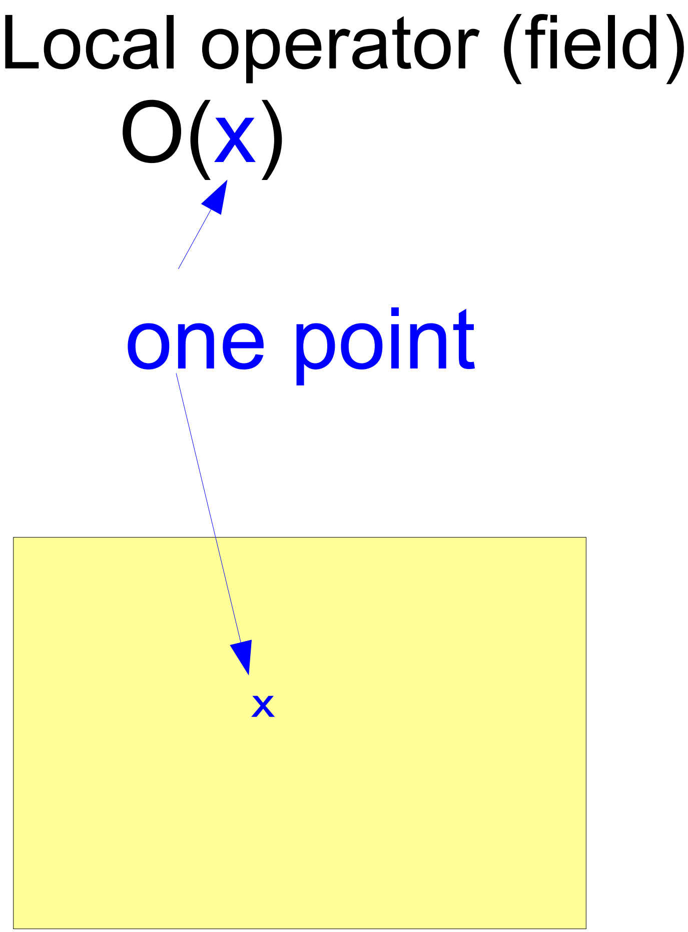
BPS non-local operators in AdS/CFT correspondence

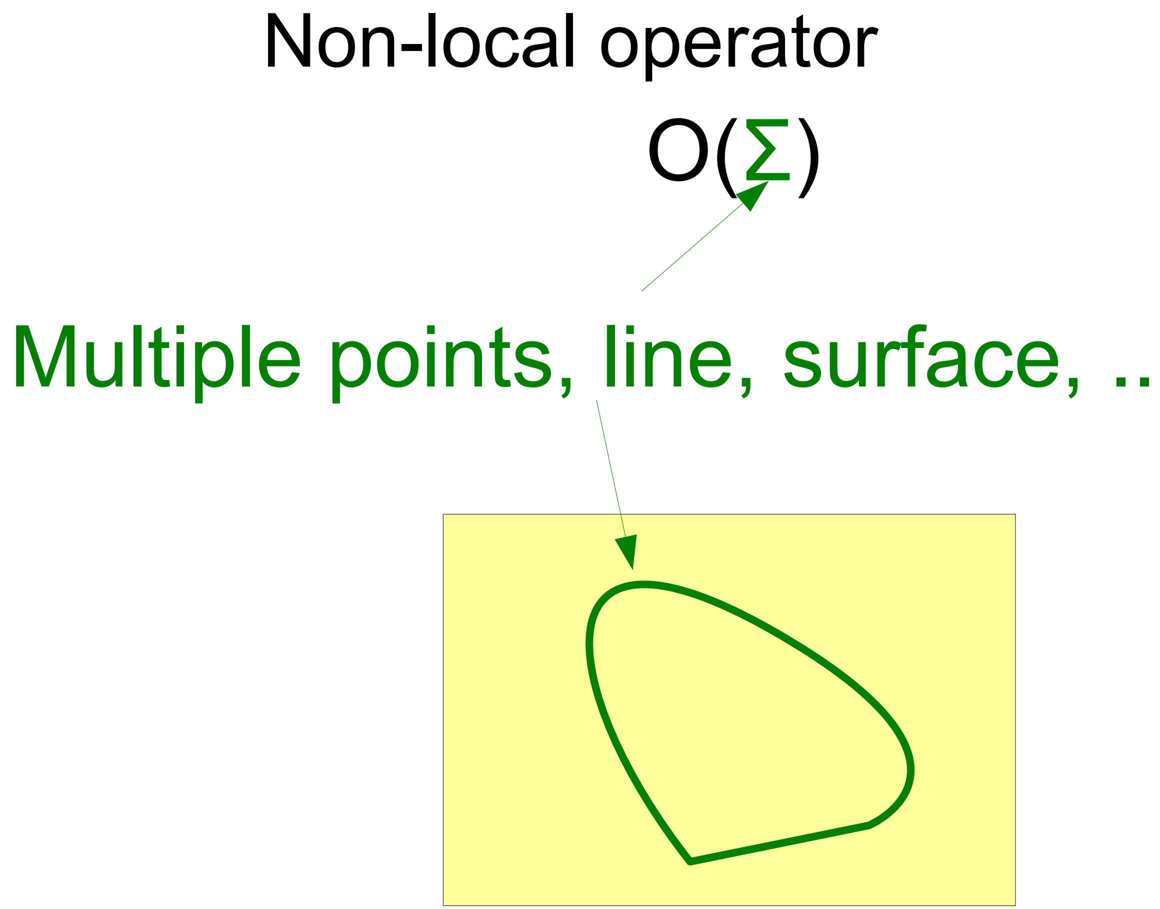
Satoshi Yamaguchi
(Seoul National University)

E. Koh, SY, arXiv:0812.1420 to appear in JHEP

Introduction

Non-local operators in quantum field theories





Example: Wilson loop

We want to address the operator localized on a sub-manifold

Motivation

- Phase structure of the QFT
  (c.f. Wilson loop, 't Hooft loop)

  Potentials between test particles, test strings etc.

