

# 原子スペクトルにおける 同位体効果と 素粒子の新しい相互作用

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# Introduction

# Frontiers in particle physics

Energy frontier: LHC, ILC,...

Intensity frontier: B factory, muon,...

Cosmic frontier: CMB,...

Precision / low energy frontier

$0\nu\beta\beta$ , DM, EDM,...

Temporal variation of fundamental constants

$\alpha$ ,  $m_e/m_p$  using atomic clock

$\text{Yb}^+$  :  $\delta\nu/\nu \sim 10^{-18}$ ,  $\delta\nu \sim \text{sub Hz}$

Huntemann et al. (PTB) 2016

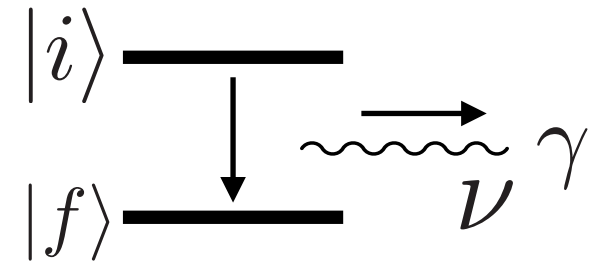
Isotope shift new neutron-electron interaction

# Isotope shift (IS)

Transition frequency difference between isotopes

$$h\nu_A = E_A^i - E_A^f$$

$$\text{IS} = \nu_{A'A} := \nu_{A'} - \nu_A$$



No IS for infinitely heavy and point-like nuclei

→  $\text{IS} = \text{MS} + \text{FS}$

Mass shift: finite mass of nuclei (reduced mass)

$$\text{MS} \propto \mu_{A'} - \mu_A \quad (\text{dominant for small } Z)$$

Field shift: finite size of nuclei

$$\text{FS} \propto \langle r^2 \rangle_{A'} - \langle r^2 \rangle_A \quad (\text{dominant for large } Z)$$

Theoretical calculation of IS: not easy

$$\text{IS} \sim O(\text{GHz}) \sim O(10 \mu\text{eV})$$

# Plan of talk

Introduction (2)

King's linearity (3)

Nonlinearities (10)

Status and prospect (4)

Summary and outlook (1)

# King's linearity

# King's linearity

King, 1963

IS of two transitions:  $t = 1, 2$

$$\nu_{A'A}^t = K_t \mu_{A'A} + F_t \langle r^2 \rangle_{A'A} \quad \begin{array}{l} \mu_{A'A} := \mu_{A'} - \mu_A \\ \langle r^2 \rangle_{A'A} := \langle r^2 \rangle_{A'} - \langle r^2 \rangle_A \end{array}$$

Modified IS:  $\tilde{\nu}_{A'A}^t := \nu_{A'A}^t / \mu_{A'A}$

$$\tilde{\nu}_{A'A}^t = \boxed{K_t} + \boxed{F_t \langle r^2 \rangle_{A'A} / \mu_{A'A}} \text{ nuclear factor}$$

electronic factors

King's linearity eliminating the nuclear factor

$$\tilde{\nu}_{A'A}^2 = K_{21} + \frac{F_2}{F_1} \tilde{\nu}_{A'A}^1 \quad K_{21} := K_2 - \frac{F_2}{F_1} K_1$$

→  $(\tilde{\nu}_{A'A}^1, \tilde{\nu}_{A'A}^2)$  on a straight line, King's plot

# IS data of Ca<sup>+</sup>

Gebert et al. PRL 115, 053003 (2015)

