Physics
OSAKA UNIVERSITY

イオンビームを用いた
高エネルギー光渦生成の基礎的研究

田中 実（阪大理）

「量子ビーム応用」領域 公募研究

領域研究会 (第 3 回領域全体会議) 2021/06/15

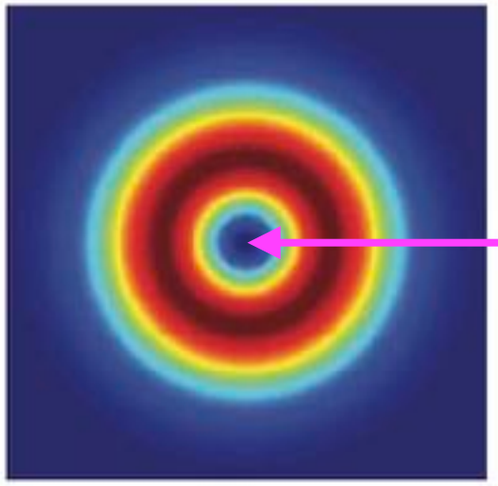
科研費
KAKENHI

Twisted photon(捩光子), optical vortex(光渦)

Orbital angular momentum (OAM) of light

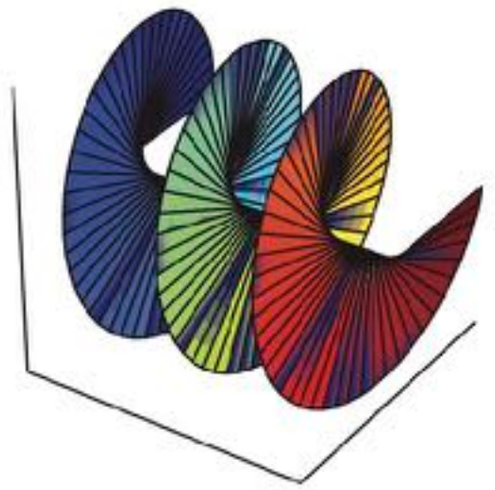
winding field phase ~ $e^{im\varphi}$

G. Molina-Terriza et al.
Nat. Phys. 3, 305 (2007)



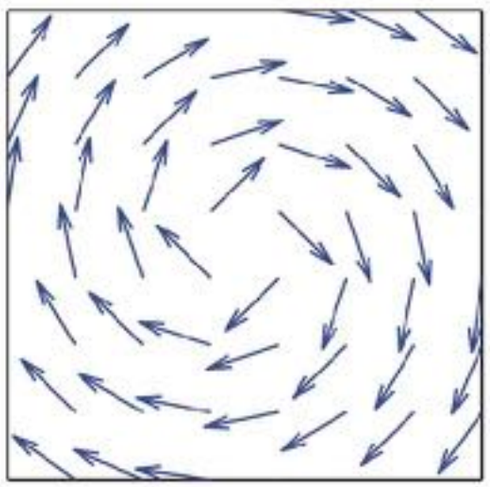
field intensity

phase singularity

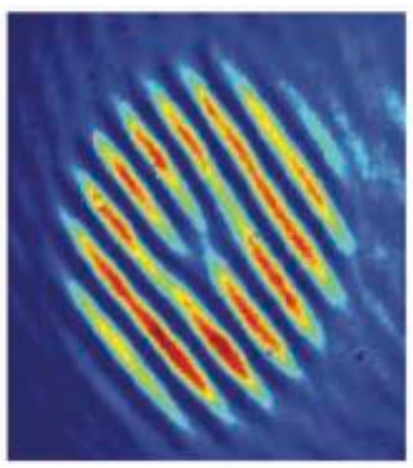


wave front

helicoid



transvers Poynting vector



interference pattern with plane wave

computer-generated hologram

Generation (and use) of twisted photons

Optical region

Y. Shen et al., Light: Sci. & App. 8, 90 (2019)

fork hologram, lens-based mode converter, etc.

(micro manipulation, imaging, data transmission, etc.)

