Multi-dimensional perspectives on the inflationary Universe

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Plan of this talk

Part I: "multi-dimensional" field space

- A brief review and the current status of inflation
- Multi-field inflation
- Sneutrino inflation scenario as a multi-field model

Part II: "multi-dimensional" test of inflation

- Observables of inflation
- Prospects from LiteBIRD
- Quantum nature of primordial fluctuation as a test of model

Part I:
“Multi-dimensional” field space

Basics of inflation

Inflation in brief

- Inflation is a superluminal expansion of the Universe at its very early stage and can solve the problems in the standard big bang cosmology (the horizon and flatness problems).

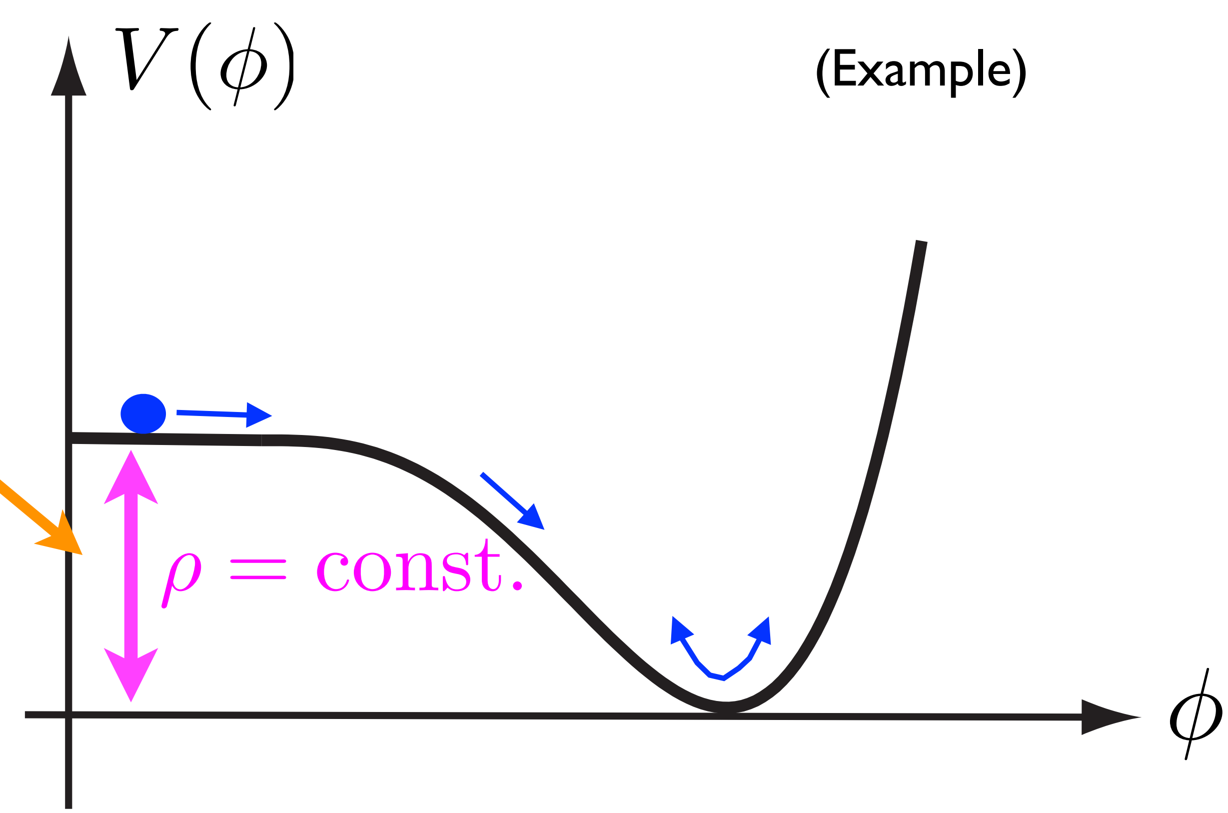
- Inflation is considered to be driven by a scalar field, called inflaton field.

The potential energy (vacuum energy) of the inflaton drives inflation.

$$\left(\frac{1}{a}\frac{da}{dt}\right)^2 = \frac{1}{3M_{\rm pl}^2}\rho$$

$$\Rightarrow \quad a \propto \exp(Ct)$$

(accelerated expansion)



Inflation in brief

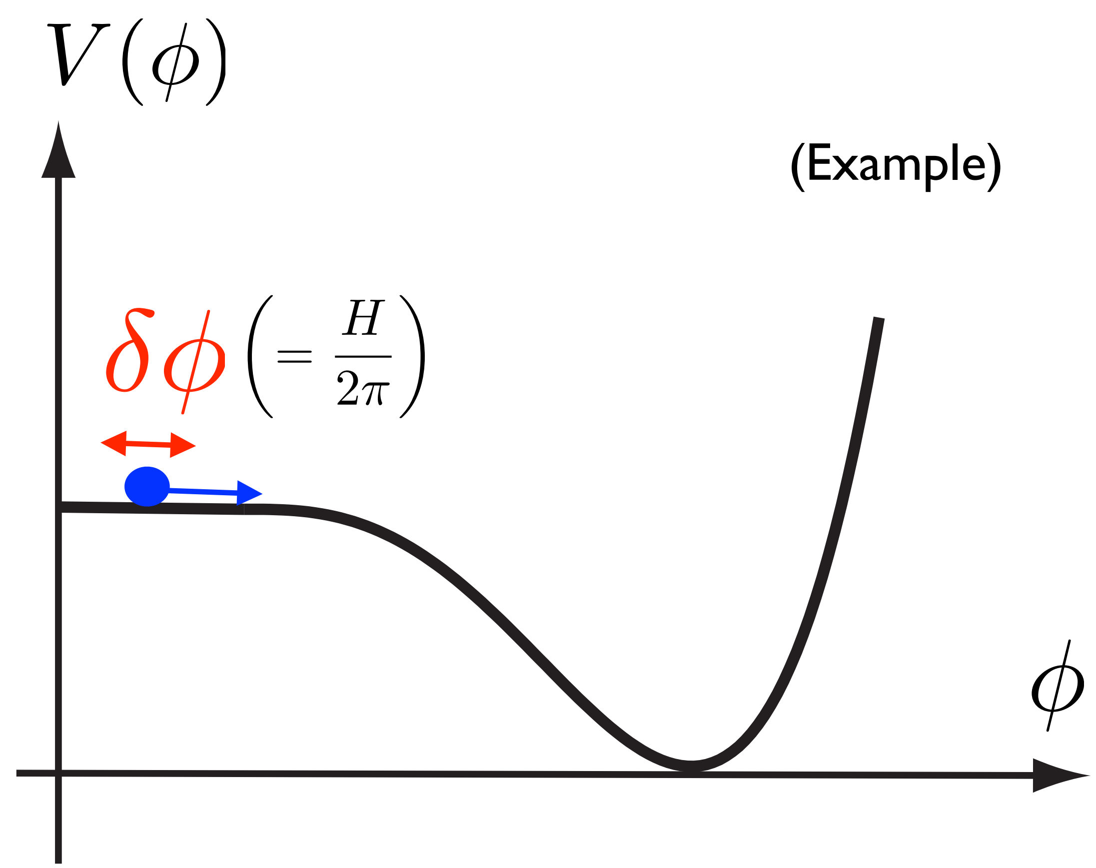
- Quantum fluctuations of the inflaton can generate primordial density fluctuations.

- Curvature perturbation (scalar mode) is generated.

(Its amplitude depends on models and their model parameters.)

- Gravitational waves (tensor mode) are also generated.

(Its amplitude depends on models and their model parameters.)

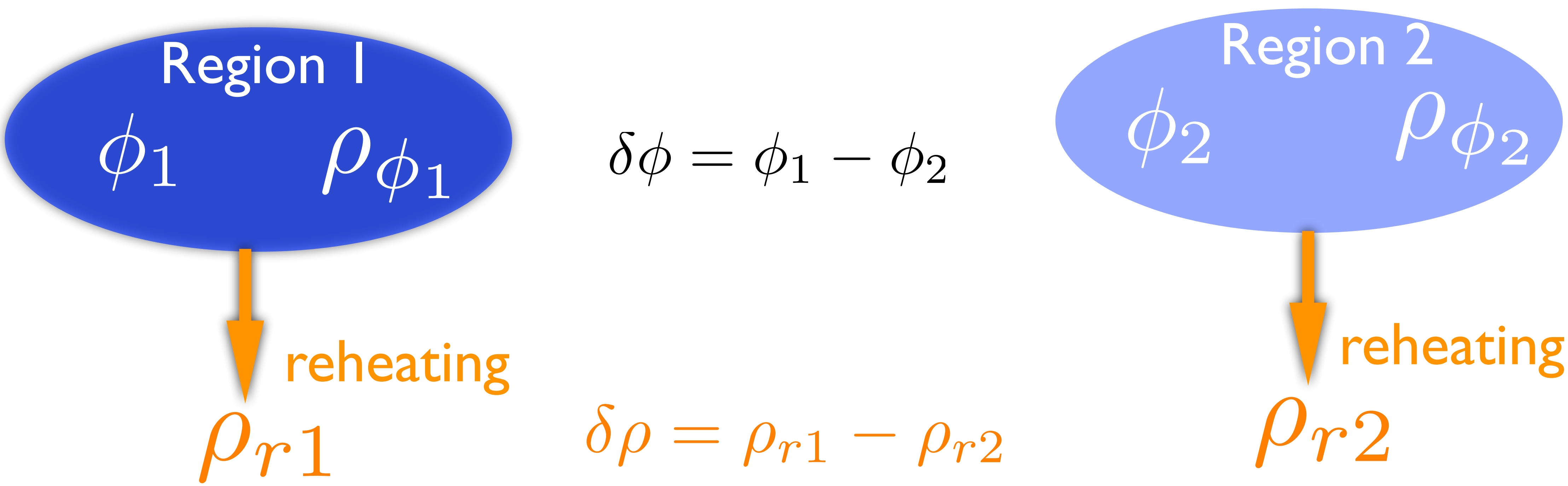


We can probe the nature of primordial perturbations with observations of CMB, large scale structure and so on.

→ We can probe the mechanism of inflation.

Intuitive estimate of primordial fluctuations

- Energy density of the inflaton (scalar field): $\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$



- The curvature perturbation can be roughly given as

$$\frac{\delta\rho}{\rho} \sim \frac{\delta a}{a} \sim H\delta t \sim H\frac{\delta t}{\delta\phi}\delta\phi \sim H\frac{\delta\phi}{\dot{\phi}} \sim \zeta \text{ (curvature perturbation)}$$

$(\rho \propto a^{-n})$ $\left(H = \frac{1}{a}\frac{da}{dt}\right)$

Fluctuations generated from inflation

- The curvature perturbation (scalar mode)

$$H^2 \simeq \frac{V(\phi)}{3M_{\rm pl}^2}$$

The Hubble parameter during inflation

(Inflationary energy scale)

Inflaton field fluctuations

Curvature perturbation: $\zeta = -\frac{H}{\dot{\phi}}\delta\phi$

$$\delta\phi = \frac{H}{2\pi}$$

Inflaton dynamics (inflaton potential)

$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$ → Slow-roll approximation: $\dot{\phi} \simeq -\frac{1}{3H}\frac{dV}{d\phi}$

- Gravitational waves spectrum (tensor mode)

Gravitational waves

$$ds^2 = -dt^2 + a(t)^2\,[1 + h_{ij}]\;dx^i dx^j$$

In Fourier space:

$$h_{ij} = \sqrt{8\pi G}\sum_{A=+,\times}\int d^3k\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; h_k(t)\; e^A_{ij}(\boldsymbol{n})$$

GW amplitude is determined by the inflationary energy scale: $h_k \sim \frac{H_{\rm inf}}{M_{\rm pl}}$

Observational probes (will be discussed more in part II)

- What we can measure in observations are the correlation functions:

Scalar mode (the curvature perturbation)

2-point function: $\left\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2) \right\rangle$ $\longrightarrow$ power spectrum

3-point function: $\left\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3) \right\rangle$ $\longrightarrow$ bi-spectrum

4-point function: $\left\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\zeta(\vec{k}_4) \right\rangle$ $\longrightarrow$ tri-spectrum

Tensor mode (gravitational waves)

2-point function: $\left\langle h(\vec{k}_1)h(\vec{k}_2) \right\rangle$ $\longrightarrow$ power spectrum

3-point function: $\left\langle h(\vec{k}_1)h(\vec{k}_2)h(\vec{k}_3) \right\rangle$ $\longrightarrow$ bi-spectrum

4-point function: $\left\langle h(\vec{k}_1)h(\vec{k}_2)h(\vec{k}_3)h(\vec{k}_4) \right\rangle$ $\longrightarrow$ tri-spectrum

Primordial power spectra

- Power spectrum for the curvature perturbation (scalar mode)

amplitude spectral index

$$\mathcal{P}_\zeta(k) = A_s(k_{\rm ref}) \left(\frac{k}{k_{\rm ref}} \right)^{n_s - 1} \qquad \left(\sim \frac{1}{M_{\rm pl}^6} \frac{V^3}{(V')^2} \right)$$

V : inflaton potential, $V' = dV/d\phi$

- Gravitational wave power spectrum (tensor mode)

$$\mathcal{P}_T(k) = A_T(k_{\rm ref}) \left(\frac{k}{k_{\rm ref}} \right)^{n_T} \qquad \left(\sim \frac{H_{\rm inf}^2}{M_{\rm pl}^2} \right)$$

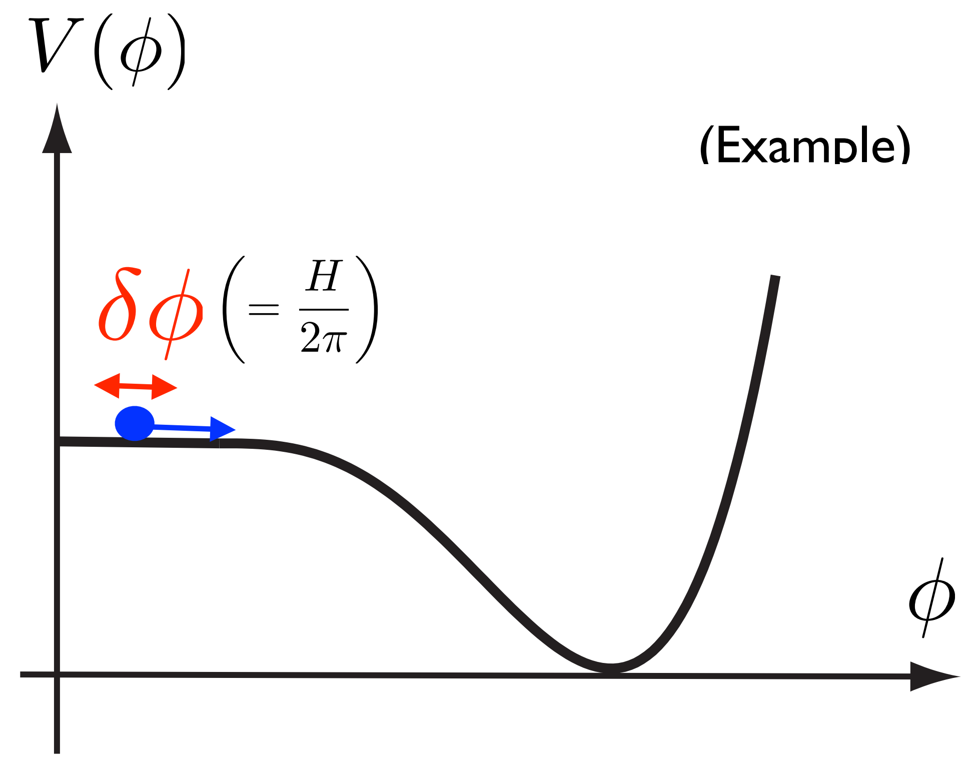
➡ Tensor-to-scalar ratio: $r \equiv \dfrac{\mathcal{P}_T}{\mathcal{P}_\zeta}$

Characterizing the observables

- Once the inflaton potential is given, one can calculate the slow-roll parameters:

$$\epsilon = \frac{1}{2} M_{\rm pl}^2 \left(\frac{V_\phi}{V} \right)^2, \qquad \eta = M_{\rm pl}^2 \frac{V_{\phi\phi}}{V}$$

where $V_\phi \equiv \frac{dV}{d\phi}, \quad V_{\phi\phi} \equiv \frac{d^2 V}{d\phi^2}$