- Understand branes in string theory through AdS/CFT

Classification

Non-local operators localized on a submanifold
can be classified by the dimension of the submanifold.

In a 4-dimensional field theory

0 dim	Local operator
1 dim	Line operator (Ex. Wilson loop) Introduce test particle
2 dim	Surface operator Introduce test string
3 dim	Interface operator (Can connect two different CFTs) Introduce test membrane (wall)

AdS/CFT correspondence

AdS / CFT

AdS = IIB String AdS5 x S5

CFT = 4dim N=4 Super YM theory SU(N)

Some objects ⟺ Local or non-local Operators

- Field fluctuation
- F-string
- D-brane probe
- NS5-brane probe
- Gravity solution

  etc.

GKPW prescription ⟺ Correlation functions

Plan of this talk

- Overview of 1/2 BPS non-local operators in AdS/CFT
  - 1/2 BPS non-local operators in N=4 SYM
  - Their gravity dual

- 1/4 or less BPS surface operators [Koh, SY '08]
  - Description in N=4 SYM
  - Gravity dual as D3-brane probe
  - Correlator with local operators

1/2 BPS (non-)local operators in 4-dim N=4 SYM

Summary

In 4-dim N=4 SYM

- 1/2 BPS local operators
- 1/2 BPS Wilson-'t Hooft loops
- 1/2 BPS surface operators
- 1/2 BPS interface operators

IIB string theory

- Field fluctuation
- F-string
- D-brane probe
- NS5-brane probe
- Gravity solution

etc.

4-dim N=4 SYM

- Field Contents

  - Vector $A_\mu, \mu = 0,1,2,3$
  - Spinors ψ
  - Scalars $\phi_i, i = 4,\cdots,9$

  Each field is an N x N Hermitian Matrix

- Action

$$S_{YM} = \frac{2N}{\lambda} \int d^4 x \, \mathrm{tr} \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \cdots \right]$$

$\lambda = g_{YM}^2 N$: 't Hooft coupling

Global symmetry

- Conformal symmetry

$$SO(2,4) \simeq SU(2,2)$$

- R-symmetry

$$SO(6) \simeq SU(4)$$

- Super and superconformal symmetry

(4,4) of SU(2,2) x SU(4) (complex)

$\Rightarrow$ PSU(2,2|4)

- 1/2 BPS local operators
- 1/2 BPS Wilson loops
- 1/2 BPS surface operators
- 1/2 BPS interface operators

1/2 BPS local operators

$$\Phi := \phi_4 + i\,\phi_5$$

$$\boxed{\prod_j tr[\Phi^j]^{n_j}}$$

$$J = \sum j\, n_j$$

Dilatation $\boxed{\Delta = J}$

Symmetry

Bosonic symmetry

R x SO(4) x SO(4)

$\Delta - J$ → R

Rotation around the point
Part of conformal symmetry
(→ first SO(4))

Rotation of $\phi_{6}, \ldots, \phi_9$
Part of R-symmetry
(→ second SO(4))

Also commute with half of supersymmetry
and superconformal symmetry

Gravity dual of 1/2 BPS local operators

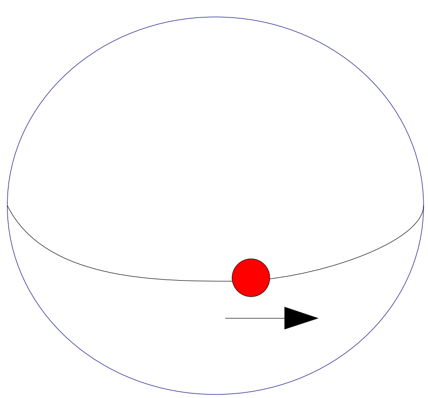
- Field fluctuation

KK reduction of IIB supergravity on AdS5 x S5

[Kim, Romans, van Nieuwenhuizen '85]

There is a field s_Δ with mass$^2 = \Delta(\Delta-4)$
with the correct symmetry

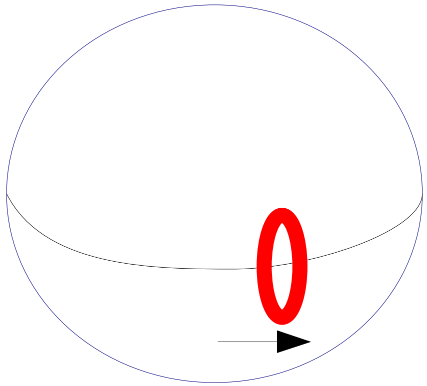
$$tr[\Phi^\Delta] \quad \Longleftrightarrow \quad s_\Delta$$



when $\Delta \ll N$

- Giant graviton [Grisaru,Myers,Tafjord '00], [Hashimoto, Hirano, Itzhaki '00], [McGreevy, Suskind, Toumbas '00]

$$\Delta \sim N$$

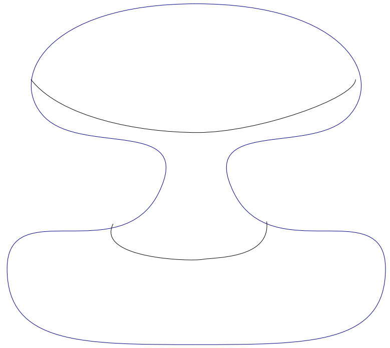


Large number of rotating gravitons puff up and become a D3-brane

- Bubbling supergravity solution

$$\Delta \sim N^2$$

[Lin, Lunin, Maldacena '04]

