Islands and Cosmology

Review talk

RIKEN
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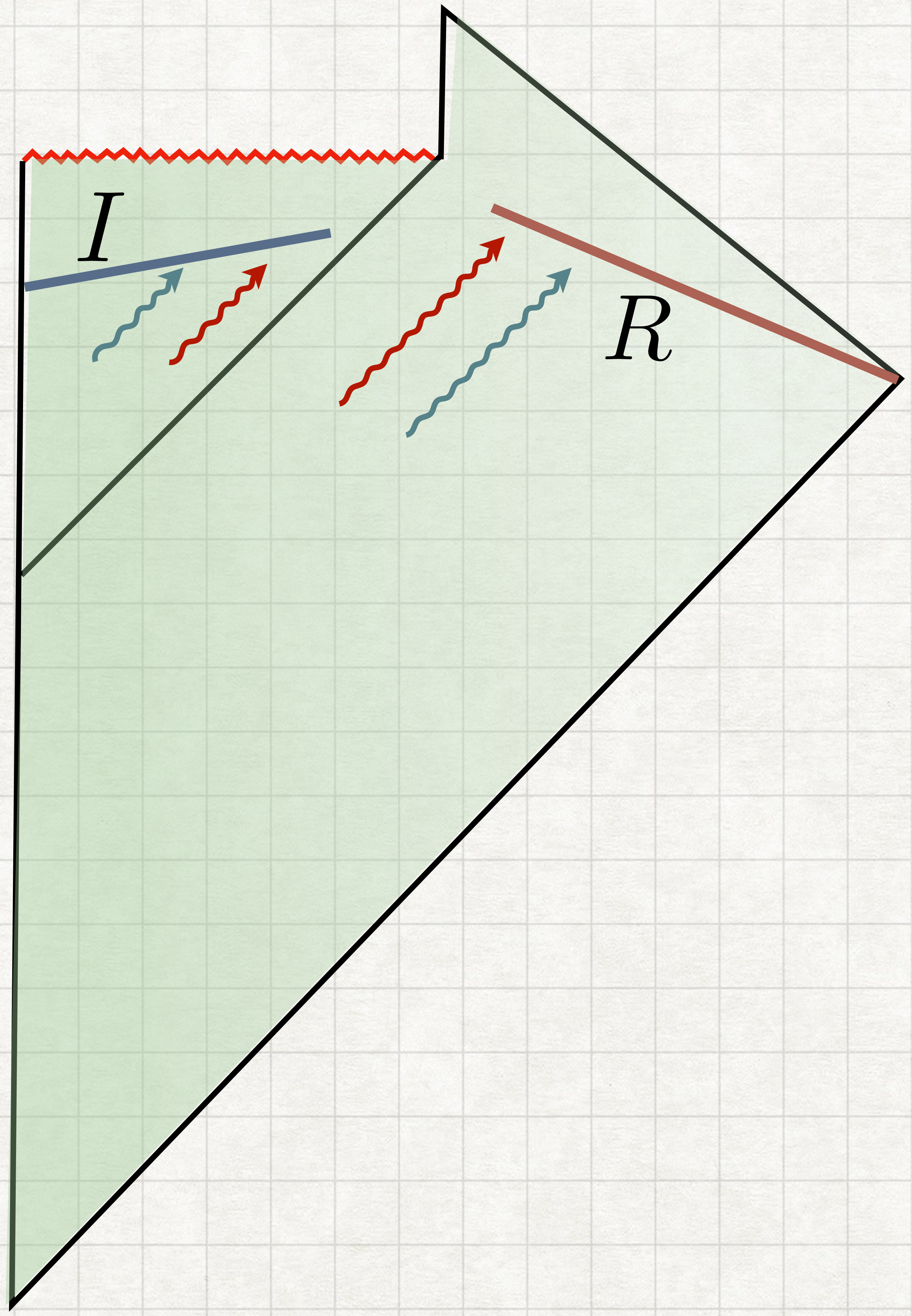
Island Formula

[Penington, Almheiri-Engelhardt-Marolf-Maxfield '19]



The entropy of Hawking radiation in quantum gravity

$$S_{\mathrm{Rad}} = \min \operatorname{ext}_I \left[\frac{\mathrm{Area}(\partial I)}{4G_N} + S_{\mathrm{matter}}\,(I \cup R) - S_{\mathrm{ct}}(\partial I) \right]$$

At late stage of the black hole evaporation,

$$S_{\mathrm{Rad}} \approx \frac{\mathrm{Area}(\mathrm{horizon})}{4G_N} = S_{\mathrm{BH}}$$

$$\to 0$$

The island region I is encoded in Hawking radiation!

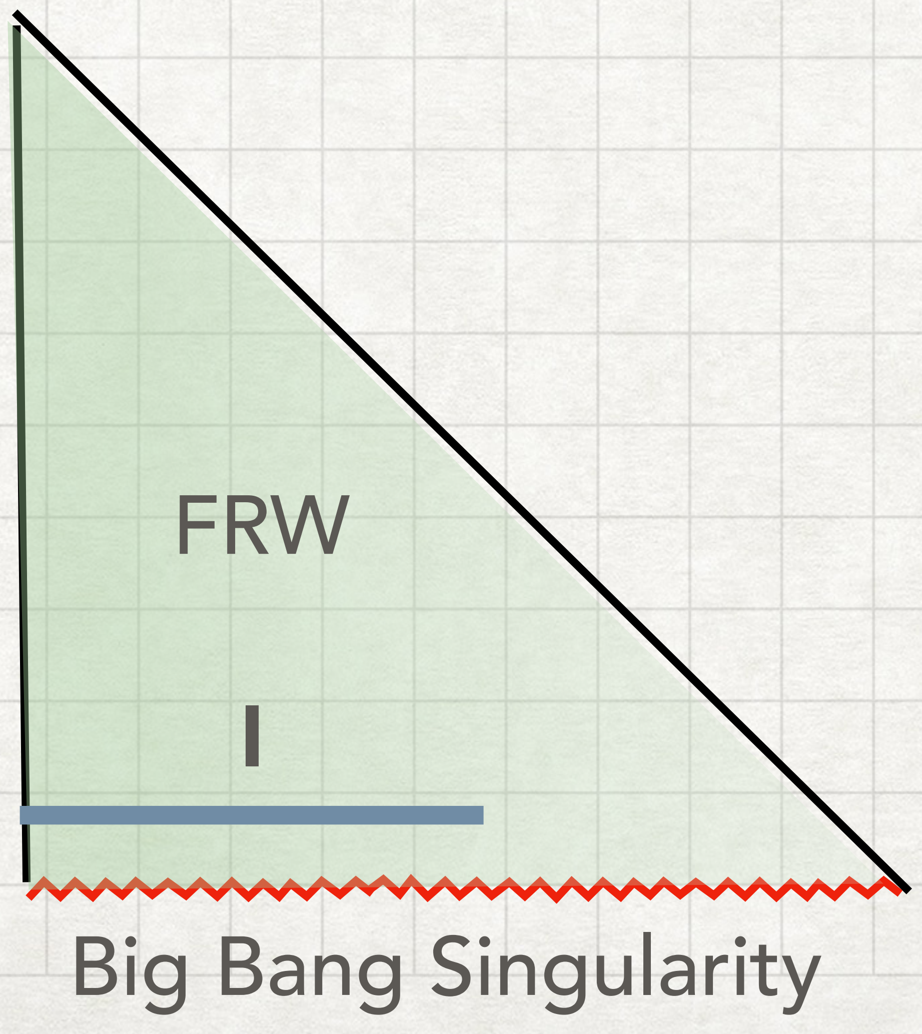
In quantum gravity, information is stored in a non-local form.

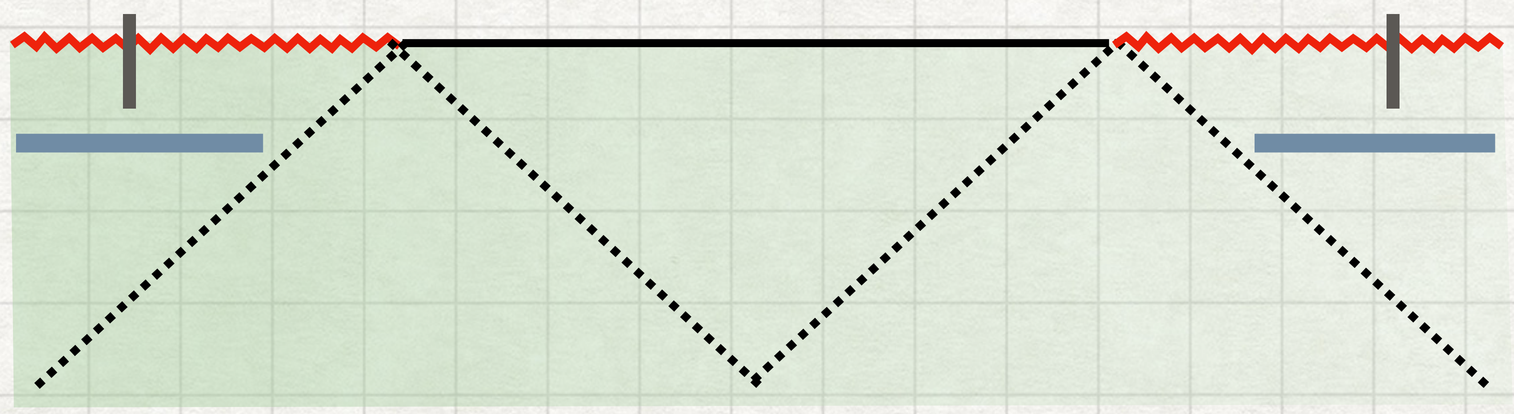
Q. Do the effects of the existence of islands (wormholes) affect us living in a low energy world
Are they observable?

- Any implications for early universe, where the effects of quantum gravity are crucial?

