QFT II. Homework Problem Set 3. (11/11/2016)

Due 11/25/2016

(1) Consider d-dimensional free complex scalar field $\phi(x)$ and the action

$$S_{0s} = \int d^d x \Big(\partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi \Big).$$

The fourier transformation of $\phi(x)$ is denoted by $\tilde{\phi}(p)$ and they are related as

$$\phi(x) = \int_p \tilde{\phi}(p) e^{-ipx}, \quad \int_p := \int \frac{d^d p}{(2\pi)^d}, \qquad \phi^*(x) = \int_p \tilde{\phi}^*(p) e^{ipx}.$$

Find the two point functions

$$\big\langle \tilde{\phi}(p) \tilde{\phi}(q) \big\rangle_0, \quad \big\langle \tilde{\phi}(p) \tilde{\phi}^*(q) \big\rangle_0.$$

(2) Consider the d-dimensional Maxwell theory. This theory contains a vector field $A_\mu(x)$ and the action is written as

$$S_{0v} = \int d^d x \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right), \quad F_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu.$$

The fourier transformation of $A_\mu(x)$ is denoted by $\tilde{A}_\mu(p)$ and they are related as

$$A_\mu(x) = \int_p \tilde{A}_\mu(p) e^{-ipx}.$$

Try to find the two point functions

$$\big\langle \tilde{A}_\mu(p) \tilde{A}_\nu(q) \big\rangle_0.$$

Do you find some problem?

(3) In order to solve the problem you found in the above, let us consider gauge fixed action (The general procedure of gauge fixing will be given later in the lecture.)

$$S_{0v'} = \int d^d x \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2\xi} (\partial^\mu A_\mu)^2 \right),$$

where ξ is a constant parameter. Find the two point functions

$$\big\langle \tilde{A}_\mu(p) \tilde{A}_\nu(q) \big\rangle_0.$$

Hint: Because of the Lorentz symmetry, this two point function has the form

$$\Big(B(p^2) \eta_{\mu\nu} + C(p^2) p_\mu p_\nu \Big) \tilde{\delta}(p+q).$$

Determine the functions $B(p^2)$ and $C(p^2)$.

(continued overleaf)

(4) Let us consider the d-dimensional (gauge fixed) scalar QED, which contains a complex scalar $\phi(x)$ and a vector field $A_\mu(x)$. The action is written as

$$S = \int d^d x \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2\xi} (\partial^\mu A_\mu)^2 + D_\mu \phi^* D^\mu \phi - m^2 \phi^* \phi \right),$$
$$D_\mu \phi := \partial_\mu \phi - ieA_\mu \phi, \quad D_\mu \phi^* := \partial_\mu \phi^* + ieA_\mu \phi^*.$$

Find the Feynman rule for connected correlation functions.

Hint: Express the line for the complex scalar by an arrowed line and for the vector field by a wavy line. There are two kinds of vertices: