Bubbling Geometries for Half BPS Wilson Lines

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S. Yamaguchi, to appear

1. Overview

AdS_5 / CFT_4

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AdS5 x S5

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4dim N=4
Super YM theory

deform

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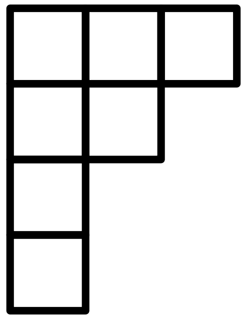
Goal

Supergravity
Solutions

?

Wilson loop
operators

$$Tr_R P\exp\left[i\oint(A+\phi)\right]$$



2. 4 dim N=4 U(N)SYM

Field Contents

Trivial reduction of 10 dim SYM

- Vector $A_\mu, \mu=0,1,2,3$
- Spinors λ 16 real components
- Scalars $\phi_i, i=4,\cdots,9$

Each field is N x N Hermitian Matrix

Symmetries

R-symmetry SO(6)=SU(4)

A_μ : Singlet λ : Spinor ϕ_i: Vector

Supersymmetry

$$\delta A_\mu = i\bar{\lambda}\Gamma_\mu \epsilon$$

$$\delta \phi_i = i\bar{\lambda}\Gamma_i \epsilon$$

$$\delta \lambda = \cdots$$

Conformal Symmetry SO(2,4)=SU(2,2)

Gauge invariant operators

½ BPS local operators

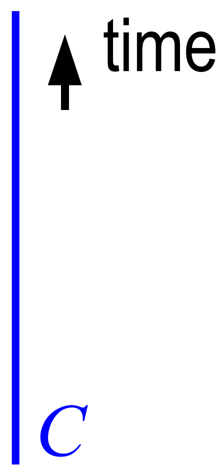
$$O_{\vec{k}}(x)=\prod_j Tr[Z^j]^{k_j}(x) \qquad Z:=\phi_4+i\phi_5$$

$$\vec{k}:=(k_1,k_2,k_3,\ldots),\, k_j\in\mathbb{Z}_{\geqslant 0}$$

This operator is invariant under the SUSY transformation

$$\delta_\epsilon O_{\vec{k}}=0, \quad \text{for} \quad \boxed{i\Gamma_{45}\epsilon=\epsilon.}$$

½ BPS straight Wilson line



$$W_{\vec{k}}=\prod_j Tr[U^j]^{k_j},$$

$$U=P\exp\int_C ds[A_\mu\dot{x}^\mu+\phi_4|\dot{x}|]$$

$$=P\exp\int dx^0[A_0+\phi_4]$$

This operator is invariant under the SUSY transformation

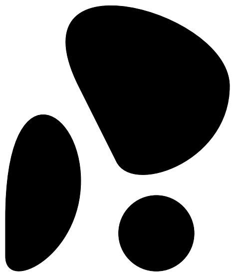
$$\delta_\epsilon W_{\vec{k}}=0, \quad \text{for} \quad \boxed{\Gamma_{04}\epsilon=\epsilon.}$$

3. LLM Solution [Lin, Lunin, Maldacena]

½ BPS local operators that preserves
SO(4) x SO(4) symmetry

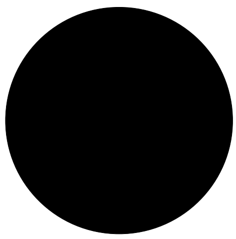
Supergravity solutions

"Droplet" $z : \mathbb{R}^2 \to \{-1, +1\}$



→ ½ BPS smooth solution

Example



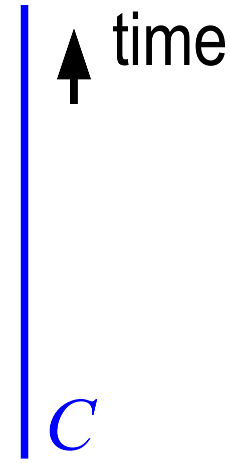
→ AdS5 x S5

Interpretation

The phase space of free fermions derived from Gaussian matrix quantum mechanics

4. Symmetry of a Straight Wilson line.

$$U := P\exp\int dx^0[A_0+\phi_4]$$



- Space-like rotation SO(3)
- Dilatation
- Time translation
- Special Conformal Symmetry time direction