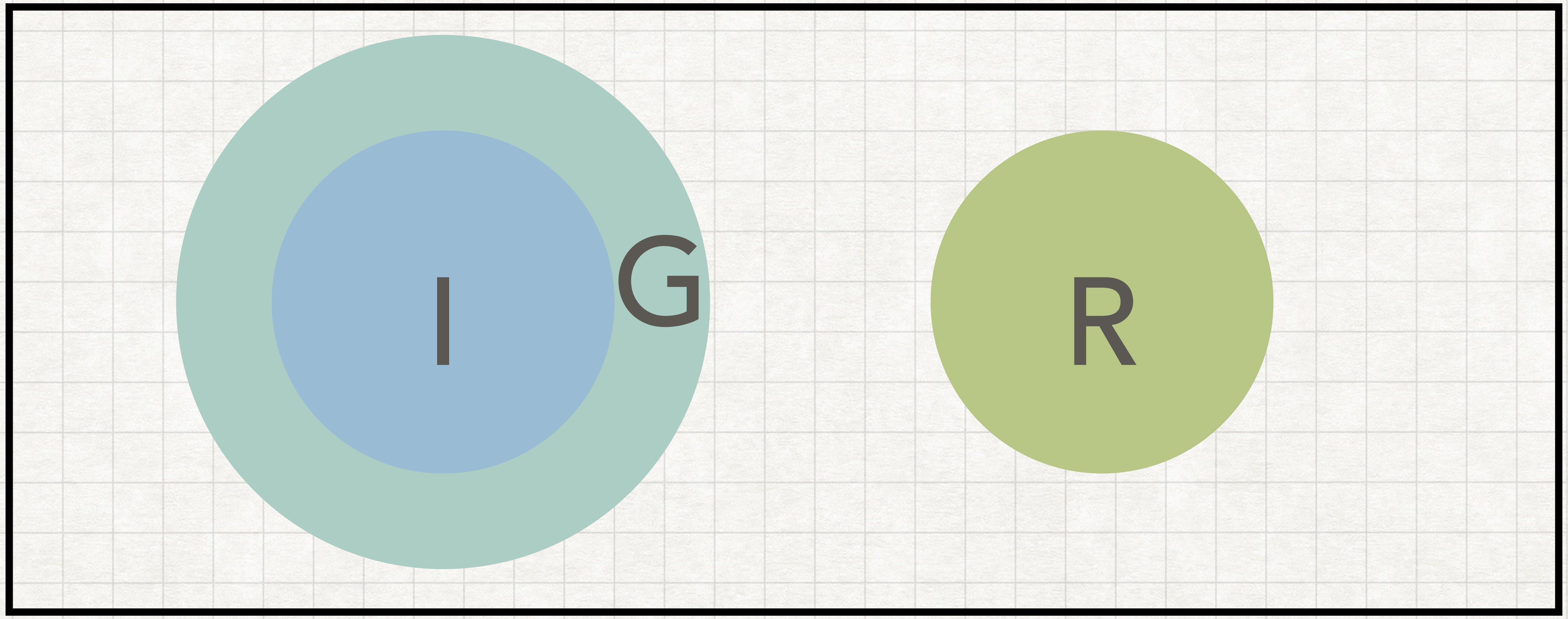
In this talk I will review one (direct) application of the island formula to cosmology, arXiv:2008.01022 "Island in cosmology".

In this paper, the Island formula is applied to the FRW universe and de Sitter spacetime.





Q. When can a region I be the island for a region R?



1. Bekenstein bound is violated for the region I

2. The island I exists in the quantum normal region

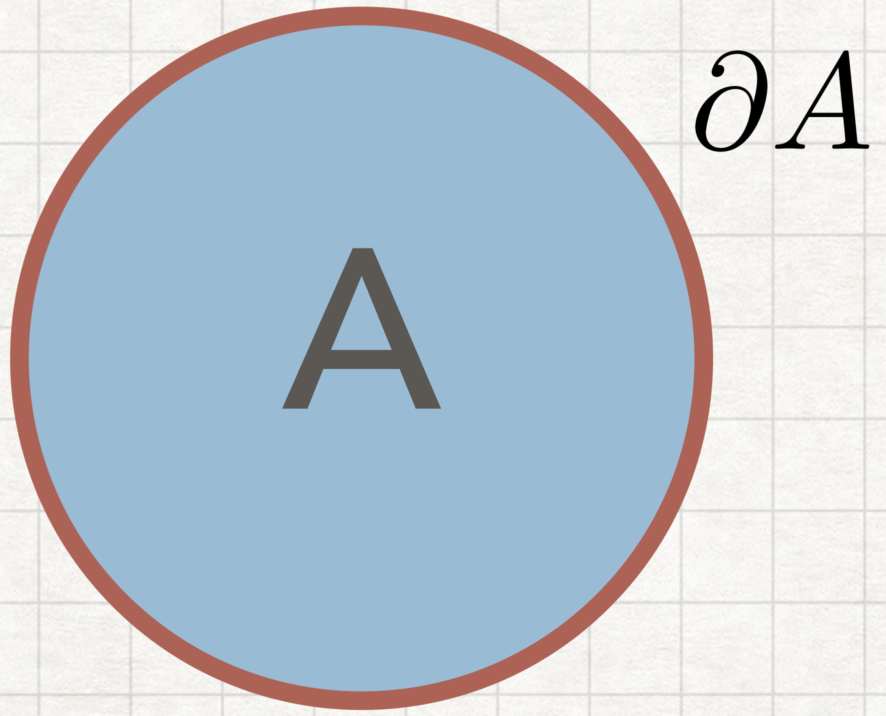
3. A region G that surrounds island I and has a common boundary with it exists in the quantum normal region

1. Bekenstein bound is violated for the region I

Bekenstein bound

For a region A defined in quantum gravity:

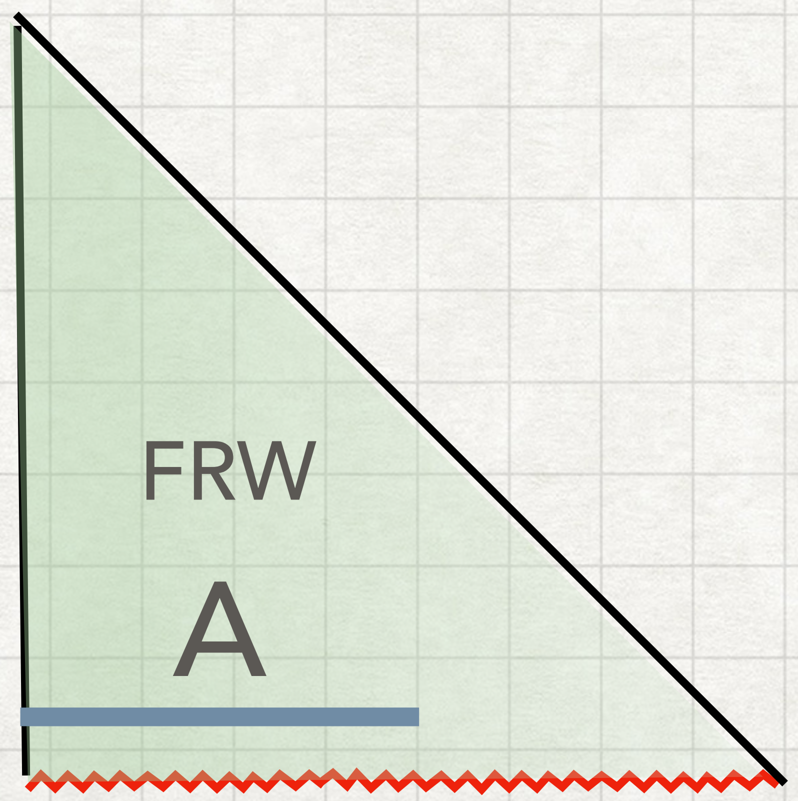
$$S_{\text{matter}}(A) \leq \frac{\text{Area}(\partial A)}{4G_N}$$