Line 1: 397 nm  $^2P_{1/2}(4p) - ^2S_{1/2}(4s)$

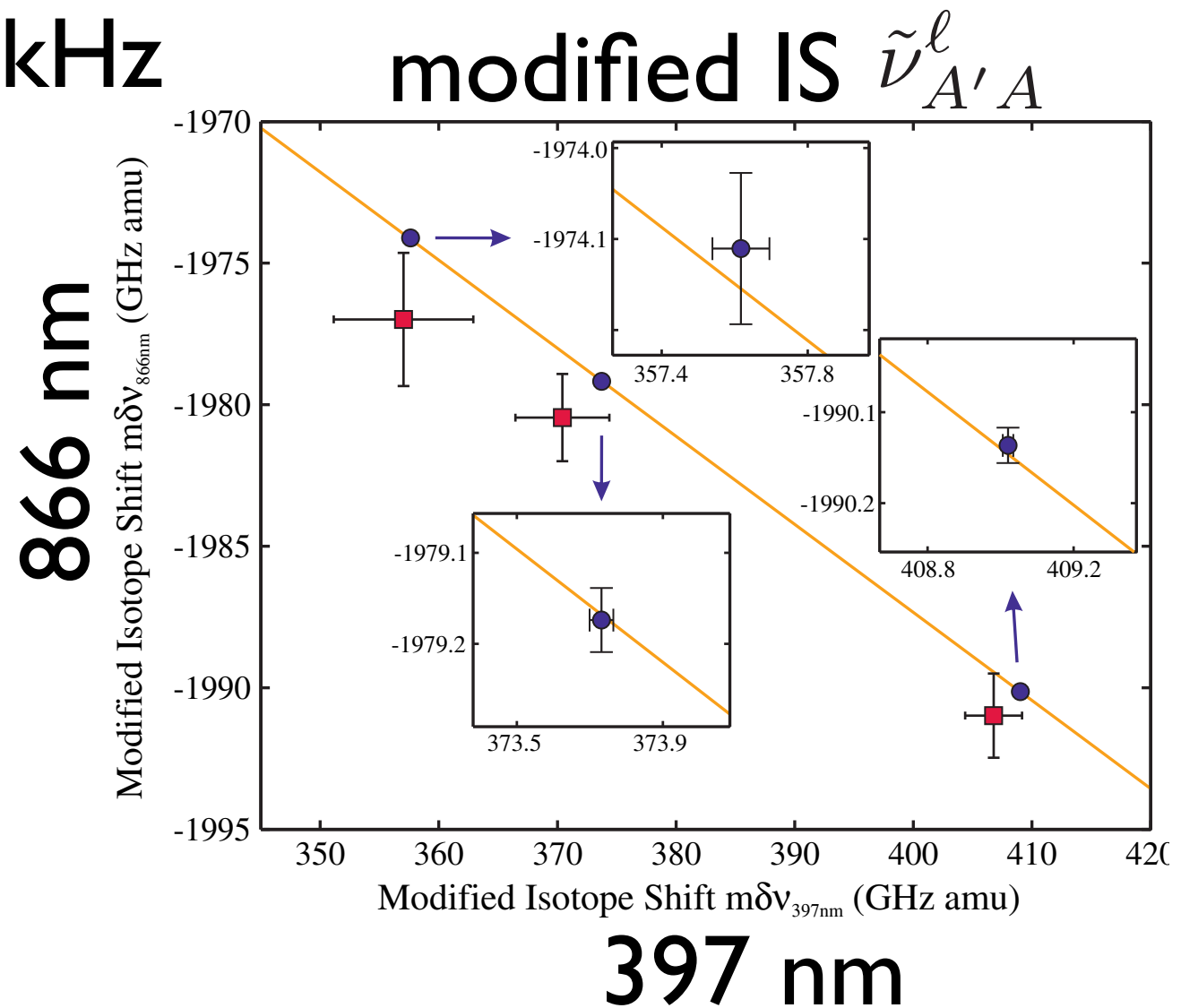
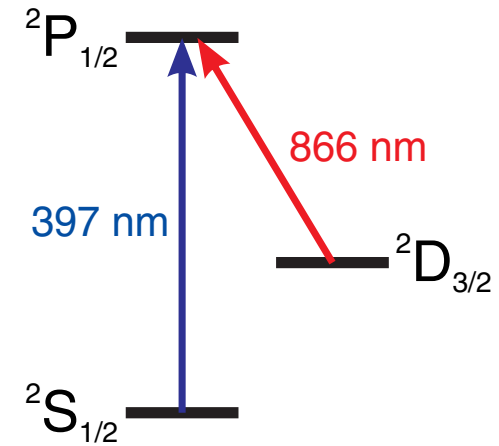
Line 2: 866 nm  $^2P_{1/2}(4p) - ^2D_{3/2}(3d)$

Isotope pairs: (42, 40), (44, 40), (48, 40)

IS precision  $\sim O(100)$  kHz

King's plot

linear within errors

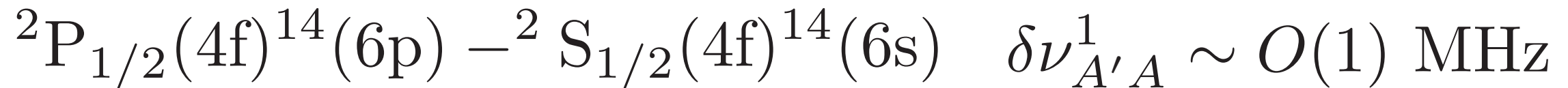




# IS data of Yb<sup>+</sup>

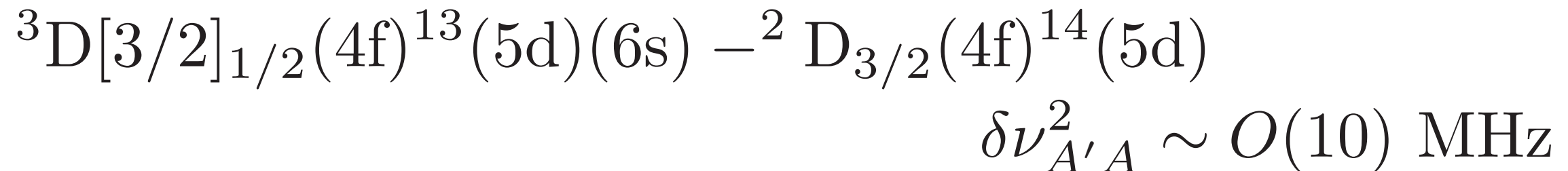
Line 1: 369 nm

Martensson-Pendrill et al. PRA49, 3351 (1994)



