Supersymmetric quantum field theory with **exotic symmetry** in 3+1 dimensions and fermionic fracton phases

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based on

SY, arXiv:2102.04768 [hep-th]

Introduction

Fracton phases

[Chamon 05], [Haah 11],...

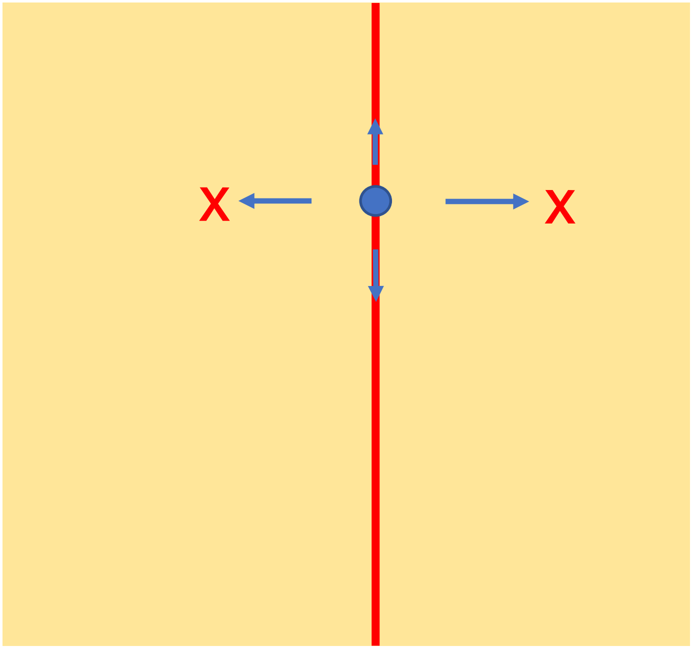
Review papers: [Nandkishore, Hermele 1803.11196], [Pretko, Chen, You 2001.01722]

Quantum system with the following exotic features.

- Exponentially large ground state degeneracy. But the residual entropy is **not extensive**.

  Eg. log(# ground states) $\propto$ (area) (cf. Blackhole entropy)

- Particle-like excitation with restricted mobility.

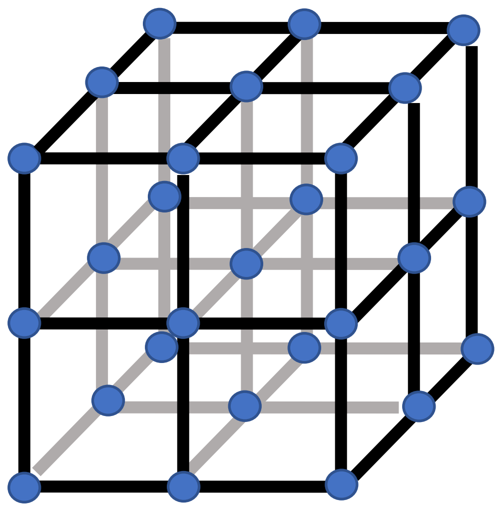


- Subsystem symmetry (≠ higher form symmetry)

  [Pretko 17],...

Fracton phases

There are a lot of examples of lattice quantum mechanics models which show fracton phases. [Haah 11], [Vijay, Haah, Fu 15, 16],...



Continuum quantum field theory?

[Pretko 17], [Ma, Hermele, Chen 18], [Bulmash, Barkeshli 18], [Seiberg 19], [Seiberg, Shao 20],...

Strict continuum limit may be difficult (eg. # ground states $\rightarrow \infty$)

But it is a nice way to consider universal features independent of the microscopic detail

We employ this continuum field theory approach.

Fracton phases

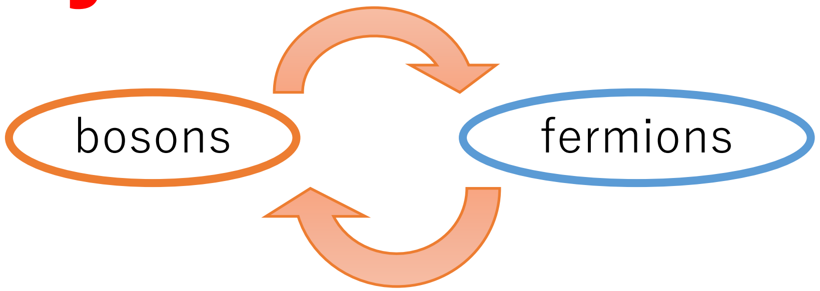
Fermions ?

No nice principles (no Lorents symmetry, no continuous rotational symmetry,...)

※ Some fermionic fracton phases are considered in [You, von Oppen 19], [Tantivasadakarn 20], [Shirley 20]

Fracton phases Fermions ?

Let us introduce **Supersymmetry**



⇒ A “natural” fermionic fracton phase.