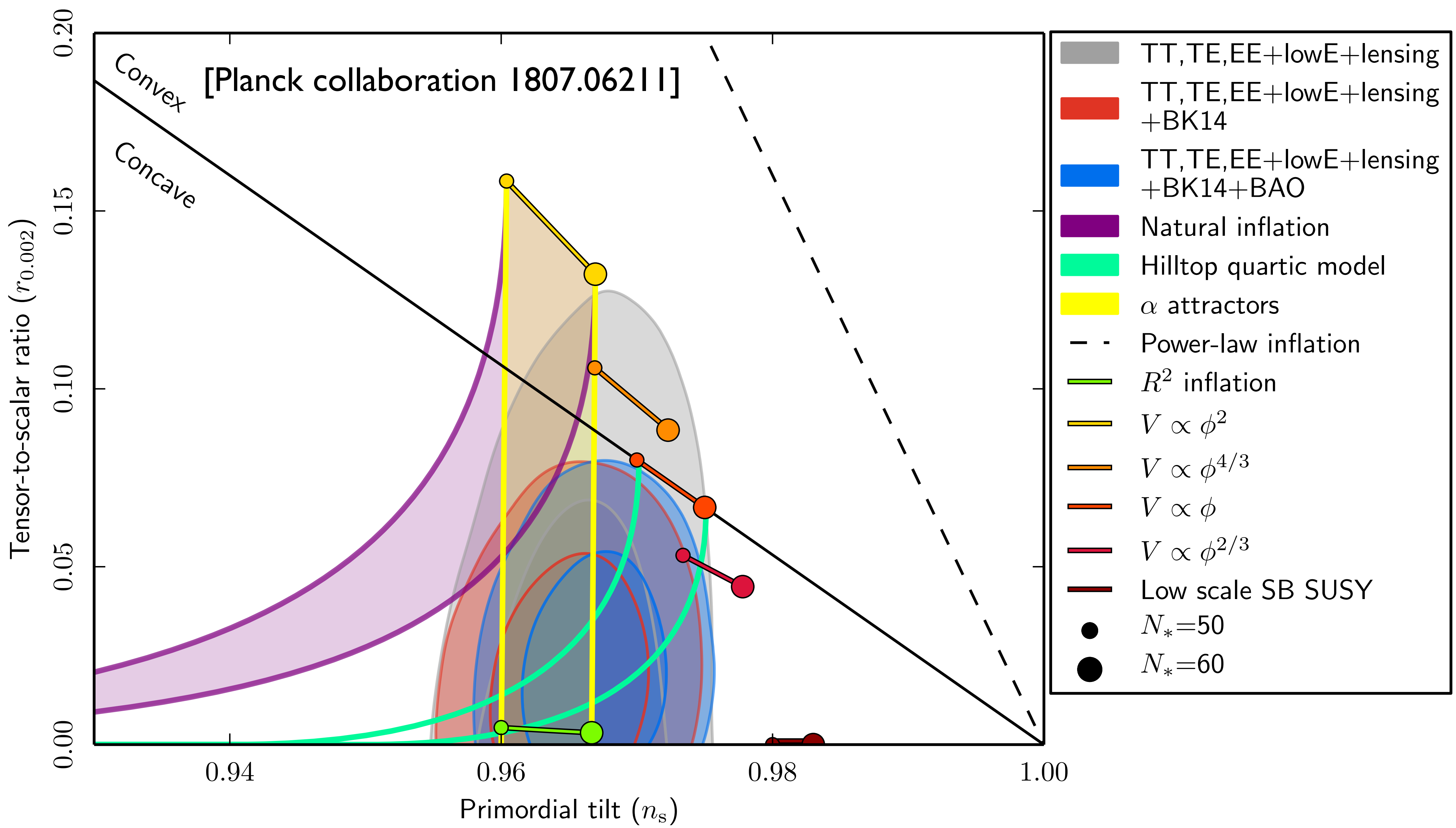


- (Scalar) spectral index: $n_s = 1 - 6\epsilon + 2\eta$ (For the standard single-field models)

- Tensor-to-scalar ratio: $r = 16\epsilon$ (For the standard single-field models)

Measurements of n_s and r give the information on the inflaton potential.

Inflationary models: current status

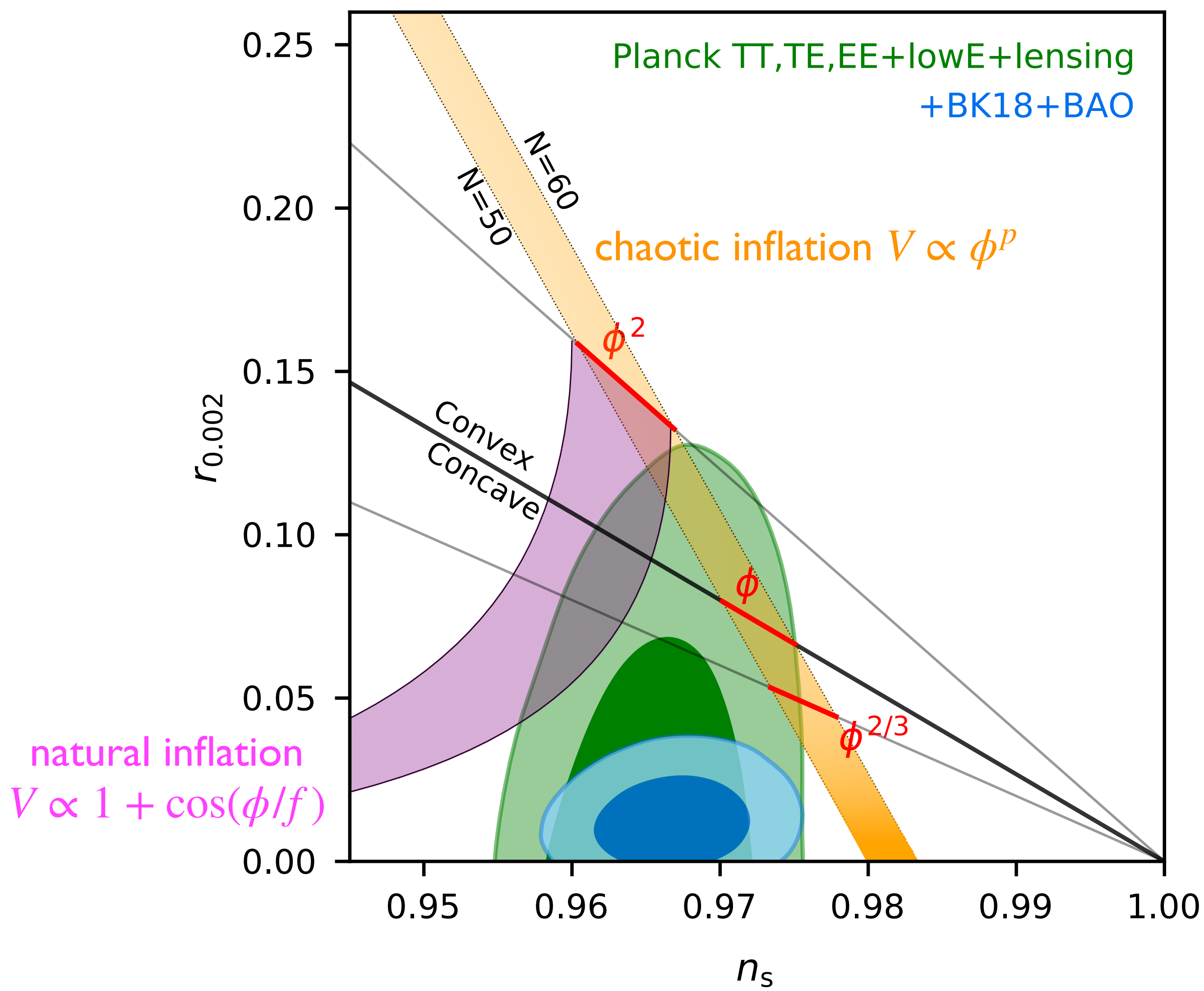


Spectral index: $n_s = 0.9649 \pm 0.0042 \;\; (68\% \text{ CL})$ [Planck 2018]

Tensor-to-scalar ratio: $r_{0.05} < 0.06 \;\; (95\% \text{ CL})$ [Planck + BK15+ …]

Inflationary models: current status

[BICEP/Keck Array collaboration 2110.00483]



Spectral index:

$$n_S = 0.967 \pm 0.0037 \ (68\,\%\ \mathrm{CL})$$

Tensor-to-scalar ratio:

$$r_{0.05} < 0.035 \ (95\,\%\ \mathrm{CL})$$

(Planck+BK18+BAO)

Constraints on inflation models are now very stringent, and many inflation models for single-field models are excluded.

(Chaotic inflation with any power and natural inflation are now excluded.)

What kind of inflation models is successful?

- Simple standard inflation models do not seem to work.

  (although some inflaton potential can be consistent with observations.)

- We can consider a different framework:

  - Extension of gravity

    A simple one: non-minimally coupled inflation models

    (e.g., Higgs inflation) [Cervantes-Cota, Dehnen astro-ph/9505069; Bezrukov, Shaposhnikov 0710.3755; …]

    (See [Kodama, TT 2112.05283] for a general non-minimal inflation)

  - Multi-dimensional field space (multi-field inflation)

    From the viewpoint of particle physics, scalar fields would be ubiquitous in the early Universe.

Example: Sneutrino inflation model

- Sneutrino (supersymmeric partner of right-handed neutrinos) can play a role of the inflaton. → Sneutrino inflation

[Murayama et al. PRL 70, 1912 (1993), …]

- There are 3 scalar fields (sneutrinos) in this set up:

$$V = \frac{1}{2}M_\phi^2\phi^2 + \frac{1}{2}M_\chi^2\chi^2 + \frac{1}{2}M_\sigma^2\sigma^2$$

- The case where only one field is relevant to the inflation is excluded.

But, in general, other field(s) can also affect the inflationary dynamics and primordial fluctuations.

→ Multi-field inflation

Multi-field inflation model

- Spectator field models

Spectator field is not responsible for the inflationary dynamics, but responsible for primordial fluctuations [e.g., curvaton model, modulated reheating model, …].

$$V(\phi,\sigma) = \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}m_\sigma^2\sigma^2$$

(some hierarchy between the masses)

inflaton

spectator field

(The properties of primordial fluctuations generated from the spectator are different from those of the inflaton.)

- Multi-inflaton models

Multiple fields can have sizable energy fraction during inflation.

$$V(\phi,\chi) = \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}m_\chi^2\chi^2$$

(both fields have almost the same masses)

inflaton

(Non-trivial trajectory in the multi-dimensional field space give different predictions for observables.)

Spectator field models

- Another scalar field other than the inflaton may be subdominant during inflation and does not affect the inflationary dynamics (such a field called "spectator field").

- Even though the spectator field does not affect the inflationary expansion, it can contribute to primordial fluctuations.

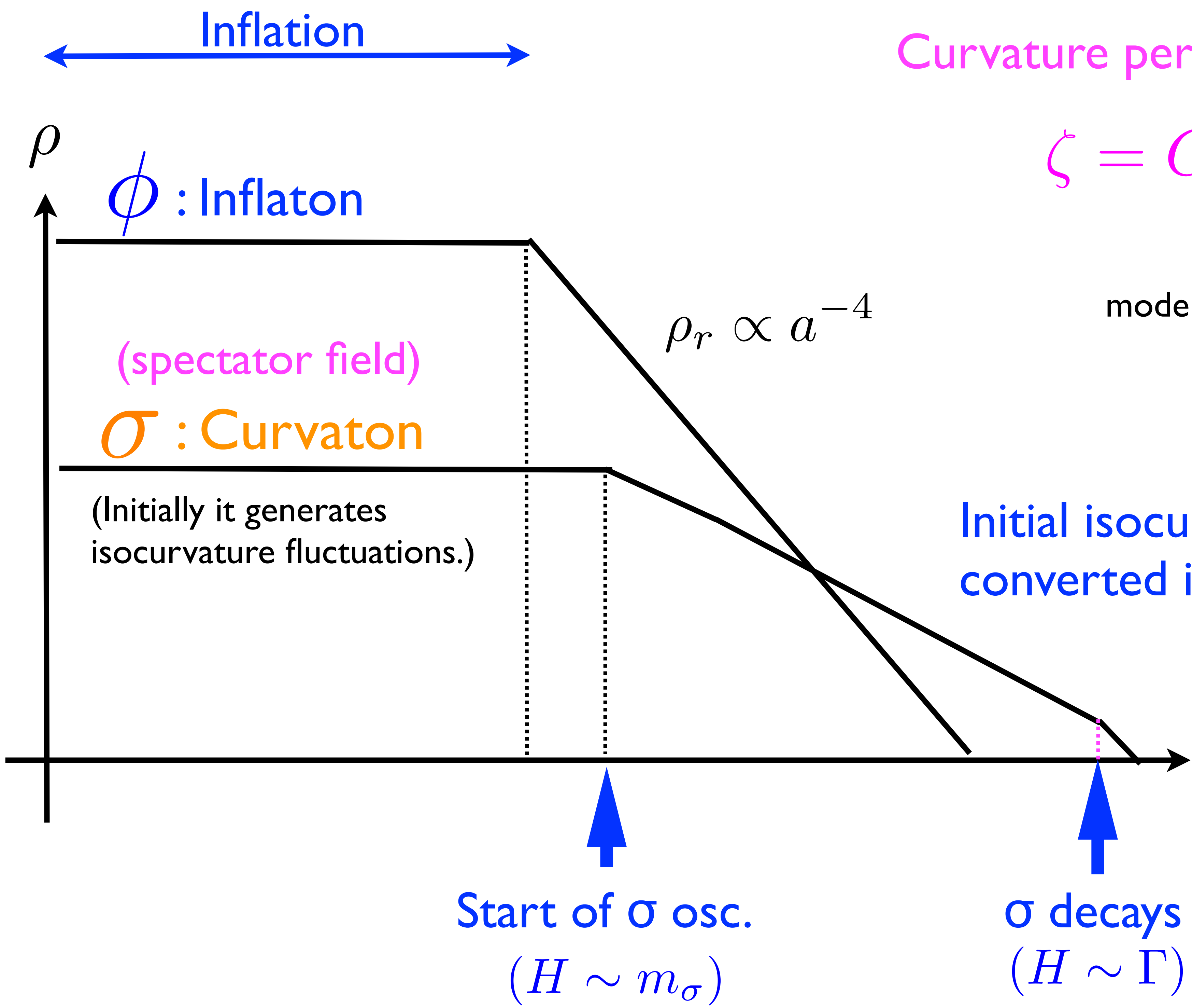
  (e.g., curvaton model, modulated reheating, …)

- Predictions for n_s and r can be modified from the original model (single-field counterpart).

- In particular, r can be suppressed in this framework.

Curvaton model

[Mollerach 1990; Linde, Mukhanov 1997; Enqvist & Sloth; Lyth & Wands; Moroi & TT, 2001]



Curvature perturbation is given by:

$$\zeta = C_\phi \delta\phi + C_\sigma \delta\sigma$$

model-dependent coefficients

Initial isocurvature fluctuations are converted into adiabatic ones.

Primordial power spectrum w/spectator field

- Assuming that both of the inflaton and the spectator contribute to primordial fluctuations,

  - Power spectrum:

inflaton

spectator field

$$\mathcal{P}_\zeta^{(\mathrm{total})} = \mathcal{P}_\zeta^{(\phi)} + \mathcal{P}_\zeta^{(\sigma)}$$
$$= A_s^{(\phi)}(k_{\mathrm{ref}}) \left(\frac{k}{k_{\mathrm{ref}}}\right)^{n_s^{(\phi)}-1} + A_s^{(\sigma)}(k_{\mathrm{ref}}) \left(\frac{k}{k_{\mathrm{ref}}}\right)^{n_s^{(\sigma)}-1}$$

  - Spectral indices:

inflaton part: $n_s^{(\phi)} - 1 = -6\epsilon + 2\eta_\phi \qquad \left(\epsilon = -\frac{\dot{H}}{H^2}, \quad \eta_\phi = \frac{V_{\phi\phi}}{3H^2}\right)$

spectator field part: $n_s^{(\sigma)} - 1 = -2\epsilon + 2\eta_\sigma \qquad \left(\eta_\sigma = \frac{V_{\sigma\sigma}}{3H^2}\right)$

Primordial power spectrum w/spectator field

- Assuming that both of the inflaton and the spectator contribute to primordial fluctuations,

  - Tensor mode power spectrum:

$$\mathcal{P}_T = \frac{8}{M_{\rm pl}^2}\left(\frac{H_{\rm inf}}{2\pi}\right)^2$$

(Same as the single-field case)

  - Tensor-to-scalar ratio:

$$R \equiv \frac{\mathcal{P}_\zeta^{(\sigma)}}{\mathcal{P}_\zeta^{(\phi)}}$$

(the ratio of two power spectra)

