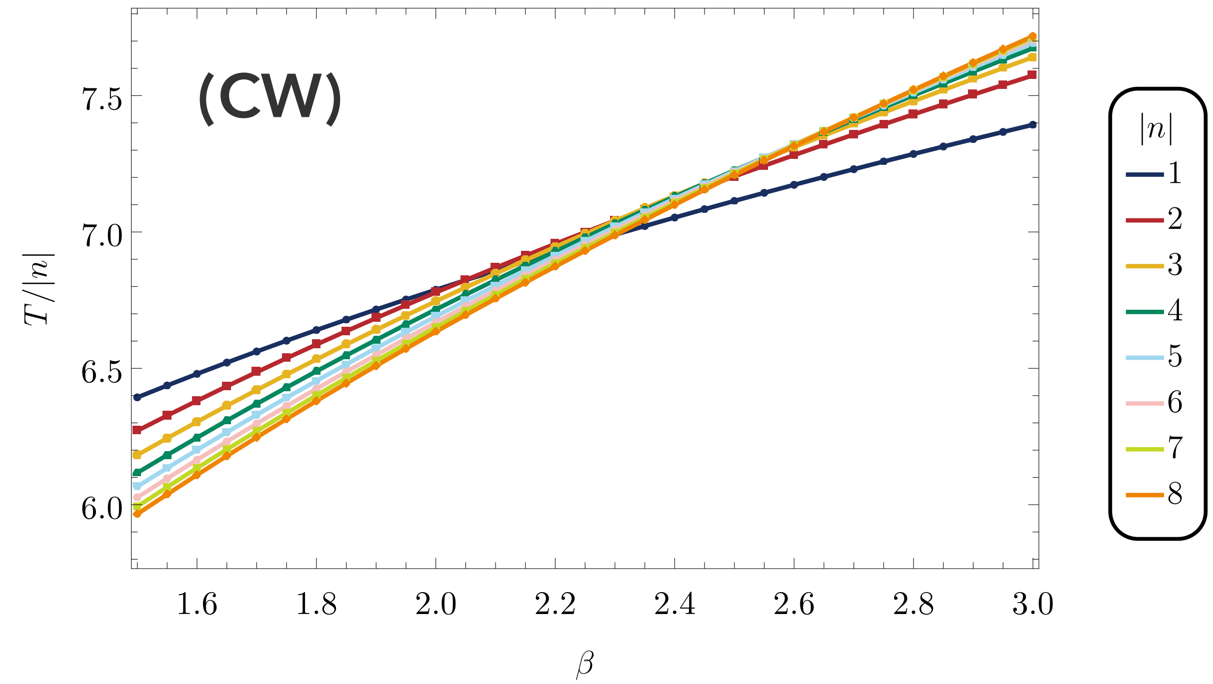
$$\Phi(x) = f(r)e^{in\theta} \qquad A_\theta(x) = n\,a(r)$$



$\longrightarrow$ winding # $= n$ & magnetic flux $\int d^2x\, B = 2\pi n$

- **String tension for QQ and CW cases:**





- **For QQ case, all lines cross at $\beta = 1$ (BPS state) while it doesn't happen for CW case.**

BPS state

[Bogomol'nyi '76]

[Prasad-Sommerfield '75]

- In Quadratic Quartic case, the energy can be rewritten by completion of square:

$$T = 2\pi|n| + 2\pi\int_0^\infty \mathrm{d}r\, r\left[\left(f' + |n|\frac{a-1}{r}f\right)^2 + \frac{n^2}{2r^2}\left(a' + \frac{r}{|n|}(f^2-1)\right)^2 + \frac{1}{2}(\beta-1)(f^2-1)^2\right]$$

- For $\beta = 1$, the last term vanishes and the EOMs reduce to

$$f' + |n|\frac{a-1}{r}f = 0 \qquad a' + \frac{r}{|n|}(f^2-1) = 0$$

BPS equations

$$\longrightarrow \quad \frac{T}{|n|} = 2\pi$$

But, CW doesn't have this property!

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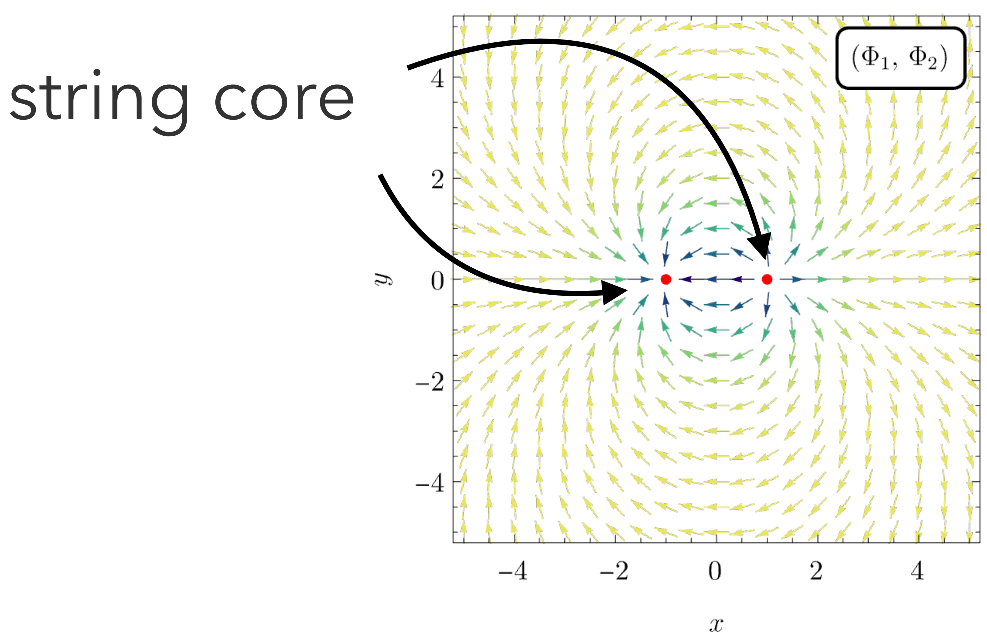
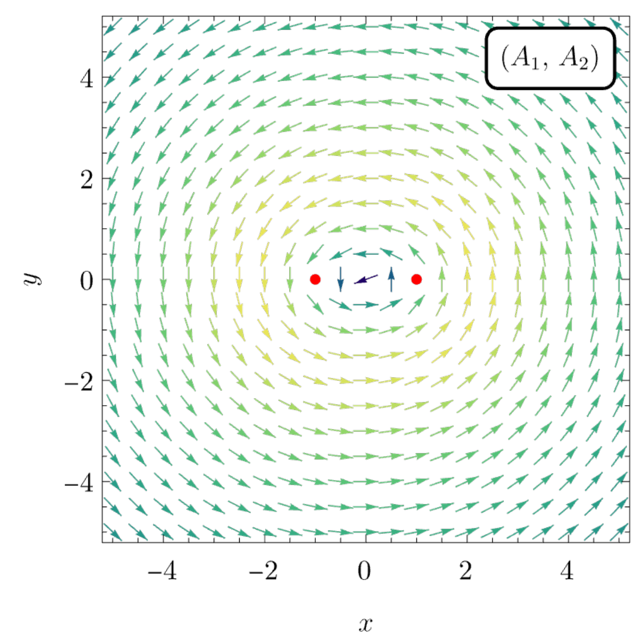
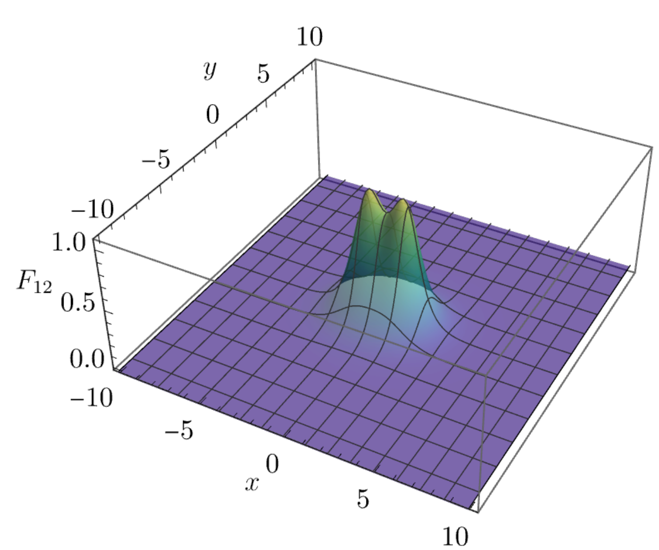
Interaction of CW-ANO string

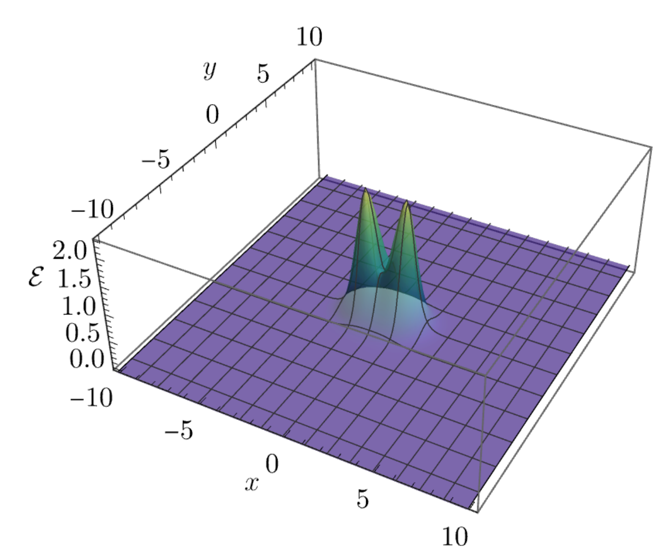
Two string system

- We put two strings on xy plane with distance d.







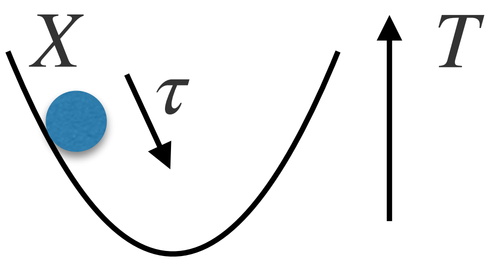


Relaxation

- **To calculate the tension of the two strings as a function of d, we adopt the so-called relaxation technique.**

- Keeping the positions of the strings fixed, introduce a fictitious time τ and change the field configurations iteratively by the flow equation:

$$\partial_\tau X = -\frac{\delta T}{\delta X} \qquad X = \Phi \text{ or } A_i$$

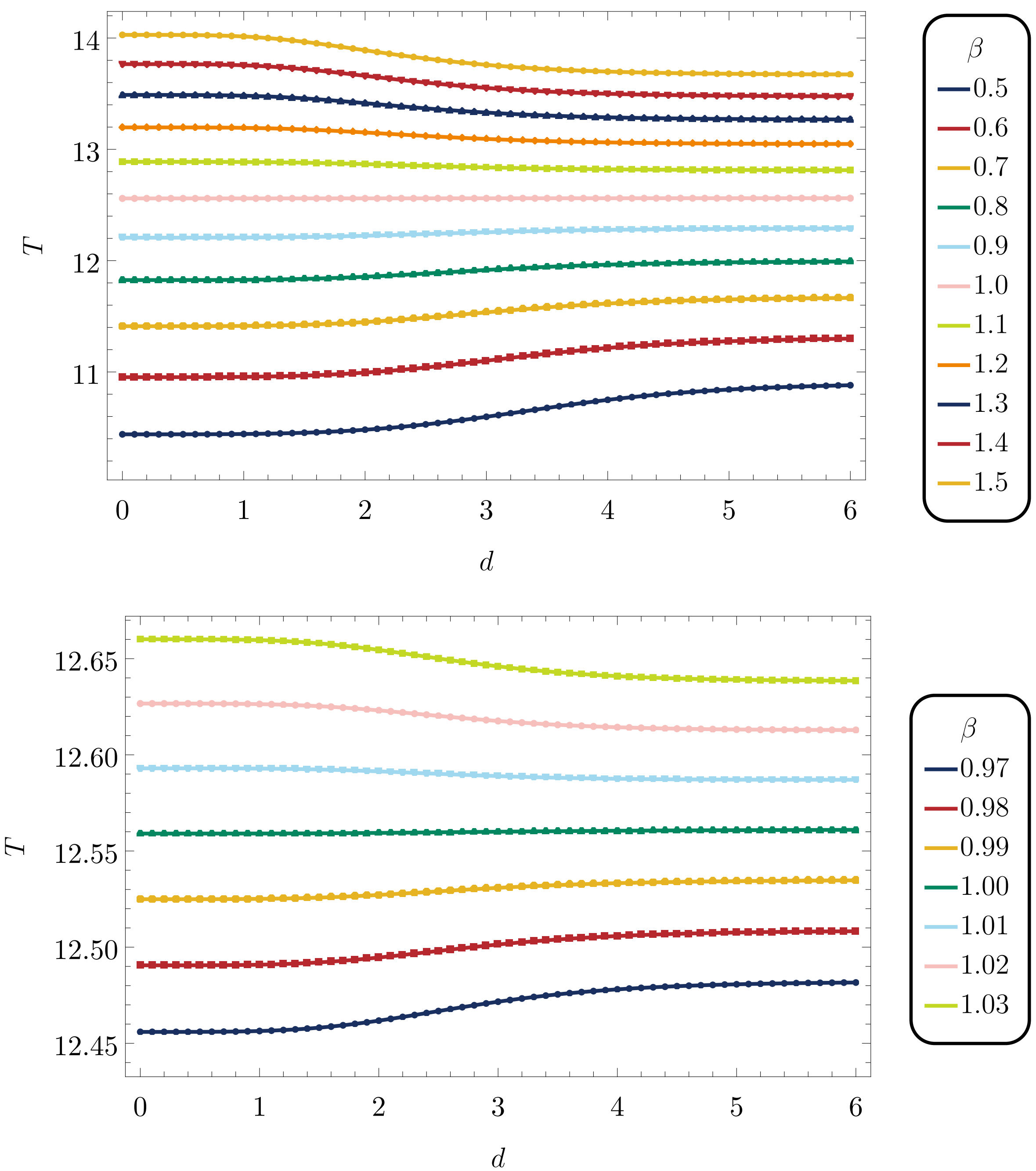


- These equations are diffusive and converge for large τ
  → minimum-tension configuration w/ fixed d