Line 2: 935 nm

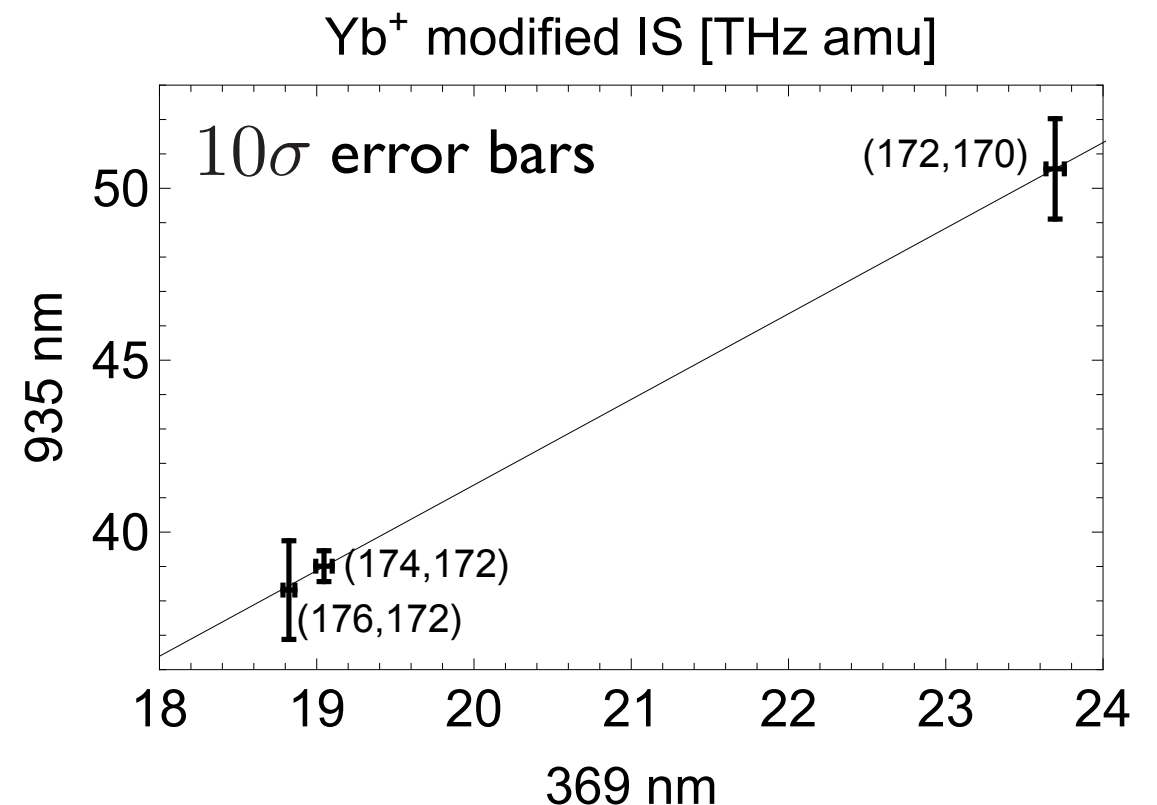
Sugiyama et al. CPEM2000



Isotope pairs: (172, 170), (174, 172), (176, 172)

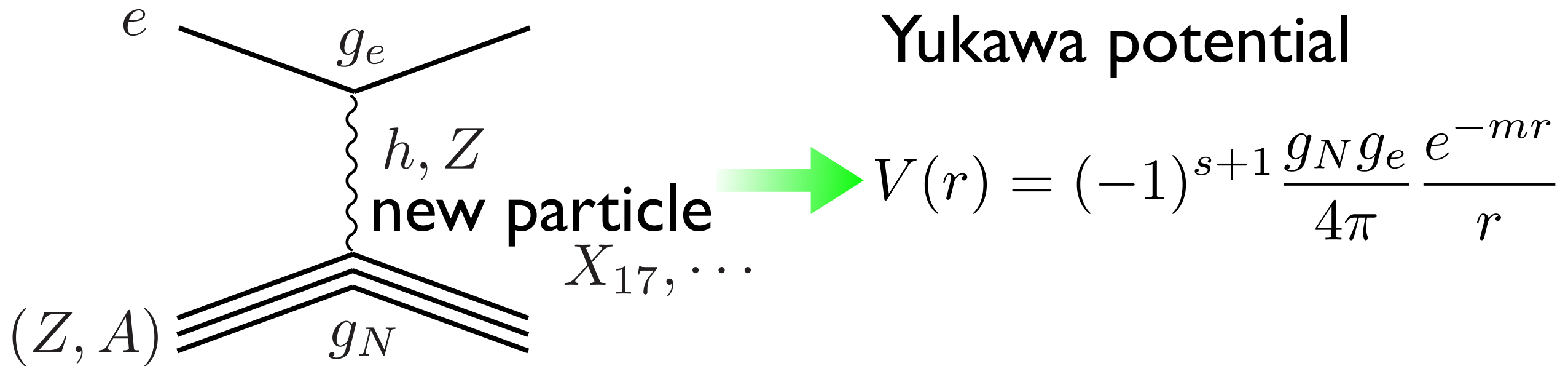
King's plot

linear within errors



# Nonlinearities

# Particle shift (PS)



## Frequency shifts by particle exchange (Yb<sup>+</sup> g.s.)

$$|\Delta\nu| \sim \begin{cases} 10^{-4} \text{ Hz} & \text{Higgs (SM)} \\ 400 \text{ Hz} & \text{Higgs (LHC bound)} \\ 800 \text{ Hz} & Z \\ 10 \text{ MHz} & X_{17} \text{ 17 MeV vector boson} \end{cases}$$

<< theoretical uncertainties

# Breakdown of the linearity by PS

Delaunay et al. arXiv:1601.05087v2

$$IS = MS + FS + PS$$

PS by new neutron-electron interaction

$$\nu_{A'A}^t = K_t \mu_{A'A} + F_t \langle r^2 \rangle_{A'A} + X_t (A' - A)$$

Generalized King's relation

$$\tilde{\nu}_{A'A}^2 = K_{21} + F_{21} \tilde{\nu}_{A'A}^1 + \varepsilon A'A \quad \text{nonlinearity}$$