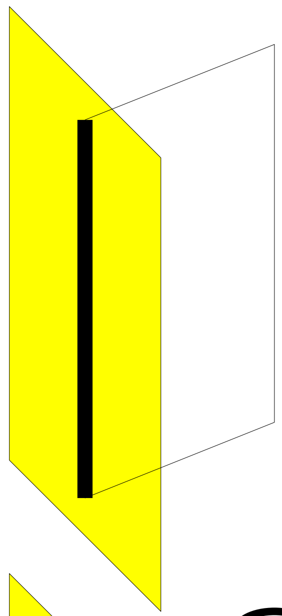
} SL(2,R)

- Part of R-symmetry SO(5)

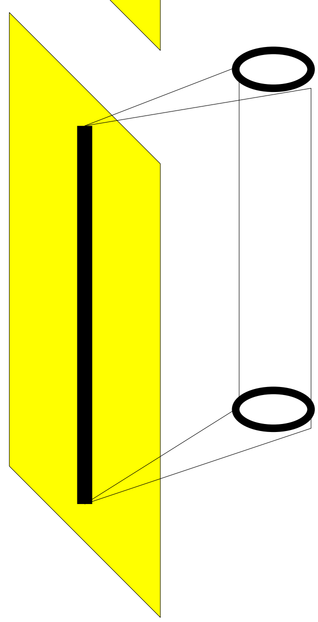
SL(2,R) x SO(3) x SO(5)

5. Probe Picture

[Ray, Yee],[Maldacena]
[Drukker,Fiol]



AdS2 F-String



AdS2 x S2 D3-brane
with electric flux

Both preserves SL(2,R)xSO(3)xSO(5) and
½ Supersymmetry

Goal: The SUGRA solution (backreaction)

↑

Stack of these branes

Note

AdS2 x S2 D3-brane

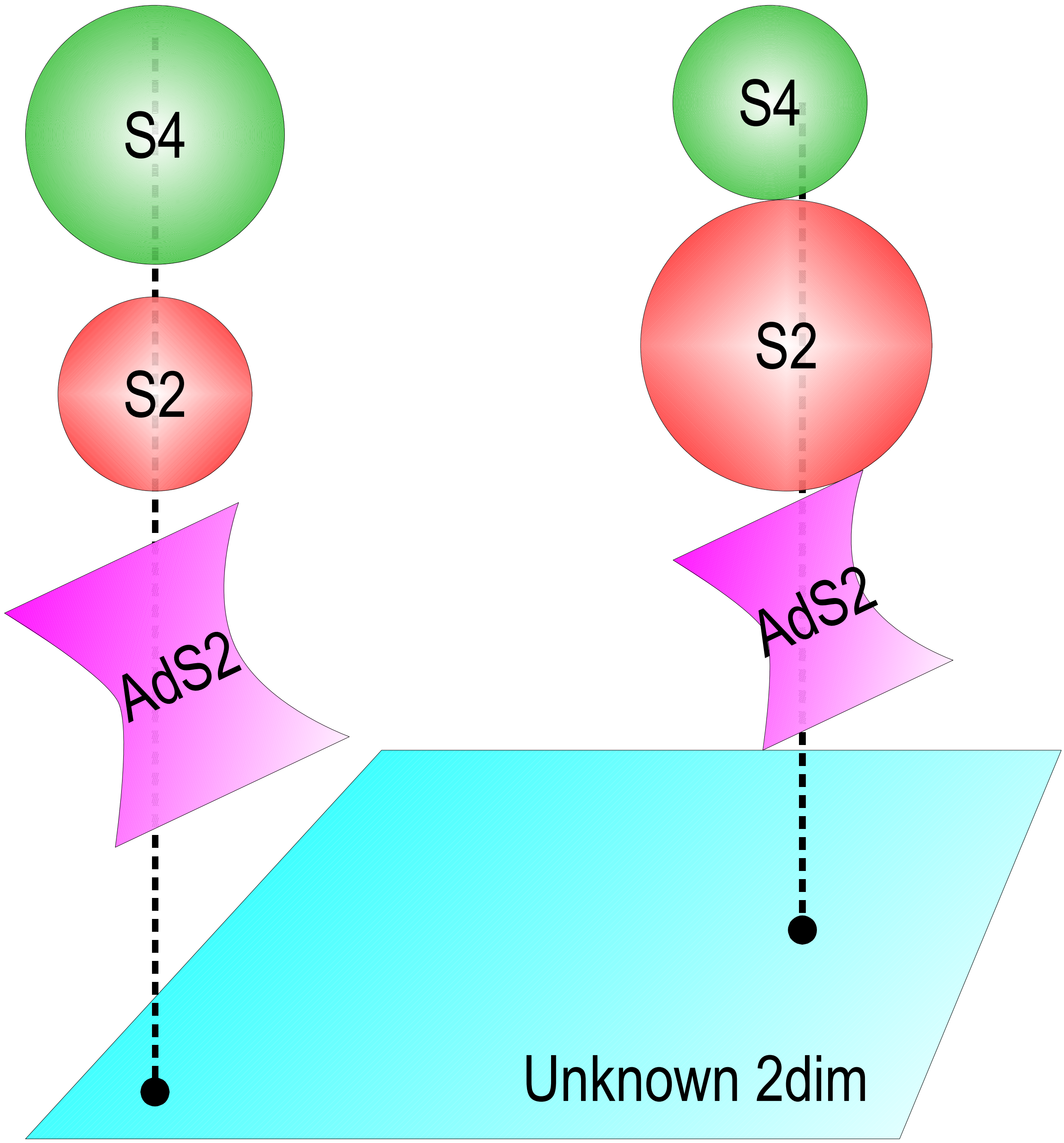
$$S_{CS} = \int F \wedge C_2, \quad C_2$$:RR 2-form