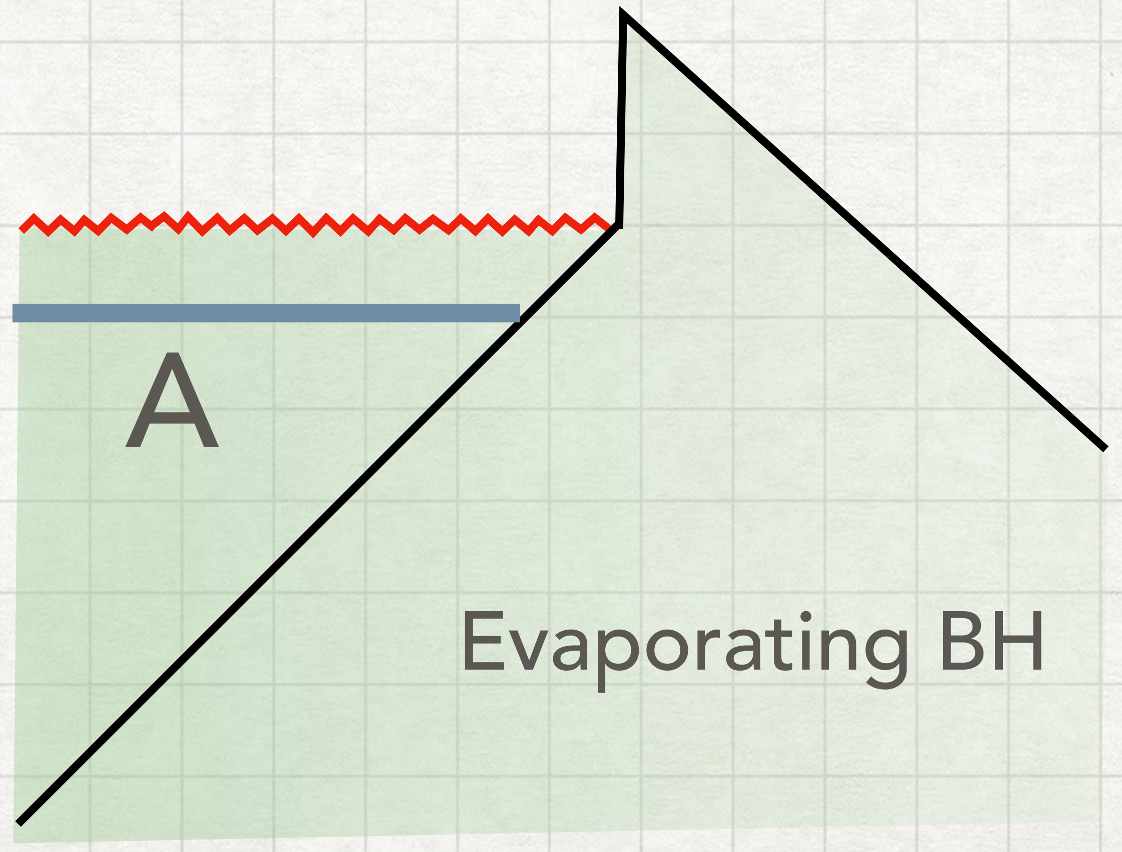


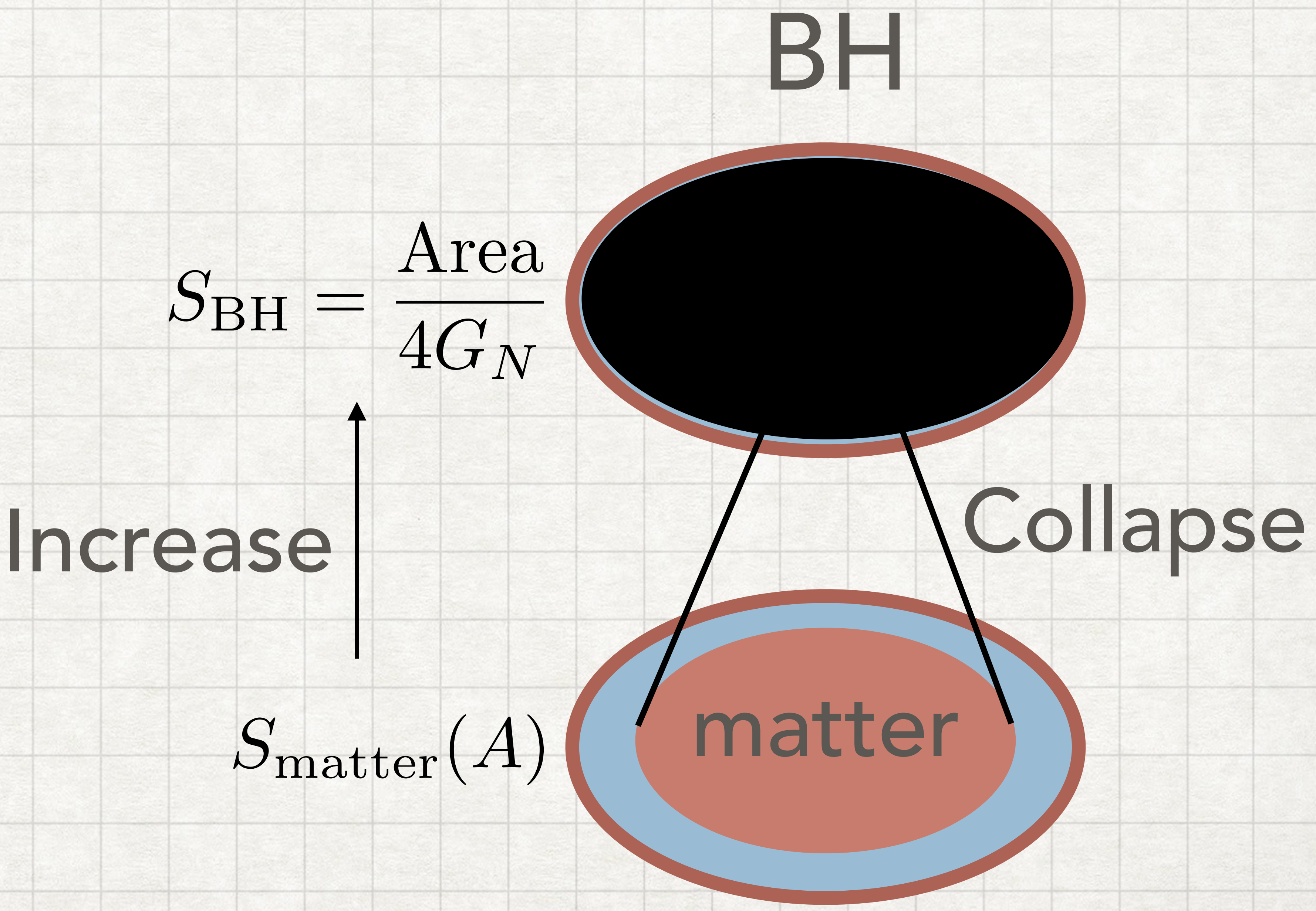
- The foundation of the holographic principle

(sometimes called "holographic bound")

- It can be violated ``near the singularity"

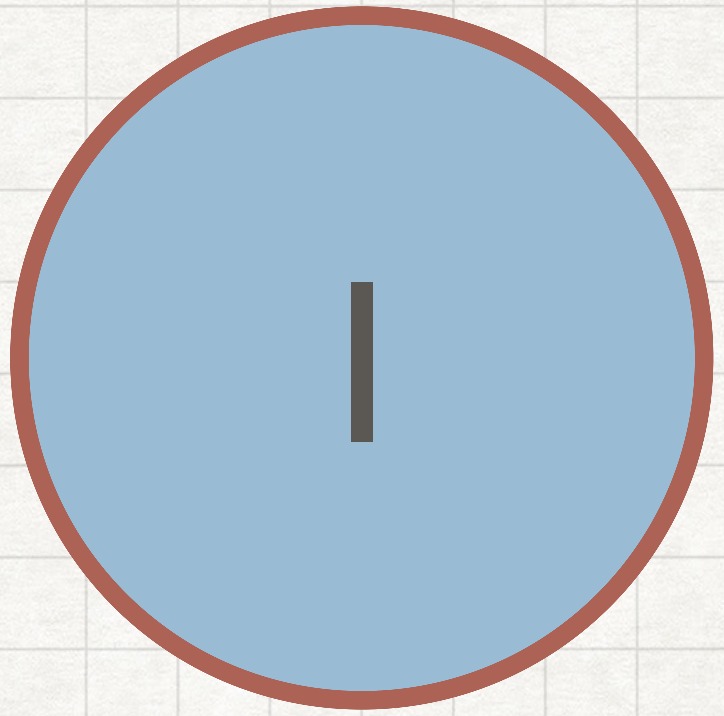






1. Bekenstein bound is violated for the region I

$$S_{\text{matter}}\ (I) \gtrsim \frac{\text{Area}(\partial I)}{4G_N}$$



Conditions for non-trivial islands to dominate

$$\frac{1}{4G_N}\text{Area}(\partial I) + S_{\text{mat}}(I \cup R) - S_{\text{ct}}(\partial I) < S_{\text{mat}}(R)$$

can be re-combined and written as the following UV finite quantities

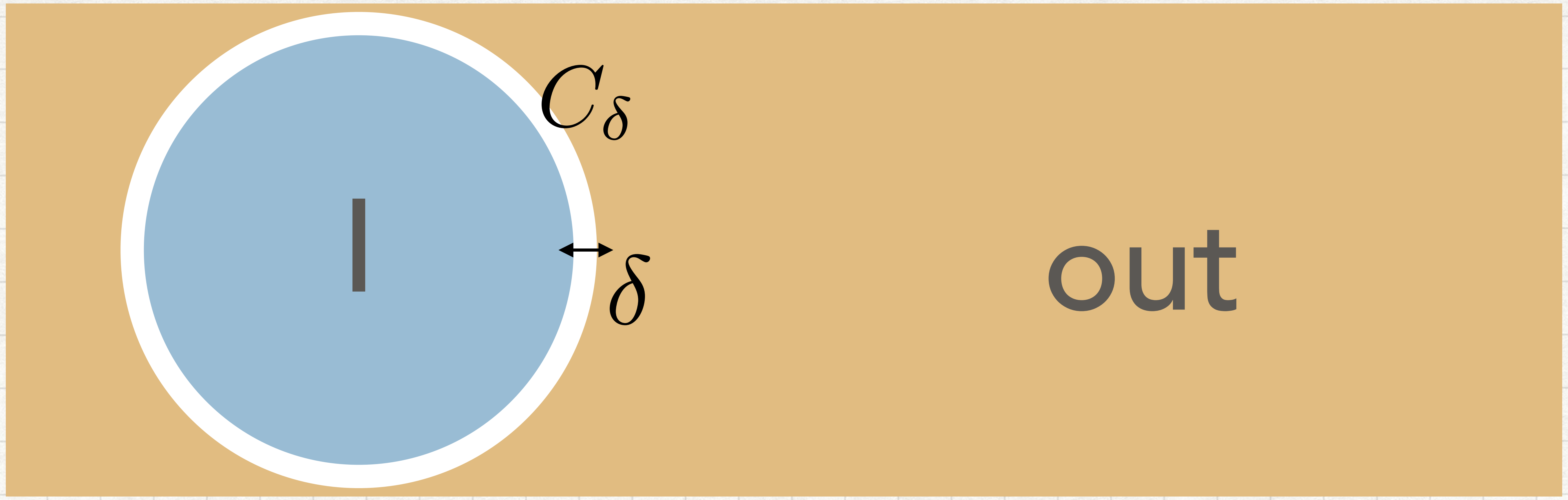
$$I_{\text{mat}}(I, R) \geq S_{\text{gen}}(I)$$

Mutual Information

$$I_{\text{mat}}\ (I, R) = S_{\text{mat}}\ (I) + S_{\text{mat}}\ (R) - S_{\text{mat}}\ (I \cup R)$$
$$= \widehat{S}_{\text{mat}}(I) + \widehat{S}_{\text{mat}}(R) - \widehat{S}_{\text{mat}}(I \cup R)$$

