QFT II. Homework Problem Set 2. (11/11/2016)

Due 10/28/2016

Let us evaluate finite dimensional integral by using Wick's theorem and the Feynman diagrams and so on.

(1) Let A_{ab}, $(a, b = 1, \ldots, N)$ a symmetric, positive definite matrix and ϕ^a, $(a = 1, \ldots, N)$ integration variables, and consider the Gaussian integral

$$Z_0 := \int D\phi e^{-\frac{1}{2}A_{ab}\phi^a\phi^b},$$

where $\int D\phi$ is defined as

$$\int D\phi := \int_{-\infty}^{+\infty} \prod_a \frac{d\phi^a}{\sqrt{2\pi}}.$$

Show

$$Z_0 = \frac{1}{\sqrt{\det A}}.$$

(2) Let J_a a "source" and consider the generating function

$$Z_0[J] := \frac{1}{Z_0} \int D\phi e^{-\frac{1}{2}A_{ab}\phi^a\phi^b + J_a\phi^a}.$$

Show

$$Z_0[J] = e^{\frac{1}{2}\Delta^{ab}J_aJ_b}, \qquad (\star)$$

where Δ^{ab} is the inverse matrix of A_{ab}.

(3) Consider expectation values

$$\langle \phi^{a_1}\phi^{a_2}\cdots\phi^{a_K}\rangle_0 := \frac{1}{Z_0} \int D\phi \phi^{a_1}\phi^{a_2}\cdots\phi^{a_K} e^{-\frac{1}{2}A_{ab}\phi^a\phi^b}.$$

Explain the Wick's theorem

$$\langle \phi^{a_1}\phi^{a_2}\cdots\phi^{a_K}\rangle_0 = \begin{cases} 0 & (K = \text{odd}), \\ \sum_{\text{pairing}(i_\ell, j_\ell)} \prod_\ell \Delta^{a_{i_\ell} a_{j_\ell}} & \end{cases}$$

using the result $(\star)$.

(4) Let $A_{ab} = \delta_{ab}$ for simplicity and consider "interacting" theory

$$S = \frac{1}{2}\phi^2 + \frac{1}{8}\lambda(\phi^2)^2, \qquad \text{where } \phi^2 := \delta_{ab}\phi^a\phi^b.$$

Consider

$$\langle \phi^{a_1}\phi^{a_2}\cdots\phi^{a_K}\rangle := \frac{1}{Z_0} \int D\phi \phi^{a_1}\phi^{a_2}\cdots\phi^{a_K} e^{-S}.$$

(i) Write down the Feynman rule.

(ii) Calculate $\langle \phi^a\phi^b\rangle$ to $O(\lambda)$.