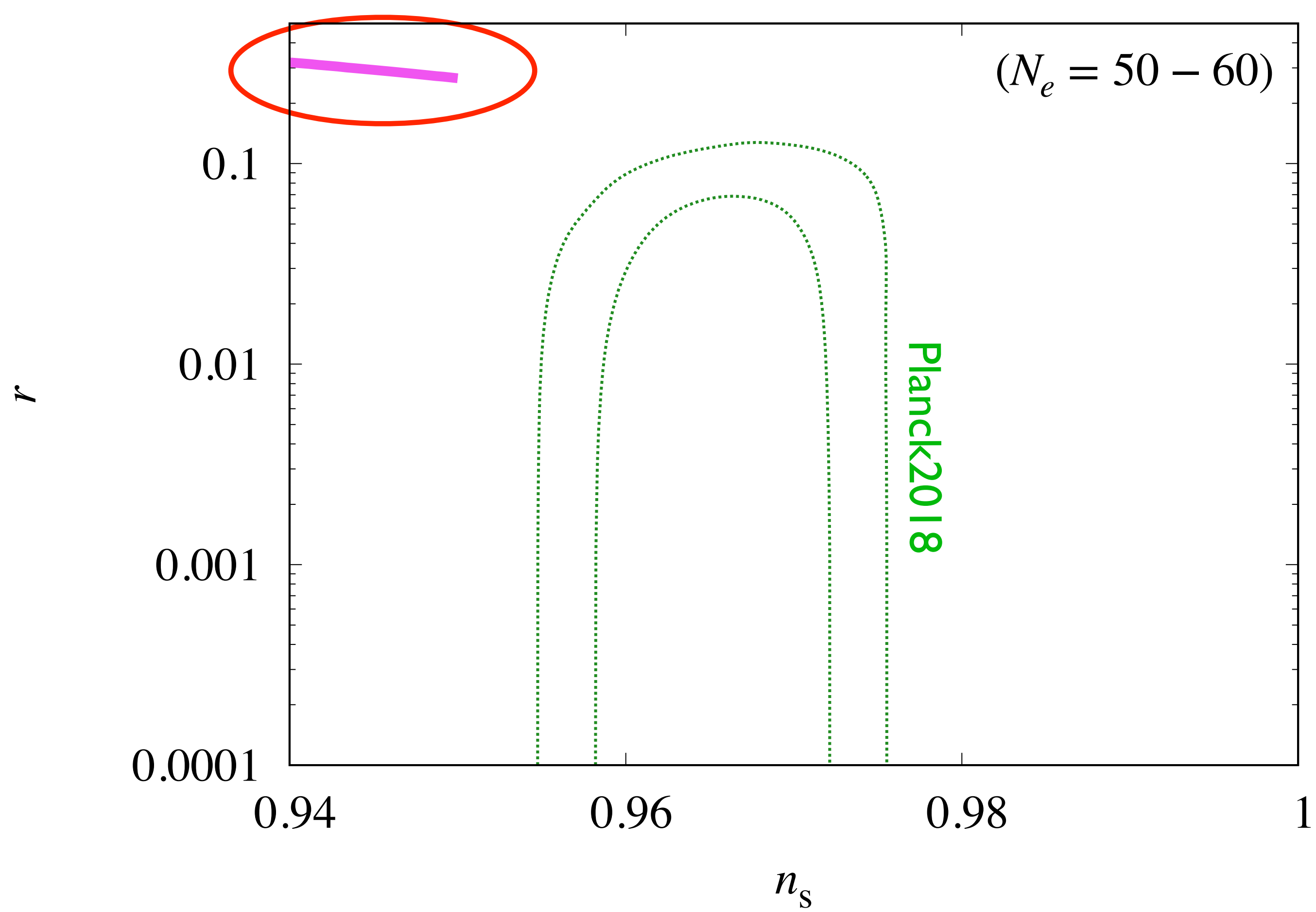
$$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_\zeta^{(\rm total)}} = \frac{\mathcal{P}_T}{\underbrace{\mathcal{P}_\zeta^{(\phi)}}_{\text{inflaton}} + \underbrace{\mathcal{P}_\zeta^{(\sigma)}}_{\text{spectator field}}} = \frac{16\epsilon}{1+R}$$

Large contribution from the spectator predicts a small r.

Predictions for n_s and r w/spectator field

[see, e.g., Enqvist, TT 1306.5958; Vennin, Koyama, Wands 1507.07575]

(Example) Chaotic inflation $\left(V(\phi) = \frac{1}{4}\lambda\phi^4\right)$ + spectator field $\left(V(\sigma) = \frac{1}{2}m_\sigma^2\sigma^2\right)$



Inflaton ~ 100%

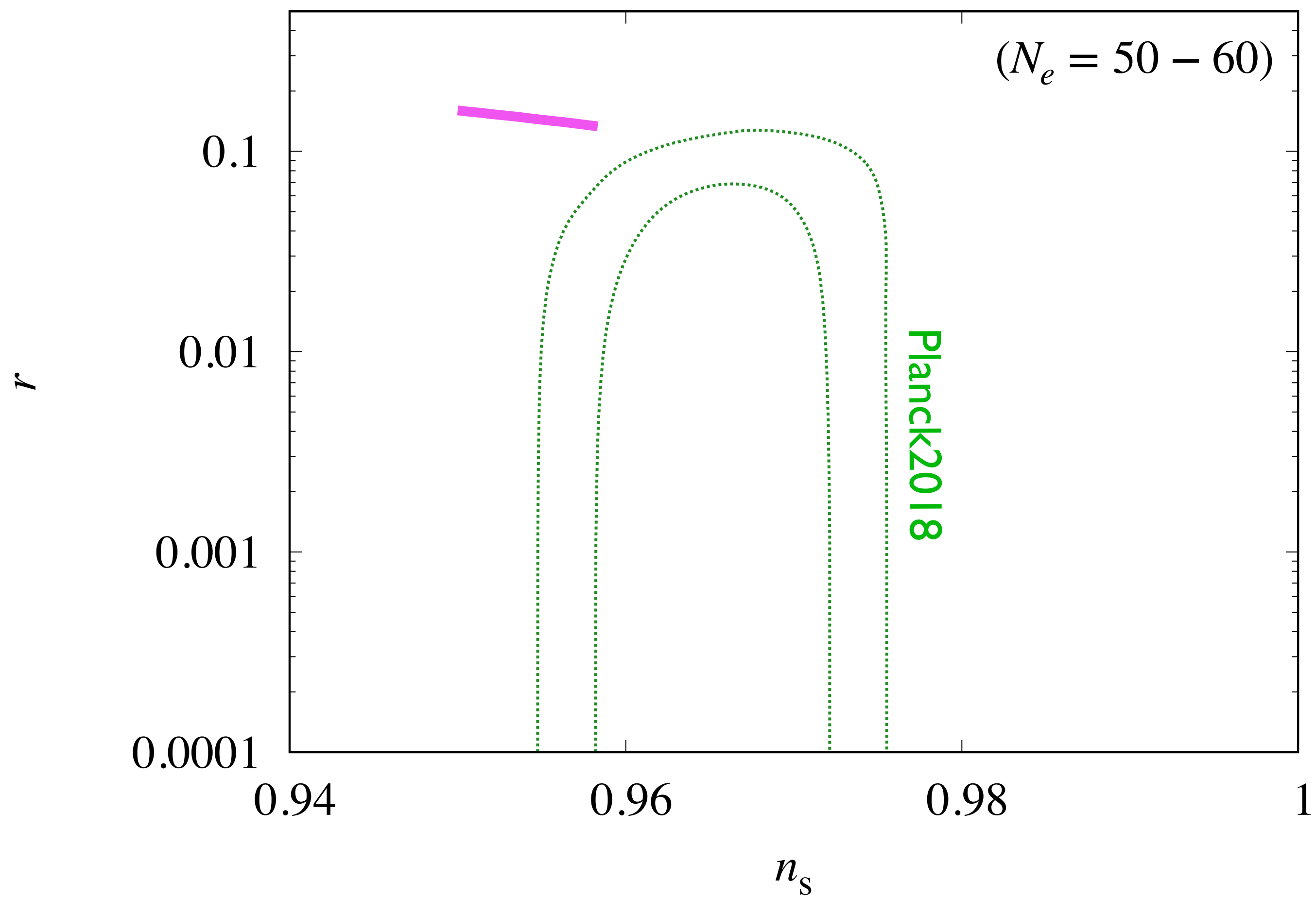
Spectator field ~ 0%

- Quartic chaotic inflation model is completely excluded.
  (when there is no fluctuations from a spectator field)

Predictions for n_s and r w/spectator field

[see, e.g., Enqvist, TT 1306.5958; Vennin, Koyama, Wands 1507.07575]

(Example) Chaotic inflation $\left(V(\phi) = \frac{1}{4}\lambda\phi^4\right)$ + spectator field $\left(V(\sigma) = \frac{1}{2}m_\sigma^2\sigma^2\right)$



Inflaton ~ 50%

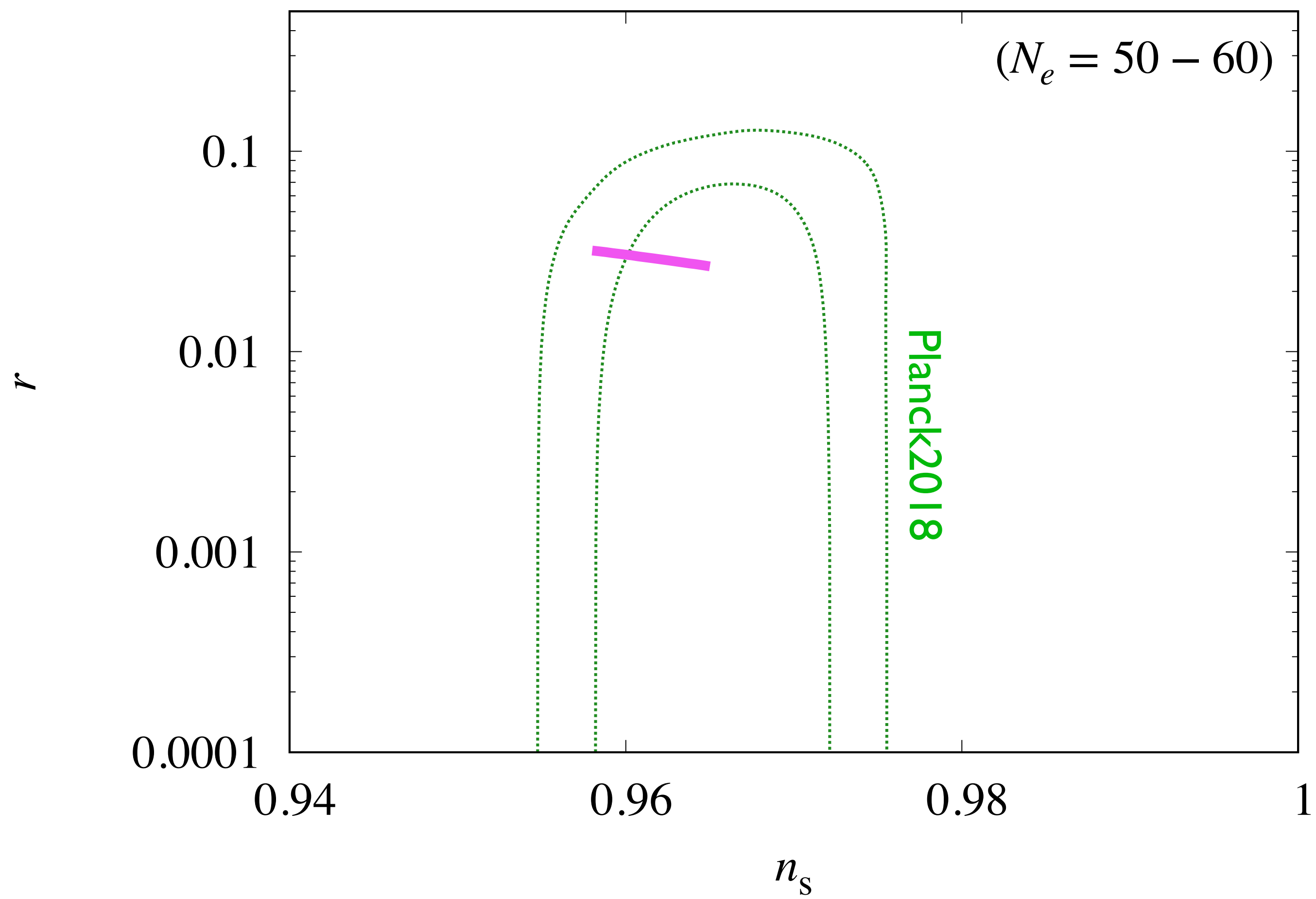
Spectator field ~ 50%

- Contribution from a spectator field changes the prediction of n_s and r.

Predictions for n_s and r w/spectator field

[see, e.g., Enqvist, TT 1306.5958; Vennin, Koyama, Wands 1507.07575]

(Example) Chaotic inflation $\left(V(\phi) = \frac{1}{4}\lambda\phi^4\right)$ + spectator field $\left(V(\sigma) = \frac{1}{2}m_\sigma^2\sigma^2\right)$



Inflaton ~ 10%

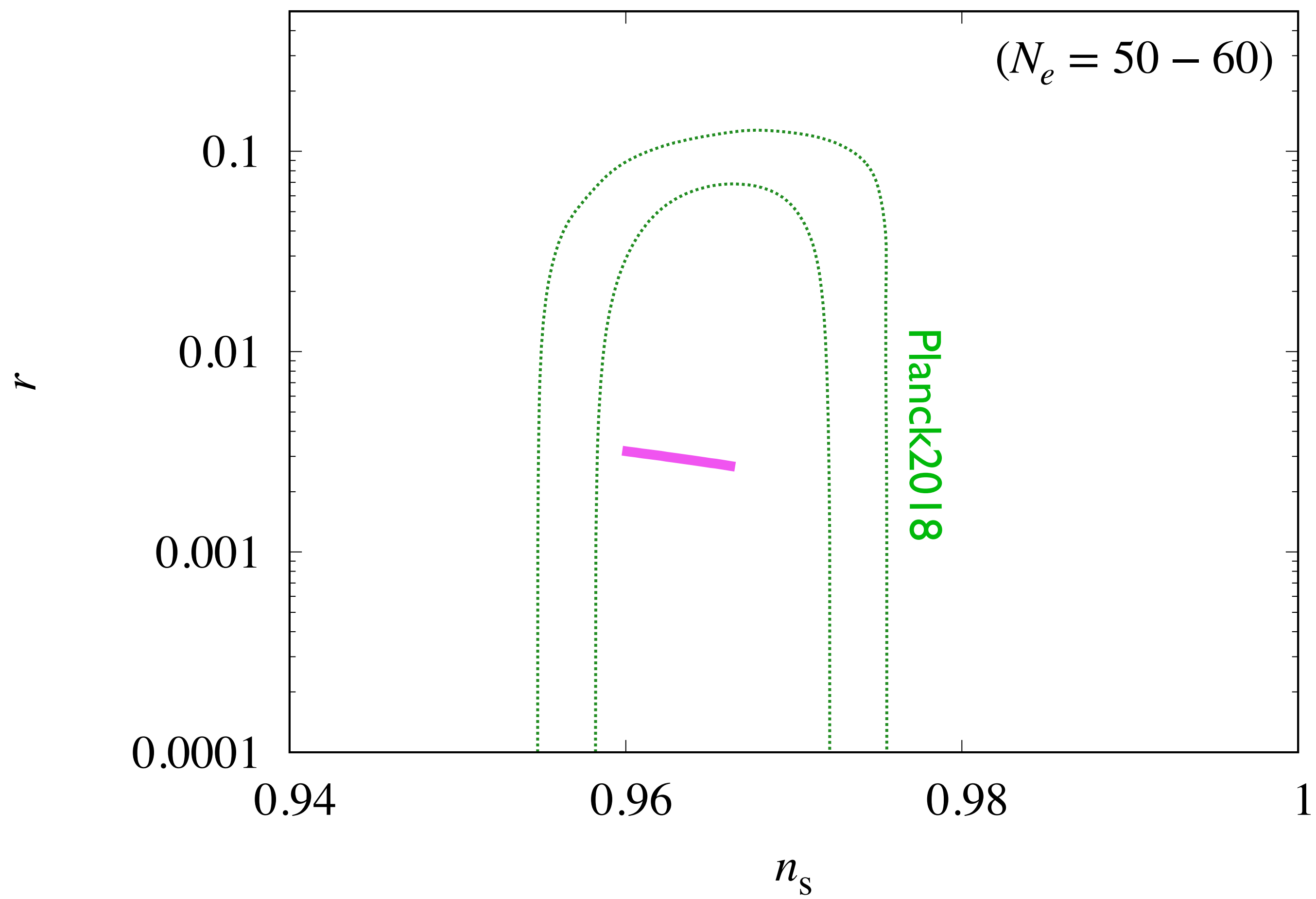
Spectator field ~ 90%

- Contribution from a spectator field changes the prediction of n_s and r.