Generalized entropy

$$\frac{1}{4G_N}\text{Area}(\partial I) + S_{\text{mat}}(I)$$



Strong subadditivity: $I(I,\mathrm{out}) \geq I(I,R) \geq S_{\mathrm{gen}}(I)$

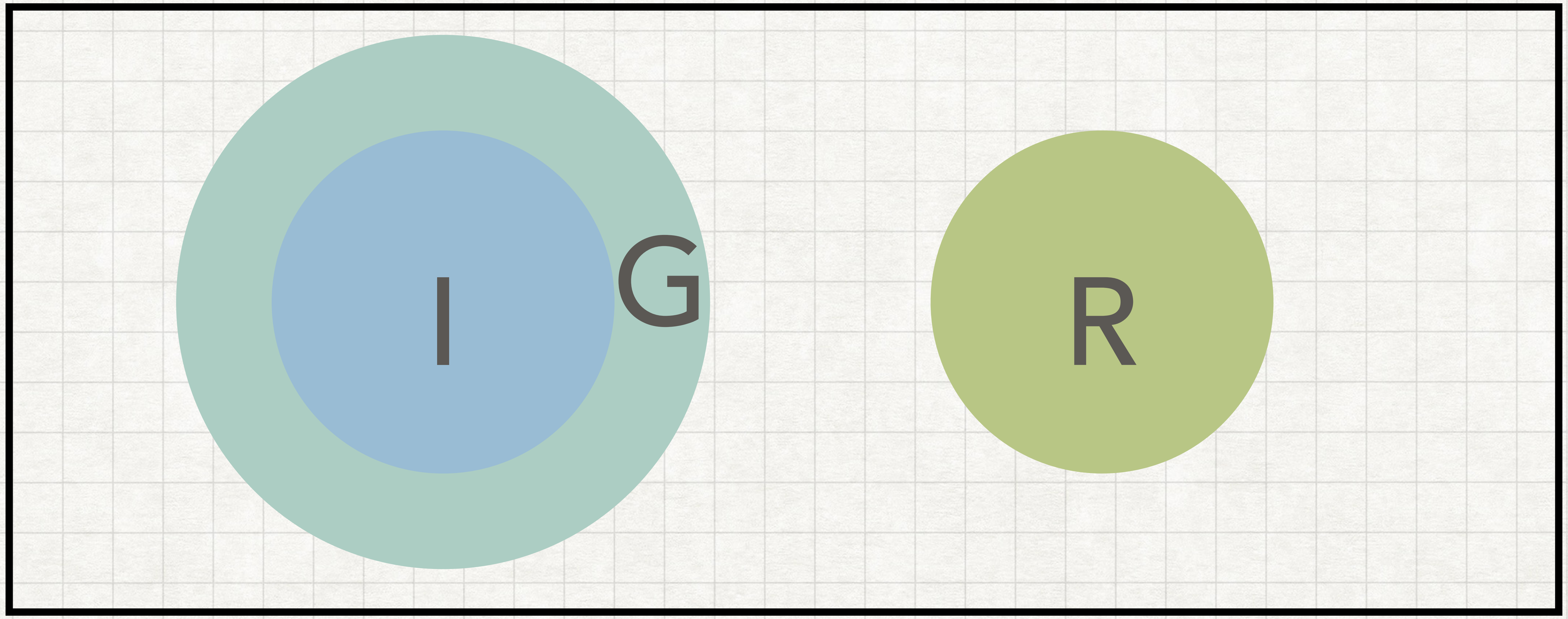
$$= I(I, I\cup C_\delta) = \widehat{S}_{\mathrm{mat}}(I) + \widehat{S}_{\mathrm{mat}}(I\cup C_\delta) - \widehat{S}_{\mathrm{mat}}(C_\delta)$$

$$\underset{\delta\to 0}{\simeq} 2\widehat{S}_{\mathrm{mat}}(I) + (\mathrm{subleading})$$

これからBekenstein boundの破れが導かれる

$$\widehat{S}_{\mathrm{mat}}(I) \gtrsim \frac{\mathrm{Area}(\partial I)}{4G_N}$$

Q. When can a region I be the island for a region R?



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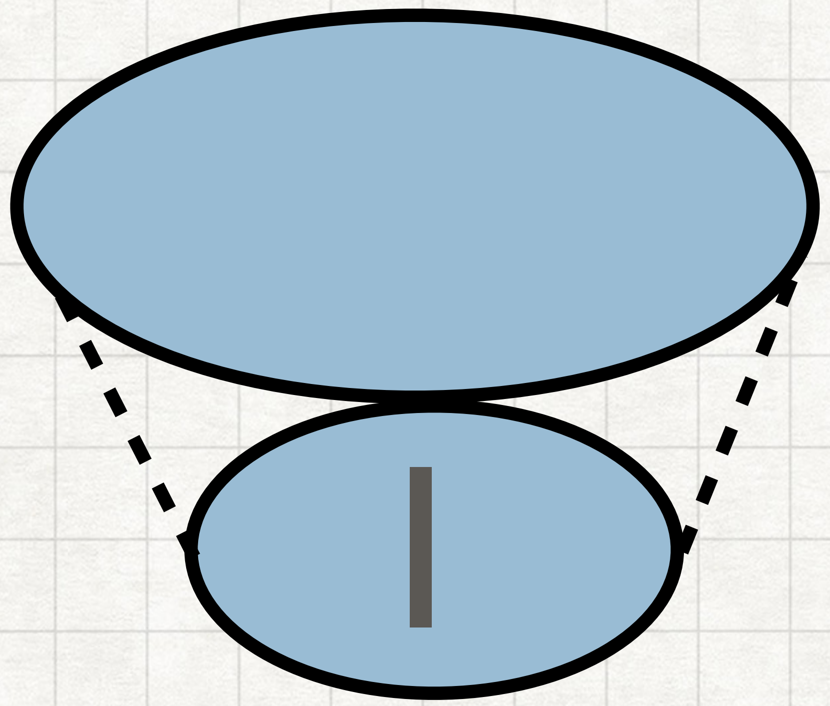
2. The island I exists in the quantum normal region

Classical normal region ("untrapped" surface)

Deformation in outward null direction $\frac{d}{d\lambda^+}\text{Area} \geq 0$

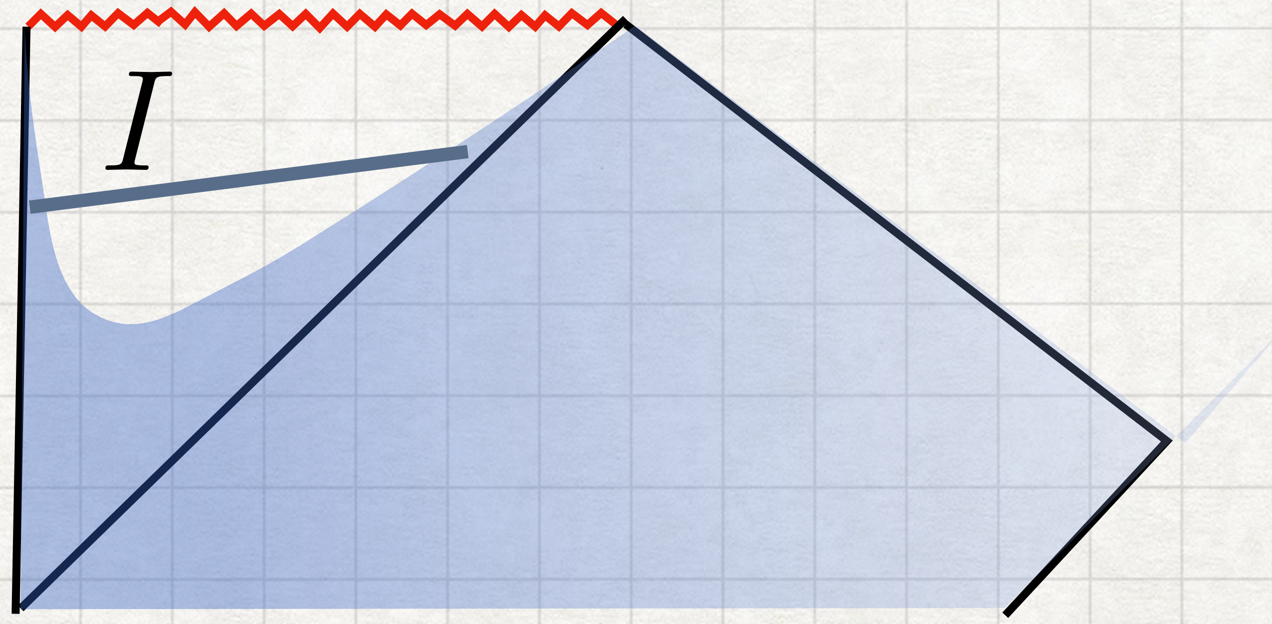
Deformation in inward null direction $\frac{d}{d\lambda^-}\text{Area} \leq 0$



Quantum normal region

Deformation in outward null direction $\frac{d}{d\lambda^+}S_{\text{gen}} \geq 0$

Deformation in inward null direction $\frac{d}{d\lambda^-}S_{\text{gen}} \leq 0$



2. The island I exists in the quantum normal region

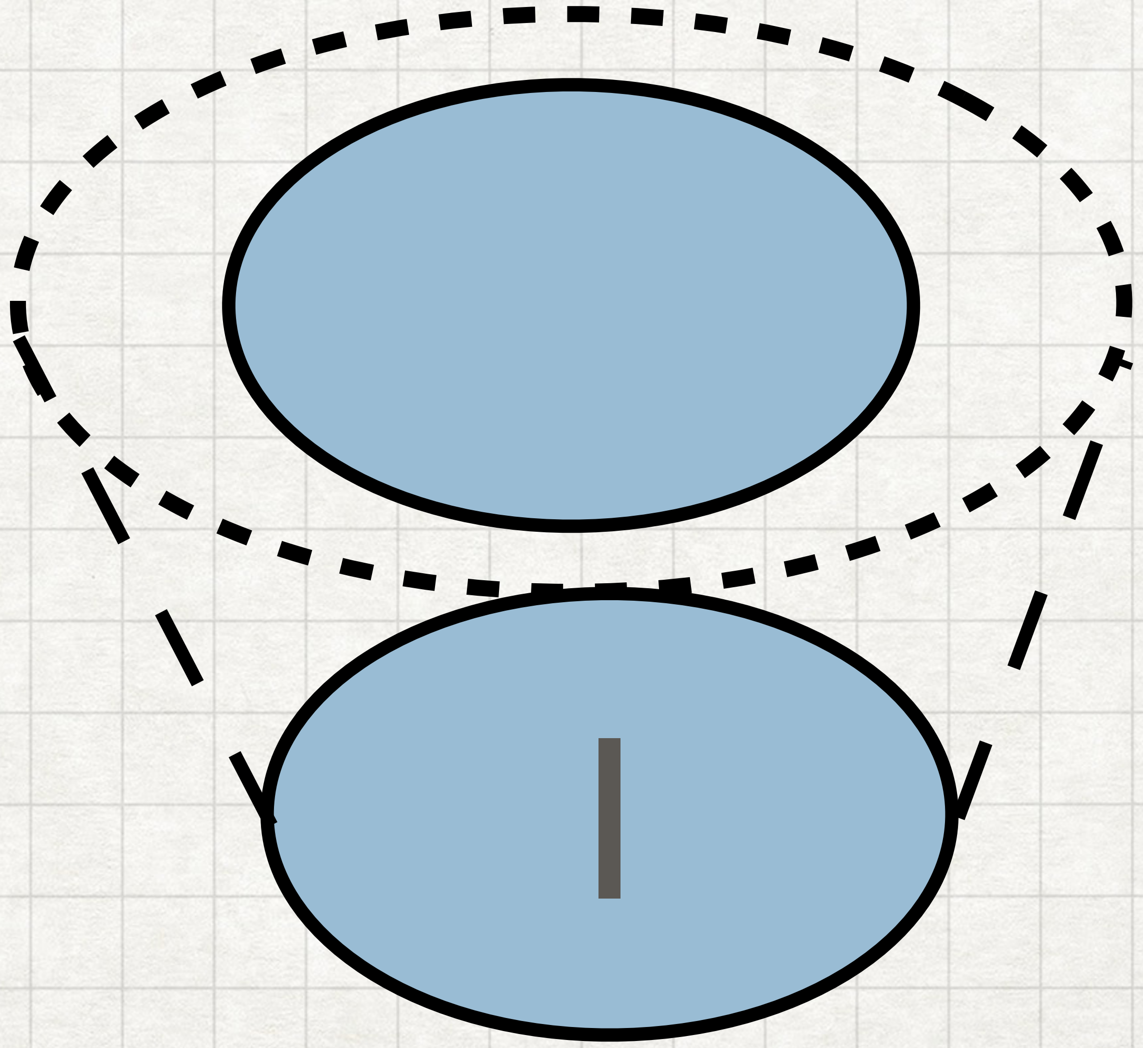
Extremal condition for the island $\quad \frac{d}{d\lambda} S_{\text{gen}}(R \cup I) = 0$