X-ray region

S. Sasaki, I. McNulty, PRL 100, 124801 (2008)
E. Hemsing et al. Nat. Phys. 9, 549 (2013)

helical undulator, FEL

Gamma-ray region (proposals)

backward Compton scattering

$$e + \gamma_{\mathrm{tw}} \to e + \gamma_{\mathrm{tw}}$$

U.D. Jentschura, V.G. Serbo, PRL 106, 013001 (2011)

nonlinear Thomson scattering

$$e + \gamma_{\mathrm{pw}} + \gamma_{\mathrm{pw}} \to e + \gamma_{\mathrm{tw}}$$

Y. Taira, T. Hayakawa, M. Katoh, Sci. Rep. 7, 5018 (2017)

resonant Rayleigh scattering with boosted ions

$$\gamma_{\mathrm{tw}} + I \to I^* \to I + \gamma_{\mathrm{tw}}$$

D. Budker et al. Ann. Phys. (Berlin) 532, 2000204 (2020)

MT, N. Sasao, arXiv:2102.00661, to appear in IJMPE
(doi:10.1142/S0218301321500403)

Ion beam vs electron beam

Ions

resonant Rayleigh scattering

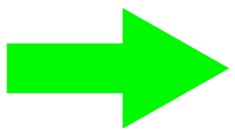
$$\sigma \propto \lambda^2 \sim \frac{1}{(Z^2\alpha^2 m_e)^2}$$

Electrons

Thomson/Compton scattering

$$\sigma \propto r_0^2 = \left(\frac{\alpha}{m_e}\right)^2$$

Larger cross section of ions



twisted gamma rays from boosted ions

Gamma factory

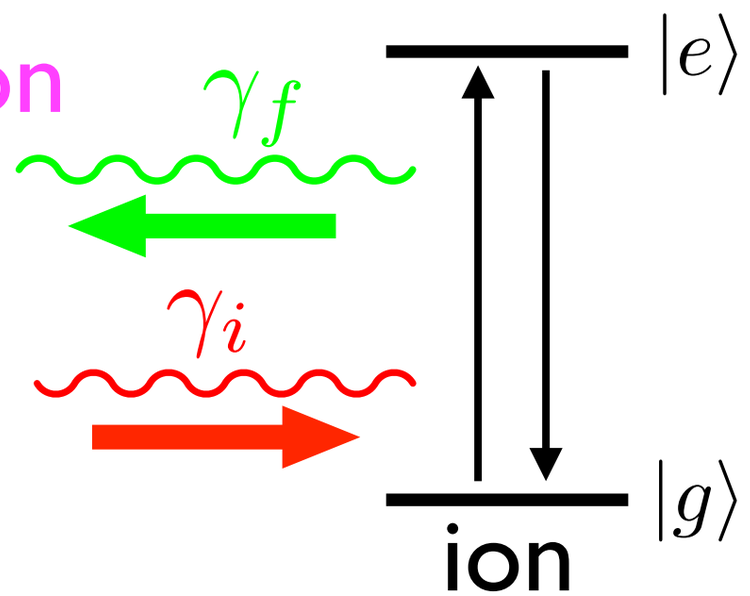
E.G.Bessonov, NIMB 309, 92 (2013)
M.W. Krasny, CERN-SPSC-2019-031; SPSC-I-253

Rayleigh scattering by boosted ion

$$\gamma_i + |g\rangle \to |e\rangle \to |g\rangle + \gamma_f$$

Lorentz boost $E = \gamma M$

e.g. $\gamma \sim 10^3$ @LHC



Level splitting: E_{eg}

binding energy of H-like ion $= (Z^2/n^2)13.6$ eV

Resonance condition: $2\gamma\omega_i \simeq E_{eg}$

$\omega_i \sim$ 1-10 eV ⟶ $Z^2/2\gamma \sim 0.1$-1

heavy ion

Up-conversion:

$$\omega_f^{\max} \simeq 2\gamma E_{eg} \simeq 4\gamma^2\omega_i \sim 0.1\text{-}1\ \mathrm{GeV}(2\gamma/10^4)^2$$

Twisted gamma rays from boosted ions

$$\gamma_{\mathrm{tw}} + I \rightarrow I^* \rightarrow I + \gamma_{\mathrm{tw}}$$

