原子スペクトルの
同位体シフトで探る
素粒子の新しい相互作用

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Introduction

Frontiers in particle physics

Energy frontier: LHC, ILC,…

Intensity frontier: B factory, muon, K, …

Cosmic frontier: CMB,…

Precision / low energy frontier

$0\nu\beta\beta$, DM, EDM,…

Temporal variation of fundamental constants

α, m_e/m_p using atomic clock

$$\mathrm{Yb}^+ : \ \delta\nu/\nu \sim 10^{-18}, \ \delta\nu \sim \mathrm{sub\ Hz}$$

Huntemann et al. (PTB) 2016

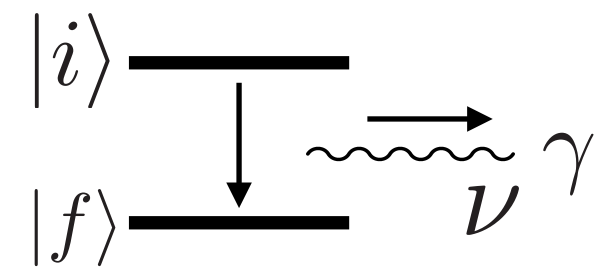
Isotope shift new neutron-electron interaction

Isotope shift (IS)

Transition frequency difference between isotopes

$$h\nu_A = E_A^i - E_A^f$$

$$\mathrm{IS} = \nu_{A'A} := \nu_{A'} - \nu_A$$



No IS for infinitely heavy and point-like nuclei

→ $\mathrm{IS} = \mathrm{MS} + \mathrm{FS}$

Mass shift: finite mass of nuclei (reduced mass)

$\mathrm{MS} \propto \mu_{A'} - \mu_A$ (dominant for Z<20)

Field shift: finite size of nuclei

$\mathrm{FS} \propto \langle r^2 \rangle_{A'} - \langle r^2 \rangle_A$ (dominant for Z>40)

Theoretical calculation of IS: not easy

$$\mathrm{IS} \sim O(\mathrm{GHz}) \sim O(10\ \mu\mathrm{eV})$$

Plan of talk

King’s linearity

King's linearity

King, 1963

IS of two transitions: $t = 1, 2$

$$\nu^t_{A'A} = K_t\, \mu_{A'A} + F_t\, \langle r^2 \rangle_{A'A}$$

$$\mu_{A'A} := \mu_{A'} - \mu_A$$

$$\langle r^2 \rangle_{A'A} := \langle r^2 \rangle_{A'} - \langle r^2 \rangle_A$$

Modified IS: $\tilde{\nu}^t_{A'A} := \nu^t_{A'A} / \mu_{A'A}$

$$\tilde{\nu}^t_{A'A} = K_t + F_t \langle r^2 \rangle_{A'A} / \mu_{A'A}$$

nuclear factor

electronic factors

King's linearity eliminating the nuclear factor

$$\tilde{\nu}^2_{A'A} = K_{21} + \frac{F_2}{F_1} \tilde{\nu}^1_{A'A}$$

$$K_{21} := K_2 - \frac{F_2}{F_1} K_1$$

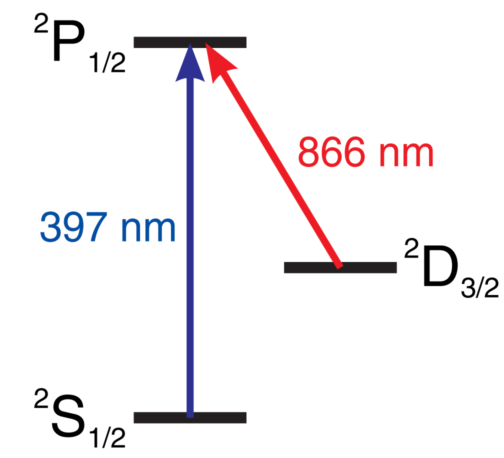
→ $(\tilde{\nu}^1_{A'A}, \tilde{\nu}^2_{A'A})$ on a straight line, King's plot

IS data of Ca^+

Gebert et al. PRL115, 053003 (2015)

Transition 1: 397 nm $^2\mathrm{P}_{1/2}(4\mathrm{p}) -^2 \mathrm{S}_{1/2}(4\mathrm{s})$

Transition 2: 866 nm $^2\mathrm{P}_{1/2}(4\mathrm{p}) -^2 \mathrm{D}_{3/2}(3\mathrm{d})$