$$\downarrow$$

$$\underbrace{\frac{d}{d\lambda}\left[S_{\text{mat}}(I) + \frac{\text{Area}(\partial I)}{4G_N} - S_{\text{ct}}(\partial I)\right]}_{= S_{\text{gen}}(I)} = \underbrace{\frac{d}{d\lambda}\left[S_{\text{mat}}(I) + S_{\text{mat}}(R) - S_{\text{mat}}(I \cup R)\right]}_{= I(I,R)} = 0$$

For the outward null deformation, the r.h.s. is positive due to the monotonicity of the mutual information

$$\pm \frac{d}{d\lambda_{\pm}} S_{\text{gen}}\,(I) \geq 0$$



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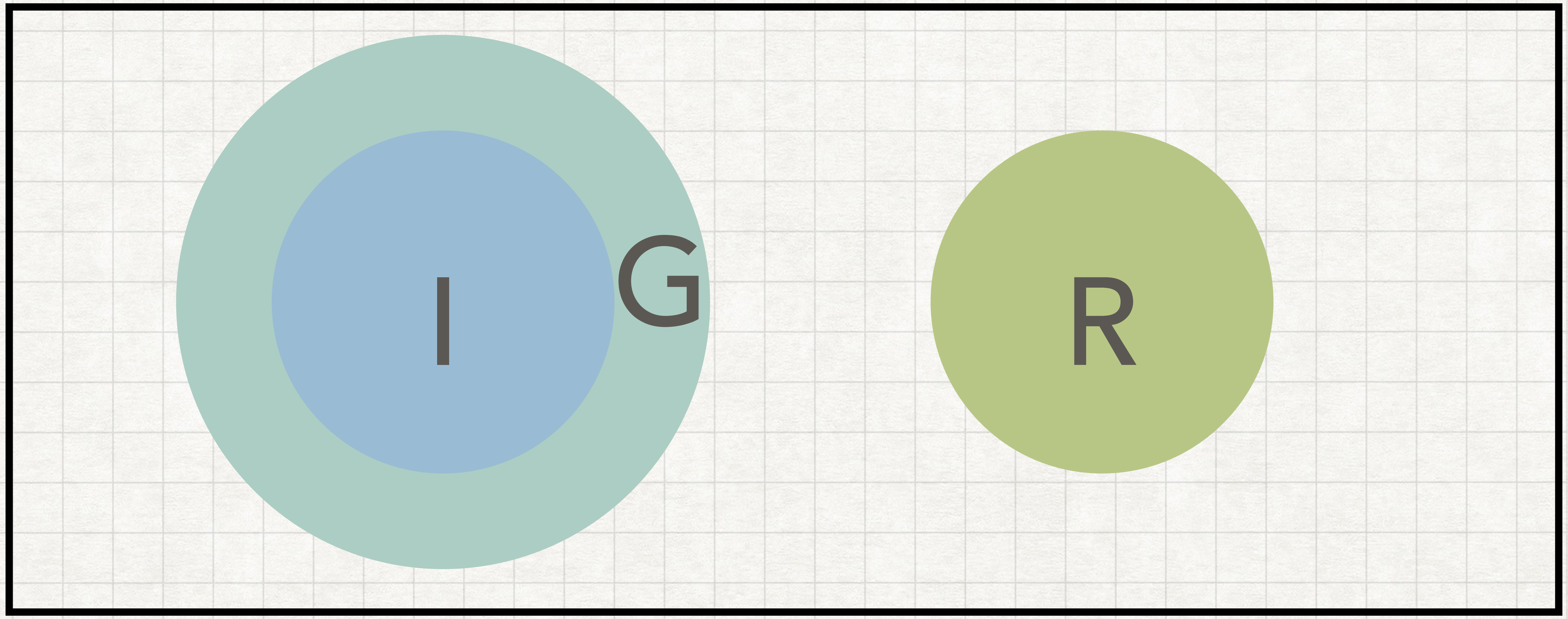

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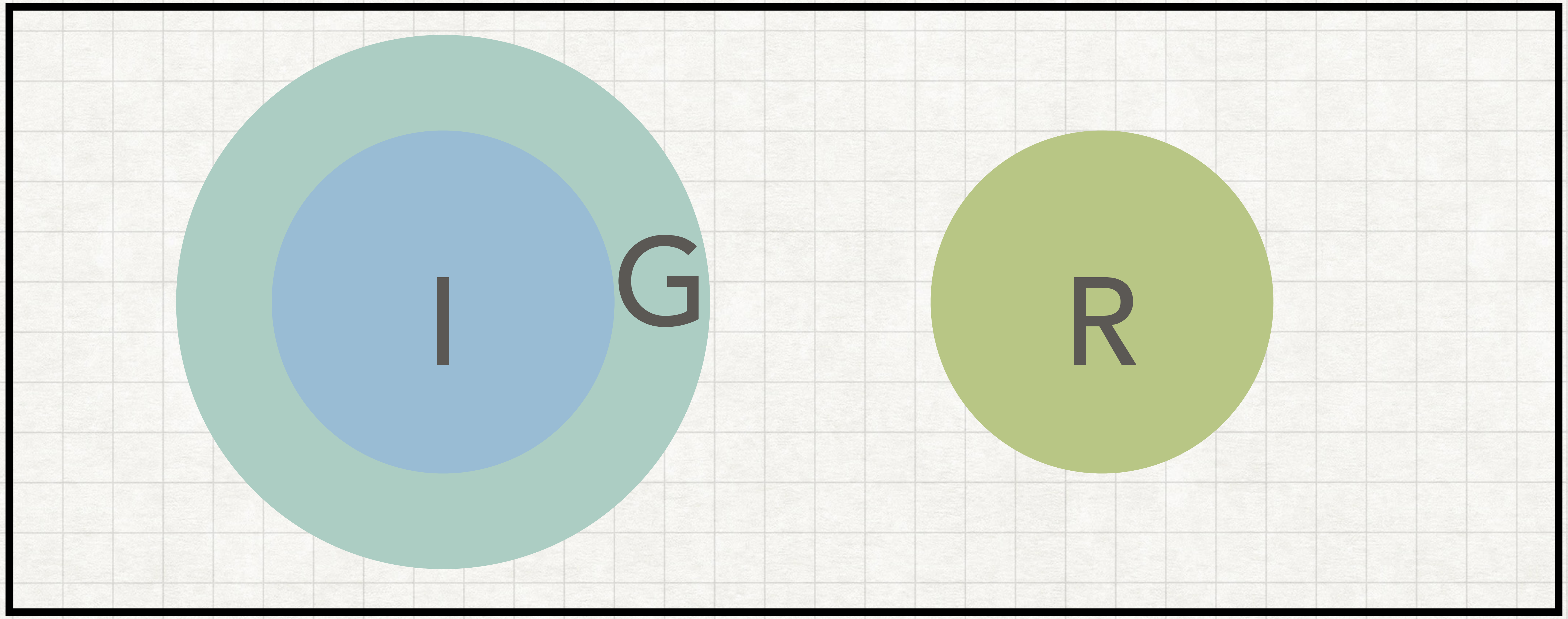
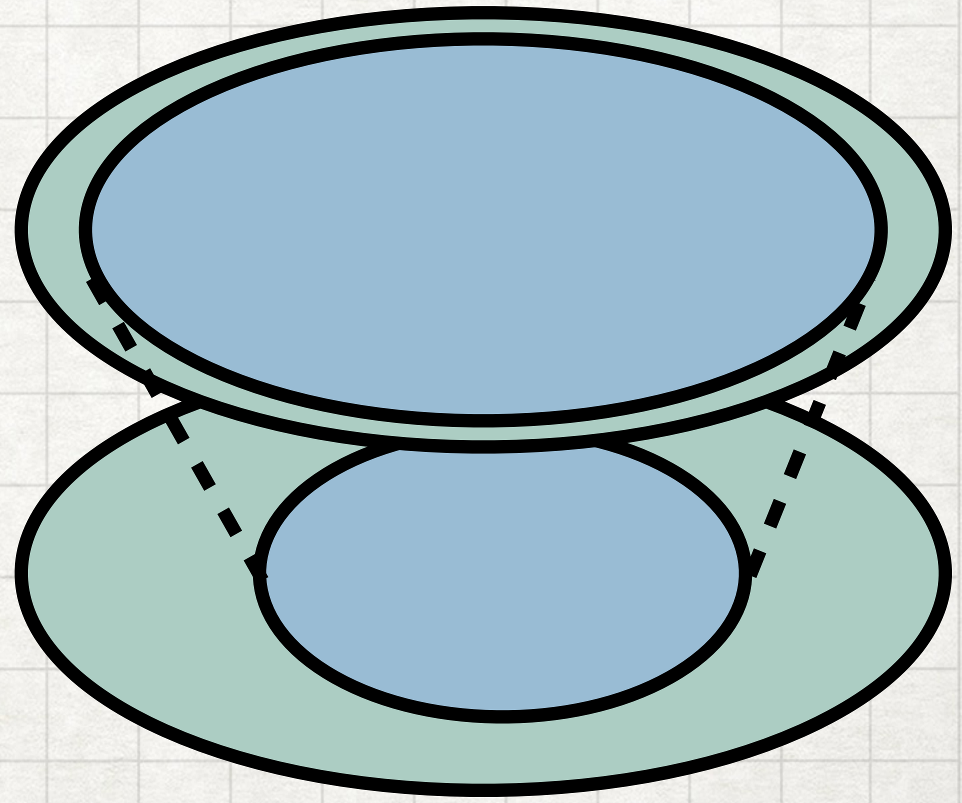