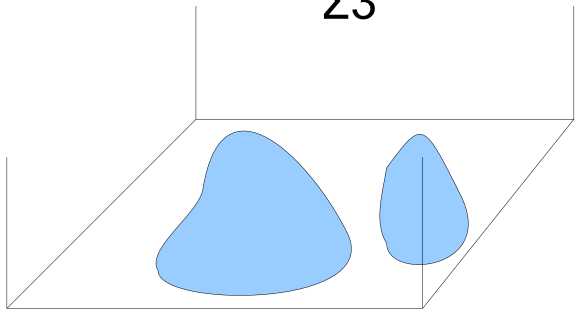


Large number of D3-branes get togather

↓

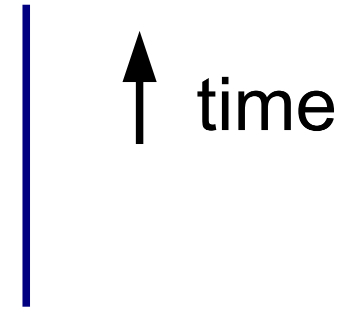
Back-reaction to the geometry

R x S3 x S3 x Σ3



- 1/2 BPS local operators
- 1/2 BPS Wilson loops
- 1/2 BPS surface operators
- 1/2 BPS interface operators

1/2 BPS Wilson line



$$Tr_R[P\exp\int dx^0 i[A_0+\phi_4]]$$

R: representation of SU(N)

Bosonic symmetry

SO(1,2) x SO(3) x SO(5)

SO(1,2): time translation, dilatation, SCT time direction

SO(3): Rotation in the space

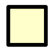
SO(5): rotation of $\phi_{6,}\ldots,\phi_9$

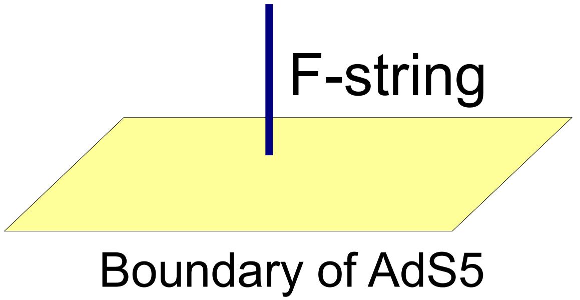
Gravity dual of half-BPS Wilson line

- Fundamental string [Rey, Yee '98], [Maldacena '98]

  AdS2 shape

  Fundamental representation

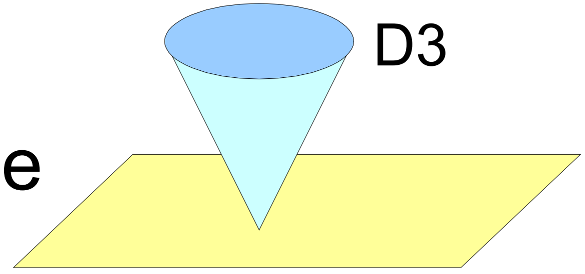




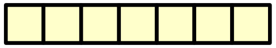
- D3-brane [Drukker, Fiol '05]

  If large number of F-strings get together, they puff up and become a D3-brane with electric flux

  AdS2 x S2 shape



  Symmetric representation

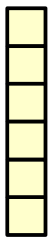


- D5-brane

[Hartnoll, Prem Kumar '06],
[SY '06], [Gomis, Passerini '06]

If large number of F-strings get together in another way,
they puff up and become a D5-brane
with electric flux

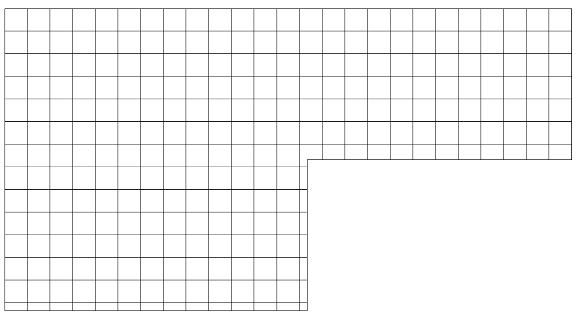
AdS2 x S4 shape

⇔ Anti-symmetric representation 

- Bubbling geometry

[SY '06], [Lunin '06], [D'Hoker, Estes, Gutperle '07]

[Okuda,Trancanelli '08], [Gomis,Matsuura,Okuda,Trancanelli '08]



If the representation becomes larger, a lot of D3-branes or D5-branes get together and the back-reaction has to be considered

AdS2 x S2 x S4 x Σ2

with smooth metric, RR5-form, RR3-form, NSNS-3-form, dilaton

Symmetry: SO(1,2) x SO(3) x SO(5)

- 1/2 BPS local operators
- 1/2 BPS Wilson loops
- 1/2 BPS surface operators
- 1/2 BPS interface operators

Disorder type operator:

Function(al) of fundamental fields

→ Boundary conditions of the fundamental fields

Example: 2 dim massless compact free boson

$$S=\frac{1}{2\pi}\int d^2 z\,\partial_z\phi\,\partial_{\bar{z}}\phi \qquad \phi\simeq\phi+2\pi R$$

Vertex operator for winding modes cannot be written as a function of the boson

It is defined by the boundary condition (or OPE)

$$\phi(z)O(0)\sim\frac{wR}{2\mathrm{i}}(\log z-\log\bar{z})O(0)$$

Lesson from this example

Classical solution with singularity

$$\phi(z) = \frac{wR}{2i}\left(\log z - \log \bar{z}\right)$$

defines a operator localized at the singular locus

- Correlation functions are defined by path-integral with this boundary condition