Strategy

Eg. 3+1 dim φ theory [You, Bi, Pretko 19], [Gorantla, Lam, Seiberg, Shao 20]

(One of the simplest QFT with subsystem symmetry)

$\sim$ resemble [GLSS]

1+1 dim massless free scalar

supersymmetrize

same procedure

1+1 dim massless free scalar + massless Majorana fermion

Supersymmetric φ theory

$= \varphi$ theory + (a fermionic system)

Summary of results

In 3+1 dimensions

- Supersymmetric φ theory
  - action
  - log(# ground states) $\propto$ (Area)
  - (Self-duality)
- Supersymmetric tensor gauge theory
  - action
  - BPS fractons as defect

Plan

- φ theory
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φ theory

φ theory action

[You, Bi, Pretko 19], [Gorantla, Lam, Seiberg, Shao 20]

3 + 1 dim

$x, y, z \quad t$

periodic boundary condtion

$\phi(x, y, z, t)$: real bosonic field with periodicity $\phi \sim \phi + 2\pi$

$$\mathcal{L} = \frac{1}{2}(\partial_t \phi)^2 - \frac{1}{2}(\partial_x \partial_y \partial_z \phi)^2 = 2\partial_+ \phi \partial_- \phi$$

$$\partial_\pm := \frac{1}{2}(\partial_t \pm \partial_x \partial_y \partial_z)$$

At least superficial resembulance to 1+1 dim free scalar in which $\partial_\pm := \frac{1}{2}(\partial_t \pm \partial_x)$

Subsystem symmetry (momentum and winding quadrupole symmetry)

$$\mathcal{L} = 2\partial_+\phi\partial_-\phi$$ ⇒ E.O.M. $$\partial_+\partial_-\phi = 0$$

※Leibniz rule is not simple $(\partial_\pm A)B + A(\partial_\pm B) = (\text{total derivative}) \neq \partial_\pm(AB)$

⇒ In 2+1 dim ϕ theory the expression is a bit different

Currents: $J_\pm := \partial_\pm\phi$

⇒ $$\partial_+ J_- = \partial_- J_+ = 0$$

Conservation law!

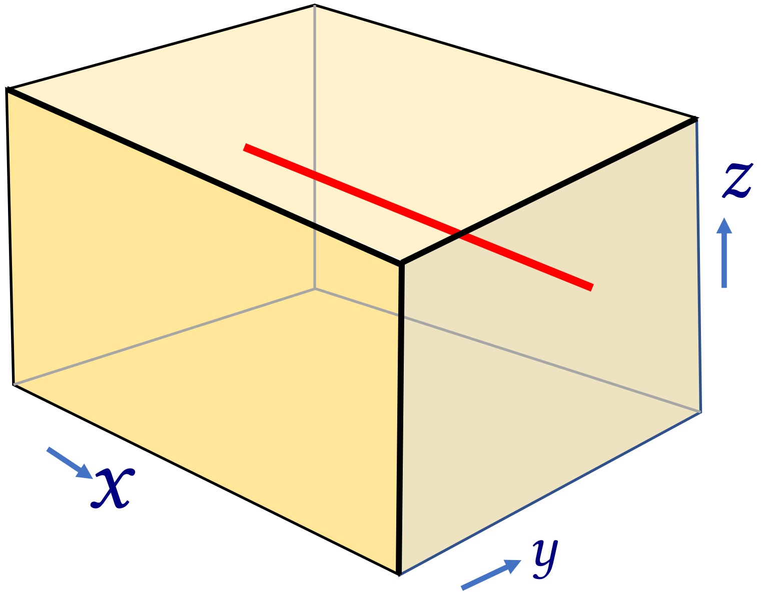
Subsystem symmetry

$$\partial_+ J_- = \partial_- J_+ = 0 \quad \Longleftrightarrow \quad \partial_t J_\pm = \pm \partial_x \partial_y \partial_z J_\pm$$

$$\Rightarrow \quad Q^{yz}_{\pm}(y, z, \not{t}) = \int dx J_\pm(x, y, z, t)$$

is conserved (time independent)!

such charges are conserved within each line parallel to x-axis.