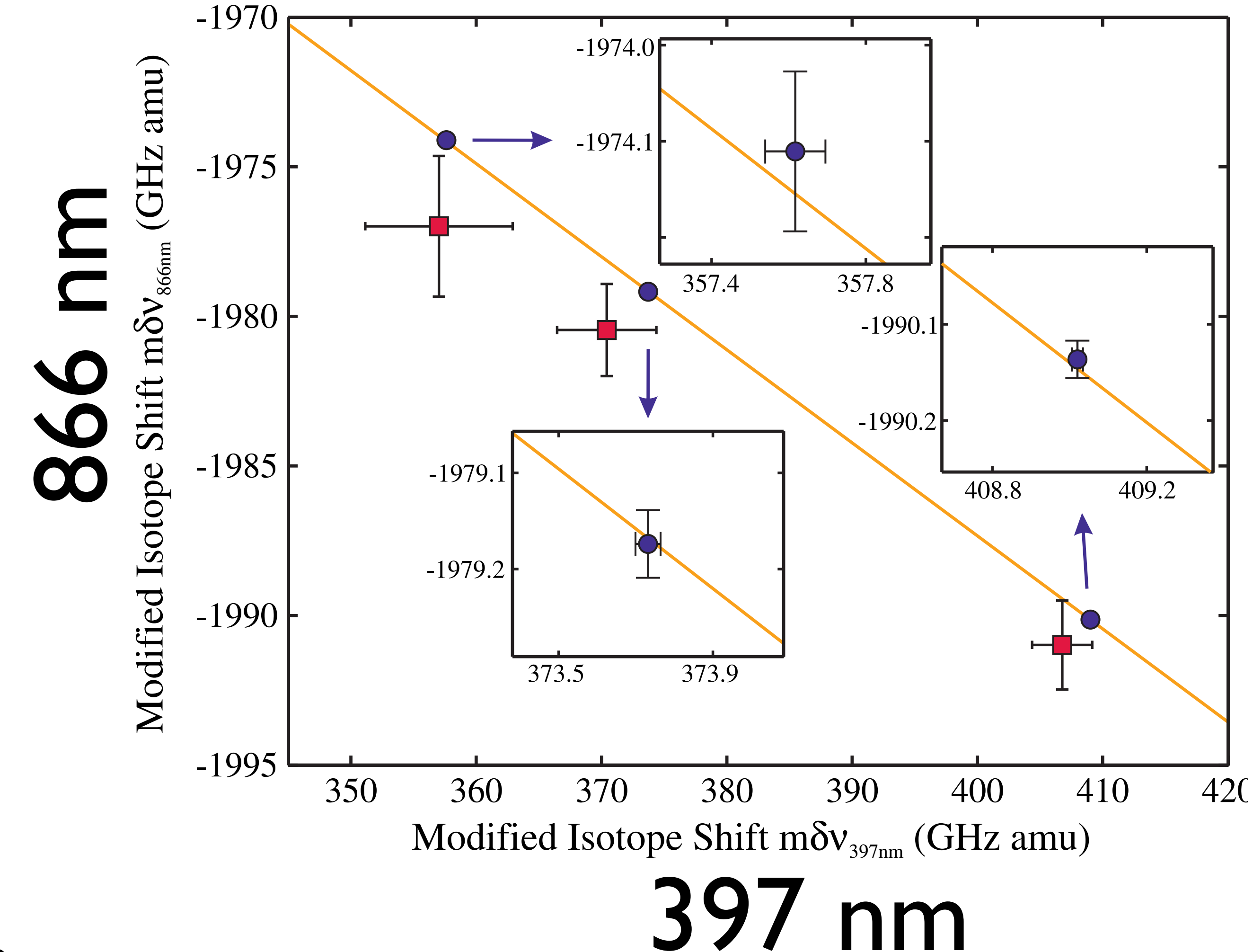
Isotope pairs: (42, 40), (44,40), (48,40)

IS precision ~ O(100) kHz

King's plot

linear within errors

modified IS $\tilde{\nu}^t_{A'A}$



IS data of Yb+

Transition 1: 369 nm Martensson-Pendrill et al. PRA49, 3351 (1994)

$$^2\mathrm{P}_{1/2}(4\mathrm{f})^{14}(6\mathrm{p}) -{}^2\mathrm{S}_{1/2}(4\mathrm{f})^{14}(6\mathrm{s}) \quad \delta\nu^1_{A'A} \sim O(1)\ \mathrm{MHz}$$

Transition 2: 935 nm Sugiyama et al. CPEM2000

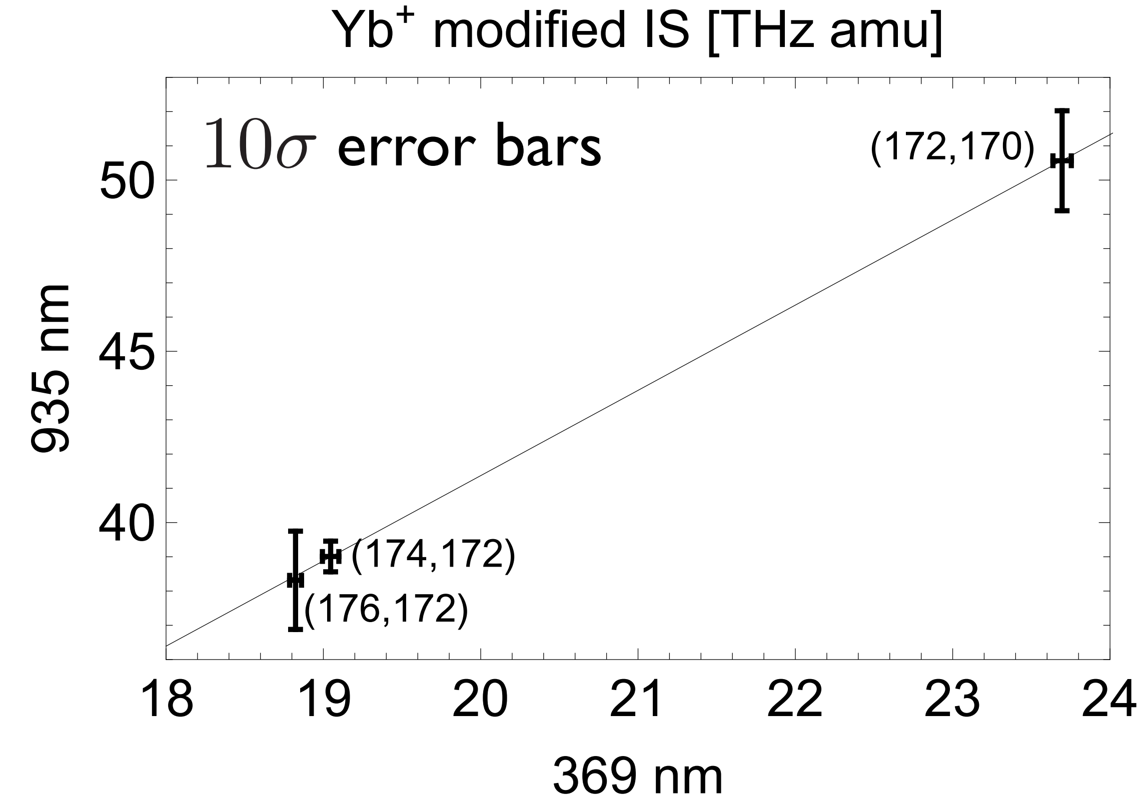
$$^3\mathrm{D}[3/2]_{1/2}(4\mathrm{f})^{13}(5\mathrm{d})(6\mathrm{s}) -{}^2\mathrm{D}_{3/2}(4\mathrm{f})^{14}(5\mathrm{d})$$

$$\delta\nu^2_{A'A} \sim O(10)\ \mathrm{MHz}$$

Isotope pairs: (172, 170), (174,172), (176,172)

