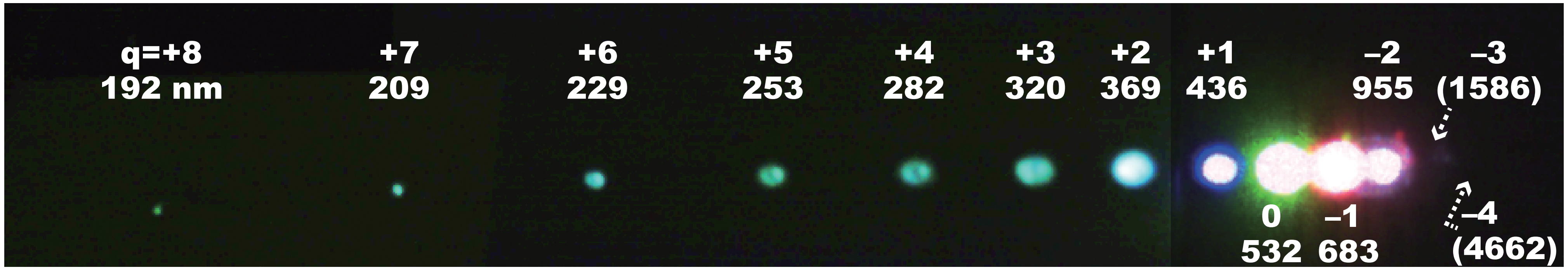
Physics
OSAKA UNIVERSITY

原子・分子過程による
ニュートリノ物理

田中　実

大阪大学



2014/11/19 @ 首都大学東京

SPAN project

SPectroscopy with Atomic Neutrino

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INTRODUCTION

What we know about neutrino mass and mixing

Masses:

$$\Delta m^2_{21} = 7.54 \times 10^{-5}\ \mathrm{eV}^2\,, \quad |\Delta m^2_{31(32)}| = 2.47\ (2.46) \times 10^{-3}\ \mathrm{eV}^2$$

Fogli et al. (2012)

$$\sum m_\nu \leqslant 0.58\ \mathrm{eV}$$ Jarosik et al. (2011)

Mixing: $U = V_{\mathrm{PMNS}}\, P$

$$V_{\mathrm{PMNS}} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}$$

$P = \mathrm{diag.}(1, e^{i\alpha}, e^{i\beta})$ Majorana phases

Bilenky, Hosek, Petcov; Doi, Kotani, Nishiura, Okuda, Takasugi; Schechter, Valle

$$s^2_{12} \simeq 0.31,\ s^2_{23} \simeq 0.39,\ s^2_{13} \simeq 0.024$$

Fogli et al. (2012)

Unknown properties of neutrinos

Absolute mass

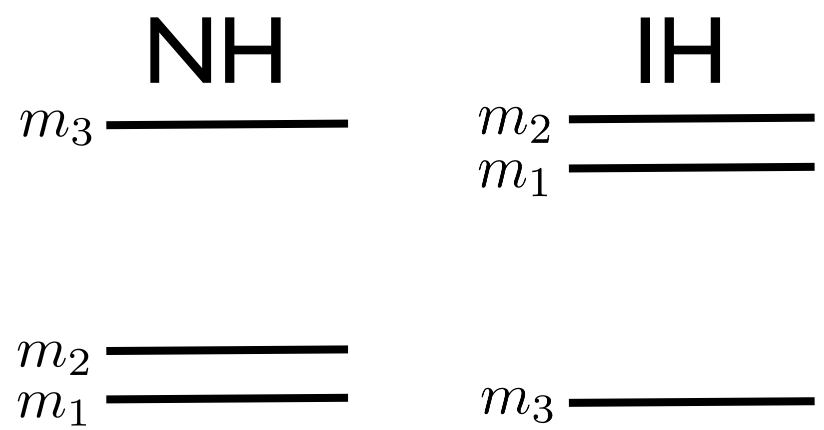
$$m_{1(3)} < 0.19\,\mathrm{eV}\,,\;\; 0.050\,\mathrm{eV} < m_{3(2)} < 0.58\,\mathrm{eV}$$

Mass type

Dirac or Majorana

Hierarchy pattern

normal or inverted



CP violation

one Dirac phase, two Majorana phases

δ $\alpha,\ \beta$

Neutrino experiments

Conventional approach $E \gtrsim O(10\mathrm{keV})$ big science

Neutrino oscillation: SK, T2K, reactors,...

$\Delta m^2,\ \theta_{ij},$ NH or IH, δ

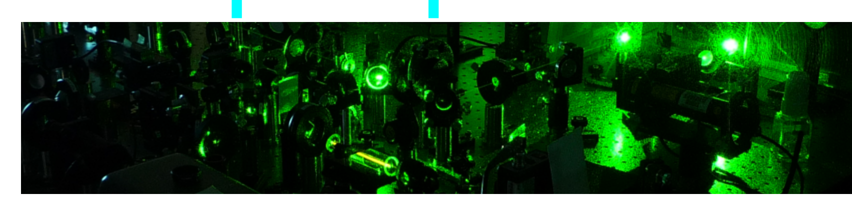
Neutrinoless double beta decays

Dirac or Majorana, effective mass $\left|\sum_i m_i U_{ei}^2\right|^2$

Beta decay endpoint: KATRIN

absolute mass

Our approach $E \lesssim O(\mathrm{eV})$ tabletop experiment



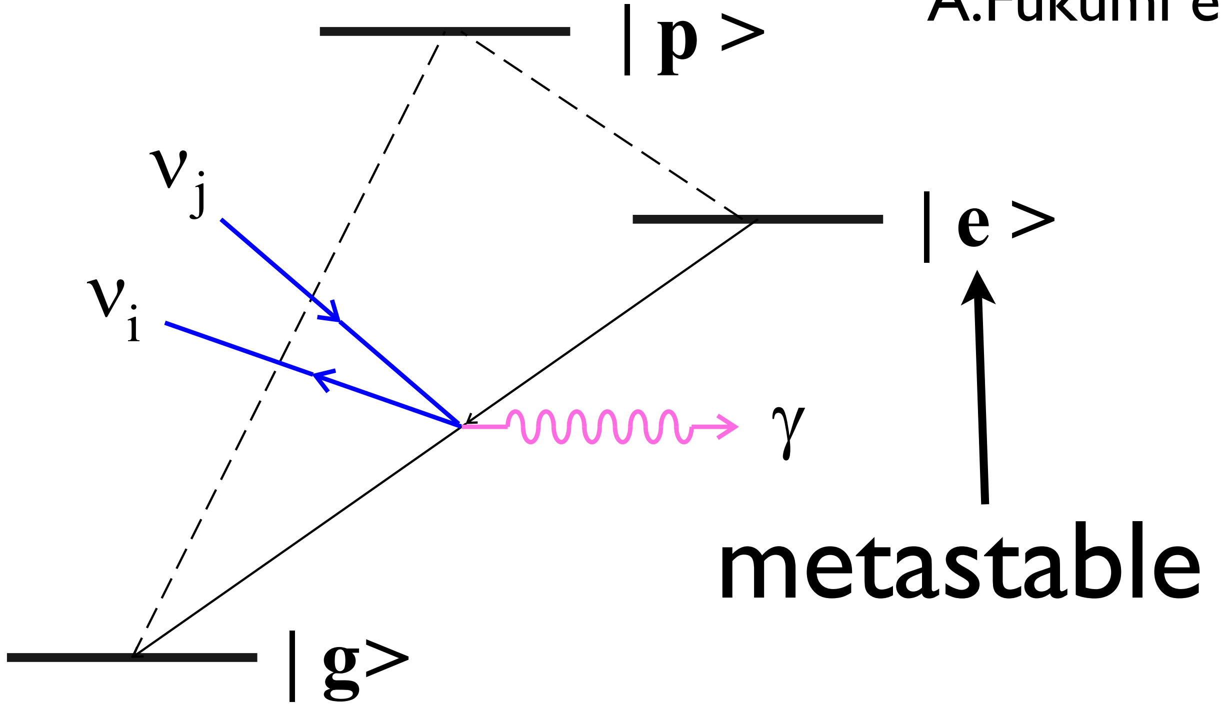
Atomic/molecular processes

absolute mass, NH or IH, D or M, $\delta,\ \alpha,\ \beta$

RENP

Radiative Emission of Neutrino Pair (RENP)

A.Fukumi et al. PTEP (2012) 04D002, arXiv:1211.4904



$$|e\rangle \to |g\rangle + \gamma + \nu_i \bar{\nu}_j$$

Λ-type level structure

Ba, Xe, Ca+, Yb,...

H_2, O_2, I_2, ...

Atomic/molecular energy scale ~ eV or less

close to the neutrino mass scale

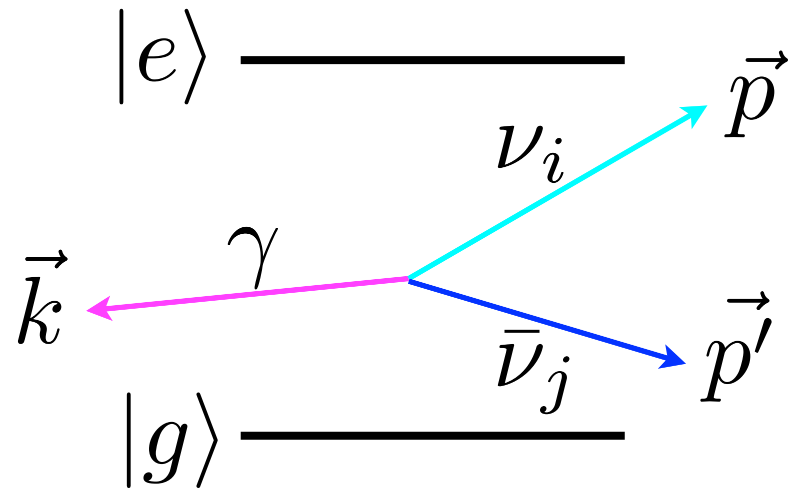
cf. nuclear processes ~ MeV

Rate $\sim \alpha G_F^2 E^5 \sim 1/(10^{33}\,\mathrm{s})$

Enhancement mechanism?

Macrocoherence

Yoshimura et al. (2008)



position of atom

$$\propto e^{-i(\vec{k}+\vec{p}+\vec{p}')\cdot\vec{x}_a}(2\pi)\delta(\epsilon_{eg}-\omega-E_p-E_{p'})$$

$$(\epsilon_{eg}=\epsilon_e-\epsilon_g,\ \omega=|\vec{k}|)$$

Macroscopic target of N atoms, volume V (n=N/V)

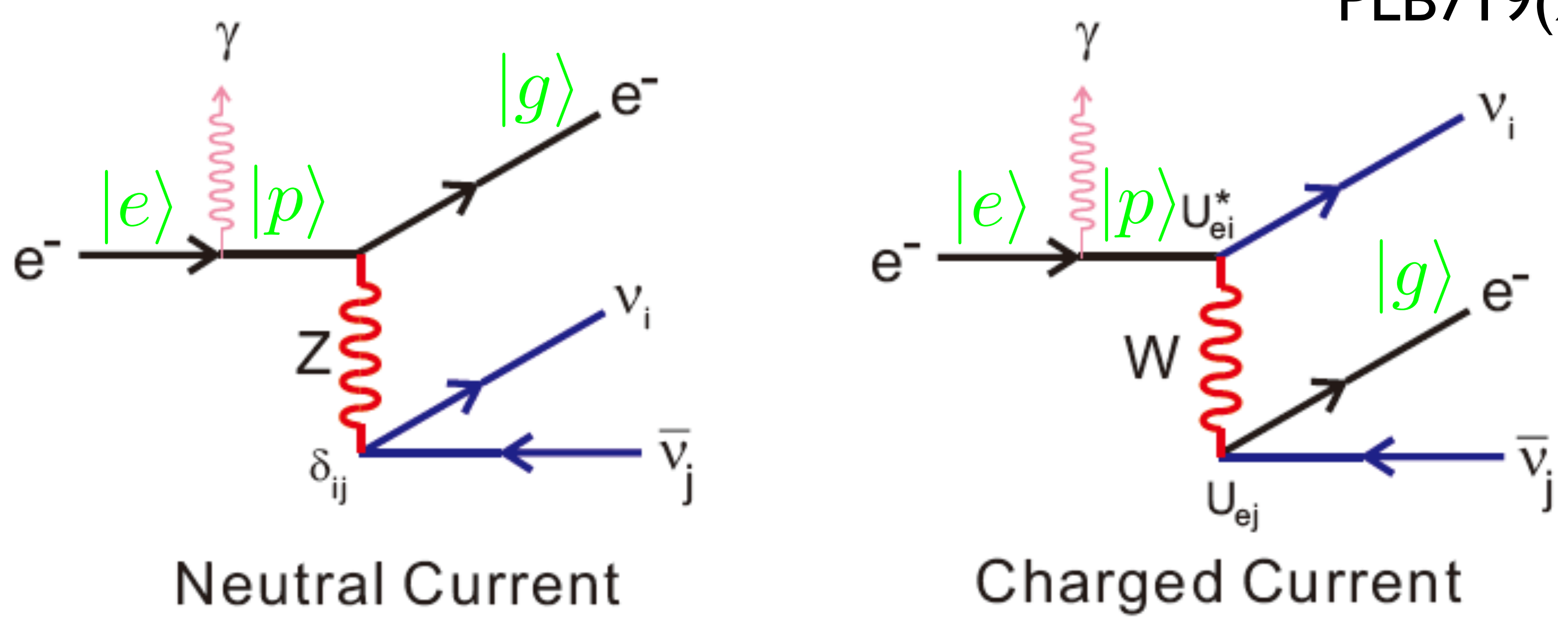
total amp. $\propto \sum_a e^{-i(\vec{k}+\vec{p}+\vec{p}')\cdot\vec{x}_a} \simeq \frac{N}{V}(2\pi)^3\delta^3(\vec{k}+\vec{p}+\vec{p}')$

$$d\Gamma \propto \boxed{n^2 V}(2\pi)^4\delta^4(q-p-p') \qquad q^\mu=(\epsilon_{eg}-\omega,-\vec{k})$$

macrocoherent amplification

Neutrino emission from valence electron

D.N. Dinh, S.T. Petcov, N. Sasao, M.T., M.Yoshimura
PLB719(2013)154, arXiv:1209.4808



$$\mathcal{H}_W = \frac{G_F}{\sqrt{2}} \sum_{i,j} \bar{\nu}_j \gamma_\mu (1-\gamma_5)\nu_i \, \bar{e}\gamma^\mu (C^V_{ji} - C^A_{ji}\gamma_5) e$$

$$C^V_{ji} = U^*_{ej}U_{ei} + (-1/2 + 2\sin^2\theta_W)\delta_{ji}, \;\; C^A_{ji} = U^*_{ej}U_{ei} - \delta_{ji}/2$$