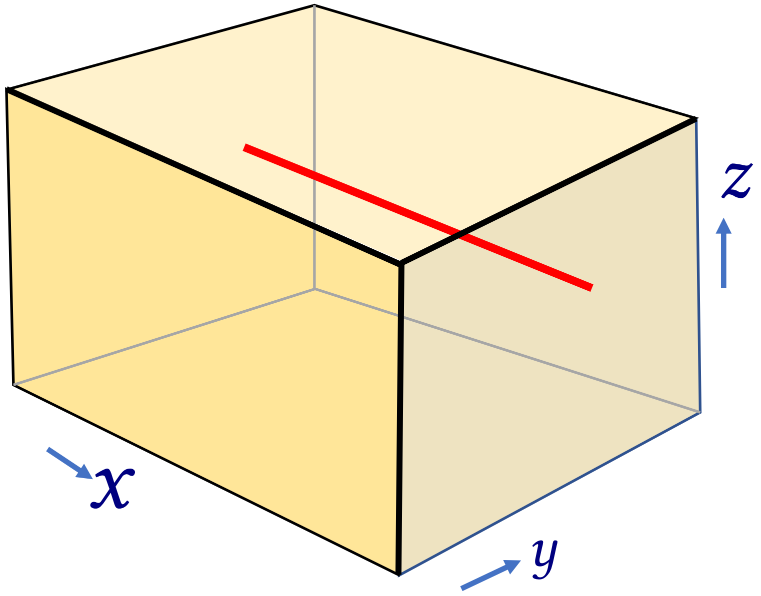


Subsystem symmetry

Subsystem symmetry

Once regularized,

(# $Q_{\pm}^{yz}(y,z)$) ∝(# lines parallel to x-axis)
∝(Area of yz-plane)



There are also $Q_{\pm}^{xy}(x,y),\ Q_{\pm}^{zx}(z,x)$

conserved charges ∝ (Area)

Summary of this section

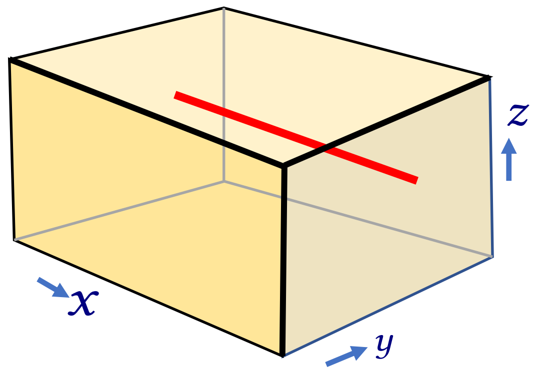
- φ theory: 3+1 dim QFT

$$\mathcal{L} = 2\partial_+\phi\partial_-\phi \qquad \partial_\pm := \frac{1}{2}(\partial_t \pm \partial_x\partial_y\partial_z)$$

- Resemble to 1+1 dim massless free scalar

- Subsystem symmetry

conserved charges ∝ (Area)



Plan

- φ theory
- Results in supersymmetric φ theory
- Superfield formalism
- Supersymmetric tensor gauge theory
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Results in supersymmetric φ theory

Action

Introduce $\psi_\pm$: real fermionic fields in addition to ϕ

Lagrangian density

"ψ theory"

$$\mathcal{L} = 2\partial_+\phi\partial_-\phi + i\psi_+\partial_-\psi_+ + i\psi_-\partial_+\psi_-$$

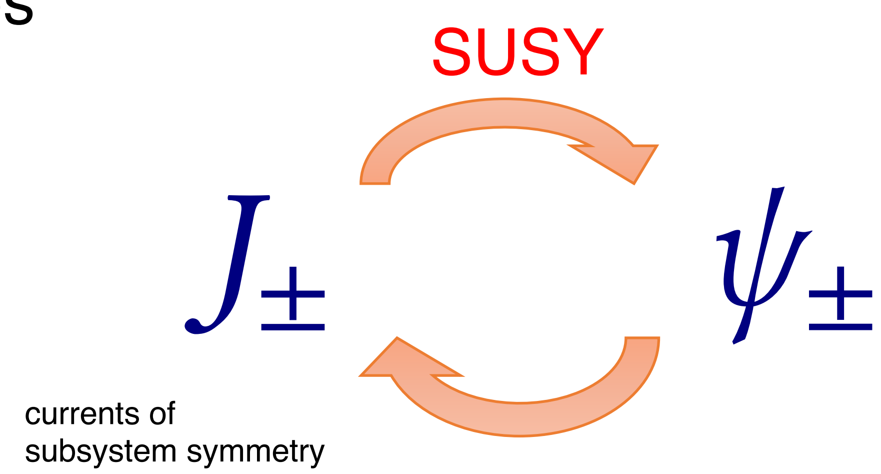
SUSY transformation $\epsilon_\pm$: real infinitesimal fermionic parameter of transformation

$$\delta\phi = i\epsilon_-\psi_+ - i\epsilon_+\psi_-,$$

$$\delta\psi_+ = -2\epsilon_-\partial_+\phi, \qquad \delta\psi_- = 2\epsilon_+\partial_-\phi$$

Same form as 1+1 dim N=(1,1) SUSY besides $\partial_\pm := \frac{1}{2}(\partial_t \pm \partial_x\partial_y\partial_z)$

Fermionic charges



$$\partial_+\psi_- = \partial_-\psi_+ = 0$$

(⇔ E.O.M. from $\mathcal{L} = i\psi_+\partial_-\psi_+ + i\psi_-\partial_+\psi_-$)