probe into new physics

PS nonlinearity

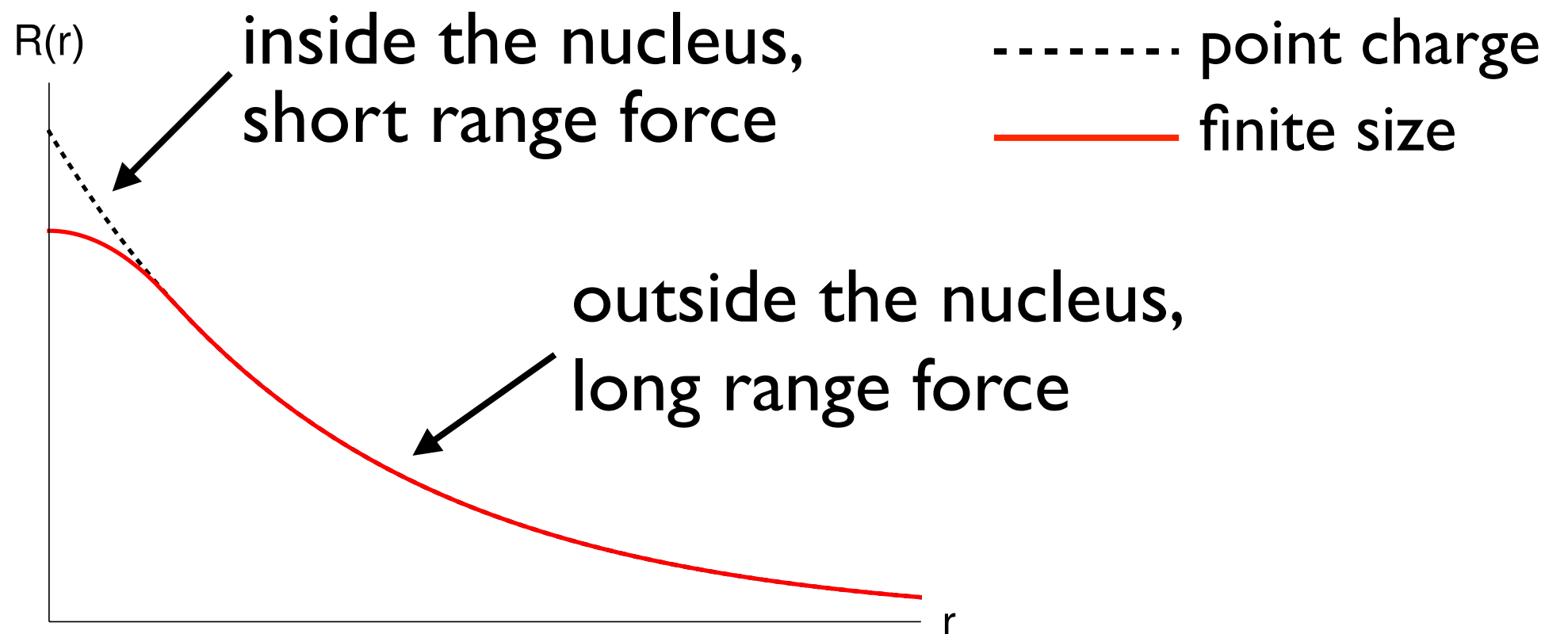
$$\varepsilon_{PS} = X_1 \left( \frac{X_2}{X_1} - \frac{F_2}{F_1} \right) \quad X_t \propto \frac{g_n g_e}{m^2} \text{ as } m \rightarrow \infty$$

# Evaluation of PS nonlinearity

## Single electron approximation

$$X_t = \frac{g_n g_e}{4\pi} \int r^2 dr \frac{e^{-mr}}{r} [R_{i_t}^2(r) - R_{f_t}^2(r)]$$

## Wave function



nuclear scale  $\longleftrightarrow$

$\longleftrightarrow$  atomic scale

# Wave function outside the nucleus

Non-relativistic (not bad for  $m \ll 100$  MeV)

Thomas-Fermi model

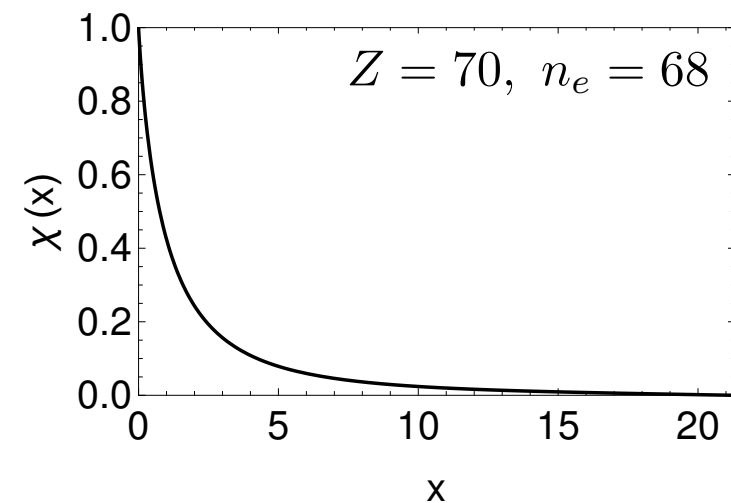
semiclassical, statistical, self-consistent field

exact in large  $Z$  limit

→ TF function

$$\frac{d^2 \chi}{dx^2} = x^{-1/2} \chi^{3/2}$$

$$\chi(0) = 1, \quad x_0 \chi'(x_0) = \frac{n_e}{Z} - 1, \quad \chi(x_0) = 0$$



One-body problem in the TF potential

$$V_{\text{TF}}(r) = -\frac{Z\alpha}{r} \chi\left(\frac{r}{b}\right) - (Z - n_e)\alpha \min\left(\frac{1}{r_0}, \frac{1}{r}\right)$$

$$b = (9\pi^2/2^7 Z)^{1/3} a_B, \quad a_B = \text{Bohr radius}$$

# Comparison to experiments

$\text{Ca}^+$	EX	TF	$\text{Yb}^+$	EX	TF
$4p \rightarrow 4s$	397	475	$6p \rightarrow 6s$	369	380
$4p \rightarrow 3d$	866	-1610	$6s \rightarrow 4f$	935	48.6

wavelength in nm

good for s and p states

poor for d and f states

# Wave function inside the nucleus

One-body problem in the nuclear potential  $V_A(r)$

$$\left[ \frac{d}{dr^2} - \frac{\ell(\ell + 1)}{r^2} + 2m_e \{E - V_A(r)\} \right] rR(r) = 0$$