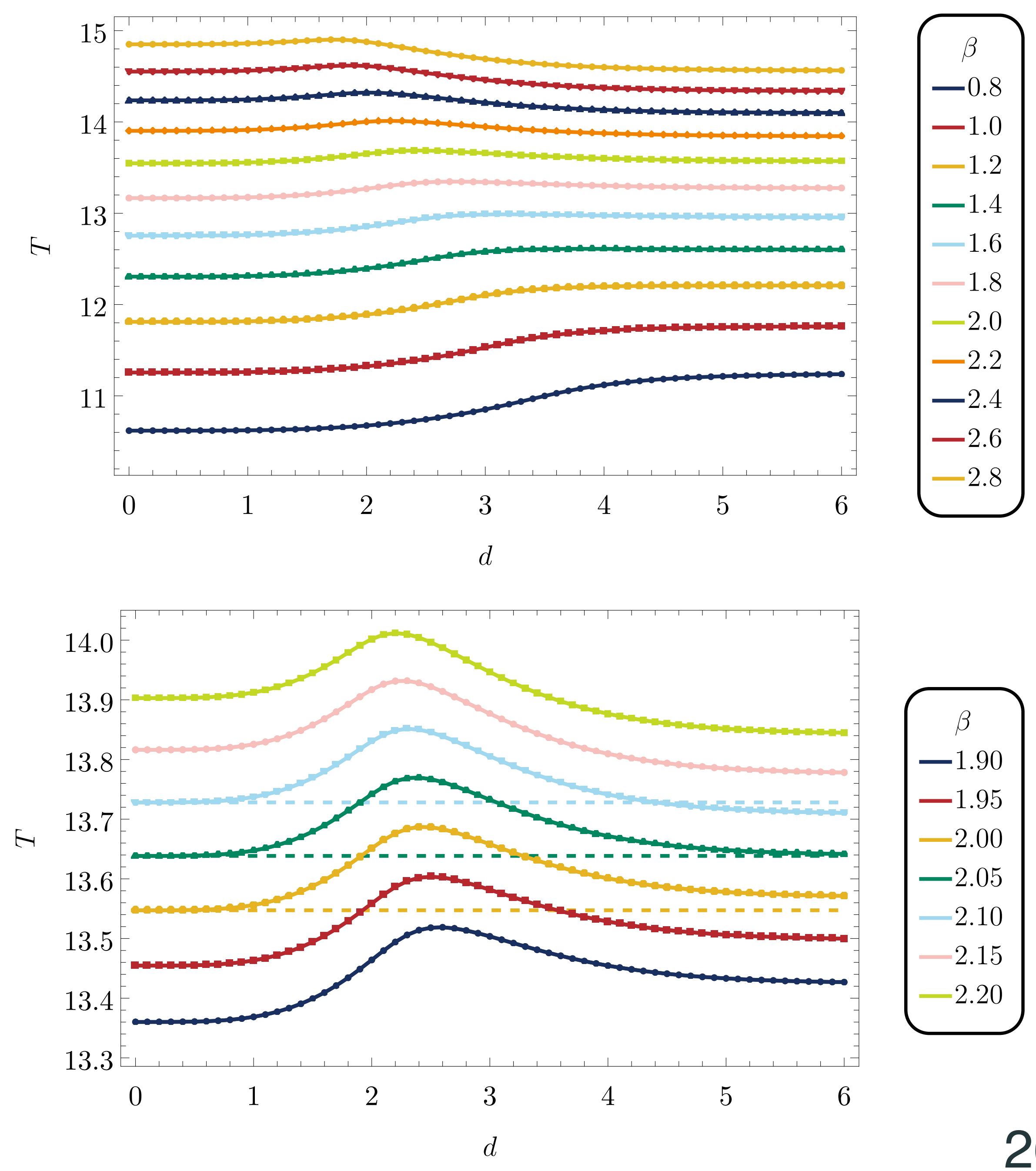
String Tension

- Tension as a function of d for different β

Quadratic-Quartic



Coleman-Weinberg



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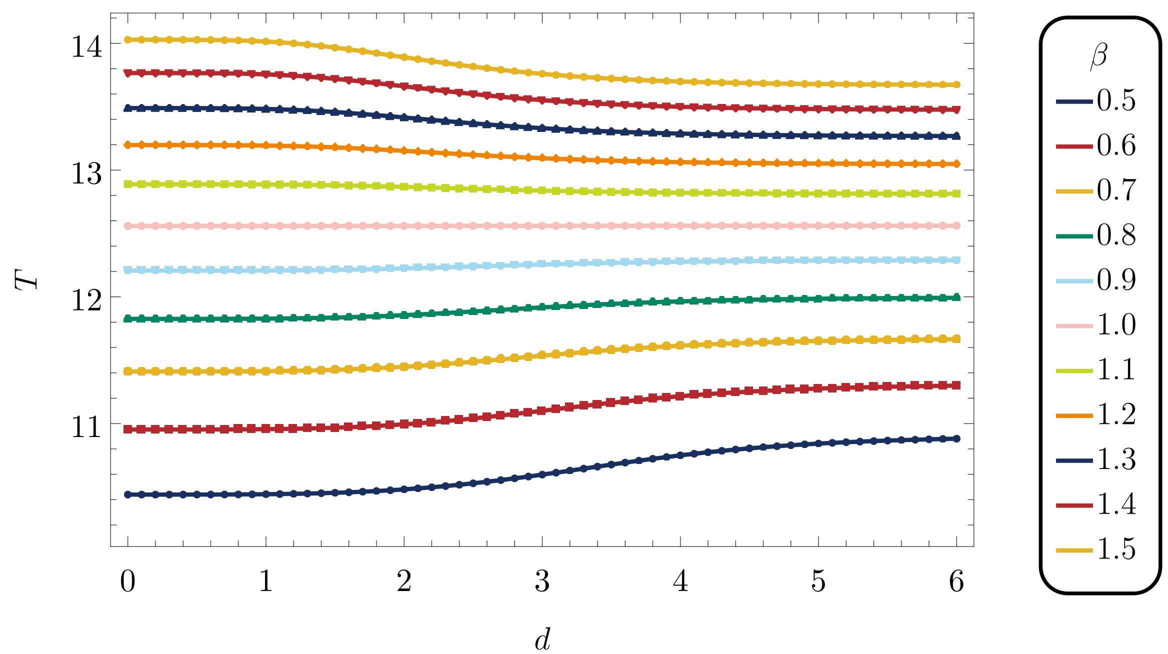