Excitation to states of larger angular momentum

Ex. $1\mathrm{s}_{1/2} \rightarrow 3\mathrm{d}_{5/2}(|m| = 5/2)$

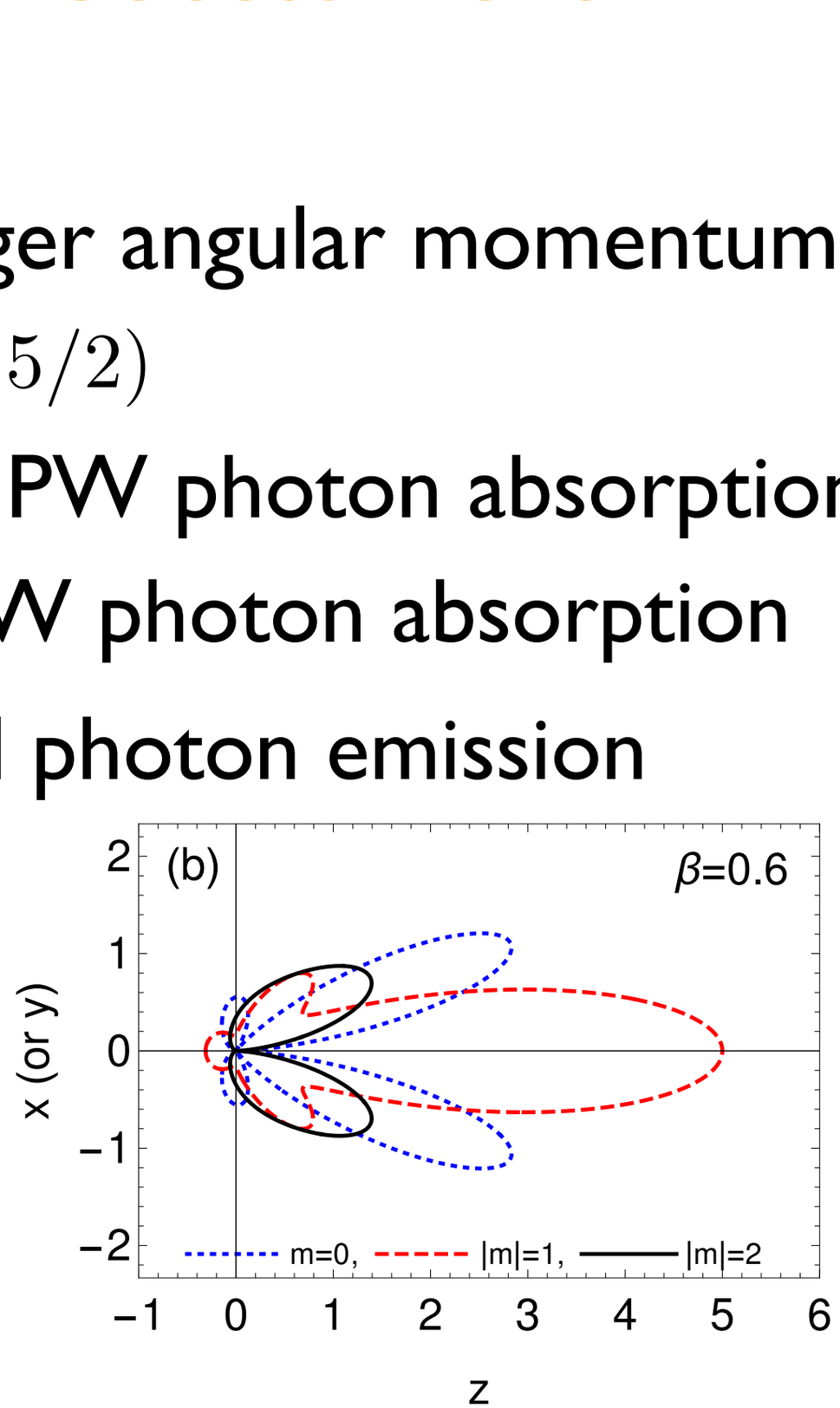
forbidden in single PW photon absorption

allowed in single TW photon absorption

Deexcitation with twisted photon emission

$$3\mathrm{d}_{5/2}(|m| = 5/2) \rightarrow 1\mathrm{s}_{1/2}$$

E2 radiation



Bessel beam

U.D. Jentschura, V.G. Serbo, PRL 106, 013001 (2011)

Twisted photon as a superposition of plane waves

PW: $A^{\mu}_{\boldsymbol{k}\lambda}(t,\boldsymbol{x}) = \varepsilon^{\mu}_{\lambda}(\boldsymbol{k}) e^{-i(\omega t - \boldsymbol{k}\cdot\boldsymbol{x})}/\sqrt{2\omega}$