Predictions for n_s and r w/spectator field

[see, e.g., Enqvist, TT 1306.5958; Vennin, Koyama, Wands 1507.07575]

(Example) Chaotic inflation $\left(V(\phi) = \frac{1}{4}\lambda\phi^4\right)$ + spectator field $\left(V(\sigma) = \frac{1}{2}m_\sigma^2\sigma^2\right)$



Inflaton ~ 1%

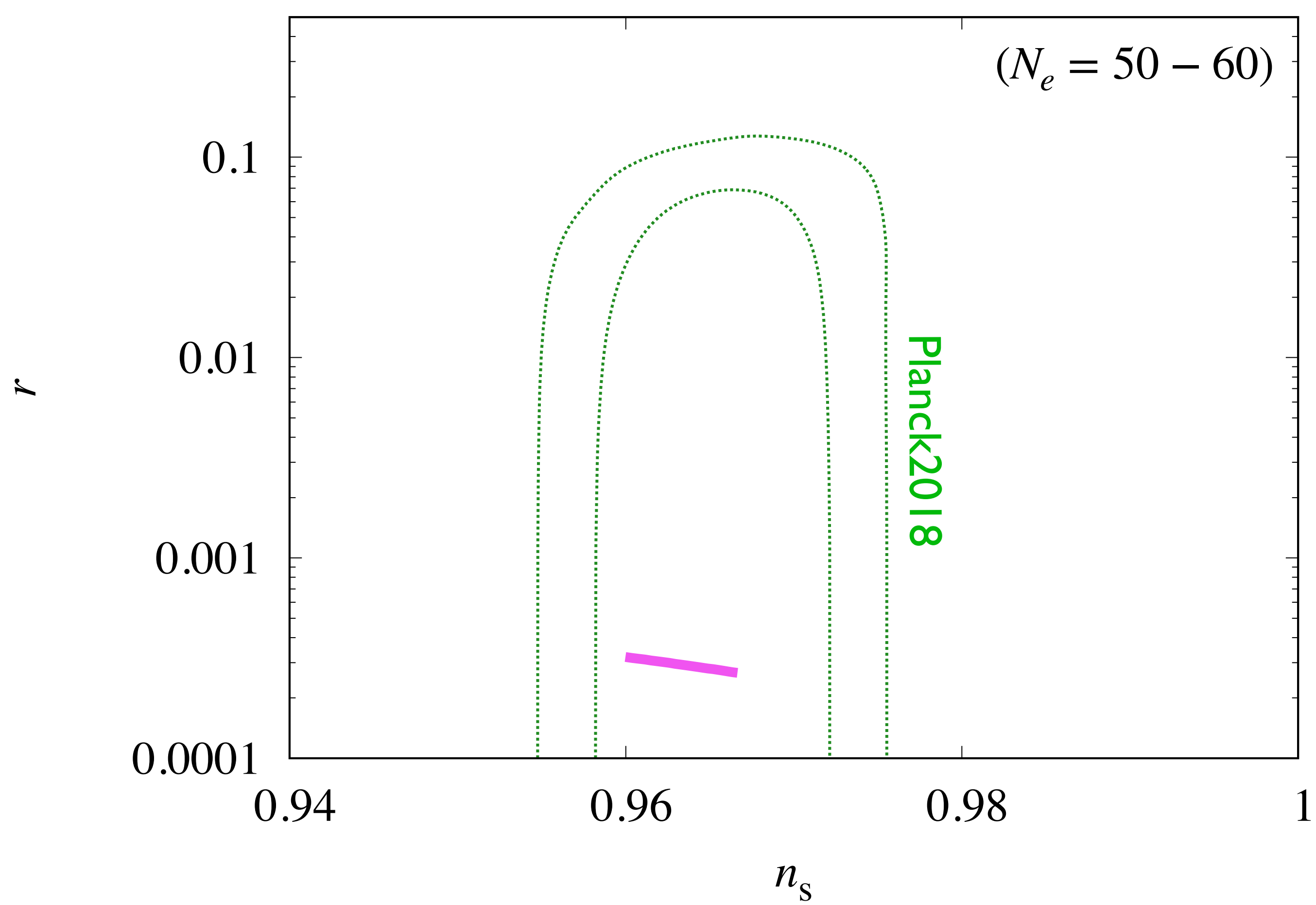
Spectator field ~ 99%

- Quartic chaotic inflation model is viable when the spectator mostly provides primordial fluctuations.

Predictions for n_s and r w/spectator field

[see, e.g., Enqvist, TT 1306.5958; Vennin, Koyama, Wands1507.07575]

(Example) Chaotic inflation $\left(V(\phi) = \frac{1}{4}\lambda\phi^4\right)$ + spectator field $\left(V(\sigma) = \frac{1}{2}m_\sigma^2\sigma^2\right)$



Inflaton ~ 0.1%

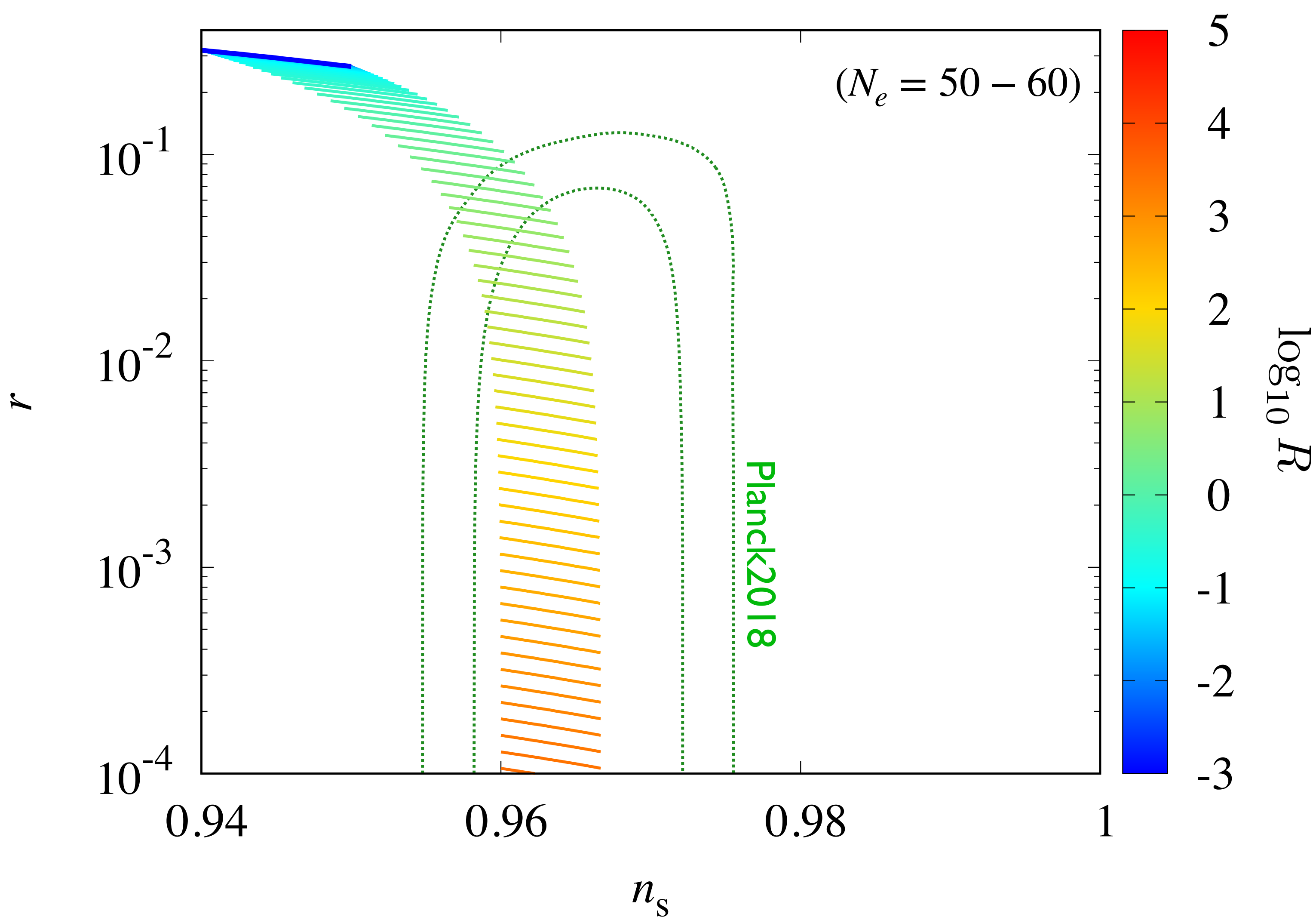
Spectator field ~ 99.9%

- Quartic chaotic inflation model is viable when the spectator mostly provides primordial fluctuations.

Predictions for n_s and r w/spectator field

[see, e.g., Enqvist, TT 1306.5958; Vennin, Koyama, Wands1507.07575]

(Example) Chaotic inflation $\left(V(\phi) = \frac{1}{4}\lambda\phi^4\right)$ + spectator field $\left(V(\sigma) = \frac{1}{2}m_\sigma^2\sigma^2\right)$

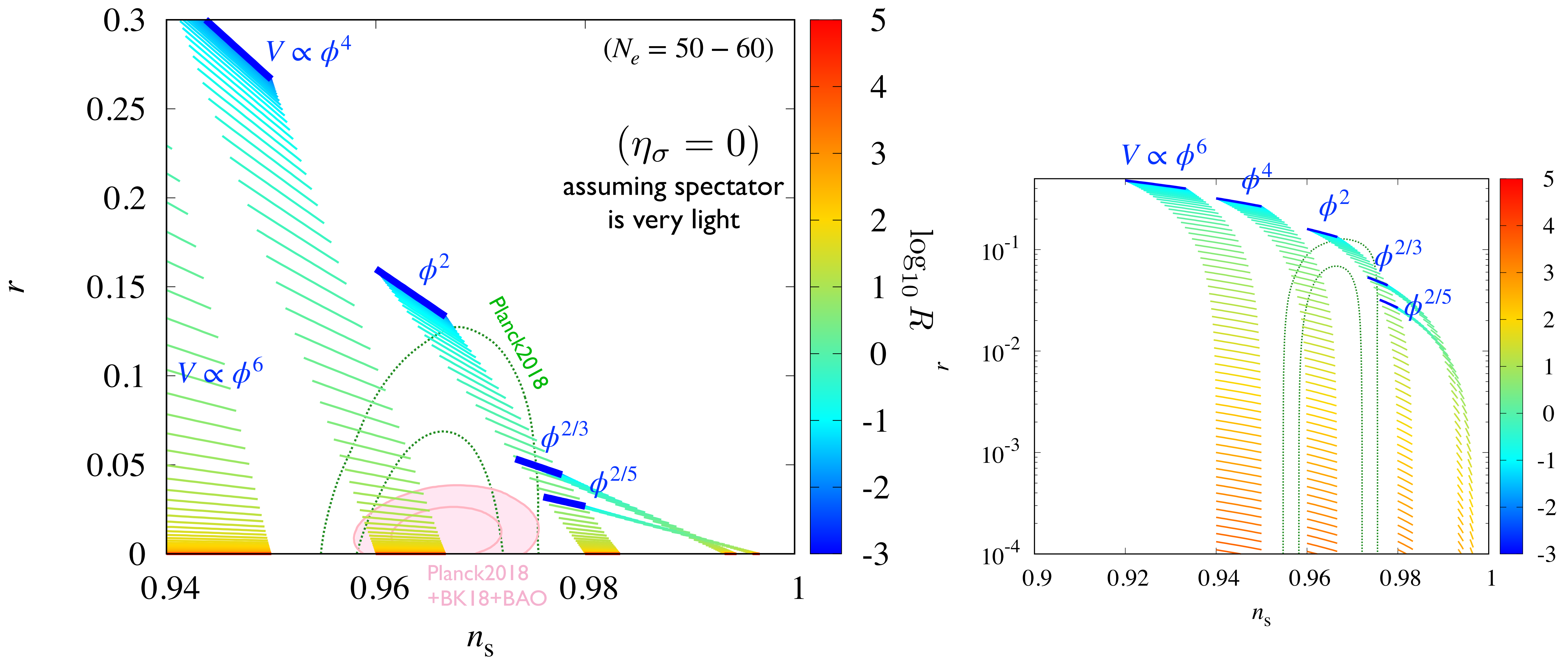


$$R \equiv \frac{\mathcal{P}_\zeta^{(\sigma)}}{\mathcal{P}_\zeta^{(\phi)}}$$

By changing the fraction of the contribution from the spectator field, the predictions for n_s and r are affected.

Predictions for n_s and r w/spectator field

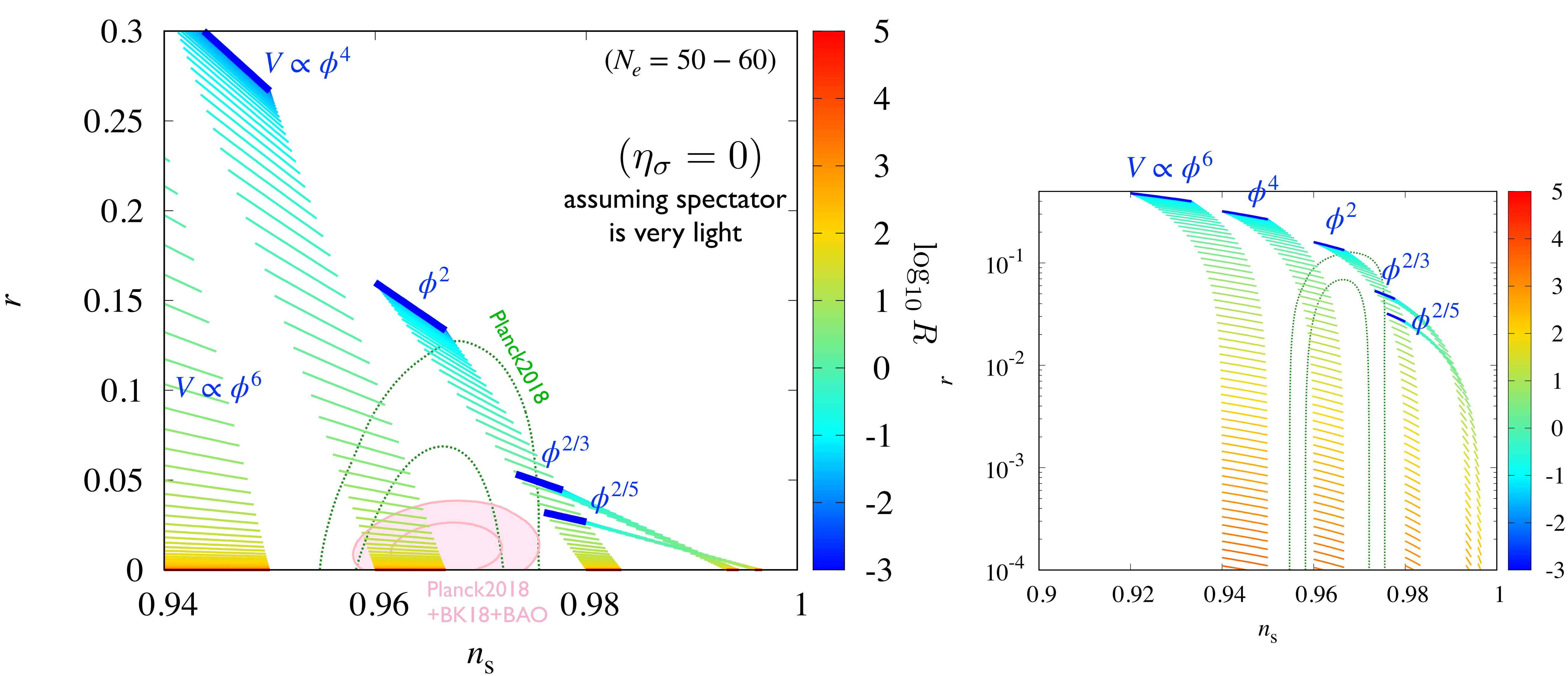
- Chaotic inflation ($V \propto \phi^n$) + spectator field model ($V = m_\sigma^2 \sigma^2 / 2$)



- Some inflation models become viable when the spectator is (partially) responsible for density perturbations.

Predictions for n_s and r w/spectator field

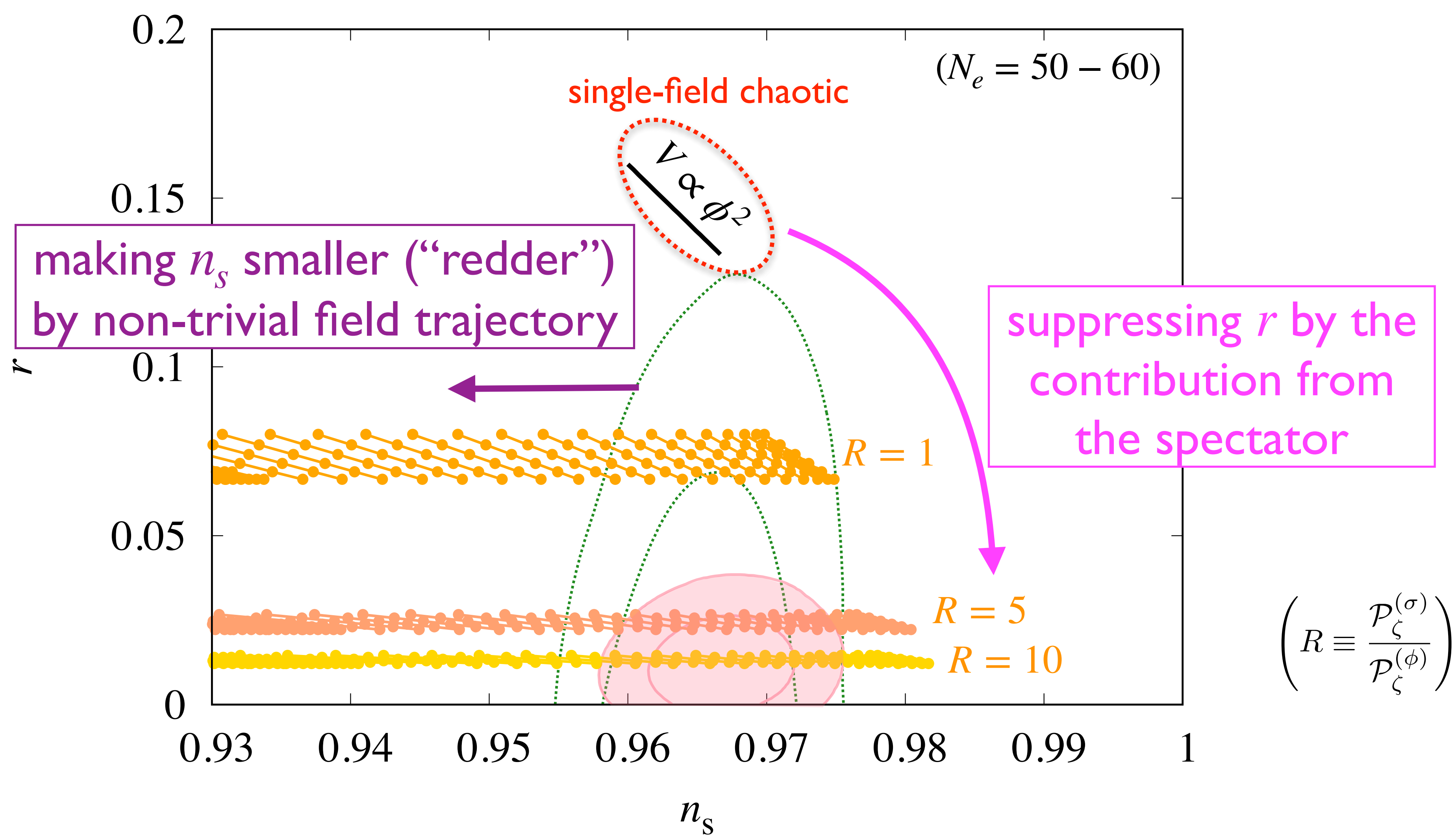
- Chaotic inflation ($V \propto \phi^n$) + spectator field model ($V = m_\sigma^2 \sigma^2/2$)



