「非可逆的対称性とその応用を目指して」

大森 寛太郎
東大 本郷

Refs.
① Review @ JPS Journal (日本物理学会誌)
KO, WIP.
② 2111.01141
Kaidi KO Zheng
③ 2111.01139
Choi, Cordova, Hsin, Lam, Shao
④ 2109.05442
Koide, Nagoya, Yamaguchi

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This talk is about "Non-invertible symmetry"

Q1: What is it?

Q2: Where is it?

(a): In 1+1 d : many examples $\begin{cases} \text{CFT} \\ \text{lattice} \\ \text{QFT} \end{cases}$

(b): $d \geq 1+1$: We have started to find! (other than TQFT).

Review

new

Q3: What is it good for?

A: So far only small pleasures are found.

Please find a striking application!

What is symmetry?

(wikipedia page reffering to)

According to Wigner, in Quantum mechanics,

S: "symmetry transformation"

$\overset{\text{def}}{\Longleftrightarrow}$ $S : \mathbb{P}\mathcal{H} \to \mathbb{P}\mathcal{H}$ $\left(\mathbb{P}\mathcal{H} = \overset{\text{Hilb. sp.}}{\mathcal{H}} / \psi \sim e^{i\alpha}\psi \right)$

① bijection

② preserves the probability

Wigner's theorem: closed Set of symmetry transformations forms a group,

and $\mathcal{H}$ is a $\underset{\text{(anti)unitary}}{\text{a}}$ projective representation of the group.

This is why we care about group theory.

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Locality

- We often care about local system: $\mathcal{H} \text{``}= \bigotimes_{x:\text{point}} \mathcal{H}_x\text{''}$. e.g. $\begin{cases} \text{QFT} \\ \text{lattice} \end{cases}$

- We also want symmetry operation local.

- A Symmetry is also often "on-site": $S = \begin{cases} \int j^0 d^3x & \text{in QFT} \\ \prod_x U_x. & \text{in lattice theory} \end{cases}$

- But not all of them: $\begin{cases} \cdot \text{ Discrete symmetry in QFT} \\ \cdot \text{ Translation on lattice} \quad S = \prod_i \mathrm{Swap}_{i,j} \\ \cdot \text{ Anomalous symmetry on lattice} \end{cases}$

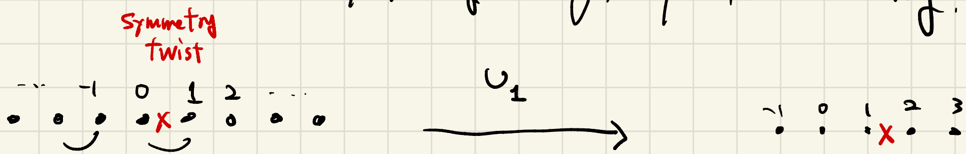
- A better notion: Topological defect operator

4 Topological defect operator

- $U(1)$: $\quad \hat{S}_\alpha[\Sigma] = e^{i\alpha\int_\Sigma j_\mu dS^\mu}$ codim 1 in spacetime

$$\hat{S}_\alpha[\quad] \times \left(\hat{S}_\alpha[\quad]\right)^{-1} = e^{i\alpha\int_V \partial^\mu j_\mu \, dvol} = 1.$$

- On site lattice symmetry: e.g. quantum Ising: $U_i = \sigma_x^i$



$$H_{i,j} = J\sum_{i,j}\sigma^z_i\sigma^z_j \qquad H^{tw}_{ij} = U_i^\dagger H_{ij} U_i = U_j H_{ij} U_j^\dagger$$

$$= -J\sum_{i,j}\sigma^z_i\sigma^z_j$$

$$U_0^\dagger\left(\sum_{i\neq 0} H_{i,i+1} + H^{tw}_{0,1}\right)U_0 = \sum_{i\neq 1} H_{i,i+1} + H^{tw}_{1,2}$$

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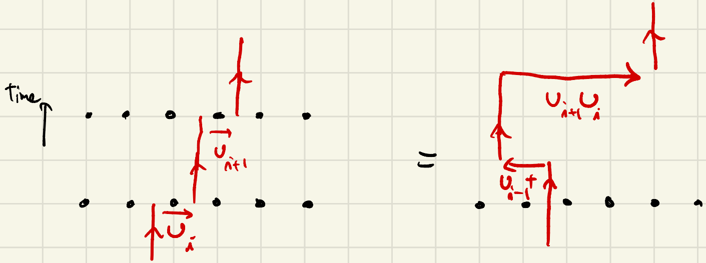
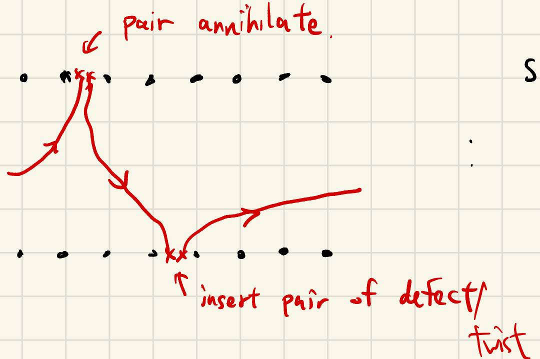


: topological.

locally realized Symmetry action : $S = \prod_i U_i$ (in periodic boundary cond.)



symmetry is achieved by

: ① Insert a pair of defects

② move them around by unitary.