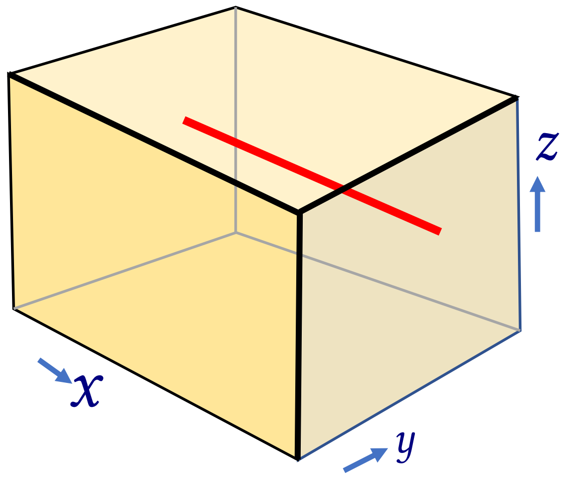
conservation law!

Fermionic charges

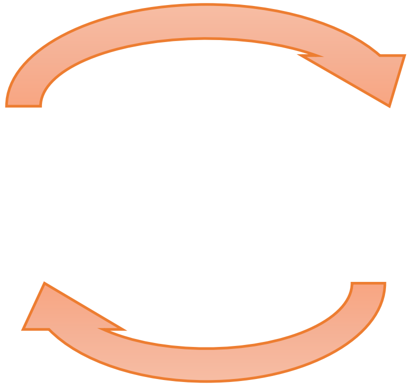
$$\partial_+\psi_- = \partial_-\psi_+ = 0$$

$$q_\pm^{yz}(y,z) = \int dx \psi_\pm(x,y,z,t)$$

is conserved!



SUSY



$Q_\pm^{yz}(y,z),\ Q_\pm^{xy}(x,y),\ Q_\pm^{zx}(z,x)$ $\qquad$ $q_\pm^{yz}(y,z),\ q_\pm^{xy}(x,y),\ q_\pm^{zx}(z,x)$

bosonic charges $\qquad$ fermionic charges

In particular, (# bosonic charges)=(# fermionic charges) in a “nice” regularization

Ground state degeneracy

(φ theory has a unique vacuum [GLSS])

(# fermionic charges)=(# bosonic charges)=: A $\propto$ (Area)

Linear combination of $q_{\pm}^{yz}(y,z),\ q_{\pm}^{xy}(x,y),\ q_{\pm}^{zx}(z,x)$

$\gamma_a,\ (a=1,\ldots,A)$ s.t. $\{\gamma_a,\gamma_b\}=2\delta_{ab}$

Clifford algebra!

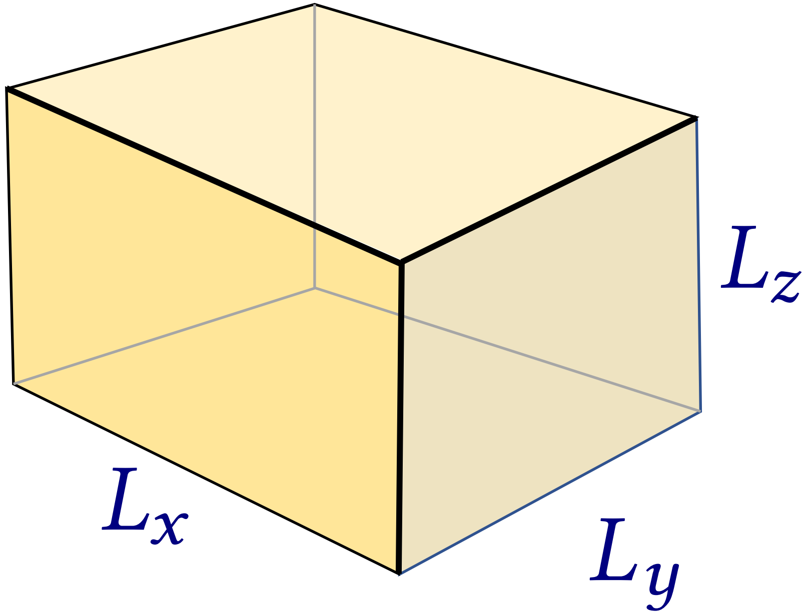
dim of irrep = $2^{A/2}$

log(# ground state) = $\dfrac{A}{2}\log 2$ $\propto$ (Area)

An attempt to regularize the ψ theory $\mathcal{L} = i\psi_+\partial_-\psi_+ + i\psi_-\partial_+\psi_-$