1/2 BPS Surface operator

[Gukov, Witten '06]

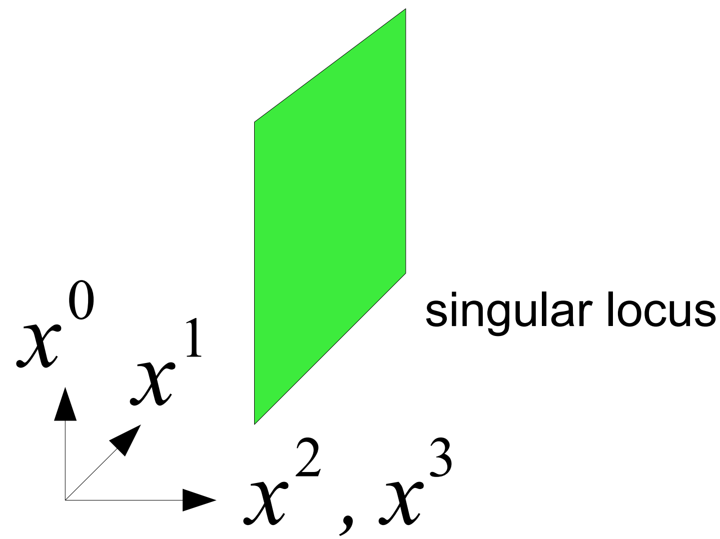
4 dim N=4 SYM

$$\Phi := \phi_4 + i\,\phi_5$$

$$z = x^2 + ix^3$$

β : constant

$$\Phi = diag\left(\frac{\beta}{z}, 0, 0, \ldots, 0\right)$$



- This configuration is a classical solution

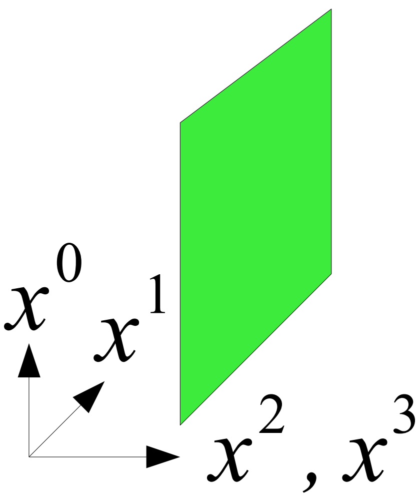
- Singular surface $z=0$ extended to x^0, x^1 direction → **Define an operator localized at $z=0$**

- Symmetry seen from the classical solution

$$\Phi = diag\left(\frac{\beta}{z}, 0, 0, \ldots, 0\right)$$

$$\Phi := \phi_4 + i\,\phi_5$$

$$z = x^2 + i x^3$$



singular locus

rotation of $\phi^6, \ldots, \phi^9$

SO(2,2) x SO(2) x SO(4)

2 dim global conformal symmetry

diagnal subgrop of

- rotation of x^2, x^3
- rotation of ϕ^4, ϕ^5

More generally

M : an integer

$N_i, \quad i = 1, \ldots, M$: partition of N $\sum_{i=1}^{M} N_i = N$

$$\Phi = \frac{1}{z} diag(\overbrace{\beta_1, \ldots, \beta_1}^{N_1}, \beta_2, \ldots, \beta_{M-1}, \overbrace{\beta_M, \ldots, \beta_M}^{N_M})$$

$$A = \frac{dz}{2\pi i z} diag(\overbrace{\alpha_1, \ldots, \alpha_1}^{N_1}, \alpha_2, \ldots, \alpha_{M-1}, \overbrace{\alpha_M, \ldots, \alpha_M}^{N_M})$$

Insertion of $\exp\left[i \sum_i \eta_i tr_{N_i} F \right]$

α_i, η_i :real

β_i :complex

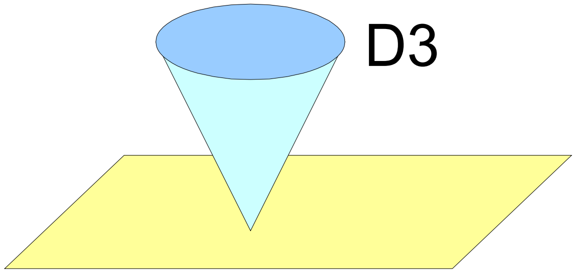
Parameters $(\beta_i, \alpha_i, \eta_i), \quad i = 1, \ldots, M$

Gravity dual of 1/2 BPS surface operator

[Constable, Erdmenger,Guralnik,Kirsch '02], [Gukov, Witten '06], [Gomis, Matsuura '07], [Drukker, Gomis, Matsuura '08], [Lin, Lunin, Maldacena '04], [Lin, Maldacena '05]

- D3-brane probe

  AdS3 x S1 shaped



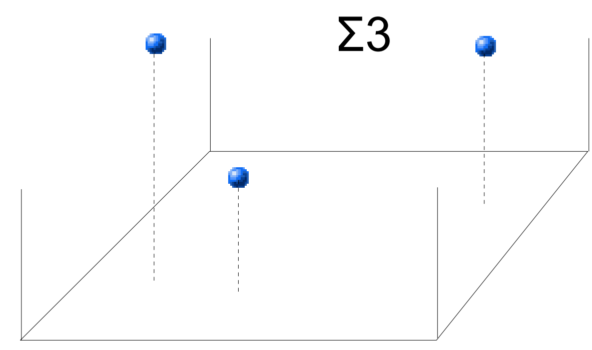
$$\Phi = diag\left(\frac{\beta}{z}, 0, 0, \ldots, 0\right)$$

- Bubbling geometry

$N_i \simeq N$ Large number of D3-brane get together and back-reaction cannot be ignored