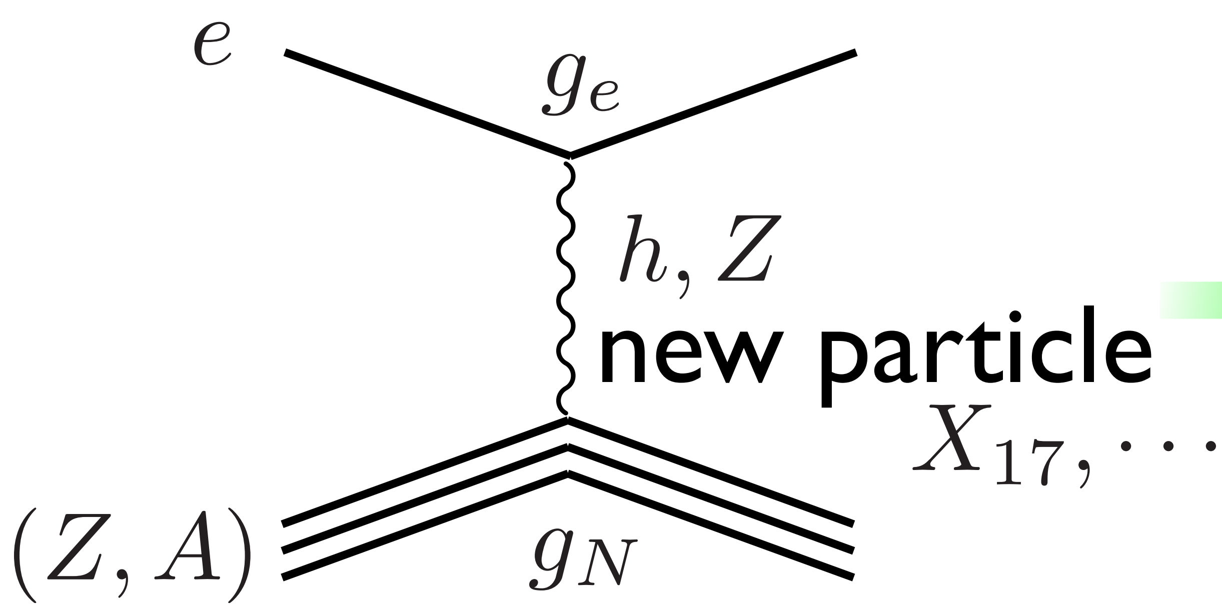
King's plot

linear within errors



Nonlinearities

Particle shift (PS)



Yukawa potential

$$V(r) = (-1)^{s+1} \frac{g_N g_e}{4\pi} \frac{e^{-mr}}{r}$$

Frequency shifts by particle exchange (Yb$^+$ g.s.)

$$|\Delta\nu| \sim \begin{cases} 10^{-4}\ \mathrm{Hz} & \text{Higgs (SM)} \\ 400\ \mathrm{Hz} & \text{Higgs (LHC bound)} \\ 800\ \mathrm{Hz} & Z \\ 10\ \mathrm{MHz} & X_{17}\ \text{17 MeV vector boson} \end{cases}$$

<< theoretical uncertainties

Breakdown of the linearity by PS

Delaunay et al. arXiv:1601.05087v2

$$\mathrm{IS} = \mathrm{MS} + \mathrm{FS} + \mathrm{PS}$$

PS by new neutron-electron interaction

$$\nu^t_{A'A} = K_t\, \mu_{A'A} + F_t\, \langle r^2 \rangle_{A'A} + X_t (A' - A)$$

Generalized King's relation

$$\tilde{\nu}^2_{A'A} = K_{21} + F_{21} \tilde{\nu}^1_{A'A} + \varepsilon\, A'A$$