-But, unfortunately, $V \propto \phi^2$ model cannot be revived even in this framework…

However, three field model works [Morishita, TT, Yokoyama, 2203.09698]

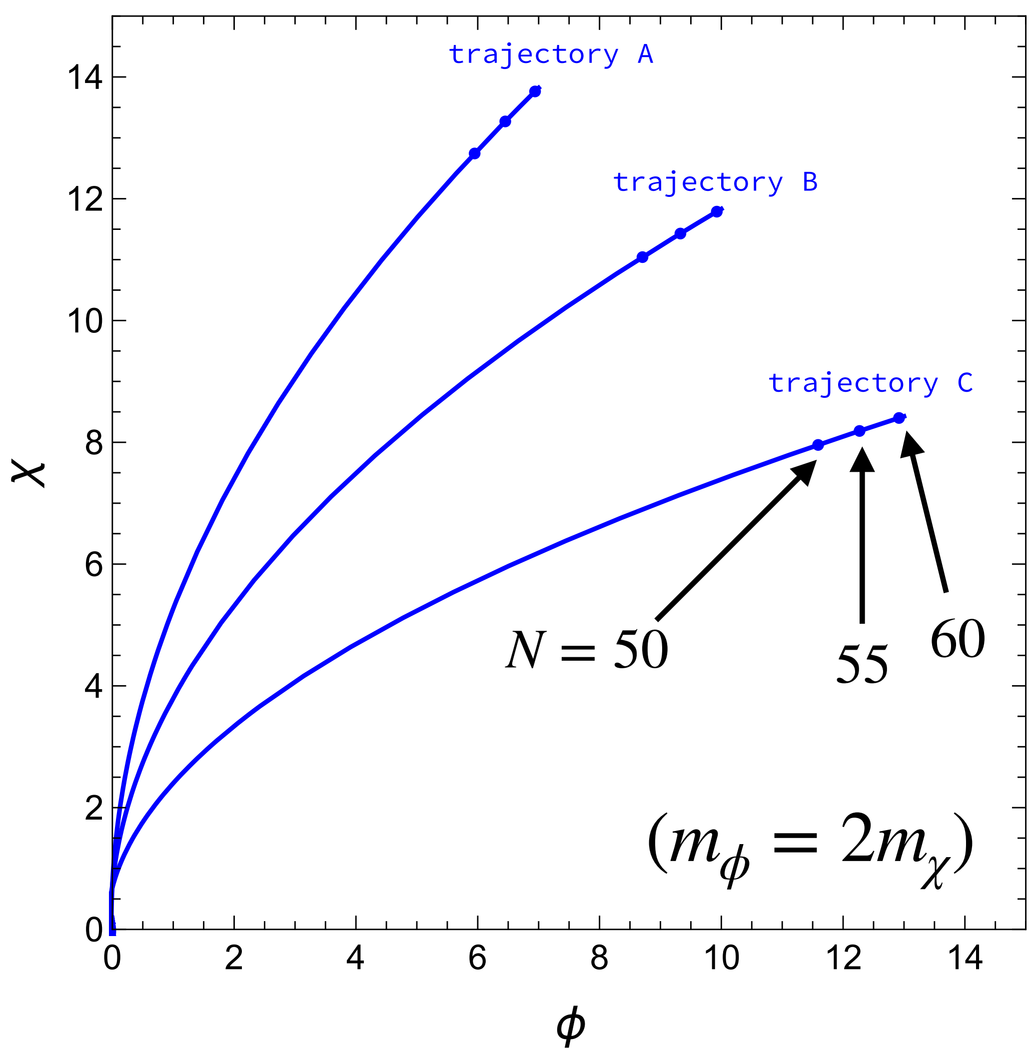
- Double chaotic inflation ($V \propto m_\phi^2\phi^2 + m_\chi^2\chi^2$) + spectator field ($V \propto m_\sigma^2\sigma^2$)



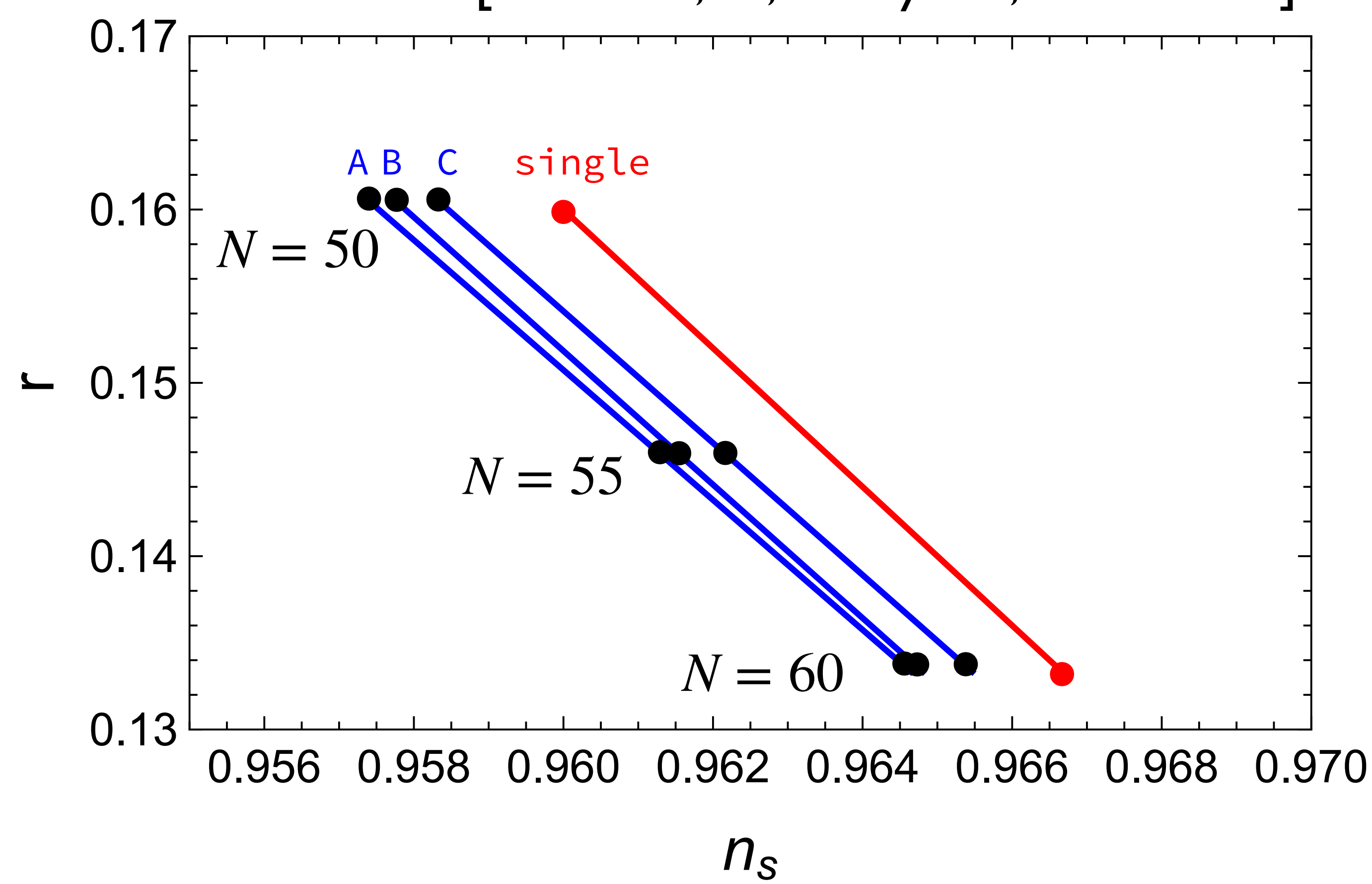
- In particular, ϕ^2 chaotic inflation model can also become viable.

Field trajectory during inflation changes the prediction for n_s

- Field trajectory in the $\phi - \chi$ plane



[Morishita, TT, Yokoyama, 2203.09698]

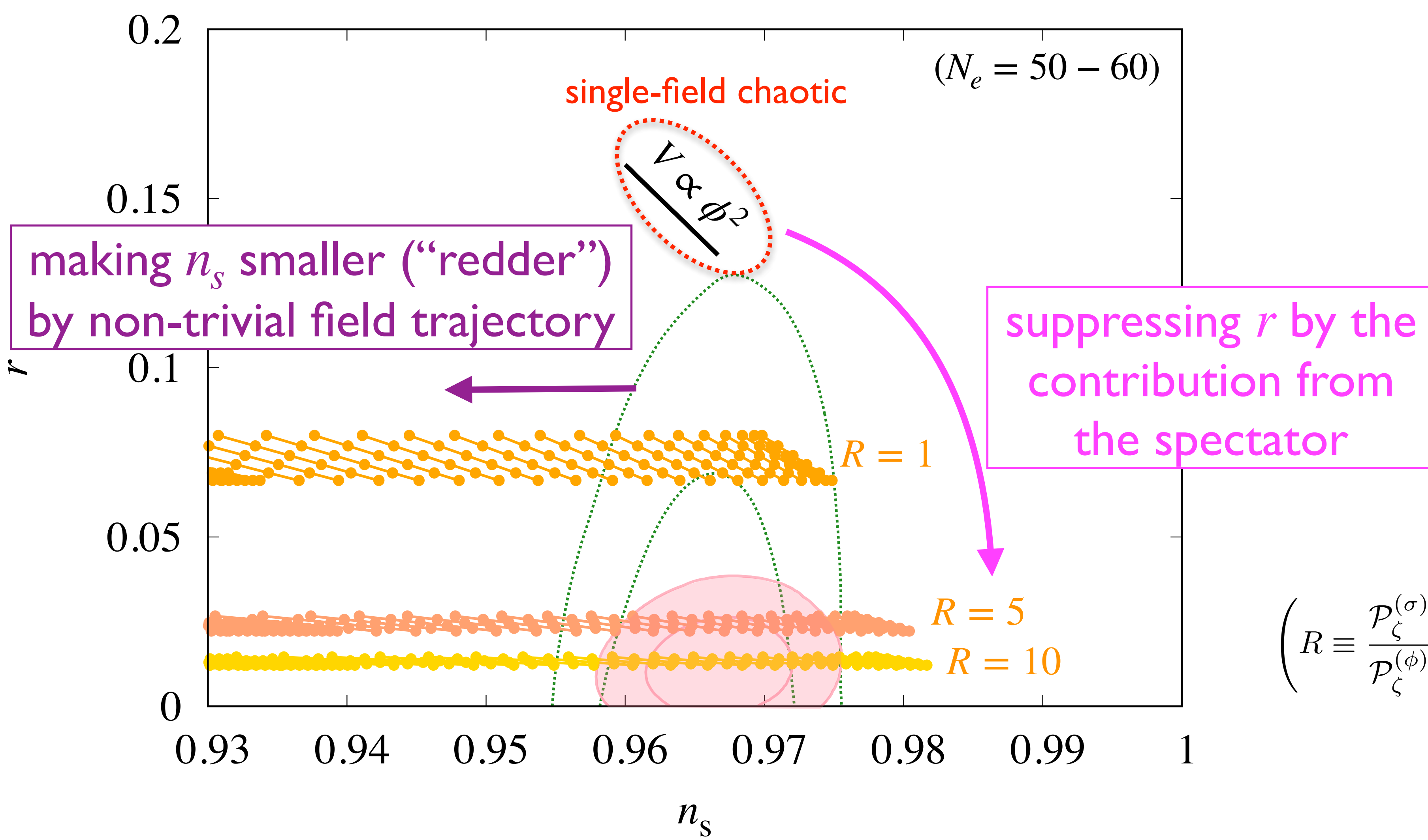


- Non-trivial field trajectory (more curved trajectory) makes the spectrum "redder" (n_s becomes smaller).

Three field model works

[Morishita, TT, Yokoyama, 2203.09698]

- Double chaotic inflation ($V \propto m_\phi^2\phi^2 + m_\chi^2\chi^2$) + spectator field ($V \propto m_\sigma^2\sigma^2$)



$\left(R \equiv \frac{\mathcal{P}_\zeta^{(\sigma)}}{\mathcal{P}_\zeta^{(\phi)}}\right)$

- In particular, ϕ^2 chaotic inflation model can also become viable.

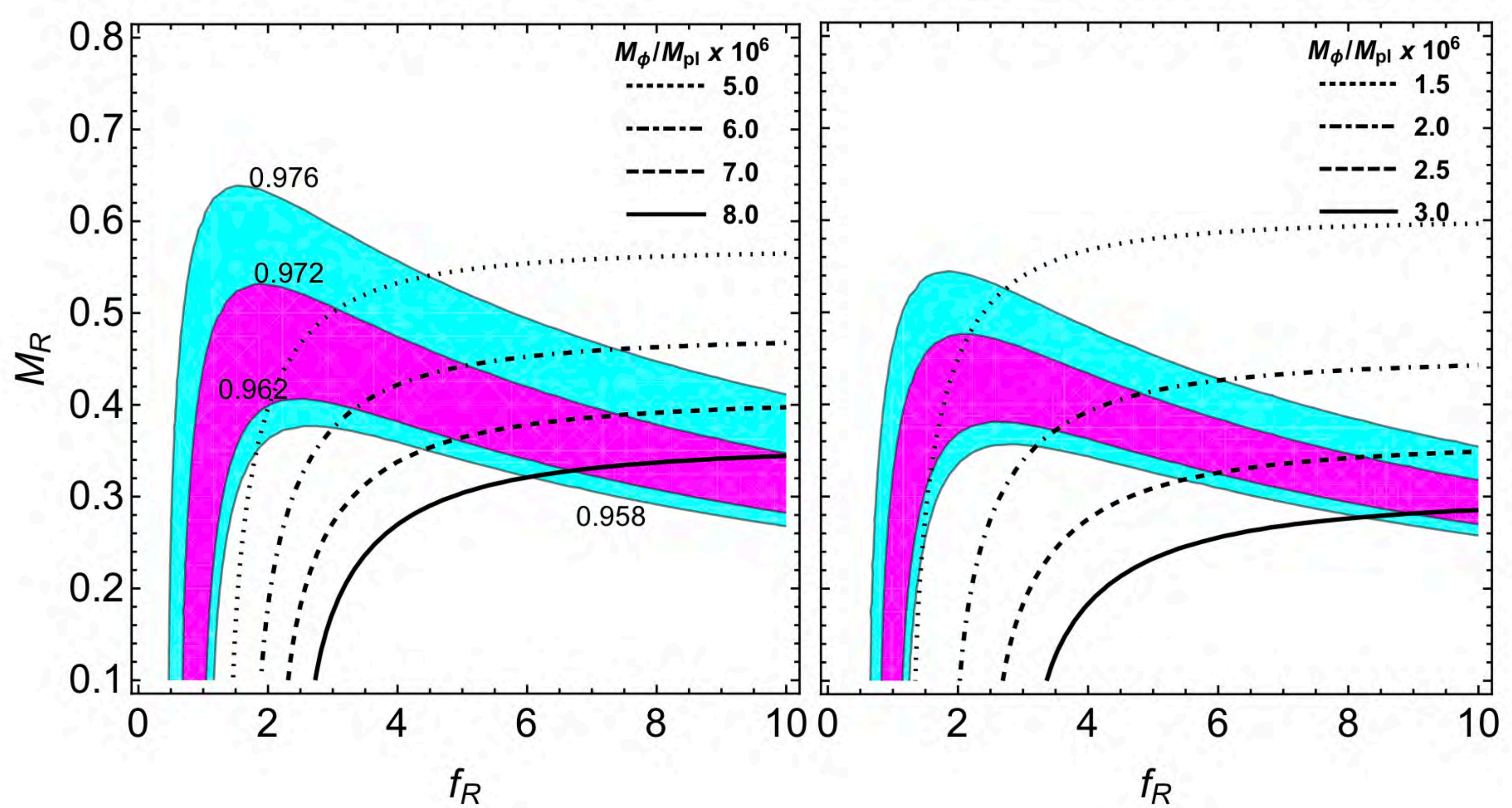
- Three field (inflatons+curvaton) case may be motivated. → Sneutrino inflation

[TT, Yamada, Yokoyama 2208.08296]

Sneutrino as inflatons + curvaton [TT, Yamada, Yokoyama 2208.08296]