$$\pm \frac{d}{d\lambda_\pm} S_{\text{gen}}(G) \leq 0$$

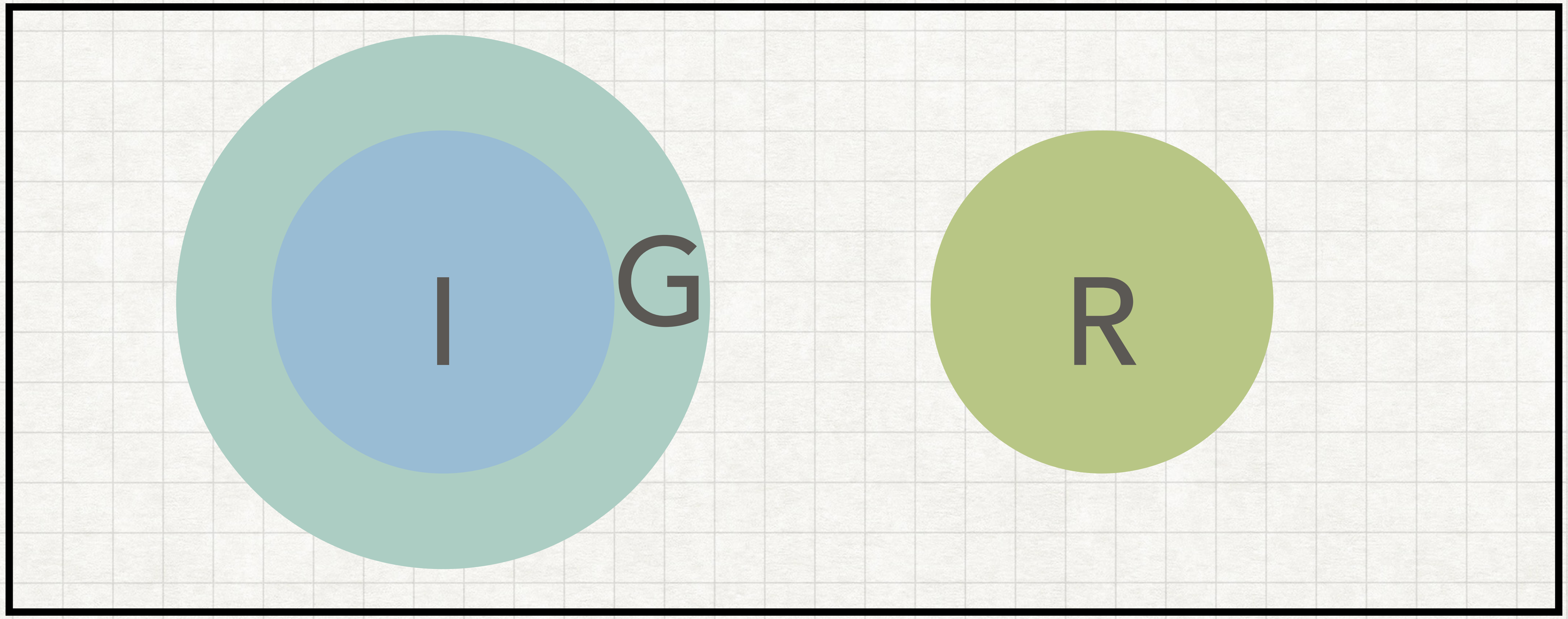
Derived from the extremal condition of the island & SSA of entropy



Outside of a sphere usually has a larger area when it is deformed “inward" (outward with respect to the sphere)

→Requires a large increase of the matter entropy in region G to compensate for the decrease in area

Q. When can a region I be the island for a region R?



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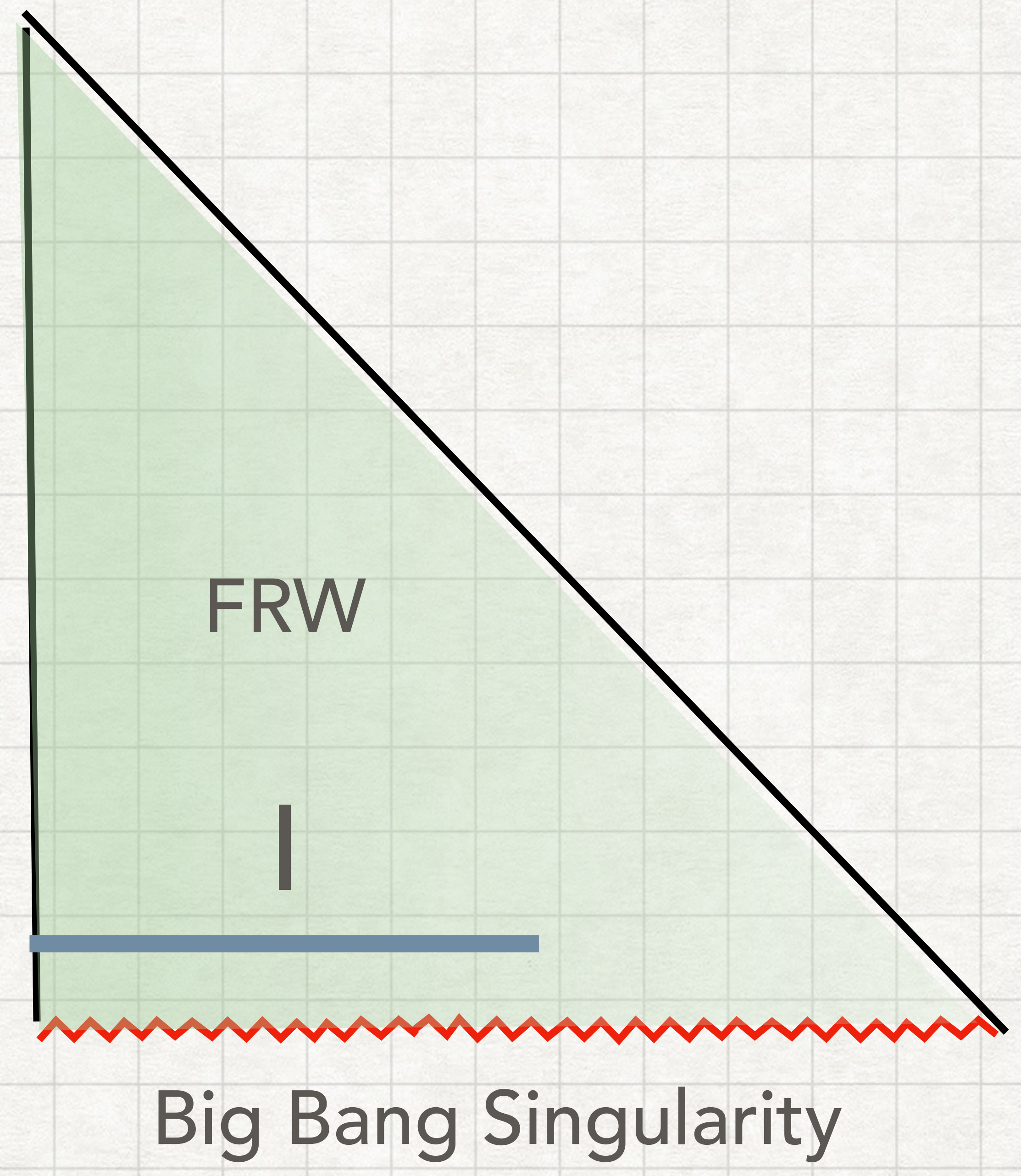
Spatially Flat FRW (Friedman-Robertson-Walker) Universe with $\Lambda = 0$



$$ds^2 = -dt^2 + a(t)^2 dx^2 = a(\eta)^2(-d\eta^2 + dx^2)$$

Matter is in a thermal state due to the thermal flux from the Big Bang singularity.