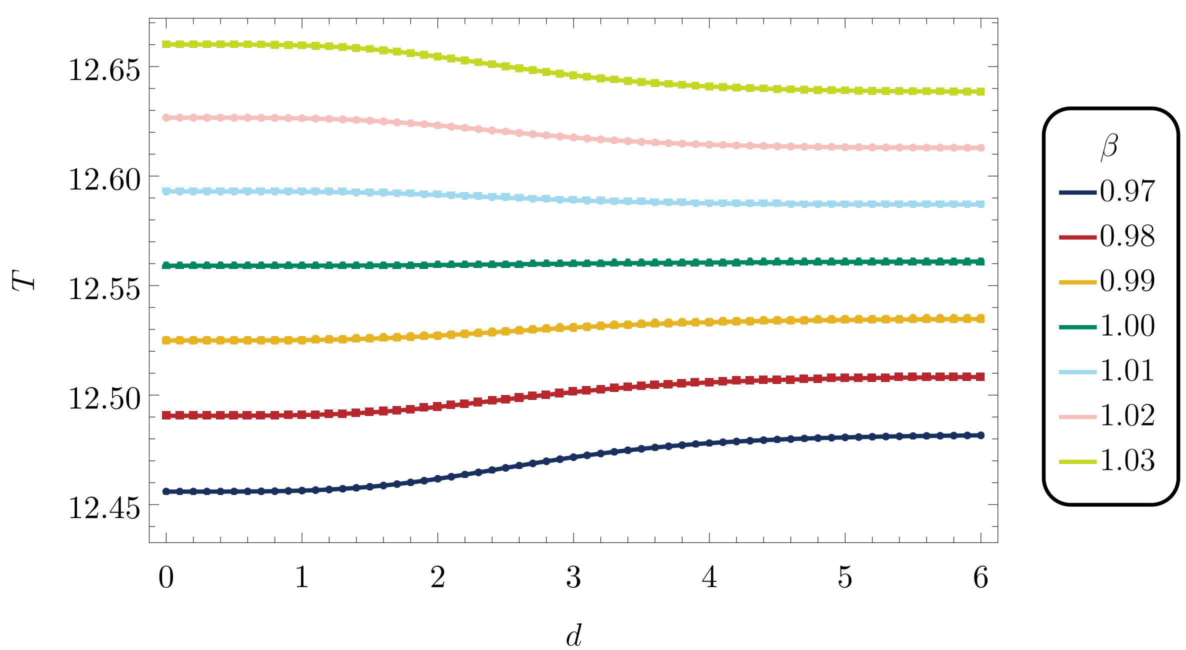

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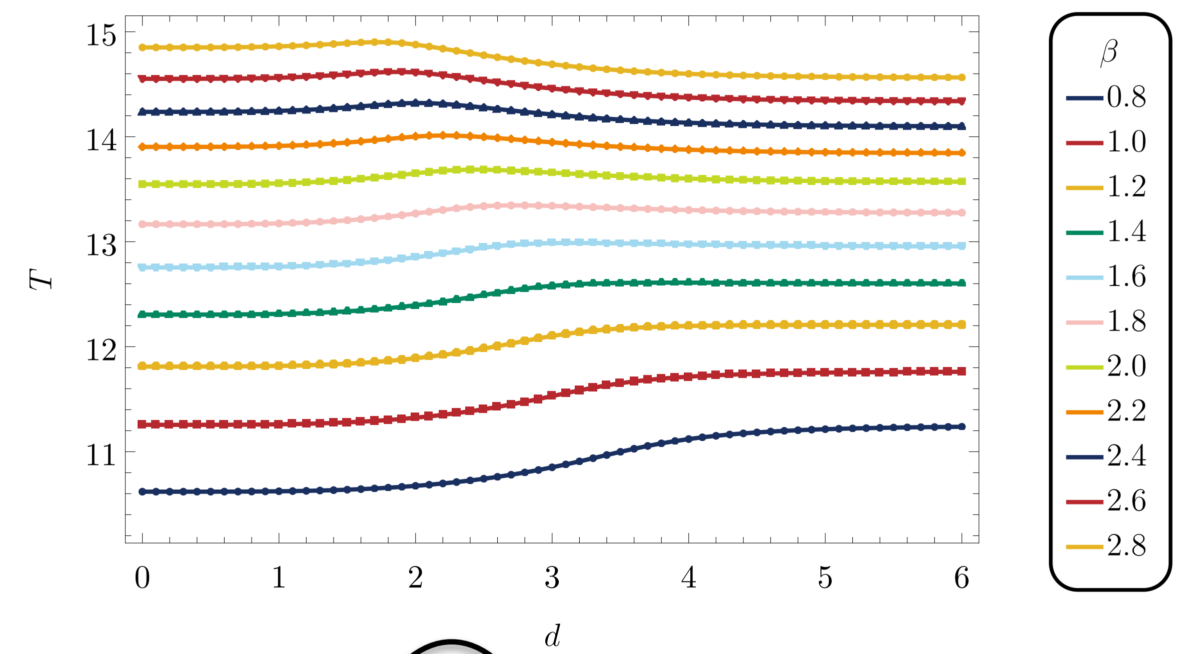
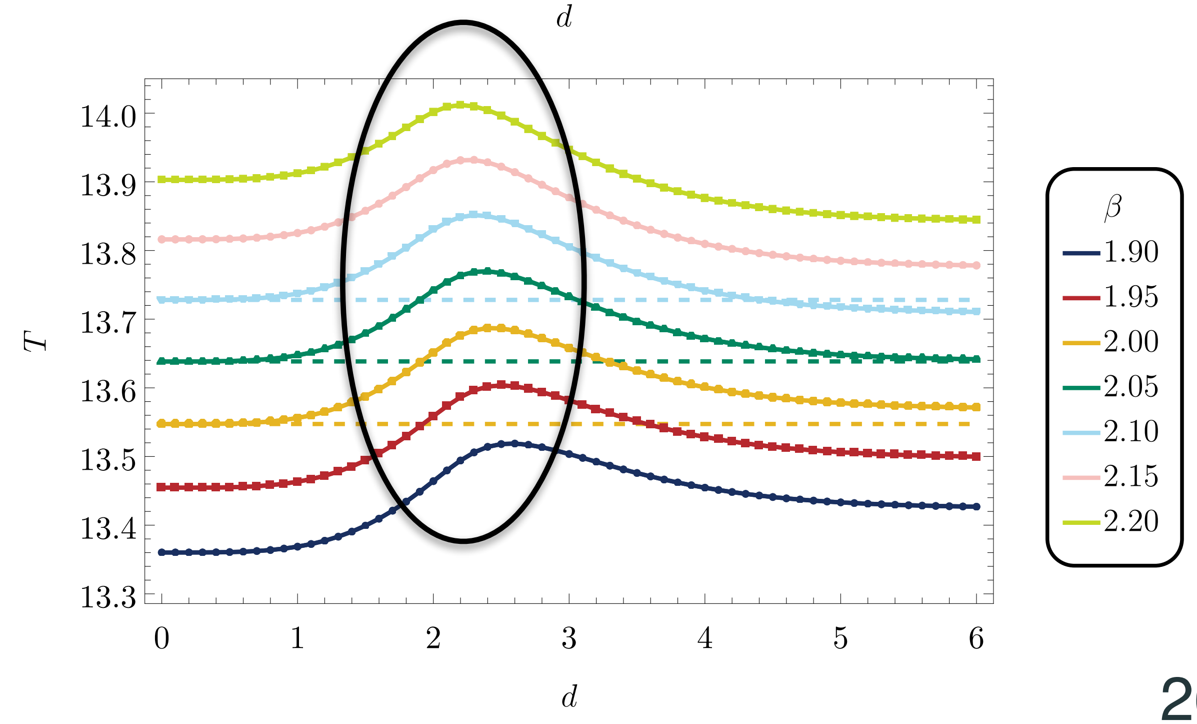
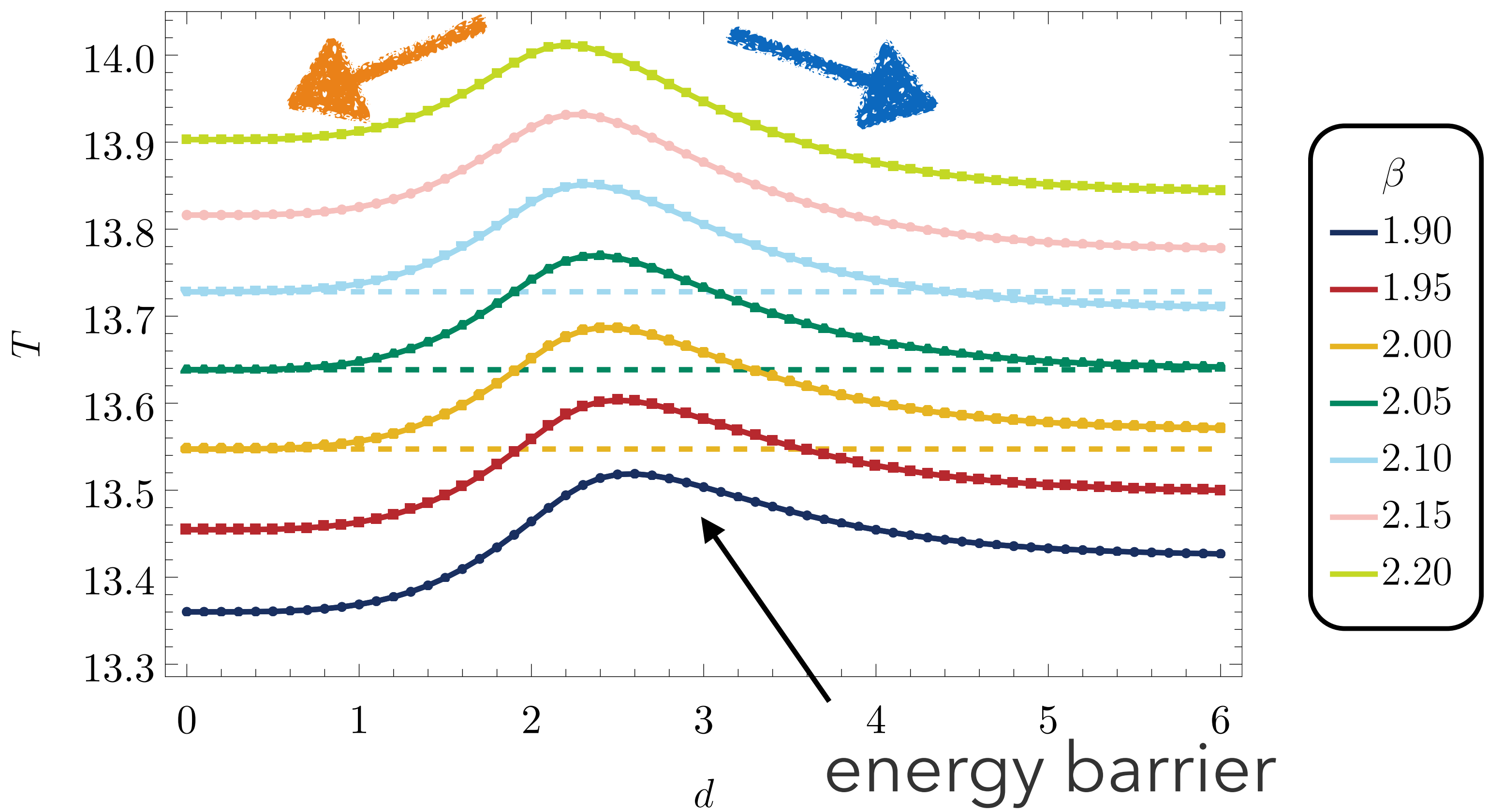




Energy barrier

- Tension as a function of d for different β

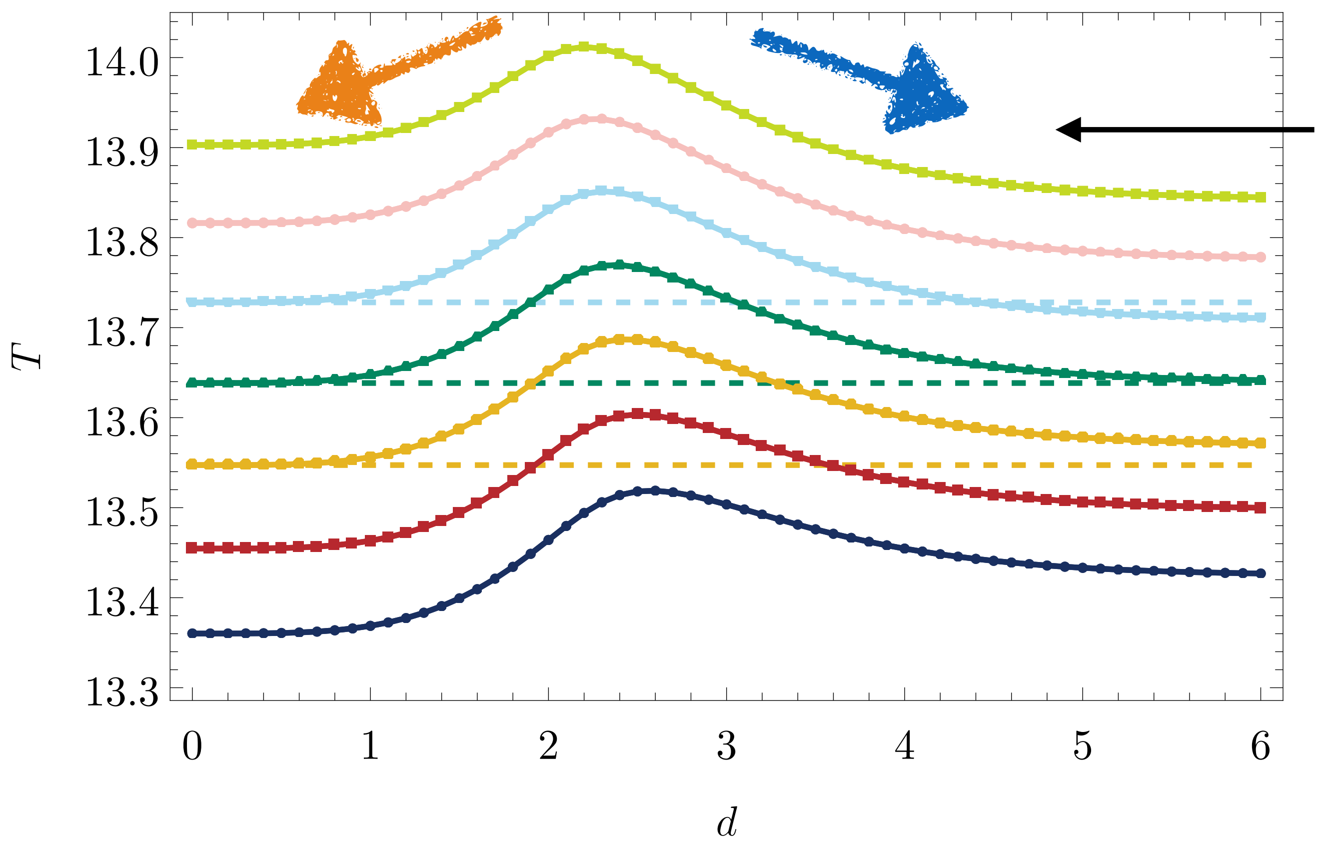
Coleman-Weinberg



- **Energy barrier** appears in CW case for $\beta > 1$!!

$\left\{\begin{matrix}\text{attractive} \\ \text{repulsive}\end{matrix}\right\}$ **for** $\left\{\begin{matrix}\text{short} \\ \text{large}\end{matrix}\right\}$ **distance**

Reason?



This repulsive behavior is easy to understand.

Each vortex has the asymptotic behavior:

$$\begin{cases} \delta f \simeq r^{-1/2} \exp\left[-\sqrt{2\beta} r\right] \\ \delta a \simeq r^{1/2} \exp\left[-\sqrt{2} r\right] \end{cases}$$

The gauge field is dominant at large d for $\beta > 1$.

→ **The gauge field mediates the repulsive force.**

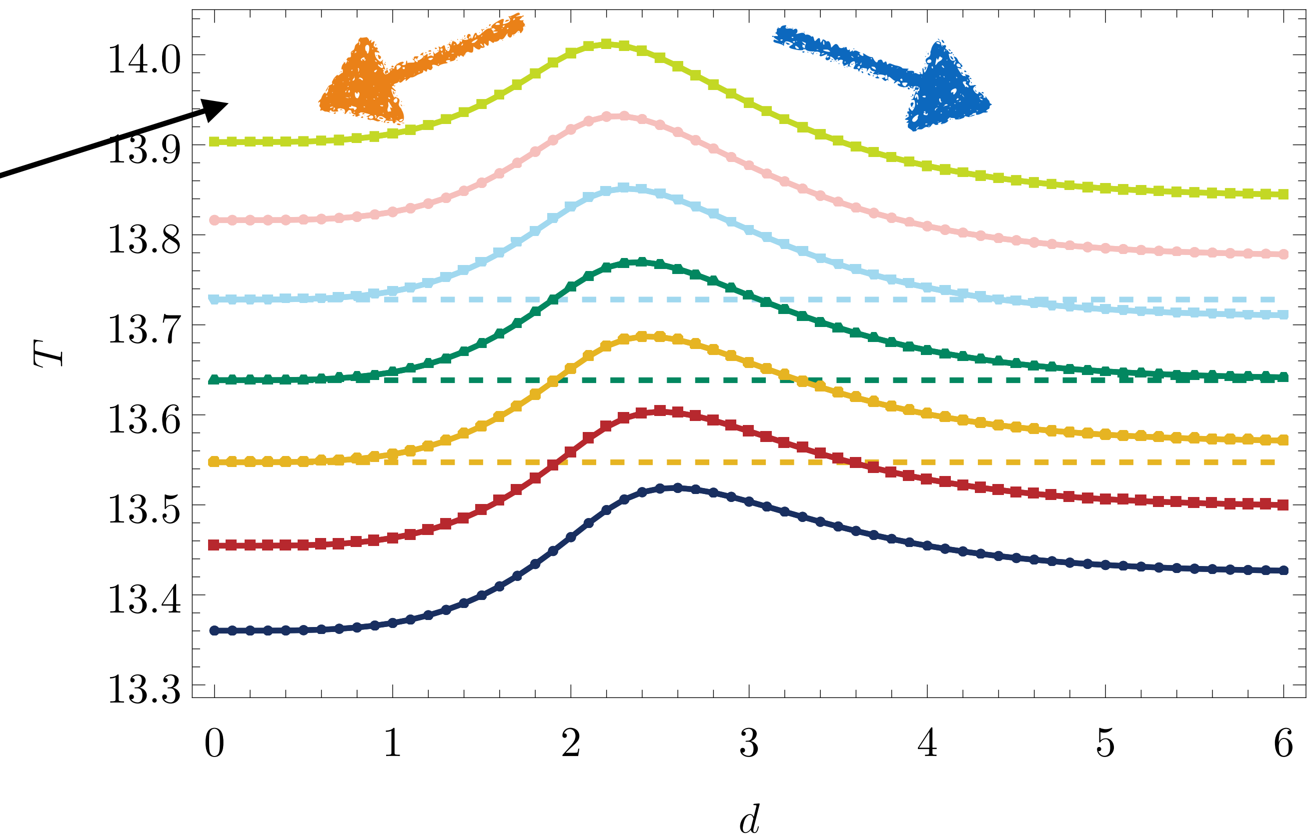
Reason?

On the other hand, this attractive behavior is difficult.

Numerical simulation saids $T(d)$ increases w/

$$T(d) \propto d^4$$

for small d independently of β.