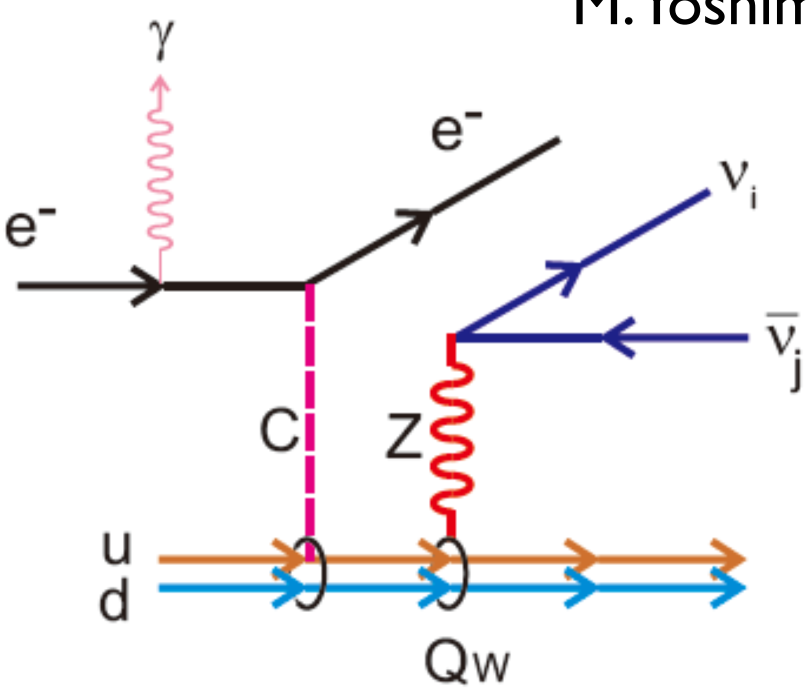
Atomic matrix element in the NR approximation

$$\langle g|\bar{e}\gamma^\mu e|p\rangle \simeq (\langle g|e^\dagger e|p\rangle, \mathbf{0}) = 0$$

$$\langle g|\bar{e}\gamma^\mu \gamma_5 e|p\rangle \simeq (0, 2\langle g|\boldsymbol{s}|p\rangle)$$ → spin current

Neutrino emission from nucleus

M.Yoshimura and N. Sasao, PRD89, 053013(2014), arXiv:1310.6472



flavor diagonal

no PMNS

weak charge: $Q_W \simeq -(\#\text{ of neutrons})$

cf. atomic parity violation

$$\mathcal{H}_W = 4\frac{G_F}{\sqrt{2}}\sum_{i,q}\bar{\nu}_i\gamma_\mu(1-\gamma_5)\nu_i\,\bar{q}\gamma_\mu(v_q - a_q\gamma_5)q$$

Nuclear matrix element in the NR limit

$$\langle N|\sum_q 4v_q\bar{q}\gamma^\mu q|N\rangle \simeq (Q_W, \mathbf{0})$$

nuclear monopole $\propto Q_W^2 Z^{8/3}$ enhancement

RENP spectrum

Energy-momentum conservation
due to the macro-coherence
→ familiar 3-body decay kinematics

Six (or three) thresholds of the photon energy

$$\omega_{ij} = \frac{\epsilon_{eg}}{2} - \frac{(m_i + m_j)^2}{2\epsilon_{eg}} \qquad i, j = 1, 2, 3$$

$\epsilon_{eg} = \epsilon_e - \epsilon_g$ atomic energy diff.

Required energy resolution $\sim O(10^{-6})\,\mathrm{eV}$

typical laser linewidth

$$\Delta\omega_{\mathrm{trig.}} \lesssim 1\,\mathrm{GHz} \sim O(10^{-6})\,\mathrm{eV}$$

RENP rate formula

dynamical factor

$$\Gamma_{\gamma 2\nu}(\omega, t) = \Gamma_0 I(\omega)\eta_\omega(t)$$

spectral function

overall rate

Overall rate

macro-coherence

~ field energy density

$$\Gamma_0^{\rm SC} \sim \frac{3n^2 V G_F^2 \gamma_{pg}\epsilon_{eg} n}{2\epsilon_{pg}^3} \sim 1\ {\rm mHz}\,(n/10^{21}{\rm cm}^{-3})^3(V/10^2{\rm cm}^3)$$

$\gamma_{pg} : |p\rangle \to |g\rangle$ **rate**

$$\Gamma_0^M \sim Q_W^2 Z^{8/3} \times \Gamma_0^S \sim 100\ {\rm kHz}$$

Spectral function (spin current)

$$I(\omega) = F(\omega)/(\epsilon_{pg} - \omega)^2$$

$$F(\omega) = \sum_{ij} \Delta_{ij}(B_{ij} I_{ij}(\omega) - \delta_M B_{ij}^M m_i m_j)\theta(\omega_{ij} - \omega)$$

$$\Delta_{ij}^2 = 1 - 2\frac{m_i^2 + m_j^2}{q^2} + \frac{(m_i^2 - m_j^2)^2}{q^4} \qquad q^2 = (p_i + p_j)^2$$

$$I_{ij}(\omega) = \frac{q^2}{6}\left[2 - \frac{m_i^2 + m_j^2}{q^2} - \frac{(m_i^2 - m_j^2)^2}{q^4}\right] + \frac{\omega^2}{9}\left[1 + \frac{m_i^2 + m_j^2}{q^2} - 2\frac{(m_i^2 - m_j^2)^2}{q^4}\right]$$

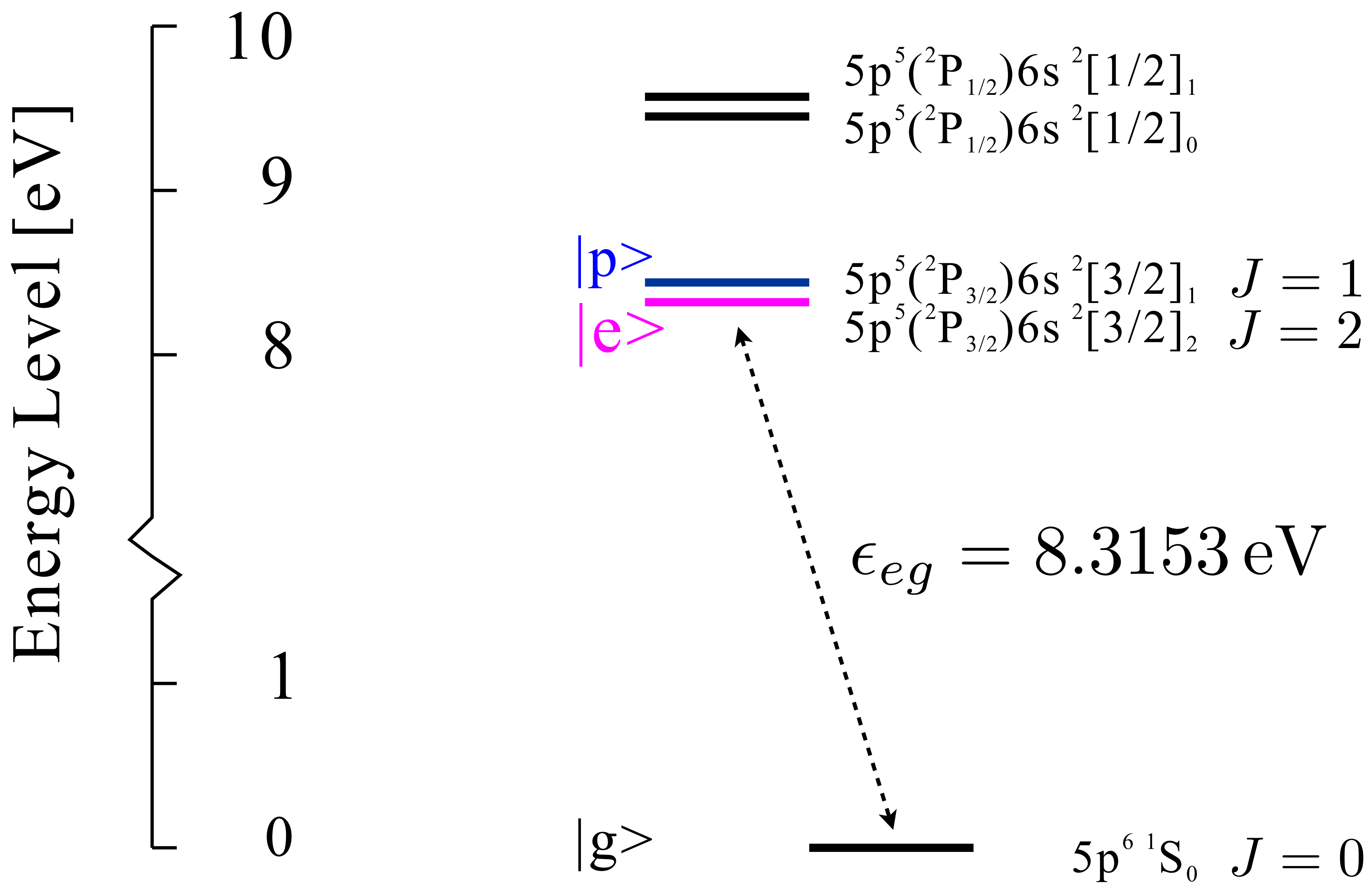
$\delta_M = 0(1)$ for Dirac(Majorana)

$$B_{ij} = |U_{ei}^* U_{ej} - \delta_{ij}/2|^2, B_{ij}^M = \Re[(U_{ei}^* U_{ej} - \delta_{ij}/2)^2]$$

Dynamical factor

$$\sim |\text{coherence} \times \text{field}|^2$$

Xe (gas target)

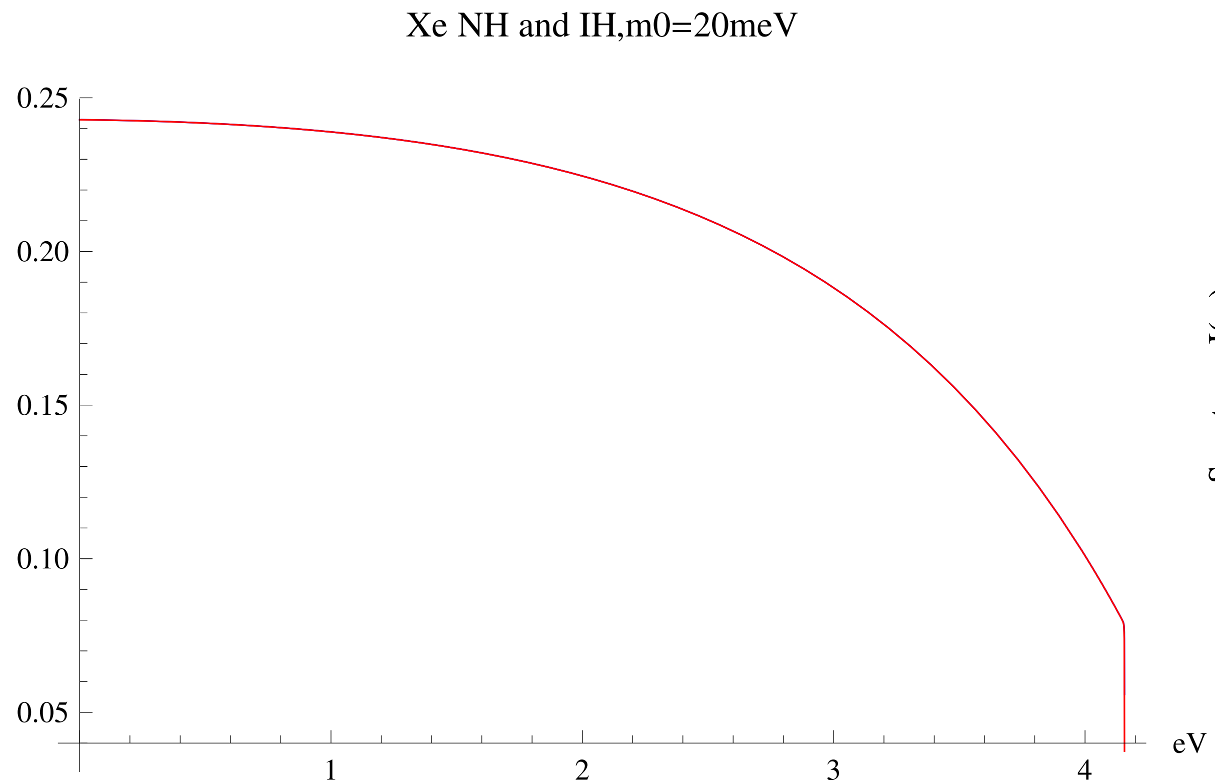


$$|e\rangle \leftrightarrow |p\rangle \quad \mathrm{M1}$$

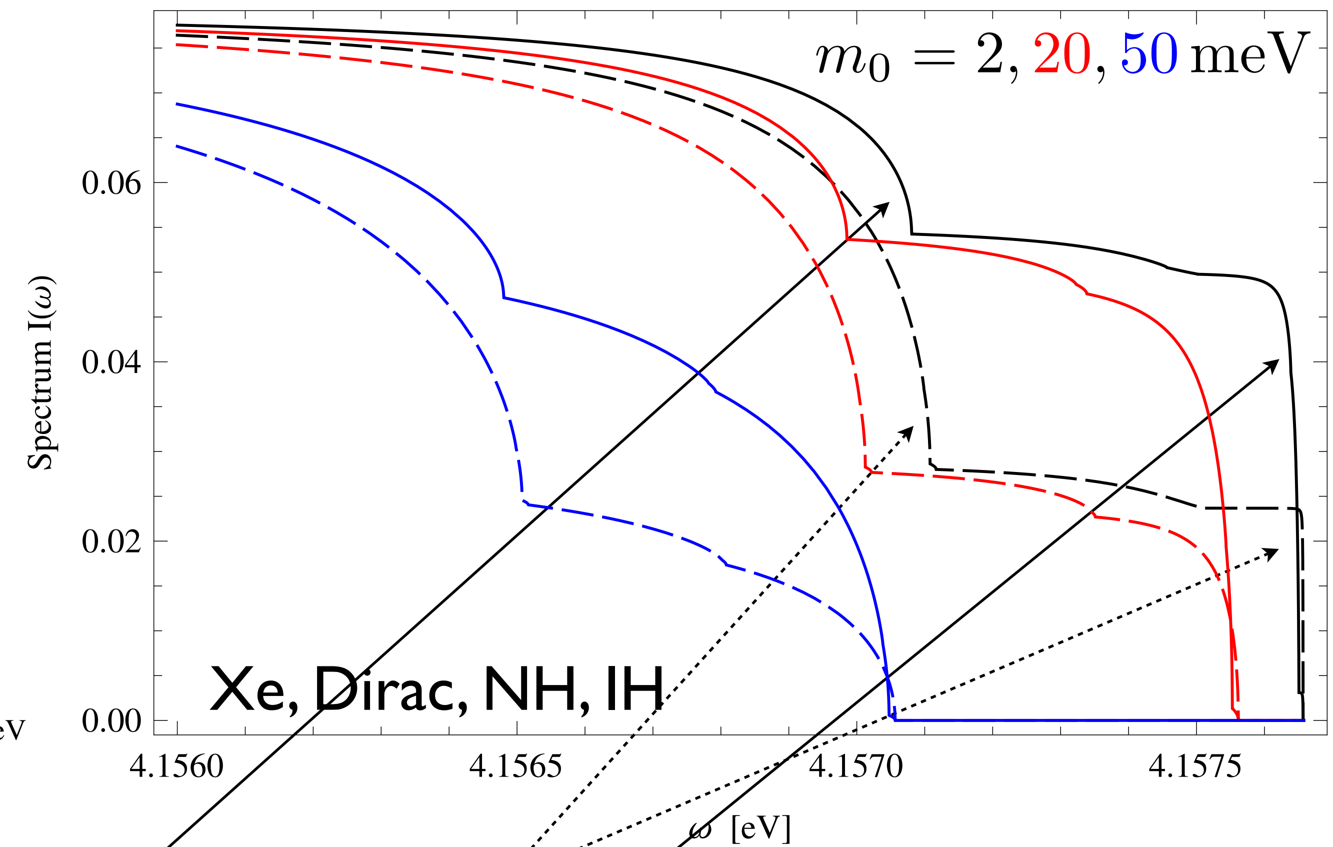
$$|p\rangle \leftrightarrow |g\rangle \quad \mathrm{E1}$$

Photon spectrum (spin current)