The SUGRA solution has RR 2-form excitation

6. Ansatz

SL(2,R) SO(3) SO(5)

$$ds^2 = e^{2A}(AdS_2) + e^{2B}(S^2) + e^{2C}(S^4) + \underbrace{ds_2^2}_{\text{Unknown}}$$



Field strength

	AdS2	2dim	S2	S4
3-form	vol(AdS2) ∧	1-form		
		1-form ∧	vol(S2)	
5-form	vol(AdS2) ∧	1-form ∧	vol(S2)	
		1-form ∧		vol(S4)
1-form		1-form		

We can reduce every unknown quantities to 2 dimensional fields

Truncation

Background D3-brane

$$G_5 = J \wedge vol(AdS_2) \wedge vol(S^2) + \tilde{J} \wedge vol(S^4)$$

F-string

$$H_3 = F \wedge vol(AdS_2)$$

A equation of motion implies

$$G_3 = K \wedge vol(S^2)$$

Other fields can be put as 0 consistently

Unknown quantities

2dim metric	ds_2^2
4 scalars	A, B, C, ϕ
4 1-forms	$J, \tilde{J}, F, K$

7. The Structure of the Geometry

SUSY $\delta \psi_{\mu} = 0, \ \ \delta \lambda = 0$

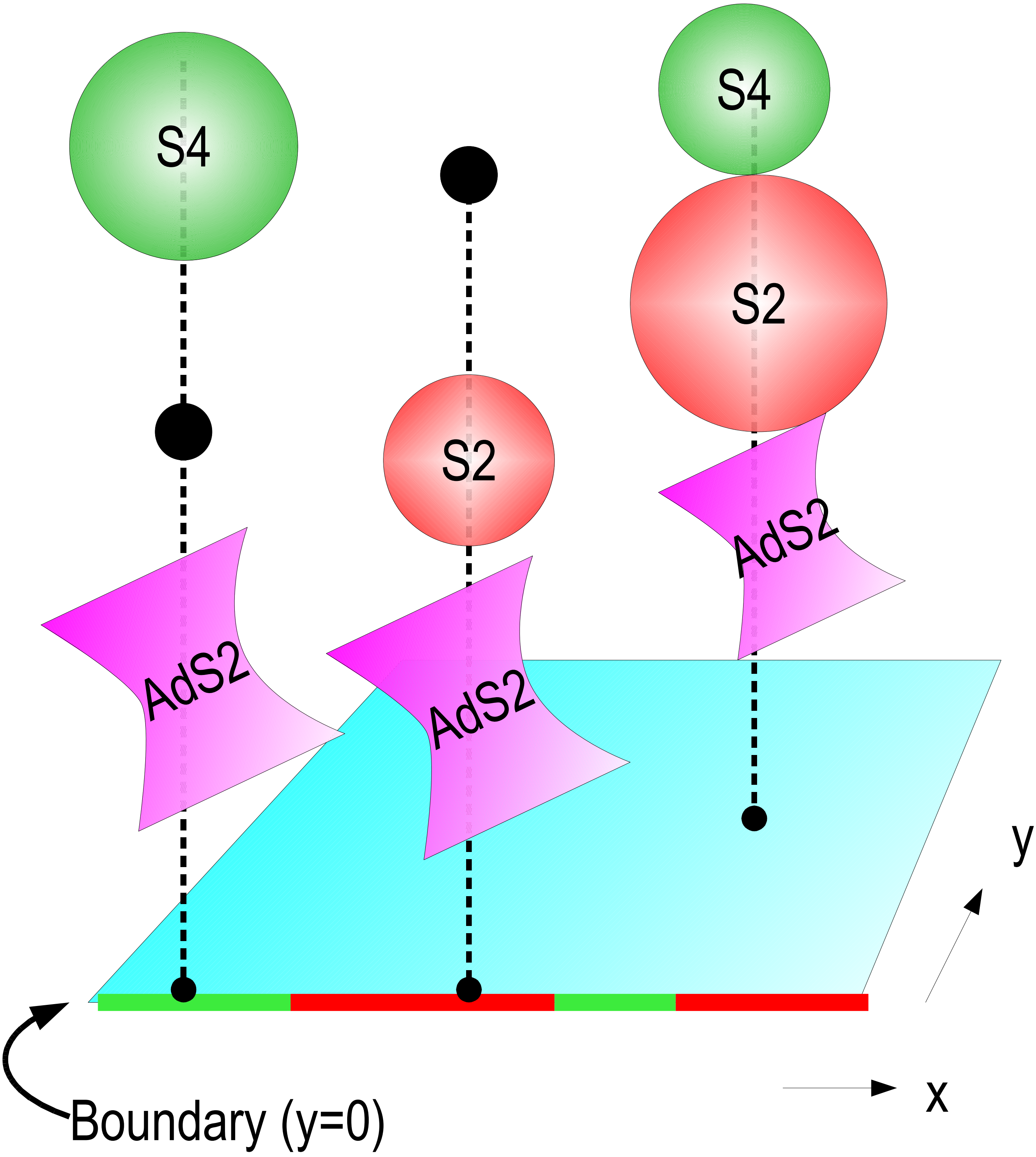
Necessary conditions

$$y := \exp(B + C)$$

Product of radi of S2 and S4

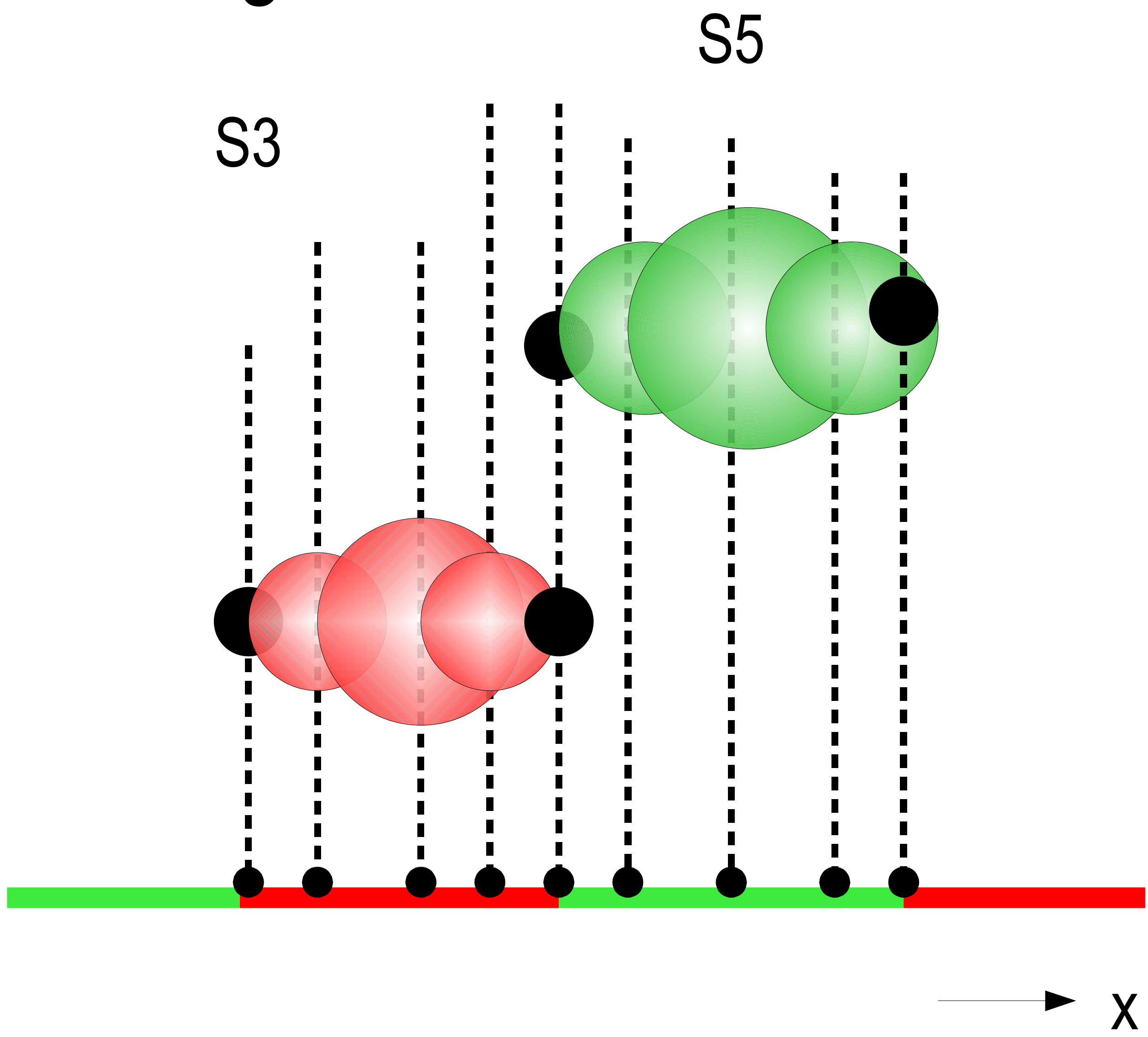
$$ds^2 = e^{2A}(AdS_2) + e^{2B}(S^2) + e^{2C}(S^4) + ds_2^2$$