- Constraint on the spectral index: $0.957 < n_s < 0.976$ (from Planck)

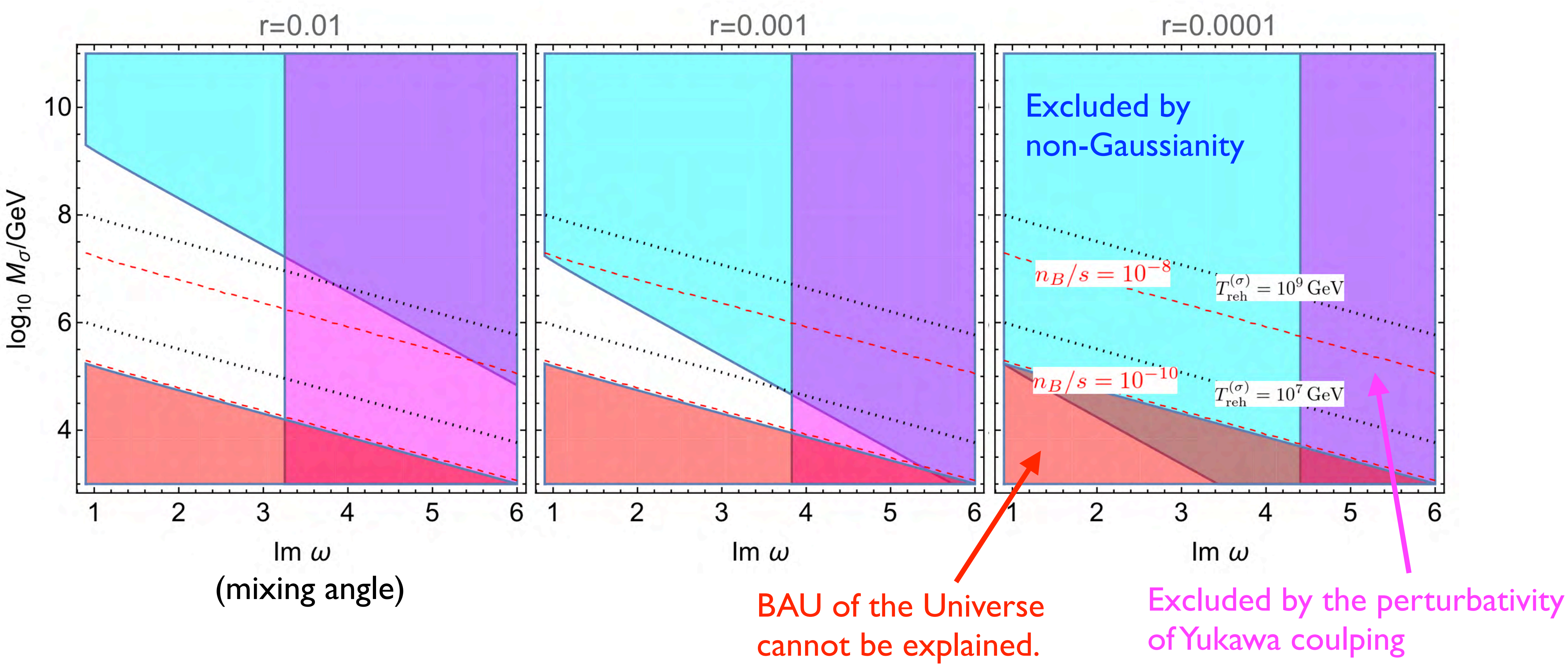
→ gives allowed ranges for $M_R\,(\equiv M_\chi/M_\phi)$ and $f_R\,(\equiv \chi_*/\phi_*)$



Broad range of f_R and $M_R \sim \mathcal{O}(0.1)$ are allowed.

Sneutrino as inflatons + curvaton [TT, Yamada, Yokoyama 2208.08296]

- Constraints from primordial fluctuations and BAU of the Universe



BAU of the Universe can be realized when $r > \mathcal{O}(10^{-4})$ in this setup.

→ Future observations of CMB B-mode (e.g., LiteBIRD) can probe.

Part II:
“Multi-dimensional” test of inflation

Observables to probe the inflation

- Curvature perturbation ζ (scalar mode)

  - Power spectrum:

$$\langle\zeta(\boldsymbol{k}_1)\zeta(\boldsymbol{k}_2)\rangle = (2\pi)^3\delta(\boldsymbol{k}_1+\boldsymbol{k}_2)P_\zeta(k_1)$$

  -Bispectrum:

$$\langle\zeta(\boldsymbol{k}_1)\zeta(\boldsymbol{k}_2)\zeta(\boldsymbol{k}_3)\rangle = (2\pi)^3\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)B_\zeta(k_1,k_2,k_3)$$

  -Trispectrum:

$$\langle\zeta(\boldsymbol{k}_1)\zeta(\boldsymbol{k}_2)\zeta(\boldsymbol{k}_3)\zeta(\boldsymbol{k}_4)\rangle = (2\pi)^3\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3+\boldsymbol{k}_4)T_\zeta(k_1,k_2,k_3,k_4)$$

$$\vdots$$

Observables to probe the inflation

- Curvature perturbation ζ (scalar mode)

- Power spectrum:

$$\mathcal{P}_\zeta(k) = \boxed{A_s(k_{\rm ref})} \left(\frac{k}{k_{\rm ref}}\right)^{\boxed{n_s}-1+\frac{1}{2}\boxed{\alpha_s}\ln(k/k_{\rm ref})+\frac{1}{6}\boxed{\beta_s}\ln^2(k/k_{\rm ref})}$$

-Bispectrum:

$$B_\zeta(k_1,k_2,k_3) = \boxed{\frac{6}{5}f_{\rm NL}}(P_\zeta(k_1)P_\zeta(k_2)+P_\zeta(k_2)P_\zeta(k_3)+P_\zeta(k_3)P_\zeta(k_1))$$

(local type)

-Trispectrum:

$$T_\zeta(k_1,k_2,k_3,k_4) = \boxed{\tau_{\rm NL}}(P_\zeta(k_{13})P_\zeta(k_3)P_\zeta(k_4)+11\ \text{perms.})$$

$k_{13} = k_1 + k_3$

$$+\boxed{\frac{54}{25}g_{\rm NL}}(P_\zeta(k_2)P_\zeta(k_3)P_\zeta(k_4)+3\ \text{perms.})$$

(local type)

Observables to probe the inflation

- Curvature perturbation ζ (scalar mode)

Well measured

Constrained

(poorly) constrained

No real constraint

- Power spectrum:

Amplitude A_s ,

Spectral index n_s , Running(s) α_s, β_s

-Bispectrum:

Nonlinearity parameter $f_{\rm NL}$ (for various "shapes")

Scale dependence $n_{f_{\rm NL}}$

-Trispectrum:

Nonlinearity parameter $\tau_{\rm NL}, g_{\rm NL}$ (for various "shapes"),

Scale dependence $n_{\tau_{\rm NL}}, n_{g_{\rm NL}}$

Observables to probe the inflation

- Tensor mode (gravitational waves) h

- Power spectrum:

$$\langle h(\boldsymbol{k}_1)h(\boldsymbol{k}_2)\rangle = (2\pi)^3\delta(\boldsymbol{k}_1+\boldsymbol{k}_2)P_T(k_1)$$

-Bispectrum:

$$\langle h(\boldsymbol{k}_1)h(\boldsymbol{k}_2)h(\boldsymbol{k}_3)\rangle = (2\pi)^3\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)B_T(k_1,k_2,k_3)$$

-Trispectrum:

$$\langle h(\boldsymbol{k}_1)h(\boldsymbol{k}_2)h(\boldsymbol{k}_3)h(\boldsymbol{k}_4)\rangle = (2\pi)^3\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3+\boldsymbol{k}_4)T_T(k_1,k_2,k_3,k_4)$$

$$\vdots$$

Observables to probe the inflation

- Tensor mode (gravitational waves) h

  - Power spectrum:

    Amplitude A_T (tensor-to-scalar ratio r)

    Spectral index n_T , Running(s) α_T

  -Bispectrum:

    Tensor nonlinearity parameter $f_{\rm NL}^{\rm tens}$

    Scale-dependence $n_{f_{\rm NL}^{\rm tens}}$

  -Trispectrum:

    Tensor nonlinearity parameter $\tau_{\rm NL}^{\rm tens}, g_{\rm NL}^{\rm tens}$

    Scale-dependence $n_{\tau_{\rm NL}^{\rm tens}}, n_{g_{\rm NL}^{\rm tens}}$

    $\vdots$

Well measured

Constrained

(poorly) constrained

No real constraint
(but possible future constraint)

“Multi-dimensional” probes of inflation

	Amplitude	scale dependence	running(s)
Scalar power spectrum	A_s	n_s	α_s, β_s
Tensor (GW) power spectrum	r	n_T	
(scalar) Non-Gaussianity (bispectrum)	$f_{\rm NL}$	$n_{f_{\rm NL}}$	
(scalar) Non-Gaussianity (trispectrum)	$g_{\rm NL}, \tau_{\rm NL}$		
(tensor) Non-Gaussianity (bispectrum)	$f_{\rm NL}^{\rm (tens)}$		
(tensor) Non-Gaussianity (trispectrum)			

well determined — relatively well constrained — poorly constrained

(Near) future prospects

- Some observables can be measured (constrained) more precisely.

  - Runnings of (scalar) spectral index α_s, β_s

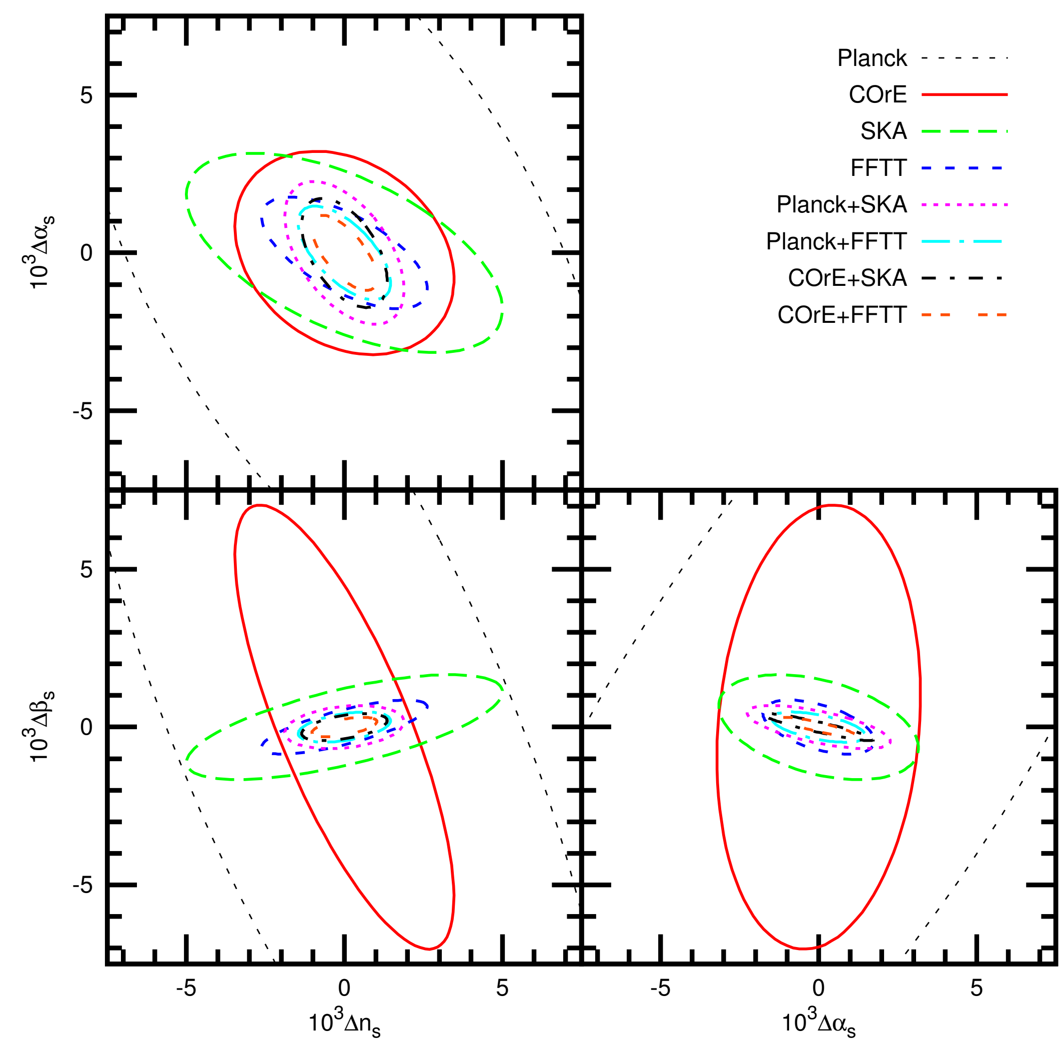
    (Euclid, Roman telescope, LSST, SKAO, …)

21cm minihalos can probe the runnings as

$$\alpha_s \sim \mathcal{O}(10^{-3}), \quad \beta_s \sim \mathcal{O}(10^{-4})$$

	$10^{-3}\Delta n_s$	$10^{-3}\Delta\alpha_s$	$10^{-3}\Delta\beta_s$
Planck	7.7	10.7	15.1
COrE	3.2	2.9	6.5
SKA	4.6	2.9	1.5
FFTT	2.4	1.6	0.79
Planck+SKA	1.7	2.0	0.63
Planck+FFTT	1.3	1.3	0.44
COrE+SKA	1.2	1.6	0.39
COrE+FFTT	0.95	1.1	0.28

[Sekiguchi, TT, Tashiro, Yokoyama 1705.00405]



n_T is important to test single-field/multi-field models

	Amplitude	scale dependence	running(s)
Scalar power spectrum	A_s	n_s	α_s, β_s
Tensor (GW) power spectrum	r	$\boxed{n_T}$	
(scalar) Non-Gaussianity (bispectrum)	$f_{\rm NL}$	$n_{f_{\rm NL}}$	
(scalar) Non-Gaussianity (trispectrum)	$g_{\rm NL}, \tau_{\rm NL}$		
(tensor) Non-Gaussianity (bispectrum)	$f_{\rm NL}^{\rm (tens)}$		
(tensor) Non-Gaussianity (trispectrum)			

well determined — relatively well constrained — poorly constrained

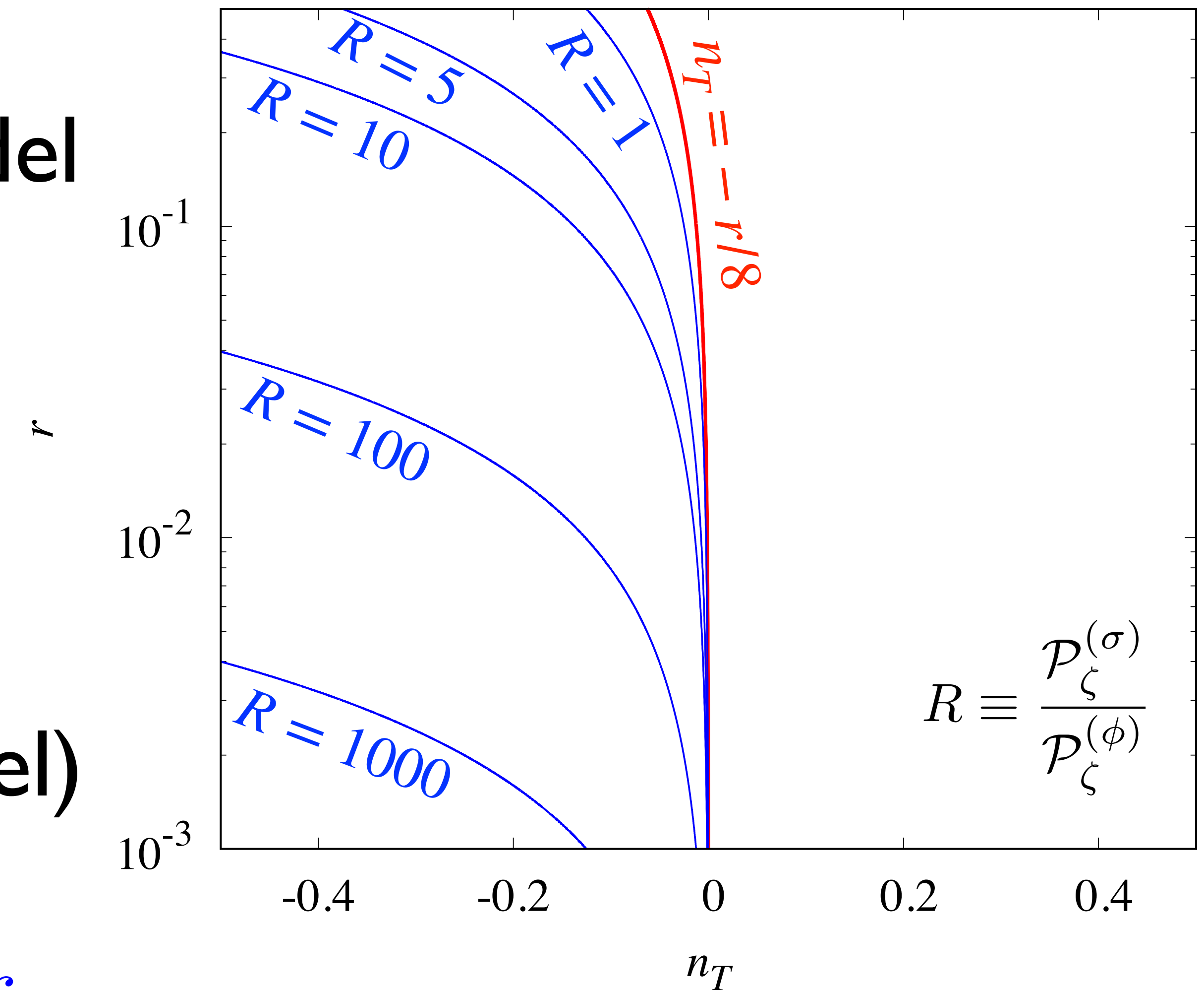
Consistency test of inflation: r and n_T