AdS3 x S3 x S1 x Σ3

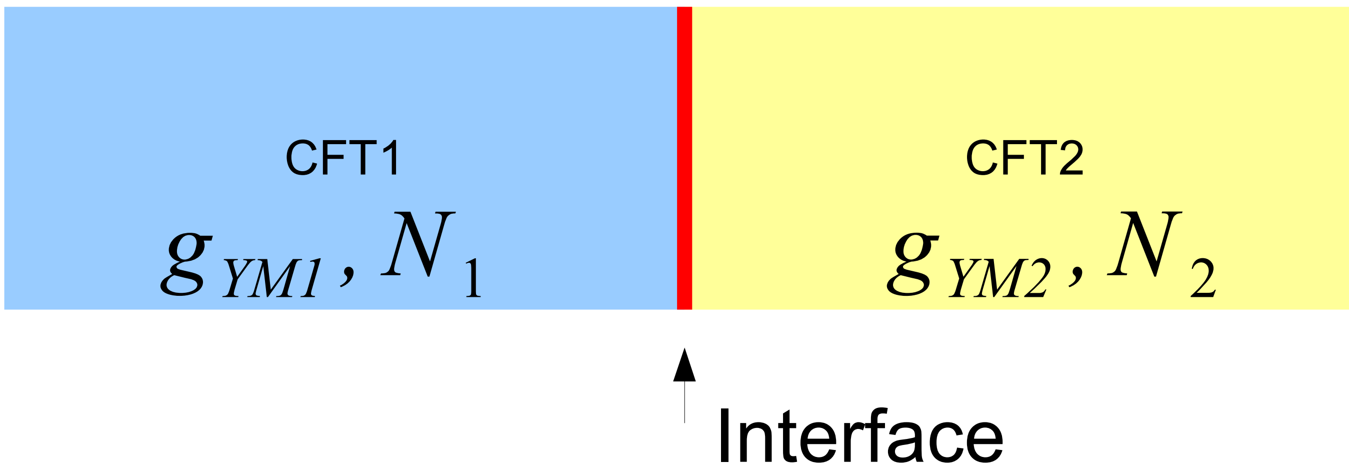
SO(2,2) x SO(4) x SO(2)



- 1/2 BPS local operators
- 1/2 BPS Wilson loops
- 1/2 BPS surface operators
- 1/2 BPS interface operators

1/2 BPS Interface

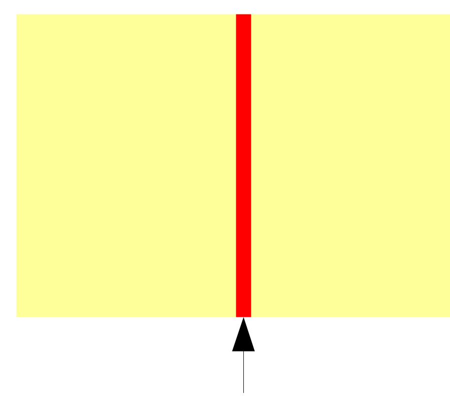
The bulk theory of each side may be different!



- example1 :   CFT1=CFT2

  [DeWolfe, Freedman, Ooguri]

  Introduce fundamental hypermultiplet
  localized at the interface

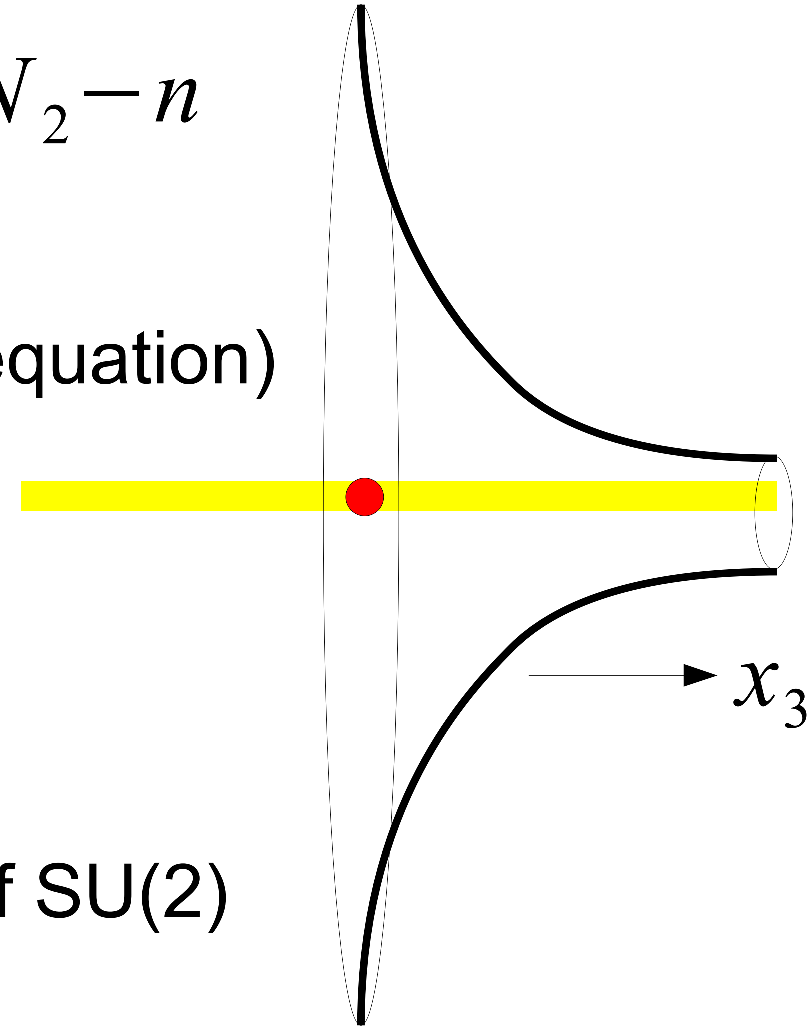


- example2: “Nahm pole” $N_1 = N_2 - n$

  [Constable, Myers, Tafjord '99]