$$S_{\text{matter}}(I) \approx s_{th}\text{Vol}(I)$$

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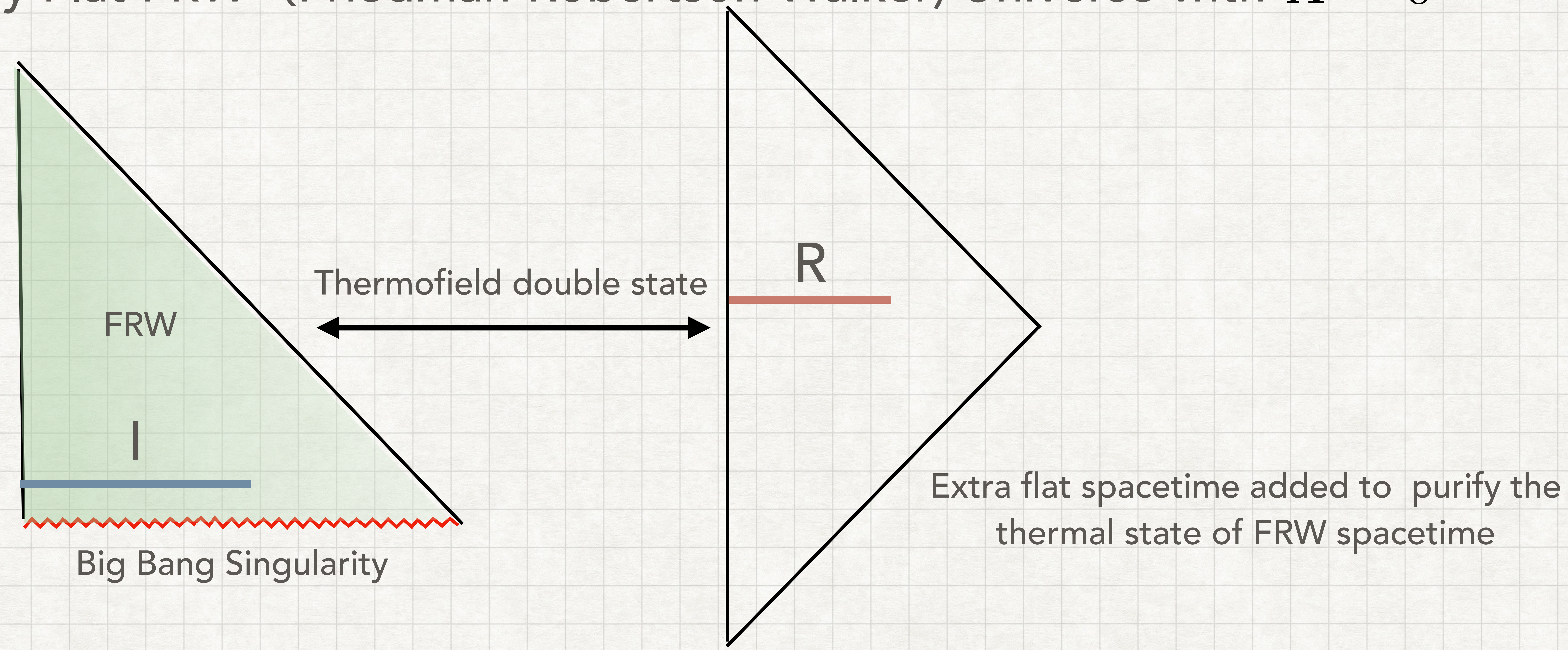

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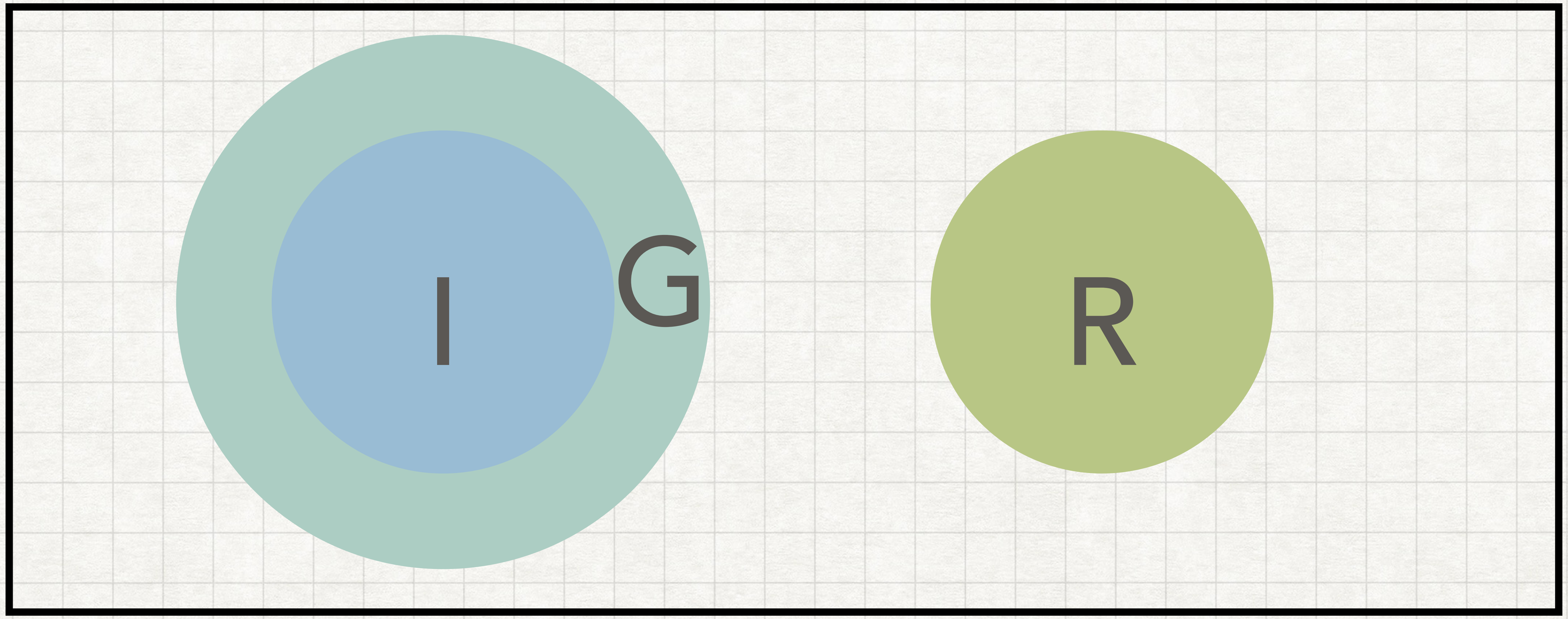

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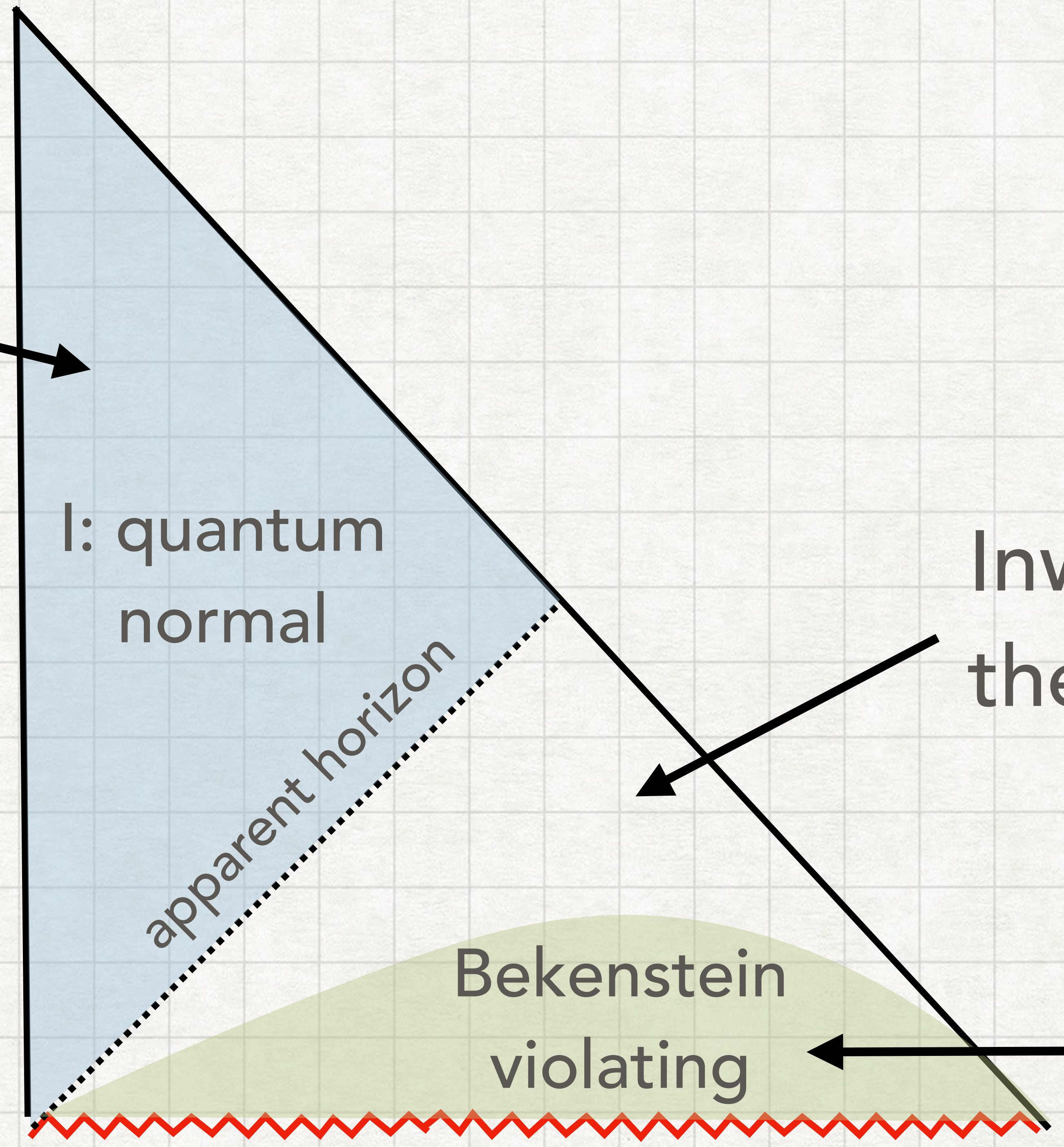
Spatially Flat FRW (Friedman-Robertson-Walker) Universe with $\Lambda = 0$

Quantum normal region:
the effect of the expansion of
the universe is not significant.