Twisted photon (Bessel beam):

$$A^{\mu}_{m k_T k_z \lambda}(t,\boldsymbol{x}) := \int a_{m k_T}(\boldsymbol{k}_T) A^{\mu}_{\boldsymbol{k}\lambda}(t,\boldsymbol{x}) dk_T^2/(2\pi)^2$$

$$a_{m k_T}(\boldsymbol{k}_T) := (-i)^m e^{im\varphi_k} \sqrt{2\pi/k_T}\, \delta(|\boldsymbol{k}_T| - k_T)$$

$$A^{\mu}_{m k_T k_z \lambda}(t,\boldsymbol{x}) = -i\lambda\sqrt{k_T/4\pi\omega}\, e^{-i(\omega t - k_z z)} \Big[e^{i(m-\lambda)\varphi} \cos^2\frac{\theta_k}{2} J_{m-\lambda}(k_T\rho)\eta^{\mu}_{\lambda}$$

$$- e^{i(m+\lambda)\varphi} \sin^2\frac{\theta_k}{2} J_{m+\lambda}(k_T\rho)\eta^{\mu}_{-\lambda} + \frac{i}{\sqrt{2}} e^{im\varphi} \sin\theta_k J_m(k_T\rho)\eta^{\mu}_0 \Big]$$

$$\eta^{\mu}_{\lambda} := (0, -\lambda, -i, 0)/\sqrt{2}\,,\ \eta^{\mu}_0 := (0,0,0,1)$$

$$\sin\theta_k := |\boldsymbol{k}_T|/|\boldsymbol{k}|$$ pitch angle

Heavy ion excitation by twisted photons

Relativistic effects ~ $O(Z\alpha)$

Dirac theory of Hydrogen-like ion:

interaction hamiltonian $H_I = e\,\boldsymbol{\alpha}\cdot\boldsymbol{A}$

wave function

$$\psi(\boldsymbol{x}) = \begin{pmatrix} \dfrac{G(r)}{r}\mathcal{Y}^{j_3}_{j\ell_A}(\theta,\varphi) \\ i\dfrac{F(r)}{r}\mathcal{Y}^{j_3}_{j\ell_B}(\theta,\varphi) \end{pmatrix}$$

transition matrix element: $|i\rangle \to |f\rangle$

$$\mathcal{M}_{fi} = e\int d^3x\,\psi^\dagger_f(\boldsymbol{x})\boldsymbol{\alpha}\psi_i(\boldsymbol{x})\cdot\boldsymbol{A}(t,\boldsymbol{x})e^{i\omega t}$$

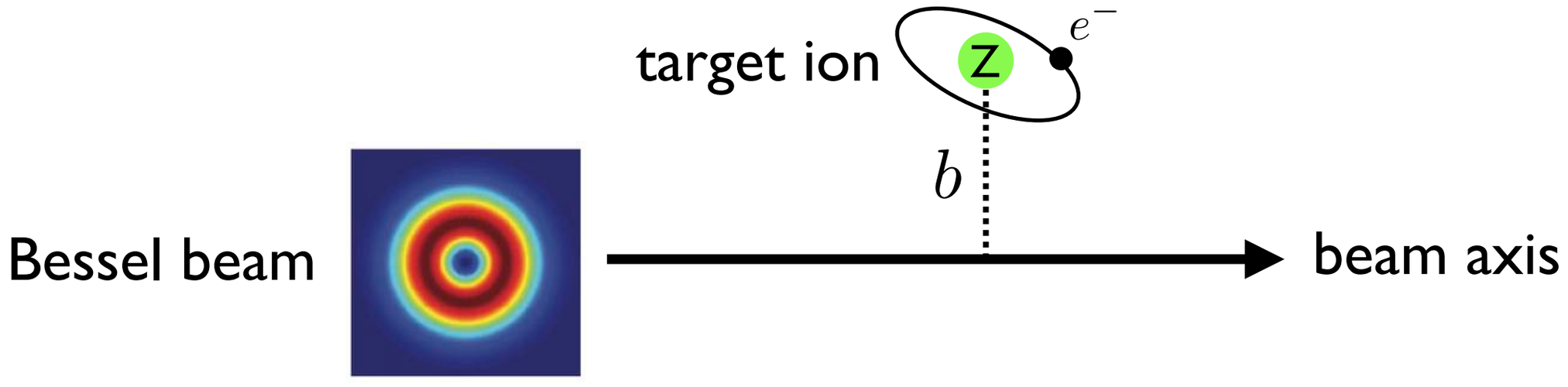
H.M. Scholz-Marggraf et al. PRA90, 013425(2014)