Global shape



Threshold region



The threshold weight factors

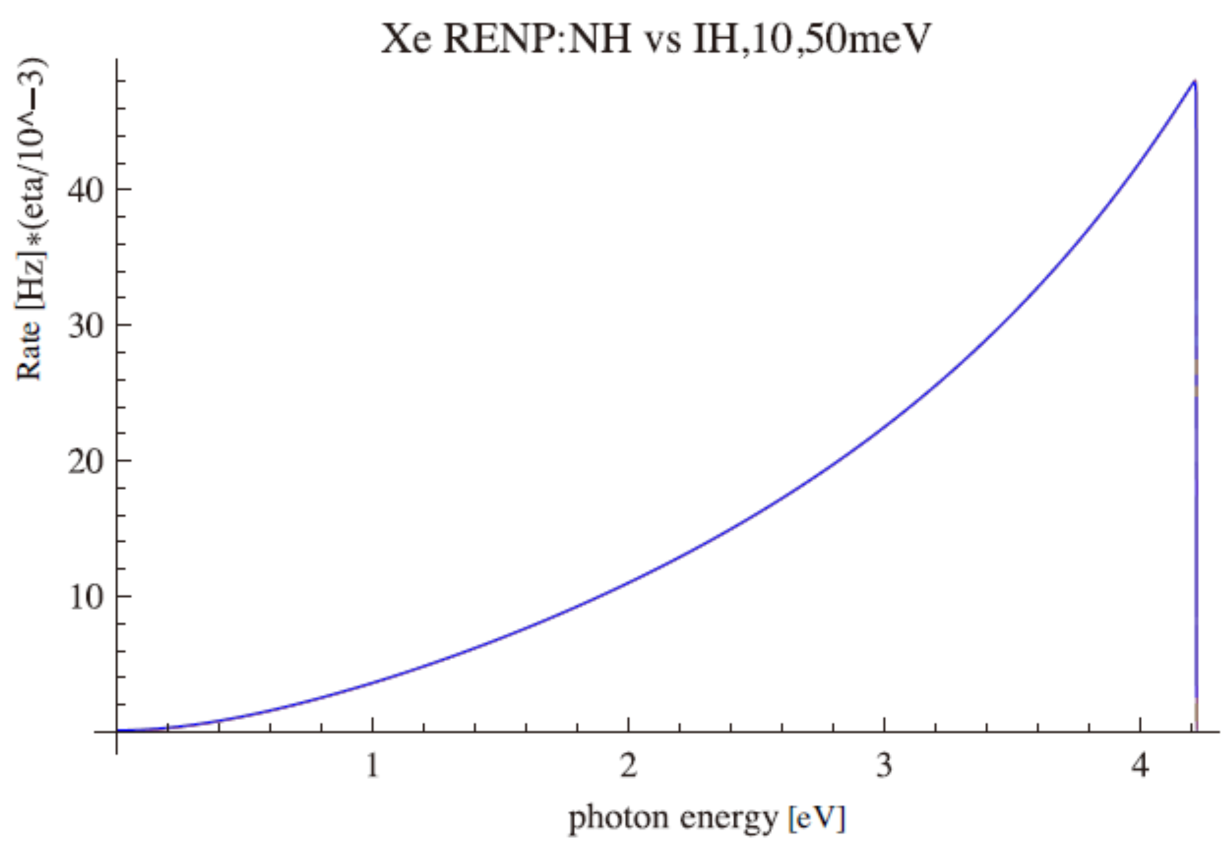
B_{11}	B_{22}	B_{33}	$B_{12}+B_{21}$	$B_{23}+B_{32}$	$B_{31}+B_{13}$
$(c_{12}^2 c_{13}^2 - 1/2)^2$	$(s_{12}^2 c_{13}^2 - 1/2)^2$	$(s_{13}^2 - 1/2)^2$	$2c_{12}^2 s_{12}^2 c_{13}^4$	$2s_{12}^2 c_{13}^2 s_{13}^2$	$2c_{12}^2 c_{13}^2 s_{13}^2$
0.0311	0.0401	0.227	0.405	0.0144	0.0325

Photon spectrum (nuclear monopole)