3 dim cubic lattice with periodic boundary condition

$$L_x \times L_y \times L_z$$



Real fermion $c_{\vec{n}}$ at each site $\vec{n} = (n_x, n_y, n_z)$

$n_i \in \mathbb{Z}$

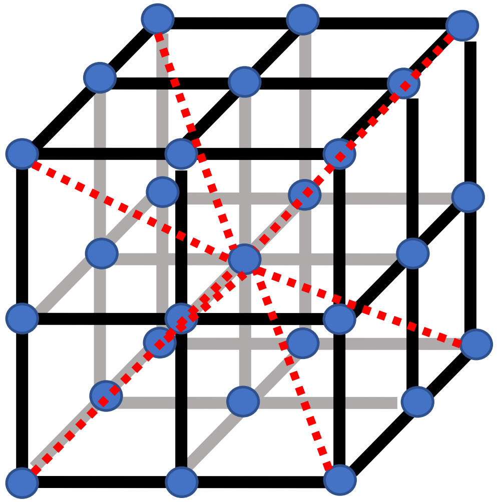
$$\{c_{\vec{n}}, c_{\vec{n}'}\} = \delta_{\vec{n},\vec{n}'}$$

$$H = \sum_{\vec{n}} c_{\vec{n}} i\Delta_{xyz} c_{\vec{n}}$$

$$\Delta_{xyz}c_{\vec{n}} := \frac{1}{8}\big(c_{(n_x+1,n_y+1,n_z+1)} - c_{(n_x-1,n_y+1,n_z+1)} - c_{(n_x+1,n_y-1,n_z+1)} + c_{(n_x-1,n_y-1,n_z+1)} - c_{(n_x+1,n_y+1,n_z-1)} + c_{(n_x-1,n_y+1,n_z-1)} + c_{(n_x+1,n_y-1,n_z-1)} - c_{(n_x-1,n_y-1,n_z-1)}\big)$$

$$\to a^3\partial_x\partial_y\partial_z c$$

naively



Solved

$$H = \sum_{\vec{n}} c_{\vec{n}} i \Delta_{xyz} c_{\vec{n}}$$

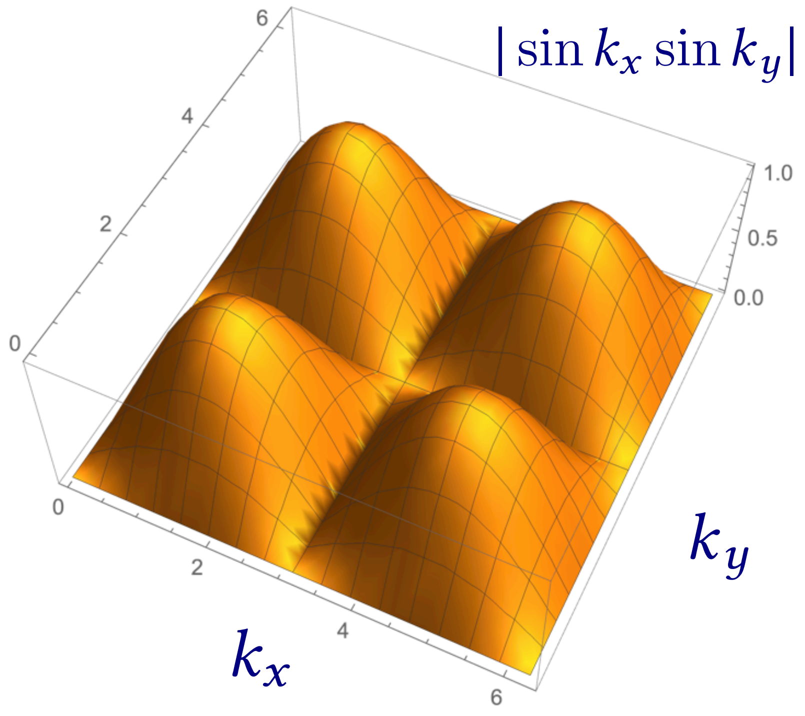
$$c_{\vec{n}} = \frac{1}{\sqrt{L_x L_y L_z}} \sum_{\vec{k}} b_{\vec{k}} e^{i\vec{k}\cdot\vec{n}}$$

$$\vec{k} = (k_x, k_y, k_z),\ k_i \in \frac{2\pi}{L_i}\mathbb{Z},\ k_i \sim k_i + 2\pi$$