nonlinearity

probe into new physics

PS nonlinearity

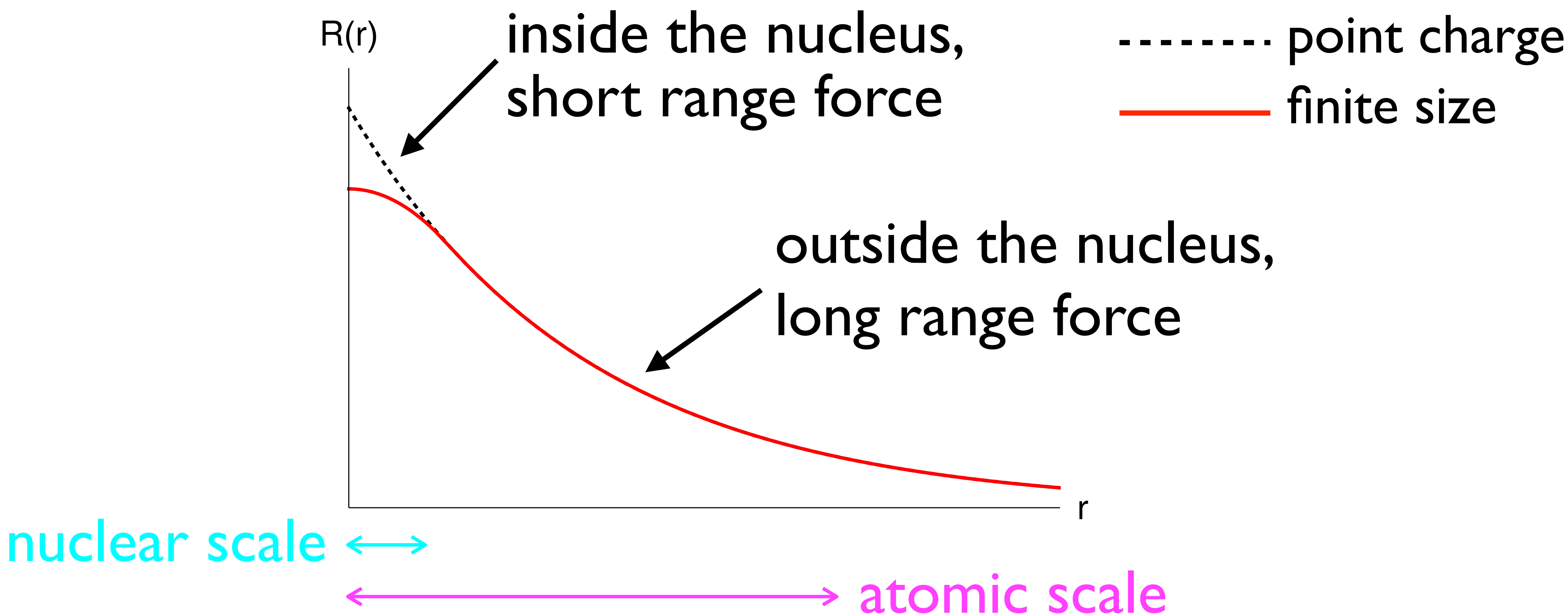
$$\varepsilon_{\mathrm{PS}} = X_1 \left(\frac{X_2}{X_1} - \frac{F_2}{F_1} \right) \qquad X_t \propto \frac{g_n g_e}{m^2} \text{ as } m \to \infty$$

Evaluation of PS nonlinearity

Single electron approximation

$$X_t = \frac{g_n g_e}{4\pi} \int r^2 dr \frac{e^{-mr}}{r} \left[R_{i_t}^2(r) - R_{f_t}^2(r) \right]$$

Wave function



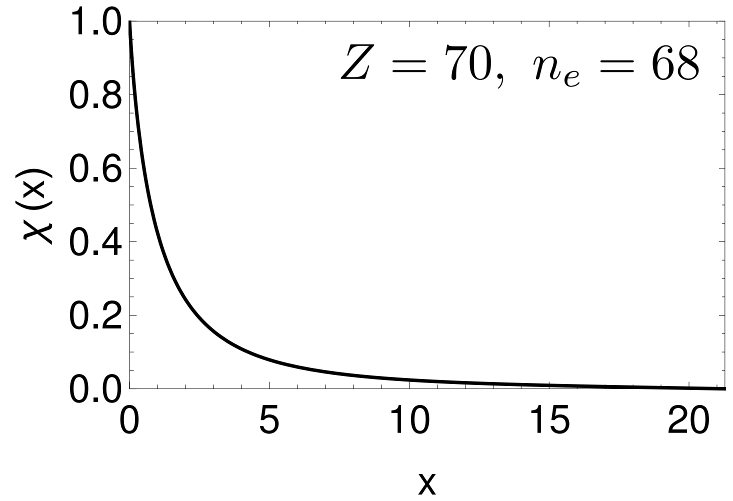
Wave function outside the nucleus

Non-relativistic (not bad for m<<100 MeV)

Thomas-Fermi model

semiclassical, statistical, self-consistent field

exact in large Z limit



→ TF function

$$\frac{d^2\chi}{dx^2} = x^{-1/2}\chi^{3/2}$$

$$\chi(0) = 1,\ x_0\chi'(x_0) = \frac{n_e}{Z} - 1,\ \chi(x_0) = 0$$

One-body problem in the TF potential

$$V_{\mathrm{TF}}(r) = -\frac{Z\alpha}{r}\chi\left(\frac{r}{b}\right) - (Z - n_e)\alpha \min\left(\frac{1}{r_0}, \frac{1}{r}\right)$$

$$b = (9\pi^2/2^7 Z)^{1/3} a_B,\ a_B = \text{Bohr radius}$$

Comparison to experiments

Ca^+	EX	TF
4p→4s	397	475
4p→3d	866	-1610

Yb^+	EX	TF
6p→6s	369	380
6s→4f	935	48.6

wavelength in nm

good for s and p states

poor for d and f states

Wave function inside the nucleus

One-body problem in the nuclear potential $V_A(r)$

$$\left[\frac{d}{dr^2} - \frac{\ell(\ell+1)}{r^2} + 2m_e\{E - V_A(r)\}\right] rR(r) = 0$$

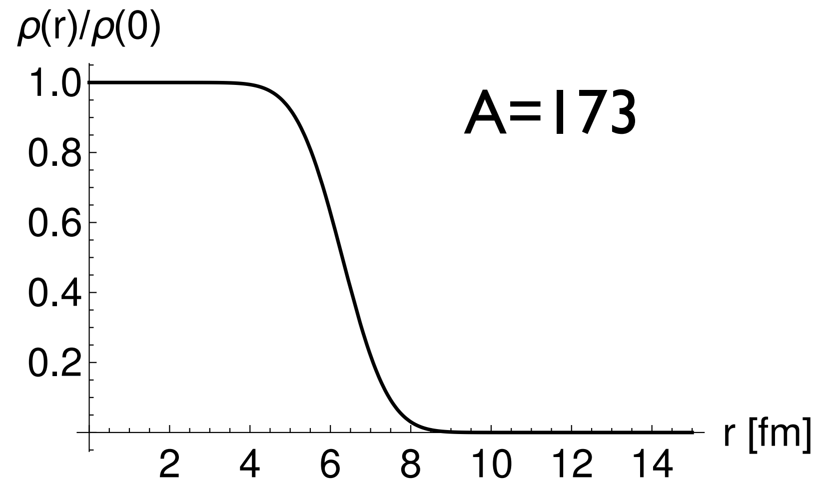
$\ell = \text{angular momentum}$

Series expansion: $V_A(r) = \sum_{i=0} v_i r^i, \ v_1 = 0$

$$R(r) = \sum_{i=0} \chi_i^\ell r^{\ell+i}$$

$\longrightarrow \chi_1^\ell = 0, \ \chi_2^\ell / \chi_0^\ell = m_e v_0 / (2\ell + 3)$

