NATIONAL SCIENCE CENTRE
POLAND

UNIVERSITY OF WARSAW

On the phenomenology of sphaleron-induced processes at the LHC and beyond

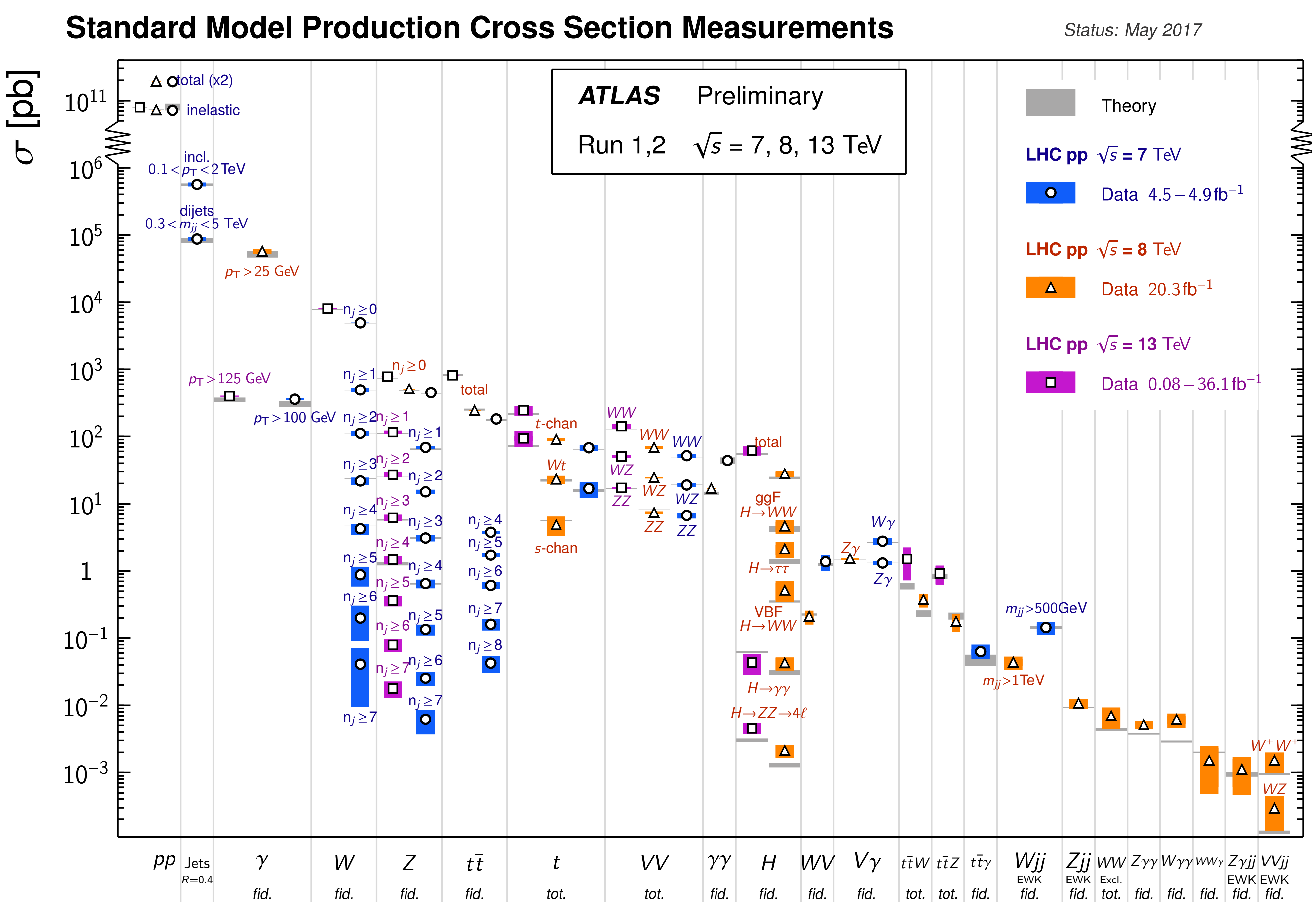
Kazuki Sakurai

(University of Warsaw)

2021/1/26 Seminar @ Osaka University

Perturbative sector of EW theory is very well tested!

- Remarkable agreement between experimental results and perturbative calculations.



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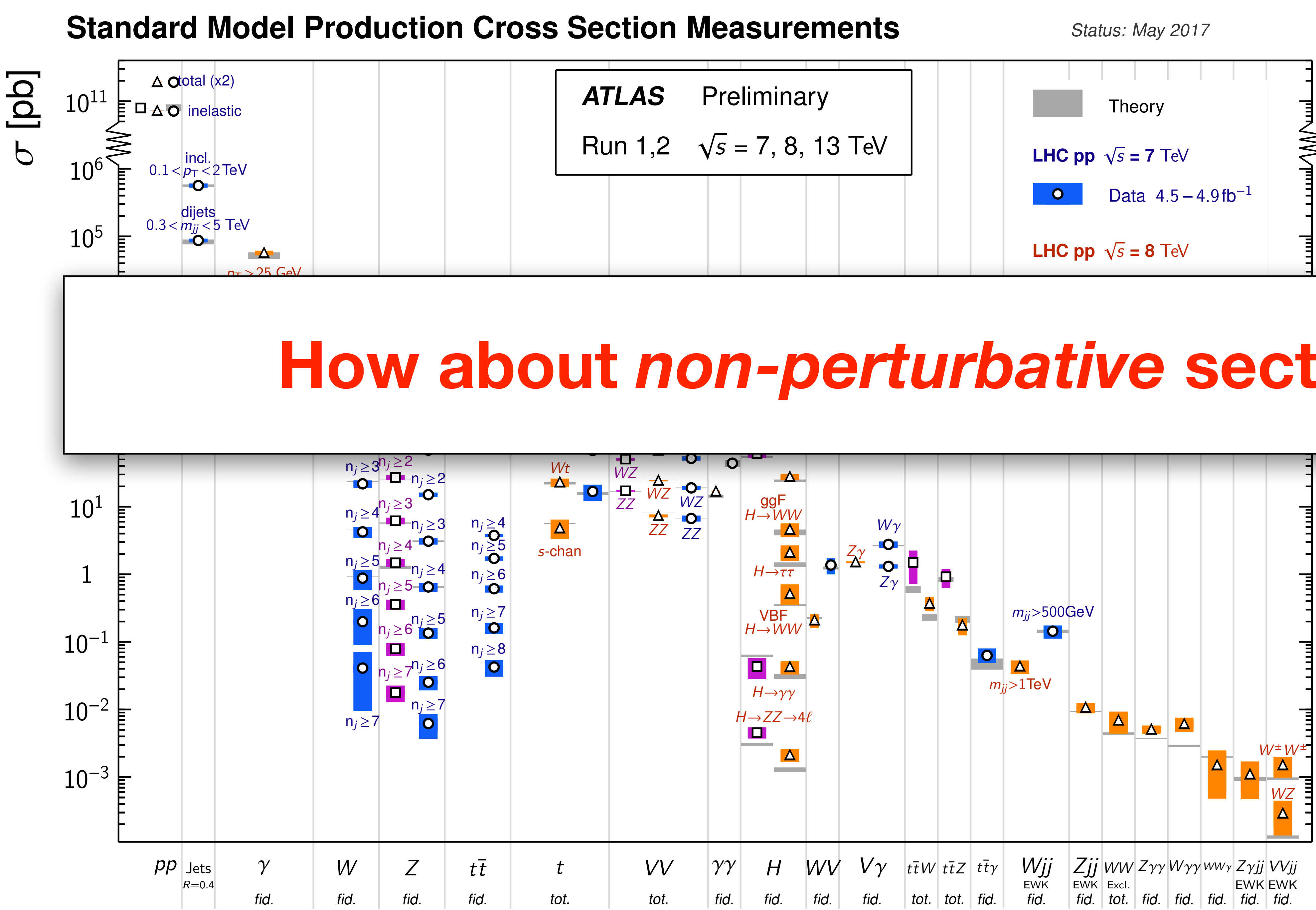

How about *non-perturbative* sector?

Vacua of EW theory

action: $S_{EW} = \frac{1}{2g^2} \int \mathrm{Tr}\left(F_{\mu\nu} F^{\mu\nu}\right)$ $\quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$

gauge trans.: $A_\mu \to U^\dagger A_\mu U + U^\dagger \partial_\mu U$ $\quad S_{EW} \to S_{EW}$

a vacuum: $A_\mu = 0 \quad \leftrightarrow \quad A_\mu = \underline{U^\dagger \partial_\mu U}$

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- The vacuum configurations are not unique. The choices are as many as

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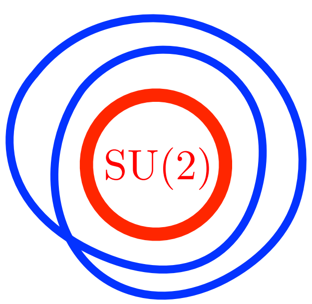

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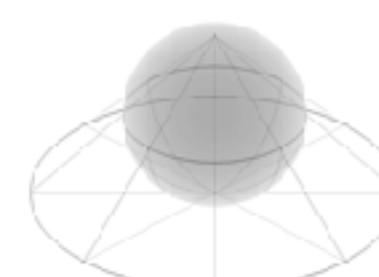
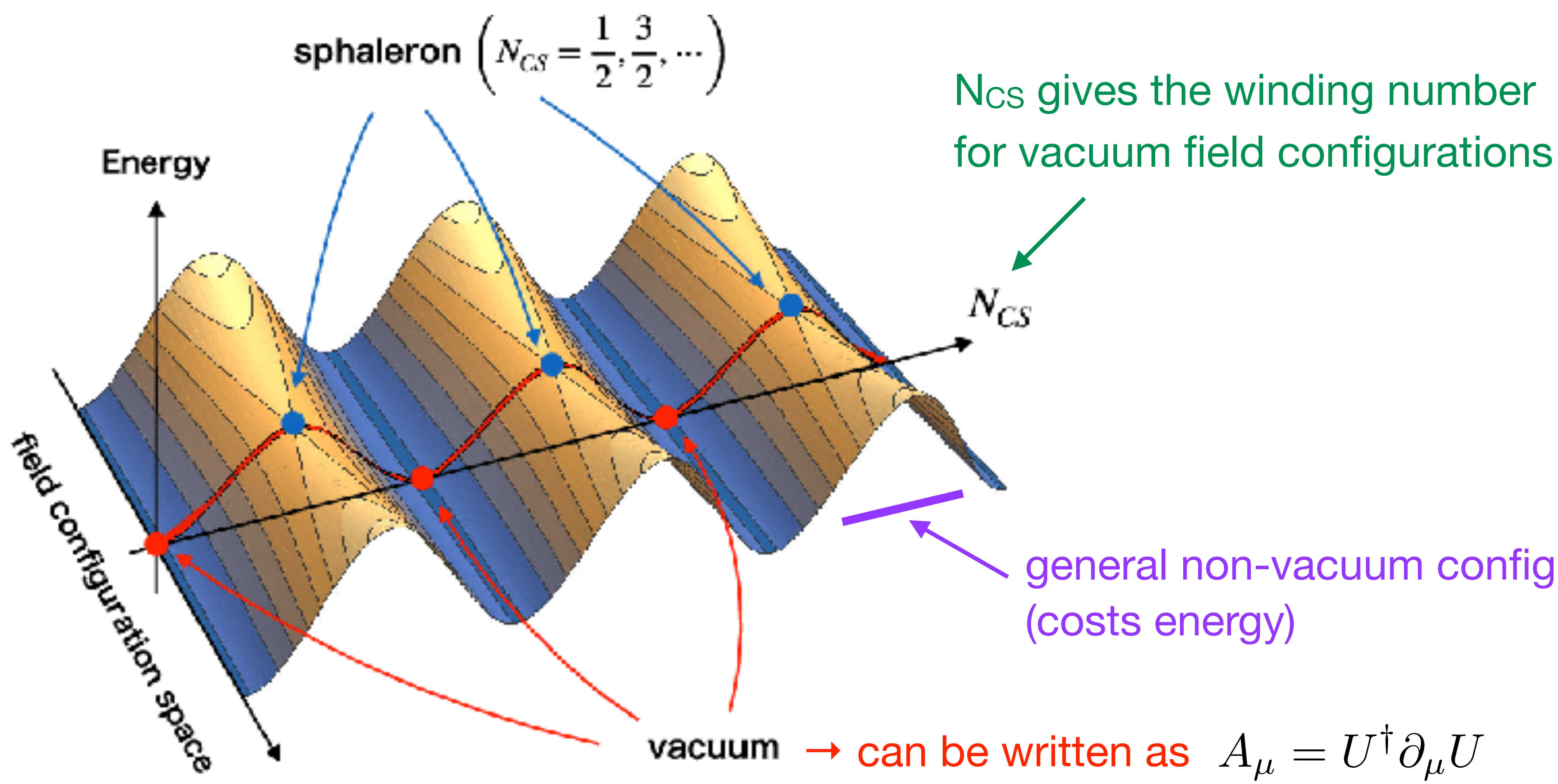
Let's impose a condition

$U(\infty) \to \mathbf{1}$





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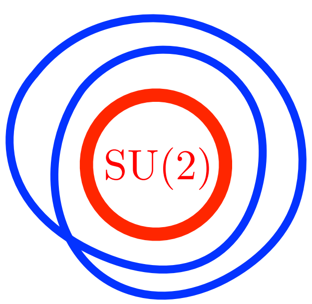

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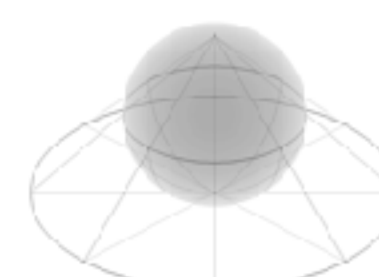
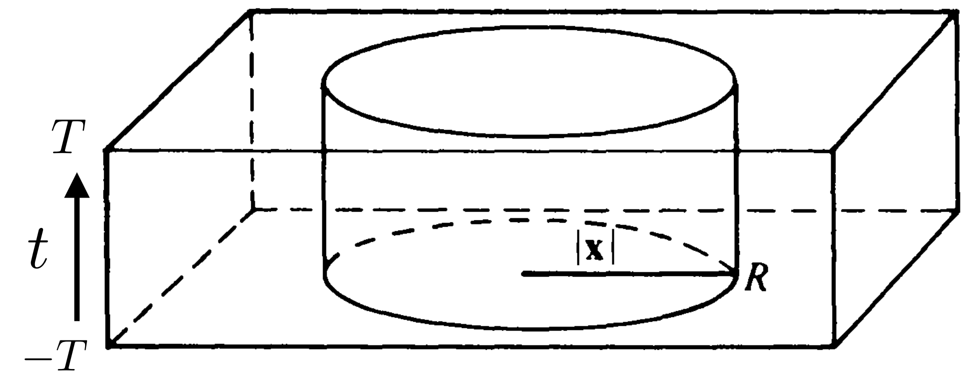


- Consider *physical* field configurations in Euclidean space-**time**, which gives a **finite action**.

$(A_\mu = U^\dagger \partial_\mu U)$

outside the cylinder, $F_{\mu\nu} = 0$.



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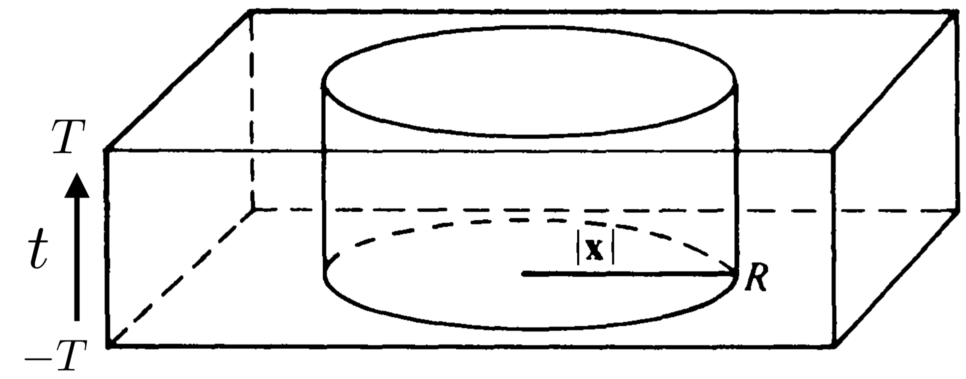

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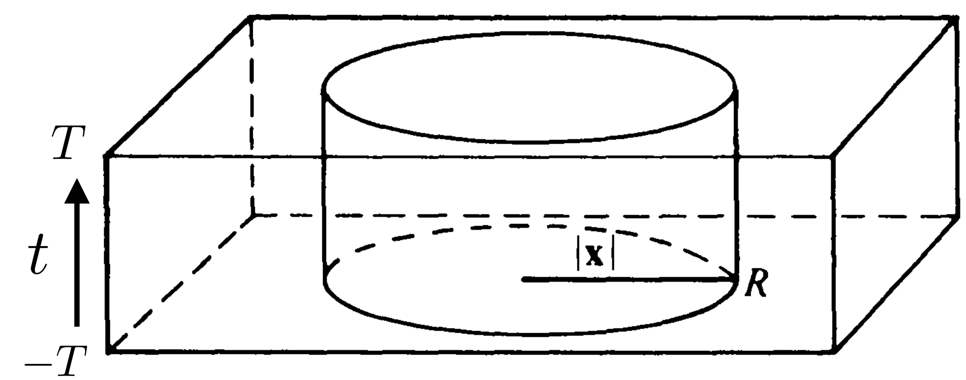

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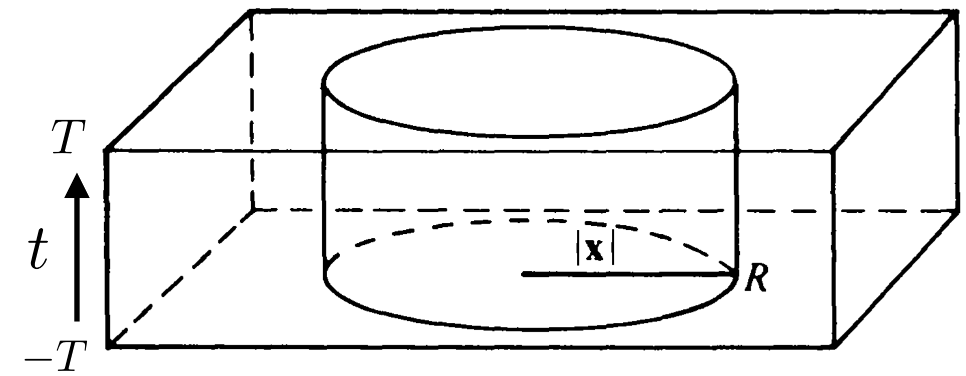

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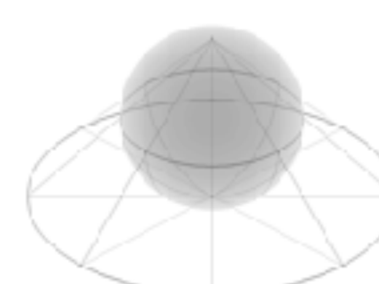
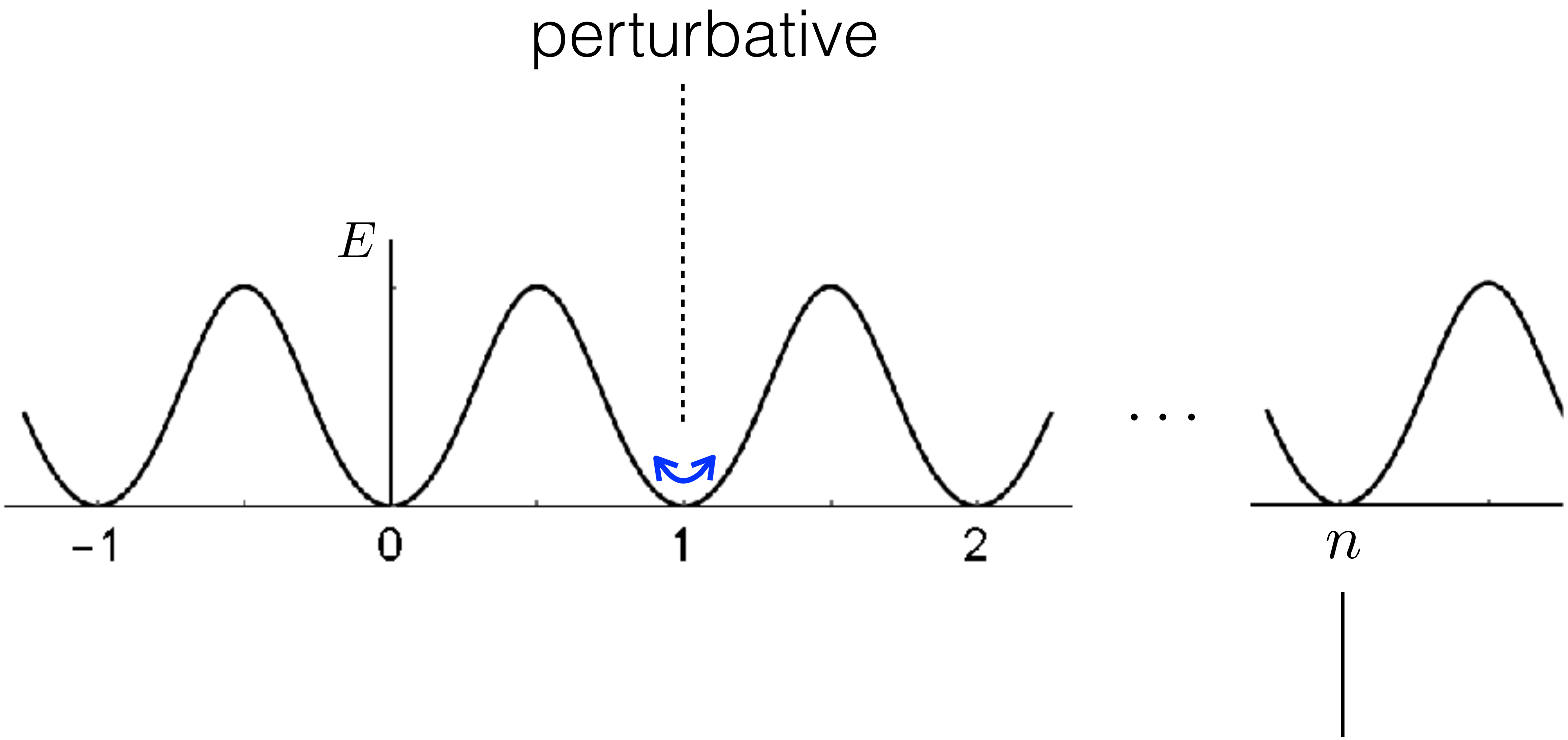
- Everywhere outside the cylinder, $\underline{U(t, \mathbf{x}) = U(-T, \mathbf{x}) = \mathbf{1}}$

- The field confing at t=T is characterised by a map $U_{ij}(\mathbf{x})$

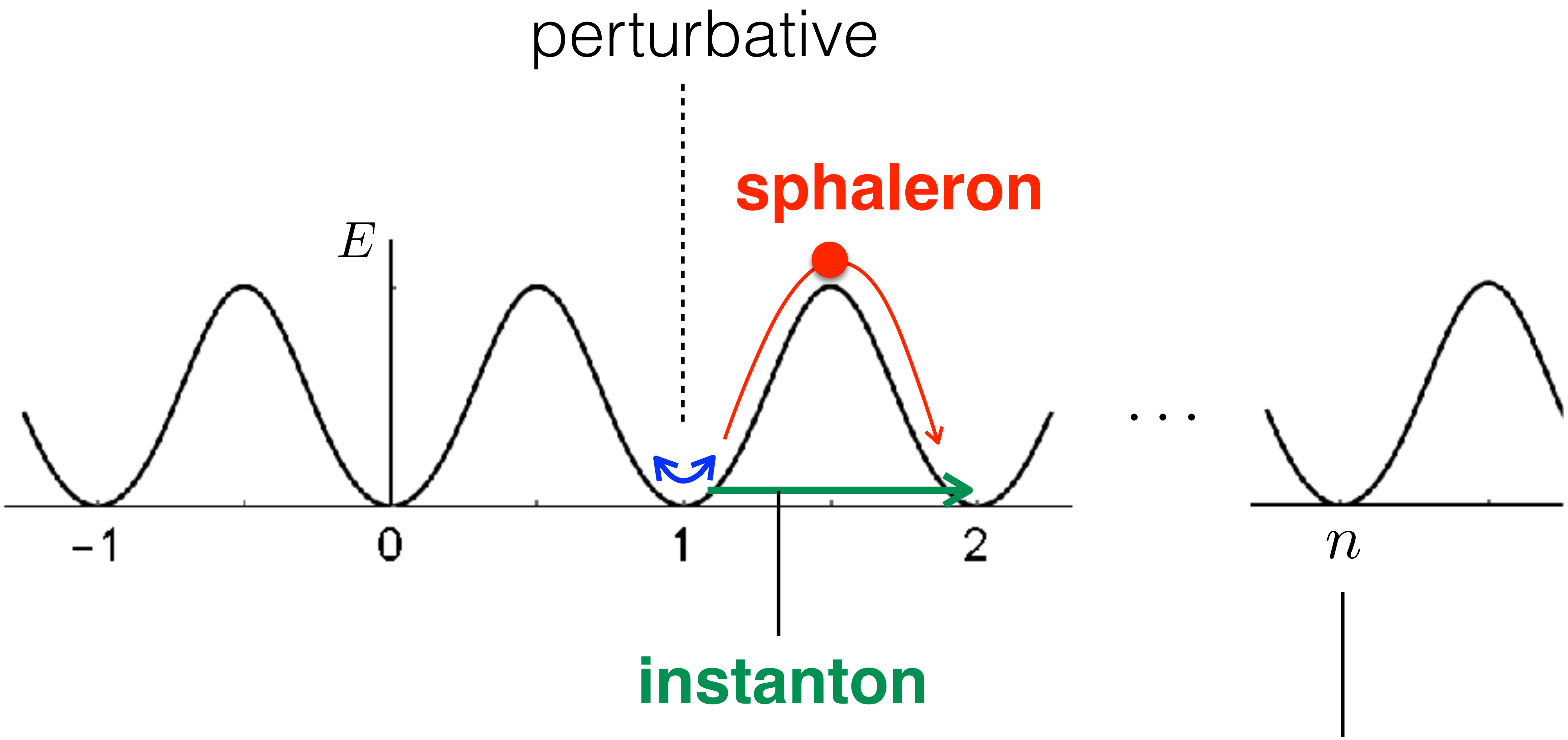
$$\mathrm{SU}(2) \simeq S^3 \xleftarrow{\text{map}} S^3$$





$$A_{n,\mu}(\mathbf{x}) = U_n(\mathbf{x})^\dagger \partial_\mu U_n(\mathbf{x})$$

$$U_n(\mathbf{x}) = \exp\left(in\pi \frac{\mathbf{x}\cdot\sigma}{\sqrt{\mathbf{x}^2 - \rho^2}} \right)$$



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- What do such processes look like?
- How large is the event rate?

finite energy condition:

$F_{\mu\nu}(\mathbf{x}) \to 0$ for $\mathbf{x} \to \infty$

Define a (gauge dependent) current K as

$$K_\mu = \frac{1}{8\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathrm{tr}A^\nu(\partial^\rho A^\sigma + \frac{2}{3}A^\rho A^\sigma)$$

Then it follows

$$\int K_0(A_n(\mathbf{x}))d^3x = n\,, \qquad \frac{1}{16\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu} = \partial^\mu K_\mu\,, \qquad \tilde{F}^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}\,,$$

therefore

$$\int \frac{1}{16\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}d^4x = \int \partial^\mu K_\mu d^3x dt = \left[\int K_0(t,\mathbf{x})d^3x\right]_{t=-\infty}^{t=\infty}$$

$$= n(t=\infty) - n(t=-\infty) = \Delta n$$

The change of the winding number is related to the integral of FF~.