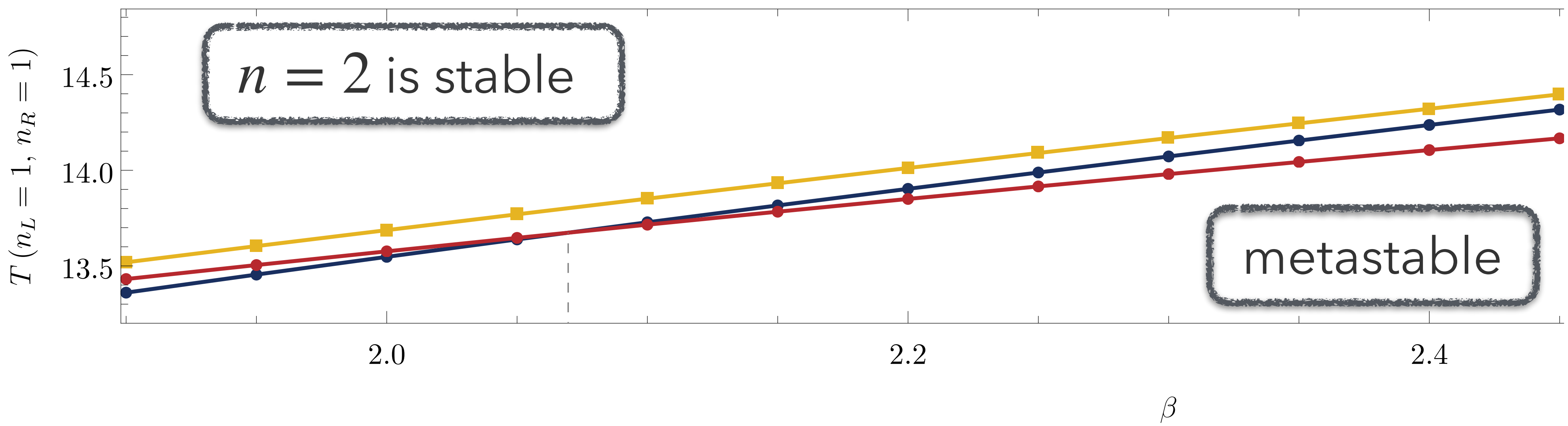


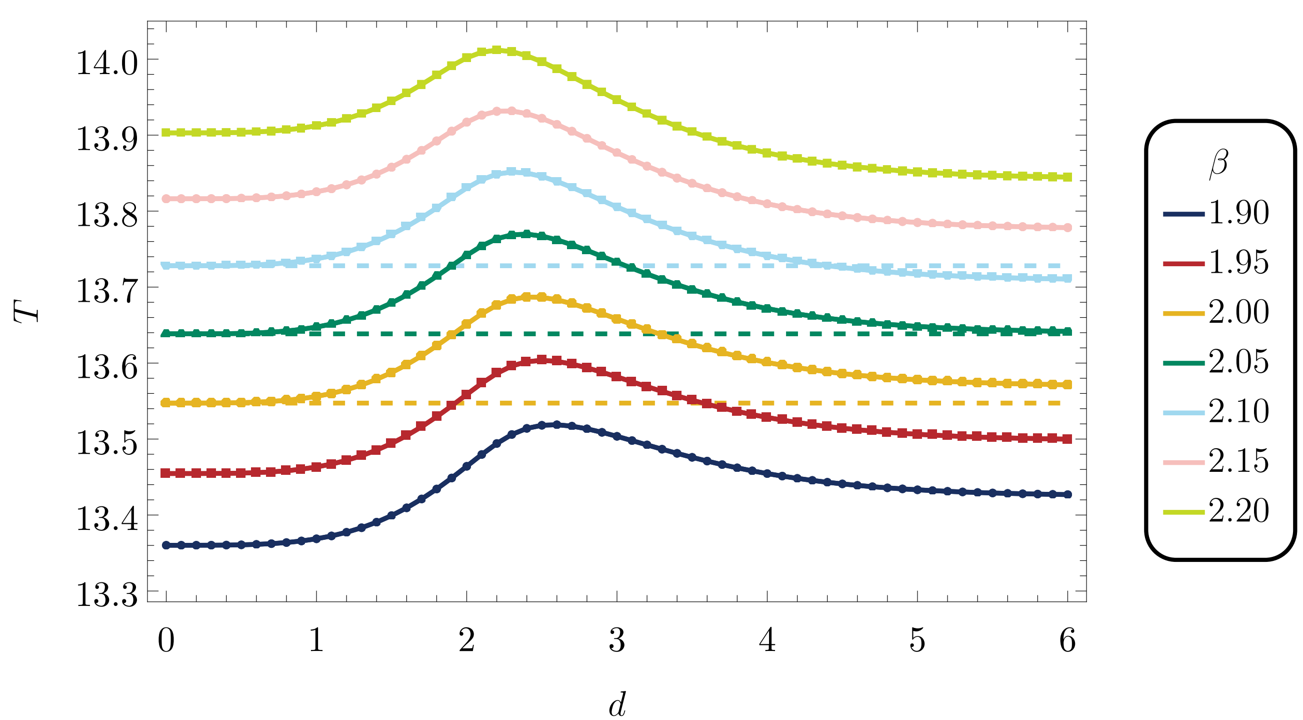
But it is difficult to understand analytically!

As shown later, the flatter structure of the potential seems crucial…

Stability-metastability

- We can read off the stability of the vortex of $n = 2$ state.





Stability-metastability

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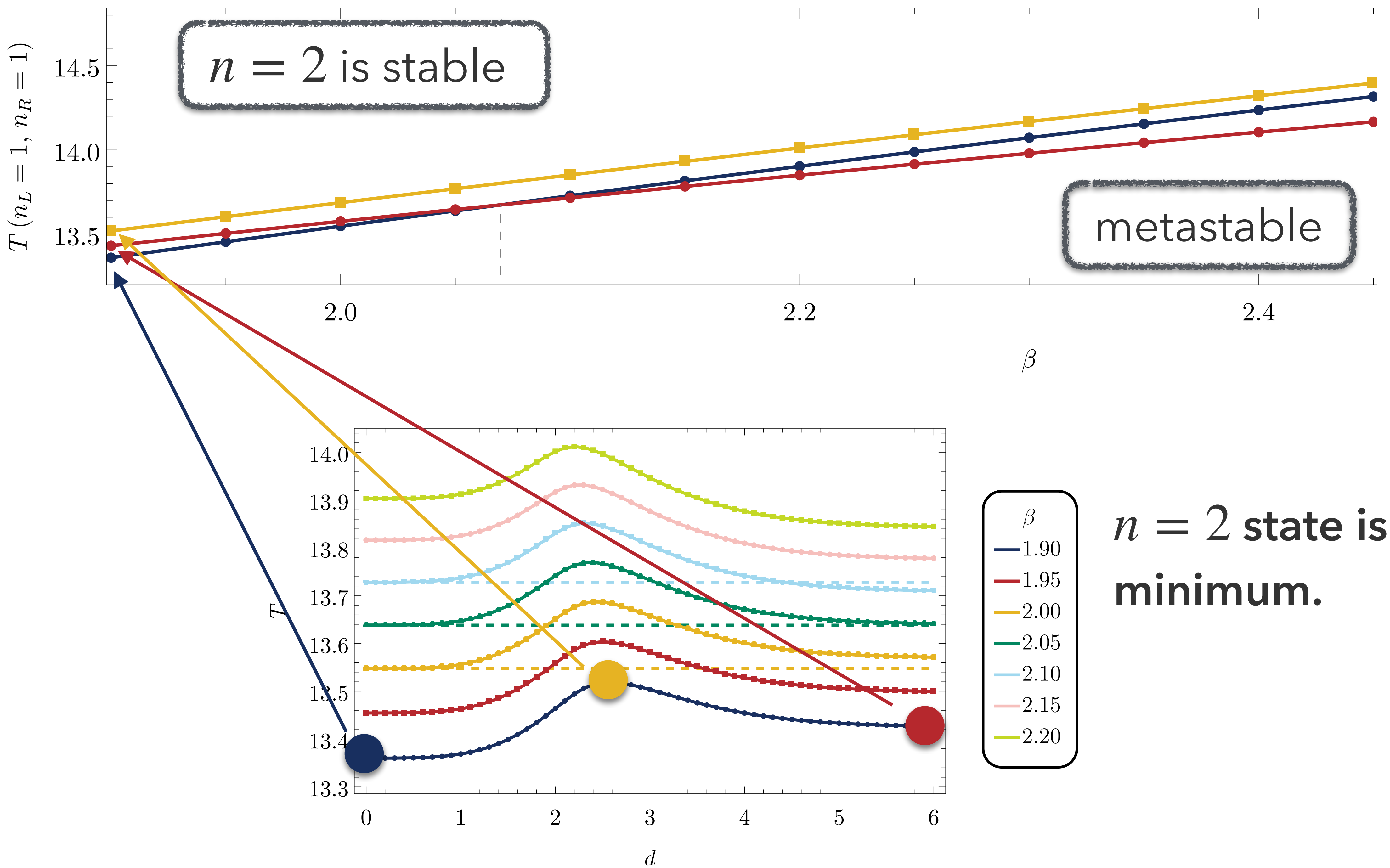
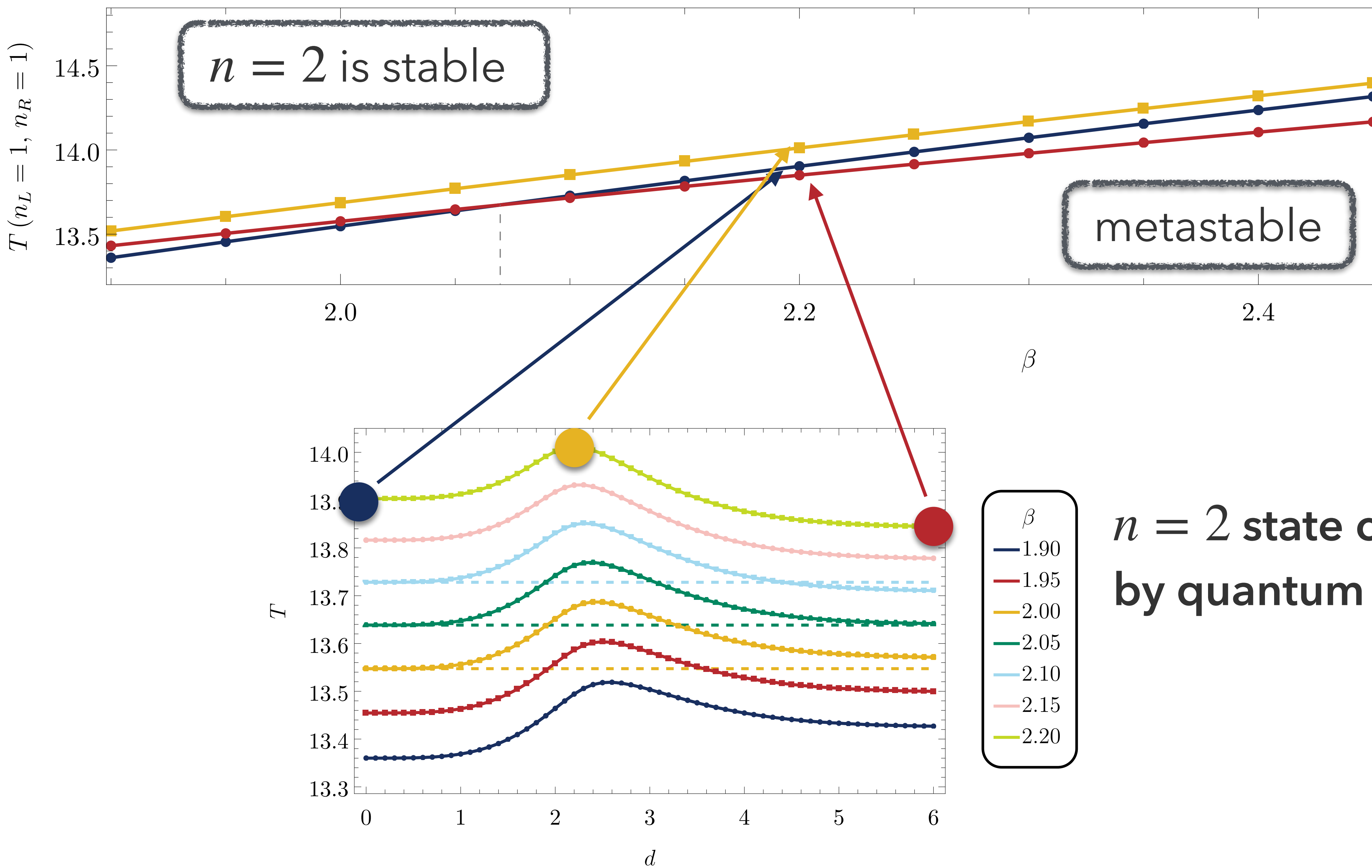


$n = 2$ state is absolute minimum.