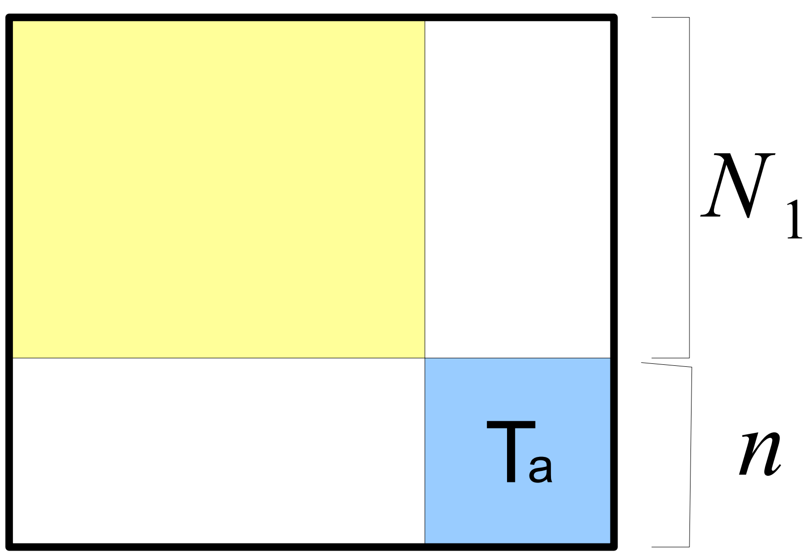
  Classical solution

  (SUSY → Nahm equation)



$$\phi_{3+a} = \frac{1}{x_3} T_a$$

T_a n-dimensional representation of SU(2)



- Symmetry

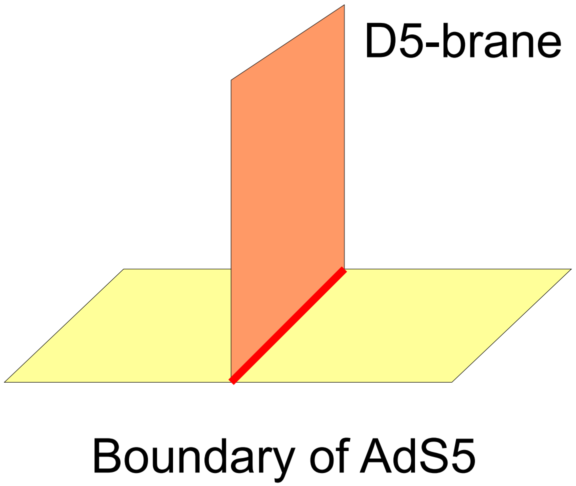
SO(2,3) x SO(3) x SO(3)

3 dim confomal

Rotation of ϕ_4, ϕ_5, ϕ_6

Rotation of ϕ_7, ϕ_8, ϕ_9

Gravity dual of 1/2 BPS interface

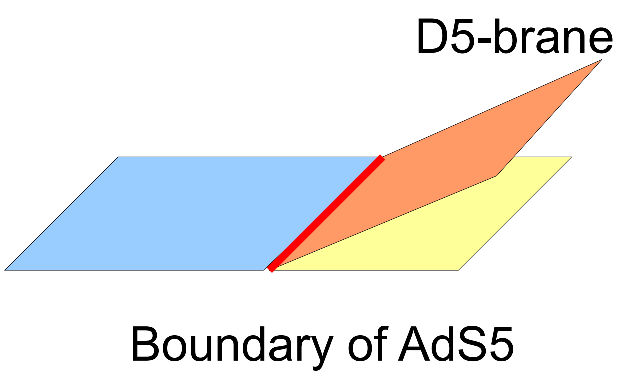


[Karch, Randall]

- D5-brane probe

  AdS4 x S2 shaped

  ⇔ Example1 fundamental hypermultiplet



- D5-brane probe

Tilted AdS4 x S2 shaped

n Magnetic flux

⇔ Example2 Nahm pole with n-dim irreducible representation

- Bubbling geometry

[Gomis, Romelsberger], [D'Hoker, Estes, Gutperle '07]

AdS4 x S2 x S2 x Σ2

with metric and various flux

SO(2,3) x SO(3) x SO(3) symmetry

Large class of interfaces

Even the case that more than two CFTs are connected at a interface is realized

BPS surface operators

Summary of the result

[Koh, SY]

- 1/4 or less BPS surface operators

- Identify gravity dual

- Check the supersymmetry
  in both the gauge theory side and gravity side

- Calculate the correlation functions with local operators
  in both sides and see they agree

How they agree between weak and strong coupling ?

1/2 BPS surface operator

4 dim N=4 SYM

$$\Phi := \phi_4 + i\,\phi_5$$

$$\Phi = diag\left(\frac{\beta}{z^1}, 0,0,\ldots,0\right)$$

$$z^1 = x^2 + ix^3$$

β : constant

- Supersymmetry

$$\delta\psi = D_\mu \phi_I \Gamma^{\mu I} \epsilon = 0 \quad \Rightarrow \quad (1+\Gamma^{2345})\epsilon = 0$$

1/2 BPS

Holomorphy is important to preserve the supersymmetry!

- Dilatation symmetry

Φ has conformal dimension 1

Degree (-1) is important to preserve the dilatation symmetry

1/4 BPS surface operator

$$\Phi \sim \frac{1}{\sqrt{z^1 z^2}}$$

Multi-valued

$$z^1 = x^2 + ix^3$$
$$z^2 = x^0 + ix^1$$

Well-defined ??