- The change of N_{CS} is related to the following quantity:

$$\Delta N_{CS} = \frac{g^2}{16\pi^2} \int \mathrm{Tr}\left[F_{\mu\nu}\tilde{F}^{\mu\nu}\right] d^4x$$

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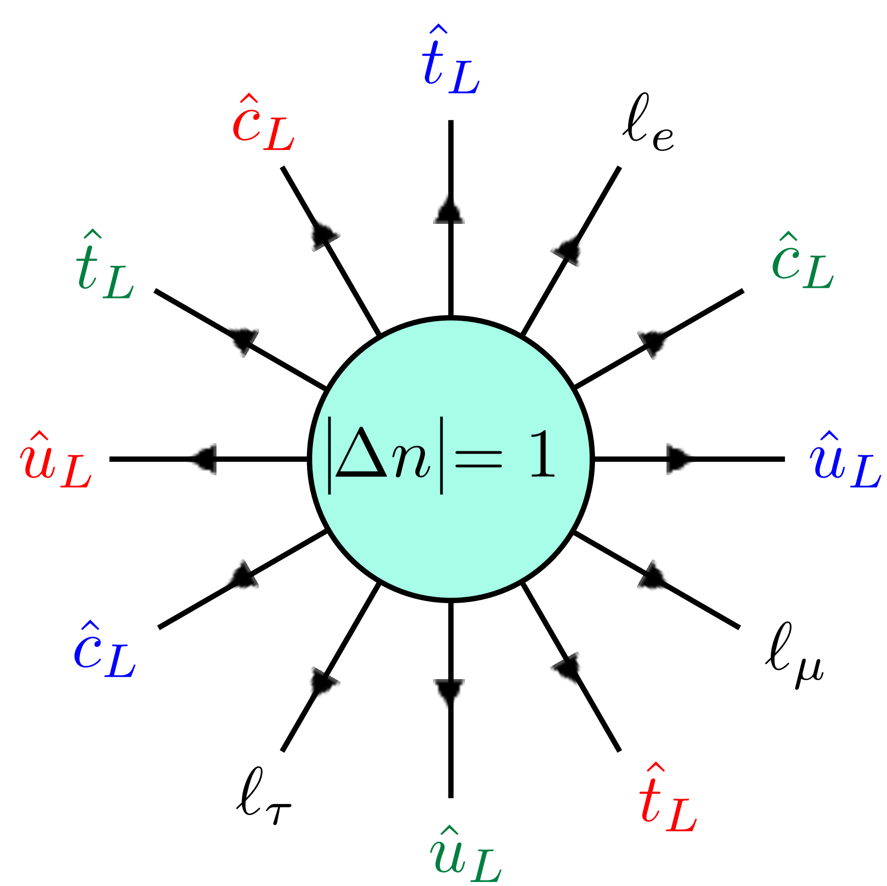
- ΔN$_{CS}$ is related to the change of SU(2) charged fermion numbers.

$$\Delta B = \Delta L = 3\Delta N_{CS}$$

$$\Delta(B+L) \neq 0$$

$$\Delta(B-L) = 0$$



|ΔN$_{CS}$| = 1 transition creates 12 fermions altogether!

Party at collider!

Instanton rate

The tunnelling rate can be estimated using the WKB approximation as

$$\langle n|n+\Delta n\rangle \sim e^{-\hat{S}_E}$$

S_E is the Euclidean action at the stationary point, which is given by

$$\hat{S}_E = \frac{1}{2g^2}\int FF d^4x$$

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$$\begin{aligned} \hat{S}_E &= \frac{1}{2g^2} \int F F d^4x \\ &= \frac{1}{2g^2} \left| \int F \widetilde{F} d^4x \right| \\ &= \frac{8\pi^2}{g^2} |\Delta n| \end{aligned}$$

Note that:

$$\int (F \pm \tilde{F})^2 d^4x \geq 0$$

$$\Longrightarrow \quad \int F F d^4x \geq \left| \int F \tilde{F} d^4x \right|$$

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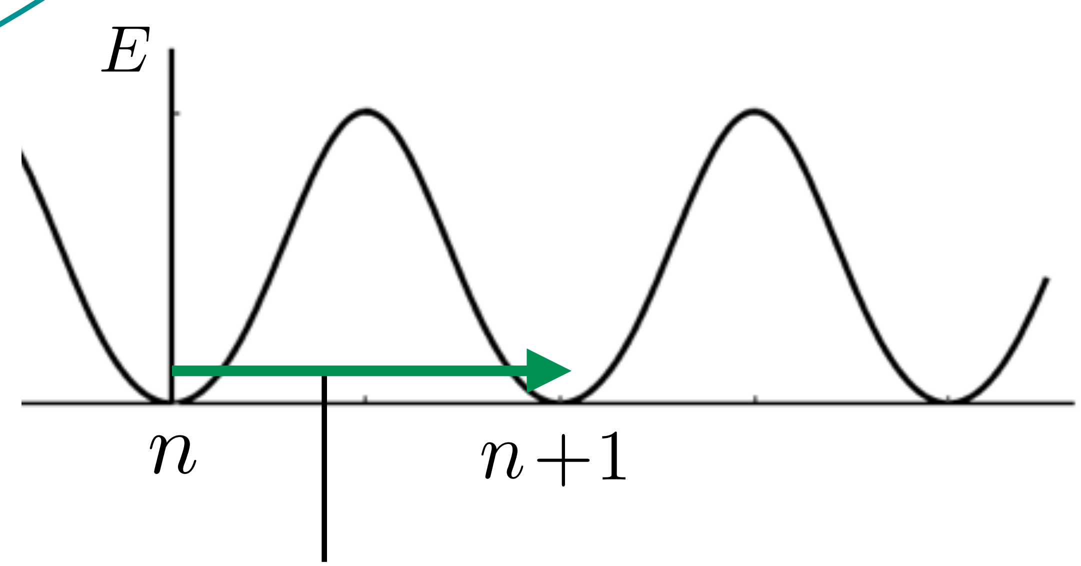
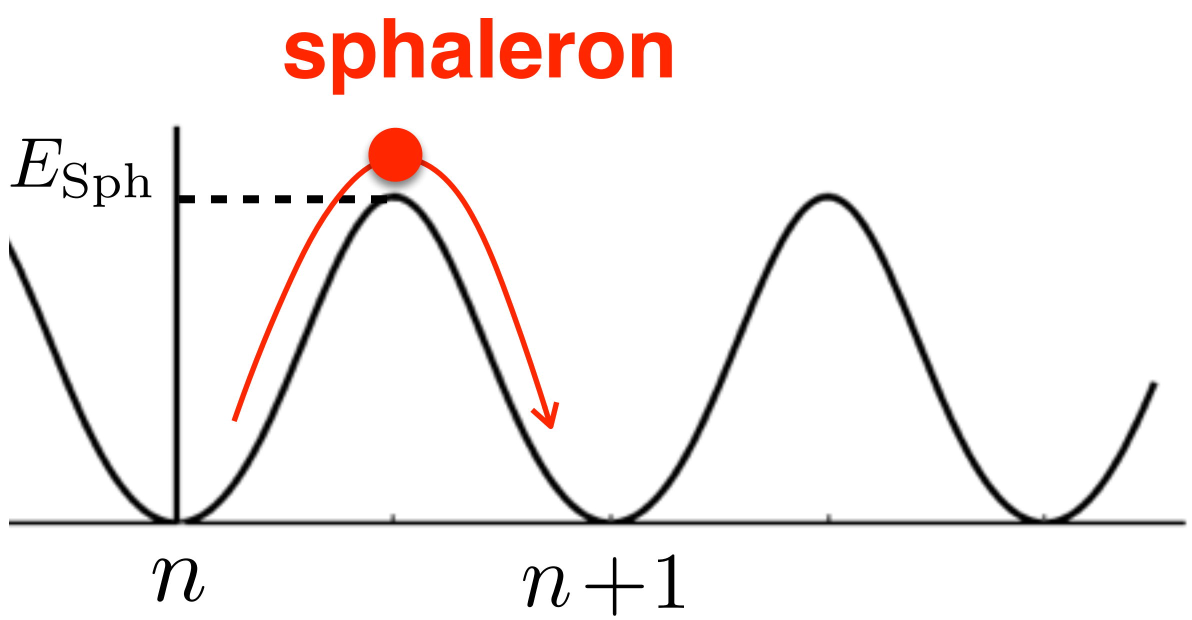


$$e^{-\frac{4\pi}{\alpha_W}} \sim 10^{-170}$$

The tunnelling rate is unobservably small



The barrier hight was calculated by
F.R.Klinkhamer and N.S.Manton (1984)

$$E_{\rm Sph} = \frac{2m_W}{\alpha_W} B\left(\frac{m_H}{m_W}\right)$$

$$\simeq 9\,\mathrm{TeV} \quad (\text{for } m_H = 125\mathrm{GeV})$$

- At high temperature, the sphaleron rate may be unsuppressed.

$$\Gamma \propto \exp\left(-\frac{E_{\rm Sph}(T)}{T}\right)$$

It plays an important role in baryo(lepto)genesis.

What happens for the high energy (zero temperature) case?

- The LO Matrix Element in the ***instanton* background**

$$i\mathcal{M} \sim \int \mathcal{D}q\mathcal{D}W\mathcal{D}\phi\, q(x_1)\cdots q(x_{12})W(y_1)\cdots W(y_{n_W})\phi(z_1)\cdots\phi(z_{n_h})\exp(-S_E)\Big|_{\mathrm{LSZ}}$$

[Ringwald ’90, Espinosa ’90]

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- Evaluate it at the instanton configuration:

[Ringwald '90, Espinosa '90]

$$W^{\mu a}_{\text{inst}} \simeq \frac{2\rho^2}{g} U_{ab} \frac{\bar{\eta}_{b\mu\nu}(x-x_0)_\nu}{(x-x_0)^2[(x-x_0)^2+\rho^2]} \qquad \phi_{\text{inst}}(x) \simeq v\left[\frac{(x-x_0)^2}{(x-x_0)^2+\rho^2}\right]^{1/2}$$

orientation position size

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- Integration over orientation, position, size and phase-space:

$$\sigma(n_W, n_h) \sim \int |\mathcal{M}|^2 \cdot d\Phi_{\rm PS}$$

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- Result

$$\sigma_{\mathrm{LO}}(n_W, n_h) \sim \mathscr{G}^2 2^n v^{-2n}\left[\frac{\Gamma(n+103/12)}{\Gamma(103/12)}\right]^2 \frac{1}{n_B!\,n_H!} \qquad \mathscr{G} = 1.6\times 10^{-101}\ \mathrm{GeV}^{-14}$$

$$\times \int \prod_{i=1}^{10} \frac{d^3 p_i}{(2\pi)^3 2E_i} E_i \prod_{j=1}^{n_B} \frac{d^3 p_j}{(2\pi)^3 2E_j} \frac{2(4E_j^2 - m_W^2)}{m_W^2} \prod_{k=1}^{n_H} \frac{d^3 p_k}{(2\pi)^3 2E_k} (2\pi)^4 \delta^{(4)}\left(P_{\mathrm{in}} - \sum_{i=1}^{10} p_i - \sum_{j=1}^{n_B} p_j - \sum_{k=1}^{n_H} p_k\right)$$

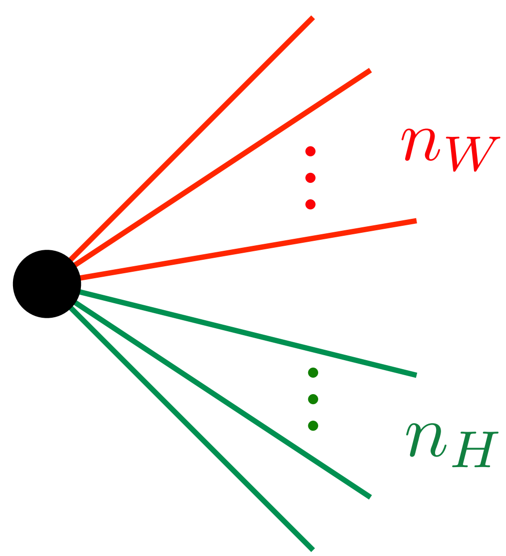
The cross-section grows exponentially as increasing # of EW and H bosons!

$$i\mathcal{M} \sim \int \mathcal{D}q\mathcal{D}W\mathcal{D}\phi\, q(x_1)\cdots q(x_{12})W(y_1)\cdots W(y_{n_W})\phi(z_1)\cdots\phi(z_{n_h})\exp(-S_E)\Big|_{\text{LSZ}}$$

$$A^{\text{inst}\,a}_{\mu}(x_i) \xrightarrow{\textbf{FT}} \frac{4i\pi^2\rho^2}{g}\,\frac{\bar{\eta}^a_{\mu\nu}p_i^\nu}{p_i^2(p_i^2+m_W^2)}\,e^{ip_ix_0} \xrightarrow{\textbf{LSZ}} \boxed{\frac{4i\pi^2\rho^2}{g}\,\frac{\bar{\eta}^a_{\mu\nu}p_i^\nu}{p_i^2}\,e^{ip_ix_0}}$$

$$H^{\text{inst}}(x_j) \to -\frac{2\pi^2\rho^2 v}{(p_j^2+m_H^2)}\,e^{ip_jx_0} \to \boxed{-2\pi^2\rho^2 v\, e^{ip_jx_0}}$$

- **Multi-particle interaction under the instanton BG is (almost) a point-like vertex**



$$i\mathcal{M} \sim \left[\frac{4i\pi^2\rho^2}{g}\,\frac{\bar{\eta}^a_{\mu\nu}p_i^\nu}{p_i^2}\,e^{ip_ix_0}\right]^{n_W}\left[-2\pi^2\rho^2 v\, e^{ip_jx_0}\right]^{n_H}$$

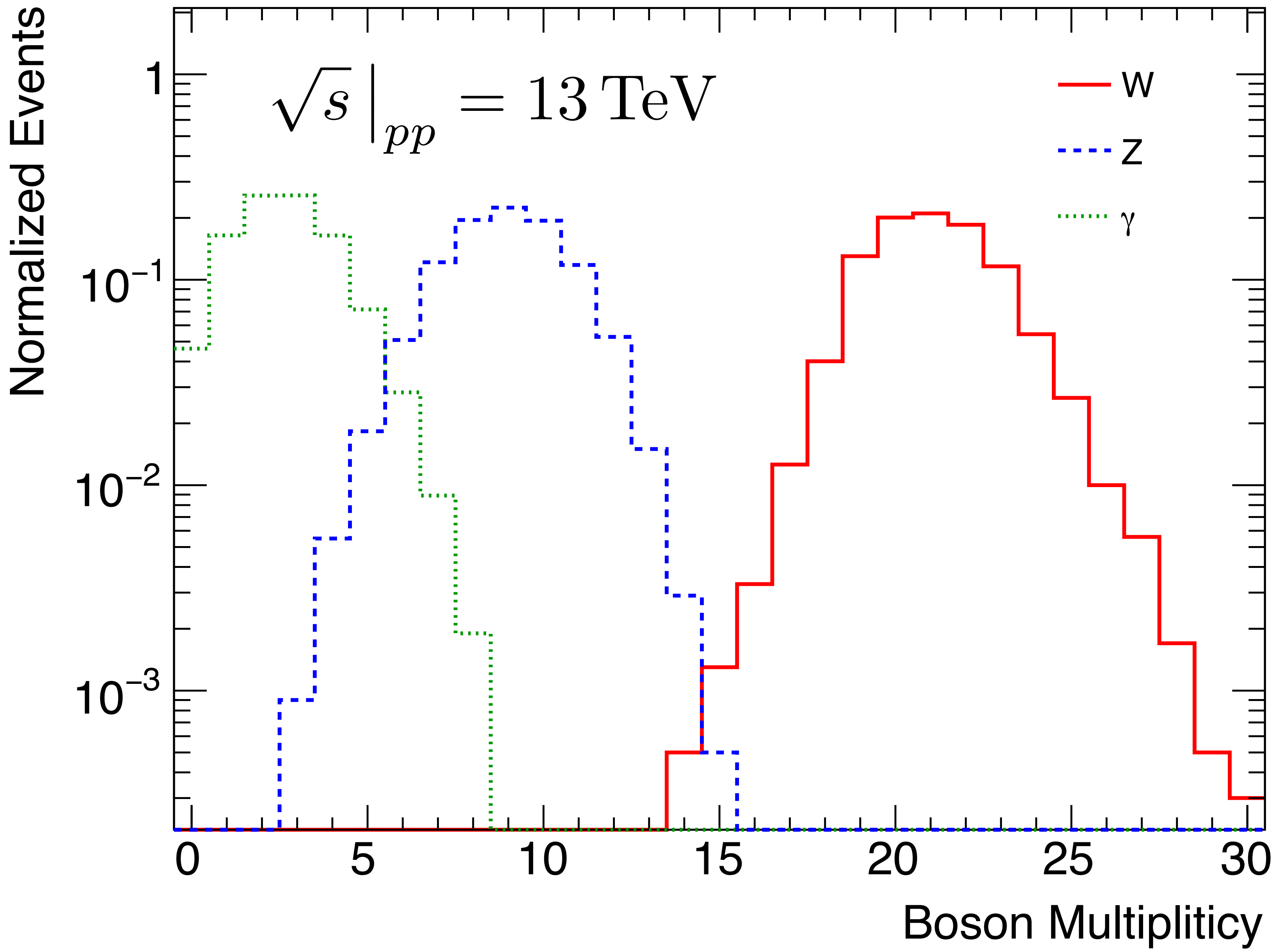
$$\Phi_n(Q) \sim (Q^2)^{n-2}$$

n-body phase-space

Such a vertex is highly unrenormalisable and high energy behaviour is not regulated.

Enhancement at large *nW* and *nH*.

- The MC Event Generator (**HERBVI**) was developed by Gibbs and Webber based on the LO ME formula:



$$\langle n_B \rangle \sim \frac{4\pi}{3\alpha_W}\left(\frac{9}{8}\epsilon^{\frac{4}{3}} + \mathcal{O}(\epsilon^2)\right)$$

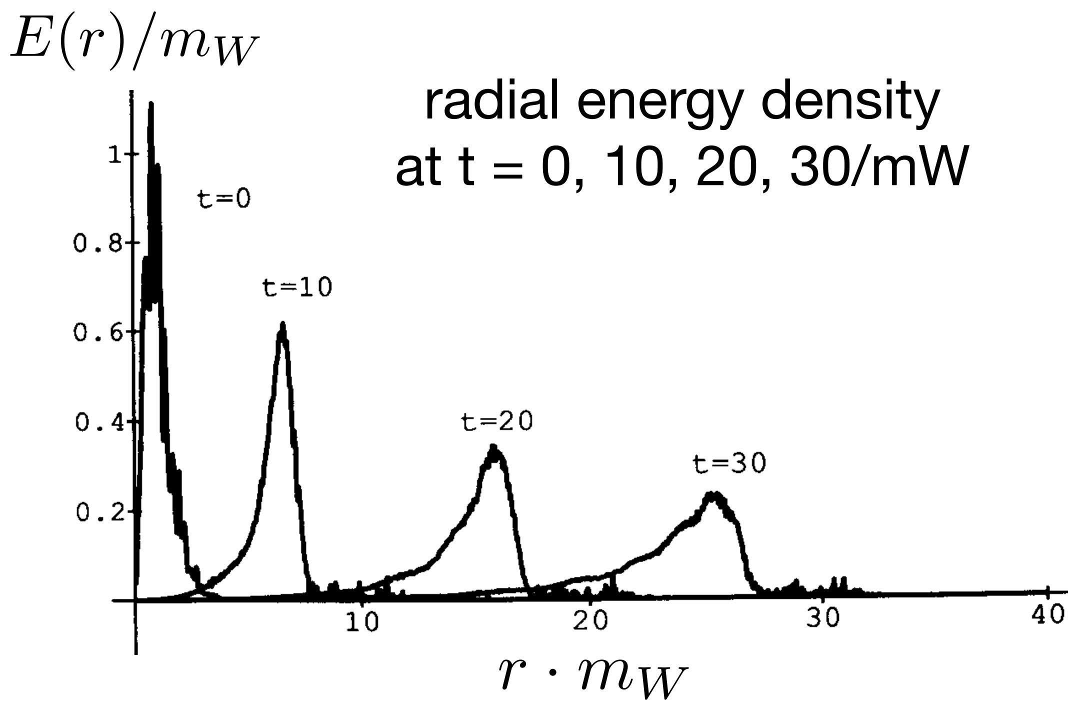
$$\langle n_h \rangle \sim \frac{4\pi}{\alpha_W}\frac{3}{32}\epsilon^2$$

$$\epsilon \equiv \frac{\alpha_W}{\sqrt{6}\pi}\frac{\sqrt{\hat{s}}}{m_W}$$

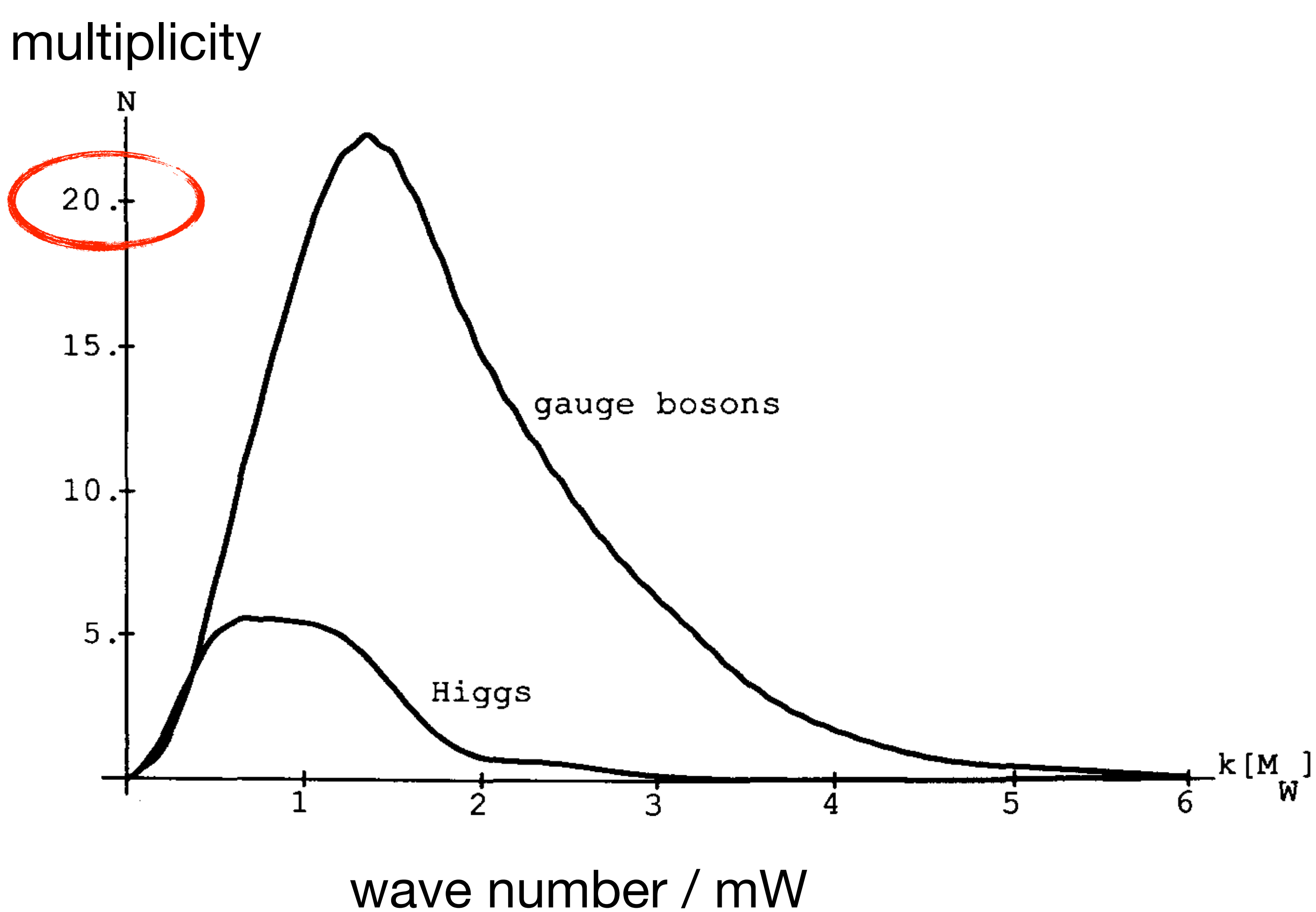
O(30) EW gauge bosons are produced!

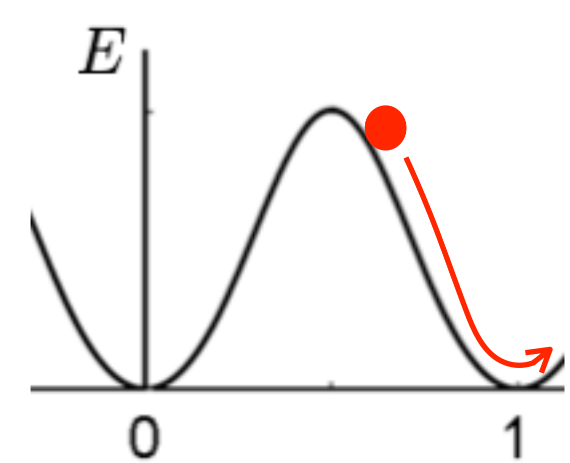
Festival at collider!