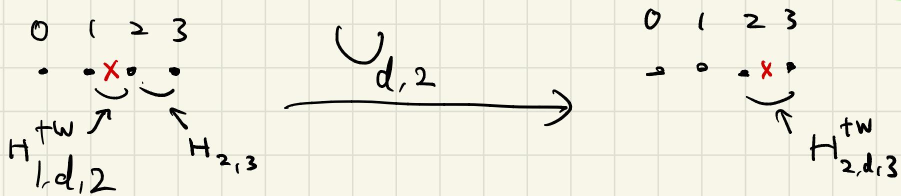
③ pair annihilation

- Symmetry in local system $\longleftrightarrow$ topological defect operator

6 Non-invertible defect in $1+1$d critical Ising

- What if defect itself support some degrees of freedom?

- Suppose: A defect $\times$ has a spin on it s_d, and coupling $H^{tw}_{i,d,j}$ with neighboring sites!
  Further, demand that you can move the defect by a unitary:



$$U^{\dagger}_{d,2}\left(H^{tw}_{1,d,2}+H_{2,3}+\sum_{j=1}^{3}H^{\text{single}}_{j}\right)U_{d,2}=H_{1,2}+H^{tw}_{2,d,3}+\sum_{j=1}^{3}H^{\text{single}}_{j}$$

- E.g. critical Ising model: $H_{crit}=-\sum_{i=1}^{L}\left(\sigma^z_i\,\sigma^z_{i+1}+\sigma^x_i\right)\quad(L+1\sim 1)$.

- $s=0,1$

  $\langle s_d', s_i' | U_{e,i} | s_d\, s_i\rangle=\delta_{s_i,s_e'}(-1)^{s_i'(s_e+s_i)}$, $\quad H^{tw}_{i,d,i+1}=\sigma^z_i\sigma^z_d+\sigma^x_d\,\sigma^z_{i+1}$

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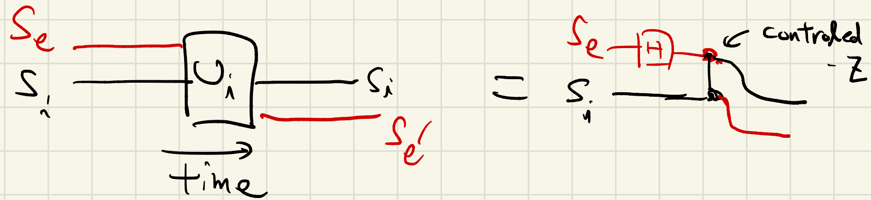
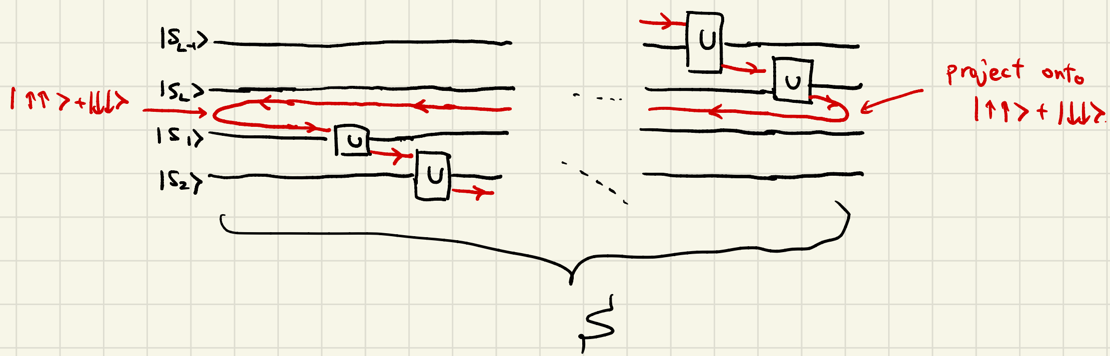


[Aasen Mong Fendley '16, references therein].

