QFT II. Homework Problem Set 6. (1/20/2017)

Due 2/10/2017

Consider the 4-dimensional ϕ^4 theory,

$$S = \int d^4x \left[\frac{1}{2} Z_\phi \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} Z_m m^2 \phi^2 - \frac{1}{4!} Z_\lambda \lambda \phi^4 \right].$$

Use the dimensional regularization and $\overline{\text{MS}}$ scheme, find $\beta(\alpha)$ to $O(\alpha^2)$. Here $\alpha = \frac{\lambda}{(4\pi)^2}$.

Hint:

(1) Write down the action in $d = 4 - \epsilon$ dimensions, with dimensionless coupling λ and dimension 1 constant $\tilde{\mu}$

(2) Find the Feynman rule.

(3) Calculate $\Pi(p^2)$ to $O(\lambda)$. It is enough to find the divergent part. Determine Z_ϕ, Z_m to $O(\alpha)$ following $\overline{\text{MS}}$ scheme.

(4) Calculate $V_4(p_1, \ldots, p_4)$ to $O(\lambda^2)$. It is enough to find the divergent part. Determine Z_λ to $O(\alpha)$ following $\overline{\text{MS}}$ scheme.

(5) Find the relation between bare parameter α_0 and α.

(6) Use the fact that $\frac{d}{d \ln \mu} \alpha_0 = 0$ and find $\frac{d}{d \ln \mu} \alpha = \beta(\alpha)$ in $\epsilon \to 0$ limit.