Nuclear charge distribution



Helm distribution

Helm 1956

Gaussian smearing of uniform sphere

$$\rho_A(r) = \int d^3r' \frac{3}{4\pi r_A^3}\theta(r_A - r') \frac{1}{(2\pi s^2)^{3/2}} e^{-|\boldsymbol{r}-\boldsymbol{r}'|^2/(2s^2)}$$

$$r_A^2 = c^2 + 7\pi^2 a^2/3 - 5s^2, \;\; s \simeq 0.9 \text{ fm}$$

$$a \simeq 0.52 \text{ fm}, \;\; c \simeq 1.23A - 0.60 \text{ fm}$$

Lewin, Smith1996

$$\langle r^2 \rangle = \frac{3}{5}(r_A^2 + 5s^2), \;\; \langle r^4 \rangle = \frac{3}{7}(r_A^4 + 14r_A^2 s^2 + 35s^2)$$

$$v_0 = \frac{3Z\alpha}{2r_A}\left[\left(1 - \frac{s^2}{r_A^2}\right)\mathrm{Erf}\left(\frac{r_A}{\sqrt{2}s}\right) + \sqrt{\frac{2}{\pi}}\frac{s}{r_A} e^{-r_A^2/(2s^2)}\right]$$

$v_1 = 0$ no cusp at the origin

Seltzer moment expansion of field shift

Seltzer 1969

$$\mathrm{FS} = Z\alpha \int d^3 r_N \int d^3 r_e \frac{\rho_{A'A}(r_N)\rho_{if}(r_e)}{|\boldsymbol{r}_N - \boldsymbol{r}_e|}$$

$$\rho_{A'A}(r) := \rho_{A'}(r) - \rho_A(r)$$

$$\rho_{if}(r) := R_i^2(r) - R_f^2(r) = r^{2\ell} \sum_{k=0} \xi_k^\ell r^k, \ \ell = \min(\ell_i, \ell_f)$$

$$= Z\alpha \sum_{k=0} \frac{\xi_k^\ell}{(2\ell + k + 3)(2\ell + k + 2)} \langle r^{2\ell+k+2} \rangle_{A'A}$$

$$\langle r^n \rangle_{A'A} := \langle r^n \rangle_{A'} - \langle r^n \rangle_A$$

$$= F_t \langle r^2 \rangle_{A'A} + \cdots, \ F_t = \frac{Z\alpha}{6} \xi_0^0$$

Note: $\xi_1^\ell = 0$ **no cubic term**

Heavy particle limit

$$ma_B \gg Z\,,\;\; a_B = \text{Bohr radius} \sim (4\ \text{keV})^{-1}$$

$$F_t, X_t \propto |\psi_{i_t}(0)|^2 - |\psi_{f_t}(0)|^2 \longrightarrow \lim_{m\to\infty}\left(\frac{X_2}{X_1} - \frac{F_2}{F_1}\right) = 0$$

Asymptotic behavior of PS

$$X_t \propto \int dr r^2 \rho_{i_t f_t}(r) \frac{e^{-mr}}{r} = \frac{1}{m^2}\sum_{k=0}(2\ell + k + 1)!\frac{\xi_k^\ell}{m^{2\ell+k}} + \cdots$$

$\xi_1^0 = 0$ for nucl. charge distribution without cusp

$$\frac{X_2}{X_1} - \frac{F_2}{F_1} \sim O\left(\frac{1}{m^2}\right) \longrightarrow \varepsilon_{\rm PS} \sim O\left(\frac{1}{m^4}\right)$$

less sensitive to heavier particles

cf. Berengut et al. arXiv:1704.05068 $\varepsilon_{\rm PS} \propto 1/m^3$

Field shift nonlinearity

One of the sources of nonlinearity in QED

$$\mathrm{FS} = F_\ell \langle r^2 \rangle_{A'A} + G_t \langle r^4 \rangle_{A'A}$$

$$\tilde{\nu}^2_{A'A} = K_{21} + F_{21}\tilde{\nu}^1_{A'A} + \varepsilon\, A'A$$

→ $\varepsilon = \varepsilon_{\mathrm{PS}} + \varepsilon_{\mathrm{FS}}$

Wave function inside the nucleus is relevant.