Real time evolution [Heilmund, Kripfganz '91]



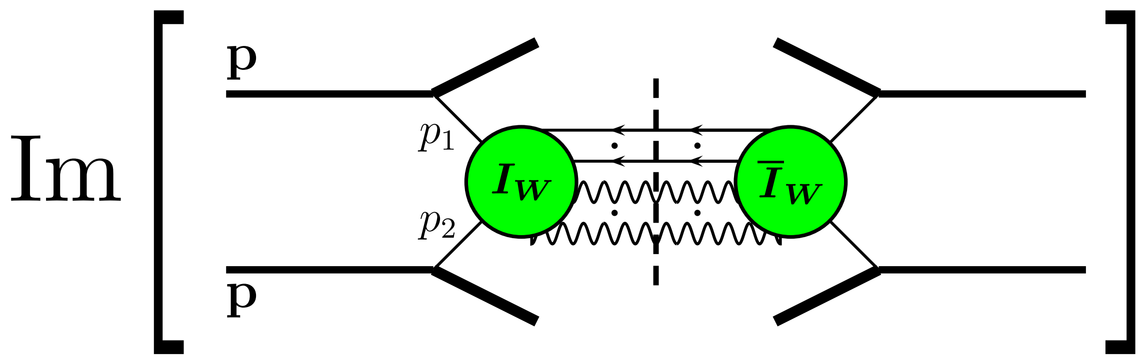
- Prepare an *almost* sphaleron configuration, deviated slightly to the unstable direction.
- Evolve it with EoM and observe the field rump dissipates.
- Fourier expand (expansion in terms of fee particle modes) the final state and count the number of W and H bosons.





- The inclusive cross-section can be estimated using the dispersion relations (optical theorem).

[Khoze, Ringwald '91]



$$\hat{\sigma}^{\rm BV}_{\rm incl} \equiv \sum_{n_W, n_h} \hat{\sigma}^{\rm BV}_{n_W, n_h} = \frac{P(\epsilon)}{m_W^2} \exp\left[-\frac{4\pi}{\alpha_W} F(\epsilon)\right]$$

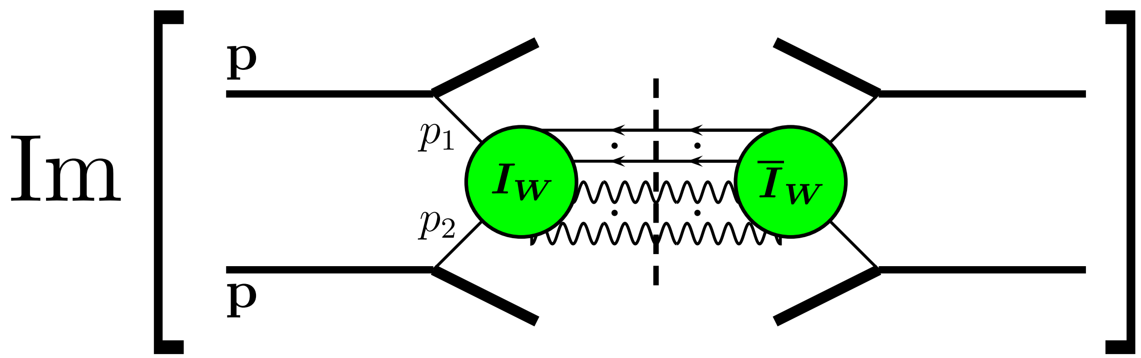
$$F(\epsilon) = 1 - \frac{9}{8}\epsilon^{4/3} + \frac{9}{16}\epsilon^2 - \frac{3}{32}\left(4 - 3\frac{m_h^2}{m_W^2}\right)\epsilon^{8/3}\ln\frac{1}{\epsilon} + \mathcal{O}(\epsilon^{8/3}\cdot\text{const})$$

$$P(\epsilon) = \frac{\pi^{15/2}}{1024}\left(\frac{3}{2}\right)^{\frac{2}{3}} d^2 \left(\frac{4\pi}{\alpha_W}\right)^{7/2} \epsilon^{\frac{74}{9}} \left[1 + \mathcal{O}\left(\epsilon^{2/3}\right)\right] \qquad d \simeq 0.15$$

$$\epsilon \equiv \frac{\sqrt{\hat{s}}}{M_0} \qquad M_0 \equiv \sqrt{6}\pi\frac{m_W}{\alpha_W} \simeq 18.3\ \text{TeV}$$

- The inclusive cross-section can be estimated using the dispersion relations (optical theorem).

[Khoze, Ringwald ’91]



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‘holy grail’ function

$$\boxed{F(\epsilon)} = 1 - \frac{9}{8}\epsilon^{4/3} + \frac{9}{16}\epsilon^2 - \frac{3}{32}\left(4 - 3\frac{m_h^2}{m_W^2}\right)\epsilon^{8/3}\ln\frac{1}{\epsilon} + \mathcal{O}(\epsilon^{8/3}\cdot {\rm const})$$

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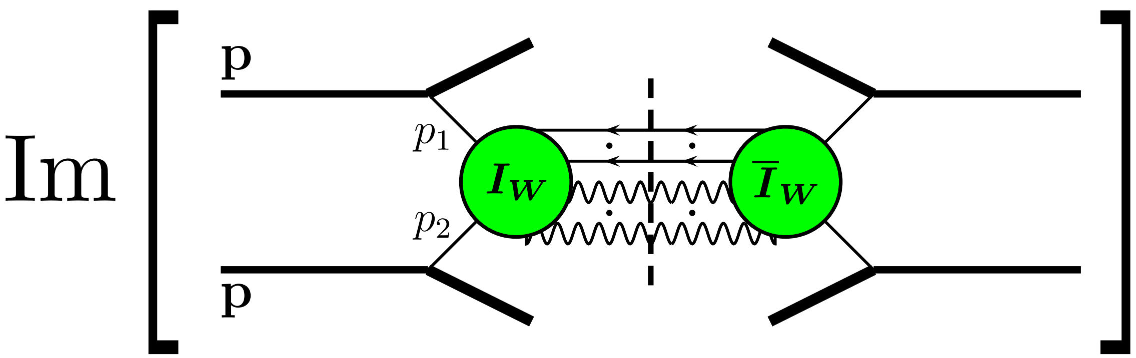

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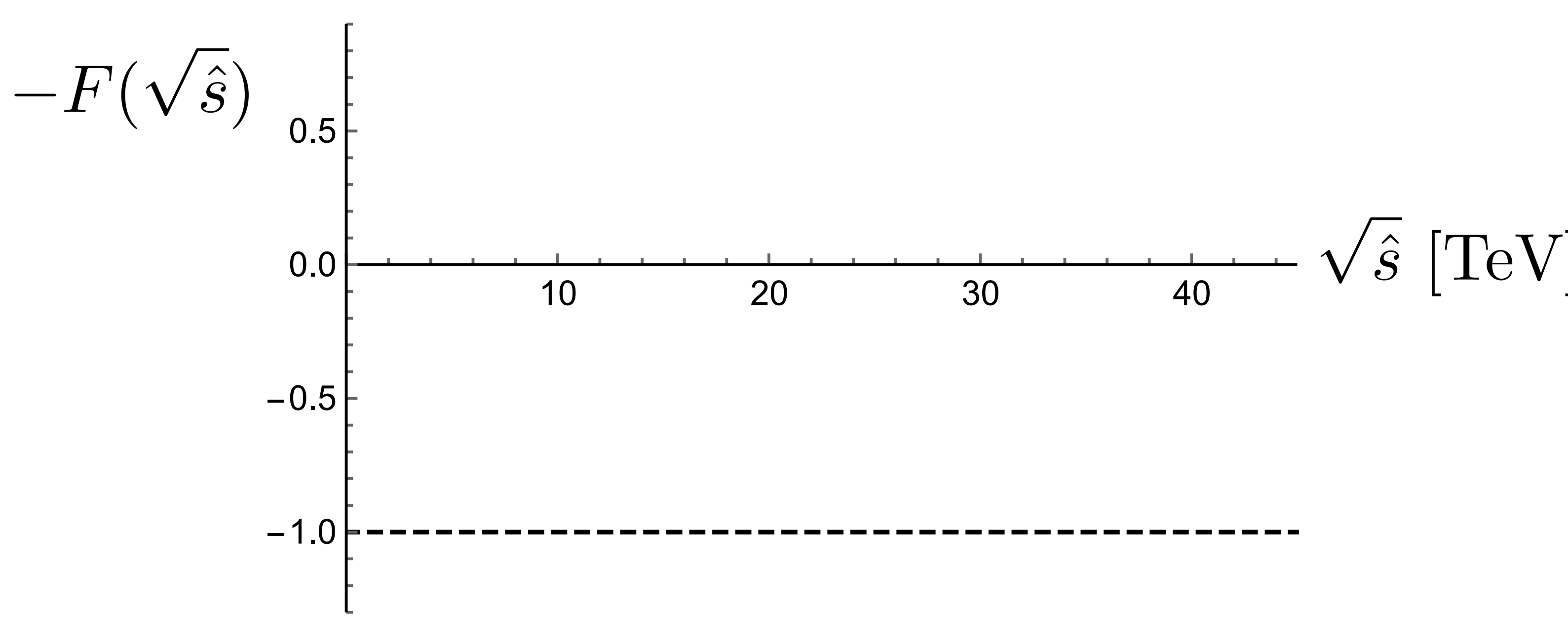

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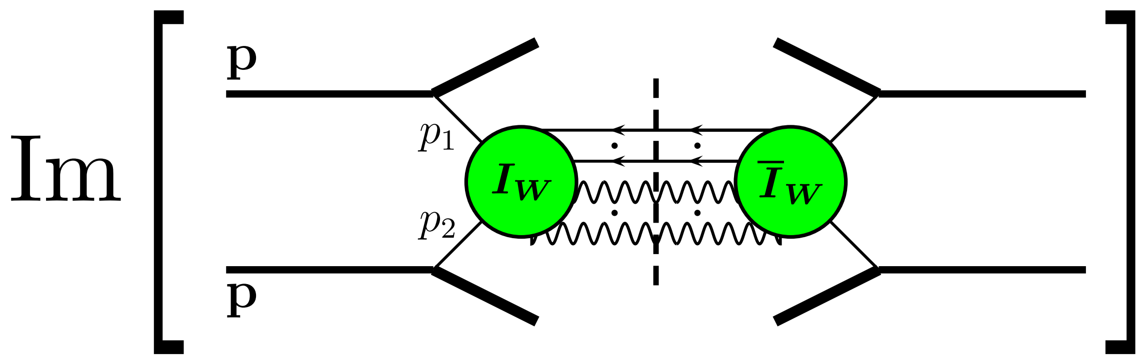
[Khoze, Ringwald '91]



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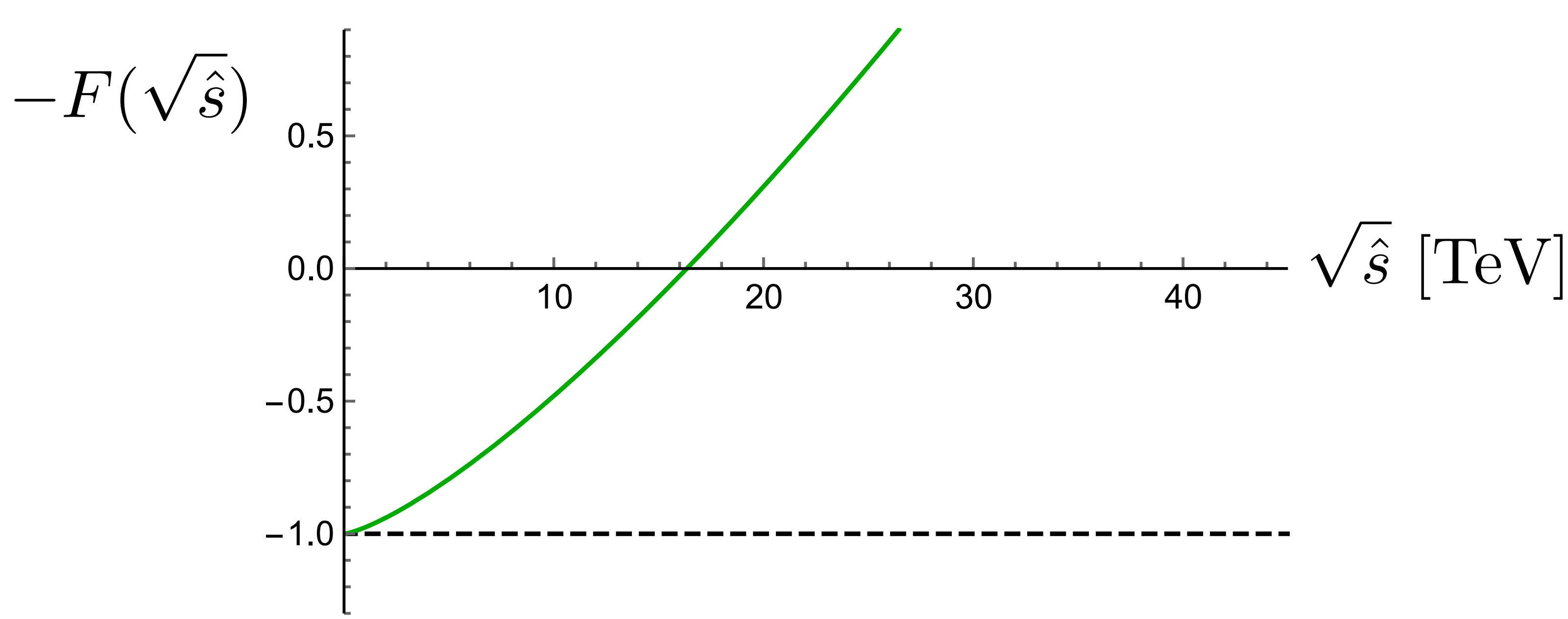

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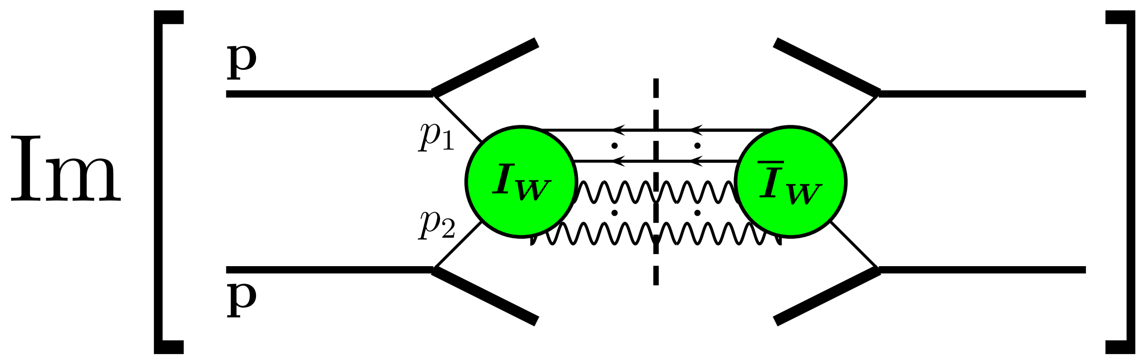

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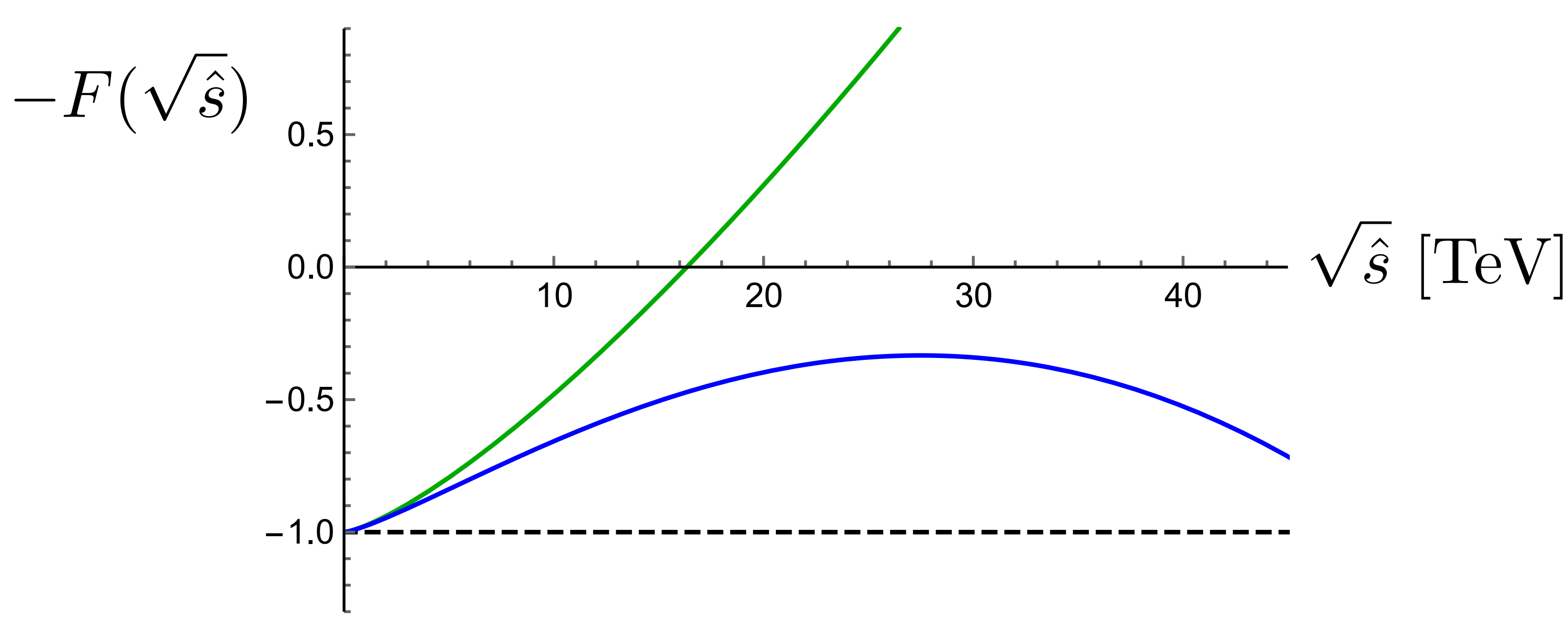

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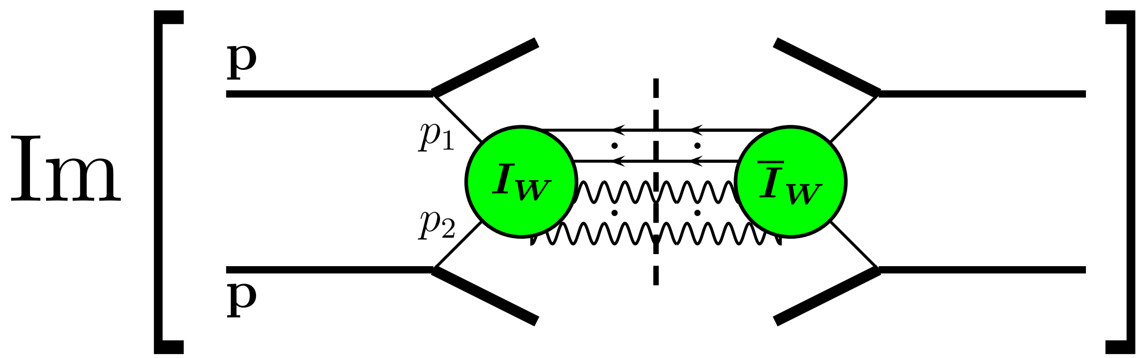

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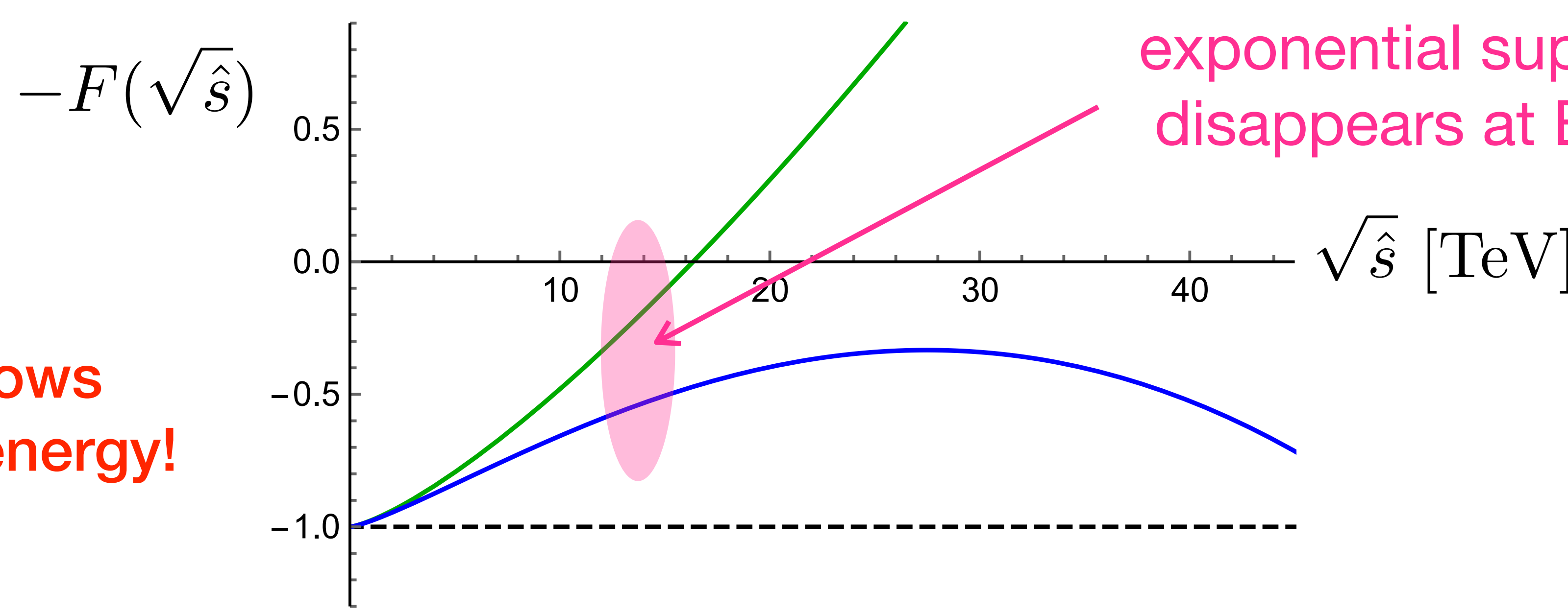


exponential suppression disappears at E ~ E_{sph}?

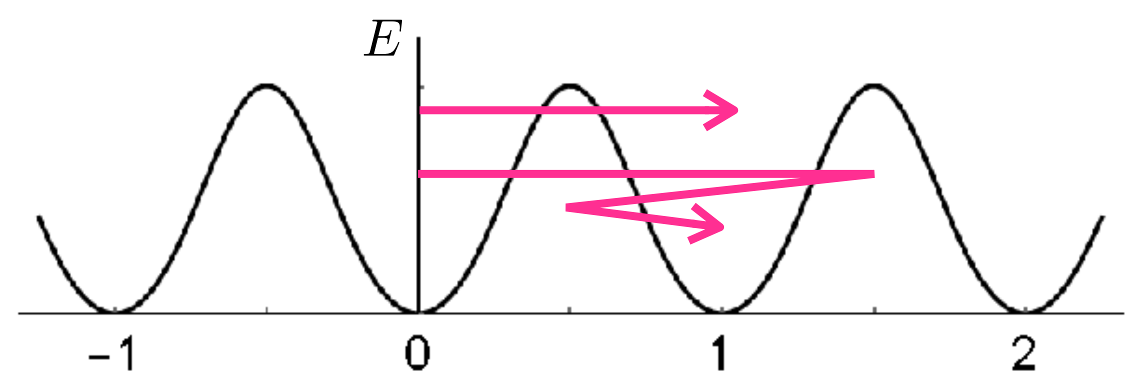
Cross-section grows exponentially with energy!

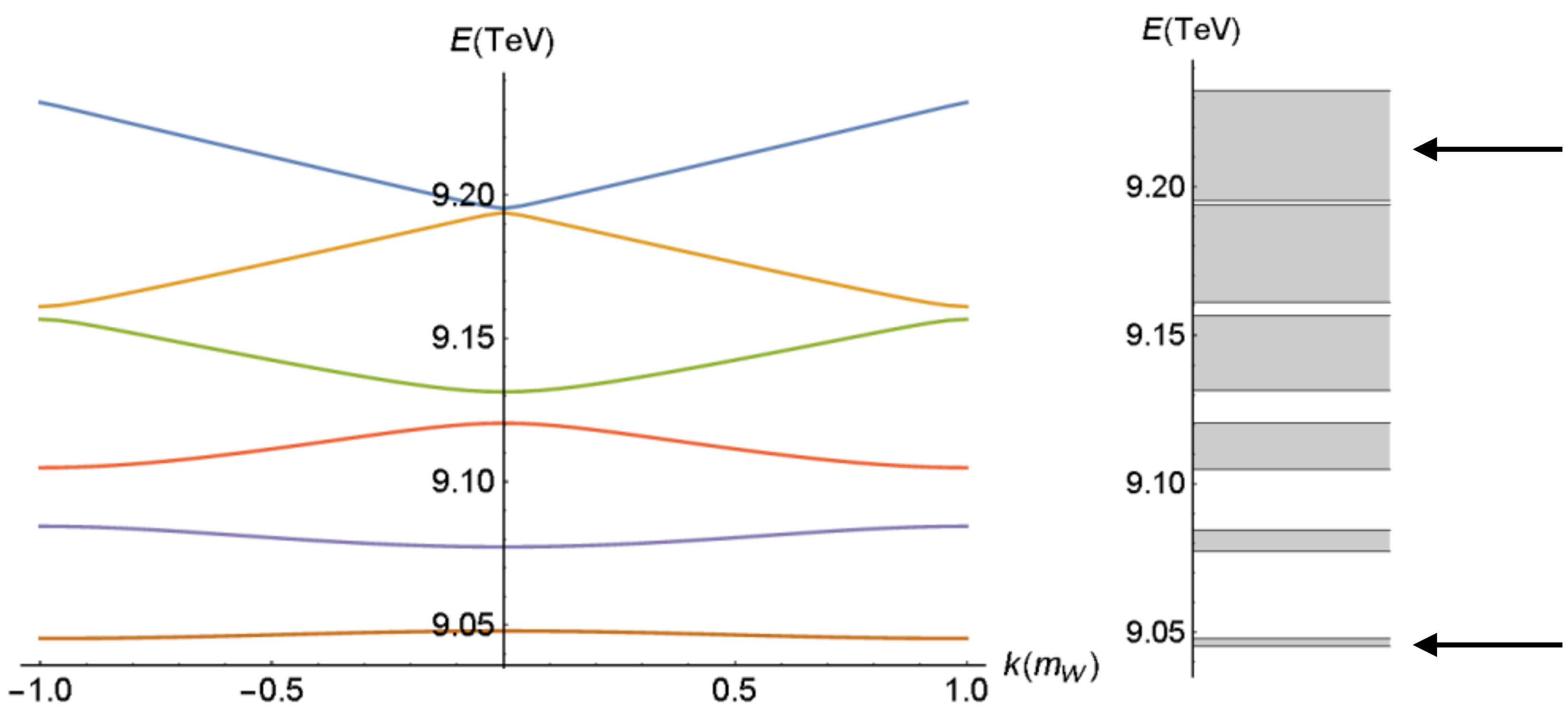
- More recently (2015), it has been pointed out that at zero temperature instanton rate may be able to overcome the exponential suppression for $E > E_{sph}$ ~ 9 TeV, if the periodicity of the EW potential is taken into account, due to *resonant tunnelling*.