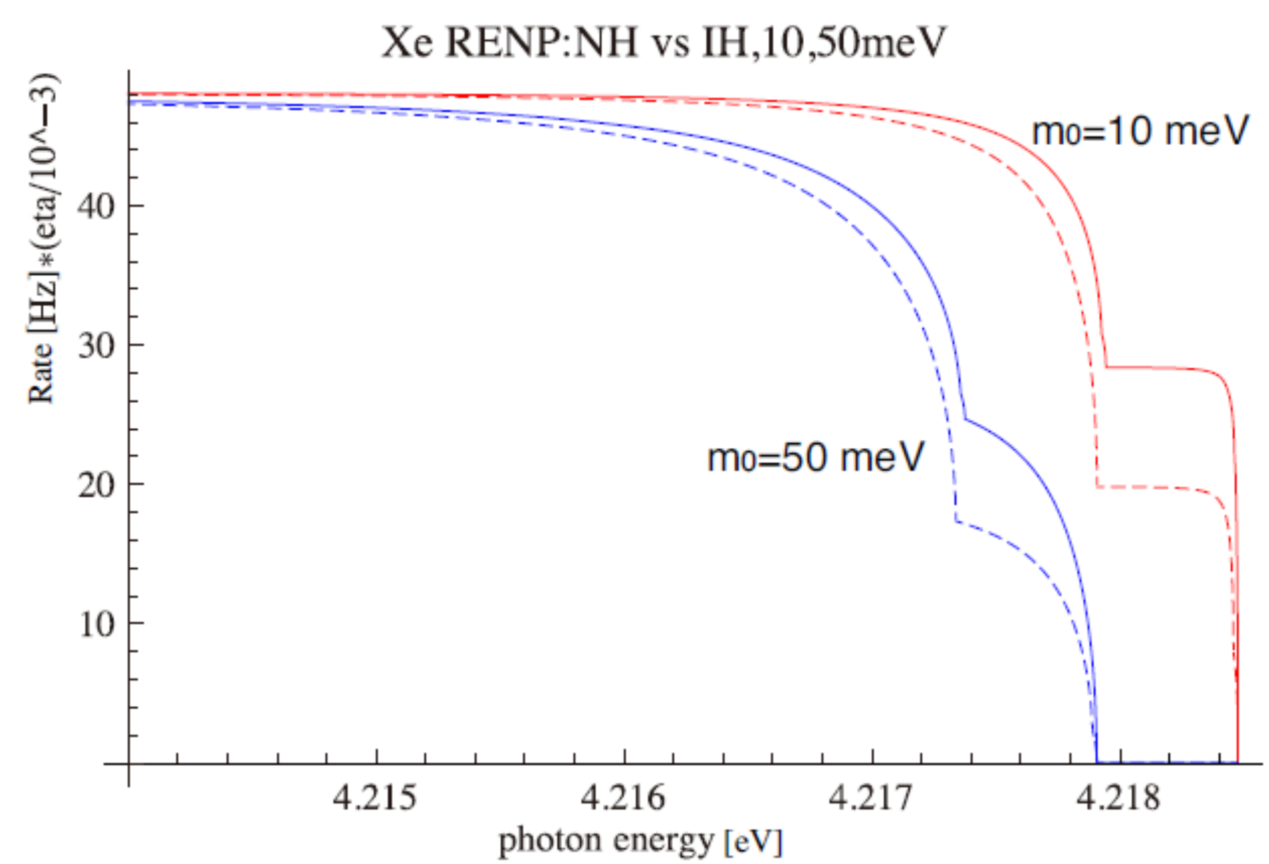
Xe $^3\mathrm{P}_1$ 8.4365 eV

$$n = 7 \times 10^{19}\ \mathrm{cm}^{-3} \quad V = 100\ \mathrm{cm}^3$$

Global shape

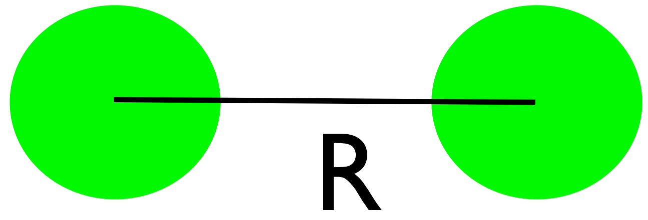


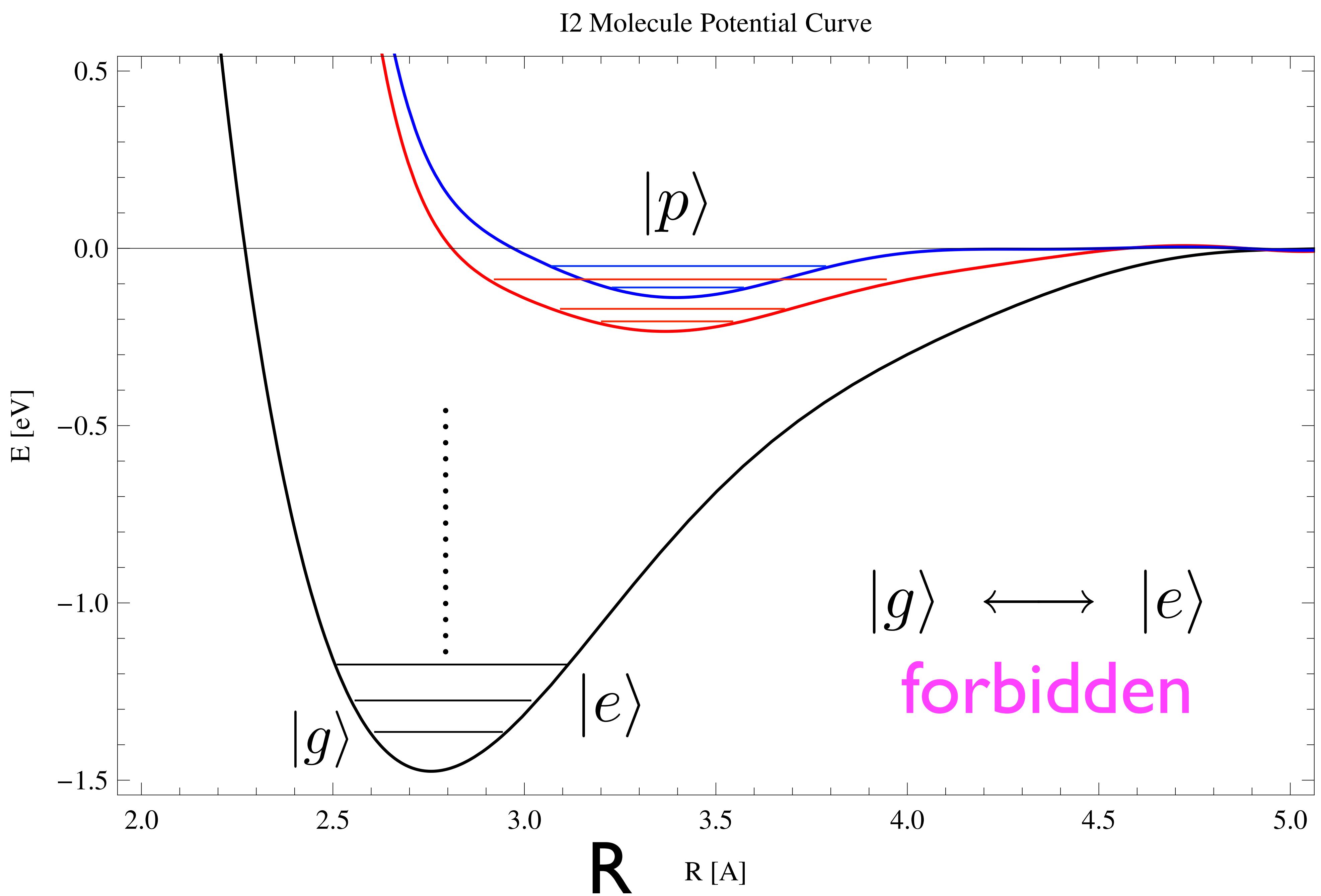
Threshold region



Homonuclear diatomic molecule

Potential curves

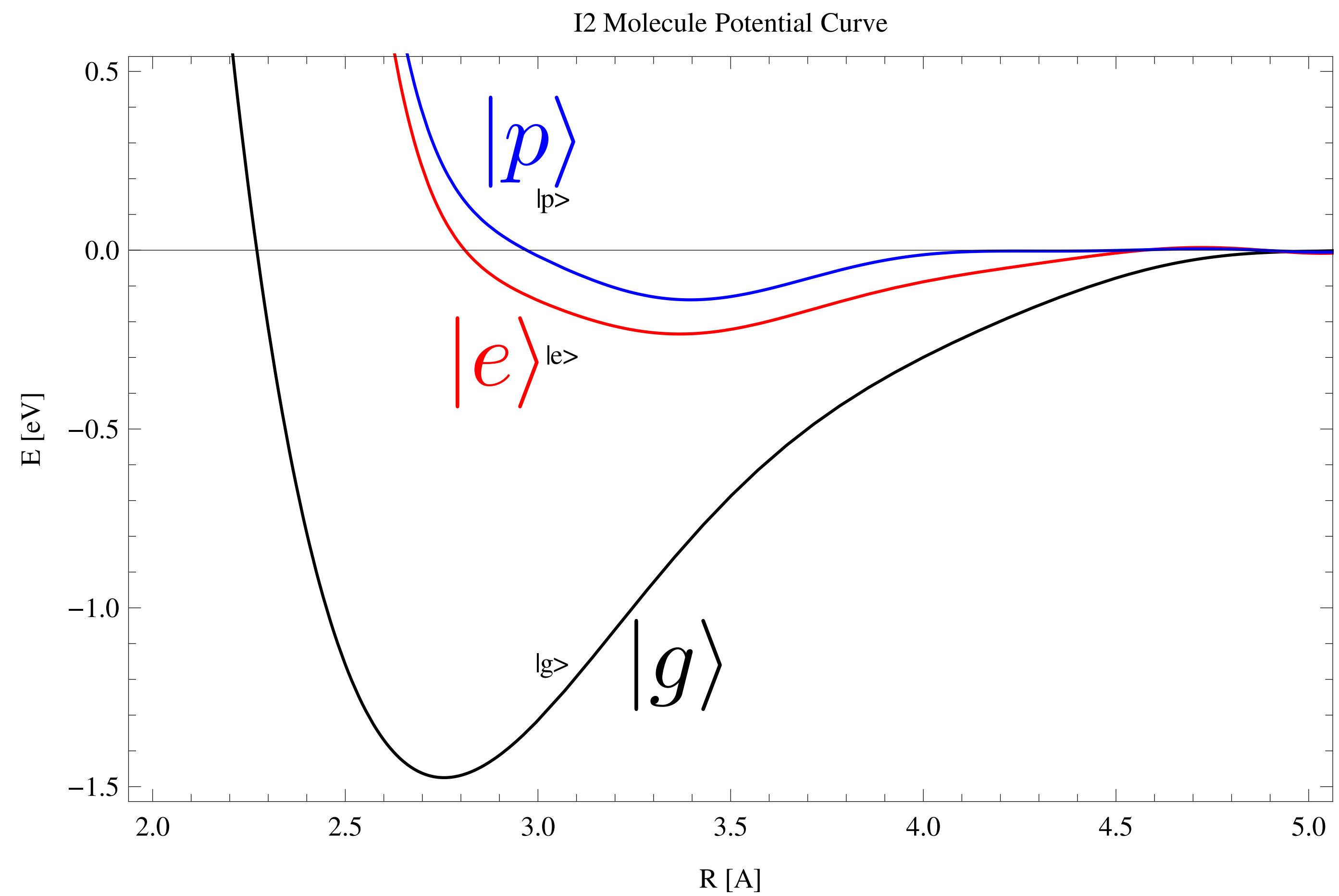


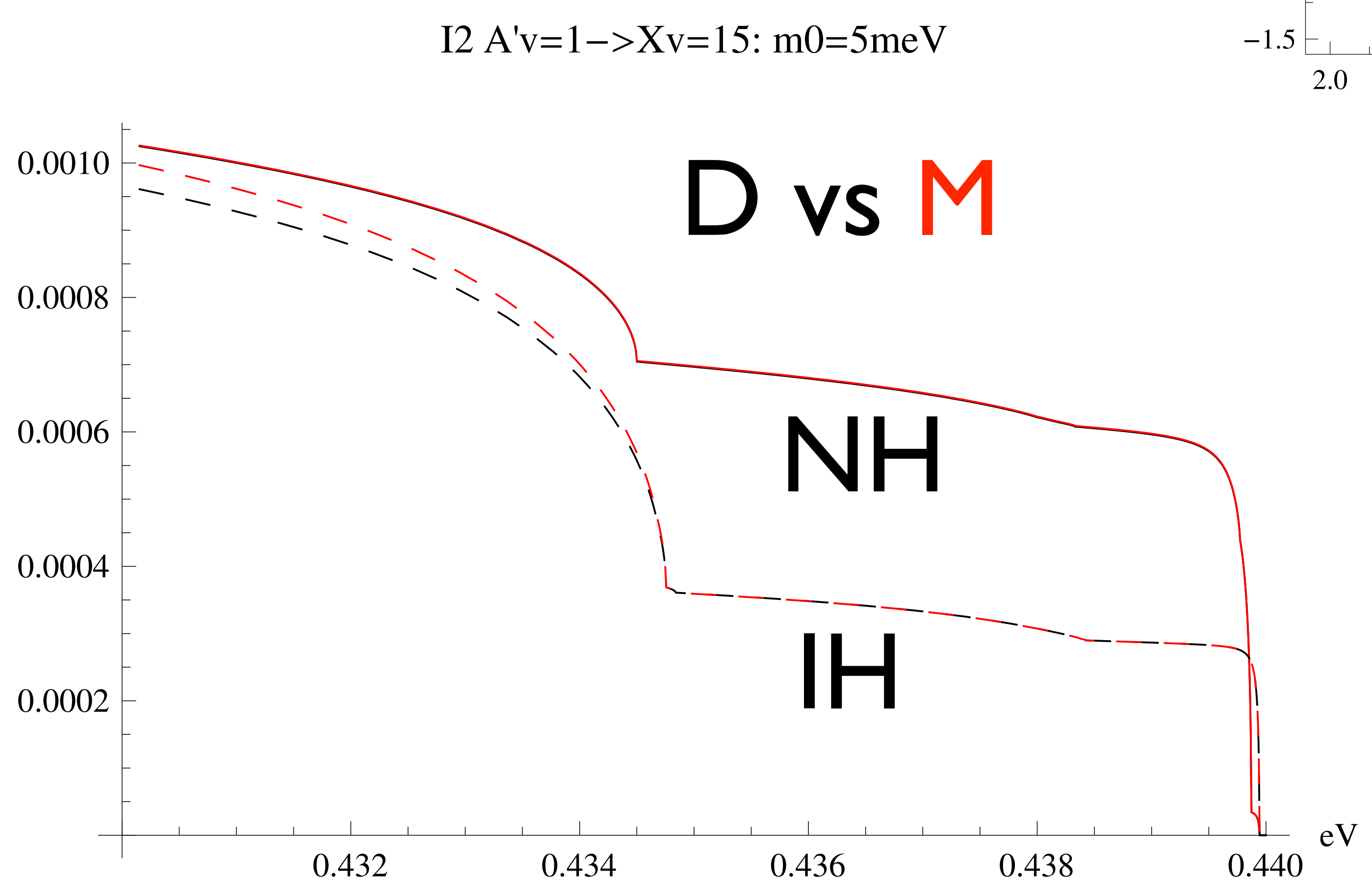


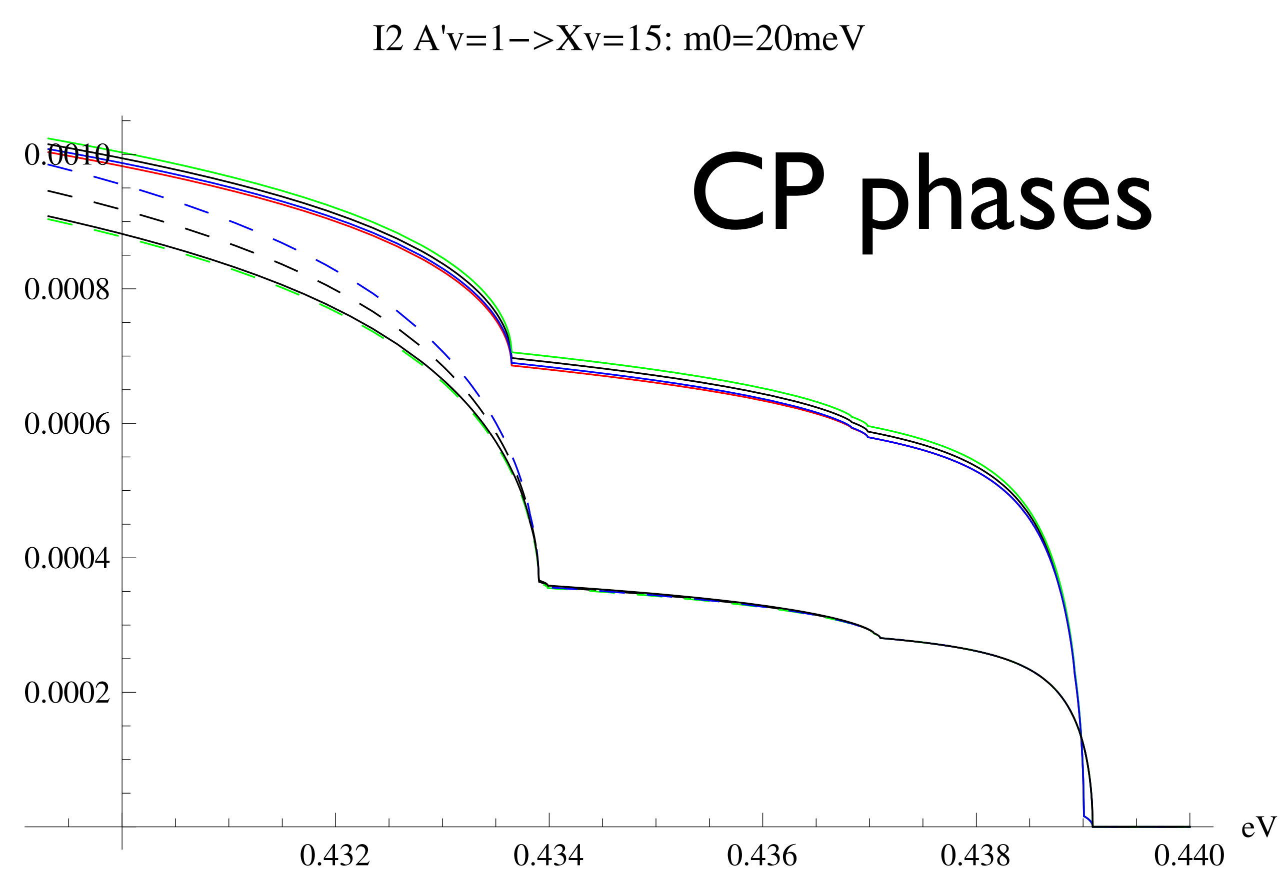
I2 molecule

potential curves

$\epsilon_{eg} \sim 1\,\mathrm{eV}$







D-M diff. < 10%

CNB

Cosmic Neutrino Background (CNB)

Big bang cosmology

Standard model of particle physics

→ CNB

CNB at present: $f(\boldsymbol{p}) = [\exp(|\boldsymbol{p}|/T_\nu - \xi) + 1]^{-1}$

(not) Fermi-Dirac dist. $|\boldsymbol{p}| = \sqrt{E^2 - m_\nu^2}$

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma \simeq 1.945 \text{ K} \simeq 0.17 \text{ meV}$$

→ $n_\nu \simeq 6 \times 56 \text{ cm}^{-3}$ Detection?

RENP in CNB

$$|e\rangle \to |g\rangle + \gamma + \nu_i \bar{\nu}_j$$

Pauli exclusion

$$d\Gamma \propto |\mathcal{M}|^2 \left[1 - f_i(p)\right] \left[1 - \bar{f}_j(p')\right]$$

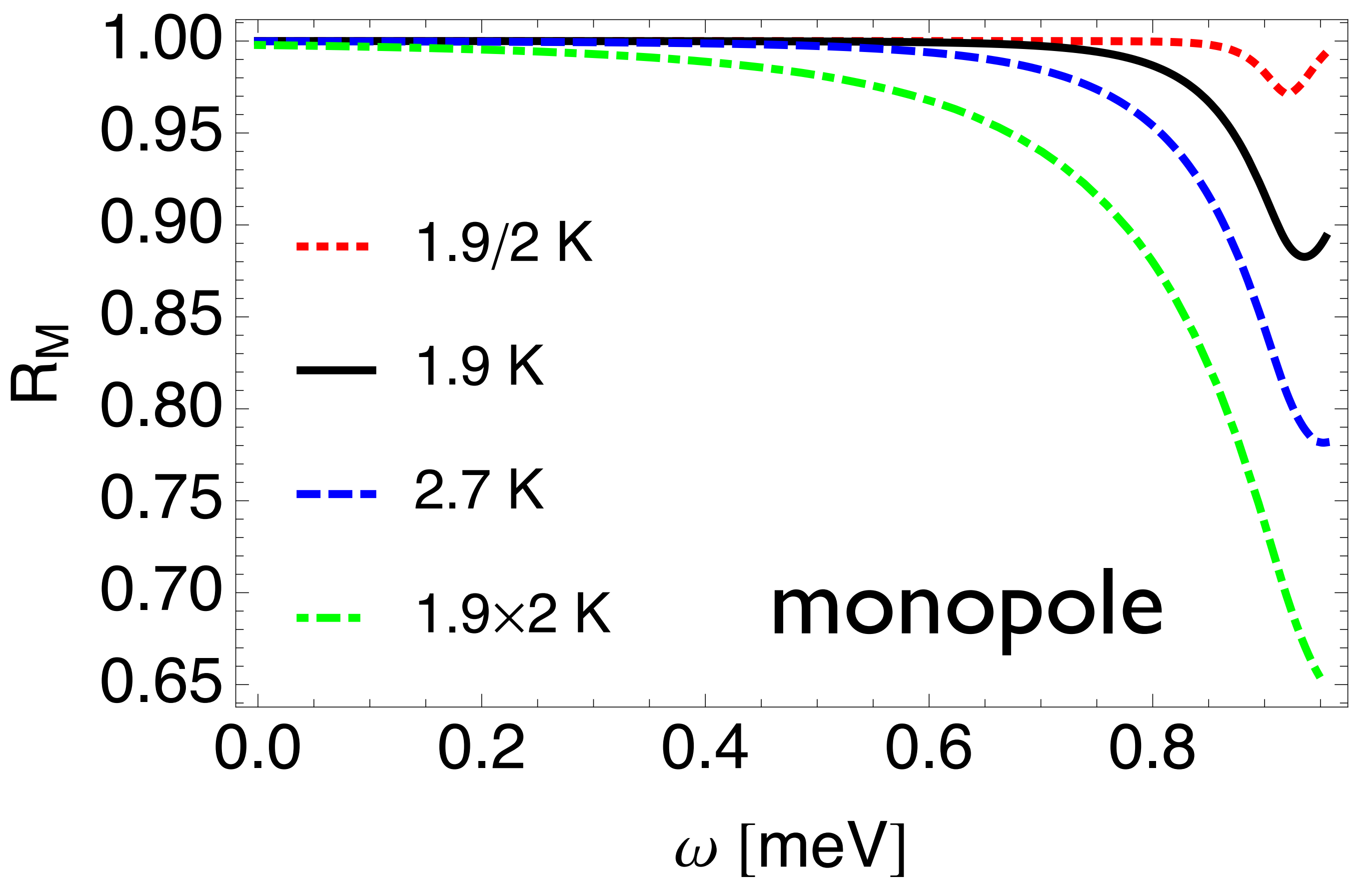
→ spectral distortion

Distortion factor

$$R_X(\omega) \equiv \frac{\Gamma_X(\omega, T_\nu)}{\Gamma_X(\omega, 0)}$$

$$X = \begin{cases} M & \text{nuclear monopole} \\ S & \text{valence } e \text{ spin current} \end{cases}$$

