QFT II. Homework Problem Set 4. (11/18/2016)

Due 12/9/2016

Consider the fields:

- $\phi_i(x),\ (i = 1, \ldots, N), \chi_a(x),\ (a = 1, \ldots, L)$ are real scalar fields. $\phi^2 := \phi_i\phi_i,\ \chi^2 := \chi_a\chi_a$.
- $\varphi_i(x)$ $(i = 1, \ldots, N)$ are complex scalar fields. $|\varphi|^2 = \varphi_i^*\varphi_i$.
- $M(x) = (M_{ij})(x),\ (i, j = 1, \ldots, N), M_{ij}^*(x) = M_{ji}(x)$ are hermitian matrix valued scalar fields.
- $A_\mu^a(x),\ (a = 1, \ldots, K)$ are vector fields.

m^2, μ^2, λ, g are constants. Find the propagators and the vertices in the momentum space Feynman rules of the following theories.

(1)
$$S = \int d^dx \left(\frac{1}{2}\partial_\mu\phi_i\partial^\mu\phi_i - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{8}(\phi^2)^2 \right).$$

(2)
$$S = \int d^dx \left(\frac{1}{2}\partial_\mu\phi_i\partial^\mu\phi_i - \frac{1}{2}m^2\phi^2 + \frac{1}{2}\partial_\mu\chi_a\partial^\mu\chi_a - \frac{1}{2}\mu^2\chi^2 - \frac{\lambda}{4}\chi^2\phi^2 \right).$$

(3)
$$S = \int d^dx \left(\frac{1}{2}\partial_\mu\phi_i\partial^\mu\phi_i - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4}\phi^2\partial_\mu\phi_i\partial^\mu\phi_i \right).$$

(4)
$$S = \int d^dx \,\mathrm{Tr} \left(\frac{1}{2}\partial_\mu M\partial^\mu M - \frac{1}{2}\mu^2 M^2 + \frac{g}{3}M^3 \right),$$

where products of M's and $\partial_\mu M$'s are products as matrices.

(5)
$$S = \int d^dx \left[\partial_\mu\varphi_i^*\partial^\mu\varphi_i - m^2|\varphi|^2 + \mathrm{Tr}\left(\frac{1}{2}\partial_\mu M\partial^\mu M - \frac{1}{2}\mu^2 M^2 \right) + g\varphi_i^* M_{ij}\varphi_i \right].$$

(6)
$$S = \int d^dx \left(-\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} - \frac{1}{2\xi}\partial^\mu A_\mu^a\partial^\nu A_\nu^a \right),$$
$$F_{\mu\nu}^a := \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f_{abc}A_\mu^b A_\nu^c,$$
$$f_{abc} : \text{ constants, totally anti-symmetric.}$$