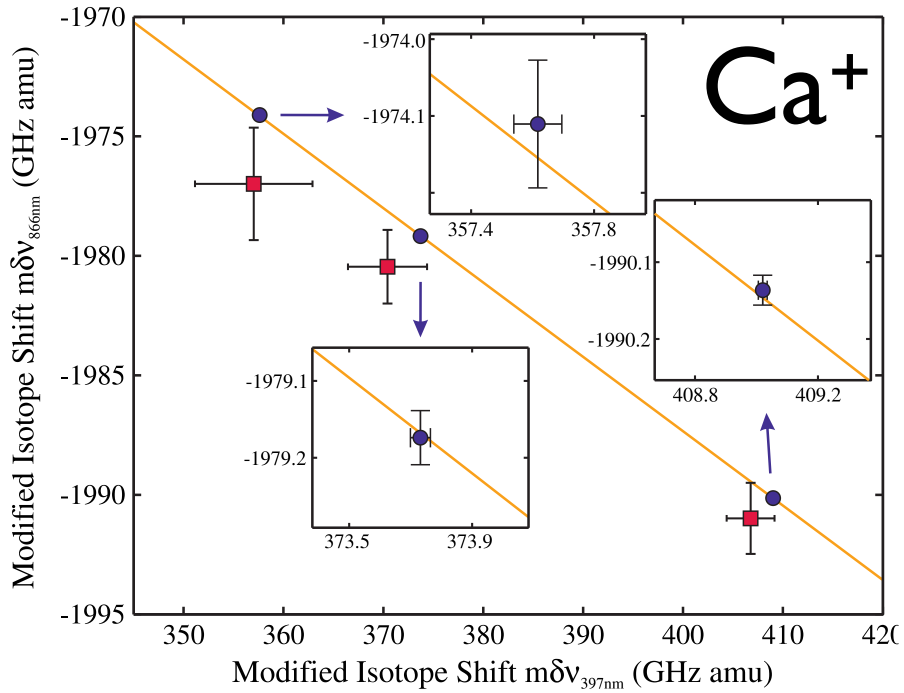
p state dominant: Ca^+ 4p, Yb^+ 6p

$$\varepsilon_{\mathrm{FS}} \propto Z|\psi'_{np}(0)|^2 \frac{d}{dA}\langle r^4 \rangle_A + \cdots$$

Status and prospect

Present constraint and future prospect

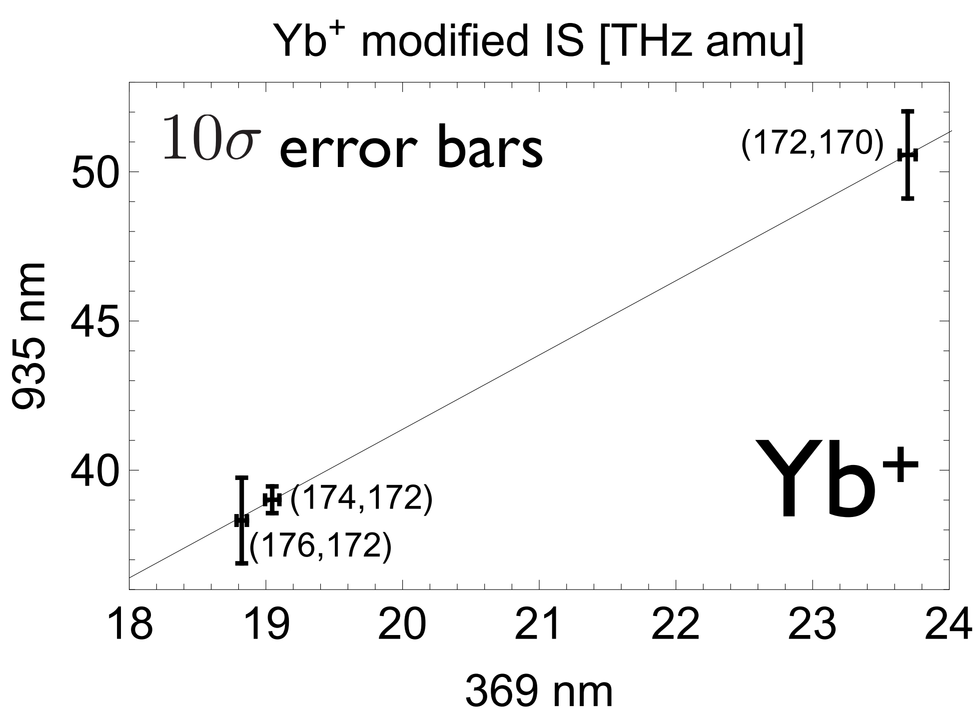
Data fitting with $\tilde{\nu}^2_{A'A} = K_{21} + F_{21}\tilde{\nu}^1_{A'A} + \varepsilon\, A'A$