$\ell = \text{angular momentum}$

Series expansion:  $V_A(r) = \sum_{i=0} v_i r^i, v_1 = 0$

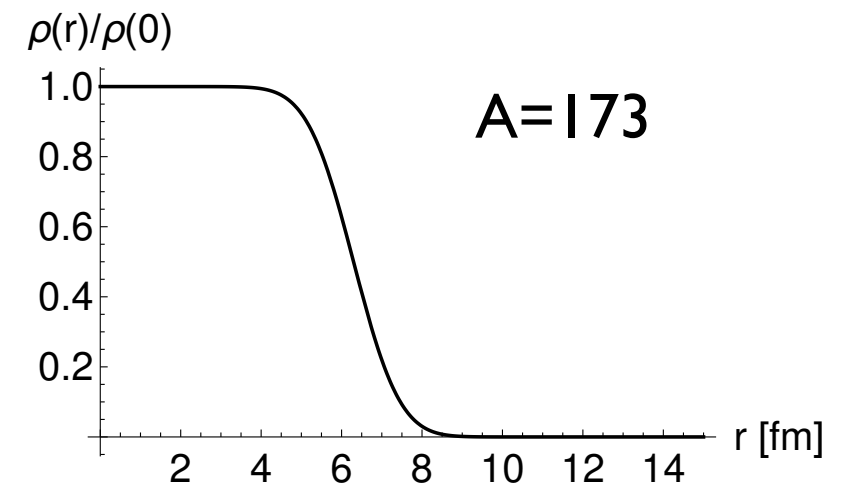
$$R(r) = \sum_{i=0} \chi_i^\ell r^{\ell+i}$$

$$\rightarrow \chi_1^\ell = 0, \chi_2^\ell / \chi_0^\ell = m_e v_0 / (2\ell + 3)$$



# Nuclear charge distribution

Helm distribution Helm 1956



Gaussian smearing of uniform sphere

$$\rho_A(r) = \int d^3 r' \frac{3}{4\pi r_A^3} \theta(r_A - r') \frac{1}{(2\pi s^2)^{3/2}} e^{-|\mathbf{r}-\mathbf{r}'|^2/(2s^2)}$$

$$r_A^2 = c^2 + 7\pi^2 a^2/3 - 5s^2, \quad s \simeq 0.9 \text{ fm}$$

$$a \simeq 0.52 \text{ fm}, \quad c \simeq 1.23A - 0.60 \text{ fm} \quad \text{Lewin, Smith 1996}$$

$$\langle r^2 \rangle = \frac{3}{5}(r_A^2 + 5s^2), \quad \langle r^4 \rangle = \frac{3}{7}(r_A^4 + 14r_A^2 s^2 + 35s^4)$$

$$v_0 = \frac{3Z\alpha}{2r_A} \left[ \left(1 - \frac{s^2}{r_A^2}\right) \text{Erf} \left( \frac{r_A}{\sqrt{2}s} \right) + \sqrt{\frac{2}{\pi}} \frac{s}{r_A} e^{-r_A^2/(2s^2)} \right]$$

$v_1 = 0$  no cusp at the origin

# Seltzer moment expansion of field shift

Seltzer 1969

$$\text{FS} = 4\pi Z\alpha \int_0^\infty dr_N r_N^2 \rho_{A'A}(r_N) \int_0^{r_N} dr_e \left( r_e - \frac{r_e^2}{r_N} \right) \rho_{if}(r_e)$$

$$\rho_{A'A}(r) := \rho_{A'}(r) - \rho_A(r)$$

$$\rho_{if}(r) := R_i^2(r) - R_f^2(r) = r^{2\ell} \sum_{k=0} \xi_k^\ell r^k, \quad \ell = \min(\ell_i, \ell_f)$$

$$= Z\alpha \sum_{k=0} \frac{\xi_k^\ell}{(2\ell + k + 3)(2\ell + k + 2)} \langle r^{2\ell+k+2} \rangle_{A'A}$$

$$\langle r^n \rangle_{A'A} := \langle r^n \rangle_{A'} - \langle r^n \rangle_A$$

$$= F_t \langle r^2 \rangle_{A'A} + \dots, \quad F_t = \frac{Z\alpha}{6} \xi_0^0$$

**Note:**  $\xi_1^\ell = 0$  no cubic term

# Heavy particle limit

$$ma_B \gg Z, \quad a_B = \text{Bohr radius} \sim (4 \text{ keV})^{-1}$$

$$F_t, X_t \propto |\psi_{i_t}(0)|^2 - |\psi_{f_t}(0)|^2 \longrightarrow \lim_{m \rightarrow \infty} \left( \frac{X_2}{X_1} - \frac{F_2}{F_1} \right) = 0$$

## Asymptotic behavior of PS

$$X_t \propto \int dr r^2 \rho_{i_t f_t}(r) \frac{e^{-mr}}{r} = \frac{1}{m^2} \sum_{k=0} (2\ell + k + 1)! \frac{\xi_k^\ell}{m^{2\ell+k}} + \dots$$

$\xi_1^0 = 0$  for nucl. charge distribution without cusp

$$\frac{X_2}{X_1} - \frac{F_2}{F_1} \sim O\left(\frac{1}{m^2}\right) \longrightarrow \epsilon_{\text{PS}} \sim O\left(\frac{1}{m^4}\right)$$

less sensitive to heavier particles


cf. Berengut et al. arXiv:1704.05068  $\epsilon_{\text{PS}} \propto 1/m^3$

# Field shift nonlinearity

One of the sources of nonlinearity in QED

$$\text{FS} = F_\ell \langle r^2 \rangle_{A'A} + G_t \langle r^4 \rangle_{A'A}$$

$$\tilde{\nu}_{A'A}^2 = K_{21} + F_{21} \tilde{\nu}_{A'A}^1 + \varepsilon_{A'A}$$

  $\varepsilon = \varepsilon_{\text{PS}} + \varepsilon_{\text{FS}}$

Wave function inside the nucleus is relevant.