larger rate $i = j$



level splitting

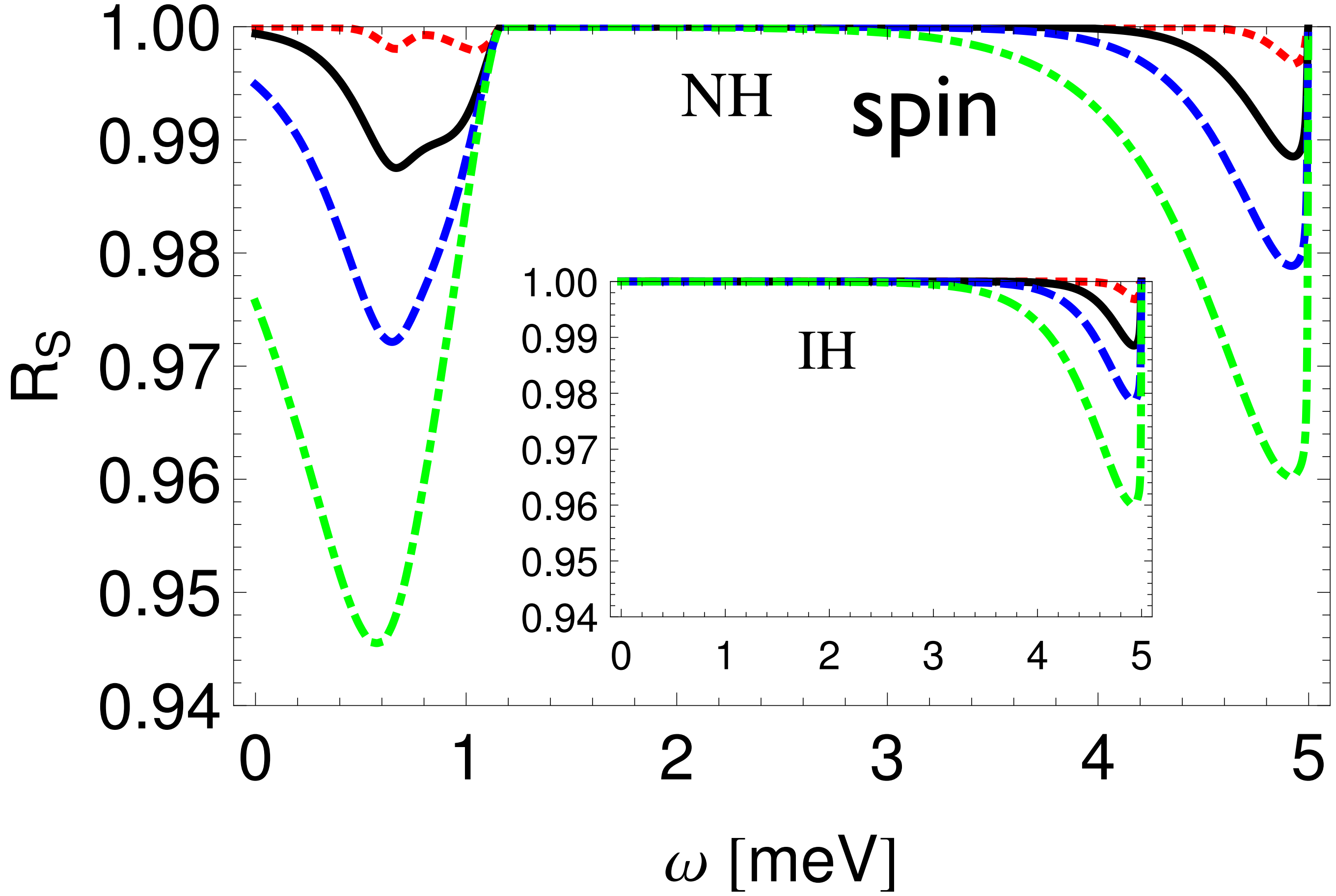
$\epsilon_{eg} = 11 \text{ meV}$

smallest neutrino mass

$m_0 = 5 \text{ meV}$

chemical potential

$\xi_i \equiv \mu_i / T_\nu = 0$

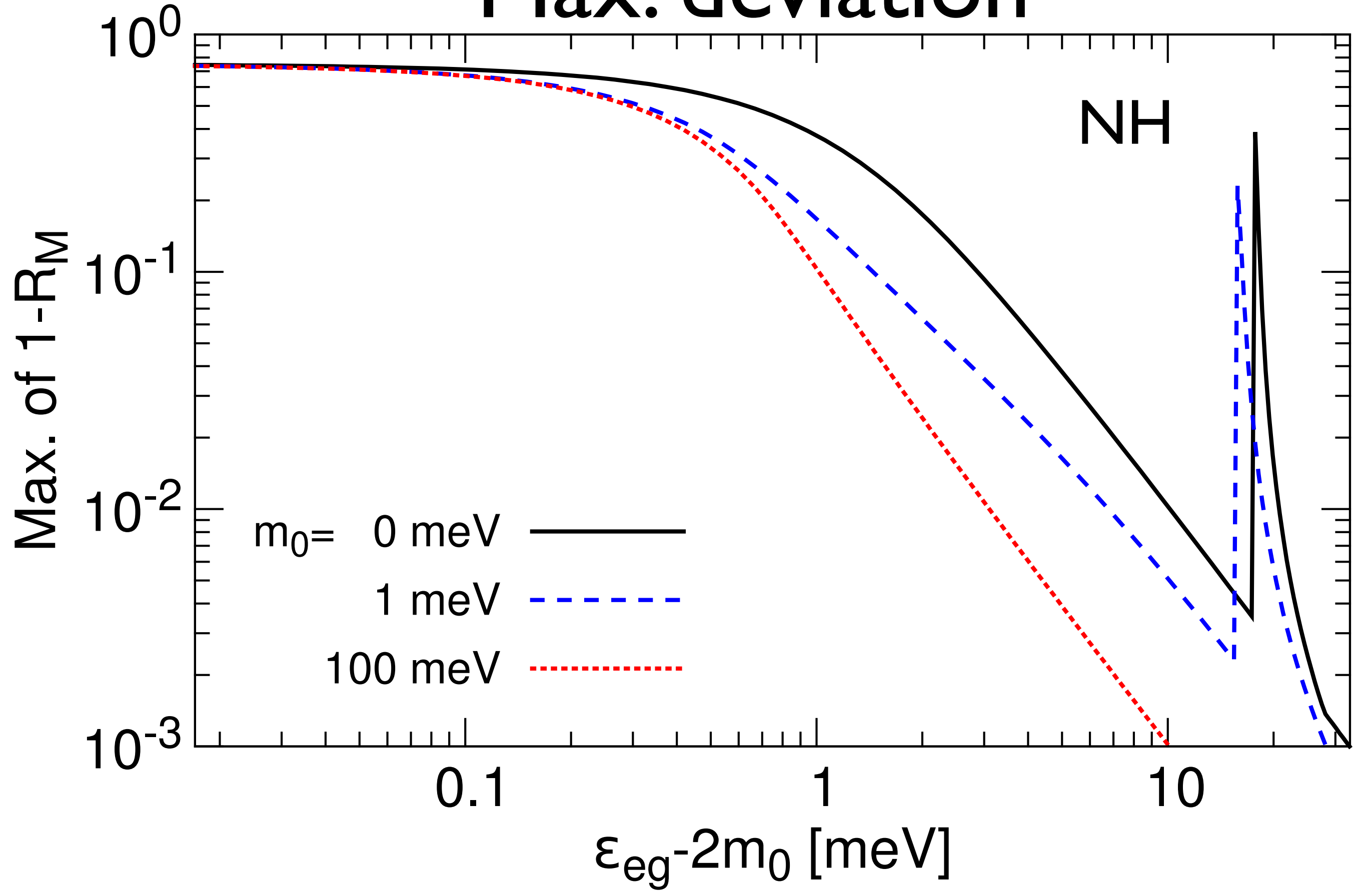


$\epsilon_{eg} = 1 \text{ meV}$

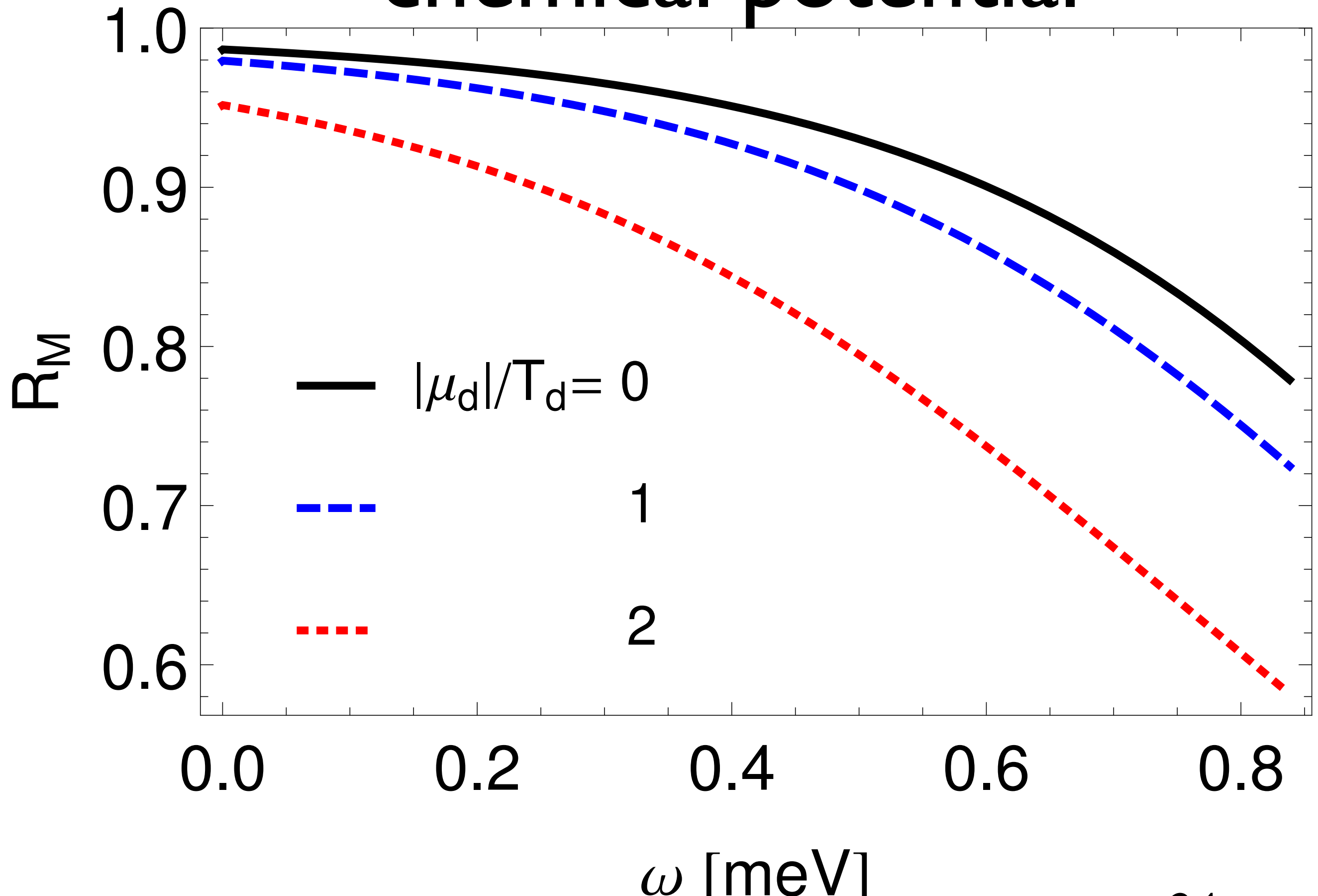
$m_0 = 0.1 \text{ meV}$

$\xi_i = 0$

Max. deviation



chemical potential



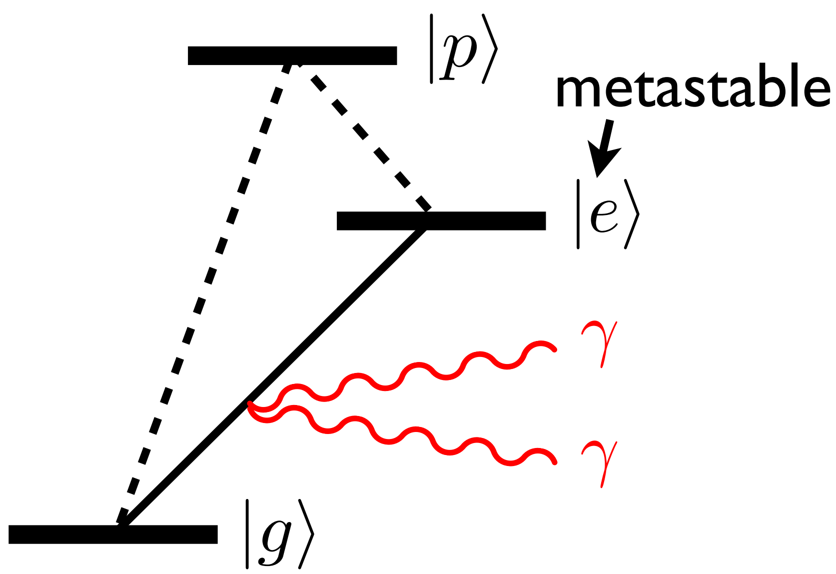
$$\epsilon_{eg} = 10T_\nu \simeq 1.7 \text{ meV}$$

$$m_0 = 0$$

PSR

Paired Super-Radiance (PSR)

M.Yoshimura, N. Sasao, MT, PRA86, 013812 (2012)



$$|e\rangle \to |g\rangle + \gamma + \gamma$$

Prototype for RENP

proof-of-concept for the macrocoherence

Preparation of initial state for RENP

coherence generation ρ_{eg}

dynamical factor $\eta_\omega(t)$

Theoretical description to be tested

Maxwell-Bloch equation

PSR equation

Effective two-level interaction Hamiltonian

$$|g\rangle, \; |e\rangle, \; \not{|p\rangle} \qquad \mathcal{H}_I = \begin{pmatrix} \alpha_{ee} & \alpha_{ge} e^{i\varepsilon_{eg} t} \\ * & \alpha_{gg} \end{pmatrix} E^2$$

$$\alpha_{ge} = \frac{2 d_{pe} d_{pg}}{\epsilon_{pg} + \epsilon_{pe}}, \qquad \alpha_{aa} = \frac{2 d_{pa}^2 \epsilon_{pa}}{\epsilon_{pa}^2 - \omega^2}, \qquad (a = g\,, e)$$

d_{pa} : dipole matrix element

Field (1+1 dim.)

$$\omega = \epsilon_{eg}/2$$

$$E = E_R\, e^{-i(\omega t - kx)} + E_L\, e^{-i(\omega t + kx)} + \text{c.c.}$$

$$k = \omega$$

$|e\rangle$

L-mover $e^{i\omega(t+x)}$ ← → R-mover $e^{i\omega(t-x)}$

$|g\rangle$

$$\sim e^{2i\omega t} = e^{i\epsilon_{eg} t}$$

macrocoherence

Bloch equation $\partial_t \rho = i[\rho, \mathcal{H}_I] + \mathrm{relaxation\ terms}$

density matrix

$$\rho = |\psi\rangle\langle\psi| = \rho_{gg}|g\rangle\langle g| + \rho_{ee}|e\rangle\langle e| + \rho_{eg}|e\rangle\langle g| + \rho_{ge}|g\rangle\langle e|$$

coherence (of an atom) $|\rho_{eg}| \leq 1/2$

Maxwell equation $(\partial_t^2 - \partial_x^2)E = -\partial_t^2 P$

macroscopic polarization $P = -\frac{\delta}{\delta E}\mathrm{tr}(\rho\mathcal{H}_I)$

Rotating wave approximation (RWA)

omitting fast oscillation terms

Slowly varying envelope approximation (SVEA)

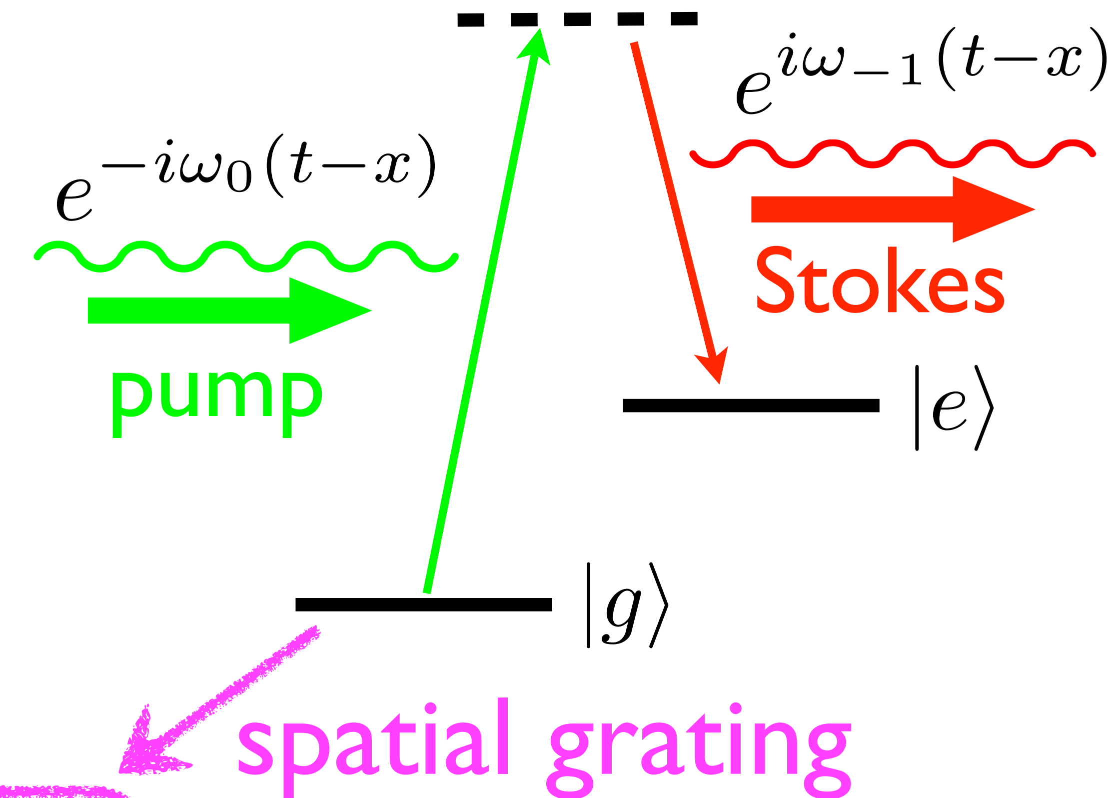
$$|\partial_{x,t}E_{R,L}| \ll \omega|E_{R,L}|,\ |\partial_{x,t}R_i^{(0,\pm)}| \ll \omega|R_i^{(0,\pm)}|$$