$$\{b_{\vec{k}}, b_{\vec{k}'}\} = \delta_{\vec{k},-\vec{k}'}$$

$$b^\dagger_{\vec{k}} = b_{-\vec{k}}$$

$$H = \sum_{\vec{k}} \sin k_x \sin k_y \sin k_z b_{-\vec{k}} b_{\vec{k}}$$



Ground state degeneracy

$$H = \sum_{\vec{k}} \sin k_x \sin k_y \sin k_z b_{-\vec{k}} b_{\vec{k}}$$

(# zero modes) = $2L_xL_y + 2L_yL_z + 2L_zL_x - 4L_x - 4L_y - 4L_z + 8$

(# ground states)= $2^{L_xL_y + L_yL_z + L_zL_x - 2L_x - 2L_y - 2L_z + 4}$

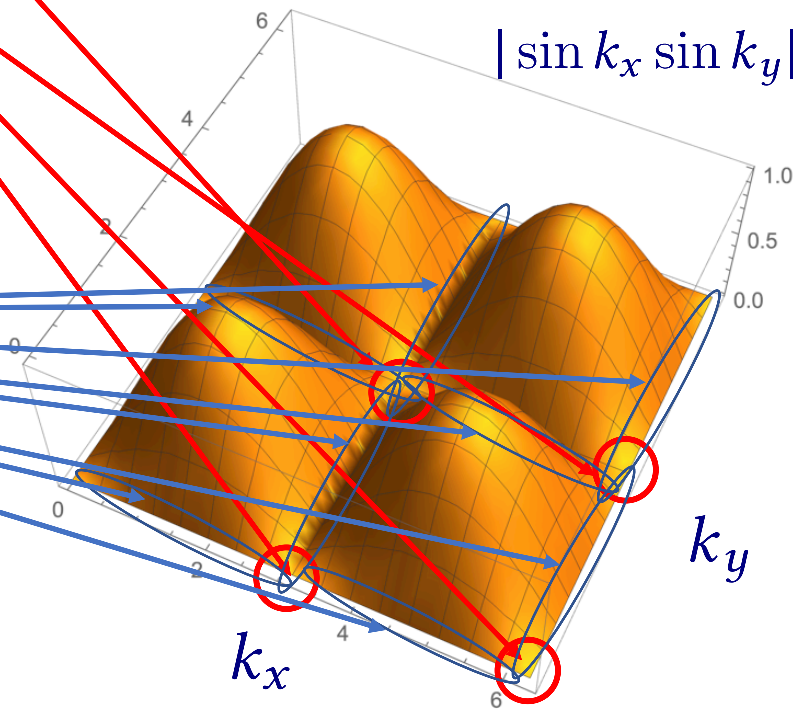
Continuum limit?

In addition to large wave length point $\vec{k} \sim (0,0,0)$

there are 7 points of $(\sin k_x, \sin k_y, \sin k_z) \sim (0,0,0)$ "doublers"

4 copies of ψ theory? It may be still too naive.

In low energy, not only modes around 8 doublers, but also modes around zero modes survive. It is not easy to take "continuum limit" of this lattice theory.



Summary of this section

- Supersymmetric φ theory = φ theory + ψ theory

$$\mathcal{L} = 2\partial_+\phi\partial_-\phi + i\psi_+\partial_-\psi_+ + i\psi_-\partial_+\psi_-$$

- Subsystem symmetry: bosonic charges ⇔ fermionic charges

log(# ground states) ∝ (Area)

- An attempt to regularize the ψ theory

It is an open problem.

Plan

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Superfield formalism

Idea of superfield

Introduce real fermionic coordinates $\theta^+, \ \theta^-$ in addition to x, y, z, t

$\Phi(x, y, z, t, \theta^+, \theta^-)$ "superfield"

$$S = \int d^4x \int d^2\theta \mathscr{L}(\Phi, \text{derivatives})$$

$$S = \int d^4x \int d^2\theta \mathscr{L}(\Phi, \text{derivatives})$$

Our superfield formalism is almost parallel to 1+1 dim N=(1,1) superfield besides an obstacle.

$\mathscr{L}$ must be quadratic in superfields in order to have supersymmetry.

Warm up 1 ϕ : local field Translation $\delta_x \phi = \epsilon \partial_x \phi$

How to write down an invariant action under translation?

$\phi_1,\ \phi_2$ local field ➡ $\phi_3 := \phi_1 \phi_2$, $\partial_\mu \phi_1$ are local fields

$$\delta_x \phi_3 = (\delta_x \phi_1)\phi_2 + \phi_1(\delta_x \phi_2) = \epsilon(\partial_x \phi_1)\phi_2 + \epsilon\phi_1(\partial_x \phi_2) = \epsilon\partial_x(\phi_1\phi_2) = \epsilon\partial_x \phi_3$$

Leibniz rule

➡ Lagrangian density $\mathcal{L}$ is local field if it is a polynomial of local fields and their derivatives.

$\delta_x \mathcal{L} = \epsilon \partial_x \mathcal{L}$ ➡ $\mathcal{L}$ is invariant up to a total derivative.

Warm up 2

ϕ : local field

Consider transformation $\delta_{xyz}\phi = \epsilon\partial_x\partial_y\partial_z\phi$ "xyz-local field"

How to write down an invariant action under this transformation?