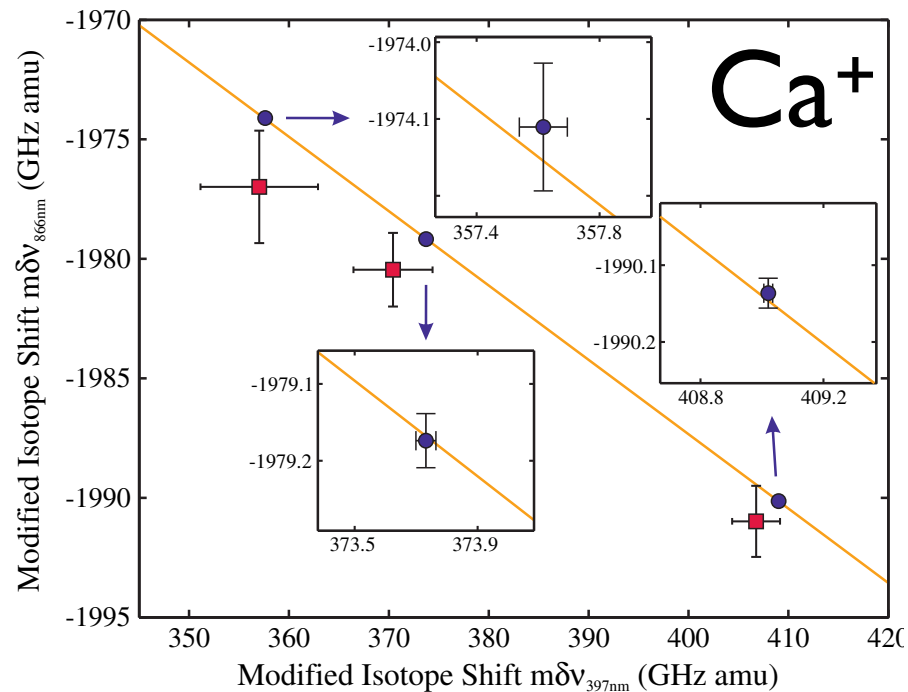
p state dominant:  $\text{Ca}^+ 4p, \text{Yb}^+ 6p$

$$\varepsilon_{\text{FS}} \propto Z |\psi'_{np}(0)|^2 \frac{d}{dA} \langle r^4 \rangle_A + \dots$$

# Status and prospect

# Present constraint and future prospect

Data fitting with  $\tilde{\nu}_{A'A}^2 = K_{21} + F_{21}\tilde{\nu}_{A'A}^1 + \varepsilon A'A$