- Standard single-field inflation model

$$n_T = -2\epsilon$$
$$r = 16\epsilon$$
$$\Rightarrow \quad n_T = -\frac{r}{8}$$

- Multi-field model (spectator model)

$$n_T = -2\epsilon$$
$$r = \frac{16\epsilon}{1+R}$$
$$\Rightarrow \quad n_T = -(1+R)\frac{r}{8}$$



R→0: (pure) inflaton case

R→∞: (pure) curvaton case

→ Tensor scale-dependence should give important test of the inflationary Universe

Expected constraints from LiteBIRD

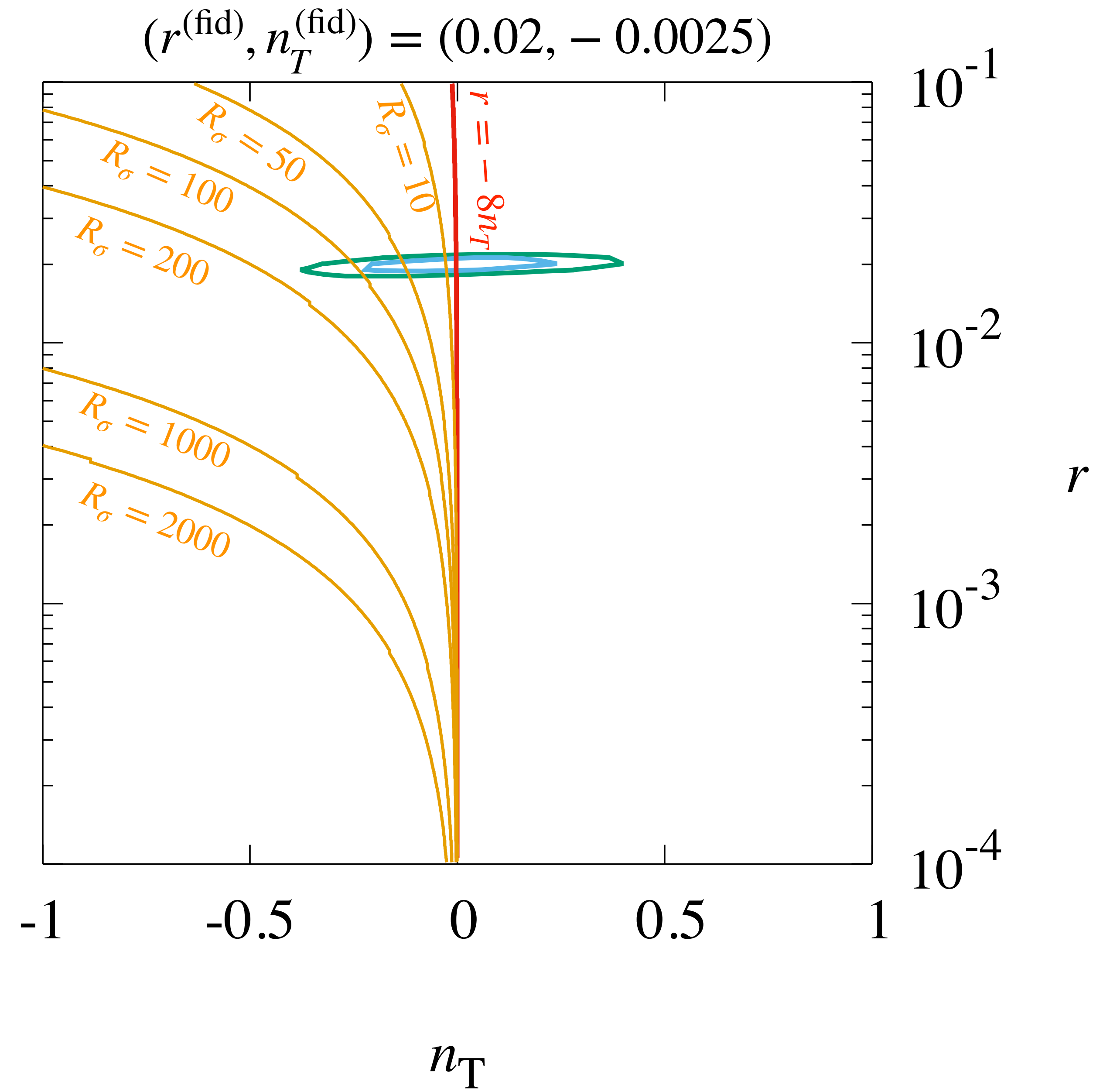
[Jinno, Kohri, Moroi, TT, Hazumi 2310.08158]

- When r is detected, n_T can also be constrained.

→ The spectator contribution can also be constrained.

(when $r = 0.02$, the spectator contribution is constrained as $R < \mathcal{O}(100)$.)

It would be difficult to check the single-field consistency relation $r = -8n_T$.)



Expected constraints from LiteBIRD

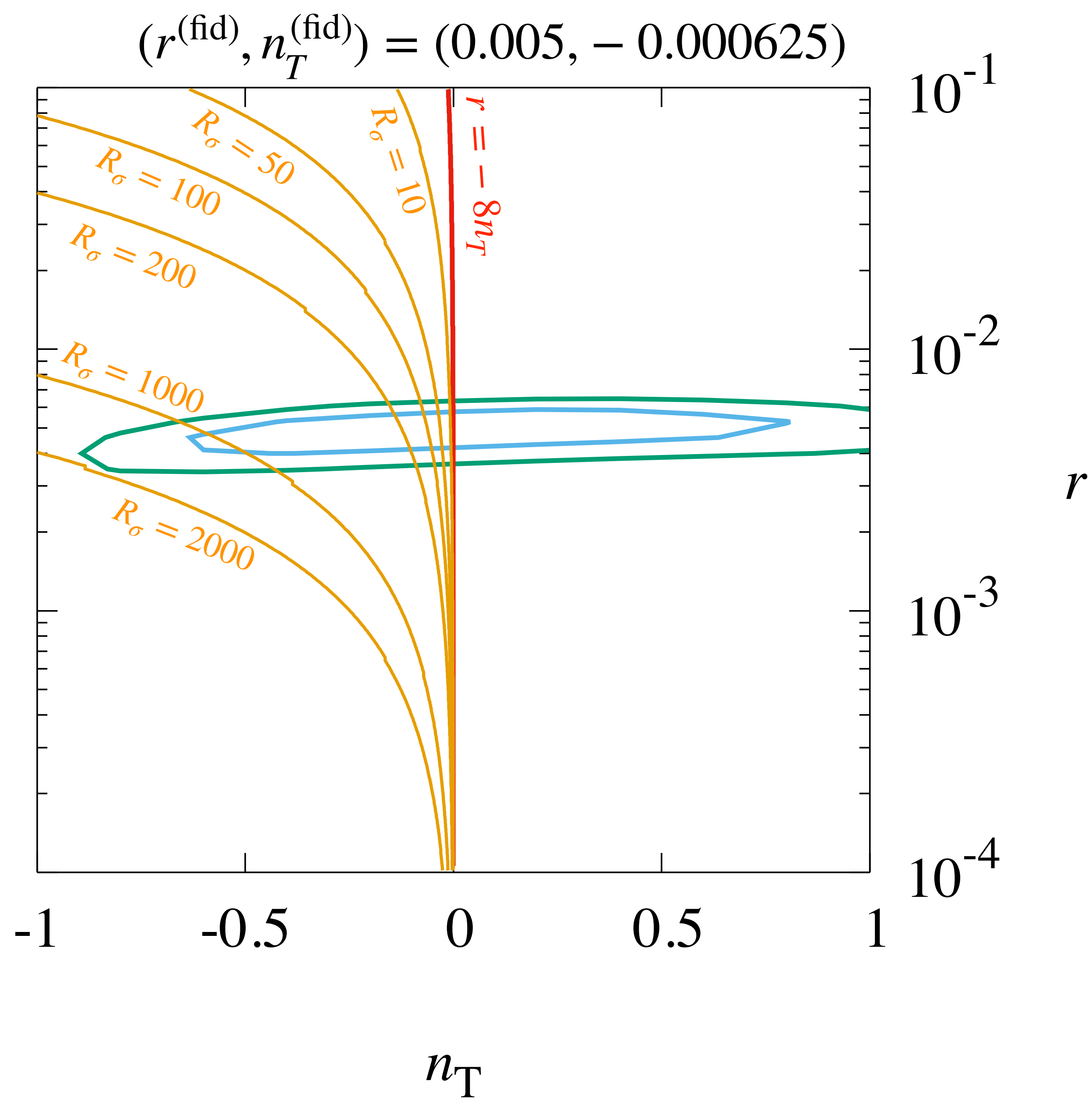
[Jinno, Kohri, Moroi, TT, Hazumi 2310.08158]

- When r is detected, n_T can also be constrained.

→ The spectator contribution can also be constrained.

(when $r = 0.005$, the spectator contribution is constrained as $R < \mathcal{O}(10^3)$.)

It would be difficult to check the single-field consistency relation $r = -8n_T$.)



Expected constraints from LiteBIRD

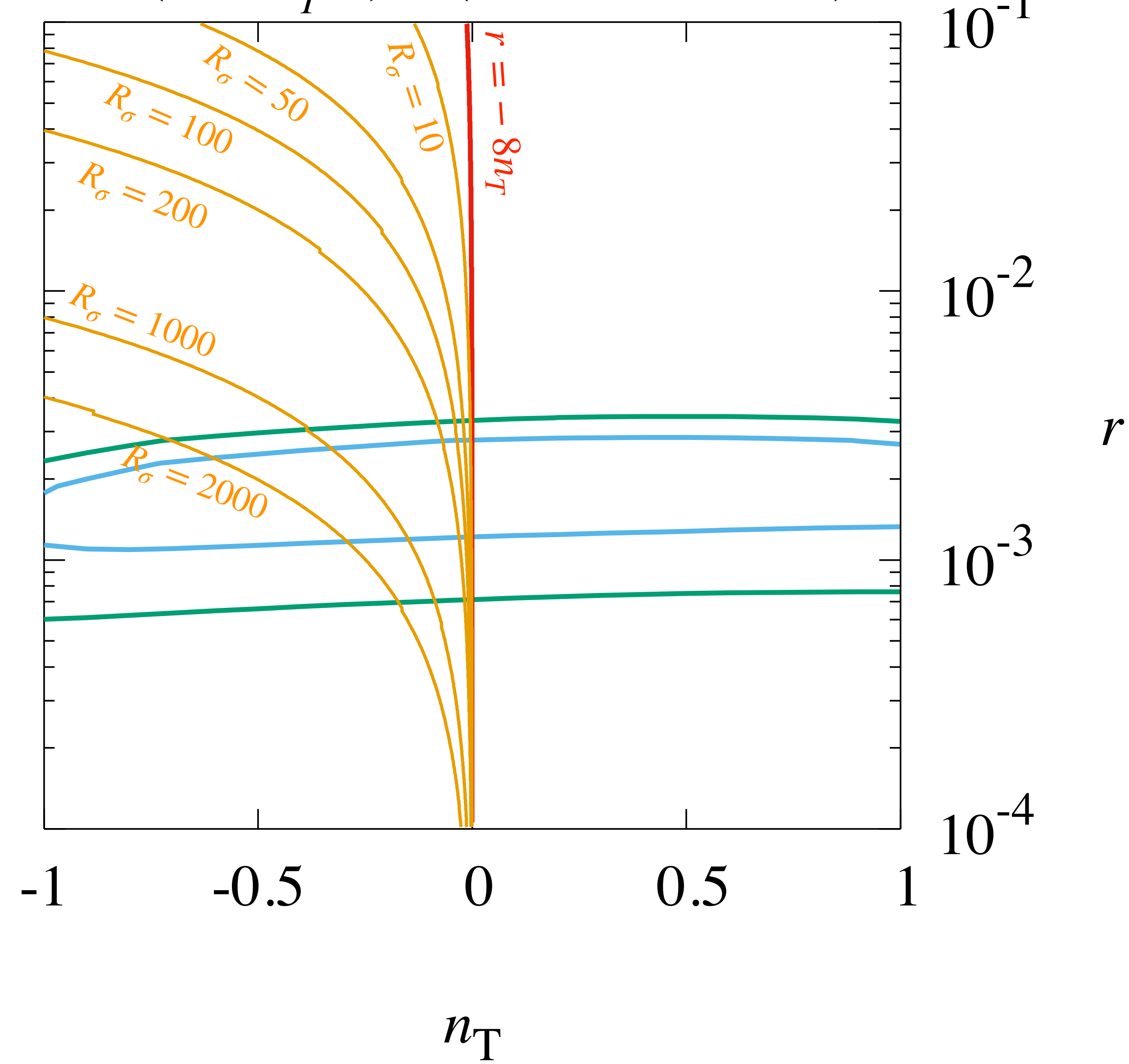
[Jinno, Kohri, Moroi, TT, Hazumi 2310.08158]

- When r is detected, but $r = 0.002$, the constraint on n_T would be very loose…

→ The spectator contribution can still be constrained, but not so severe.

(However, with the information on n_s and some assumption, we could say something (discuss a bit later).)

$(r^{(\mathrm{fid})}, n_T^{(\mathrm{fid})}) = (0.002, -0.00025)$



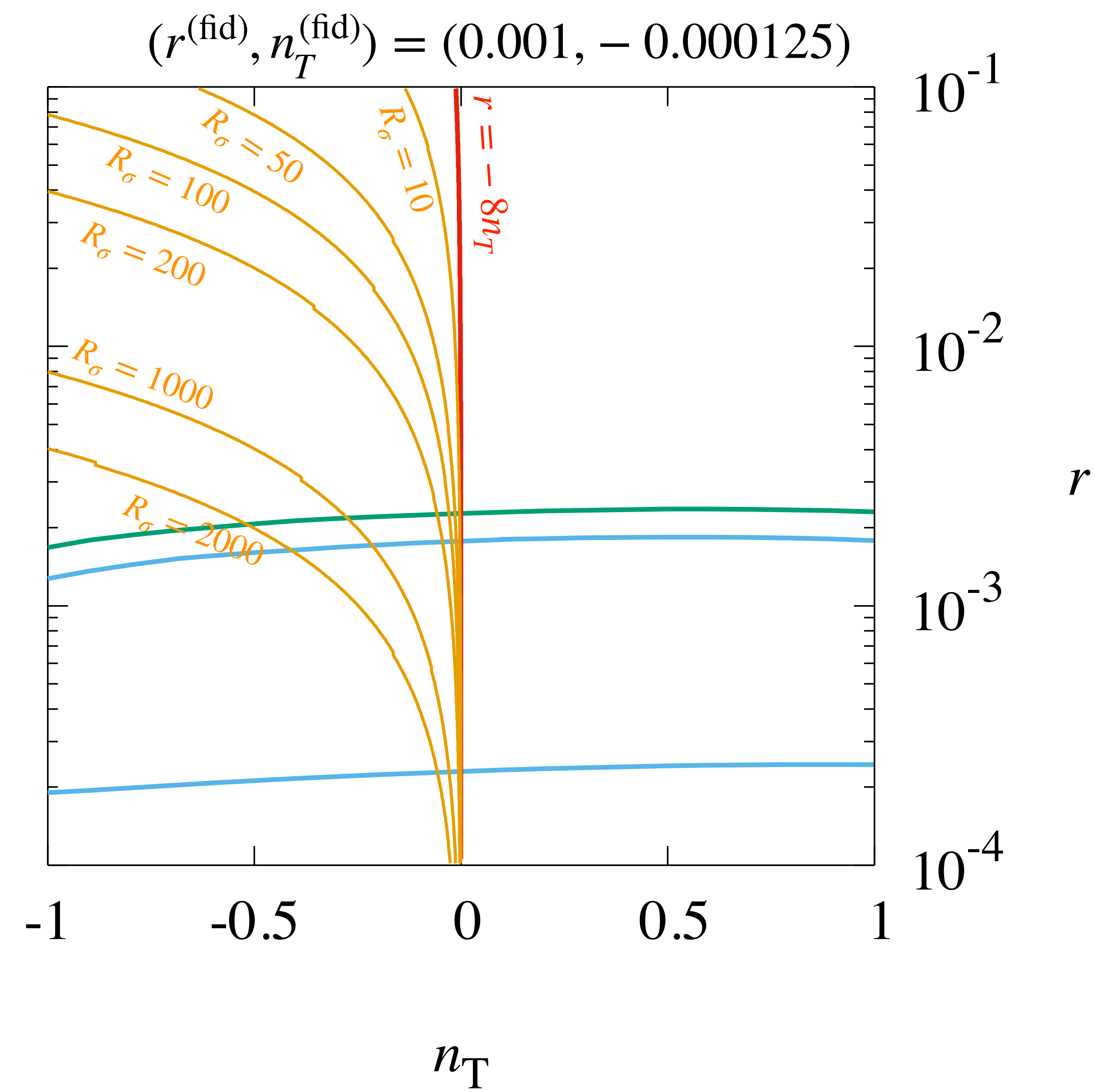
Expected constraints from LiteBIRD

[Jinno, Kohri, Moroi, TT, Hazumi 2310.08158]

- When r is marginally detected, the constraint on n_T is not so useful…

It seems difficult to constrain the spectator contribution.