$\phi_1, \ \phi_2$ xyz- local field ⇨ $\partial_\mu\phi_1$ are xyz-local fields

But $\phi_3 := \phi_1\phi_2$ is NOT an xyz-local fields

$$\delta_{xyz}\phi_3 = (\delta_{xyz}\phi_1)\phi_2 + \phi_1(\delta_{xyz}\phi_2) = \epsilon(\partial_x\partial_y\partial_z\phi_1)\phi_2 + \epsilon\phi_1(\partial_x\partial_y\partial_z\phi_2)$$
$$\neq \epsilon\partial_x\partial_y\partial_z(\phi_1\phi_2) = \epsilon\partial_x\partial_y\partial_z\phi_3$$

Warm up 2

ϕ : local field   Consider transformation $\delta_{xyz}\phi = \epsilon\partial_x\partial_y\partial_z\phi$ "xyz-local field"

$$\phi_3 := \phi_1\phi_2$$

$$\delta_{xyz}\phi_3 = (\delta_{xyz}\phi_1)\phi_2 + \phi_1(\delta_{xyz}\phi_2) = \epsilon(\partial_x\partial_y\partial_z\phi_1)\phi_2 + \epsilon\phi_1(\partial_x\partial_y\partial_z\phi_2)$$

$$= \underline{(\text{total derivative})}$$

If the Lagrangian density $\mathcal{L}$ is quadratic (or lower) polynomial of xyz-local field, $\mathcal{L}$ is invariant under this transformation up to total derivative.

Supersymmetry

$$\Phi(t, x, y, z, \theta^+, \theta^-)$$

$$\delta\Phi = (i\epsilon_- Q_+ - i\epsilon_+ Q_-)\Phi$$ "superfield"

$$Q_\pm := -i\frac{\partial}{\partial\theta^\pm} + 2\theta^\pm\partial_\pm \qquad \partial_\pm := \frac{1}{2}(\partial_t \pm \partial_x\partial_y\partial_z)$$

How to write down an invariant action under supersymmetry?

Φ_1, Φ_2 : superfield $\Rightarrow$ $\mathcal{D}_\pm\Phi_1,\ \partial_\pm\Phi_1$ are superfields

$$\mathcal{D}_\pm := -i\frac{\partial}{\partial\theta^\pm} - 2\theta^\pm\partial_\pm$$

$\Phi_1\Phi_2$ is NOT!

Supersymmetry $\delta\Phi = (i\epsilon_- Q_+ - i\epsilon_+ Q_-)\Phi$ "superfield"

Φ_1, Φ_2 : superfield $\Rightarrow$ $\delta(\Phi_1\Phi_2) = (\text{total derivative})$

If $\mathscr{L}$ is quadratic (or lower) polynomial of superfields, $\mathscr{L}$ is invariant under supersymmetry transformation up to total derivative.

Eg. $\Phi \sim \Phi + 2\pi$

$$\mathscr{L} = \frac{1}{2}\mathcal{D}_-\Phi\mathcal{D}_+\Phi$$

= Supersymmetric φ theory

open problem

How to write down interacting theories?

Summary of this section

- Supersymmetric theory

$$S = \int d^4x \int d^2\theta \mathscr{L}$$

$\mathscr{L}$: quadratic or lower in superfields.

Eg. supersymmetric φ theory

$$\mathscr{L} = \frac{1}{2}\mathcal{D}_-\Phi\mathcal{D}_+\Phi$$

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Supersymmetric tensor gauge theory

Supersymmetric φ theory $\mathscr{L} = \frac{1}{2}\mathcal{D}_-\Phi\mathcal{D}_+\Phi$

Shift symmetry $\Phi \longrightarrow \Phi + \lambda$ λ : constant

Gauge this shift symmetry!

Gauge transformation parameter $K(t, x, y, z, \theta^+, \theta^-)$

Gauge transformation $\Phi \longrightarrow \Phi' = \Phi + K$

$$\Phi \longrightarrow \Phi' = \Phi + K$$

Introduce gauge superfields $\Gamma_\pm$

Covariant derivative $\nabla_\pm \Phi = \mathcal{D}_\pm \Phi - \Gamma_\pm$

Gauge transformation $\Gamma'_\pm = \Gamma_\pm + \mathcal{D}_\pm K$

⇒ $\nabla_\pm \Phi$ is gauge invariant

Where is tensor gauge fields?

$$\Gamma_+ = \chi_+ - 2\theta^+ A_+ + \theta^-(B+\sigma) - 2i\theta^+\theta^-(\lambda_+ + \partial_+\chi_-),$$
$$\Gamma_- = \chi_- - \theta^+(B-\sigma) - 2\theta^- A_- + 2i\theta^+\theta^-(\lambda_- + \partial_-\chi_+),$$
$$K = \omega + i\theta^+\eta_+ + i\theta^-\eta_- + i\theta^+\theta^-\tau,$$

Gauge transformation of A components

$$A'_\pm = A_\pm + \partial_\pm\omega$$

$$A_t := A_+ + A_-,\ A_{xyz} := A_+ - A_- \Rightarrow A'_t = A_t + \partial_t\omega,\ A'_{xyz} = A_{xyz} + \partial_x\partial_y\partial_z\omega$$

They are tensor gauge fields considered in

[You, Bi, Pretko 19], [You, Burnell, Hughs 19], [Gorantla, Lam, Seiberg, Shao 20]

