QFT II. Homework Problem Set 1. (10/7/2016)

Due 10/21/2016

Write your name and student ID. Staple the report together.

I. Consider a real scalar field

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2.$$

(1) Find the canonical momentum Π for ϕ.

(2) Write down the Hamiltonian.

(3) Mode expansion is given by

$$\phi(x) = \int \frac{d^3\vec{p}}{(2\pi)^3 2\omega_{\vec{p}}} \left[a(\vec{p})e^{-ipx} + a^\dagger(\vec{p})e^{ipx}\right],$$

where $p^\mu = (\omega_{\vec{p}}, \vec{p})$ and $\omega_{\vec{p}} > 0$. Express $\omega_{\vec{p}}$ in terms of $\vec{p}$. (Hint: use the equation of motion)

(4) Express $a(\vec{p})$ and $a^\dagger(\vec{p})$ in terms of ϕ and its derivatives.

(5) Find the commutation relations

$$[a(\vec{p}), a(\vec{p}\,')], \quad [a(\vec{p}), a^\dagger(\vec{p}\,')].$$

Notice: The normalization of the creation-annihilation operators might be different from your familiar normalization.

II. The Lagrangian density is given by

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{g}{3!}\phi^3.$$

(1) Draw all the connected Feynman diagrams contributing to

$$\langle 0|T[\phi(x)\phi(y)]|0\rangle$$

to g^2. The states and the operators are in the Heisenberg picture unless otherwise indicated.

(2) Consider a scattering process

$$\phi(p_1) + \phi(p_2) \to \phi(p_3) + \phi(p_4).$$

Find T_{fi} to $O(g^2)$. Use the Mandelstam variables

$$s = (p_1 + p_2)^2, \quad t = (p_1 - p_3)^2, \quad u = (p_1 - p_4)^2.$$