Twisted photon amplitude

a superposition of plane wave amplitudes

$$\mathcal{M}_{fi}^{(\mathrm{tw})} = (-i)^{2m+m_i-m_f}\sqrt{\frac{k_T}{2\pi}}\, e^{i(m+m_i-m_f)\phi_b}\, J_{m+m_f-m_i}(k_T b)$$
$$\times \sum_{m'_f, m'_i} d^{j_f}_{m_f m'_f}(\theta_k) d^{j_i}_{m_i m'_i}(\theta_k) \mathcal{M}^{(\mathrm{pl})}_{m'_f m'_i}$$

impact parameter: $\boldsymbol{b} = b(\cos\phi_b, \sin\phi_b, 0)$



Cross section

Randomly distributed ions

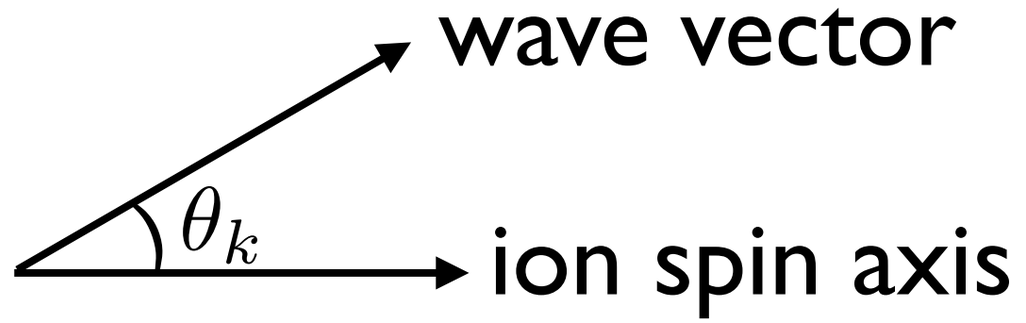
➡ average over the impact parameter $(k_T R \gg 1)$

$$\sigma = \frac{4}{(\Gamma_f + \Gamma_L)\cos\theta_k} |\mathcal{M}_{fi}^{(\mathrm{pl})}(\theta_k)|^2$$ on-resonance