Stability-metastability

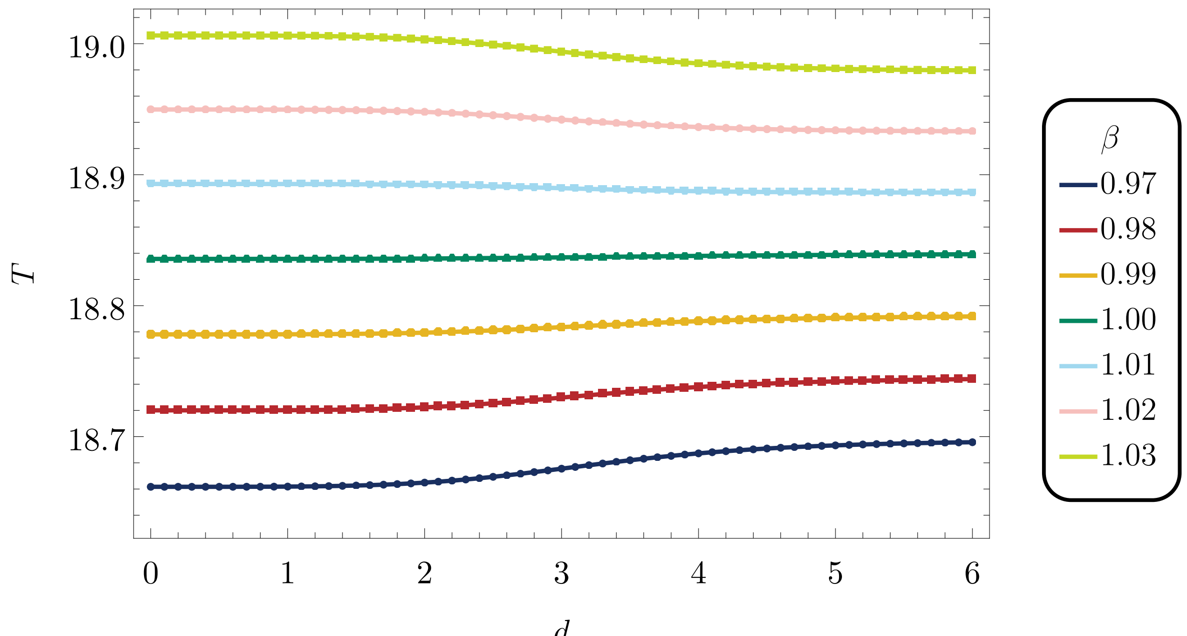
- We can read off the stability of the vortex of $n = 2$ state.



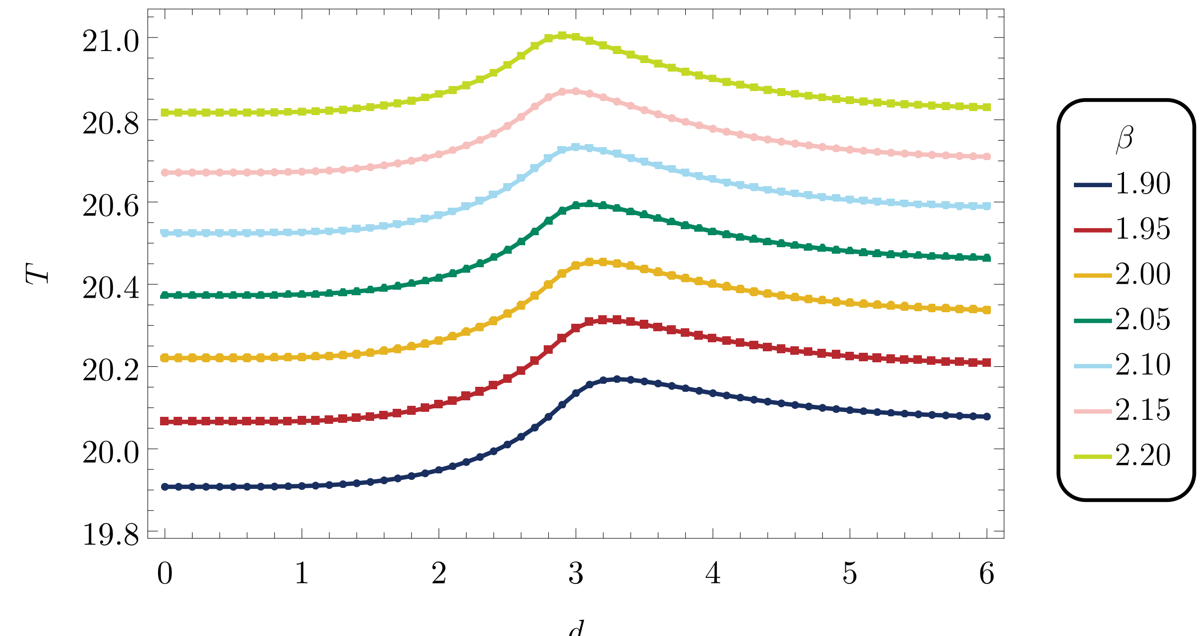
$n = 2$ state can decay by quantum tunneling.

Higher winding

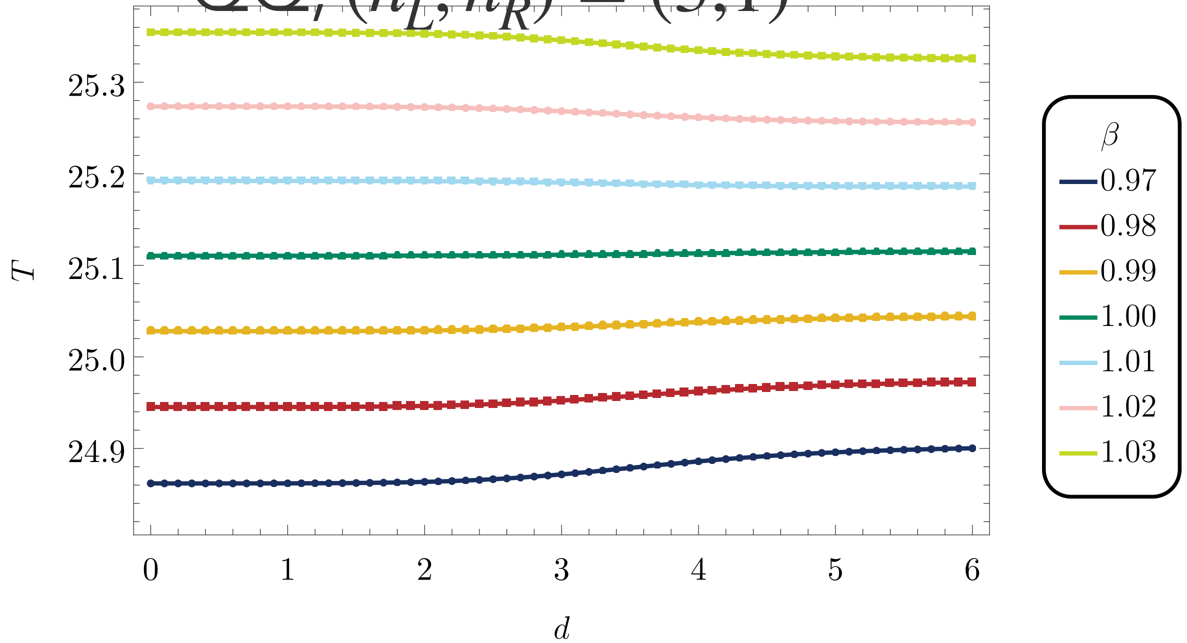
QQ, $(n_L, n_R) = (2,1)$



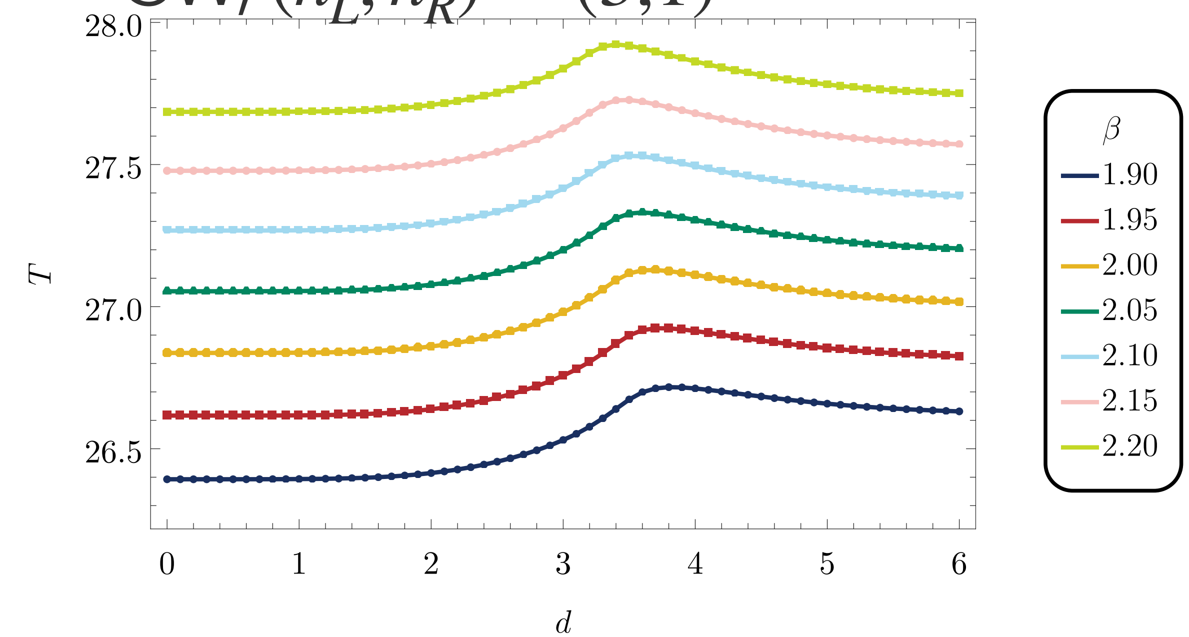
CW, $(n_L, n_R) = (2,1)$



QQ, $(n_L, n_R) = (3,1)$

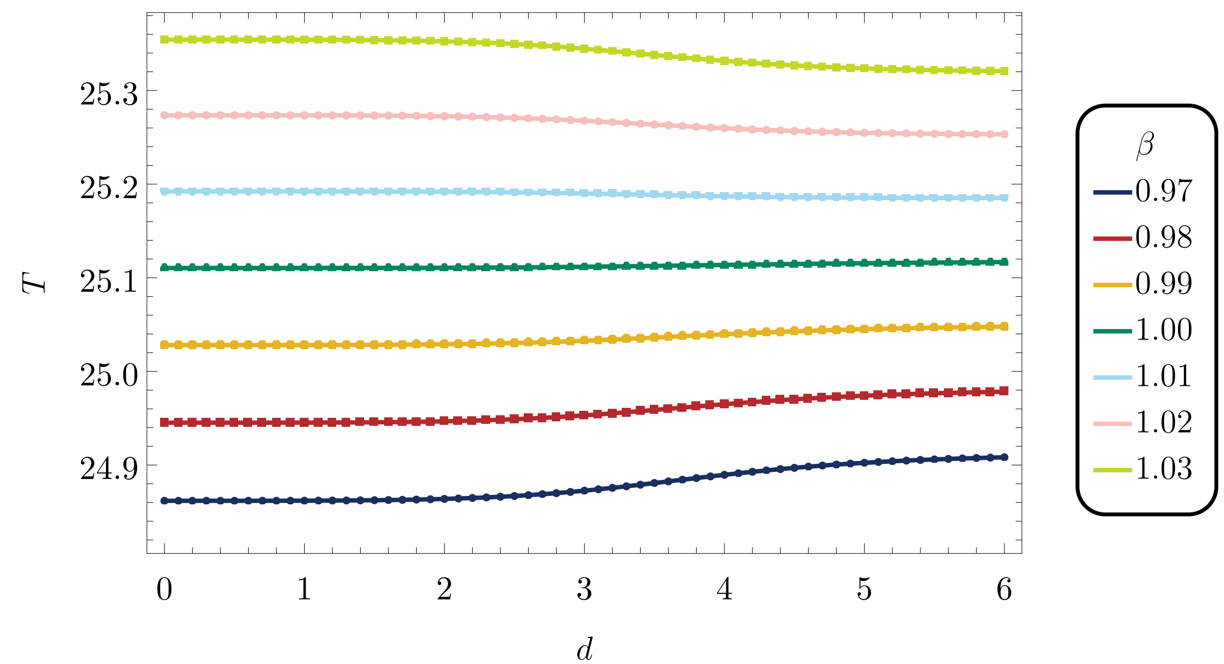


CW, $(n_L, n_R) = (3,1)$

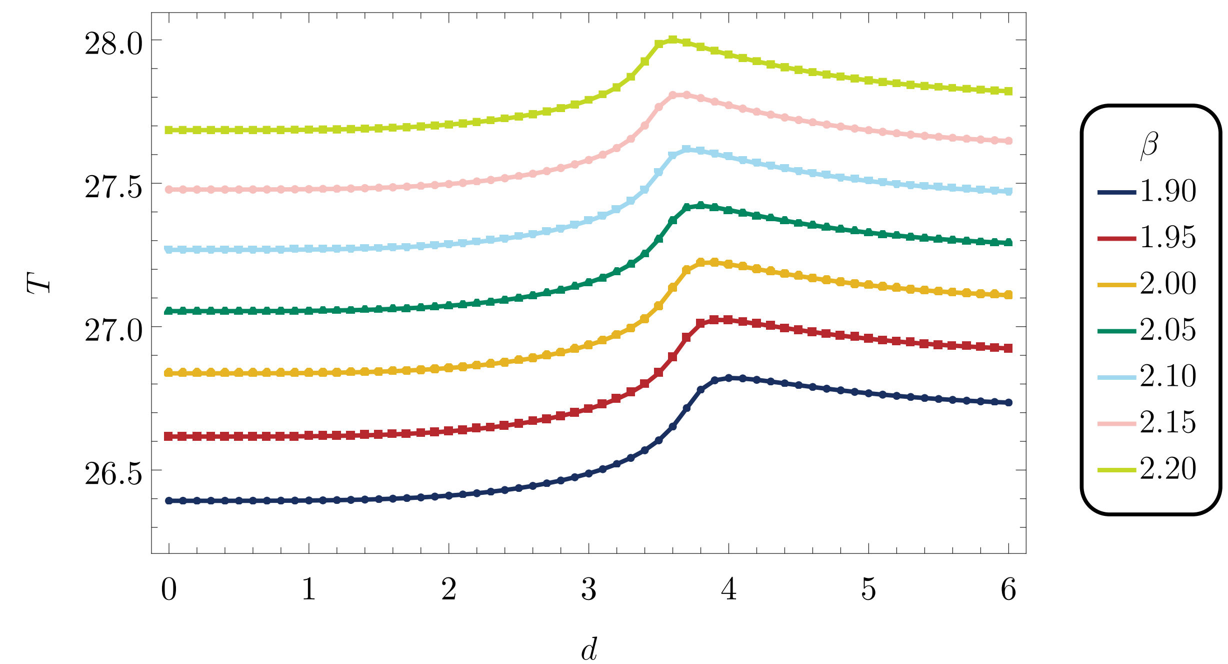


Higher winding (con't)