$$ds_2^2 = \frac{1}{e^{2B} + e^{2C}}(dy^2 + dx^2)$$

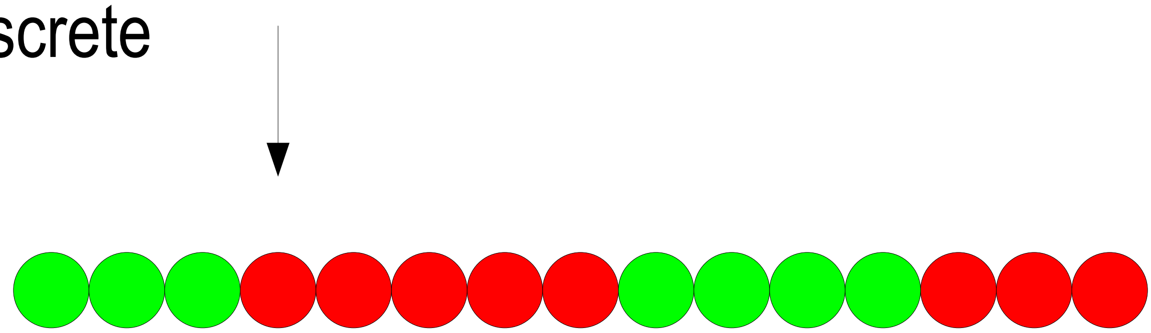


y=(radius S2) x (radius S4)

Bubbling



discrete



Maya diagram

Relation to the label of Wilson line ?