Yes, in the following way.

$$\Phi = \mathrm{diag}\left(\frac{\beta}{\sqrt{z^1 z^2}}, -\frac{\beta}{\sqrt{z^1 z^2}}, 0, \cdots, 0\right), \qquad A_\mu = 0,$$

For expample for fixed z^2, there is monodroy around $z^1 = 0$

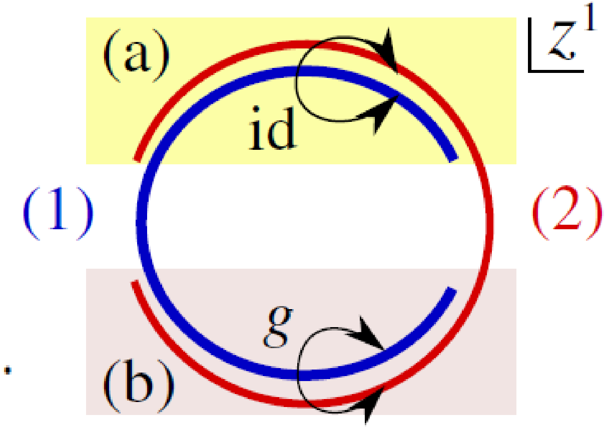
$$z^1 \to z^1 e^{2\pi i}$$

Cancel the monodromy by the gauge holonomy

Introduce two patches

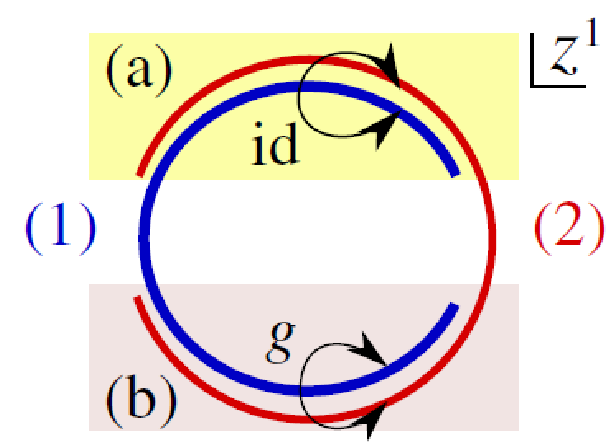
(1) $0 < \phi_1 < 2\pi$ (branch cut at $\phi_1 = \pi$).

(2) $-\pi < \phi_1 < \pi$ (branch cut at $\phi_1 = 0$).



$$z^1 = r_1 e^{i\phi_1}$$

In region (a) two patches are related by identity gauge transformation.



In region (b) two patches are related by the gauge transformation by the constant matrix g

$$g = \begin{pmatrix} i\sigma_1 & 0 \\ 0 & I_{N-2} \end{pmatrix}$$

⇒ Cancel the monodromy and become a consistent configuration

Supersymmetry in the gauge theory side

$$\delta\psi = D_\mu \phi_I \Gamma^{\mu I} \epsilon = 0$$

$$(1+\Gamma^{0145})\epsilon = (1+\Gamma^{2345})\epsilon = 0$$

1/4 BPS

Expectation value is also checked $\langle O_\Sigma \rangle = 1$

Gravity dual

= a D3-brane configuration

AdS5 x S5 coordinates $(z^1, z^2, \omega^1, \omega^2, \omega^3)$

all complex numbers

$$ds^2 = \frac{1}{\sum_a |\omega^a|^2} \left(\sum_{a=1}^{3} |d\omega^a|^2 + (\sum_a |\omega^a|^2)^2 \sum_{m=1}^{2} |dz^m|^2 \right)$$

D3-brane wrapping the surface

$$z^1 z^2 (\omega^1)^2 - \kappa^2 = 0, \quad \omega^2 = \omega^3 = 0,$$

κ : constant related to β by $\kappa = \dfrac{2\pi\beta}{\sqrt{\lambda}}$

Supersymmetry of the gravity dual

- Kappa symmetry projection
- 12 dimensional formulation

[Mikhailov '00], [Kim, Lee '06]

Correlator with chiral primary in the gauge theory side

Want to calculate some physical quantities

$$\langle O_{\Sigma} \cdot O(z) \rangle$$

(For 1/2 BPS case [Drukker, Gomis, Matsuura])

$$O(z) = C^{I_1 \cdots I_\Delta} tr[\phi_{I_1} \cdots \phi_{I_\Delta}]$$

↑ Traceless, symmetric tensor

$$\frac{\langle \mathcal{O}_\Sigma \cdot \mathcal{O}(\zeta) \rangle}{\langle \mathcal{O}_\Sigma \rangle} = \frac{1}{\langle \mathcal{O}_\Sigma \rangle} \int_{\text{boundary condition}} [DAD\psi D\phi] \, \mathcal{O}(\zeta) \, e^{-S}$$

$$\cong \mathcal{O}|_\Sigma(\zeta)$$

Classical approximation
just insert the classical solution

Only SO(4) invariant ones are non-zero

The result in the gauge theory side

(Classical approximation)

$$\frac{\langle \mathcal{O}_{\Delta,k}(\zeta)\cdot \mathcal{O}_{\Sigma}\rangle}{\langle \mathcal{O}_{\Sigma}\rangle} = \frac{(8\pi^2)^{\Delta/2}}{\lambda^{\Delta/2}\sqrt{\Delta}} C_{\Delta,k} \frac{\beta^{\Delta}}{|\zeta^1\zeta^2|^{(\Delta-k)/2}(\zeta^1\zeta^2)^{k/2}} \left(1+(-1)^{\Delta}\right)$$

Correlator with chiral primary in the gravity side

Some field fluctuation of metric and RR4-form

Chiral primary operators

GKPW: calculate the classical action of the solution with source inserted at boundary.