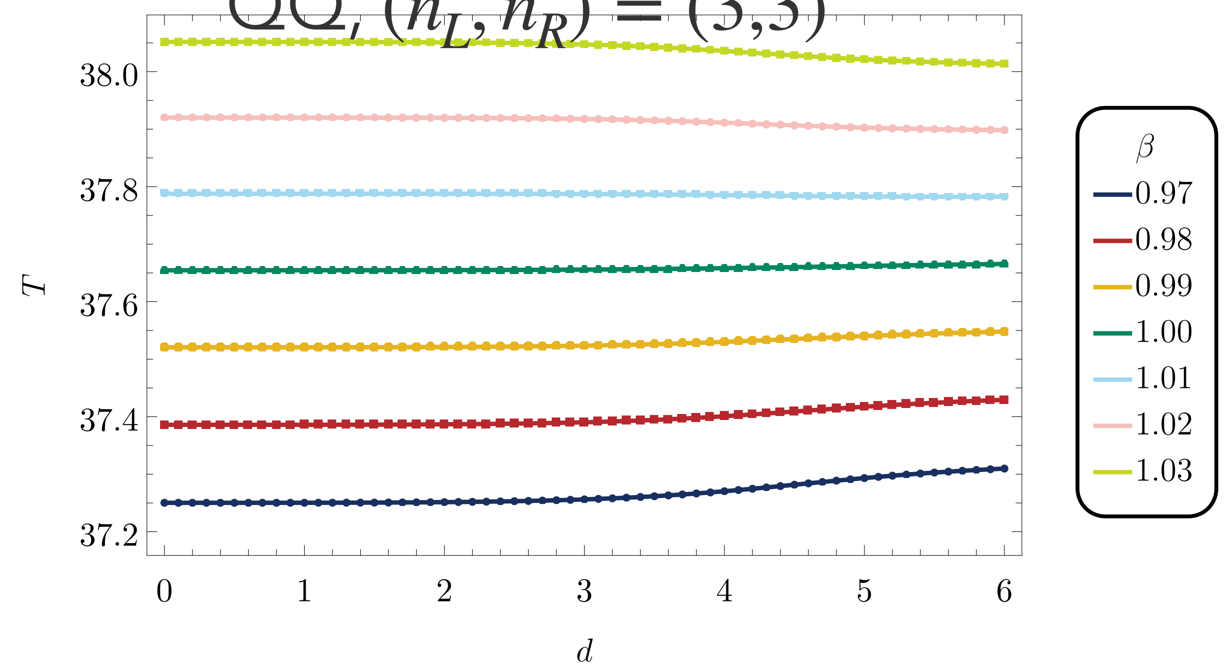
QQ, $(n_L, n_R) = (2,2)$



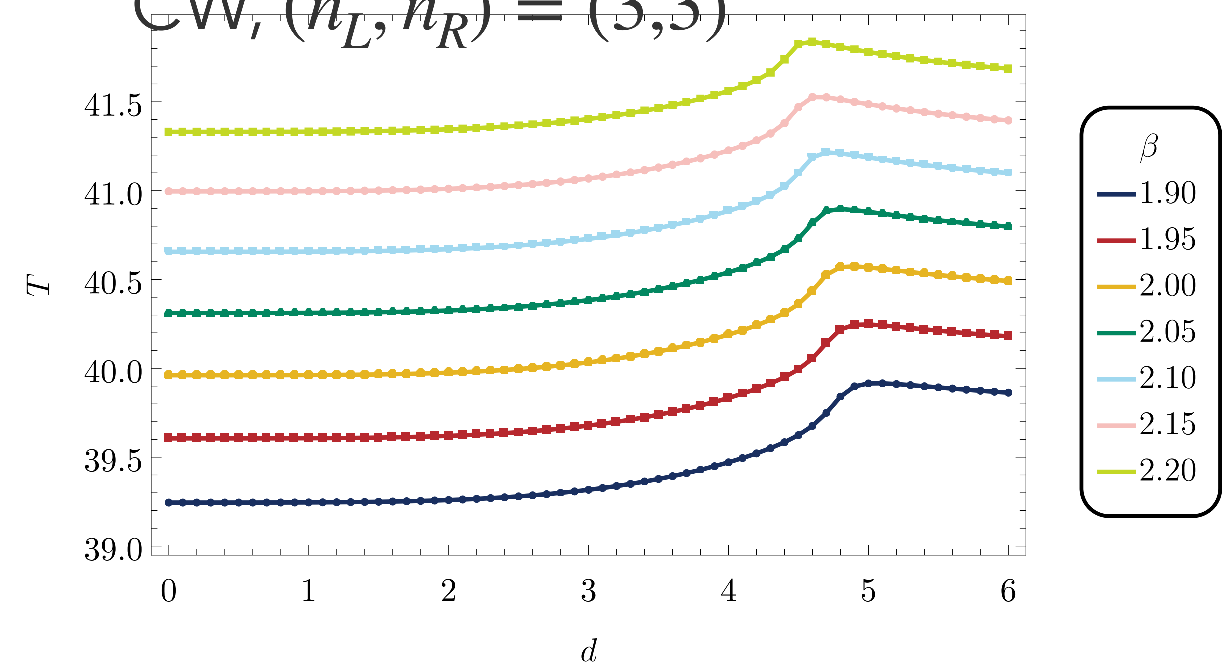
CW, $(n_L, n_R) = (2,2)$



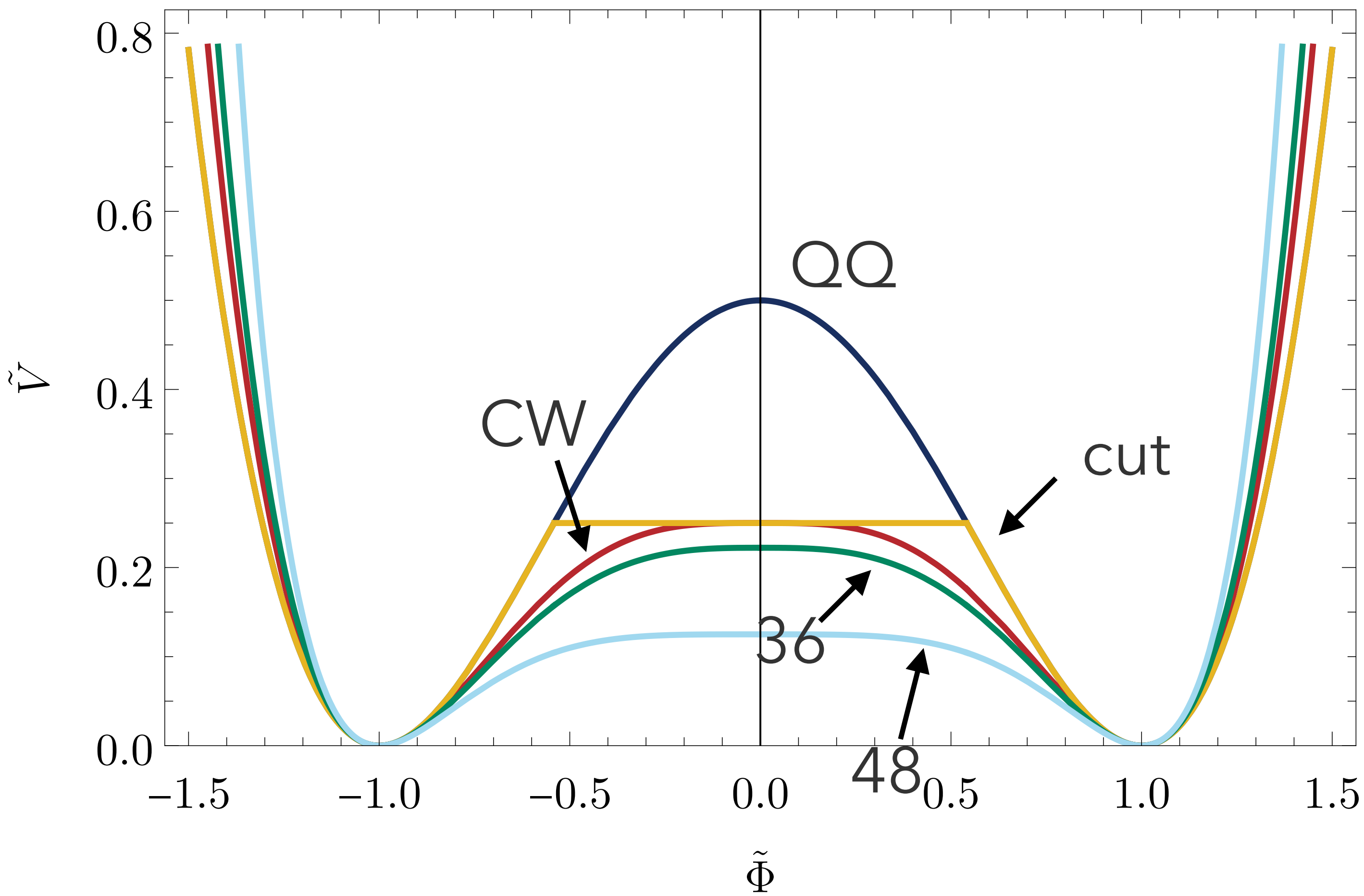
QQ, $(n_L, n_R) = (3,3)$



CW, $(n_L, n_R) = (3,3)$



Other potentials

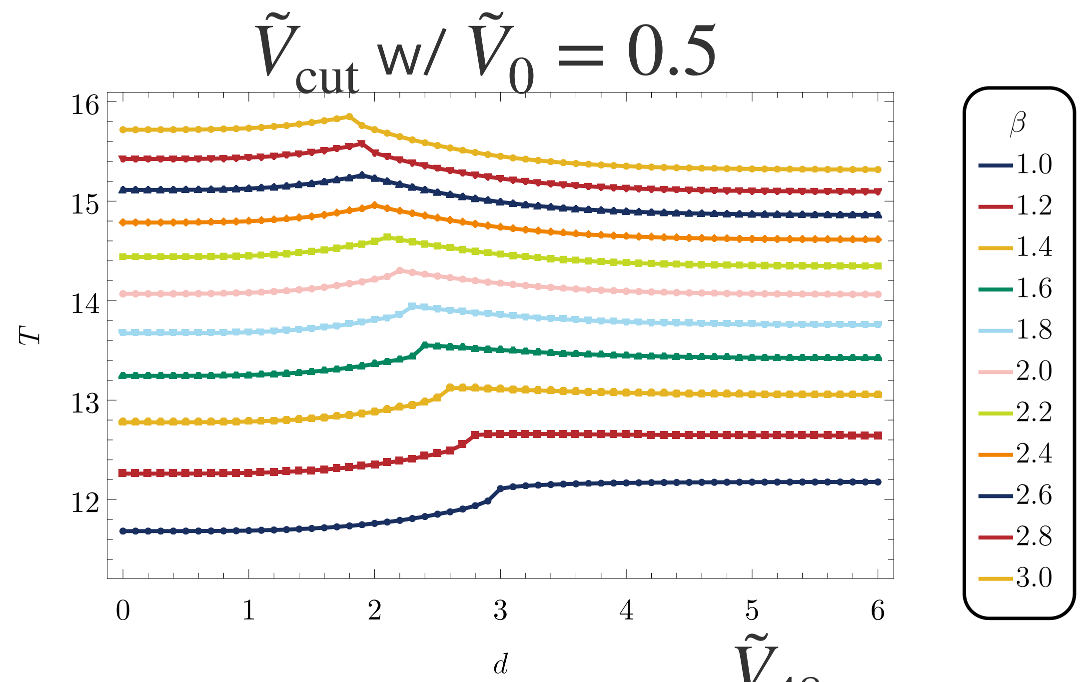


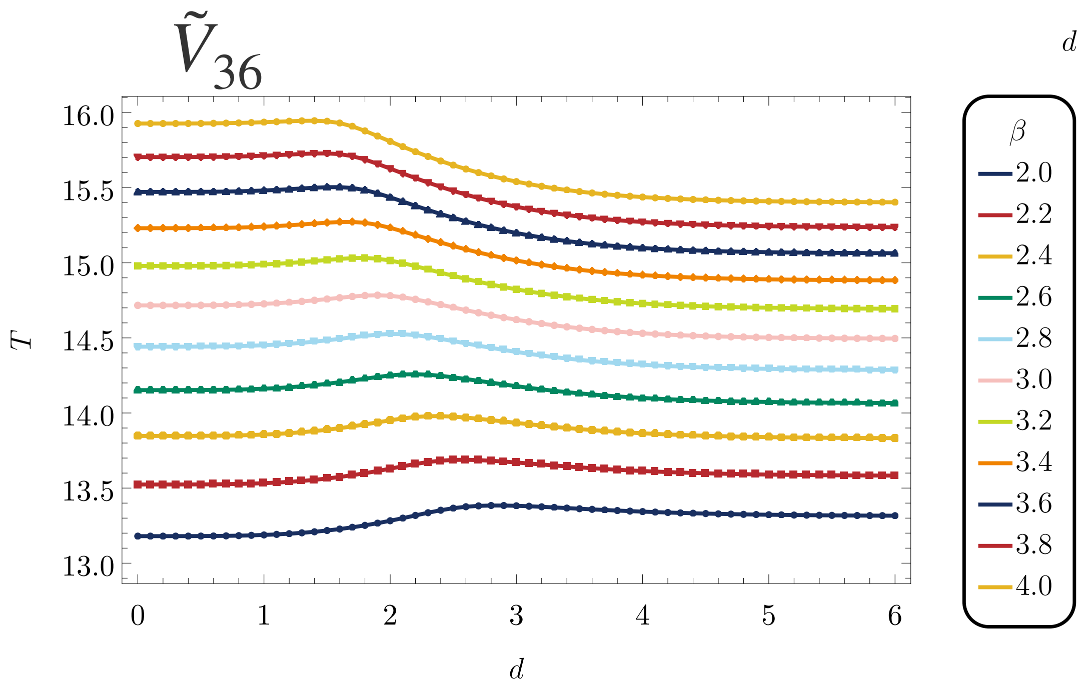
$$\tilde{V}_{\mathrm{AH-cut}} = \begin{cases} \dfrac{\beta}{2}\tilde{V}_0 & \left(|\tilde{\Phi}| < \sqrt{1-\sqrt{\tilde{V}_0}}\right) \\ \dfrac{\beta}{2}\left(|\tilde{\Phi}|^2 - 1\right)^2 & \left(|\tilde{\Phi}| > \sqrt{1-\sqrt{\tilde{V}_0}}\right) \end{cases}$$