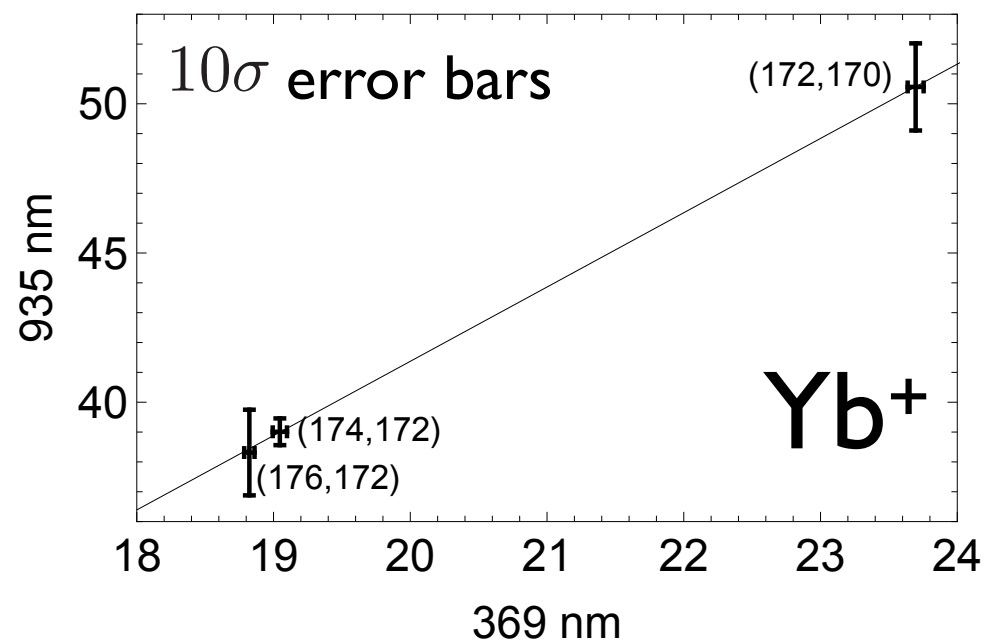


$$\varepsilon = (-2.45 \pm 4.05) \cdot 10^{-6} \text{ au}$$

future prospect  $\delta\nu = 1 \text{ Hz}$

$$|\varepsilon| < 4.5 \cdot 10^{-11}$$

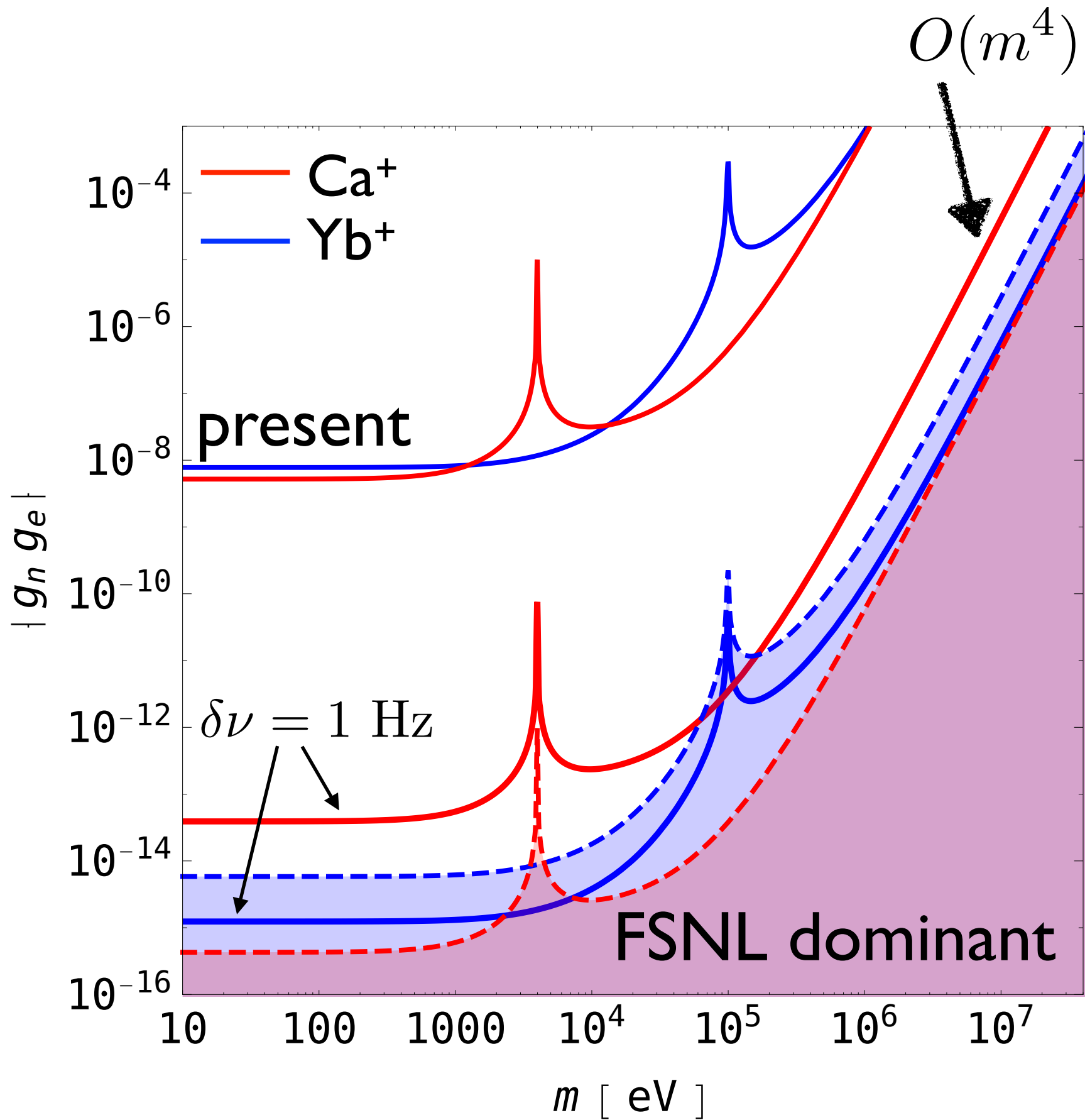
Yb<sup>+</sup> modified IS [THz amu]