$\varepsilon = (-2.45 \pm 4.05) \cdot 10^{-6}$ au

future prospect $\delta\nu = 1$ Hz

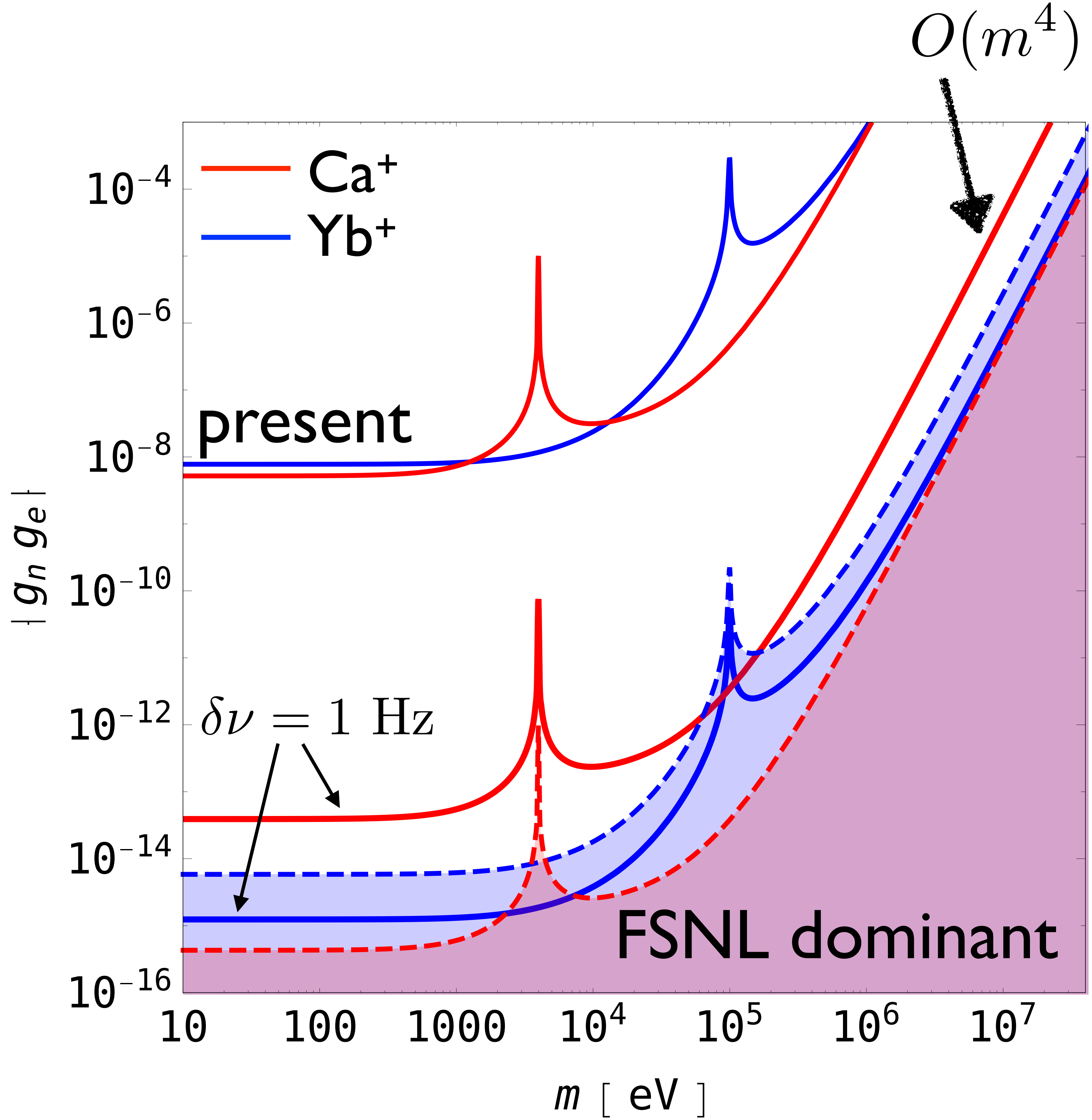
$|\varepsilon| < 4.5 \cdot 10^{-11}$



$\varepsilon = (-1.26 \pm 1.35) \cdot 10^{-4}$

future prospect $\delta\nu = 1$ Hz

$|\varepsilon| < 4.2 \cdot 10^{-11}$

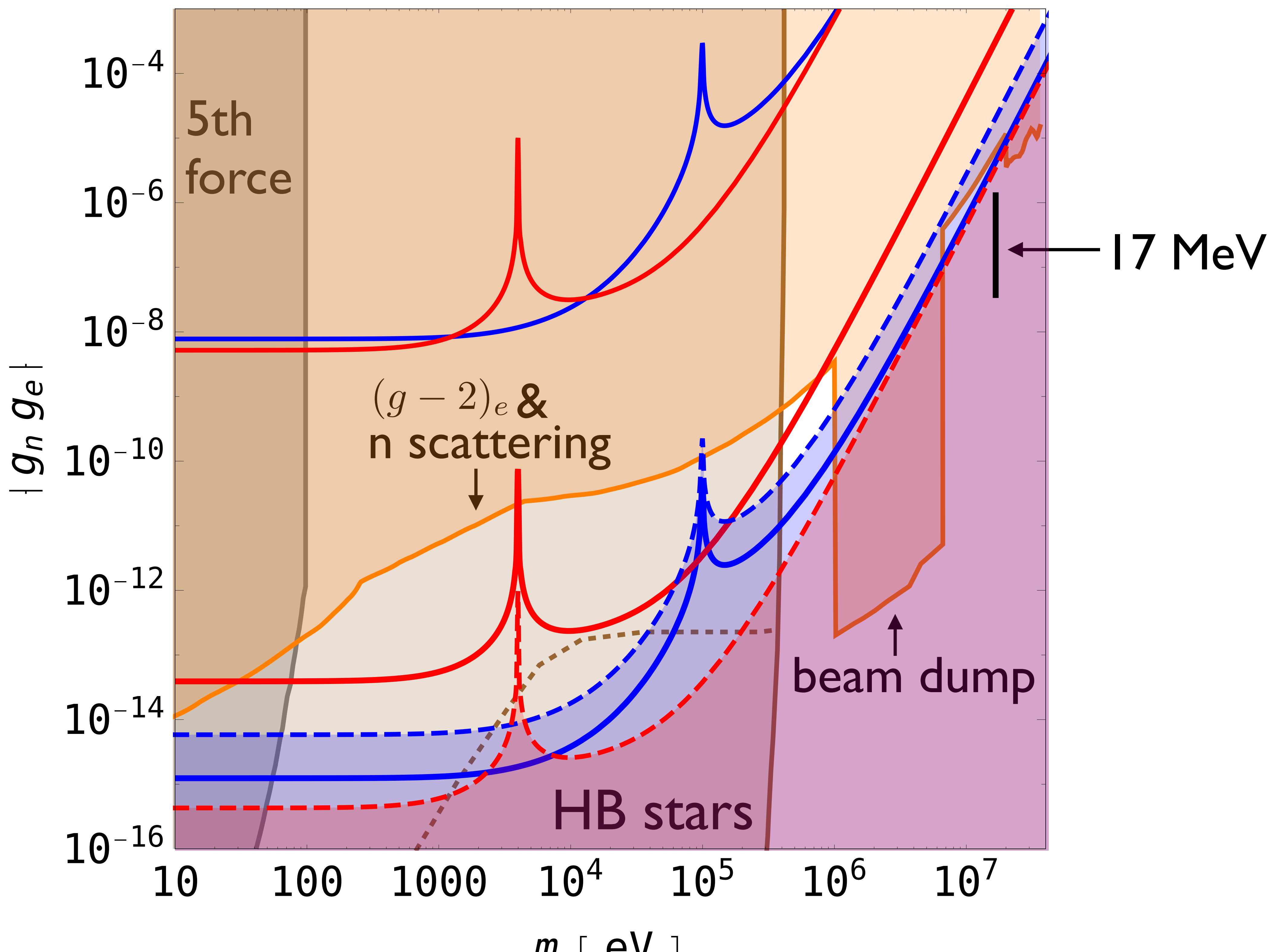


FSNL dominance:

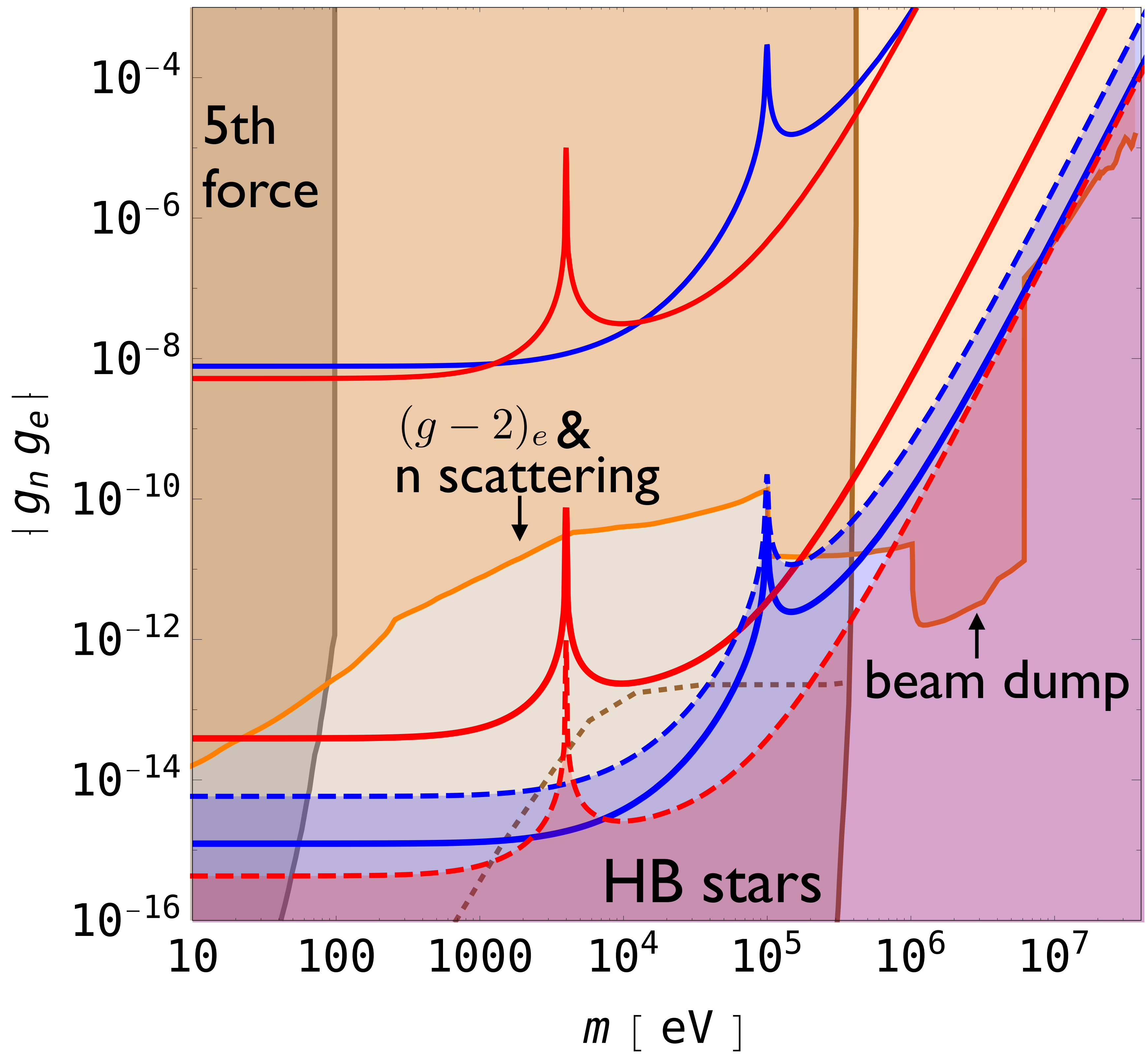
Ca^+ $\delta\nu \lesssim 0.01$ Hz

Yb^+ $\delta\nu \lesssim 4.7$ Hz

Comparison to other constraints: vector



Comparison to other constraints: scalar



Summary and outlook

Summary and outlook

- Isotope shift and King's linearity

  IS=MS+FS, $\tilde{\nu}^2_{A'A} = K_{21} + F_{21}\tilde{\nu}^1_{A'A}$

  Linear relation of modified IS of two lines

- Nonlinearity $\tilde{\nu}^2_{A'A} = K_{21} + F_{21}\tilde{\nu}^1_{A'A} + \varepsilon\, A'A$

  $\varepsilon = \varepsilon_{\mathrm{PS}} + \varepsilon_{\mathrm{FS}}$

  Particle shift nonlinearity: $\varepsilon_{\mathrm{PS}} \sim O(1/m^4)$

  sensitive for lighter particles, $m \ll 100\ \mathrm{MeV}$

  Other nonlinearities: more study needed

- Yb^+ ion trap project by Sugiyama et al. (Kyoto)

  $\delta\nu < 1\ \mathrm{Hz} \sim 100\ \mathrm{kHz}$

  possible with proved technique