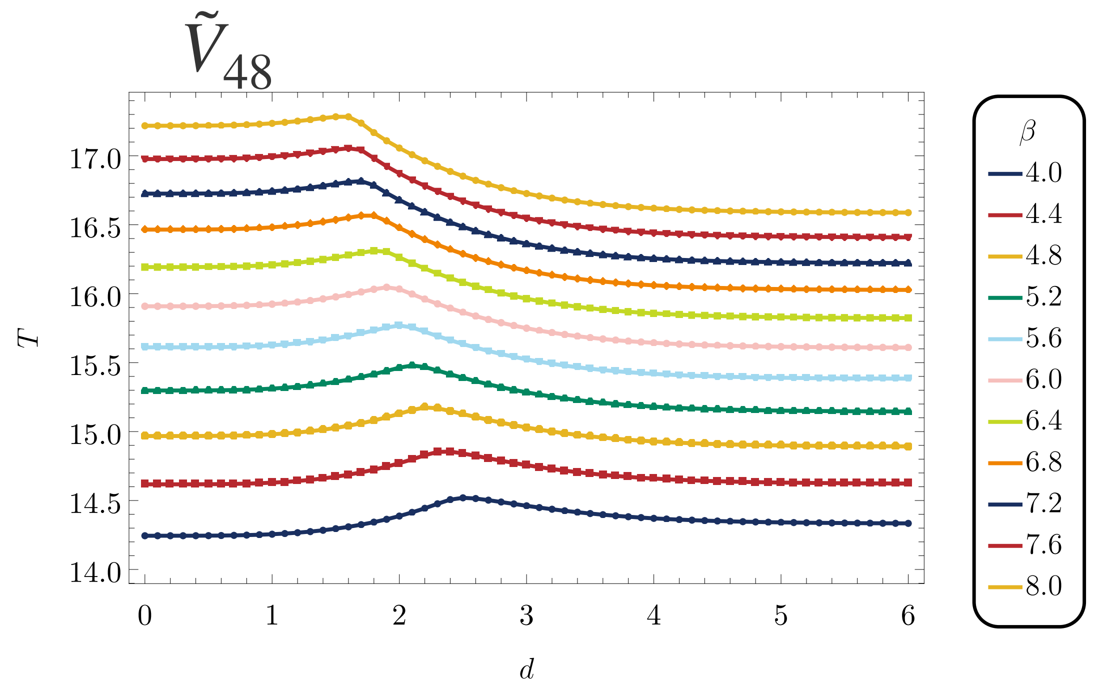
$$\tilde{V}_{\mathrm{AH-36}} = \frac{2\beta}{9}\left(|\tilde{\Phi}|^3 - 1\right)^2,$$

$$\tilde{V}_{\mathrm{AH-48}} = \frac{\beta}{8}\left(|\tilde{\Phi}|^4 - 1\right)^2.$$

Other potentials



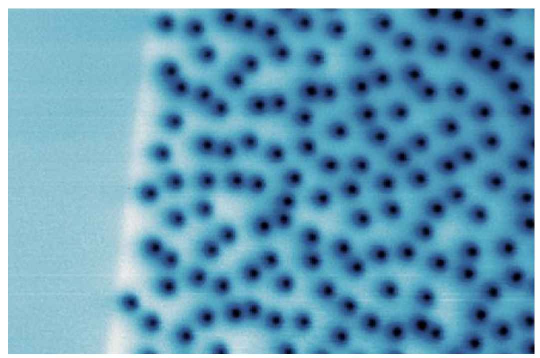




→ **It seems that the energy barrier is universal for flatter potential than Quadratic-Quartic one.**

Discussion

- **Formation of Abrikosov-like lattice in superconductor?**

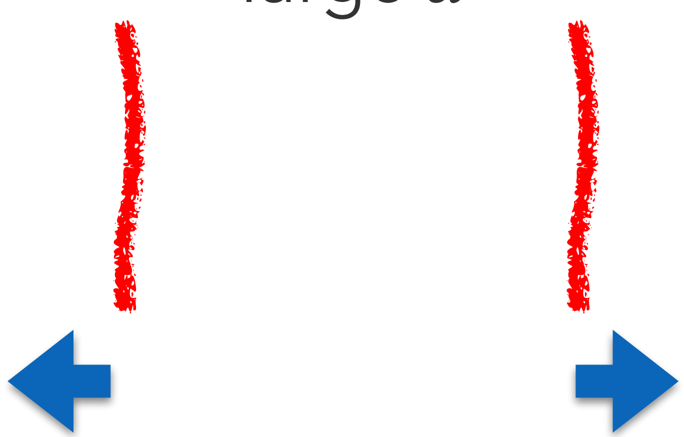


seems to depend on the distance of neighbored vortices

dilute → lattice-like structure
dense → gather!

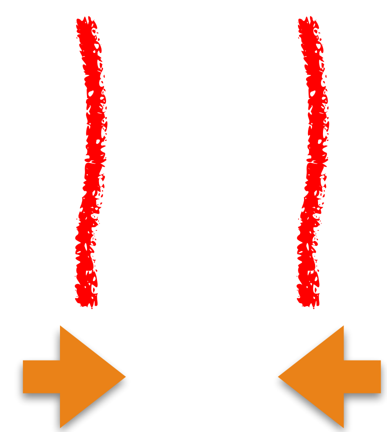
- **Cosmic string in early universe → reconnection? gravitational waves?**

large d



repulsive

small d



attractive

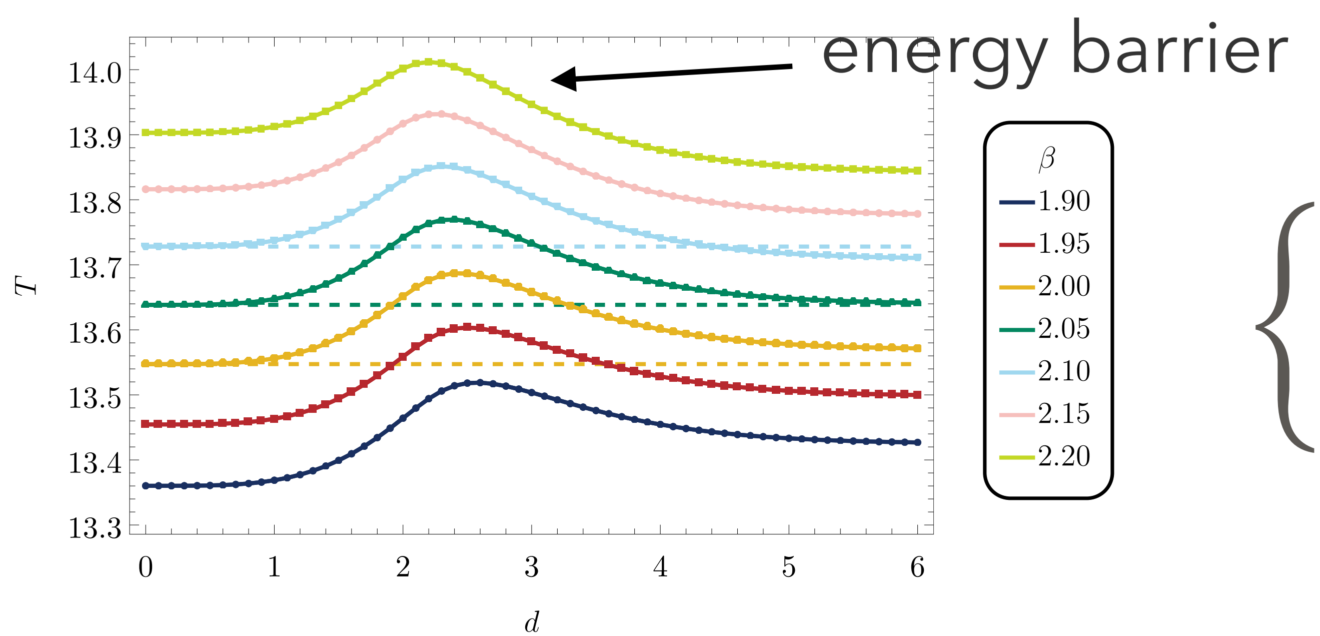
might lead to non-trivial dynamics! (future work)

Summary

- We study vortex strings in U(1) gauged model w/ Coleman-Weinberg potential (called CW-ANO string).

$$V_{\mathrm{CW}}(\Phi) = \frac{\beta}{2}\left(\log\frac{|\Phi|^2}{v^2} - \frac{1}{2}\right)|\Phi|^4$$

- In contrast to the conventional ANO string, interaction between the two CW-ANO strings has the energy barrier for $\beta > 1$.



$\left\{\begin{array}{c}\text{attractive} \\ \text{repulsive}\end{array}\right\}$ for $\left\{\begin{array}{c}\text{short} \\ \text{large}\end{array}\right\}$ distance

Backup

Bardeen's argument

[Bardeen '95]

- **Conventional potential**

$$V(\Phi) = -m^2|\Phi|^2 + \lambda(\Phi)|\Phi|^4$$

  - quantum correction for m^2

$$\delta m^2 = \Lambda^2 + m^2 \log \frac{\mu^2}{\Lambda^2} + \cdots \quad (\Lambda^2 \to 0)$$

Λ: UV cutoff scale

μ: renormalization scale

  - In scale invariant scheme (such as $\overline{MS}$), Λ^2 does not appear.
  - In the RG-running sense, this corresponds to a choice of "boundary conditions" at the cutoff scale $\mu = \Lambda$.

→ If we adopt a boundary condition that the mass vanishes at $\mu = \Lambda$, then $m^2 = 0$ at all scale. → no naturalness problem

classically scale invariance

Dimensional transmutation

[Coleman-Weinberg '73]

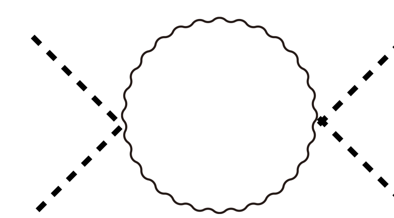
- **QCD:** $\alpha_s(\mu)^{-1} = \alpha_s(\Lambda)^{-1} + \frac{b_0}{2\pi}\log\frac{\mu}{\Lambda}$

$$\frac{\partial \alpha_s}{\partial \log \mu} = -\frac{b_0}{2\pi}\alpha_s$$

$$\alpha_s(\Lambda_{QCD})^{-1} = 0 \Leftrightarrow \Lambda_{QCD} = \Lambda \exp\left(-\frac{2\pi}{b_0 \alpha_s(\Lambda)}\right)$$

Scale Λ_{QCD} is non-perturbatively generated.