Tye, Wong [1505.03690, 1710.07223]
Qiu, Tye [1812.07181]

Resonant tunneling:

Different paths coherently interfere at particular energies, forming a conducing band structure





the band widths are unsuppressed for E > 9.1 TeV.

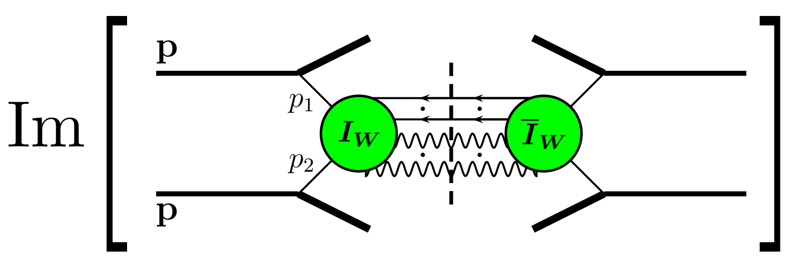
Exponentially small widths for E < 9.1 TeV corresponds to the exponential suppression in instantons

Cross-section Estimate

$$\boxed{\hat{\sigma}^{\rm BV}_{\rm incl} \equiv \sum_{n_W, n_h} \hat{\sigma}^{\rm BV}_{n_W, n_h} = \frac{P(\epsilon)}{m_W^2} \exp\left[-\frac{4\pi}{\alpha_W} \underline{F(\epsilon)} \right]} \qquad \epsilon \equiv E \cdot \frac{\alpha_W}{4\pi m_W}$$

- **Optical theorem**

  [Khoze, Ringwald '91], ...



- **Semi-Classical method**

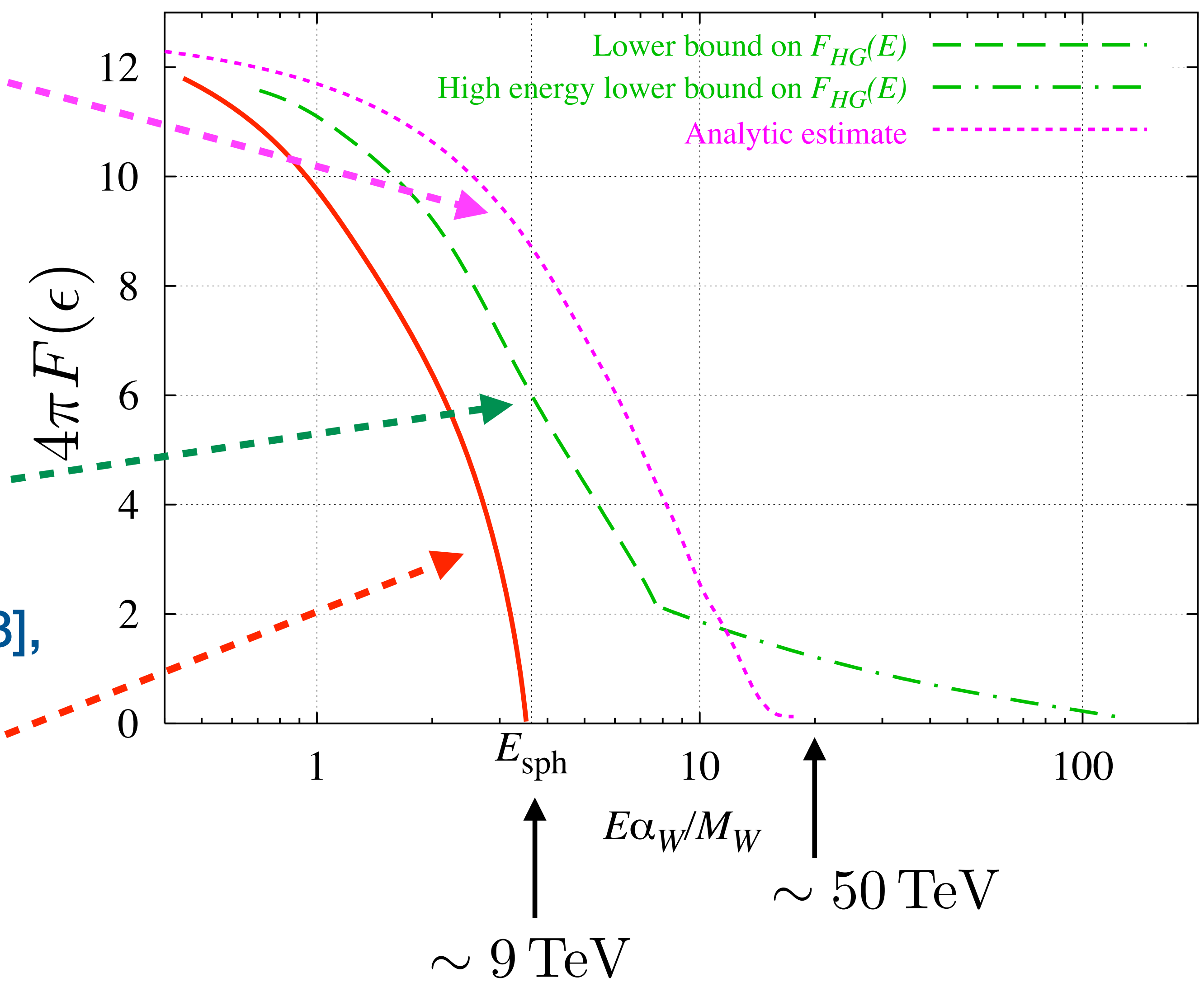
  [Bezrukov, Levkov, Rebbi, Rubakov, Tinyakov '03],

  [Rubakov, Tinyakov '92], ...

- **Resonant tunneling(?)**

  [Tye, Wong '15 '16]

[Bezrukov, Levkov, Rebbi, Rubakov, Tinyakov '03]



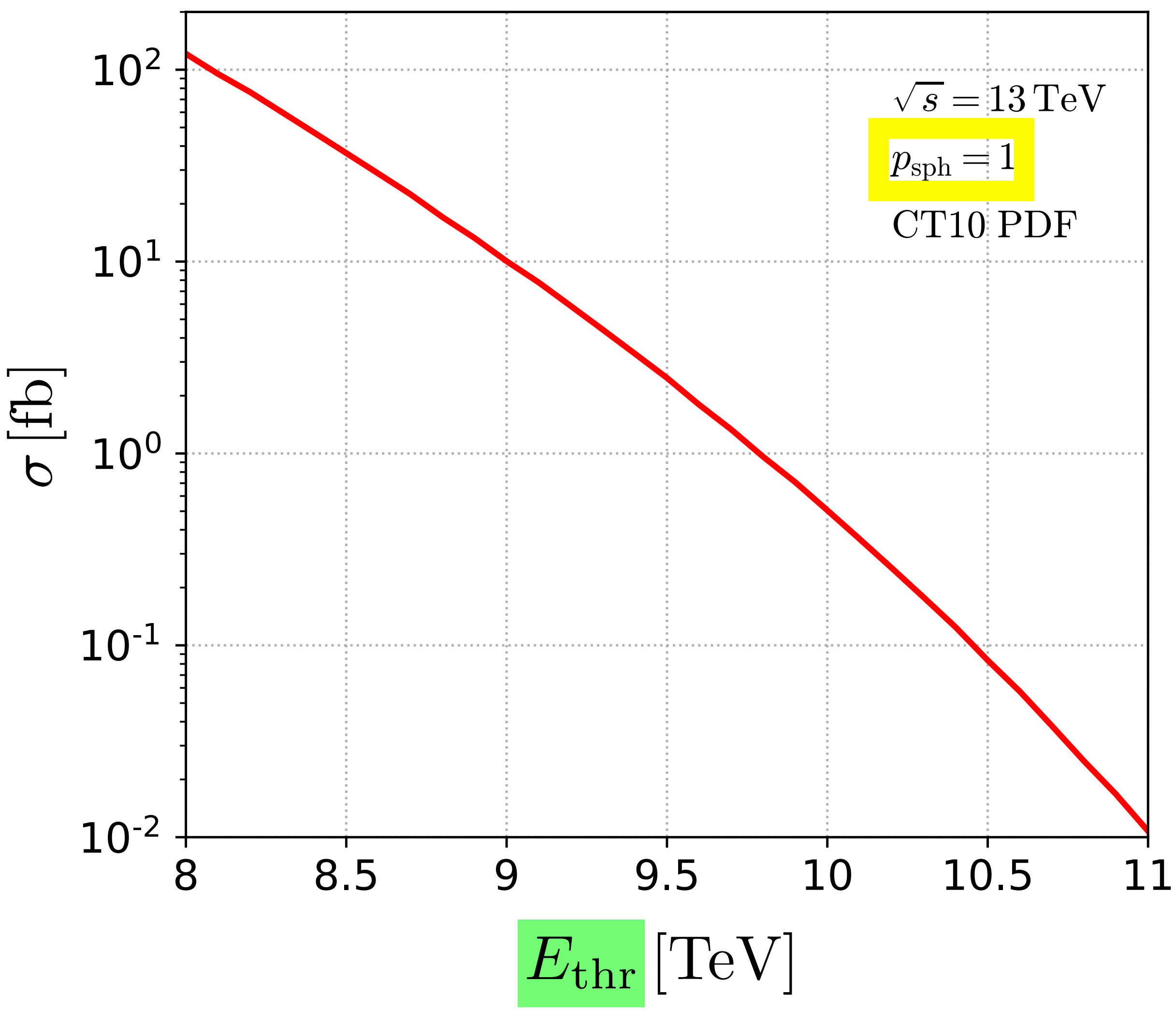
Phenomenological parametrization

partonic: $$\boxed{\hat{\sigma}_0(\sqrt{\hat{s}}) = \frac{p_{\rm sph}}{m_W^2}\Theta(\sqrt{\hat{s}} - E_{\rm thr})}$$

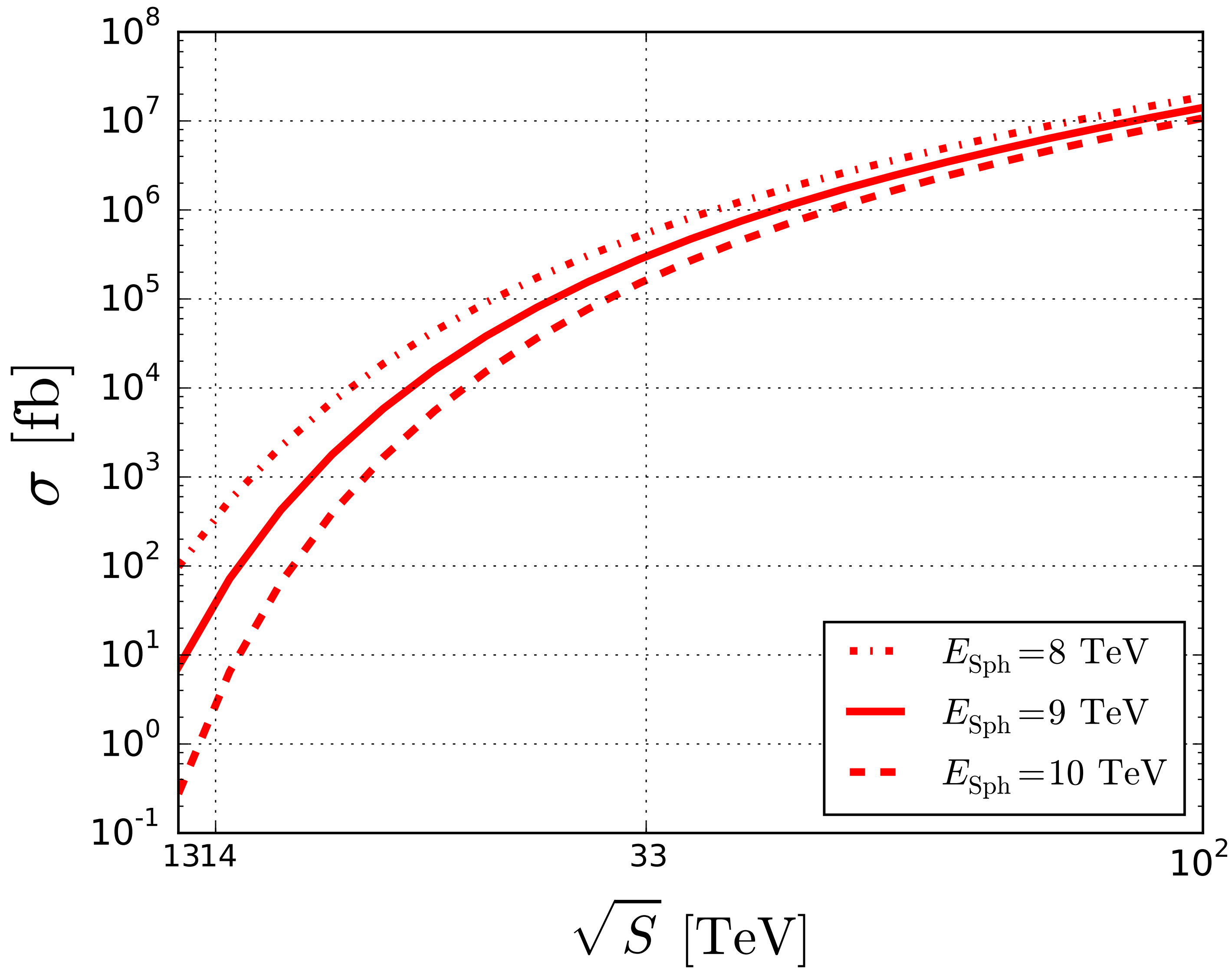
$$\hat{\sigma}^{\rm BV}_{\rm incl} \equiv \sum_{n_W, n_h} \hat{\sigma}^{\rm BV}_{n_W, n_h} = \frac{P(\epsilon)}{m_W^2} \exp\left[-\frac{4\pi}{\alpha_W} F(\epsilon)\right]$$

hadronic: $$\sigma_{pp}(\sqrt{s}) \sim \sum_{ab} \left(\frac{1}{2}\right)^2 \int dx_1 dx_2 f_a(x_1) f_b(x_2) \hat{\sigma}_0(\sqrt{s x_1 x_2})$$

[Ellis, KS, 1601.03654]



Cross Section



$p = 1$

$E_{\rm Sph} = 9\,\text{TeV}$

J. Ellis, KS [1601.03654]

	Sphaleron	gg→H
13 TeV	7.3 fb	44 x 10^3 fb
14 TeV	41 fb	50 x 10^3 fb
33 TeV	0.3 x 10^6 fb	0.2 x 10^6 fb
100 TeV	141 x 10^6 fb	0.7 x 10^6 fb

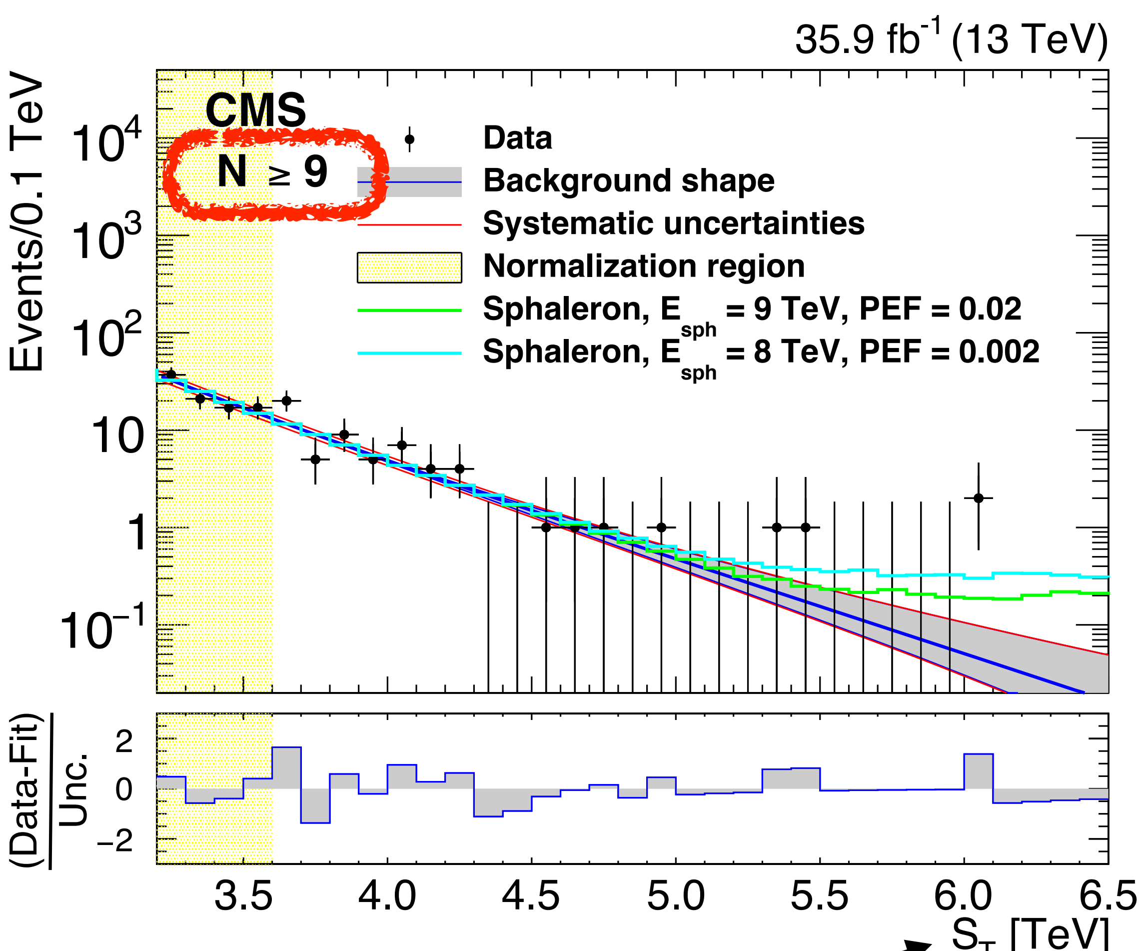
CMS-EXO-17-023

CERN-EP-2018-093
2018/11/16

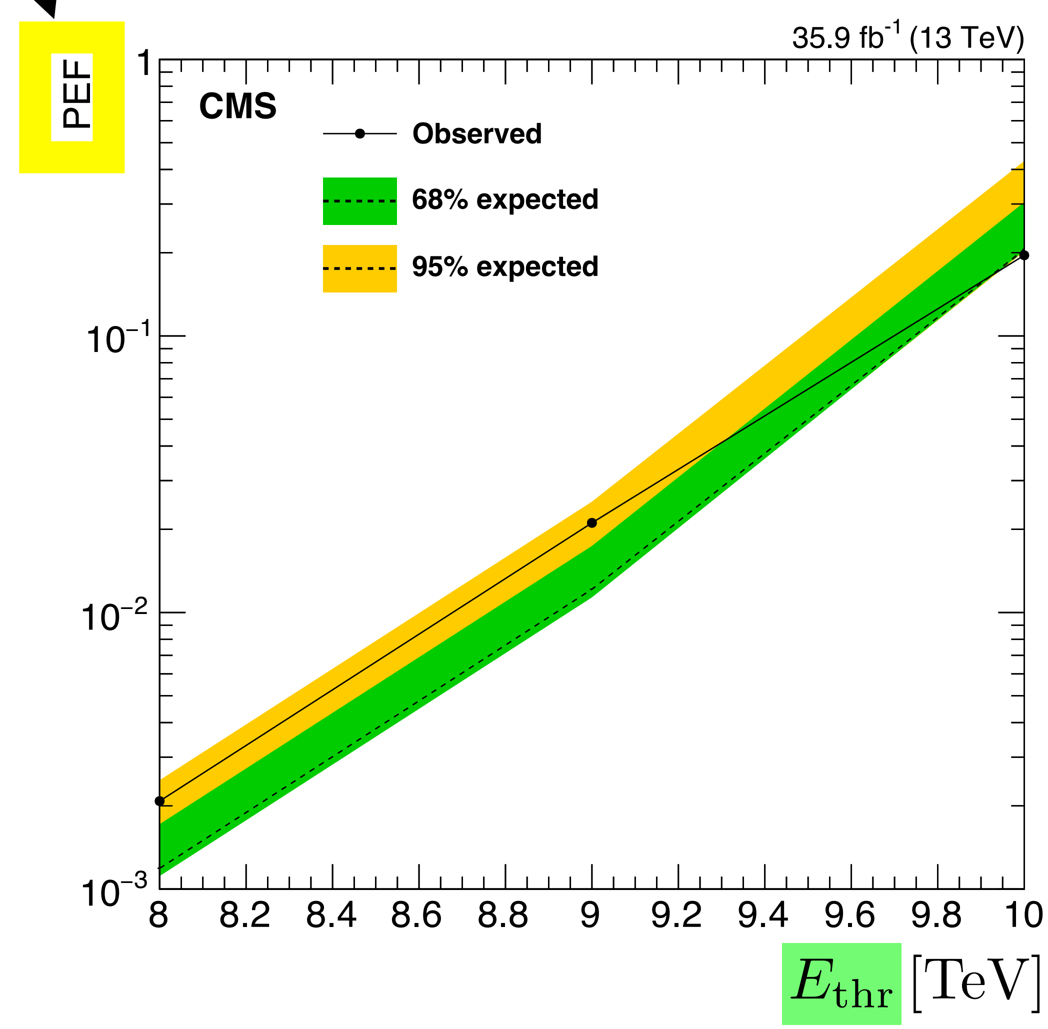
[1805.06013]

Search for black holes and sphalerons in high-multiplicity final states in proton-proton collisions at $\sqrt{s} = 13$ TeV[†]

$$\hat{\sigma}(\sqrt{\hat{s}}) = \frac{p_{\rm sph}}{m_W^2}\Theta(\sqrt{\hat{s}} - E_{\rm thr})$$



Sum of all pT in the final state



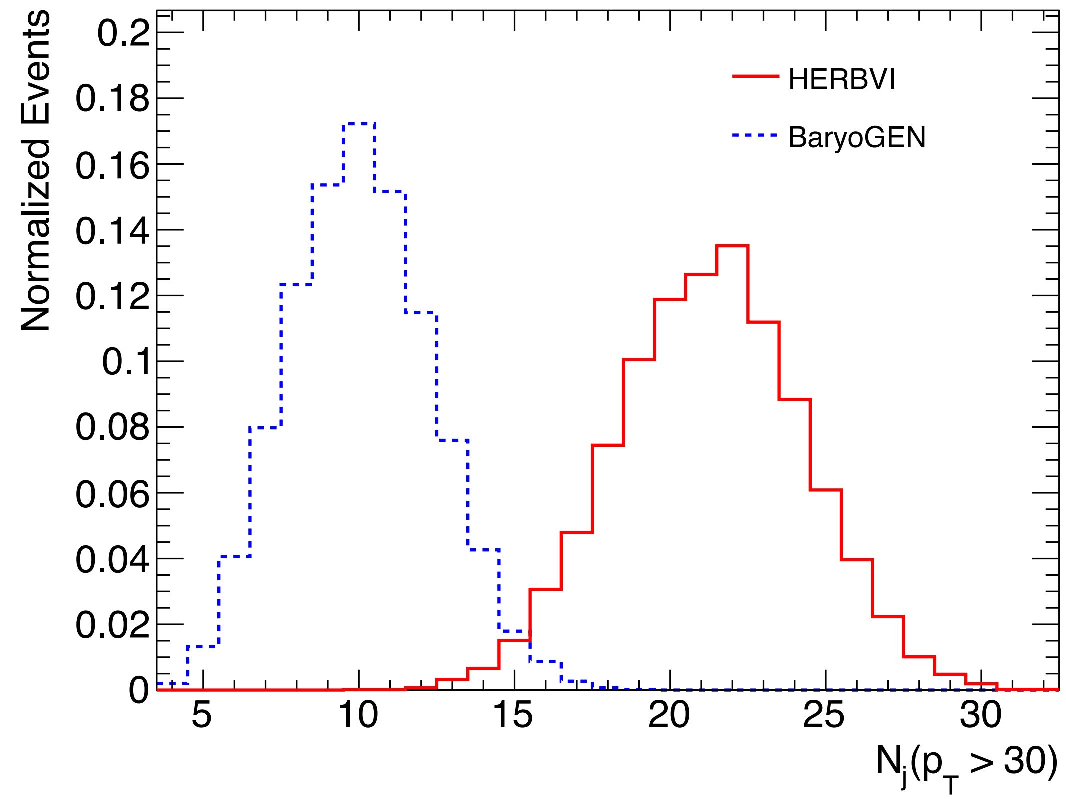
[Ringwald, KS, Webber 1809.10833]

- CMS analysis assumes sphaleron final states ***DO NOT*** involve any EW bosons.