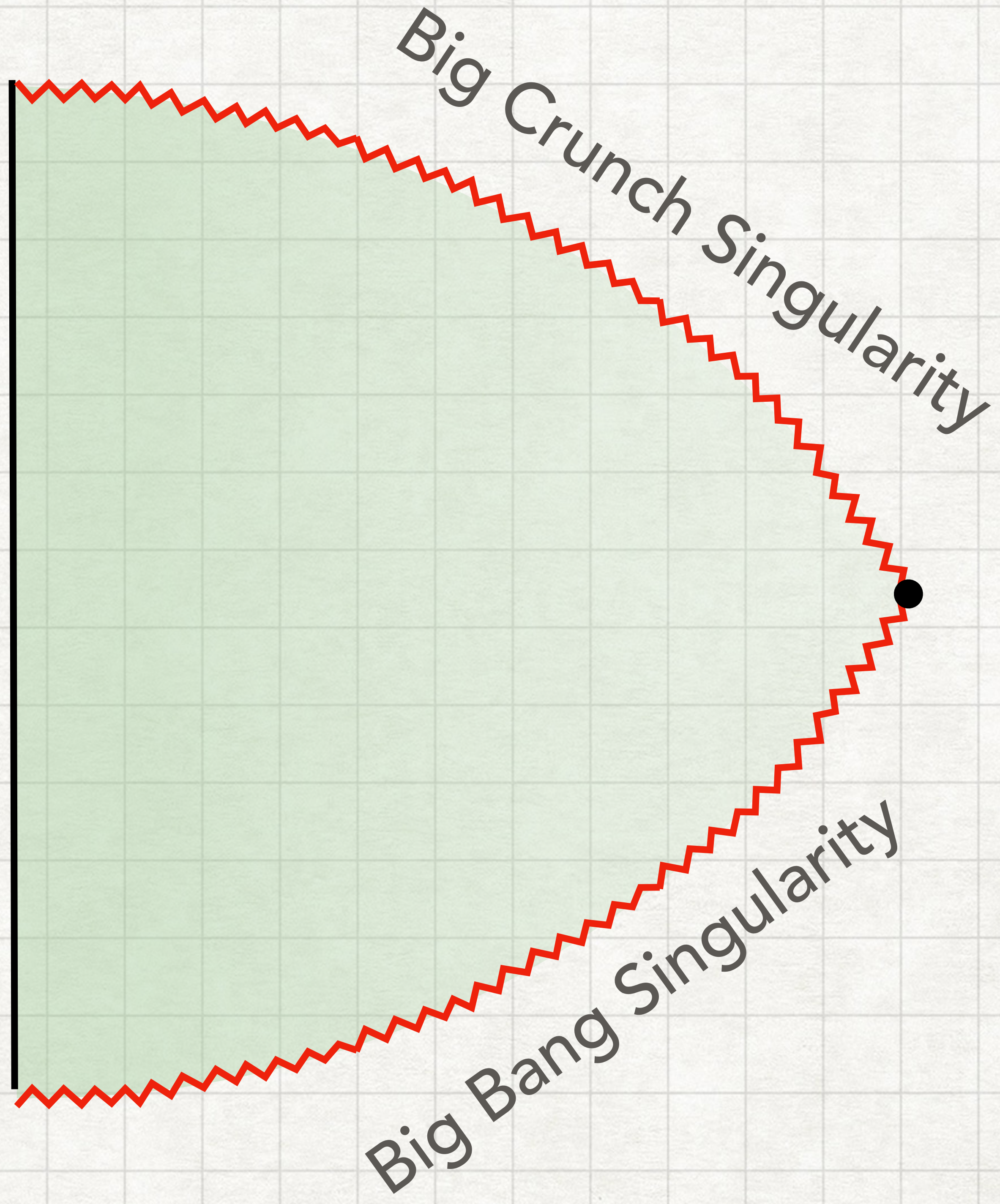


Inward null deformation increases the area due to expansion of universe

Area→0 near singularity

No island region that satisfies the conditions

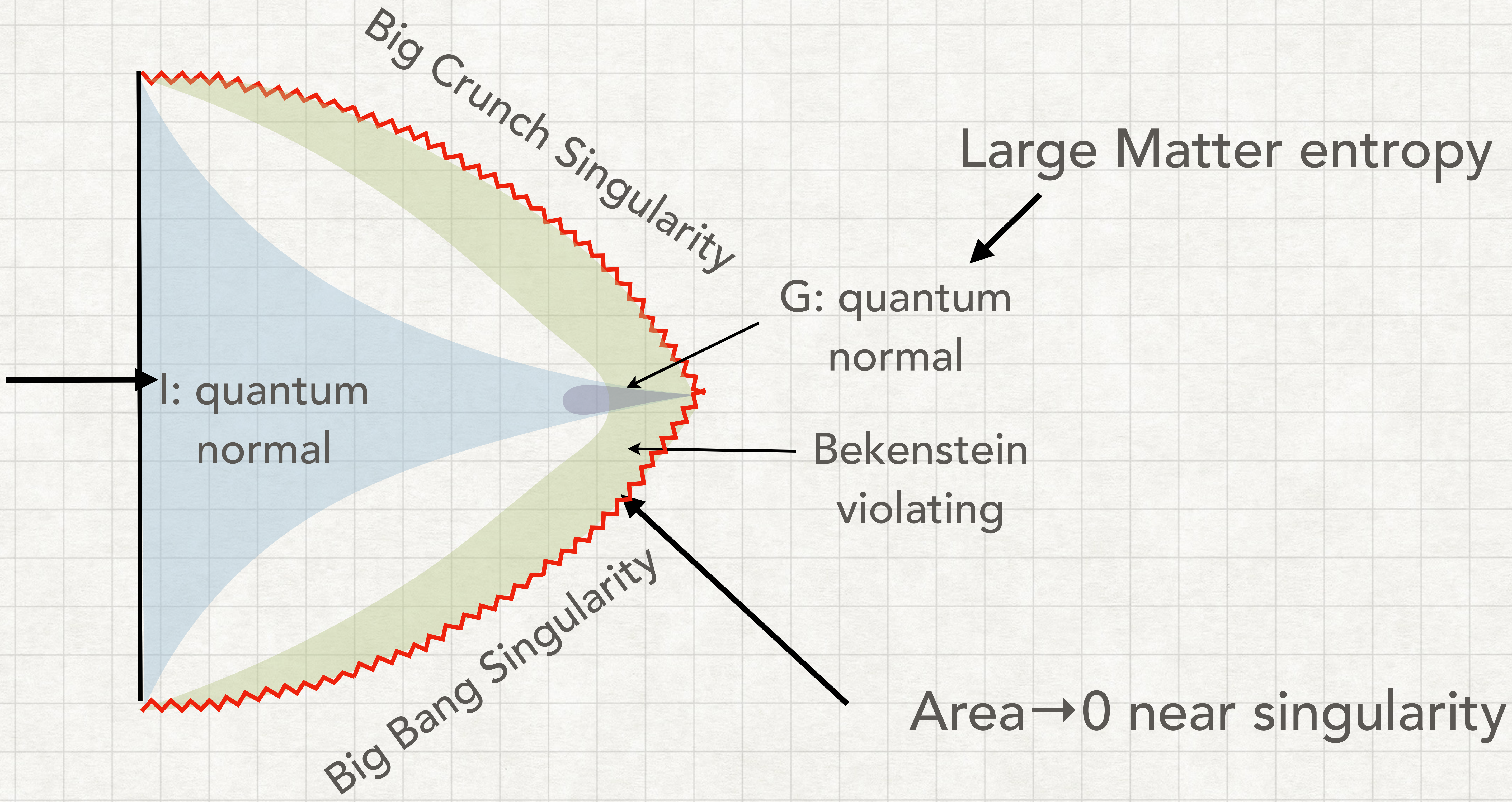
Spatially Flat FRW (Friedman-Robertson-Walker) Universe with $\Lambda < 0$



After the Big Bang, this universe expands and then re-collapses

Spatially Flat FRW (Friedman-Robertson-Walker) Universe with $\Lambda < 0$

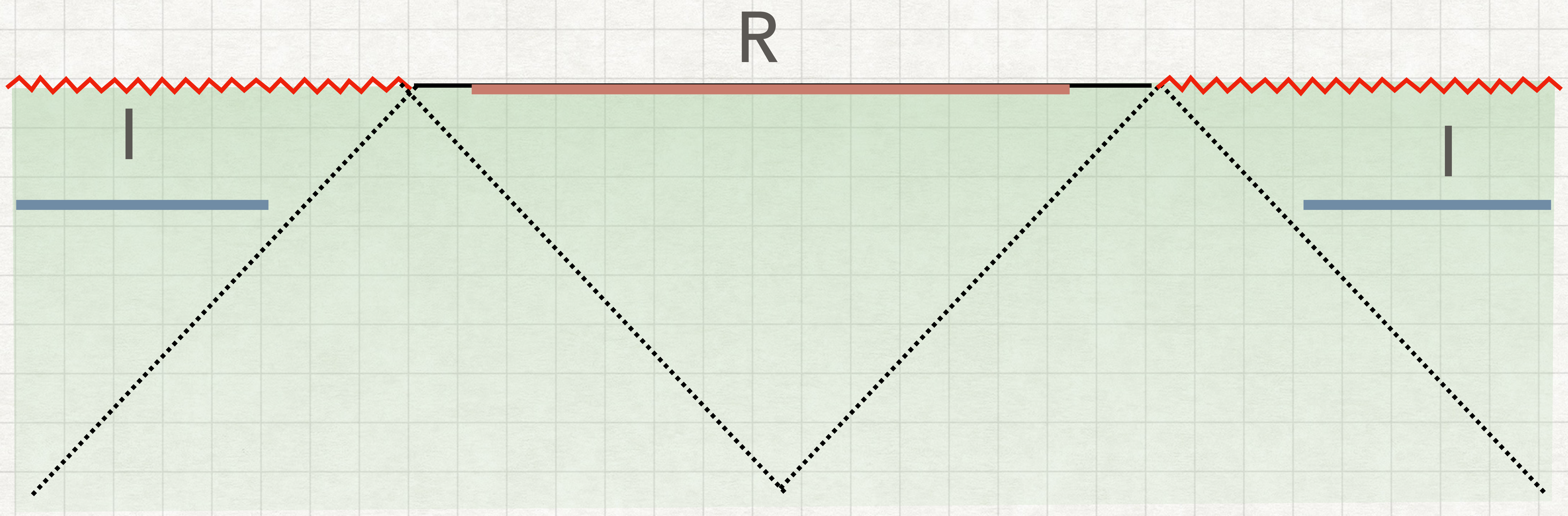
Quantum normal region:
the effect of inflation & deflation of the universe is not significant.



There is an island region that satisfies the conditions!

Re-collapsing universe can be reconstructed
from the extra flat spacetime degrees of freedom

2D de Sitter black hole



There is also an island inside a black hole
on de Sitter spacetime described by JT gravity.

See also Ugajin et al. arXiv:2008.05275 [hep-th]

We saw the most direct application of the island formula to early universe

Non-trivial island exists in recollapsing FRW universe, but their interpretation is unclear

In the future

- Can we apply the “idea” of the island formula/wormhole to early universe? &
  Can we consider observables that have imprints of quantum gravity effects?

cf.Eternal inflation & Baby universes

- Can we extend the idea of the holographic principle?