PSR with spatial gratings

How to populate $|e\rangle$

Raman scattering

$$\omega_0 - \omega_{-1} = \epsilon_{eg}$$



Generated coherence

$$\rho_{eg} = \rho_{eg}^{(0)} + \rho_{eg}^{(+)} e^{i\epsilon_{eg} x} + \rho_{eg}^{(-)} e^{-i\epsilon_{eg} x}$$

$$e^{i\omega_p(t-x)} e^{i\omega_{\bar{p}}(t-x)} = e^{i\epsilon_{eg}(t-x)}$$

$$\omega_p + \omega_{\bar{p}} = \epsilon_{eg}$$

momentum conservation in the macrocoherence → Unidirectional PSR

Para-hydrogen gas PSR experiment

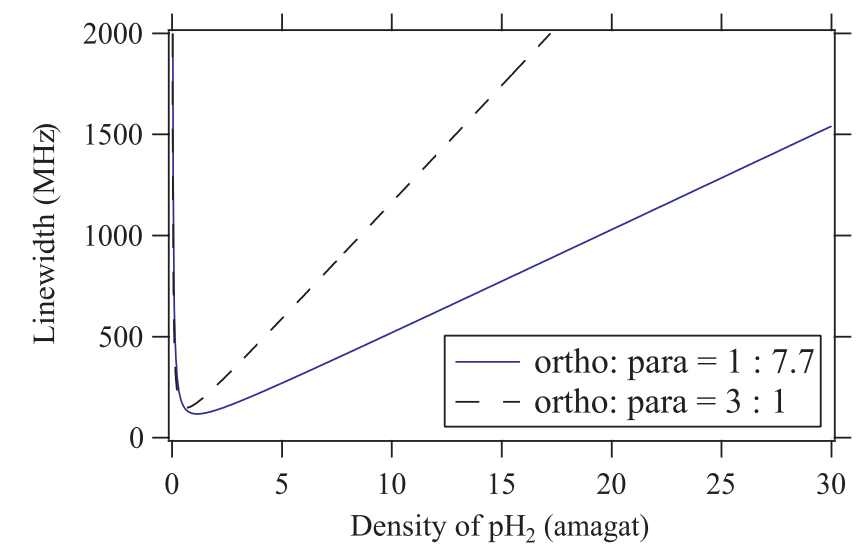
@ Okayama U

Y. Miyamoto et al., arXiv:1406.2198, to be published in PTEP

vibrational transition of p-H2

$$|e\rangle = |Xv=1\rangle \longrightarrow |g\rangle = |Xv=0\rangle$$

two-photon decay: $\tau_{2\gamma} \sim 10^{12}$ s



p-H2: nuclear spin=singlet

smaller decoherence

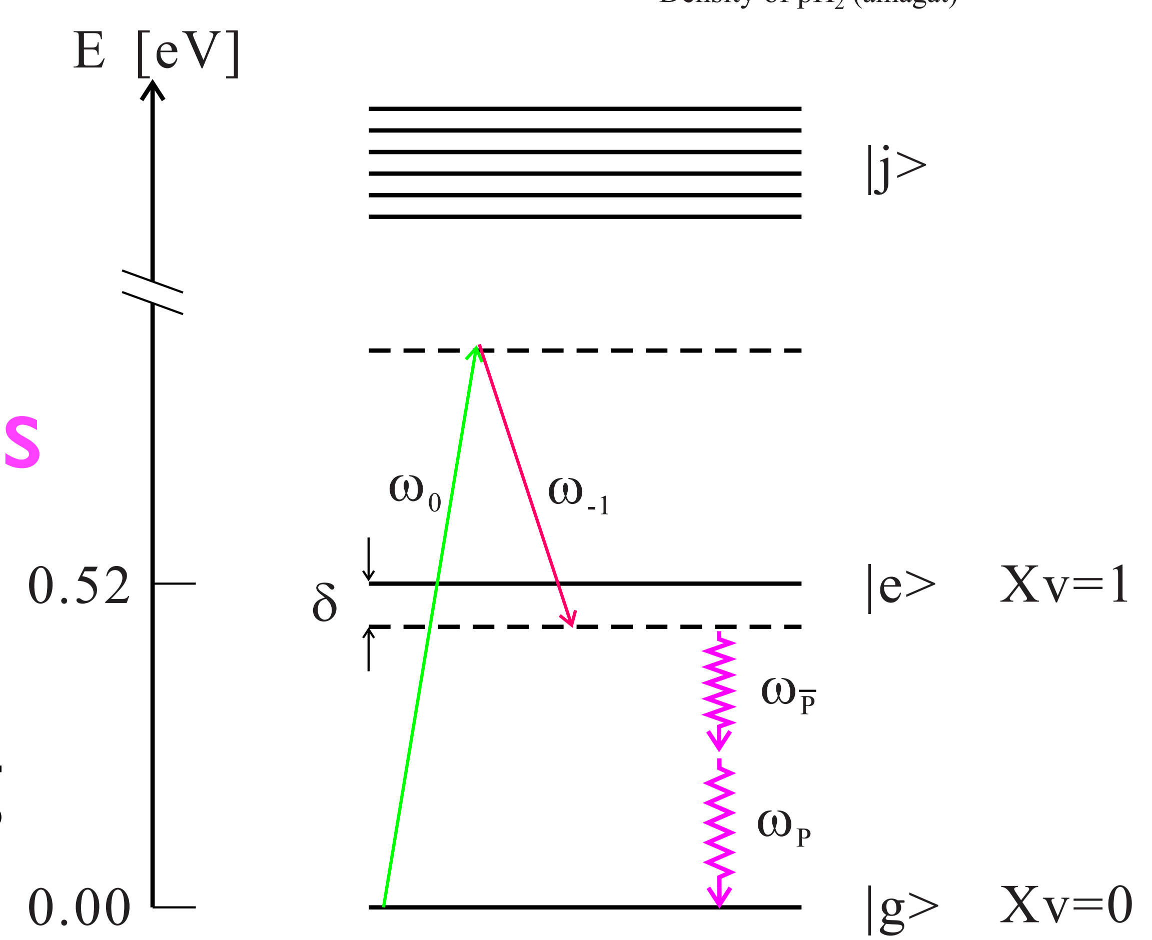
$$1/T_2 \sim 130 \text{ MHz}$$

coherence production

adiabatic Raman process

$$\Delta\omega = \omega_0 - \omega_{-1}$$
$$= \epsilon_{eg} - \delta$$
$$= \omega_p + \omega_{\bar{p}}$$

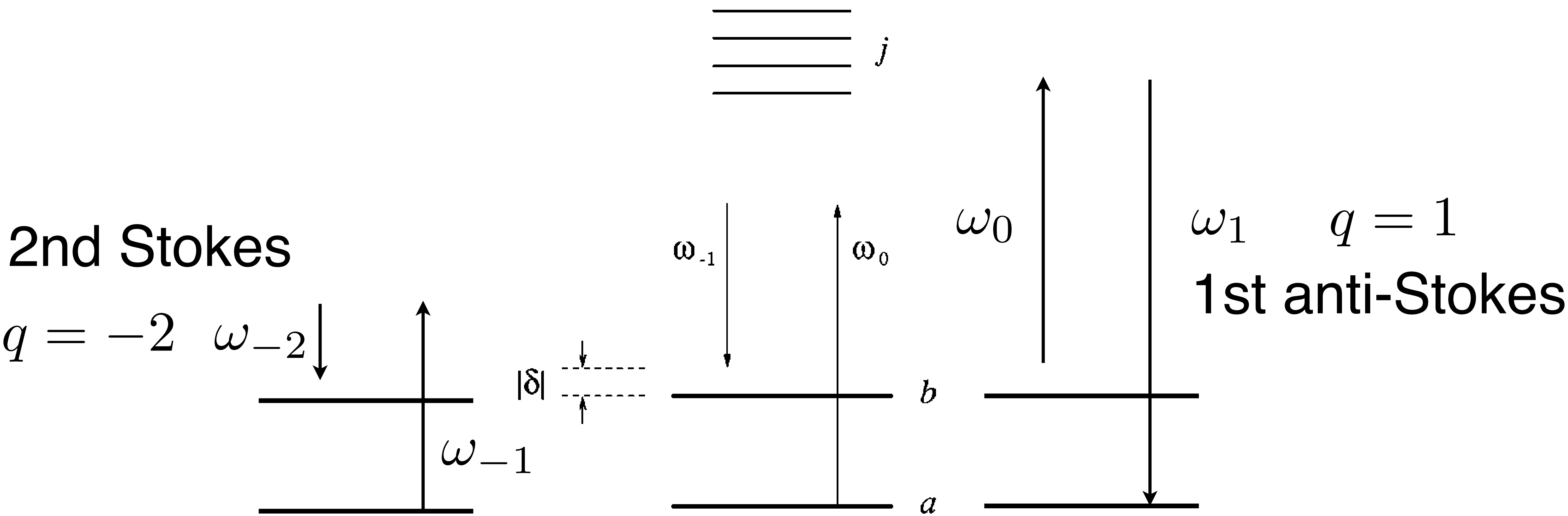
detuning



Raman sideband generation

Harris, Sokolov, Phys. Rev. A55, R4019(1997)

Kien, Liang, Katsuragawa, Ohtsuki, Hakuta, Sokolov, Phys. Rev. A60, 1562(1999)



$$\omega_q = \omega_0 + q(\omega_b - \omega_a - \delta) = \omega_0 + q\,\omega_m$$

$q \geq q_{\rm min}$ the lowest Stokes

Hamiltonian

$$H_{\text{int}} = -\sum_j E(\mu_{ja}\sigma_{ja} + \mu_{aj}\sigma_{aj} + \mu_{jb}\sigma_{jb} + \mu_{bj}\sigma_{bj})$$

$$\mu_{\alpha\beta} = \langle\alpha|d|\beta\rangle \quad \sigma_{\alpha\beta} = |\alpha\rangle\langle\beta|$$

$$E = \frac{1}{2}\sum_q (E_q e^{-i\omega_q\tau} + E_q^* e^{i\omega_q\tau})$$

Effective Hamiltonian

$|j\rangle$ far off-resonance → two-level system

$$H_{\text{eff}} = -\hbar\begin{bmatrix}\Omega_{aa} & \Omega_{ab} \\ \Omega_{ba} & \Omega_{bb} - \delta\end{bmatrix}$$

Stark shifts

$$\Omega_{aa} = \frac{1}{2} \sum_q a_q |E_q|^2 \qquad a_q = \frac{1}{2\hbar^2} \sum_j \left(\frac{|\mu_{ja}|^2}{\omega_j - \omega_a - \omega_q} + \frac{|\mu_{ja}|^2}{\omega_j - \omega_a + \omega_q} \right)$$

$$\Omega_{bb} = \frac{1}{2} \sum_q b_q |E_q|^2 \qquad b_q = \frac{1}{2\hbar^2} \sum_j \left(\frac{|\mu_{jb}|^2}{\omega_j - \omega_b - \omega_q} + \frac{|\mu_{jb}|^2}{\omega_j - \omega_b + \omega_q} \right)$$

Two-photon Rabi freq.

$$\Omega_{ab} = \Omega_{ba}^* = \frac{1}{2} \sum_q d_q E_q E_{q+1}^* \qquad d_q = \frac{1}{2\hbar^2} \sum_j \left(\frac{\mu_{aj}\mu_{jb}}{\omega_j - \omega_b - \omega_q} + \frac{\mu_{aj}\mu_{jb}}{\omega_j - \omega_a + \omega_q} \right)$$

Adiabatic eigenstate

$$|+\rangle = \cos\frac{\theta}{2} e^{i\varphi/2} |a\rangle + \sin\frac{\theta}{2} e^{-i\varphi/2} |b\rangle \xrightarrow{E \to 0} |a\rangle$$

$$\tan\theta = \frac{2|\Omega_{ab}|}{\Omega_{aa} - \Omega_{bb} + \delta} \qquad \Omega_{ab} = |\Omega_{ab}| e^{i\varphi}$$

Wave propagation

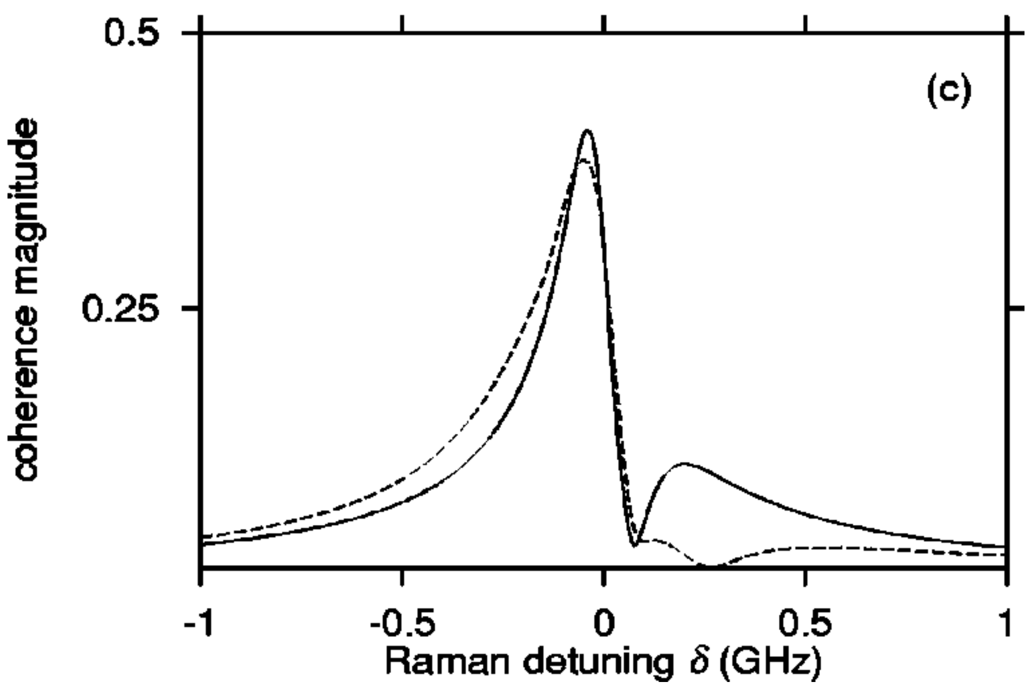
$$(\partial_t + \partial_z)E_q = in\hbar\omega_q \left(a_q\rho_{aa}E_q + b_q\rho_{bb}E_q + d_{q-1}\rho_{ba}E_{q-1} + d_q^*\rho_{ab}E_{q+1}\right)$$

Coherence $\rho_{ab} = \frac{1}{2}\sin\theta e^{i\varphi}$

molecular system of far off-resonance

$\Omega_{aa} \simeq \Omega_{bb}$ $\tan\theta \simeq 2|\Omega_{ab}|/\delta$ → $|\rho_{ab}| \simeq 1/2$