(However, with the information on n_s and some assumption, we could say something (discuss a bit later).)

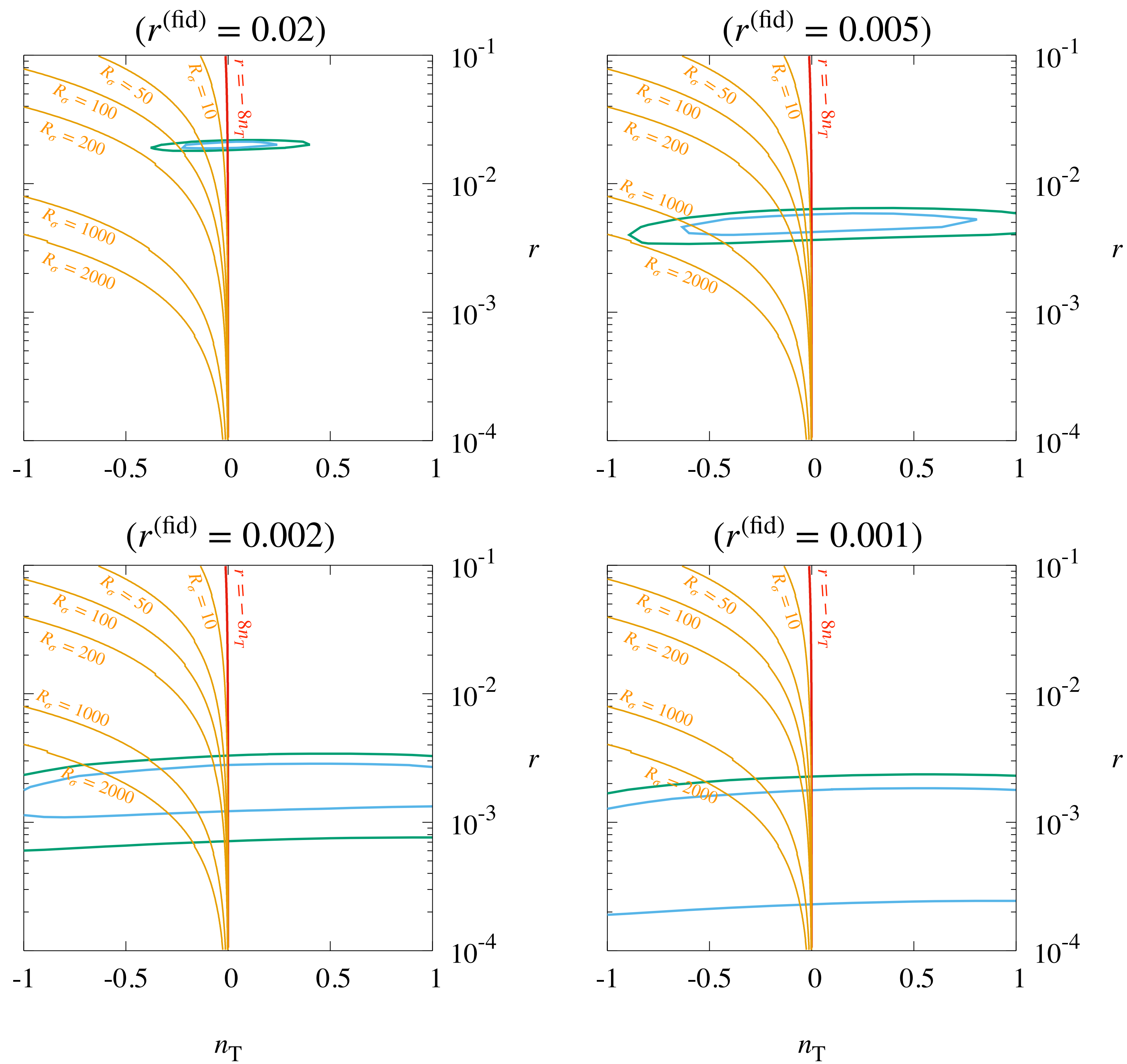


Expected constraints from LiteBIRD

[Jinno, Kohri, Moroi, TT, Hazumi 2310.08158]

- When r is detected with relatively large value, n_T can also be constrained.

→ In such a case, the spectator contribution can also be constrained.

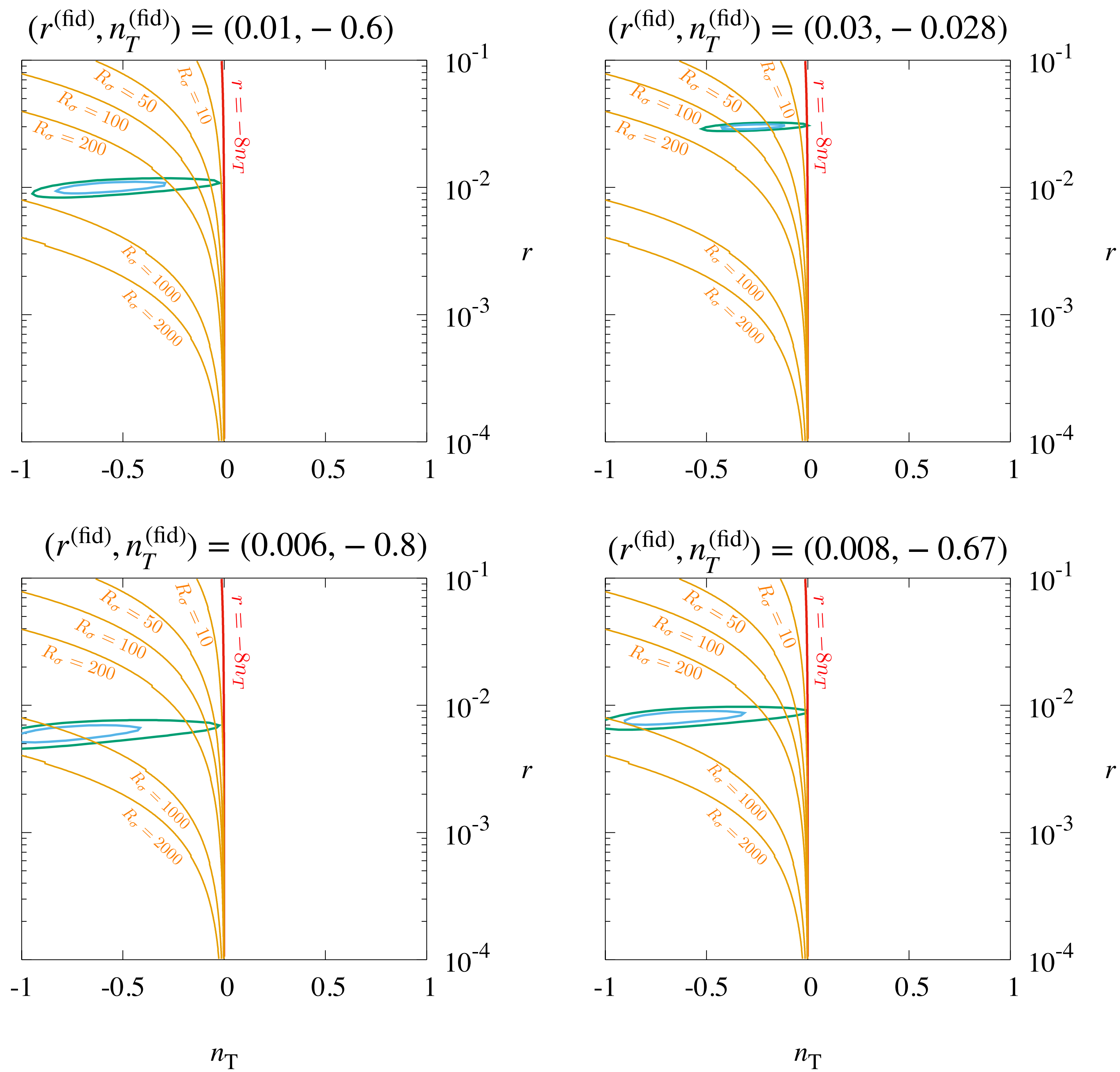


Expected constraints from LiteBIRD

[Jinno, Kohri, Moroi, TT, Hazumi 2310.08158]

- If n_T is found to be relatively large in LiteBIRD, the single-field consistency would be violated.

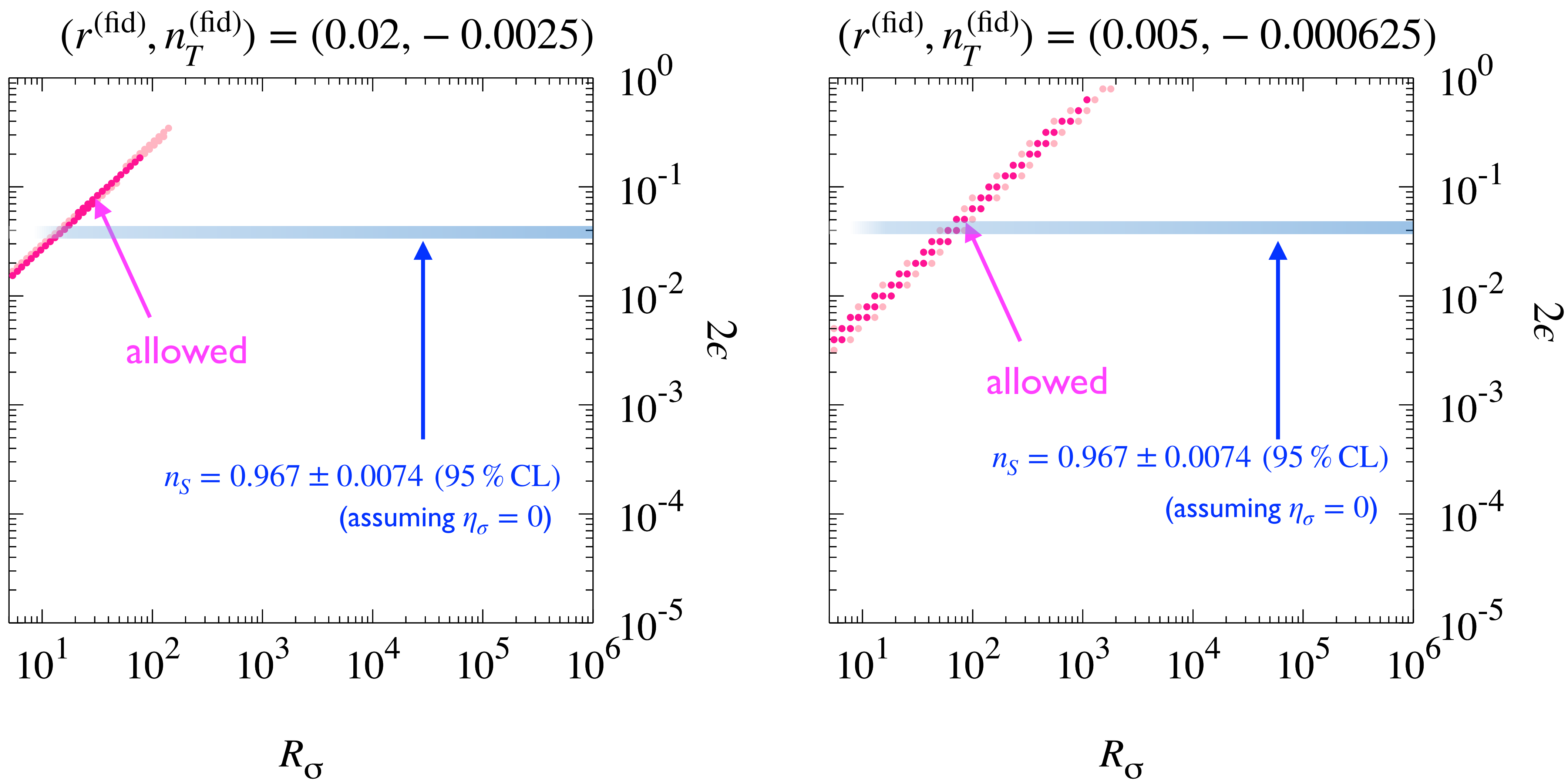
→ The single-field model is excluded.



Expected constraints from LiteBIRD

[Jinno, Kohri, Moroi, TT, Hazumi 2310.08158]

Constraints on r and n_T can be translated into those for ϵ and n_T.



Combining information from r, n_s and n_T , we can probe the spectator parameters.

Expected constraints from LiteBIRD

[Jinno, Kohri, Moroi, TT, Hazumi 2310.08158]

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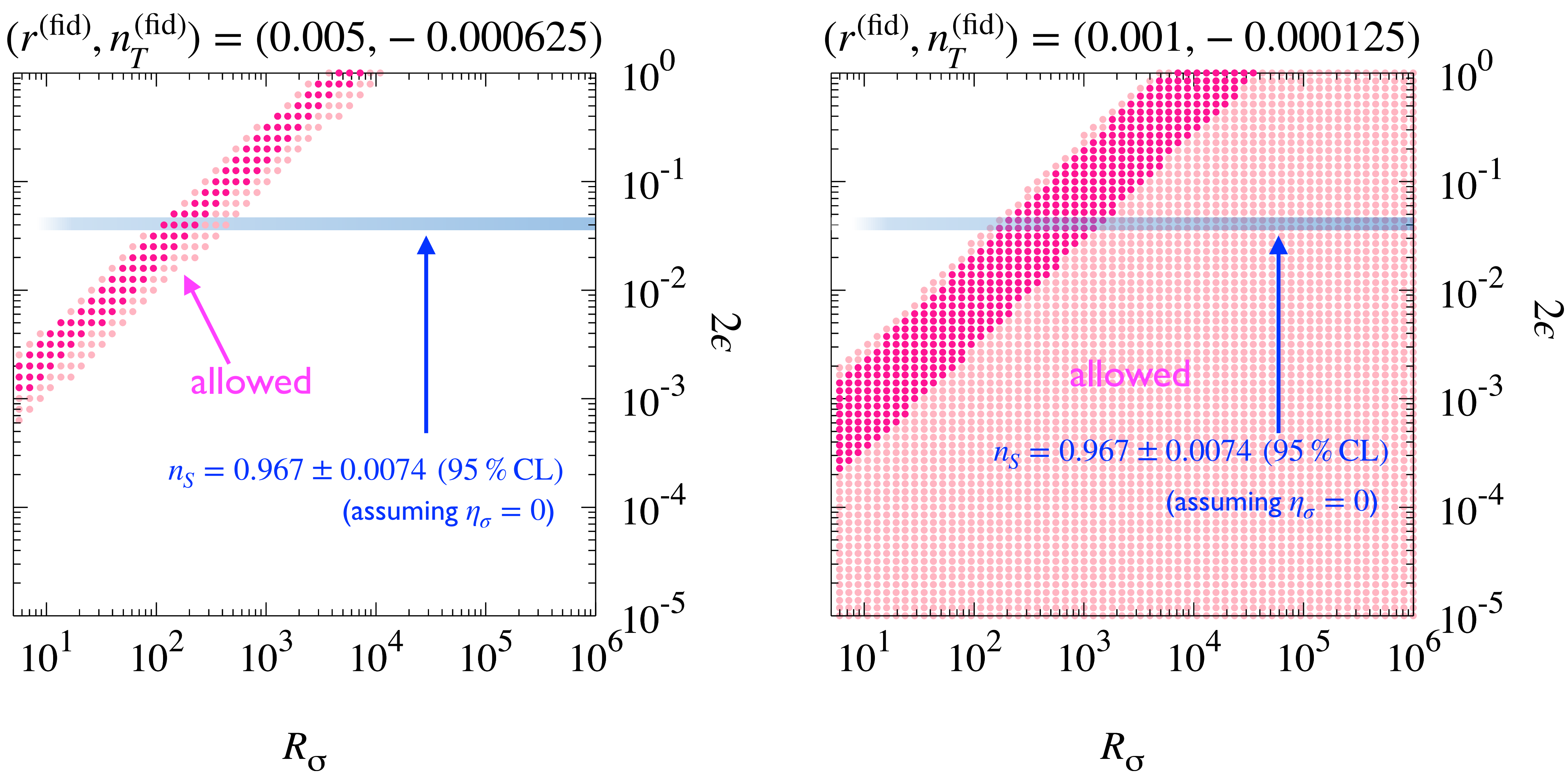

Combining information from r, n_s and n_T , we can probe the spectator parameters.

Summary

- Current cosmological observations now severely constrain the single-field inflation models, but if one considers the multi-field framework, there are still more possibilities for inflation models.

- LiteBIRD can measure/constrain the tensor-to-scalar ratio r, and possibly the tensor spectral index n_T, which can give important information on the consistency relation and multi-field inflation.

- "Multi-dimensional" observational probe can give an important insight on "multi-dimensional" field space for inflation.

- Quantum measure of primordial fluctuations such as quantum discord may give an interesting test for inflationary models.