- Further, following operation commute with H_{crit}



$$\langle\{s_i'\}|\ \$\ |\{s_i\}\rangle=\frac{1}{\sqrt{2}^L}\prod_{i=1}^{L}(-1)^{s_i'(s_{i-1}+s_i)}$$

- $\sigma_i^z\sigma_{i+1}^z\,\$=\$\,\sigma_i^x\ ,\quad \sigma_{i+1}^x\,\$=\$\,\sigma_i^z\sigma_{i+1}^z \Rightarrow \$H_{crit}=H_{crit}\$$,
- Not unitary: $\$^\dagger\$=\mathbb{1}+\prod_i\sigma_i^x=2P_{\mathbb{Z}_2\text{-singlet}}$

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- What happens :
  ① pair create defects with entanglement
  ② move the defect around S^1 with unitary
  ③ pair annihilate. → Not unitary

- (Possibly) "Non-invertible sym" $\overset{\text{def}}{\Longleftrightarrow}$ top. def. with its own degrees of freedom.

  breaks Wigner's assumption, not a group. (but a fusion cat).

- locally realized, "unitarizable" operation.

  unitary if acting on $\mathcal{H}^{\text{defect}} = \mathcal{H}^{\text{original}} \otimes (\text{extra bits})$.

- In Ising model, this non-inv. sym $\longleftrightarrow$ criticality.

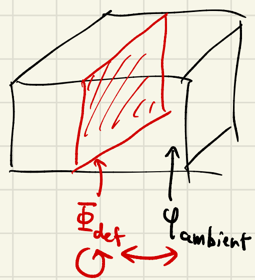
  Indeed, one can show that S-sym requirs gapless mode or ≥ 3 vacua.
  [Chang, Lin, Shao, Wang, Xi '18] (via RG flow invariance).

9] In higher dimensions

- So far it has been about 1+1 dimensions.
- In higher-dimensions, philosophy is the same:

"generalized symmetry" $\overset{\text{def}}{\Longleftrightarrow}$ topological defect op.

- Defect can have its spatial dimensions, fields on it, interaction $\begin{cases} \text{among them.} \\ \text{with bulk.} \end{cases}$



- Codimension p, invertible $\overset{\text{def}}{\Longleftrightarrow}$ "$(p-1)$-form symmetry".

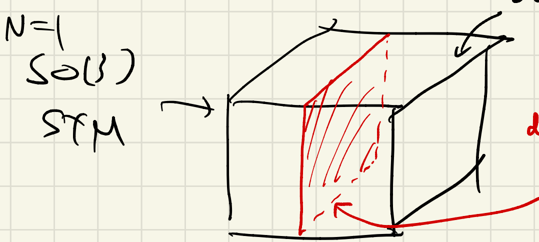
[10] Examples in 3+1 dimensions

- 3+1 d Ising gauge theory [Koide Nagoya Yamaguchi]
- 3+1 d $\mathcal{N}=1$ $SO(3)$ SYM : Classical Axial $U(1)$ $\lambda \to e^{i\alpha}\lambda$ is broken by instanton eff. [Kaidi, KO, Zheng]

  $\mathbb{Z}_4$ subgroup is "broken" to $\mathbb{Z}_2$ by "fractional" $SO(3)$ instanton.

  $\lambda \to i\lambda$

Actually it survives as a non-invertible symmetry!



SO(3) A
λ

def. fieald a : $\dfrac{U(1)^{def} \times SO(3)^{bulk}}{\mathbb{Z}_2}$ gauge field

$$S^{def} = \frac{2}{2\pi}\int a\,da.$$

- 3+1 d $\mathcal{N}=4$ $SU(N)$ SYM @ $\tau = i$ ← [Koide, Nagoya, Yamaguchi] [Kaidi, KO, Zheng]

  3+1 d free Maxwell @ $\tau = Ni$ $(N \in \mathbb{N}_{\geq 2})$ [Choi, Cordova, Hsin, Lam, Shao]

11 Summary

- In a local system, symmetries are identified with topological defect op.

  ↓ RG-inv.

- When a top. def. has its own degrees of freedom,

  we define a operation by
  1. pair create defects
  2. sweep the space by moving one of the defect by unitaries
  3. pair annihilate.

  local operations! →

- The implemented operation can be not unitary. ⟶ "noninvertible Symmetry"

- 1+1d Ising has one such sym. @ critical pt.

- Several applications in 1+1d { [Komorgolski, KO, Roumpedakis, Seifnashri '20 ; [Chang, Lin, Shao, Wang, Xi '18]

- New exmples in 3+1d, {
  - Ising gauge th on lattice
  - $\mathcal{N}=1$ $SO(3)$ SYM
  - $\mathcal{N}=4$ SYM, free maxwell

- No striking applications yet in $d > 1+1d$. But symmetry has been very important..

- New area of research, in particular $d > 1+1$.

  Please join and find new applications!!

Thank You!!