$|\rho_{ab}|$



δ

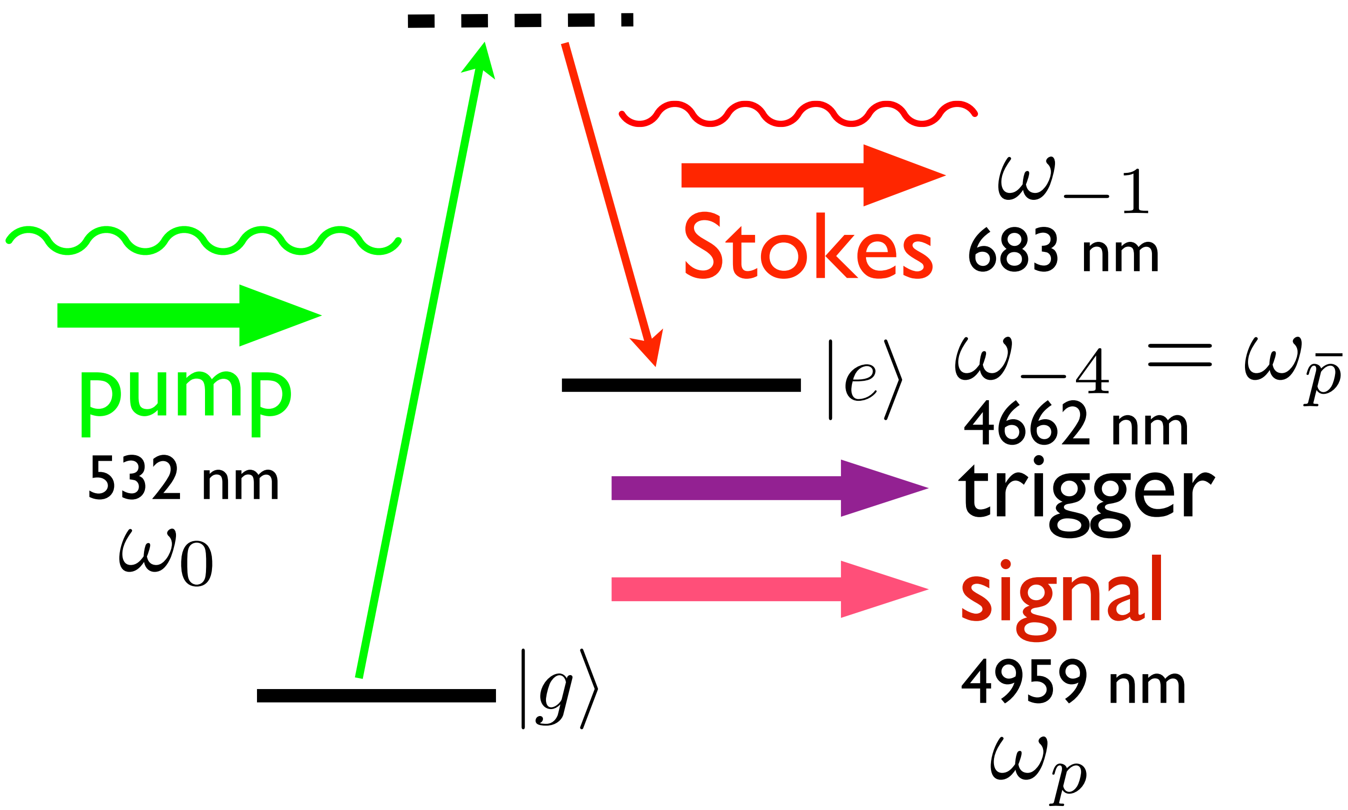
$\delta > 0\,,\ \sin\theta > 0$

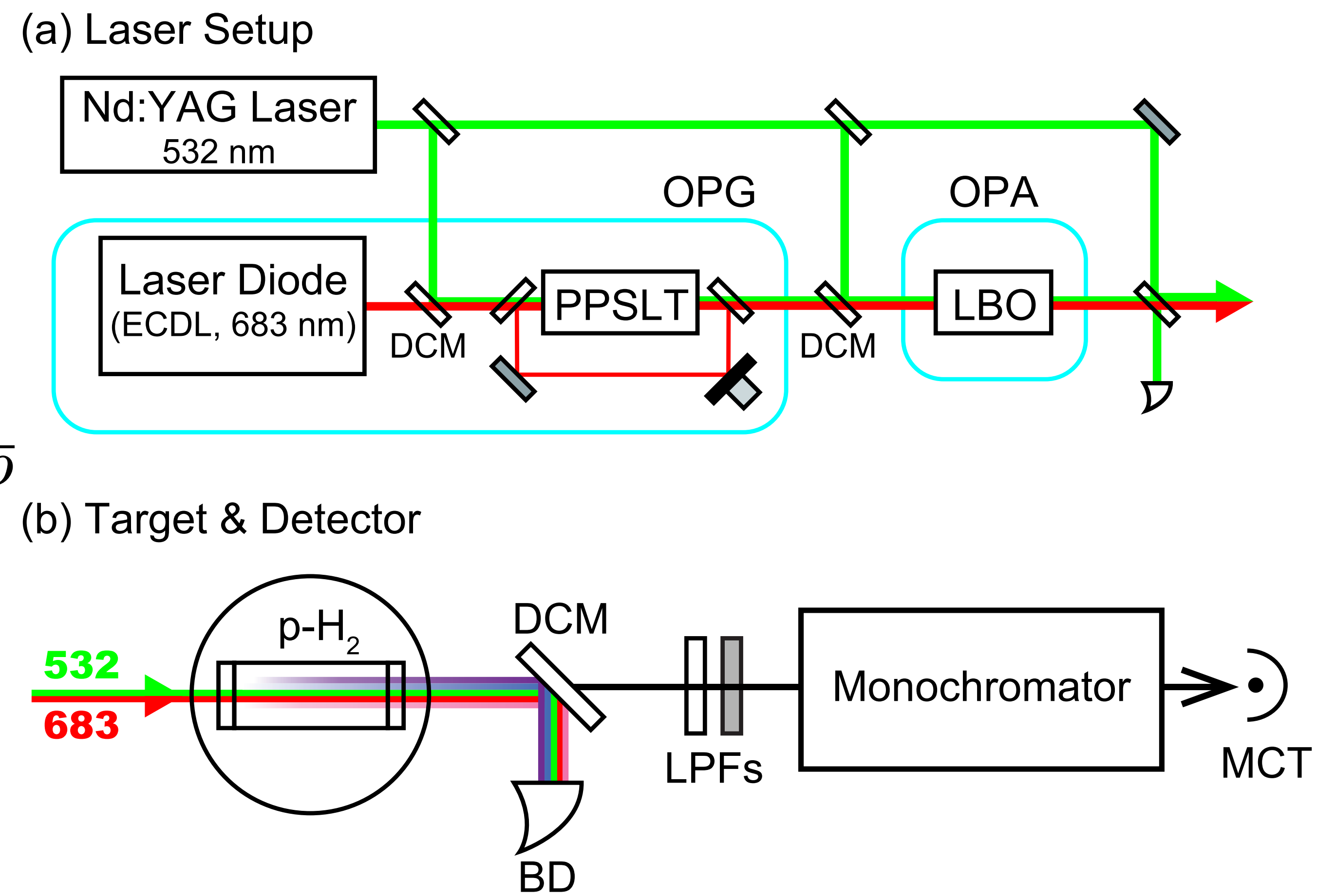
phased state

$\delta < 0\,,\ \sin\theta < 0$

antiphased state

Experimental setup





4th Stokes (q=-4) as trigger (internal trigger)

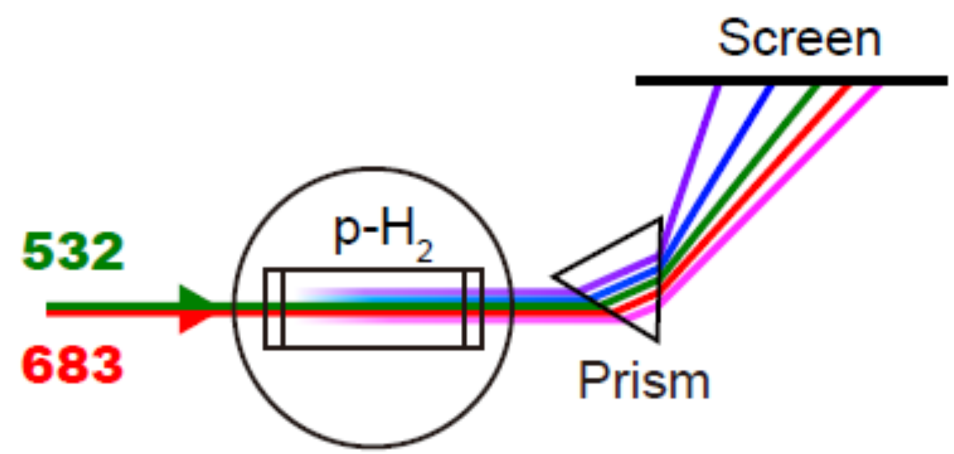
Target cell: length 15cm, diameter 2cm, 78K, 60kPa

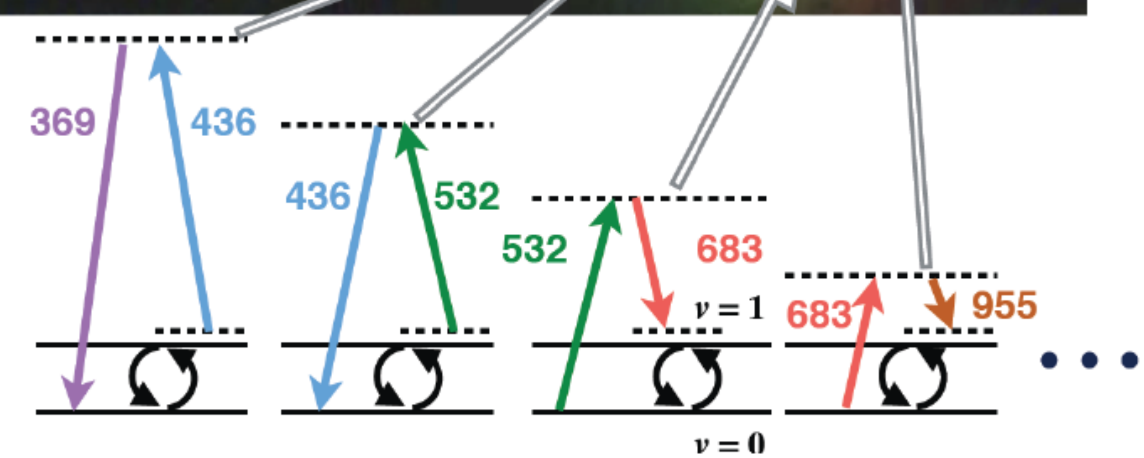
$$n = 5.6 \times 10^{19}\ \mathrm{cm}^{-3} \qquad 1/T_2 \sim 130\ \mathrm{MHz}$$

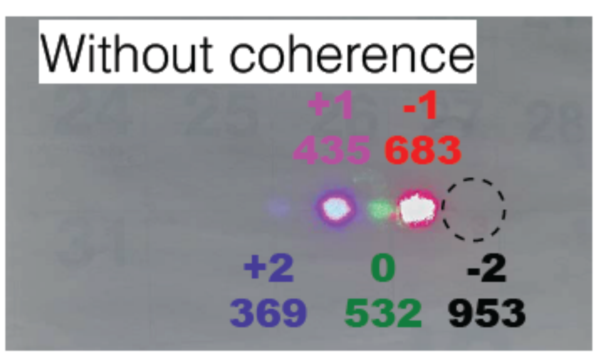
Driving lasers: $5\ \mathrm{mJ},\ 6\ \mathrm{ns},\ w_0 = 100\ \mu\mathrm{m}\ (5\ \mathrm{GW/cm^2})$

Ultra-broadband Raman sidebands

- Raman sidebands, from 192 to 4662nm, are observed: >24
- Evidence of large coherence





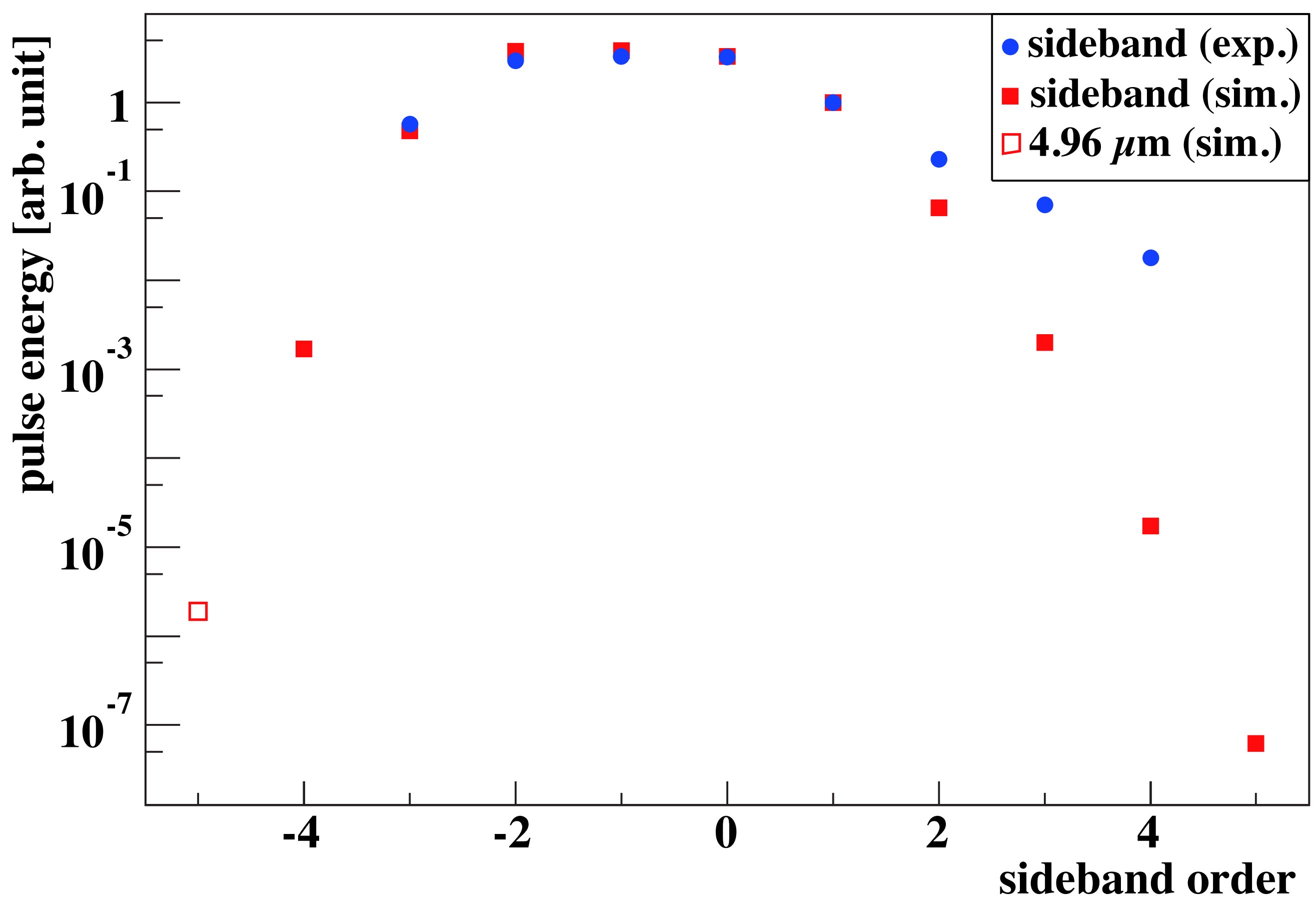


N. Sasao

Generated coherence

Maxwell-Bloch eq.