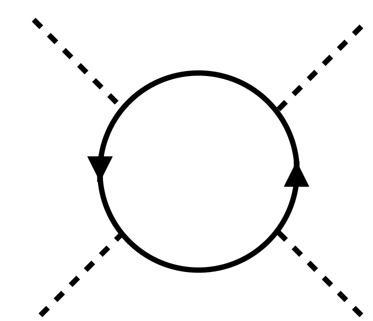
- **Coleman-Weinberg mechanism** (taking unitary gauge)



$$V_{\rm CW}(\phi) = \lambda(\phi)\phi^4$$

$$b = \frac{1}{16\pi^2}\left(\#g^4 - \#y^4\right): \beta\text{-func coeff}$$

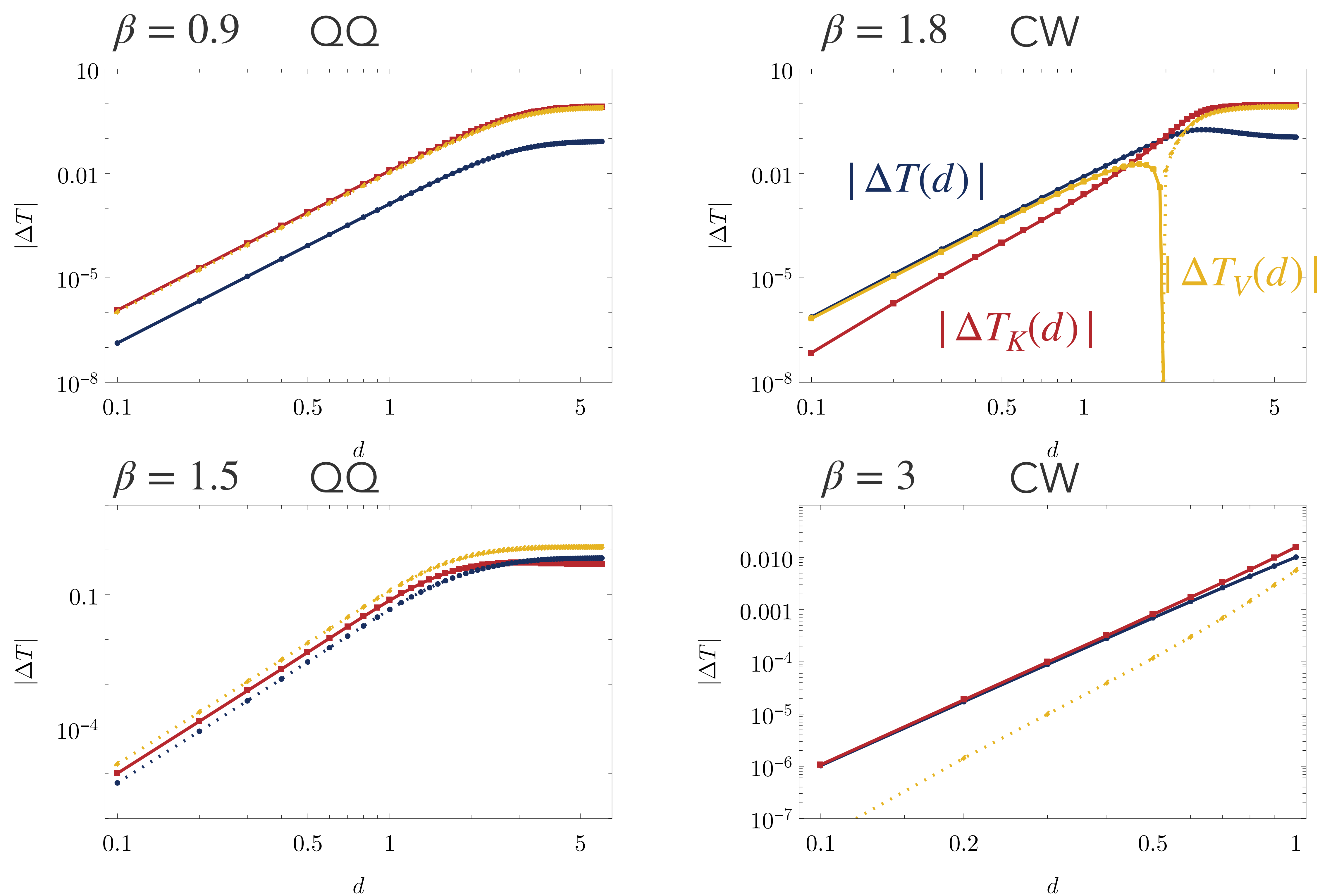
$$\lambda(\phi) = b\log\frac{\phi}{\Lambda}$$



$$V'_{\rm CW}(\phi) = 0 \Leftrightarrow \langle\phi\rangle = \Lambda\exp\left[-\left(4\frac{\lambda(\Lambda)}{b} + 1\right)\right]$$

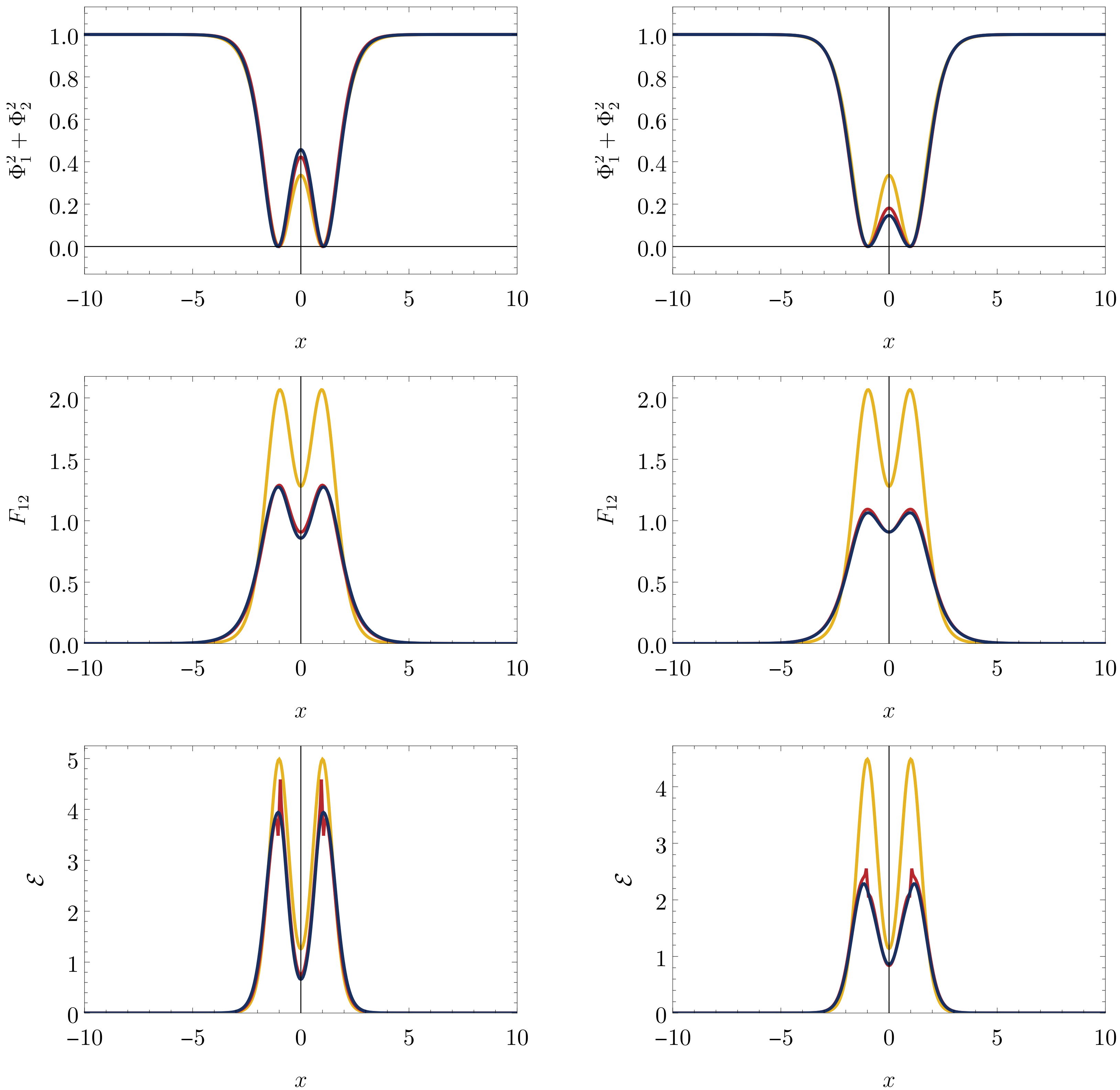
Potential minimum $\langle\phi\rangle$ is non-perturbatively generated.

Reason?



β = 0.9 QQ
β = 1.8 CW
β = 1.5 QQ
β = 3 CW
|ΔT|
d
10
0.01
10^-5
10^-8
0.1
0.5
1
5
|ΔT(d)|
|ΔT_V(d)|
|ΔT_K(d)|
10^-4
0.010
0.001
10^-6
10^-7
0.2

Relaxation



$\tau = 0, 2, 15$

$y = 0$

$\beta = 1.5$

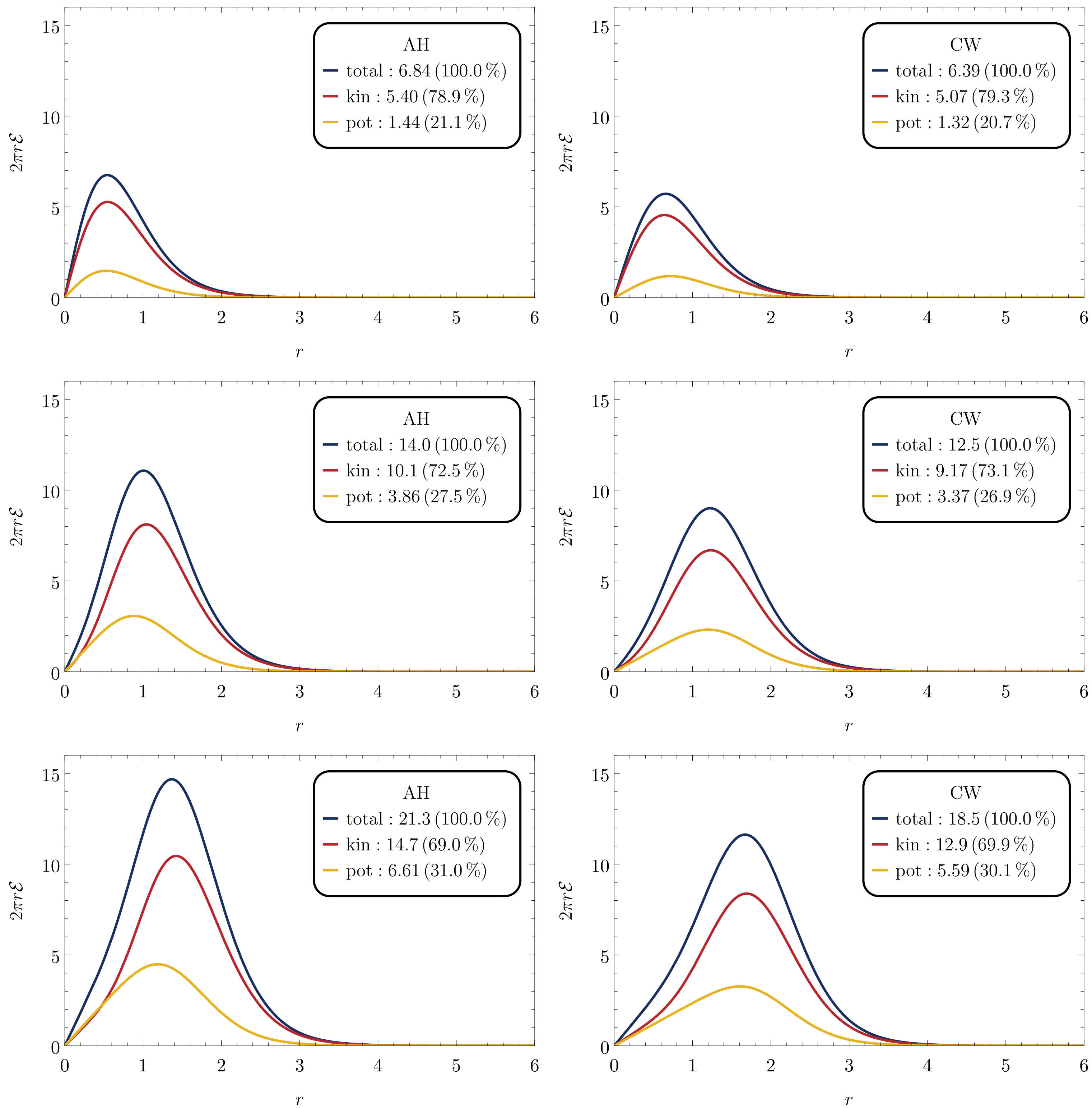
Energy decomposition

$\beta = 1.5$

n=1

n=2

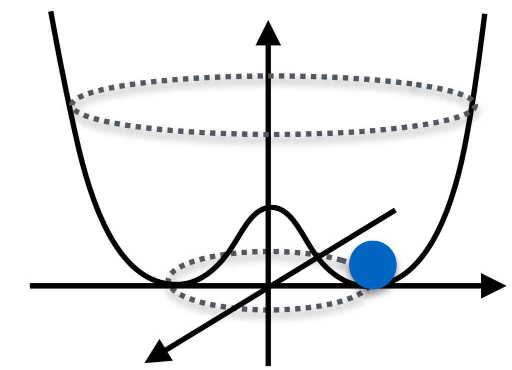
n=3



Abrikosov-Nielsen-Olesen string

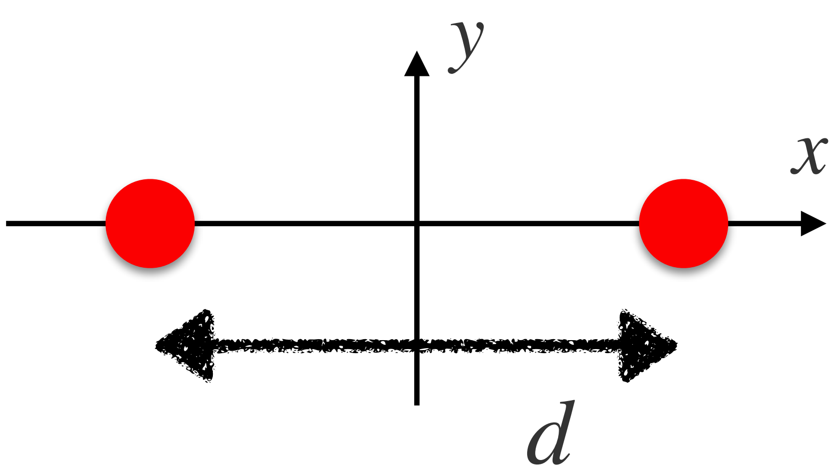
- **ANO string is a well-known example.**

~~$U(1)$~~



(3+1)D Abelian-Higgs model w/ U(1) gauge sym

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu \phi|^2 + m^2|\phi|^2 - \lambda|\phi|^4$$



(3+1)D Abelian-Higgs model w/ U(1) gauge sym

$$\beta \equiv \frac{m_\phi^2}{m_A^2} = \frac{4\lambda v^2}{2g^2v^2}$$