$\Gamma_\pm$ are SUSY completion of this tensor gauge fields

Action

$$\Sigma = \frac{i}{2}(\mathcal{D}_+\Gamma_- + \mathcal{D}_-\Gamma_+)$$

gauge invariant

$$\mathscr{L}_{\text{gauge}} = \frac{1}{g^2}\mathcal{D}_-\Sigma\mathcal{D}_+\Sigma + im\Sigma^2 - \frac{i\vartheta}{2\pi}\Sigma$$

kinetic term

gaugino mass, BF coupling

theta term

$$\mathscr{L}_{\text{matter}} = \nabla_-\Phi\nabla_+\Phi$$

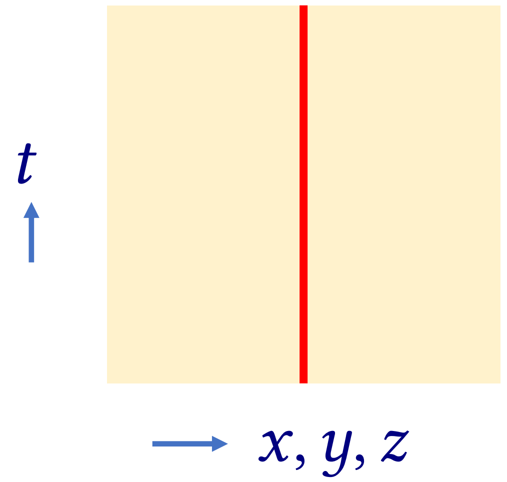
(All terms are quadratic or linear in superfields)

Fracton

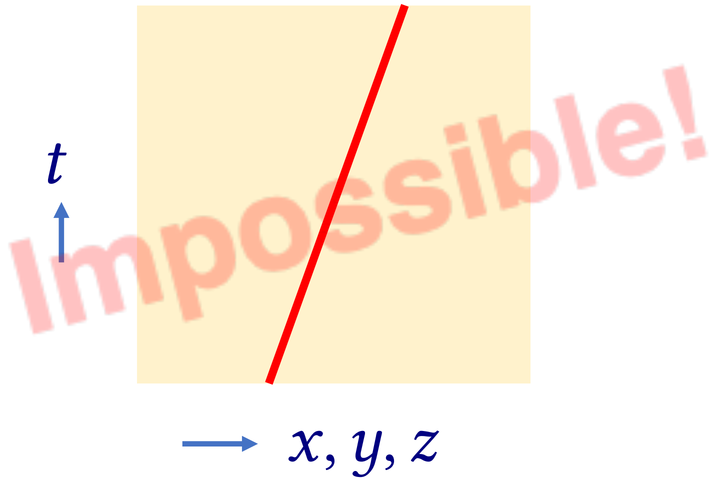
Introducing charged test "particle" (defect) at rest.

Wilson line $$W = \exp(i \int dt A_t)$$

$$A_t := A_+ + A_-$$



This particle cannot move since there is no A_x, A_y, A_z .



Fracton

It breaks all the supersymmetry.

Are there any BPS fracton?

(preserving a part of supersymmetry)

Are there any BPS fracton?

(preserving a part of supersymmetry)

YES!

$$W = \exp\left(i \int dt (A_t + \sigma)\right)$$

a scalar component in the gauge superfield

This one preserves half of the supersymmetry.

※ A combination of 4 fractons in tensor gauge theory can move.
However I could not find any BPS analogs in our supersymmetric theory.

Summary of this section

- Supersymmetric tensor gauge theory action is written.

- BPS fracton exists.

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Summary of results

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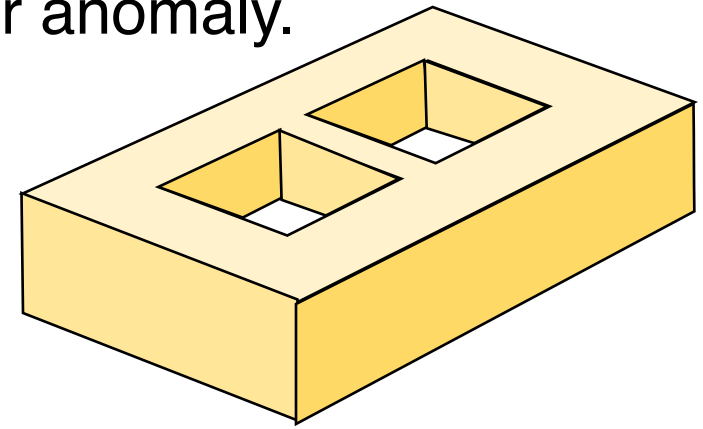
  - (Self-duality)

- Supersymmetric tensor gauge theory
  - action
  - BPS fractons as defect

Future prospects

- Lattice realization of fermionic system.
- How to write down interacting theories.
- Chiral theories similar to 1+1 dim ones and their anomaly.
  - Eg. chiral φ theory $\partial_+ \phi = 0$



- Curved space
  - Patchwork of rectangular patches ⇨ various topologies
  - N=(2,2) and twist $\rightarrow$ topological field theory