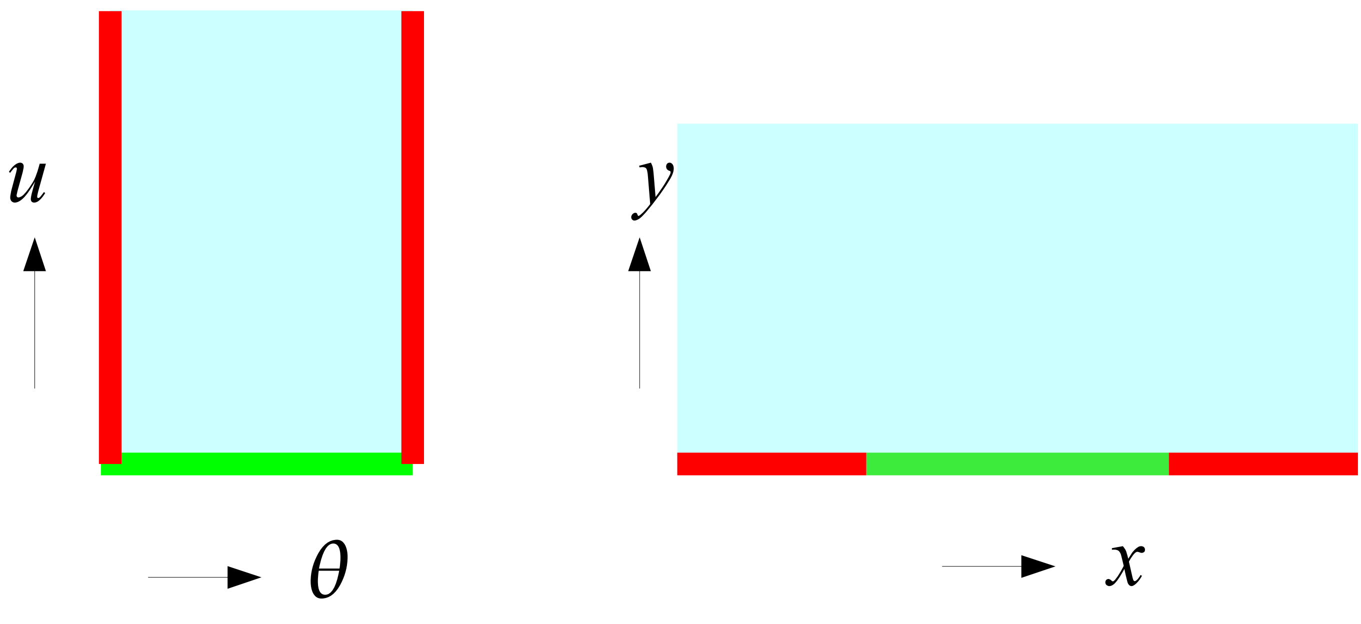
Example: AdS5 x S5 solution

$$ds^2 = R^2 \cosh^2 u (AdS_2) + R^2 \sinh^2 u (S^2) + R^2 \sin^2 \theta (S^4) + R^2 (du^2 + d\theta^2)$$

$$R^2 = \sqrt{4\pi g_s N}$$

$$u \geqslant 0, \quad 0 \leqslant \theta \leqslant \pi .$$

$$y = R^2 \sinh u \sin \theta ,$$
$$x = R^2 \cosh u \cos \theta .$$

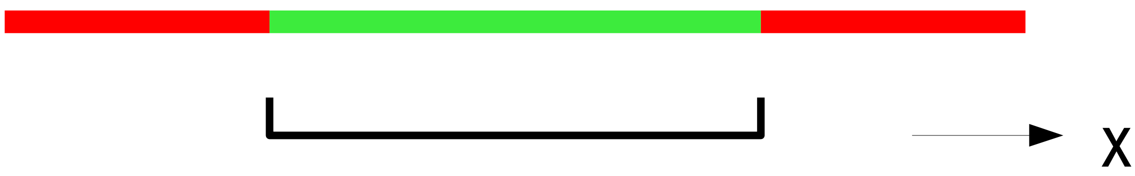


Comparing to the Gaussian matrix model

Vev of circular Wilson loops are related to Gaussian matrix model.
[Erickson, Semenoff, Zarembo], [Drukker, Gross]

Claim: The pattern of x-axis corresponds to the eigen value distribution of the matrix model

AdS5 x S5 solution

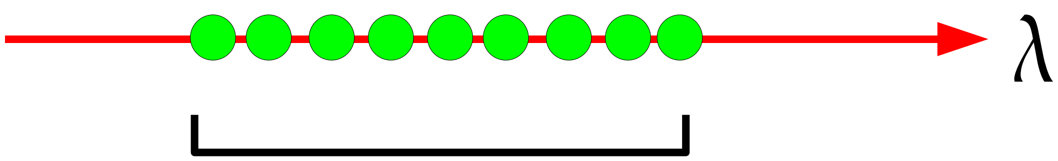


$$2\sqrt{4\pi g_s N}$$

Gaussian matrix model

$$Z = \int dM \exp\left(-\frac{1}{\hbar} Tr[M^2]\right)$$

The classical distribution of the eigen values of Gaussian matrix model



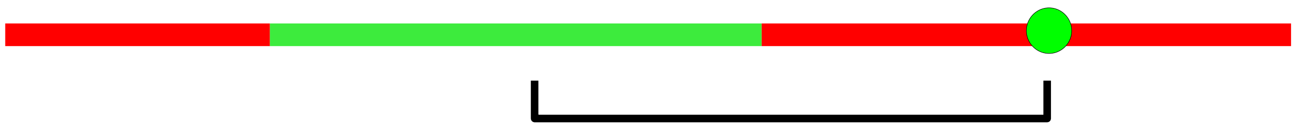
$$2\sqrt{2\hbar N}$$

Wilson line $W=\frac{1}{N}Tr[U^k],$

Geometry side

[Drukker,Fiol]