D3-brane is treated as probe

Action of the gravity side $S_{gravity} = S_{IIB\ sugra} + S_{D3}$

$$S_{D3} = S_{DBI} - S_{WZ}, \quad S_{DBI} = T_{D3} \int d^4\xi \sqrt{|\det G_{mn}|}, \quad S_{WZ} = T_{D3} \int_{\Sigma_4} C_4.$$

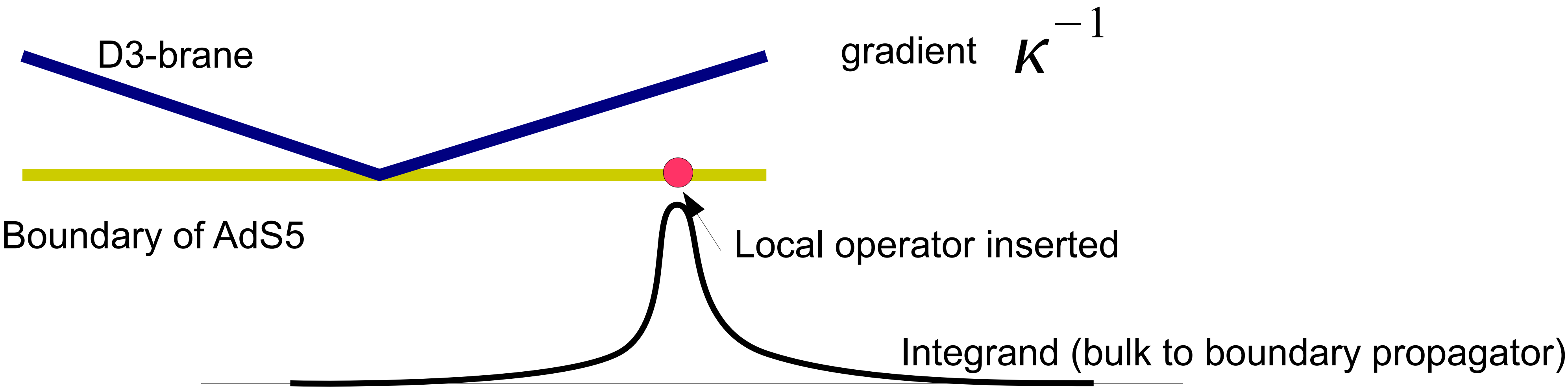
$$\frac{\langle \mathcal{O}_{\Delta,k} \cdot \mathcal{O}_\Sigma \rangle}{\langle \mathcal{O}_\Sigma \rangle} = \frac{\delta S_{gravity}}{\delta s_0(\zeta)}\Big|_{s_0=0} = \frac{\delta S_{D3}}{\delta s_0(\zeta)}\Big|_{s_0=0}$$

$$= -2\Delta T_{D3}\, c(\Delta) C_{\Delta,k} \int d^4 z \frac{\omega^{-\frac{\Delta-k}{2}}(z)\bar{\omega}^{-\frac{\Delta+k}{2}}(\bar{z})}{L^{\Delta+2}} \frac{|\zeta^m \partial_m \omega(z)|^2}{|\omega(z)|^2}$$

$$L \equiv \sum_{m=1,2} |z^m - \zeta^m|^2 + |\omega|^{-2} \qquad \omega(z) = \frac{\kappa}{\sqrt{z^1 z^2}}$$

It is not easy to evaluate exactly this integral

Approximation in the limit $\kappa \to \infty$



The integrand has a SHARP PEAK in this limit!

$$\frac{\langle \mathcal{O}_{\Delta,k} \cdot \mathcal{O}_\Sigma \rangle}{\langle \mathcal{O}_\Sigma \rangle} = -\frac{2^{\Delta/2}}{\sqrt{\Delta}} C_{\Delta,k} \frac{\kappa^\Delta}{(d_1 d_2)^{\Delta/2}} e^{-ik(\phi_1' + \phi_2')/2} (1 + (-1)^\Delta)$$

$$\zeta_j = d_j e^{i\phi'}$$

Agree with the classical calculation in the gauge theory side with the identification

$$\kappa = \frac{2\pi\beta}{\sqrt{\lambda}}$$

Correction

$$\frac{\langle \mathcal{O}_{\Delta,k} \cdot \mathcal{O}_\Sigma \rangle}{\langle \mathcal{O}_\Sigma \rangle} = I_0^{(1/4)} \left(1 + \kappa^{-2} \frac{\Delta^2 - k^2}{16(\Delta - 1)} \left(\frac{d_1^2 + d_2^2}{d_1 d_2} \right) + \cdots \right)$$

$$\kappa^{-2} = \frac{\lambda}{4\pi^2 \beta^2}$$

This expression is positive power in λ !

The situation is similar to plane wave limit of BMN

Large β mimics the perturbative expansion in λ

To compare this term with the perturbative Yang-Mills calculation is an interesting problem.

Summary

4-dim N=4 SYM

- 1/2 BPS local operators
- 1/2 BPS Wilson-'t Hooft loops
- 1/2 BPS surface operators
- 1/2 BPS interface operators

IIB string theory

- Field fluctuation
- F-string
- D-brane probe
- NS5-brane probe
- Gravity solution

  etc.

All those different pictures should be the same thing

Just classical approximation in some pictures are good and others are not.

Relation between those pictures give some hints to the quantum mechanical nature of the string theory.

- 1/4 or less BPS surface operators

- Identify gravity dual

- Check the supersymmetry
  in both the gauge theory side and gravity side

- Calculate the correlation functions with local operators
  in both sides and see they agree