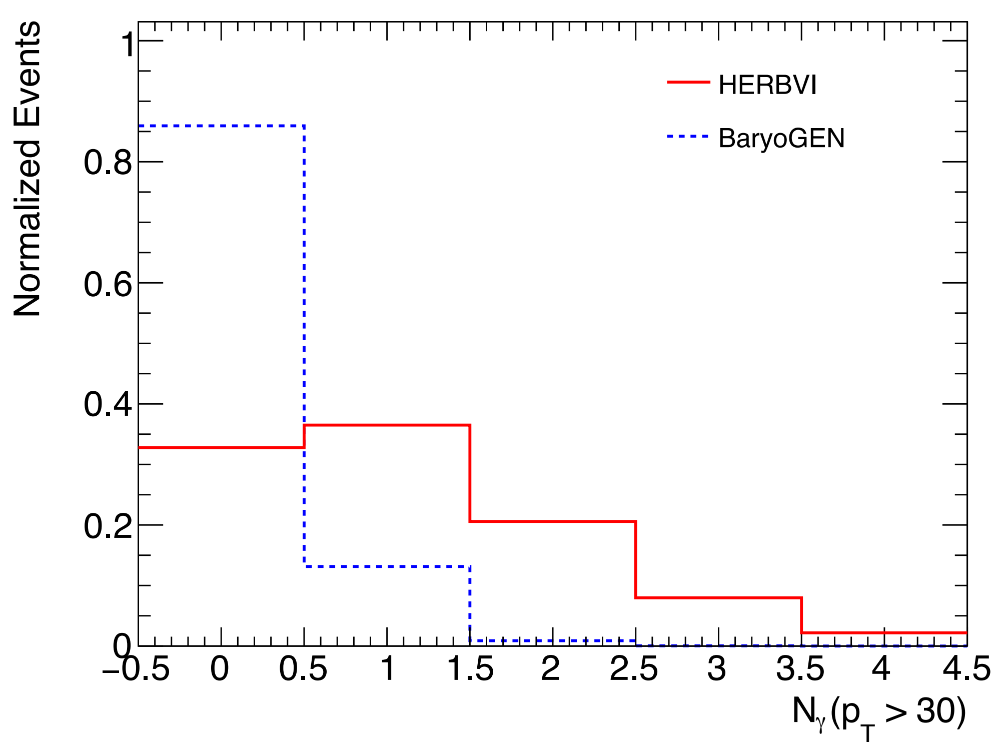
$$qq \to \begin{cases} n_q q + 3\ell & \texttt{[BaryoGEN]} \\ 7q + 3\ell + \sum n_B B & \texttt{[HERBVI]} \end{cases}$$

- **Delphes** is used for detector simulation

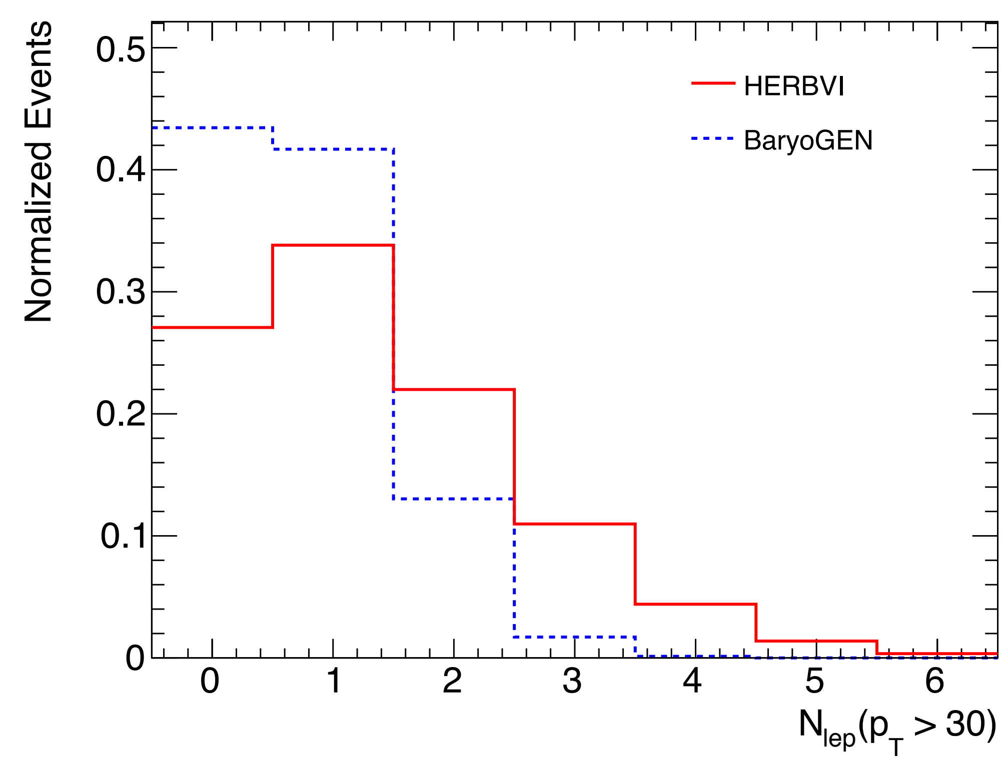
jet multiplicity



lepton multiplicity



photon multiplicity



Comparison in signal efficiencies

$$S_T \equiv E_T^{\rm miss} + \sum_i^{p_T > 70{\rm GeV}} p_T^{(i)} > S_T^{\rm min} \qquad 3.8 < S_T^{\rm min}/{\rm TeV} < 8$$

$$N(p_T > 70\,{\rm GeV}) \geq N_{\rm min} \qquad N_{\rm min} = 3, \ldots, 11$$

[Ringwald, KS, Webber 1809.10833]

$E_{\rm sph}$ [TeV]		8	8.5	9	9.5	10
multi-boson	$(N_{\rm min}, S_T^{\rm min}\,[{\rm TeV}])^*$	(11, 4.2)	(11, 4.2)	(11, 4.2)	(11, 4.2)	(11, 4.2)
	$\epsilon^{(a^*)}$ [%]	94.8	97.5	99.2	99.6	99.9
	$N_{\rm obs}^{\max(a^*)}$	3.0	3.0	3.0	3.0	3.0
Zero Boson	$(N_{\rm min}, S_T^{\rm min}\,[{\rm TeV}])^*$	(9, 5.4)	(9, 5.6)	(9, 5.6)	(8, 6.2)	(8, 6.2)
	$\epsilon^{(a^*)}$ [%]	37.7	40.5	45.3	50.5	57.5
	$N_{\rm obs}^{\max(a^*)}$	6.9	5.8	5.8	3.0	3.0

most sensitive SR

signal efficiency

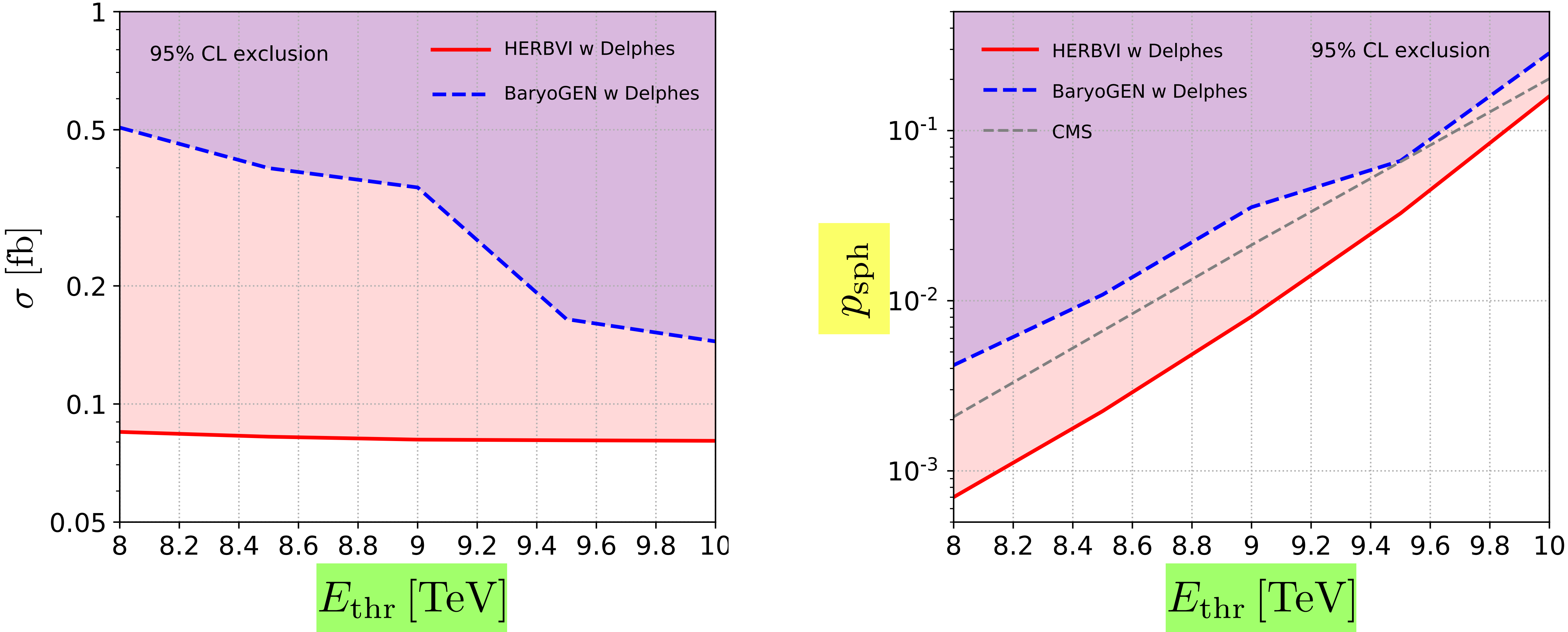
limit on signal events

signal efficiencies are much larger in the multi-boson case

Exclusion limit

[Ringwald, KS, Webber 1809.10833]

$$\hat{\sigma}(\sqrt{\hat{s}}) = \frac{p_{\rm sph}}{m_W^2}\Theta(\sqrt{\hat{s}} - E_{\rm thr})$$



$$qq \to \begin{cases} n_q q + 3\ell & (\texttt{BaryoGEN}) \\ 7q + 3\ell + \sum n_B B & (\texttt{HERBVI}) \end{cases}$$

- The limit on the multi-boson cross-section: $\sigma_{\rm sph} < 0.8\,{\rm fb}$

Implementing instanton/ sphaleron processes in Herwig-7

[A.Papaefstathiou, S.Plätzer, KS 1910.04761]

- We have implemented EW instanton/sphaleron processes at hadron collider in **HERWIG7** framework.
- comparison with other generators:

	written in	Multi-Boson	Unitarity
HERBVI	Fortran	**LO**	No
BaryoGEN	**C++**	No	No
HERWIG7	**C++**	**LO + E_freeze**	**Yes**

LO prediction cannot be trusted for $\sqrt{s} > 10$TeV.

For $\sqrt{s}$ > E_freeze, the LO Boson multiplicity distribution is calculated with $\sqrt{s}$ = E_freeze.

[A.Papaefstathiou, S.Plätzer, KS 1910.04761]

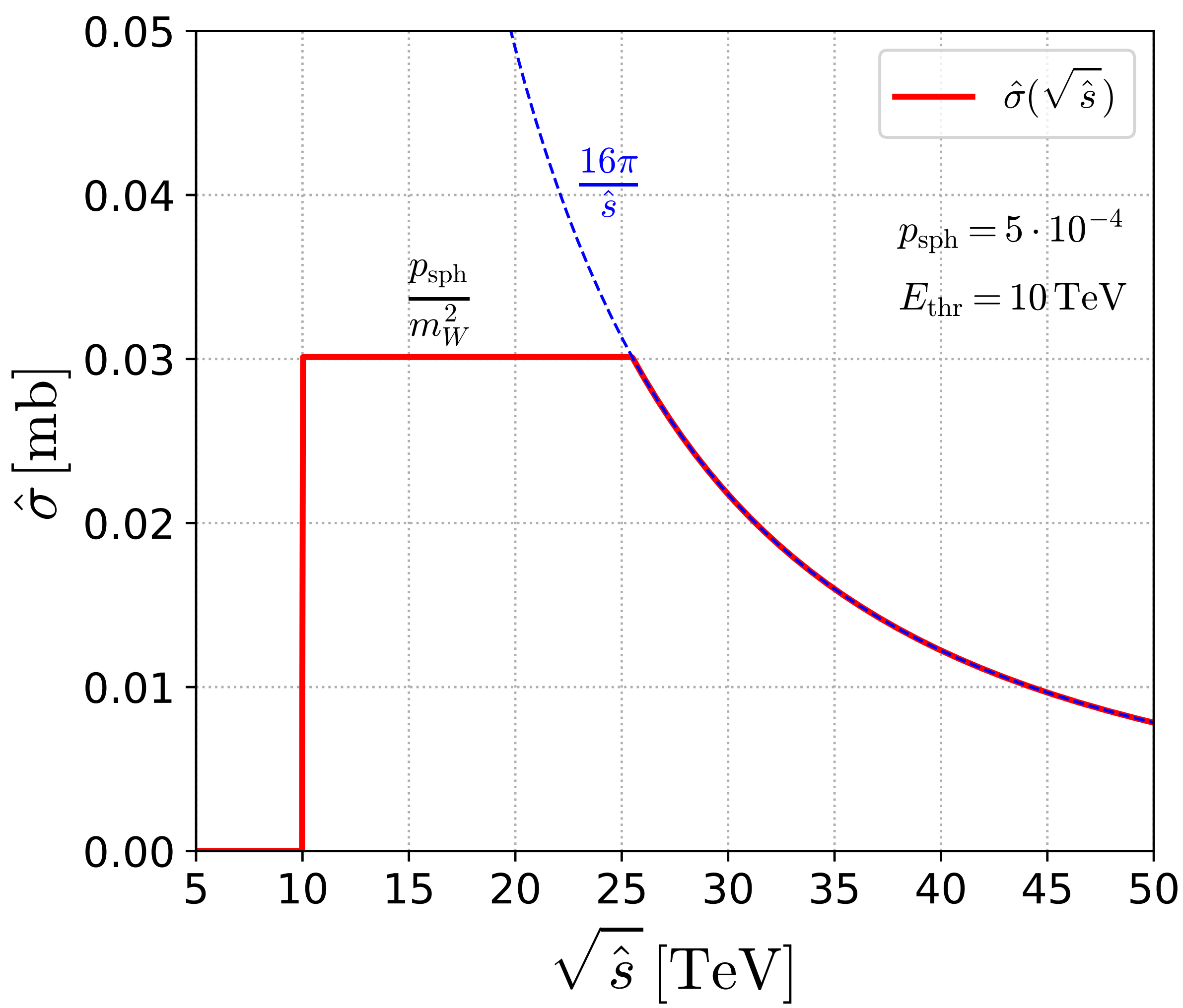
Phenomenological parametrisation for cross-section:

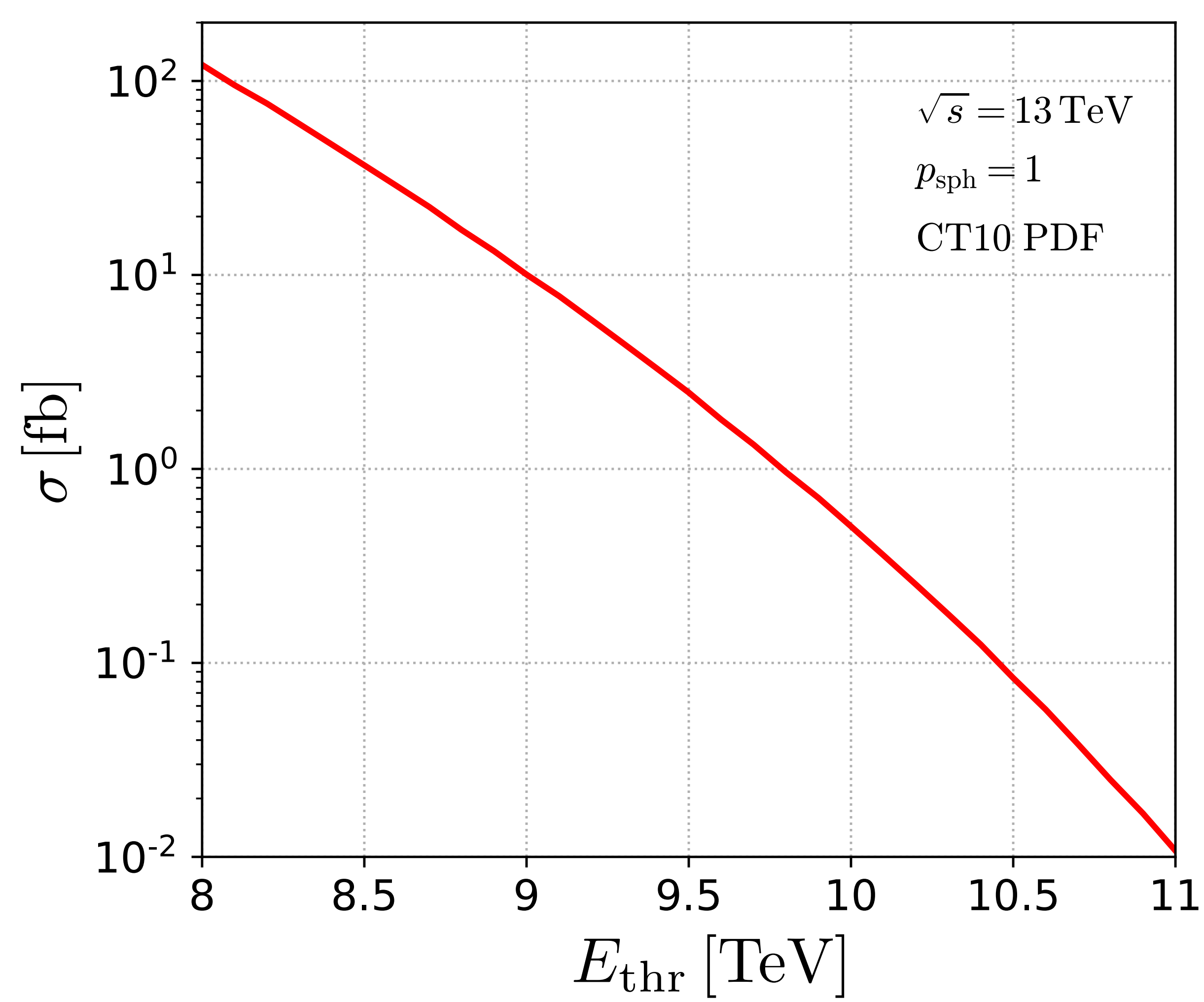
partonic: $$\hat{\sigma}_0(\sqrt{\hat{s}}) = \frac{p_{\rm sph}}{m_W^2}\Theta(\sqrt{\hat{s}} - E_{\rm thr})$$

$$\hat{\sigma}(\sqrt{\hat{s}}) = \min(\hat{\sigma}_0, \hat{\sigma}^{\rm max}_{\rm unitary}) \qquad \hat{\sigma}^{\rm max}_{\rm unitary}(\sqrt{\hat{s}}) = \frac{16\pi}{\hat{s}}$$

hadronic: $$\sigma_{pp}(\sqrt{s}) \sim \sum_{ab}\left(\frac{1}{2}\right)^2 \int dx_1 dx_2 f_a(x_1) f_b(x_2) \hat{\sigma}_0(\sqrt{s x_1 x_2})$$

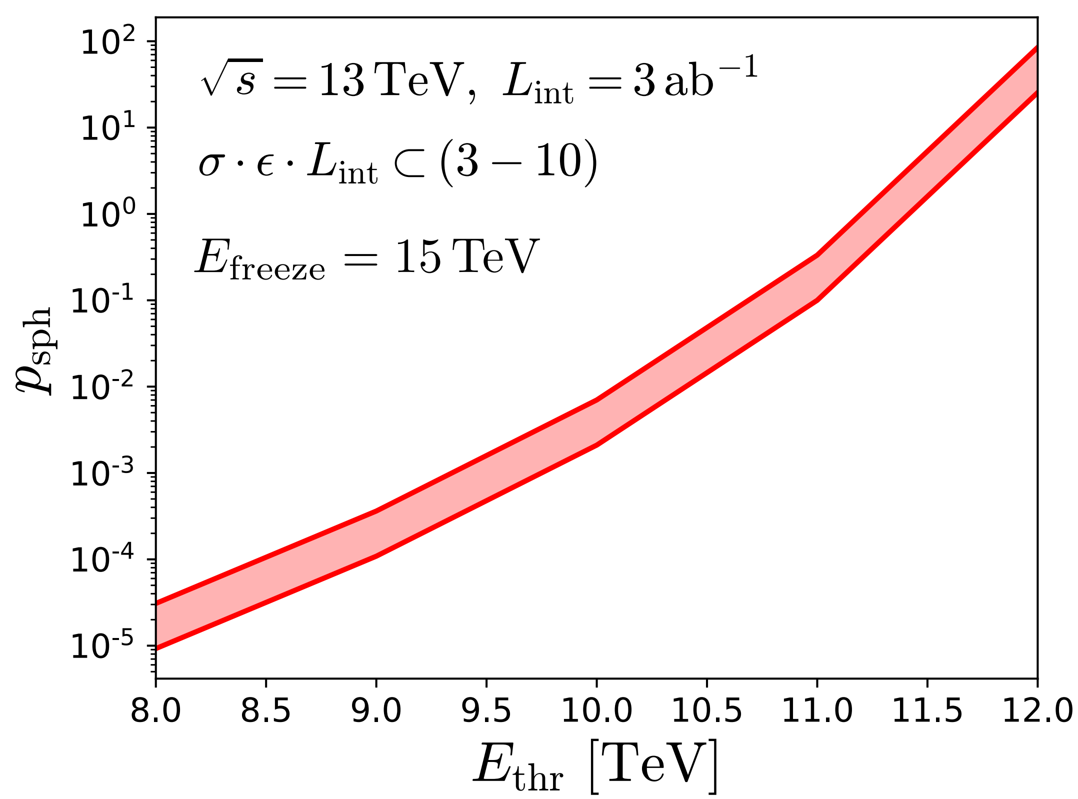
[A.Papaefstathiou, S.Plätzer, KS 1910.04761]



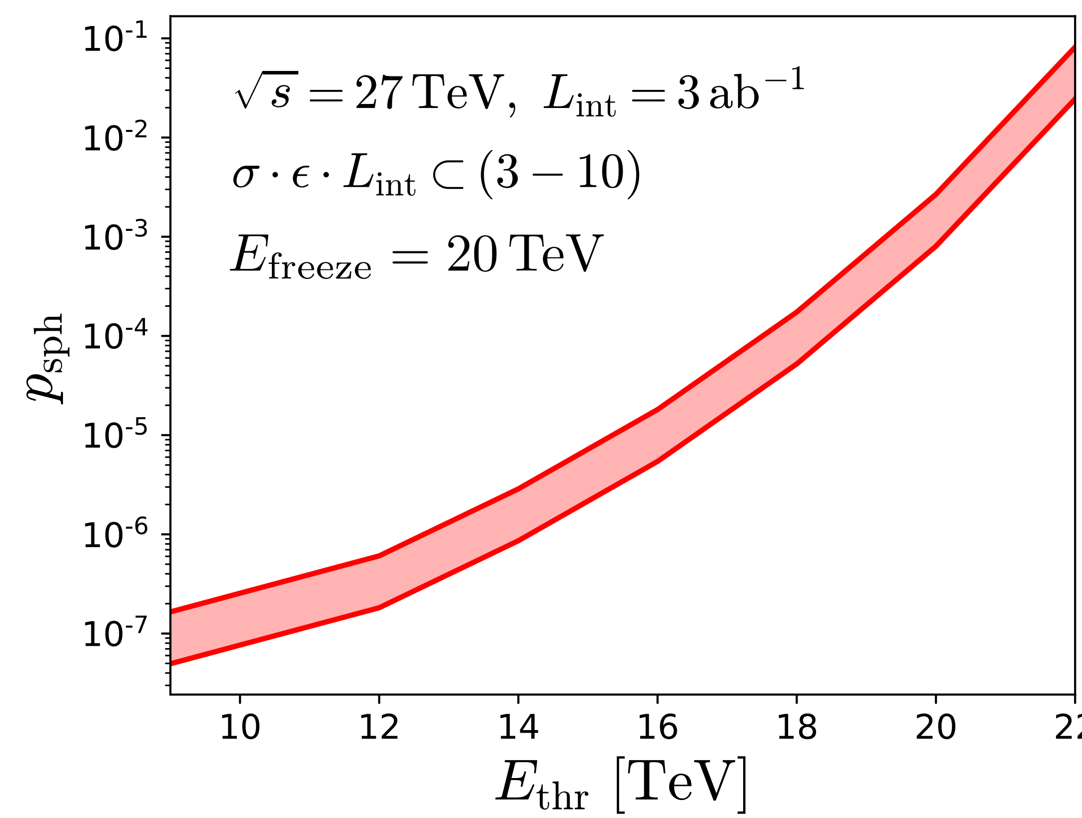


Future Projection

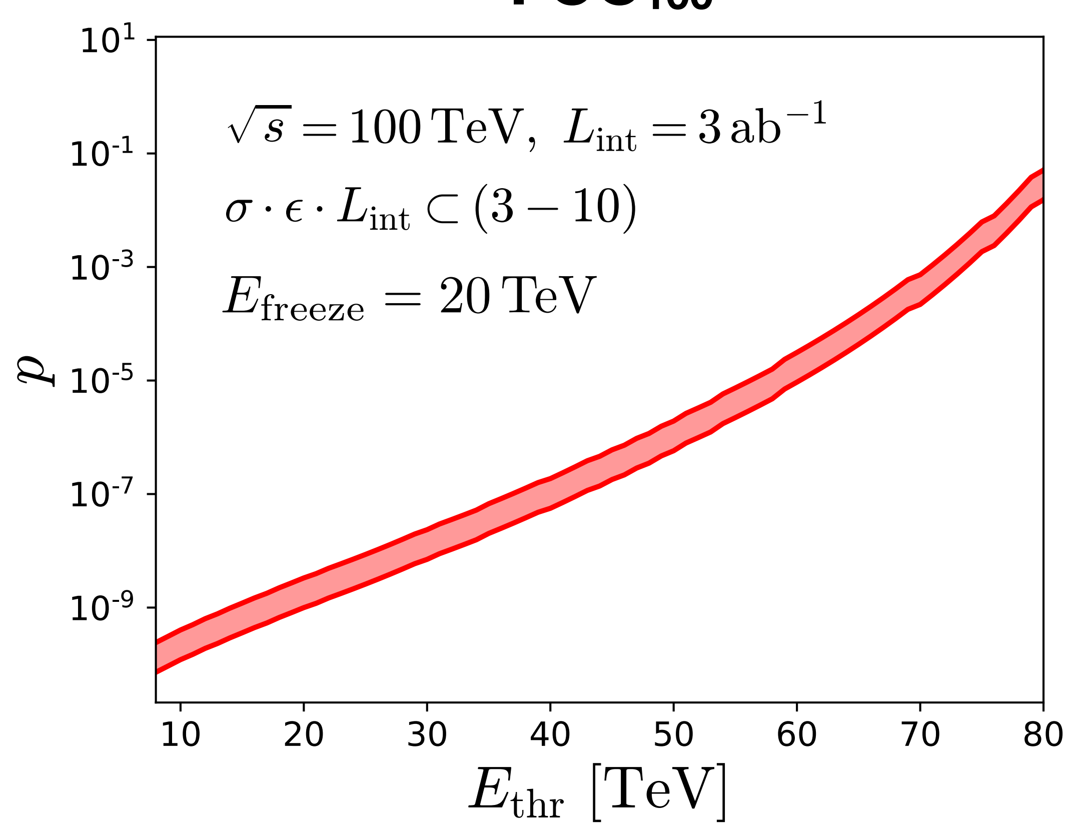
HL-LHC



HE-LHC (27 TeV)