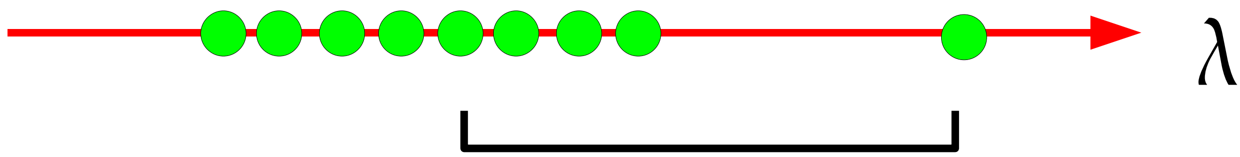
AdS2 x S2 D3-brane, k unit of electric flux



$$\sqrt{2\cdot 2\pi g_s N+\frac{1}{4}k^2(2\pi g_s)^2}$$

Matrix model side

$$\langle W\rangle=\int dM\,\frac{1}{N}Tr[e^{kM}]\exp\left(-\frac{1}{\hbar}Tr[M^2]\right),$$



$$\sqrt{2\hbar N+\frac{1}{4}k^2\hbar^2}$$

These two picture completely match
if we identify $\hbar=2\pi g_s$

Two same AdS2 x S2 D3-branes cannot coexist
Exclusion principle

Wilson line $W = \frac{1}{dim} Tr_{antisym}[U],$

Geometry side

[Yamaguchi to appear]

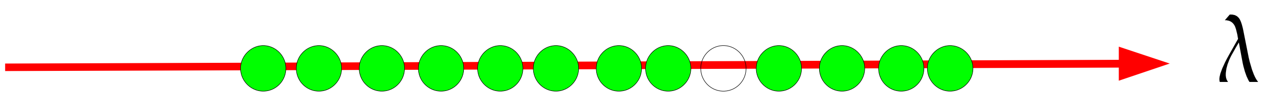
AdS2 x S4 D5-brane,

x_k

The position of D5-brane with F-string charge k

$$x_k = \sqrt{4\pi g_s N} \cos\theta, \quad k = \frac{2N}{\pi}\left(\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right),$$

Matrix model side



λ_k

k-th eigenvalue

$$\lambda_k = \sqrt{2\hbar N} \cos\theta, \quad k = \frac{2N}{\pi}\left(\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right),$$

8. Solutions in M-theory

Surface operator in 6dim(2,0)CFT
Wall operator in 3dim CFT

↓

SO(2,2) x SO(4) x SO(4) symmetry
16 SUSY

- **Anzats**

SO(2,2) SO(4) SO(4)

$$ds^2 = e^{2A}(AdS_3) + e^{2B}(S^3) + e^{2C}(S^3) + \underbrace{ds_2^2}_{\text{Unknown}}$$