$$\frac{\partial \rho_{gg}}{\partial \tau} = i\Big(\Omega_{ge}\rho_{eg} - \Omega_{eg}\rho_{ge}\Big) + \gamma_1 \rho_{ee},$$
$$\frac{\partial \rho_{ee}}{\partial \tau} = i\Big(\Omega_{eg}\rho_{ge} - \Omega_{ge}\rho_{eg}\Big) - \gamma_1 \rho_{ee},$$
$$\frac{\partial \rho_{ge}}{\partial \tau} = i\Big(\Omega_{gg} - \Omega_{ee} + \delta\Big)\rho_{ge} + i\Omega_{ge}\Big(\rho_{ee} - \rho_{gg}\Big) - \gamma_2 \rho_{ge},$$
$$\frac{\partial E_q}{\partial \xi} = \frac{i\omega_q n}{2c}\Big\{\Big(\rho_{gg}\alpha_{gg}^{(q)} + \rho_{ee}\alpha_{ee}^{(q)}\Big)E_q + \rho_{eg}\alpha_{eg}^{(q-1)}E_{q-1} + \rho_{ge}\alpha_{ge}^{(q)}E_{q+1}\Big\},$$
$$\frac{\partial E_p}{\partial \xi} = \frac{i\omega_p n}{2c}\Big\{\Big(\rho_{gg}\alpha_{gg}^{(p)} + \rho_{ee}\alpha_{ee}^{(p)}\Big)E_p + \rho_{eg}\alpha_{ge}^{(p\bar{p})}E_{\bar{p}}^{*}\Big\}.$$

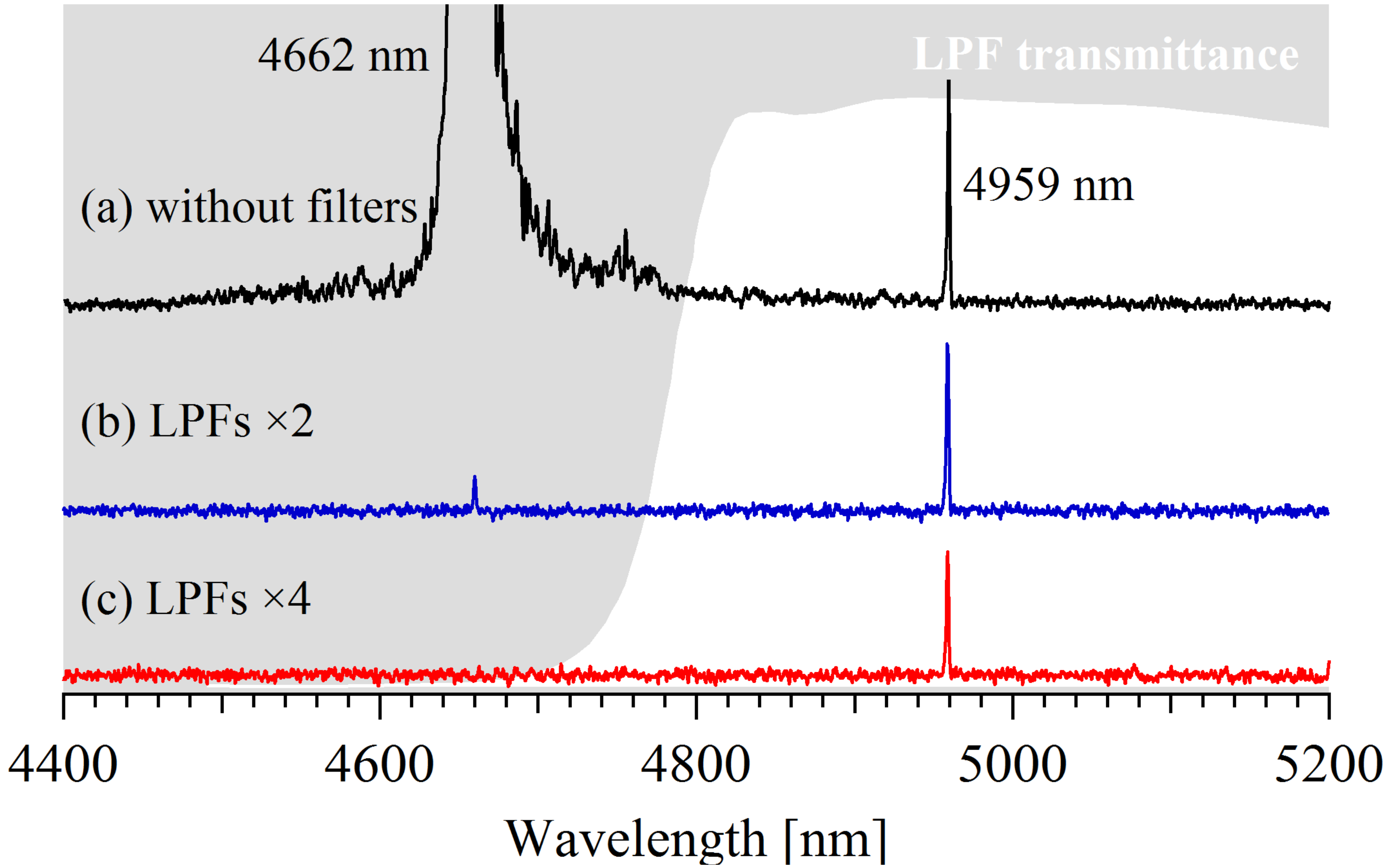


coherence estimation

$$|\rho_{eg}| \simeq 0.032$$

(6% of max.)

Observed two-photon spectrum



# of observed photons

$$4.4 \times 10^{7} / \text{pulse}$$

Estimated spontaneous rate

$$1.6 \times 10^{-8}$$

$O(10^{15})$ (or more) enhancement!

SUMMARY

Neutrino Physics with Atoms/Molecules

- ✶ RENP spectra are sensitive to unknown neutrino parameters.

  Absolute mass, Dirac or Majorana,
  NH or IH, CP

- ✶ RENP spectra are sensitive to the cosmic neutrino background.

  temperature, chemical potential.

- ✶ Macrocoherent rate amplification is essential.

  demonstrated by a QED process, PSR.

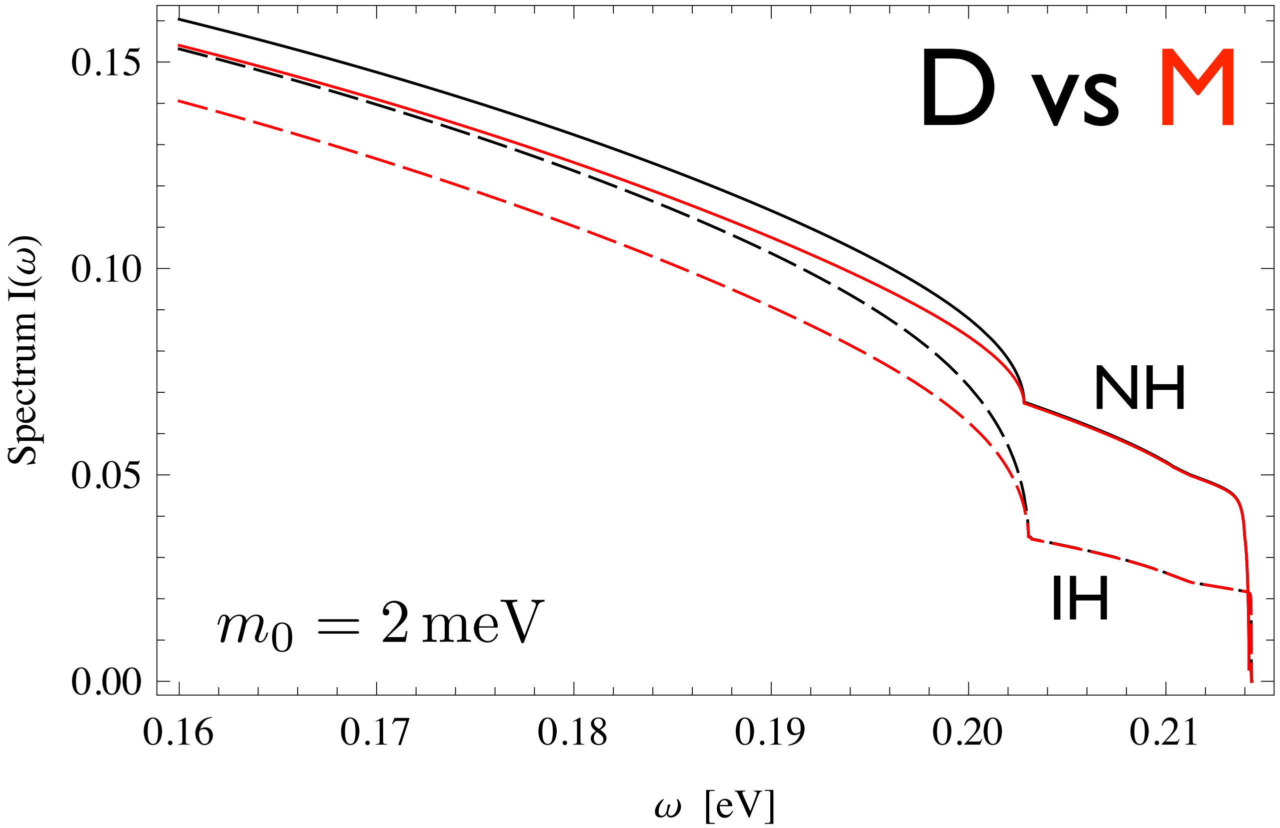
A new approach to neutrino physics

Backup Slides

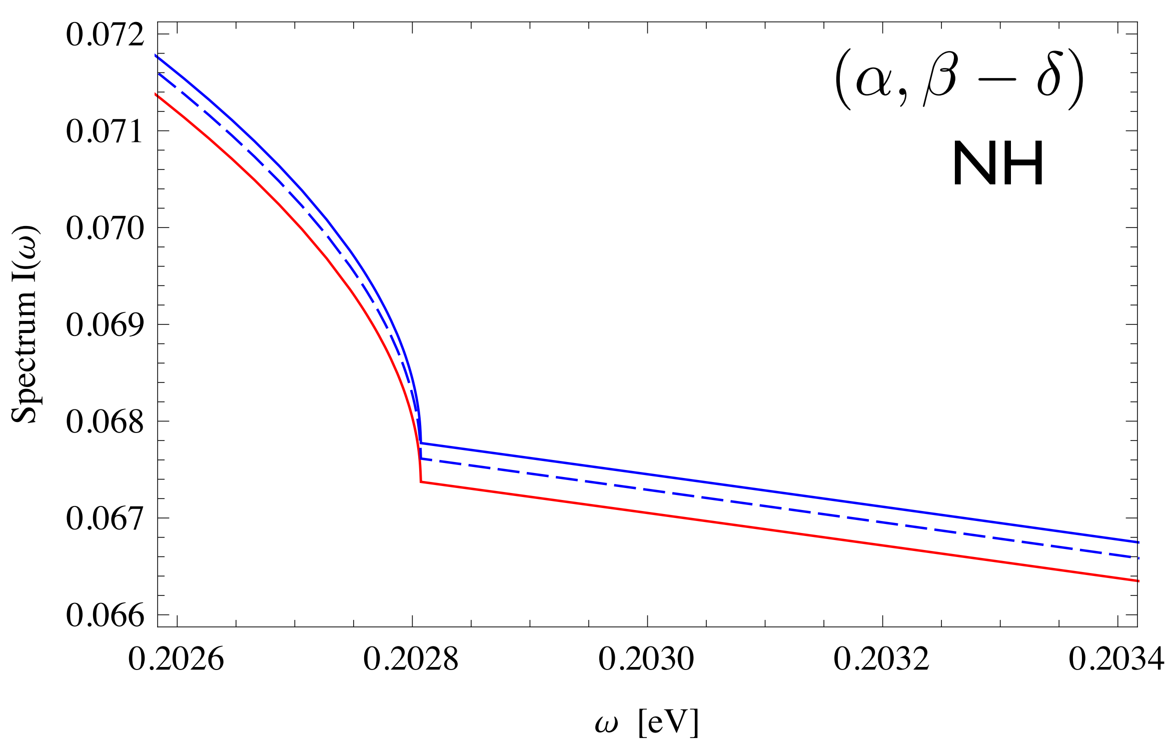
More on Dirac vs Majorana and CP phases

hypothetical atom

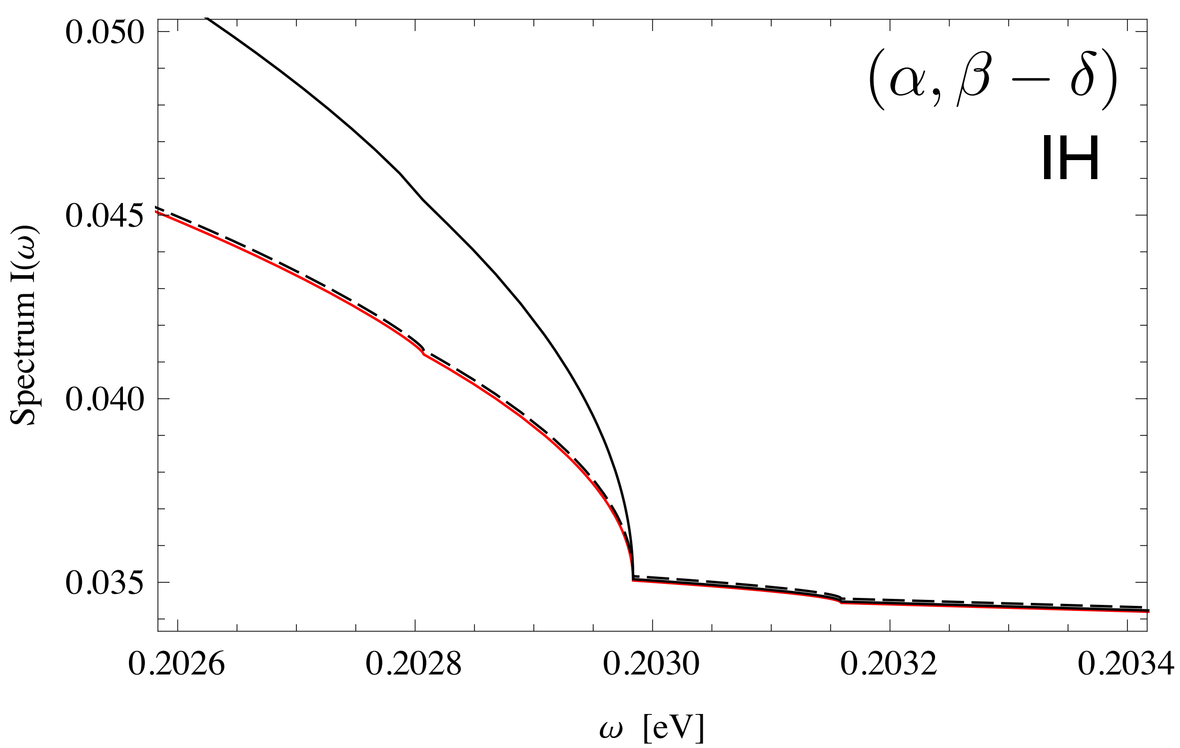
$$\epsilon_{eg} = 0.43\,\mathrm{eV}$$



CP phases (NH); Red=(0,0), Blue=(π/2,0), Blue Dashed=(0,π/2)



CP phases (IH); Red=(0,0), Black=(π/2,0), Black Dashed=(0,π/2)



Thermal history of cosmic neutrinos

$T \gtrsim 3.2\ \mathrm{MeV}$ $\nu_{e,\mu,\tau}$ in equilibrium

$T \simeq 3.2\ \mathrm{MeV}$ $\nu_{\mu,\tau}$ decoupling

$T \simeq 1.9\ \mathrm{MeV}$ ν_e decoupling

$$f_D(\boldsymbol{p}) = \left[\exp\left(\frac{\sqrt{\boldsymbol{p}^2 + m^2}}{T_D} - \xi\right) + 1\right]^{-1}$$

$T \lesssim 1.9\ \mathrm{MeV}$ free propagation

Present $a = 1$ $f(\boldsymbol{p}) = f_D(\boldsymbol{p}/a_D)$

$$f(\boldsymbol{p}) = \left[\exp\left(\frac{\sqrt{\boldsymbol{p}^2 + (m a_D)^2}}{T_D a_D} - \xi\right) + 1\right]^{-1}$$

$T_\nu = T_D a_D$ $m a_D \ll m$