Γ_f : natural width, Γ_L : laser width

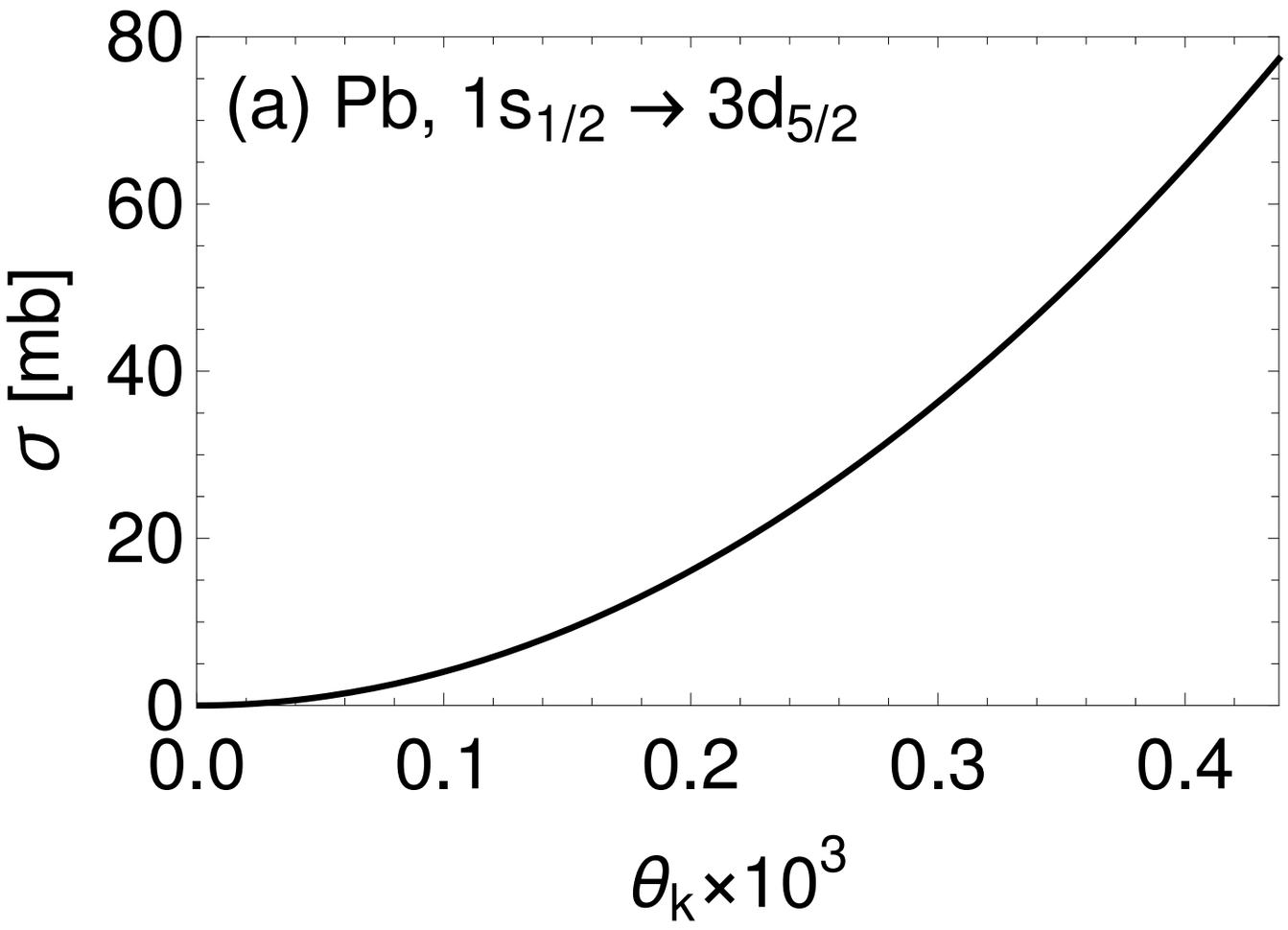
$$\mathcal{M}_{fi}^{(\mathrm{pl})}(\theta_k) = \sum_{m'_f, m'_i} d^{j_f}_{m_f m'_f}(\theta_k) d^{j_i}_{m_i m'_i}(\theta_k) \mathcal{M}^{(\mathrm{pl})}_{m'_f m'_i}$$

oblique plane wave amplitude



H-like Pb (Z=82) $1\mathrm{s}_{1/2}(m_i = 1/2) \to 3\mathrm{d}_{5/2}(m_f = 5/2)$

$$E_{eg} \simeq 91\,\mathrm{keV}\,,\ \omega_i = 10\,\mathrm{eV}\,,\ \gamma \simeq 4600\,,\ \omega_f^{\max} \simeq 830\,\mathrm{MeV}$$



$$\theta_k \lesssim 1/\gamma \ll 1$$

Deexcitation to emit twisted photons

$$\mathrm{BR}(3\mathrm{d}_{5/2} \to 1\mathrm{s}_{1/2}) \simeq 0.045$$

Summary

- ✸ Twisted photons: OAM of light
- ✸ Energy up-conversion with boosted ions

  $\omega_f^{\max} \simeq 4\gamma^2\omega_i\,,\ 2\gamma \sim 10^4$ optical → gamma ray
- ✸ Twisted photon process: forbidden → allowed
- ✸ Absorption & emission rates

  Relativistic calculation for heavy ions

  H-like Pb, typical CS x BR ~ 1 mb
- ✸ Twisted photon flux

  ~500 photons/s at the proposed Gamma factory

今後の展開

背景事象 $1\mathrm{s}_{1/2}(m_i = 1/2) \to 3\mathrm{d}_{5/2}(m_f = 3/2)$

解決法: 衝突点付近で磁場を印加

boostによるStark効果でmf=5/2を選択励起

spin rotatorが必要

2光子励起 $\gamma_{\mathrm{pw}} + \gamma_{\mathrm{pw}} + I \to I^*$, $\gamma_{\mathrm{pw}} + I \to I^* + \gamma_{\mathrm{pw}}$

cf. $e + \gamma_{\mathrm{pw}} + \gamma_{\mathrm{pw}} \to e + \gamma_{\mathrm{tw}}$

He-like, Li-like, Be-like ions

笹尾さん（岡山大）, 田代さん （東洋大）

物理応用

核子・核スピン, ガンマ線光渦検出器,
ガンマ線天体の回転, ．．．