Results

SUSY

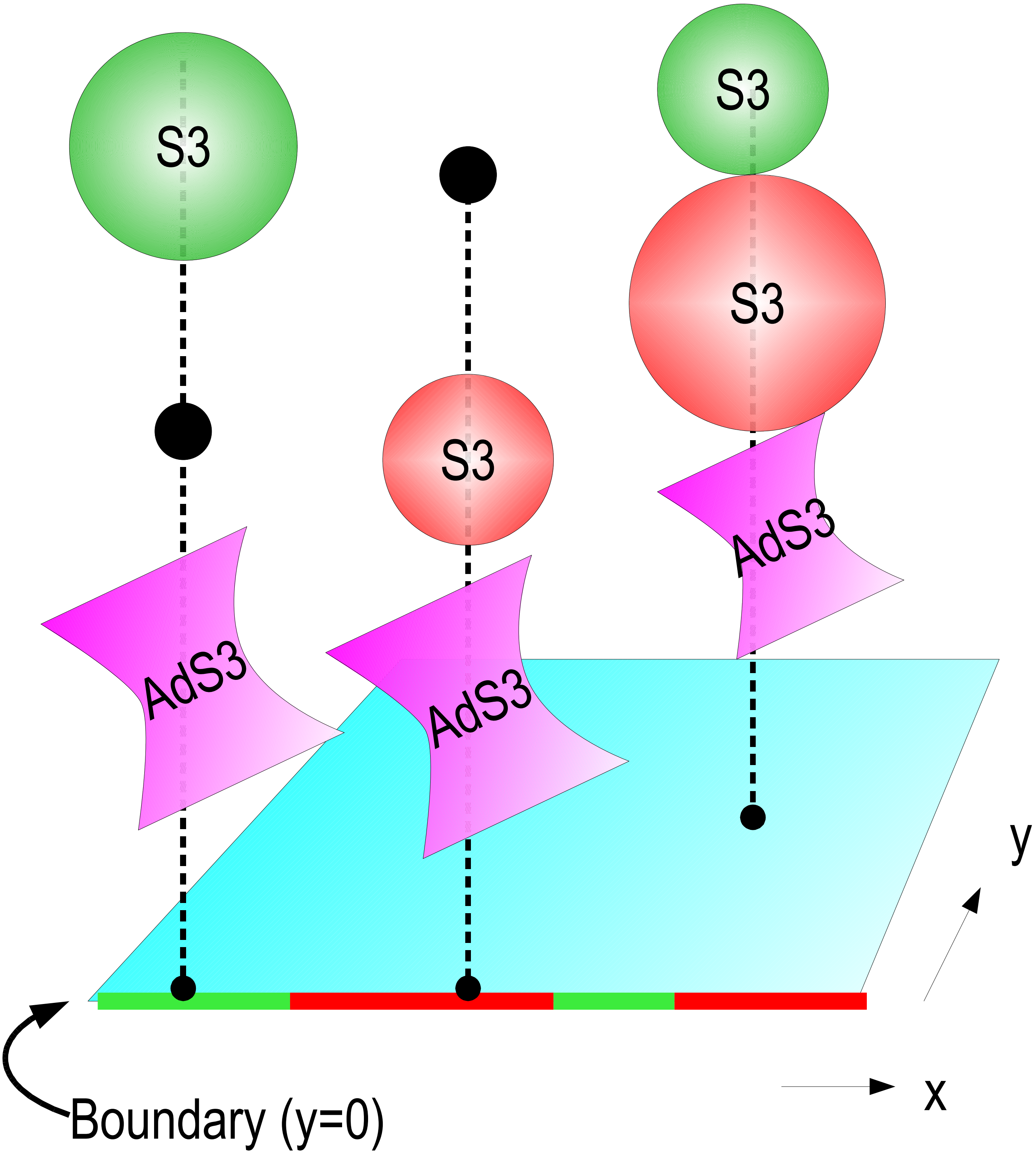
Necessary conditions

$$y := \exp(A+B+C)$$

Product of radi of AdS3, S3 and S3

$$ds^2 = e^{2A}(AdS_3) + e^{2B}(S^3) + e^{2C}(S^3) + ds_2^2$$

$$ds_2^2 = (-e^{2B+2C} + e^{2A+2B} + e^{2A+2C})^{-1} \times (dy^2 + dx^2)$$



y=(radius AdS3) x (radius S3) x (radius S3)

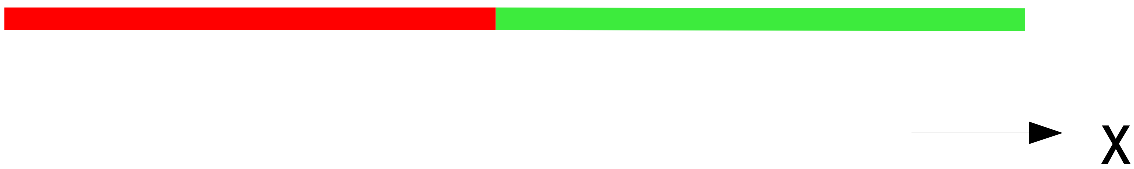
AdS3 never shrinks

A segment → S4

Examples

AdS7 x S4

AdS4 x S7



AdS3 x S3 x R4 x R

9. Conclusion

Necessary condition for supergravity solutions that preserves the same symmetry as ½ BPS Wilson line operators.

Supergravity solutions

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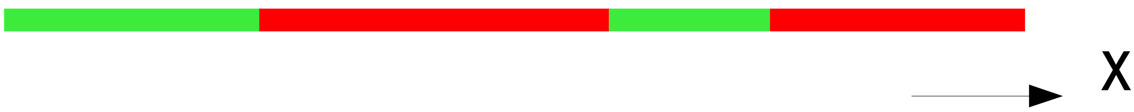
(Continuous) Maya diagram



The space of eigen values
of the Gaussian Matrix model

→ D-brane exclusion principle

The similar problem in M-theory.