FCC$_{100}$



- Event selection

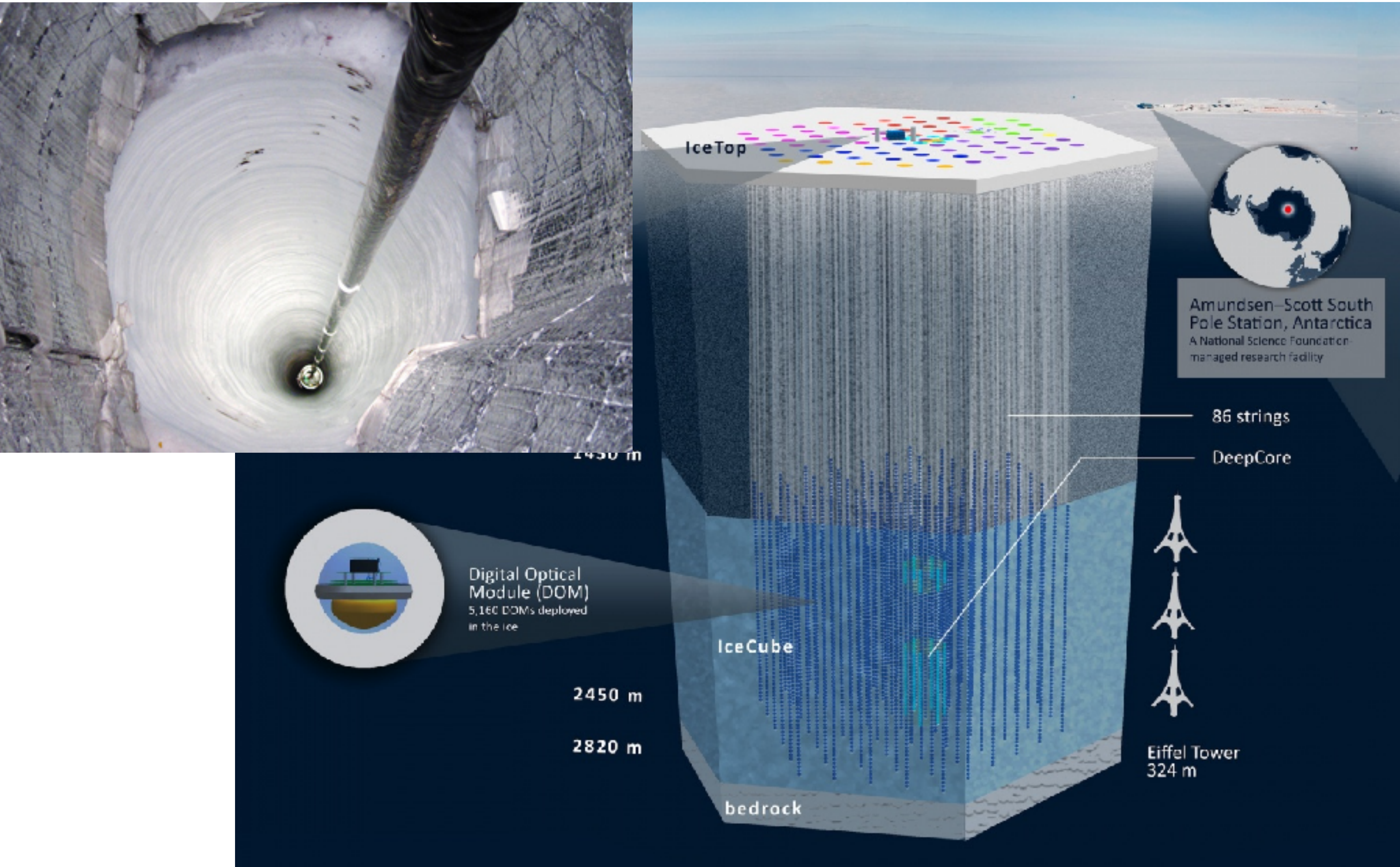
$$\begin{cases} N(p_T > 100) \geq 11,\ S_T^{100} > 4\,\mathrm{TeV} & \text{... HL-LHC} \\ N(p_T > 100) \geq 15,\ S_T^{100} > 7\,\mathrm{TeV} & \text{... HE-LHC, FCC100} \end{cases}$$

$N(p_T > 100)$: # of visible objects with p_T >100GeV

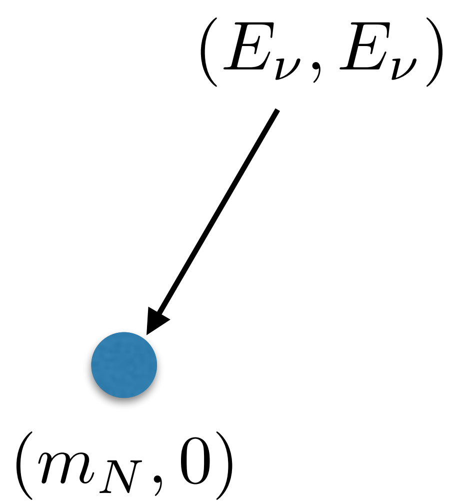
S_T^{100} : scalar p_T sum of visible objects with p_T > 100GeV

[A.Papaefstathiou, S.Plätzer, KS 1910.04761]

Sphalerons @ IceCube



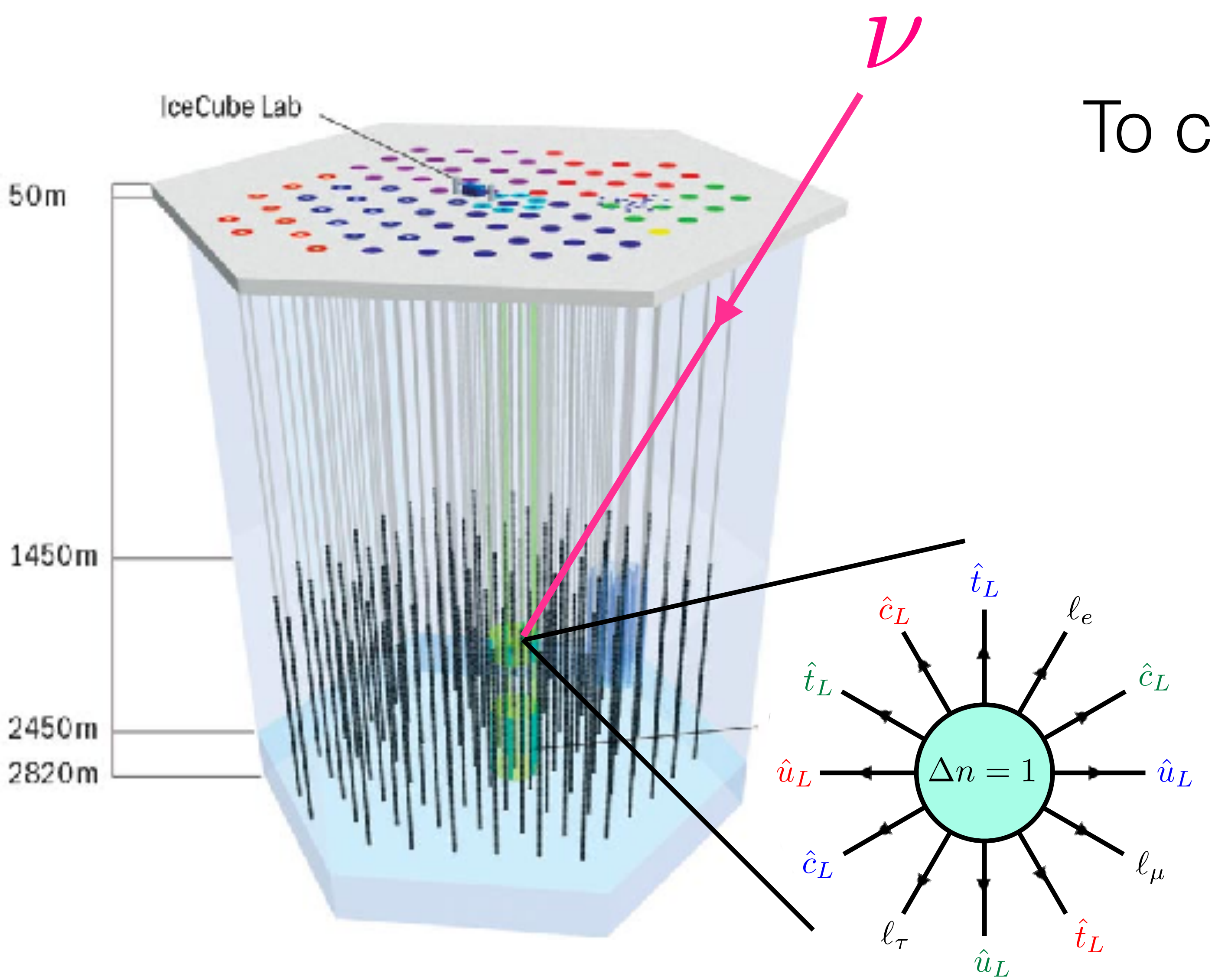
What neutrino energy is required to create a sphaleron?



$$s_{N\nu} = E^2 - p^2 = (m_N + E_N)^2 - E_N^2 \simeq 2m_N E_\nu$$

$$s_{q\nu} \simeq 2x m_N E_\nu \qquad (x = E_q/E_N)$$

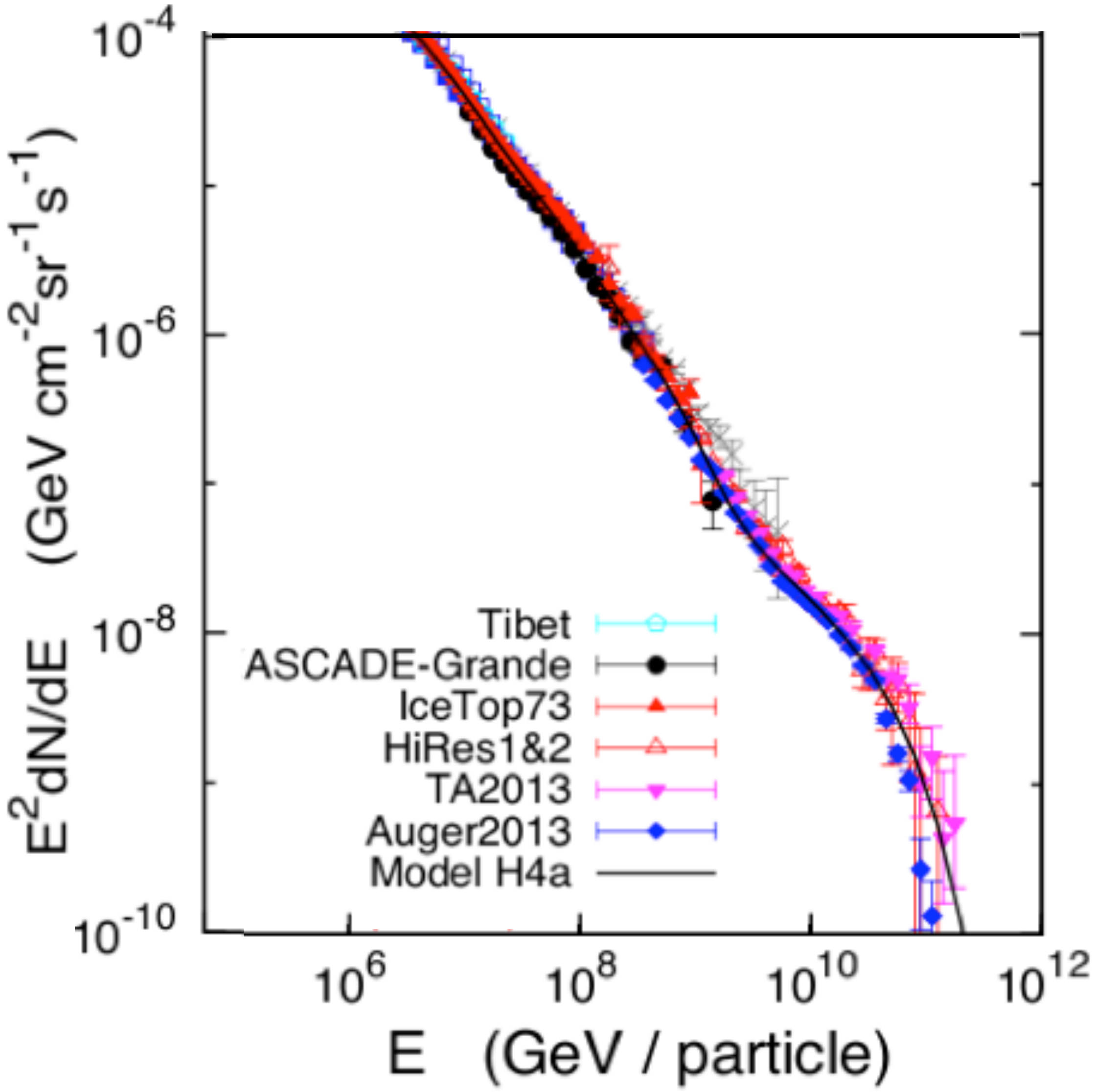
$$E_\nu \geq \frac{E_{\rm Sph}^2}{2x m_N} \simeq \frac{(9\,{\rm TeV})^2}{2x(0.94\,{\rm GeV})} \simeq \frac{4 \cdot 10^7}{x}\,{\rm GeV}$$



To create a sphaleron, one needs

$$E_\nu \gtrsim 10^{8-10}\,{\rm GeV}$$

$$({\rm for}\ 10^{-3} \lesssim x \lesssim 10^{-1})$$



Cosmic ray spectrum falling sharply above 10^{11} GeV has been observed.

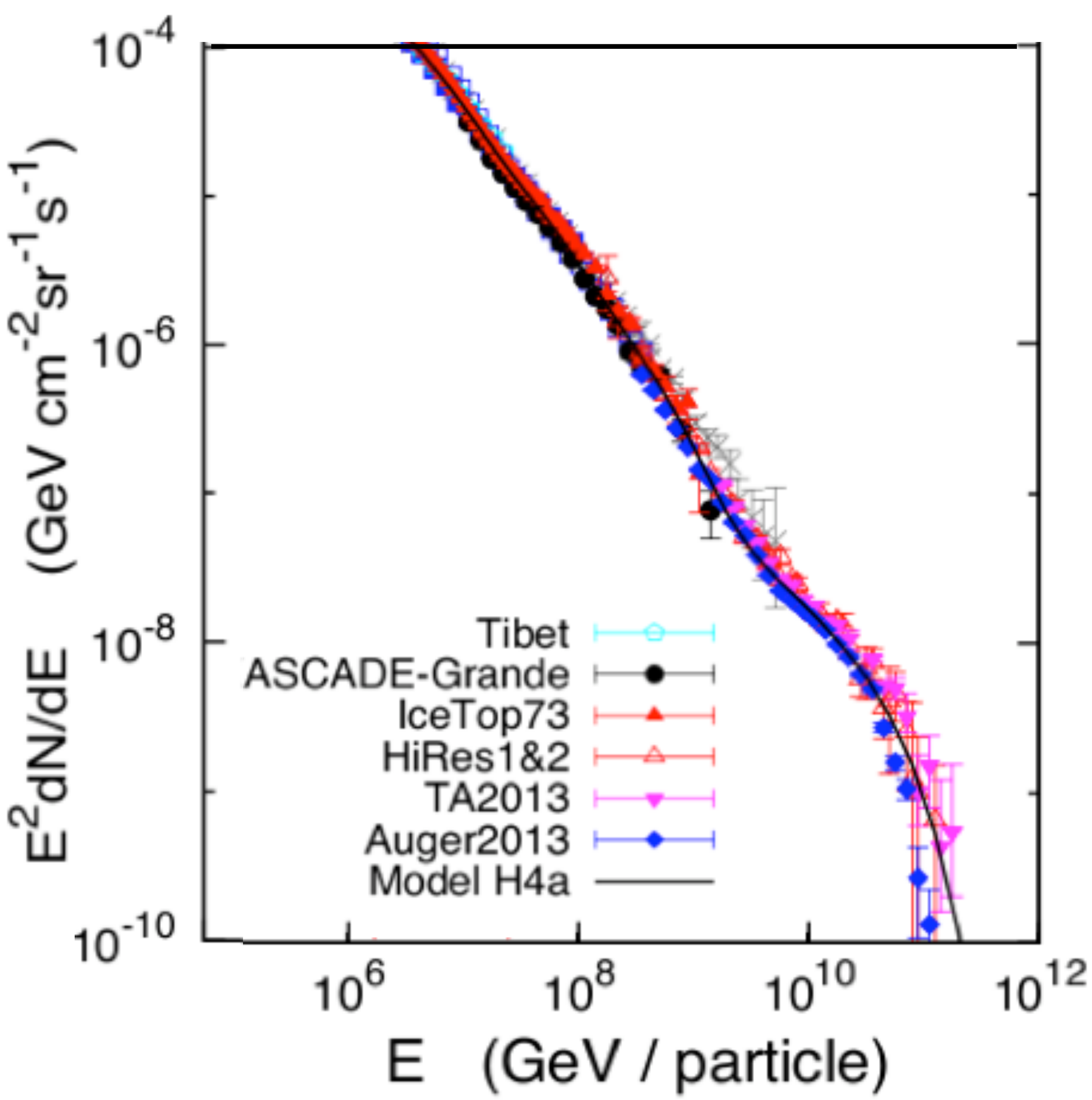
Greisen–Zatsepin–Kuzmin (GZK) process:

$$p + \gamma_{\rm CMB} \to p + \pi^0$$

$$p + \gamma_{\rm CMB} \to n + \pi^+$$

$$(p_p + p_{\gamma_{\rm CMB}})^2 \geq (m_N + m_\pi)^2 \quad (E_{\gamma_{\rm CMB}} \sim 2.6 \cdot 10^{-13}\,{\rm GeV})$$

$$\Rightarrow \quad E_p \geq \frac{(m_N + m_\pi)^2 - m_p^2}{4E_{\gamma_{\rm CMB}}} \sim 3 \cdot 10^{11}\,{\rm GeV}$$



Cosmic ray spectrum falling sharply above 10^{11} GeV has been observed.

Greisen–Zatsepin–Kuzmin (GZK) process:

$$p + \gamma_{\rm CMB} \to p + \pi^0 \quad : \pi^0 \to \gamma\gamma$$

$$p + \gamma_{\rm CMB} \to n + \pi^+ : \pi^+ \to \mu^+ \nu_\mu \to e^+ \nu_e \bar{\nu}_\mu \nu_\mu$$

These high energy neutrinos and photons should reach the earth.

$$(p_p + p_{\gamma_{\rm CMB}})^2 \geq (m_N + m_\pi)^2 \quad (E_{\gamma_{\rm CMB}} \sim 2.6 \cdot 10^{-13}\,\text{GeV})$$

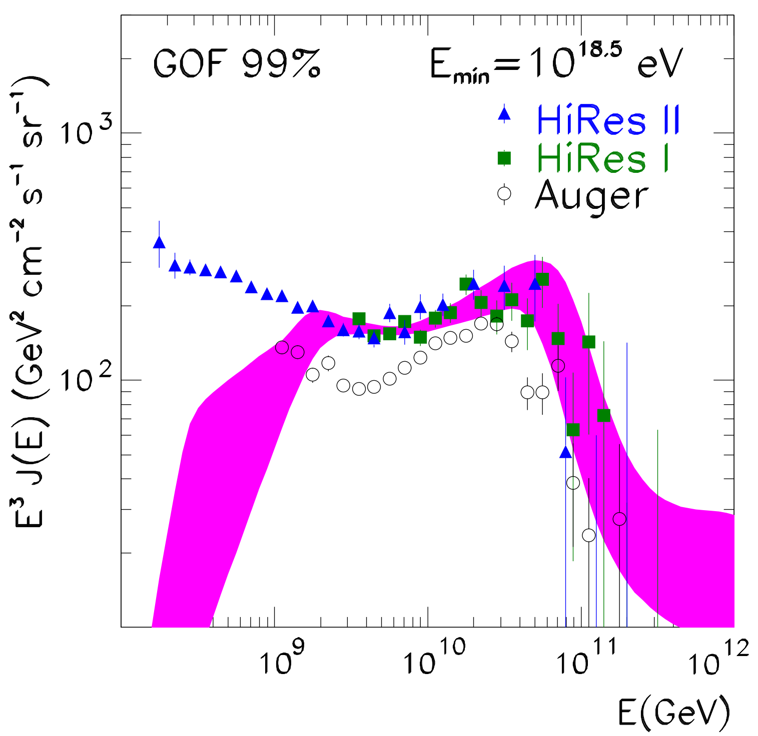
$$\Rightarrow \quad E_p \geq \frac{(m_N + m_\pi)^2 - m_p^2}{4E_{\gamma_{\rm CMB}}} \sim 3 \cdot 10^{11}\,\text{GeV}$$

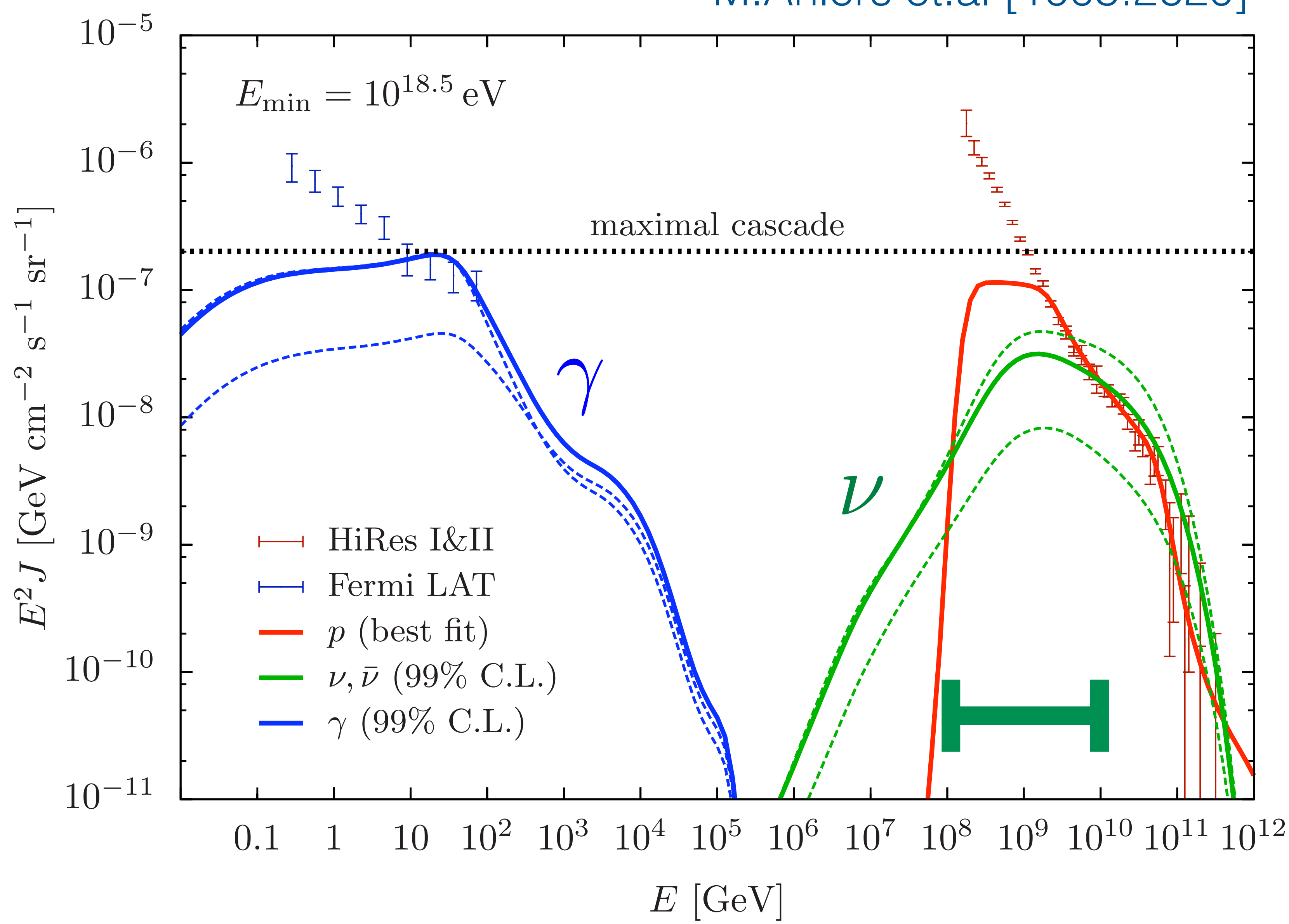
One could predict GZK neutrino and gamma ray fluxes by modelling the cosmic ray spectrum and fit it to the observed spectrum.