$$\varepsilon = (-1.26 \pm 1.35) \cdot 10^{-4}$$

future prospect  $\delta\nu = 1 \text{ Hz}$

$$|\varepsilon| < 4.2 \cdot 10^{-11}$$

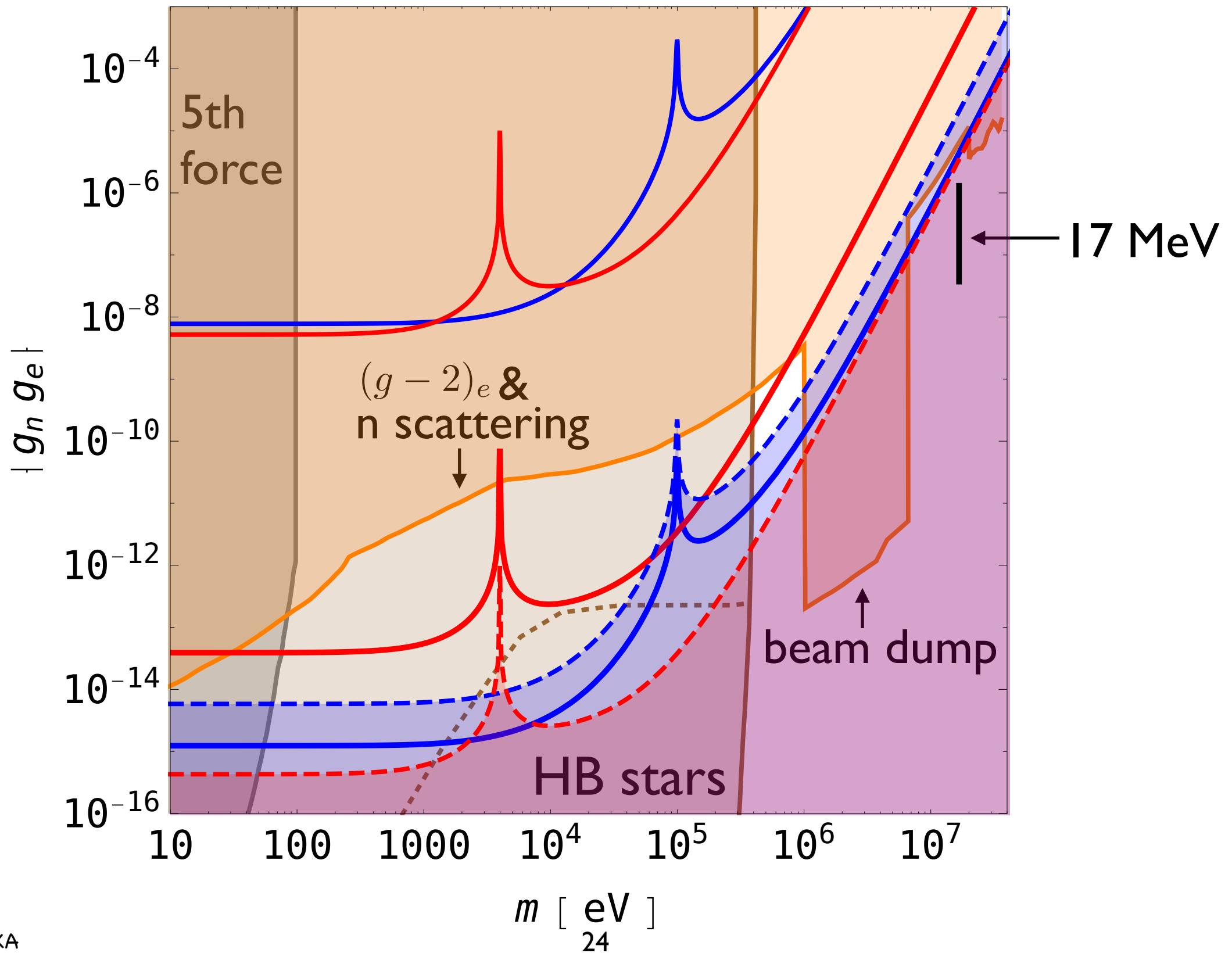


**FSNL dominance:**

$$\text{Ca}^+ \quad \delta\nu \lesssim 0.01 \text{ Hz}$$

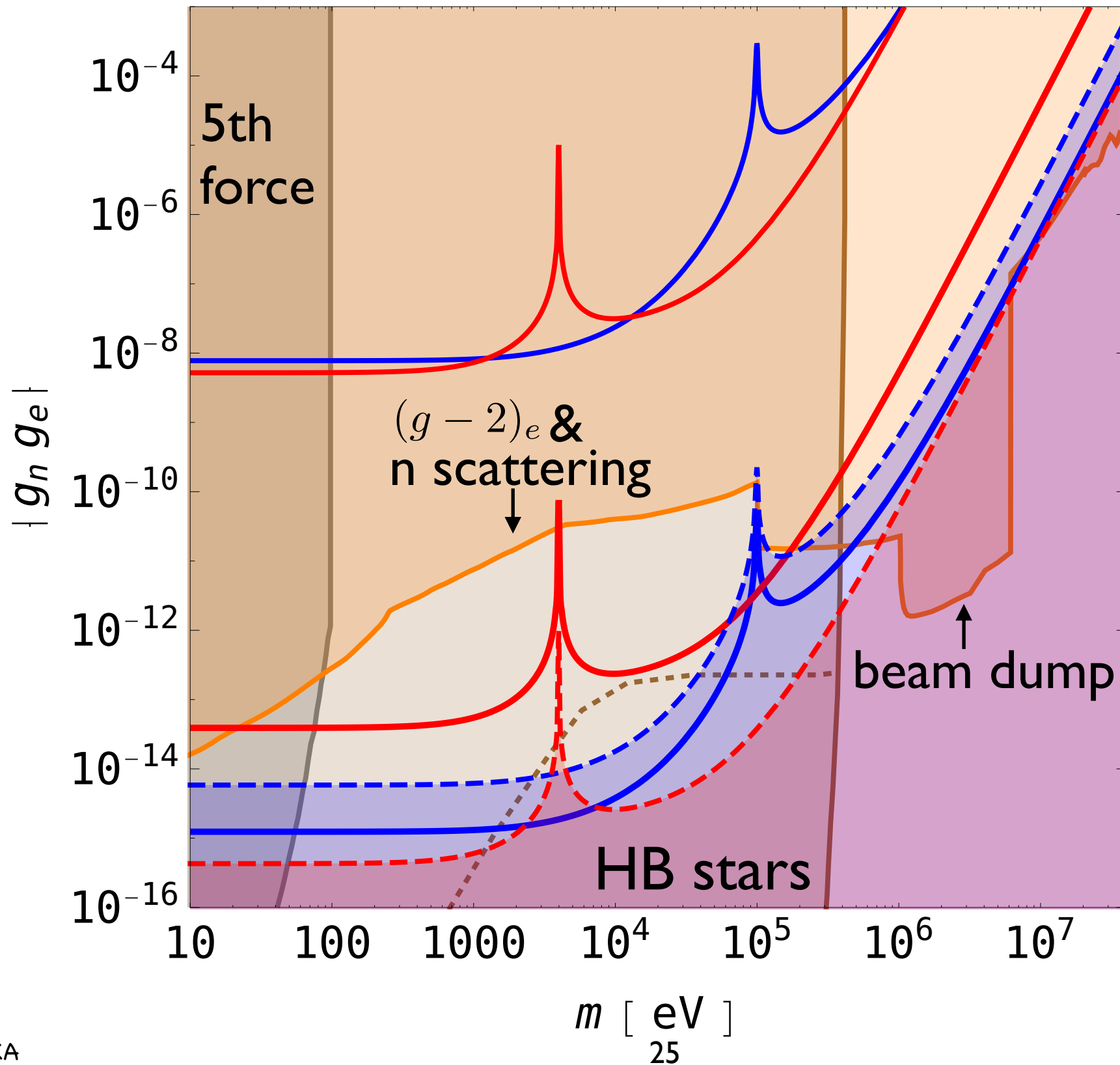
$$\text{Yb}^+ \quad \delta\nu \lesssim 4.7 \text{ Hz}$$

# Comparison to other constraints: vector





# Comparison to other constraints: scalar



# Summary and outlook

# Summary and outlook

## ■ Isotope shift and King's linearity

$$\text{IS}=\text{MS}+\text{FS}, \quad \tilde{\nu}_{A'A}^2 = K_{21} + F_{21}\tilde{\nu}_{A'A}^1$$

Linear relation of modified IS of two lines

## ■ Nonlinearity $\tilde{\nu}_{A'A}^2 = K_{21} + F_{21}\tilde{\nu}_{A'A}^1 + \varepsilon A'A$

$$\varepsilon = \varepsilon_{\text{PS}} + \varepsilon_{\text{FS}}$$

Particle shift nonlinearity:  $\varepsilon_{\text{PS}} \sim O(1/m^4)$

sensitive for lighter particles,  $m \ll 100 \text{ MeV}$

Other nonlinearities: more study needed

## ■ Yb<sup>+</sup> ion trap project by Sugiyama et al. (Kyoto)

$$\delta\nu < 1 \text{ Hz} \sim 100 \text{ kHz}$$

possible with proved technique