While neutrino energy is unchanged apart from redshift, the photons loose their energy by interacting with the intergalactic radiation fields.

$$\gamma_{\rm GZK} + \gamma \to e^+ e^-$$

M.Ahlers et.al [1005.2620]



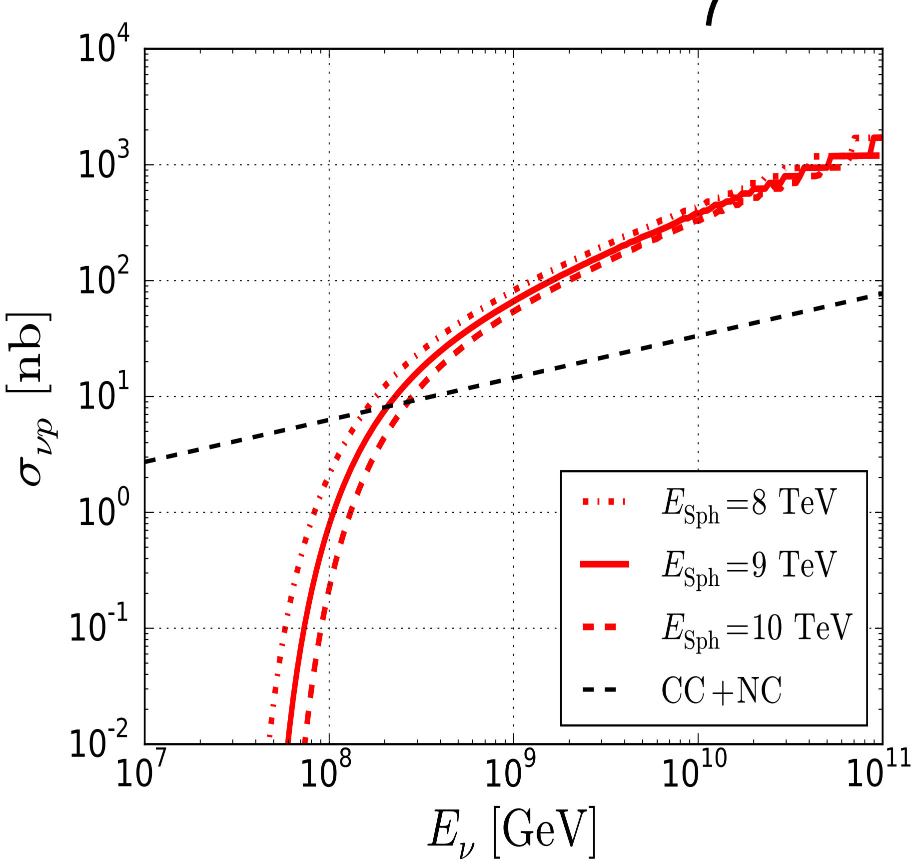


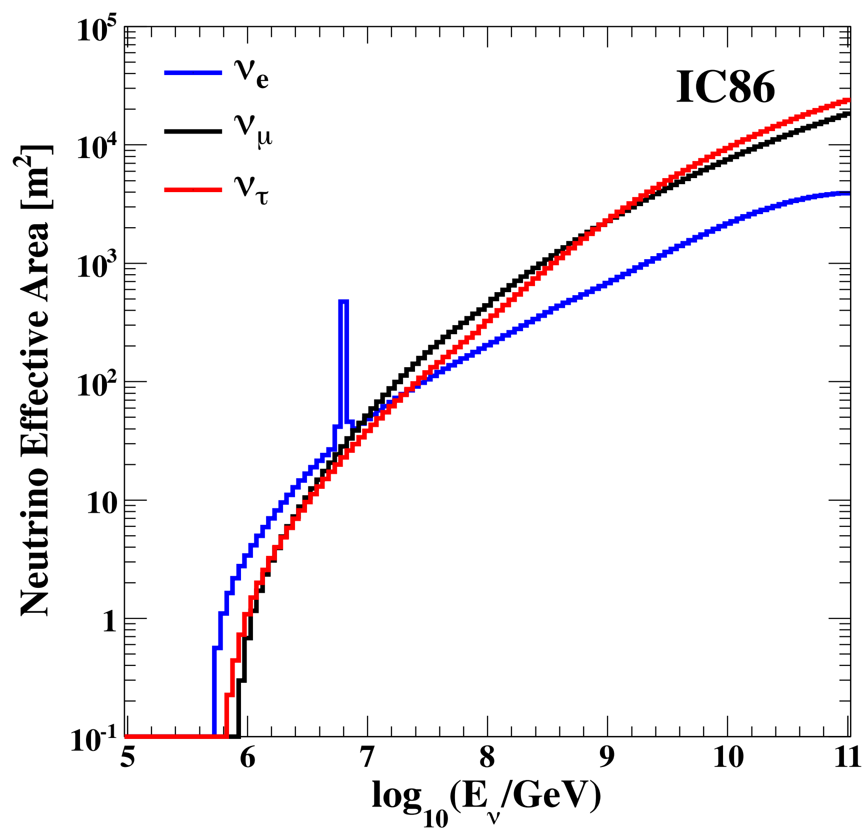
$$E_\nu^{\rm Sph} \gtrsim 10^{8-10}\,{\rm GeV}$$

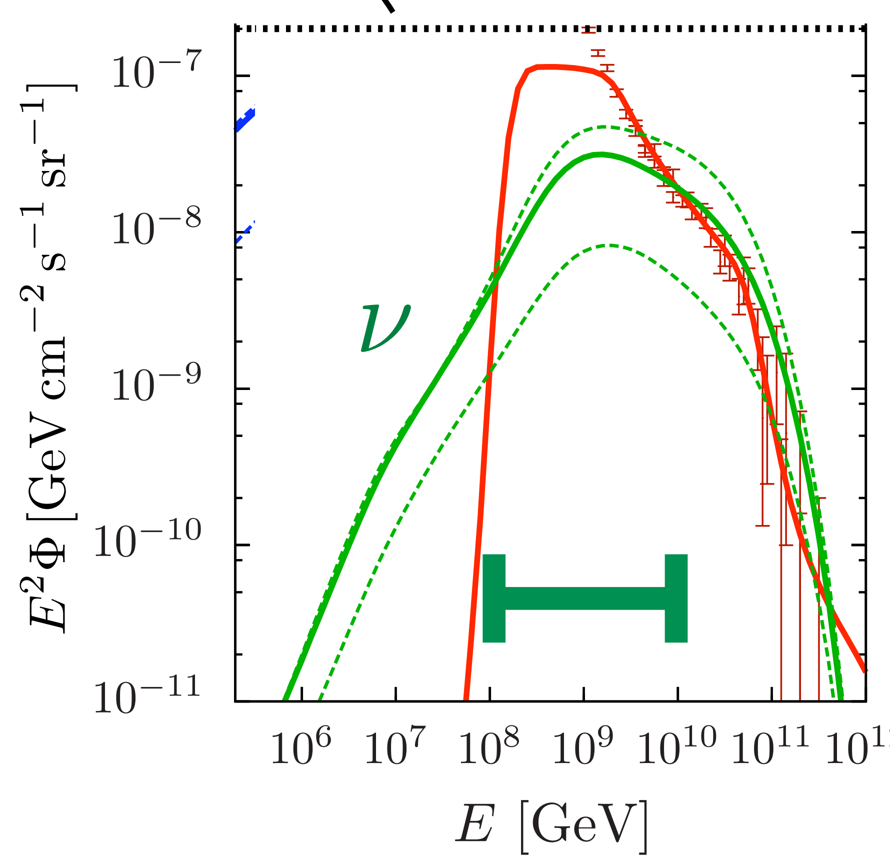
Event rate can be calculated using the energy dependent effective neutrino detection area.

$$\frac{dN_{CC/NC}}{dt} = \int_{E_{\text{three}}} dE_\nu A_{\text{eff}}(E_\nu) \frac{d^2\Phi}{dE_\nu dt}$$

$$\frac{dN_{\text{Sph}}}{dt} = \int_{E_{\text{three}}} dE_\nu \frac{\sigma_{\nu N}^{\text{Sph}}(E_\nu)}{\sigma_{\nu N}^{CC/NC}(E_\nu)} A_{\text{eff}}(E_\nu) \frac{d^2\Phi}{dE_\nu dt}$$







J.Ellis, KS, M.Spannowsky [1603.06573]

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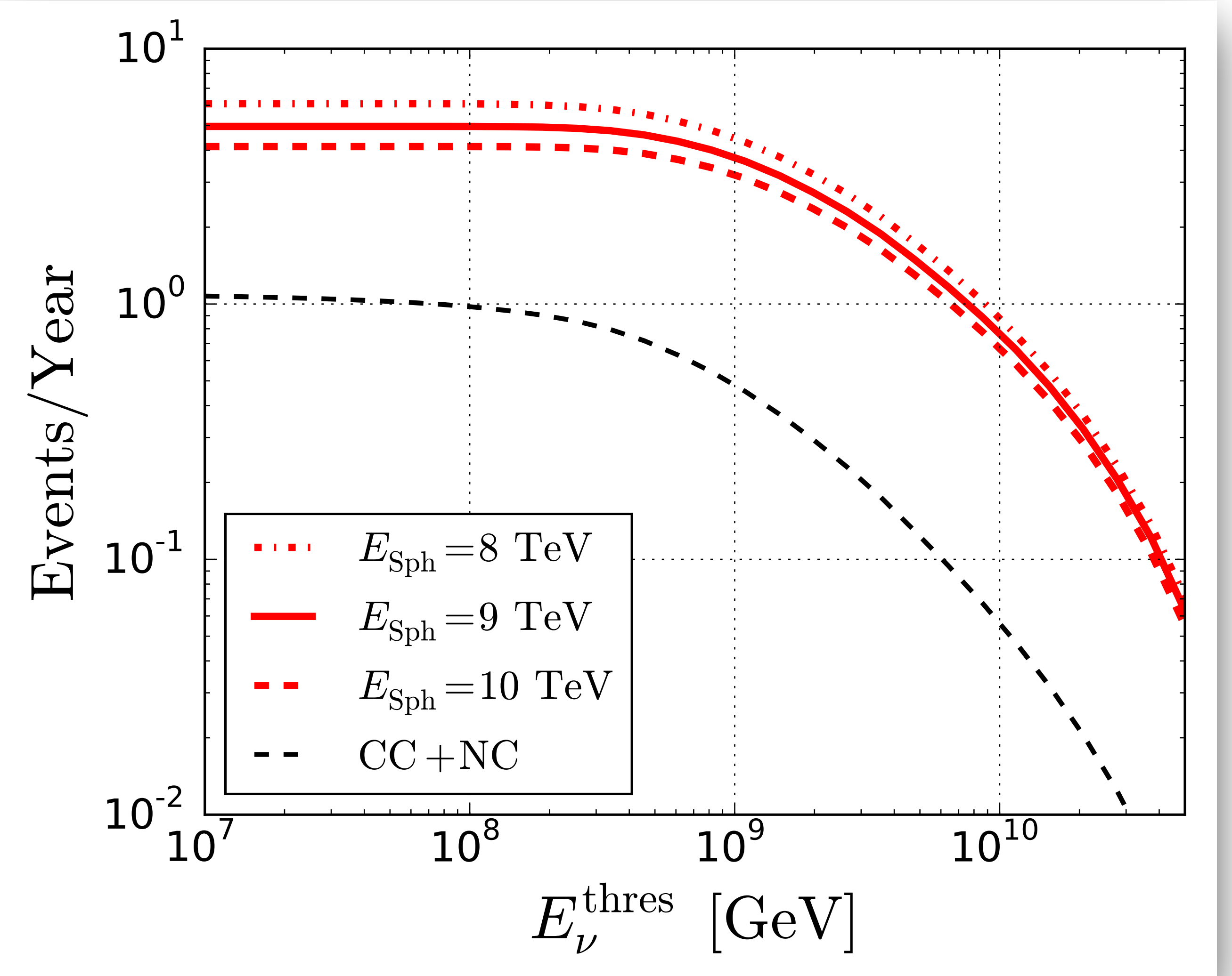
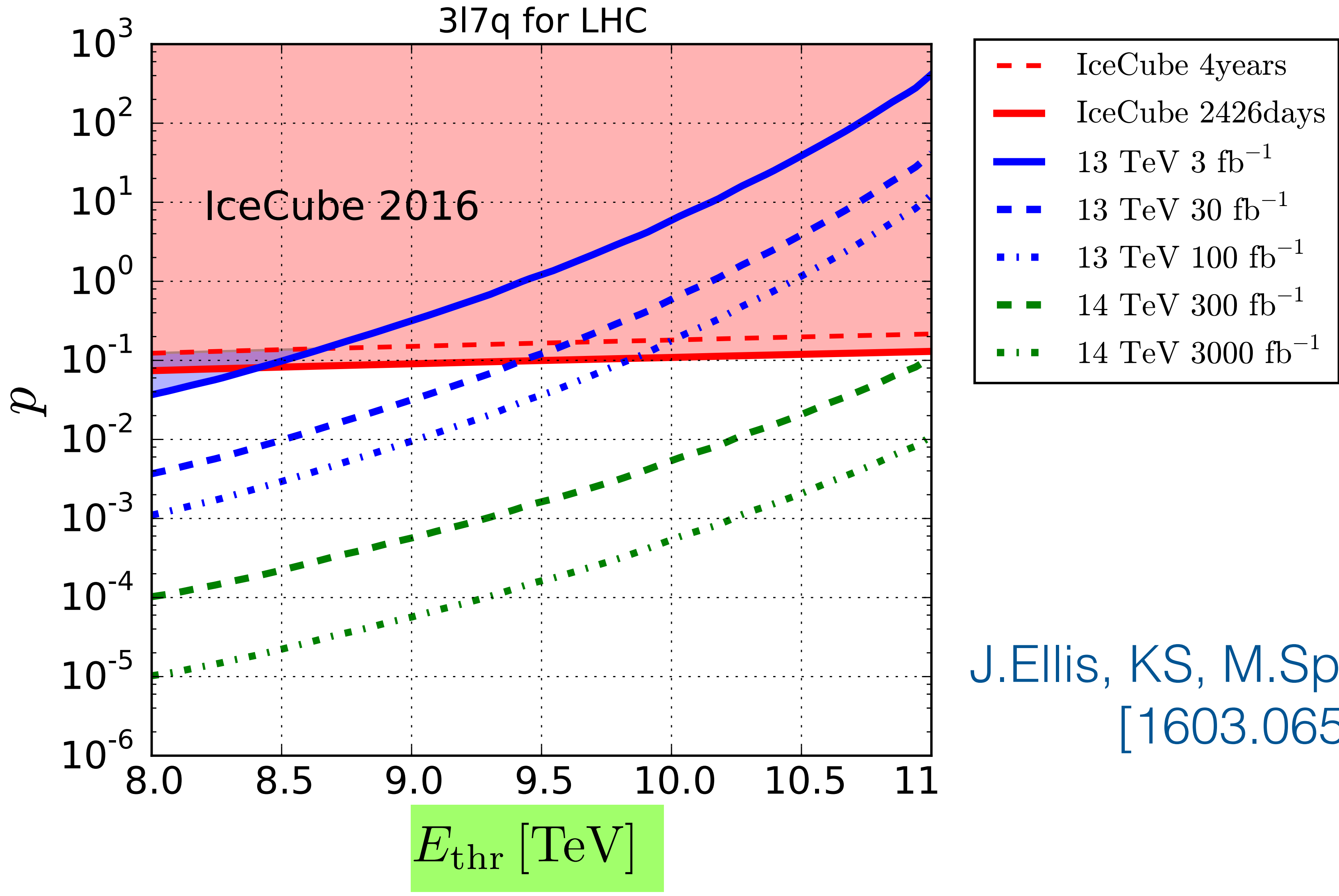
J.Ellis, KS, M.Spannowsky
[1603.06573]



Sensitivity



J.Ellis, KS, M.Spannowsky [1603.06573]

- For $E_{\rm Sph}$ ~ 9TeV, IceCube and LHC sensitivities are comparable.
- Good IceCube sensitivity persists for $E > E_{\rm Sph}$.

(because the fall of PDF is faster than that of GZK neutrino spectrum)

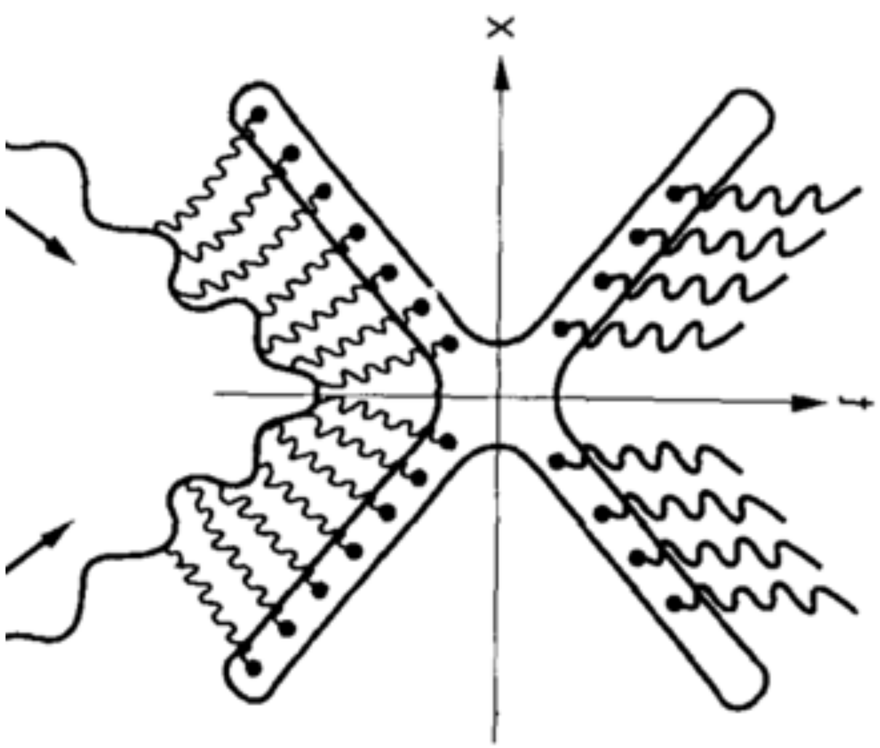
Conclusions

- The rate of zero-temperature high-energy instanton-induced process is still an open question.
- Some studies suggest that the rate of such a process may be observably large for future high-energy colliders if multiple EW bosons are produced together with fermions.
- It is important to tackle this issue from an experimental side by putting limits on the EW instanton process.
- The LHC can probe the region E_{thr} ~ 9TeV, while 100TeV collider can probe the realistic region up to E_{thr} ~ 80TeV.
- More theoretical understanding on the cross-section and final state multiplicity is necessary to fully exploit the power of future high-energy colliders.

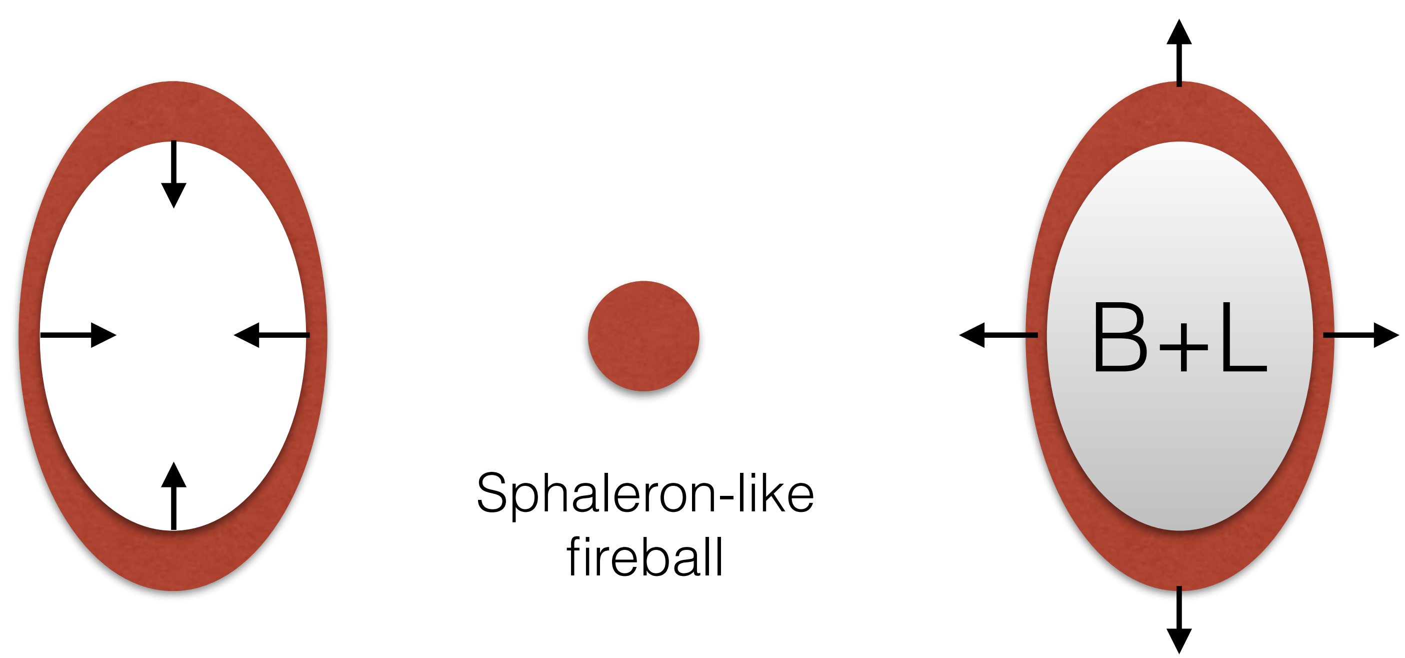
Backup

Optimistic view:

1. It is not the sphaleron which is directly created in the initial collision
2. Instantons in Minkwoski space are not point-like configurations; they are localized near the light-cone:



Cartoon of snapshots in time:



Taken from Valya Khoze's talk

Discussion

Before testing non-perturbative B+L violating processes in EW theory at future high energy collider, the following issues should be addressed:

More robust estimate of cross-section:

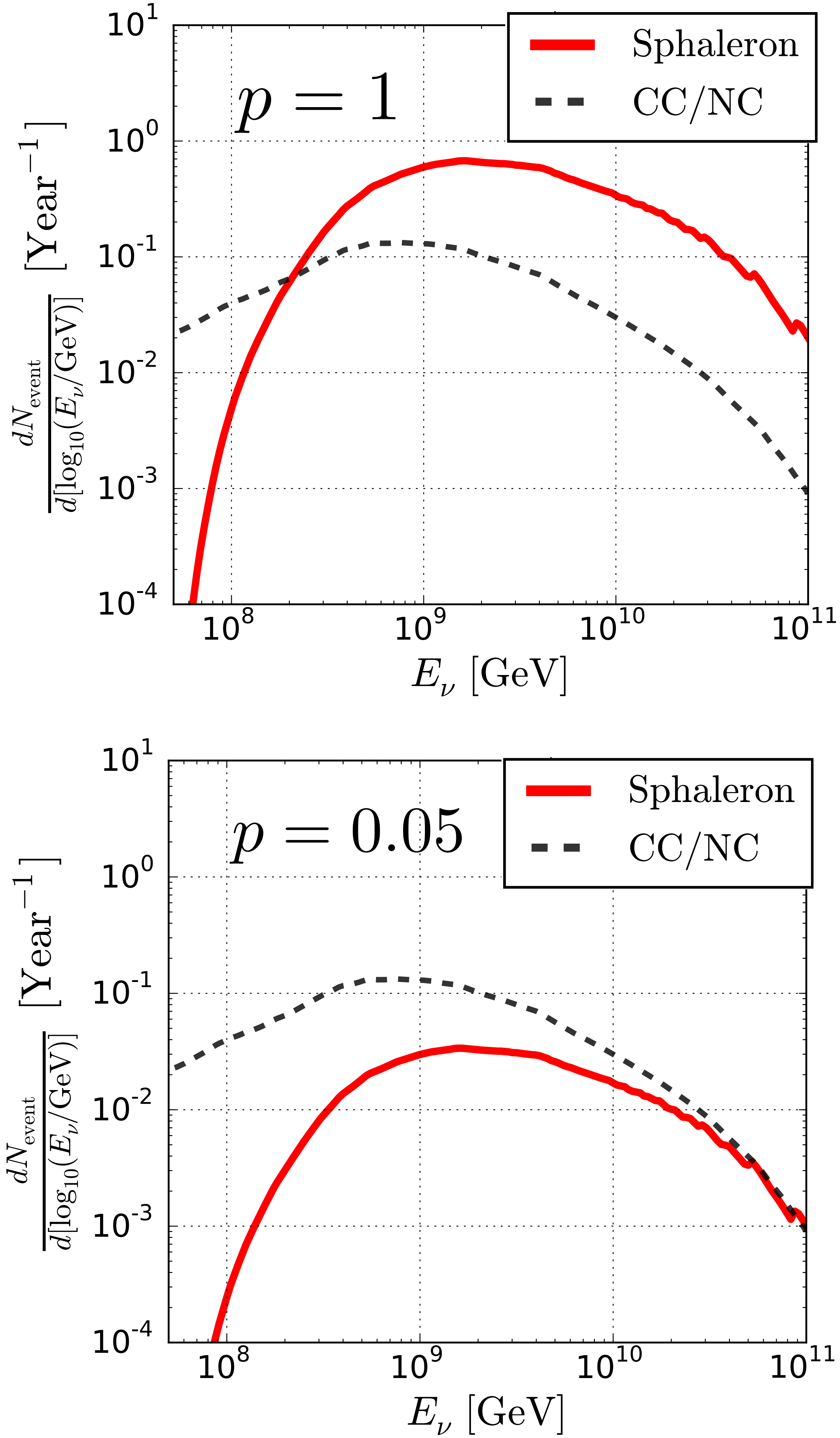
- current estimate based on instanton approach is valid only $\sqrt{\hat{s}} \ll \frac{\sqrt{6}\pi m_W}{\alpha_W} \sim 18\,\mathrm{TeV}$
- understanding a few to many suppression — Bezrukov, Levkov, Rebbi, Rubakov, Tinyakov 2003, Funakubo, Fuyuto, Senaha 2016, …
- How the instanton approach and resonant tunnelling approach are related?

Final state multiplicity:

- our best estimate of final state multiplicity is based on LO instanton approach which is valid only for the partonic energy much less than 18 TeV.

$$\sigma_{\mathrm{LO}}(n_W, n_h) \sim \mathscr{G}^2 2^n v^{-2n} \left[\frac{\Gamma(n+103/12)}{\Gamma(103/12)}\right]^2 \frac{1}{n_B!\, n_H!} \qquad \mathscr{G} = 1.6 \times 10^{-101}\,\mathrm{GeV}^{-14}$$

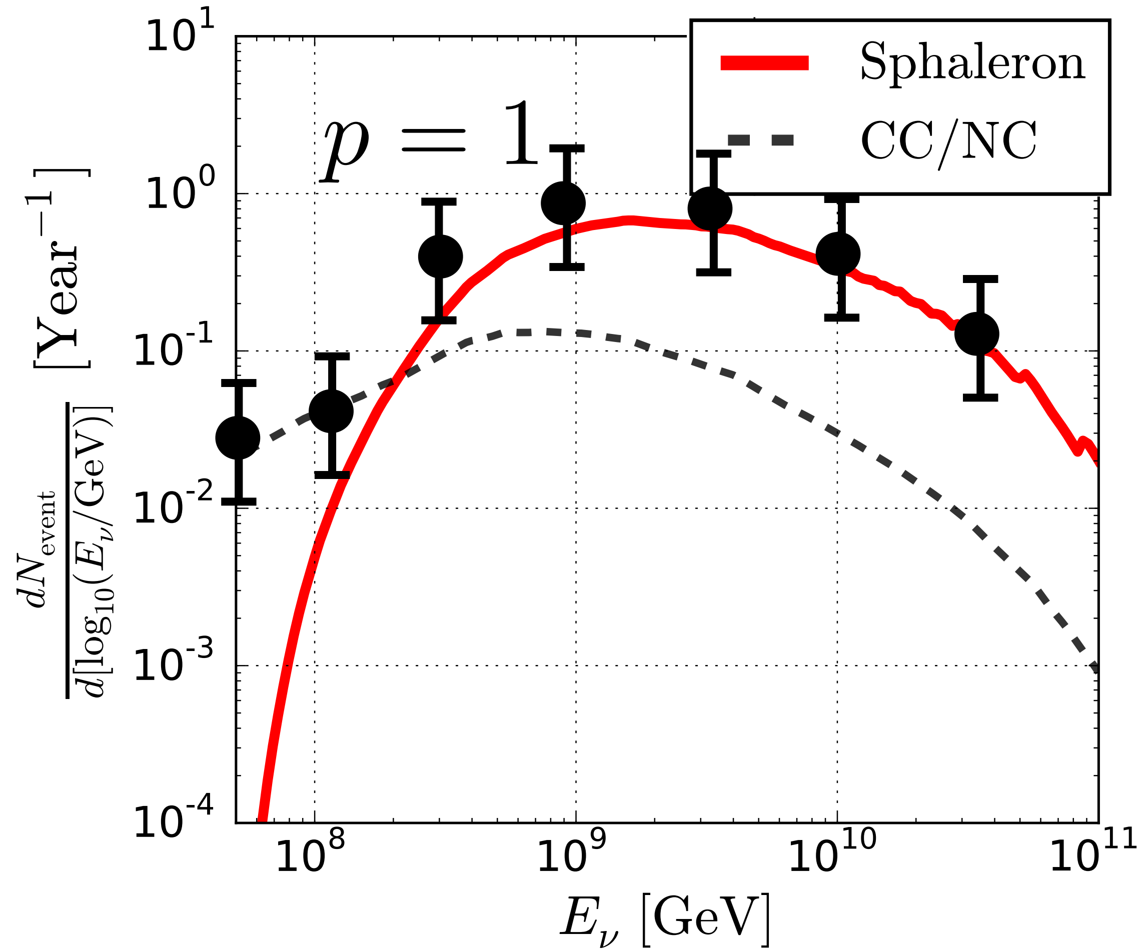
$$\times \int \prod_{i=1}^{10} \frac{\mathrm{d}^3 p_i}{(2\pi)^3 2E_i} E_i \prod_{j=1}^{n_B} \frac{\mathrm{d}^3 p_j}{(2\pi)^3 2E_j} \frac{2(4E_j^2 - m_W^2)}{m_W^2} \prod_{k=1}^{n_H} \frac{\mathrm{d}^3 p_k}{(2\pi)^3 2E_k} (2\pi)^4 \delta^{(4)}\left(P_{\mathrm{in}} - \sum_{i=1}^{10} p_i - \sum_{j=1}^{n_B} p_j - \sum_{k=1}^{n_H} p_k\right)$$

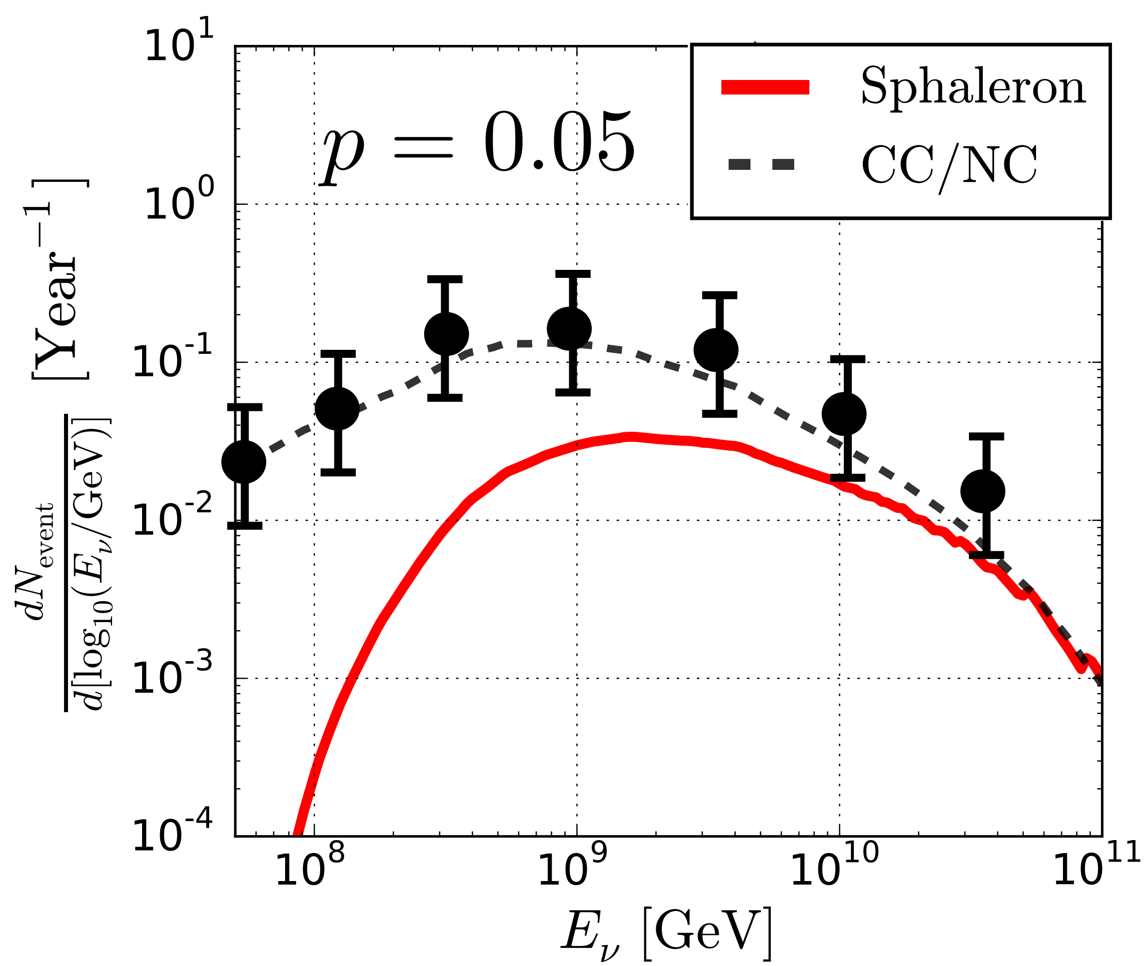


- If unknown pre-factor p is small, the sphaleron events may be hidden in the GZK neutrino events via the ordinary EW interaction.

- In this case, discrimination using the event shape is important.

How do sphaleron events look different from the ordinary neutrino events at IceCube?





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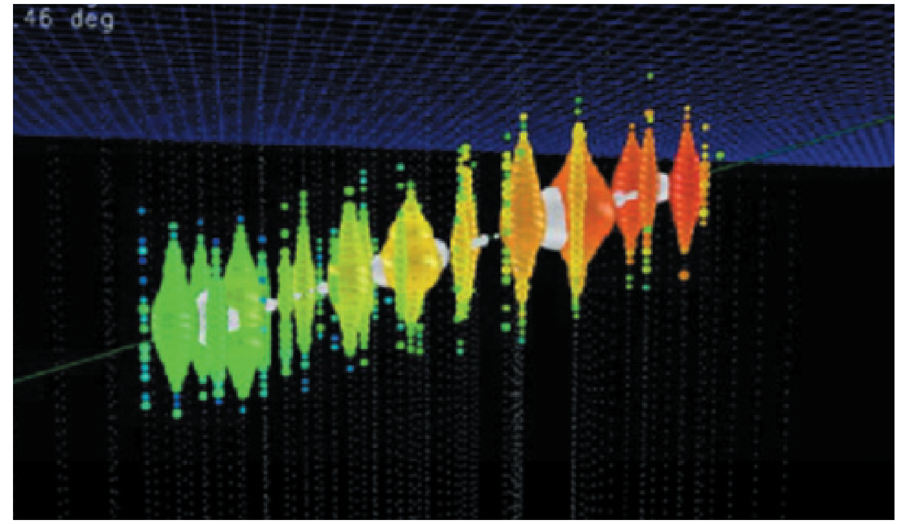
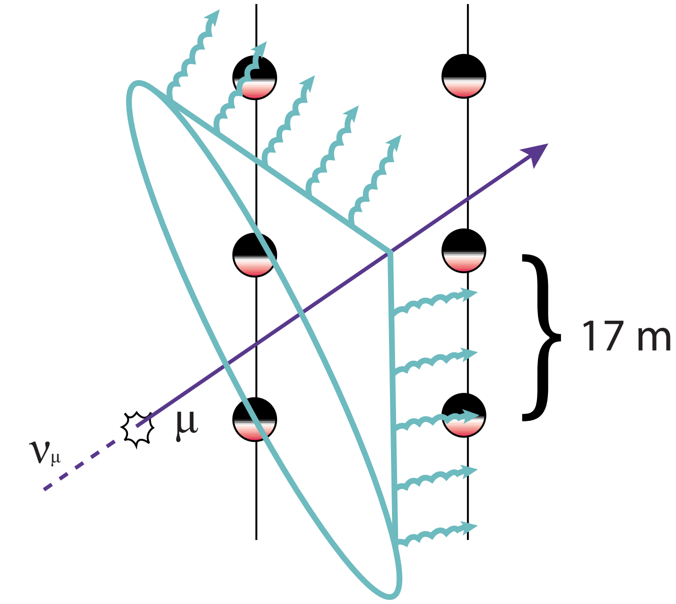
IceCube Events:

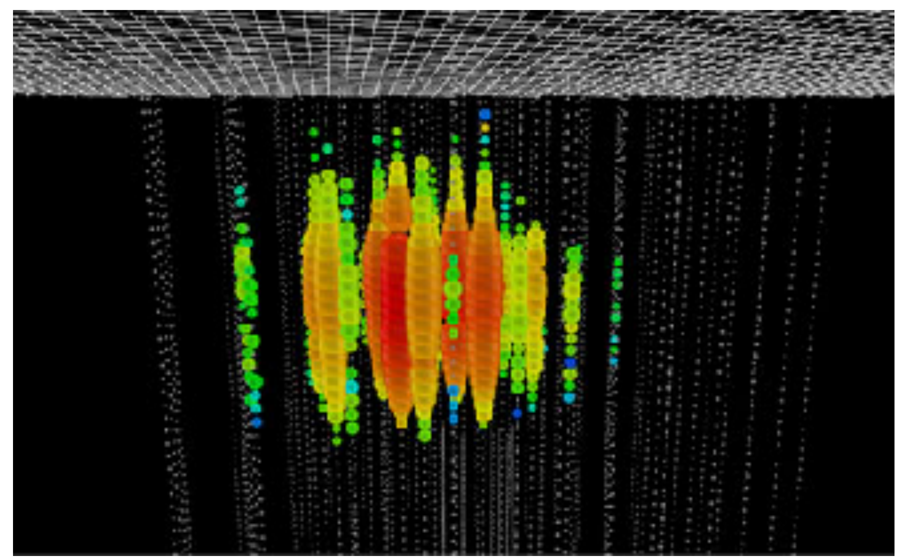
“muon bundle”

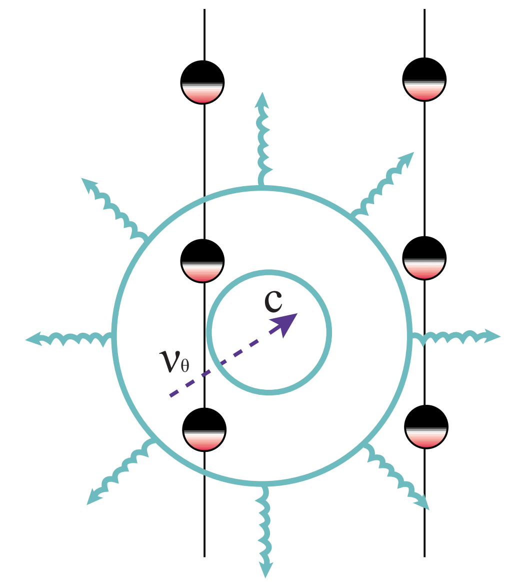




$$\nu_\mu N \to \mu X$$

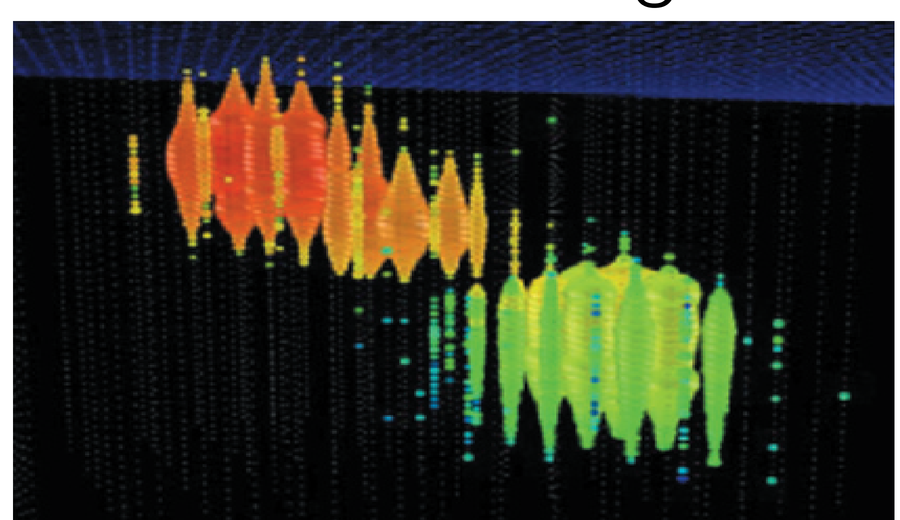
“shower”





$$\nu_e N \to eX$$
$$\nu_\tau N \to \tau X$$
$$\nu_i N \to \nu_i X$$

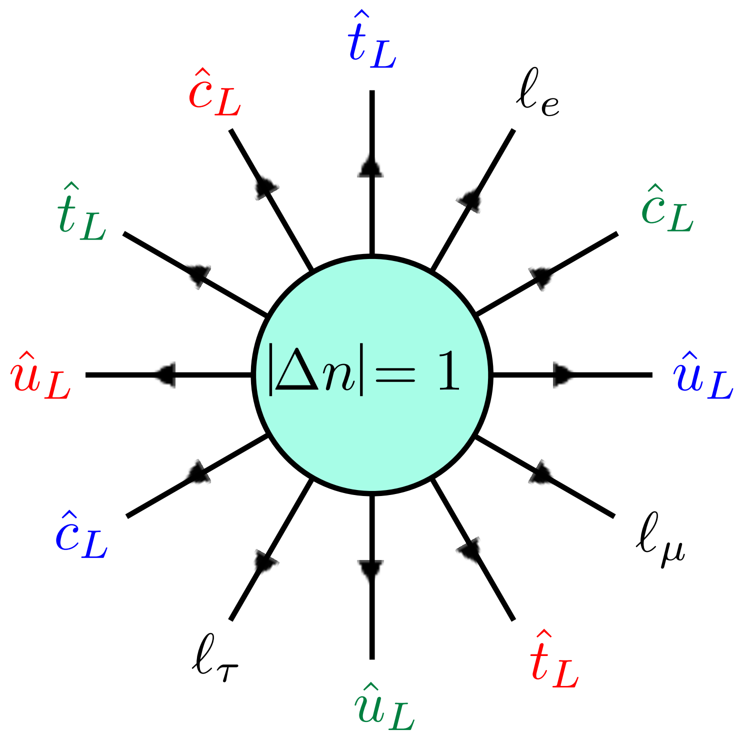
“double bang”

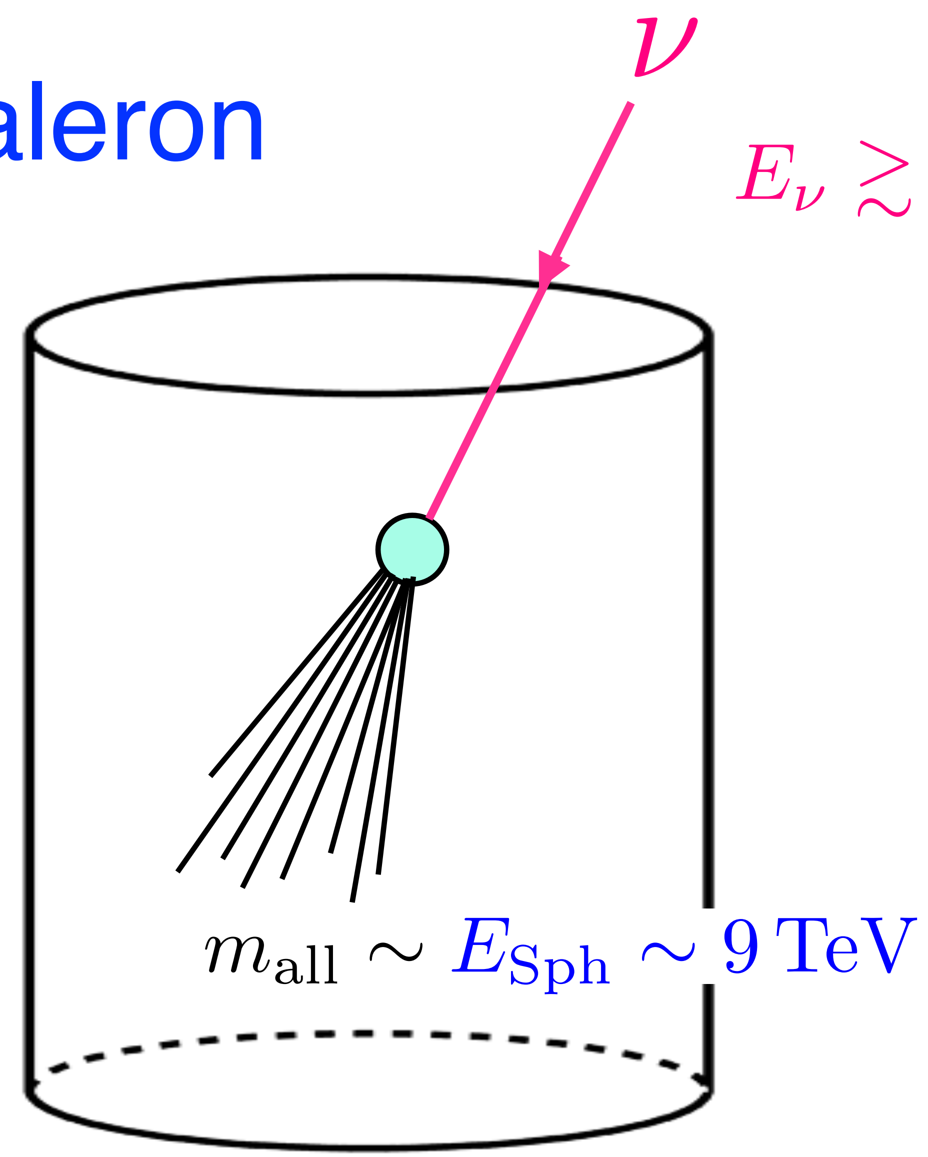


$$\nu_\tau N \to \tau X_1 \to X_1 \nu_\tau X_2$$

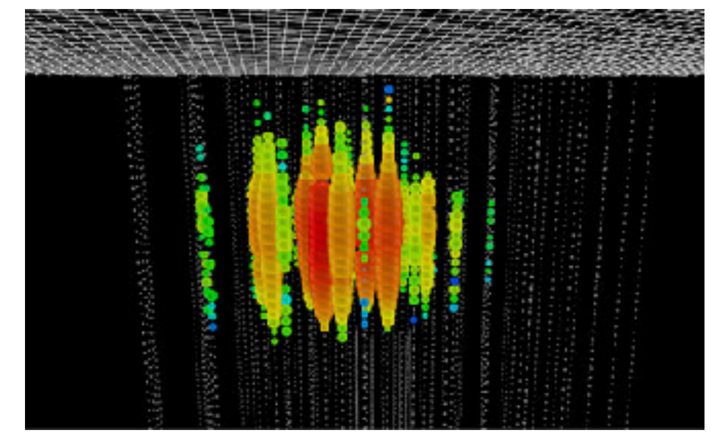
$$E_\tau \in [10^6, 10^7]\,\mathrm{GeV}$$

What does the sphaleron event look like?

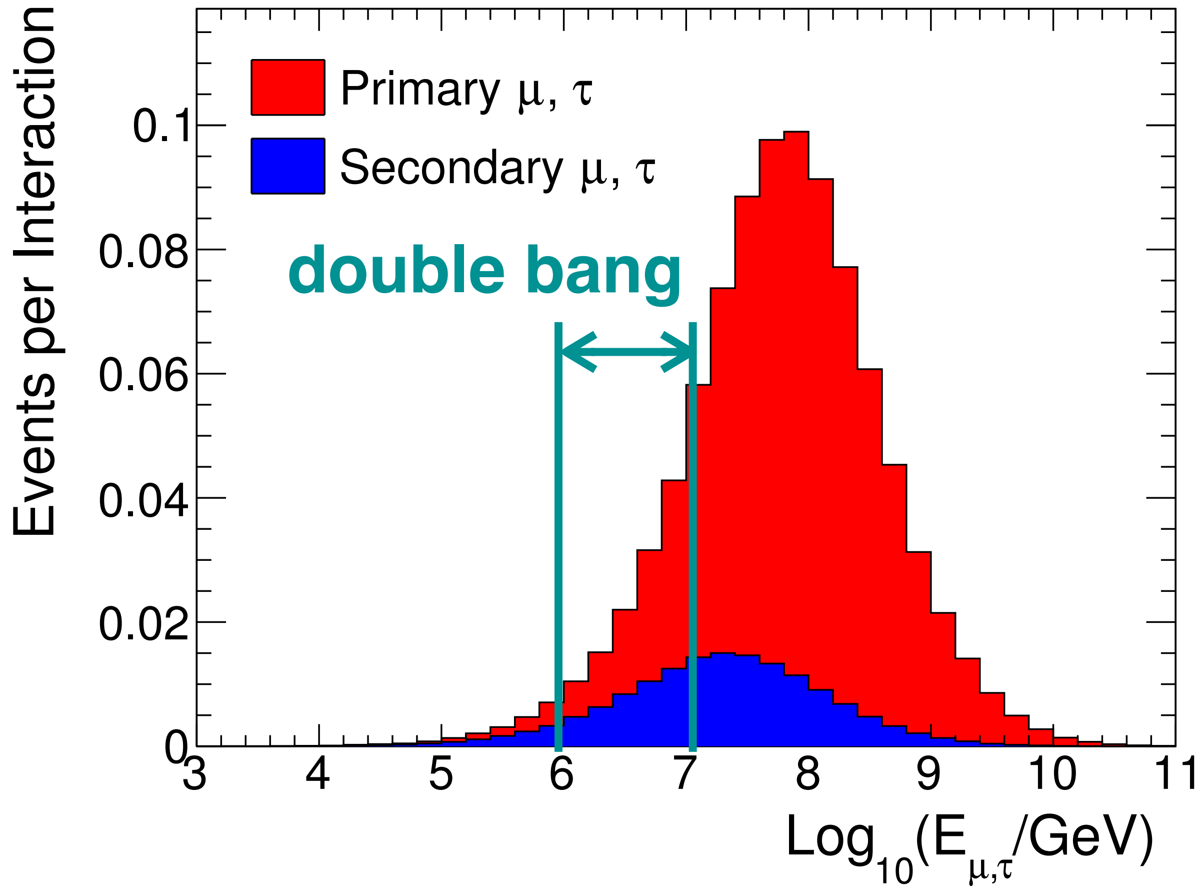




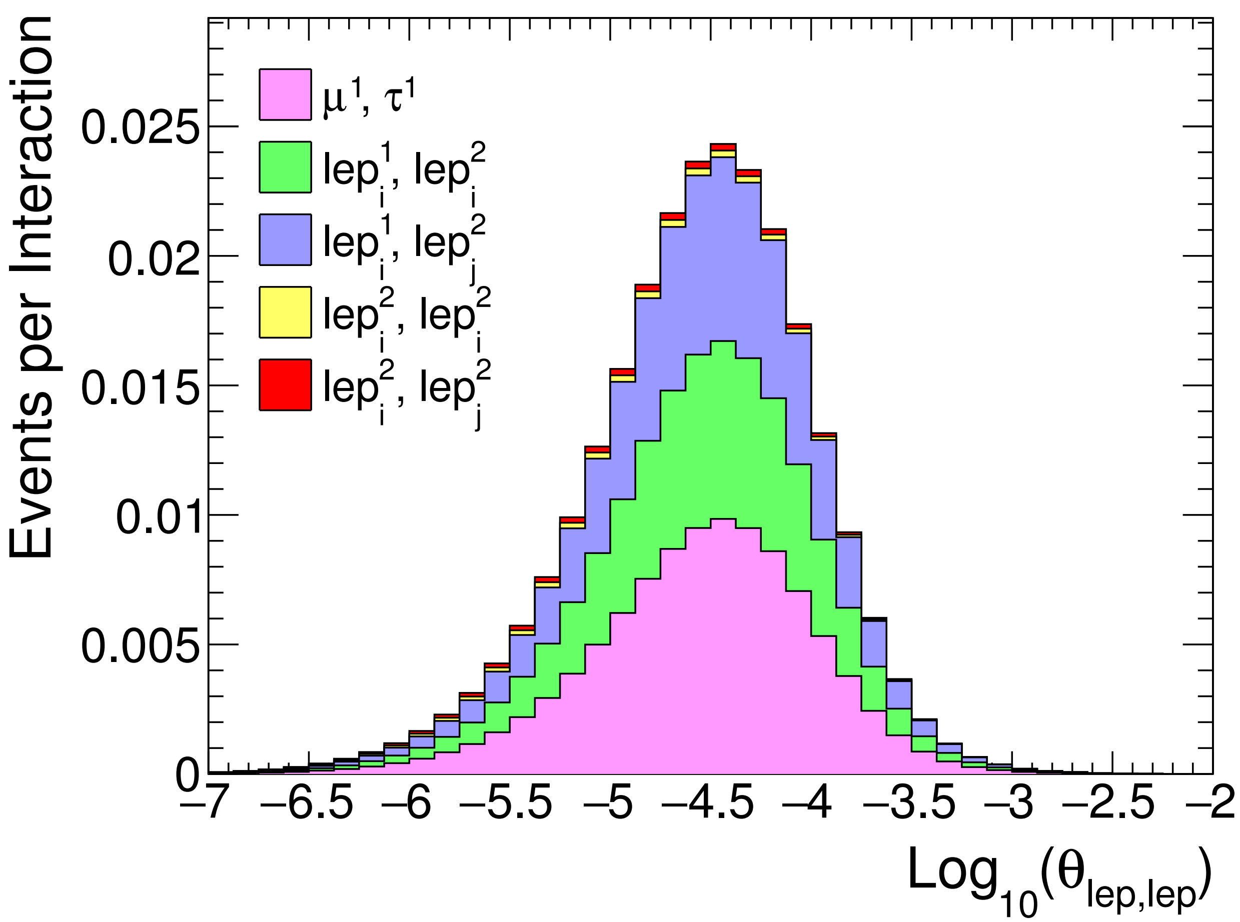
"shower"



- quarks and leptons are stopped in the ice (except for μ). ⇒ "shower"
- If μ is produced. ⇒ "bundle"
- If τ is produced with $E_\tau \in [10^6, 10^7]$ GeV. ⇒ "double bang"
- If primary μ and a μ from a top-quark decay has an opening angle with $\theta > 10^{-2}$ rad ⇒ "double bundle"??



Only 5% of the sphaleron-induced events have double bang taus.



particles are highly collimated and double bundles cannot be expected.

double bundle